Voluntary Disclosure with Evolving News*

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Abstract

We investigate a dynamic voluntary disclosure model where the value of the firm is stochastic. The manager observes the firm’s true value, whereas the market only receives a noisy signal of this process when the manager does not disclose. Disclosure is assumed to be costly and hence a real option exists as to when to begin disclosure, as the public signal may overstate the value of the firm. We show that there exists a unique threshold equilibrium of disclosure, which is characterized by the difference between the true value of the fundamentals and the market’s belief. The main results show (perhaps surprisingly) that the firm may disclose its private information even in cases in which the public signal significantly exceeds the true value of the firm. We capture this disclosure of bad news in a parsimonious model that does not rely on common assumptions of litigation risk. In the extended setting, the model captures disclosure cycles with quiet regions where the manager may discontinue disclosure. This framework can be applied to explain many puzzles, including post-earnings announcement drift and disclosure frequency.
1 Introduction

Consider the manager of a pharmaceutical firm who faces a decision of whether or not to disclose information concerning the firm’s value. The manager privately observes preliminary, unfavorable results from a phase three clinical trial for a new drug. The trial is ongoing and hence further subsequent results may prove to be more encouraging. Moreover, the firm’s research team may develop new advances concurrently that could help the firm’s overall value. The manager thus faces a real option of waiting to see if the information improves, weighed against the current market’s valuation of the firm. A rational market consequently depresses the valuation in the event of non-disclosure, leading to an implicit cost of delaying disclosure.

A similar story can be said for firms in a variety of industries. A firm’s product development and results are continuously changing over time and may improve (or decline) as new developments are made in their research and development departments. Moreover, the real costs of disclosure are nontrivial—arranging press releases, conference calls, certification, and meetings with analysts—all impose time costs on the manager and monetary costs on the firm. Similarly, the disclosure may be relevant to proprietary information that could be adopted by competitor firms. Consequently, the manager may delay disclosure to avoid real, proprietary, or certification costs. A rational market must evaluate the firm in any event and does so with the available public information. The manager thus faces an option value of delaying disclosure; if she waits, the market may overestimate the firm’s true value, while saving the costs of disclosure. However, the market may alternatively underestimate the firm’s value. If this disparity is sufficiently large, the manager finds it worthwhile to disclose.

We seek to capture this phenomena in a dynamic voluntary disclosure model where the arrival of news follows a Brownian diffusion process. We assume the manager is perfectly informed of the firm’s true (fundamental) value at any point in time and that investors observe only a noisy signal of this process. For parsimony, we assume in the baseline model that the manager can only make one non-manipulable disclosure decision, after which, all private information becomes public for the remainder of the game. This simple setting gives us a wealth of results that help to explain numerous empirical regularities. We show that there is always a positive amount of delay in the disclosure time, even though we assume the cost of disclosure is always less than the non-disclosure penalty imposed by investors (hereafter referred to as the “non-disclosure discount”). The equilibrium of the game is characterized by a unique threshold level of disclosure such that the manager only discloses when the difference between the manager’s private information and the market belief crosses this threshold. One of the main results
of this study shows conditions under which the manager discloses even when the public signal exceeds the value of the manager’s private information (i.e. disclosure during overestimation, disclosure of “bad news”), which has heretofore not been captured in the theoretical voluntary disclosure literature. There is substantial empirical evidence that managers disclose bad news more often than good news \(^1\), although no model to date has shown this ubiquitous phenomenon without an assumption of litigation risk in the event of non-disclosure (e.g. Daughety and Reinganum (2008), Dye (2014), and Marinovic and Varas (2015)).

At first glance, this result may seem paradoxical as, intuitively, disclosure during a period of overestimation should make the manager worse-off if the resulting price is lower. However, we find that the opposite is true when the optimal threshold is above the firm’s true value. When the manager discloses, the investors no longer price the firm with a truncated distribution, that is, the investors no longer impose the non-disclosure discount. Because the non-disclosure discount is greater than the cost of disclosure, the downward adjustment from the disclosure is outweighed by the upward adjustment from removal of the non-disclosure discount. The net result is a price increase even though the manager has disclosed a value lower than the filtered value.

The baseline model captures the main insights of costly, dynamic voluntary disclosure, however, we enrich this simple setting to also characterize quiet periods in dynamic disclosure. In the extended model, the information structure is the same except the manager can discontinue disclosure at any point, that is, the manager owns the private information again. In this setting, the manager can alternate between hiding her private information and disclosure, and can repeat this cycle infinitely often. For tractability, we assume the market is composed of two types of investors–informed and uninformed. During non-disclosure, or “quiet”, periods, both types of investors only observe the same public signal. However, during disclosure periods, only the informed investors observe the manager’s private information. The equilibrium is again characterized by a unique threshold level defined by the difference between the manager’s private information and the market belief. The manager begins disclosing when this threshold is reached, at which point the market valuation is adjusted with the arrival of news. Upon discontinuing disclosure, informed investors no longer receive additional information and the firm is priced as though all investors observe the public signal. The manager engages in this quiet period when the “shadow” price–the firm’s value in the case of non-disclosure–again crosses the threshold (from the bottom) to the acceptable non-disclosure level. This setting gives way

\(^1\) For example, see Skinner (1994), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), and Kross et al. (2011), among others.
to empirical predictions concerning disclosure frequency in the face of real or propriety costs.

The extended model also captures post-earnings announcement drift (PEAD), which is a well-documented phenomenon in accounting and finance. Investors tend to underreact to the information content of earnings disclosures, thereby generating return continuation. Our model predicts two conditions under which PEAD can emerge. First, the correlation between shocks to the noisy signal and the manager’s private information should be sufficiently negative. This leads investors to update their beliefs downward after observing positive shocks in the public signal. Moreover, this implies that the belief difference reverts to zero (unbiased estimation) more slowly, i.e. disclosing bad news tends to increase investors’ beliefs and vice versa. Second, there should exist a sufficiently small mass of informed investors. Only a small mass of informed investors react to a disclosure of bad (good) news, and thus the overestimation (underestimation) at the time of disclosure is not fully adjusted into the firm value, which leads to the continuation of price drift following disclosure.

This study builds from the static costly voluntary disclosure setting introduced in Jovanivc (1982) and Verrechia (1983). The dynamic disclosure papers most closely related to the model here are Marinovic and Varas (2015) and Acharya, DeMarzo, and Kremer (2011). Marinovic and Varas (2015) investigate a continuous-time voluntary disclosure model where disclosure of good news is costly. They assume that the firm faces a litigation risk when bad news is withheld and consequently the manager discloses with positive probability when the firm value is low. They find that the manager, due to the disclosure cost, does not disclose good news unless the firm’s market valuation is sufficiently low. The model here differs from Marinovic and Varas (2015) primarily in that litigation risk is a fundamental feature of their setting, whereas we are interested in the disclosure of bad news without this friction.\(^2\)

This study is also related to the literature on voluntary disclosure where the manager may not be informed of the firm’s value, as introduced (in a static setting) by Dye (1985) and Jung and Kwon (1988). Acharya, DeMarzo, and Kremer (2011) investigate a model where an exogenous correlated signal is publicly revealed at a known time. Their results show clustering of announcements in bad times, where the manager discloses immediately if the public signal is sufficiently low. Guttman, Kremer, and Skrzypacz (2014) consider a two-period model where the manager may receive independent signals of firm value in each period. They show that the market value of the firm is higher if the signal is disclosed in the second period rather than the first. The model here differs from these settings in that we assume the manager receives

\(^2\) In Marinovic and Varas (2015), the manager never discloses bad news when there is no risk of litigation.
information with probability one, disclosure is costly, and that the firm value is stochastic.

Moreover, Daley and Green (2012) present a dynamic adverse selection model where information about an asset for sale is revealed to the market through a Brownian process. They capture no-trade periods when investor belief is intermediate and partial sale when the belief is low. The manager in our setting has a utility which is affected by the market’s belief or valuation at every moment in time, and hence is compelled to disclose when the market’s belief is sufficiently low.

Lastly, we build from the methods developed by Scheinkman and Xiong (2003), who investigate a continuous-time trading model between agents with heterogeneous beliefs.

The paper is organized as follows. The next section introduces the baselines model, while section 3 presents the solution. Section 4 investigates the extended model where uninformed traders are present. The final section concludes. All proofs are relegated to the Appendix.

2 Baseline Model

In the baseline model, the manager perfectly and privately observes the firm’s fundamental value, $f_t$. We assume that the firm’s value is driven by the following process:

$$df_t = \lambda (\bar{f} - f_t) dt + \sigma f dZ^f_t,$$

where $\bar{f}$ is the long-run mean, $\lambda$ is the rate of mean-reversion, and $\sigma f$ is the constant volatility, all of which are commonly known by investors. The firm produces a continuous stream of dividends which is publicly observable and satisfies:

$$dD_t = f_t dt + \sigma D dZ^D_t,$$

where $\sigma D$ is the constant volatility parameter and $Z^D_t$ is a standard Brownian motion. We assume that $dZ^D$ and $dZ^f$ are correlated through $\phi dt$. The shock to firm’s dividend, $Z^D_t$, can be thought of as a sum of a macroeconomic or industry shock and an idiosyncratic shock, which is related to the shock to the firm’s true dividend growth $Z^f$. Dividends here are conveyed as a noisy signal of the firm’s fundamental value, but this can also be interpreted as other information transmitted to the investors from external news sources or from analyst reports. In the baseline model, we assume there is a continuum of identical, risk-neutral investors with unit mass. The market value of the firm at every point in time is set by the investors, which is given by their belief of the fundamental value conditional on the disclosure threshold and the fact that the manager has not disclosed by that time.
The manager must make a disclosure decision at every point before she has not disclosed. If the manager discloses, the firm’s fundamental value becomes publicly observable for the remainder of the game. This assumption is intended for parsimony and the results are not qualitatively affected if we allow the manager unlimited disclosure opportunities and the option to withhold information following disclosure (see the extended model in Section 4.) As is commonly assumed in dynamic voluntary disclosure models, the risk-neutral manager is concerned with the firm’s market valuation at every point in time. The manager’s utility function is

\[ W_0 = \int_0^\tau V(\hat{f}_t)e^{-\delta t} dt + \int_\tau^\infty V(f_t)e^{-\delta t} dt, \]  

(3)

where \( V(\cdot) \) is the investors’ valuation of the firm, \( \hat{f}_t \) is the investors’ belief of the true firm value during non-disclosure, \( \tau \) is the time of disclosure, and \( \delta \) is the manager’s rate of time preference, which may differ from the investors’ discount rate, \( r \). After the disclosure decision, the costs of disclosure are borne entirely by the firm (i.e. post-disclosure firm value takes into account this cost) but the results would be unchanged if rather the manager incurred a portion or all of these costs privately. The manager is concerned with the market price at all times as it is often the case that an executive’s compensation includes bonuses which are determined in part by share price. Moreover, executives often hold options, restricted stock, or shares of the firm.

3 Model Solution

We show here that the manager’s optimal disclosure time is characterized by the difference between the true value and the market’s belief. We begin first with a few preliminary results concerning the evolution of the investors’ beliefs.

3.1 Evolution of Beliefs

Prior to disclosure, investors observe only the public signal. The dynamics of the investors’ belief is derived as:

\[ d\hat{f}_t = \lambda(\bar{f} - \hat{f}_t)dt + \frac{\gamma + \phi \sigma_f \sigma_D}{\sigma_D^2} (dD_t - \hat{f}_t dt), \]  

(4)

where \( \gamma \) is the variance of the stationary solution:

\[ \gamma = \frac{\rho - \lambda - \phi \sigma_f / \sigma_D}{1/\sigma_D^2}, \]  

(5)
and
\[ \rho = \sqrt{(\lambda + \phi \sigma_f / \sigma_D)^2 + (1 - \phi^2) \sigma_f^2 / \sigma_D^2}. \]  \tag{6} 

The investors observe the innovation in dividends and update their beliefs given this filtered value. The filtered value consequently follows a mean-reverting process with the same mean-reversion rate and long-run mean as the true fundamental. As we would expect, a greater correlation between \( dZ^D \) and \( dZ^f \) implies that the investors put more weight on the public signal. We find the following property of \( \gamma \):

**Proposition 1** The stationary variance \( \gamma \) decreases with \( \lambda \) and \( \phi \).

This is unsurprising as we expect less dispersion in the investors’ beliefs with a more persistent drift of the fundamental value and less dispersion when the signal is more informative. We introduce some notation which will help in characterizing the manager’s strategy. Let \( g \) denote the difference between the true value and the investors’ estimate, \( g_t = f_t - \hat{f}_t \). Then, we see that this difference has the following dynamics:

\[ dg_t = -\rho g_t dt + \sigma_g dZ^g_t, \]  \tag{7} 

where

\[ \sigma_g = \sigma_D \sqrt{\sigma_f^2 / \sigma_D^2 + (\lambda - \rho)^2} \]  \tag{8} 

\[ dZ^g_t = \frac{1}{\sigma_g} \left( \sigma_f dZ^f_t + (\lambda - \rho) \sigma_D dZ^D_t \right). \]  \tag{9} 

From (7), we see that the belief difference also follows a mean-reverting process, where the long-run mean is equal to zero and the rate of mean reversion, \( \rho \), is defined as in (6). We find the following property for \( \rho \):

**Proposition 2** The rate of mean reversion for the belief difference, \( \rho \), increases with \( \phi \).

We see that the difference in beliefs, \( g \), quickly reverts to zero when the signal, \( D_t \), and the true dividend growth, \( f_t \), are highly correlated. That is, the investors’ filtering of the fundamental from the noisy signal is more likely to be correct when there is a higher correlation between the two. Moreover, we have that \( \sigma_g > \sigma_f \), i.e. the belief difference is more volatile than the true dividend growth. Establishing the evolution of beliefs as the difference in the investors’ belief and the investors’ belief is useful in the following section where the manager’s optimal strategy is derived.
3.2 Disclosure Option Value and Optimal Boundary

In this section, we derive the equilibrium disclosure option value and the disclosure boundary. This allows us to characterize the unique threshold level of disclosure for the manager. Recall that the manager has one irreversible disclosure decision. A natural strategy for the manager is to disclose when the difference between the true value and the investors’ belief is sufficiently large. This threshold belief difference determines the disclosure time and occurs when \( g \) hits an endogenously determined upper boundary. In the case where the belief difference, \( g \), is negative, the investors are overestimating the firm’s true fundamental, and so the manager (usually) finds it optimal to delay disclosure. As investors begin to underestimate the value of the firm’s fundamental, the difference between fundamental and the belief grows larger until it reaches the threshold level \( \bar{g} \) and manager is compelled to disclose. During the non-disclosure period, the investors are aware of the manager’s strategy, and hence the fact that the manager has not disclosed indicates to the investors that they are either overestimating the firm’s fundamental or their belief is below \( f \) but not enough to induce disclosure (in the case where the optimal \( \bar{g} \) is positive). This tilts the investors’ belief of the true value downward and results in an undervaluation of the firm, or a non-disclosure “discount”. The cost of keeping quiet is thus the potentially lower belief by investors, while the benefit is the option value that investors may begin to over-value the firm and foregoing disclosure costs. At the equilibrium boundary difference, \( \bar{g} \), the marginal cost of changing the boundary is equal to the marginal benefit.

Two surprising results emerge from this analysis. The first regards the lack of unraveling. In the solution, we assume that the cost of disclosure, \( c \), is always strictly less than the investors’ non-disclosure discount. In a static setting, unraveling would occur as all firms would find it unprofitable to withhold disclosure. However, we do not get unraveling in this case as the manager’s real option of waiting outweighs the non-disclosure discount when the belief differential is not sufficiently large. The second interesting result is with regard to the optimal boundary. In some cases, we find that the equilibrium threshold is negative, that is, the manager begins to disclose information even when the public belief is above the true value of the firm’s fundamental.

Prior to disclosure, the investors discount the future dividends flow at rate \( r \) using the filtered value and the non-disclosure information—the boundary \( \bar{g} \) and the fact that the manager has not yet disclosed. The investors’ belief of the firm’s value prior to disclosure is given by:

\[
V_t = E_t^f \left[ \int_t^\infty e^{-r(s-t)}dD_s | f_t - \hat{f}_t < \bar{g} \right],
\]

(10)
where $E^I_t[\cdot]$ represents the investors’ time $t$ conditional expectation. The best unconditional estimate of $f_t$ would just be a normal distribution with mean $\hat{f}_t$ and variance $\gamma$. However, conditional on the manager not having disclosed by time $t$, the resulting posterior belief is a downward adjustment where the investors’ best estimate is the mean of a truncated normal distribution:

$$E^I_t[f_t|f_t - \hat{f}_t < \bar{g}] = \hat{f}_t - \sqrt{\gamma} \phi \left( \frac{\bar{g}}{\sqrt{\gamma}} \right) N \left( \frac{\bar{g}}{\sqrt{\gamma}} \right)^{-1} = \hat{f}_t - q(\bar{g}), \quad (11)$$

where $q(\cdot)$ is the non-disclosure discount and a function of the optimal disclosure boundary. The next result establishes some properties of $q(\cdot)$:

**Proposition 3** The non-disclosure discount $q(\bar{g})$ is positive and decreasing in $\bar{g}$.

We see that investors punish the firm when the manager has not disclosed with a lower valuation of the fundamental. Moreover, the investor imposes a greater penalty when the optimal threshold is lower. Proposition 3 implies that investors discount their estimate of the firm’s fundamental value more heavily when the optimal threshold $\bar{g}$ is negative. This is counter-intuitive as we would expect investors to be less punitive in cases where the belief difference required for disclosure is smaller. However, from the perspective of investors, non-disclosure when $\bar{g} < 0$ puts an upper limit of $\hat{f} + \bar{g}$ on the distribution of the true firm’s fundamental. Hence, a negative disclosure threshold implies that the true dividend growth, $f$, is below the investors’ belief, $\hat{f}$, for sure. Consequently, investors know for certainty that they are overestimating the firm’s fundamental and this leads to a greater punishment in the case of non-disclosure. Figure 1 illustrates investor’s adjustment using a truncated normal distribution.

***Figure 1 Here***

At any time $t$ prior to disclosure, the market valuation of the firm is given as:

$$V_t = \frac{\hat{f} - q(\bar{g})}{r} + \frac{\hat{f}_t - \bar{f}}{r + \lambda}. \quad (12)$$

We see that the fact the manager has not disclosed decreases the firm value by $q(\bar{g})/r$, which is the present value of non-disclosure discount. The investors’ strategy is to value the firm with the non-disclosure discount continuously as long as the manager has not disclosed. The manager takes into account the investors’ valuation strategy and solves the following problem:

$$W(g_t, f_t) = \max_{\tau} E^M_t \left[ \int_{t}^{\tau} e^{-\delta(s-t)}V_s ds + e^{-\delta(\tau-t)}W_{\tau} \right], \quad (13)$$
where \( W(g_t, f_t) \) is the manager’s continuation utility, \( E_t^M[\cdot] \) is the manager’s conditional expectation, \( \delta \) is the manager’s discount rate, and \( \tau \) is a stopping time such that

\[
\tau = \inf \{ s : g_s \geq \bar{g}, s > t \}.
\]  

In the non-disclosure region, i.e. when \( g_t < \bar{g} \), \( W \) is the solution to the following PDE:

\[
\delta W = \frac{\bar{f} - q(\bar{g})}{r} + \frac{f - g - \bar{f}}{r + \lambda} - \rho g W_g + \lambda (\bar{f} - f) W_f + \frac{1}{2} \left( \sigma_g^2 W_{gg} + 2 \sigma_f^2 W_{gf} + \sigma_f^2 W_{ff} \right),
\]  

since the correlation between \( Z_g \) and \( Z_f \) is \( \sigma_f / \sigma_g \). In the RHS, the first two terms are the manager’s payoff when she does not disclose. The remaining terms are the change in the manager’s utility due to a change in the difference between the fundamental value and the investors’ belief.

We can see the two benefits of delaying a disclosure. First, there is the possibility that investors may overestimate the firm’s fundamental, i.e. \( g < 0 \), so that it dominates the non-disclosure discount, \( q(\bar{g}) \). Second, even though the non-disclosure discount may dominate the investors’ overestimation at the current moment, investors may begin to overestimate the firm’s fundamental in the future, which is the option value of waiting. The disclosure option value is coming from the difference between belief, \( g \). Thus, a natural conjecture for the solution to (15) is:

\[
W(g, f) = \frac{\bar{f}}{r \delta} + \frac{f - \bar{f}}{(r + \lambda)(\delta + \lambda)} - \frac{q(\bar{g})}{r \delta} + u(g),
\]  

where \( u(\cdot) \) is the solution to:

\[
\delta u(g) = -\frac{g}{r + \lambda} - \rho gu(g)' + \frac{\sigma_g^2}{2} u(g)''.
\]  

We see that the first two terms in (16) are the manager’s utility in case the fundamental value is publicly observed. The third term is a utility discount due to non-disclosure. Finally, \( u \) is the option value of disclosure. Following Scheinkmen and Xiong (2003), we can characterize the solution of \( u \) in the following proposition.

**Proposition 4** Let

\[
h(x) = \begin{cases} 
U \left( \frac{\delta}{2 \rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2 \right) & \text{if } x \leq 0 \\
\left( \frac{2 \pi}{\Gamma(\frac{1}{2} + \frac{\delta}{2 \rho})} \right) M \left( \frac{\delta}{2 \rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2 \right) - U \left( \frac{\delta}{2 \rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2 \right) & \text{if } x > 0,
\end{cases}
\]  

where \( \Gamma(\cdot) \) is the Gamma function, and \( M(\cdot, \cdot, \cdot) \) and \( U(\cdot, \cdot, \cdot) \) are two Kummer functions described in the Appendix. Then, any solution to (17) must satisfy

\[
u(g) = \beta h(g) - \frac{g}{(r + \lambda)(\delta + \rho)}.
\]
In the Appendix, we show that \( h(x) \) is positive, increasing, and that \( \lim_{x \to -\infty} h(x) = 0 \). Thus, when \( g \) is sufficiently low (negative), i.e. investors are overestimating, the disclosure option value is just the expected value of future overestimation, \( -g/(r + \lambda)(\delta + \rho) \). As \( g \) increases, the value of delaying increases and the manager remains quiet until \( g \) hits \( \bar{g} \).

Now, we have two unknowns, \( \beta \) and \( \bar{g} \). To solve these, we need to compute investors’ valuation and the manager’s utility after a disclosure decision. Consider \( t \geq \tau \), the firm incurs the cost of disclosure, \( c \) from dividend flows. Thus, the investors’ valuation is now

\[
V_t = \frac{\bar{f} - c}{r} + \frac{f_t - \bar{f}}{r + \lambda}. \tag{20}
\]

Notice that there is no longer a non-disclosure penalty and that the valuation does not rely on the dividend signal, but rather on the firm’s fundamental and known parameters. We assume that there is a fixed cost of disclosure \( c > 0 \), and let \( \alpha \) be the difference between the non-disclosure discount and the disclosure cost, \( \alpha = q(\bar{g}) - c \). We assume \( c \) is such that \( \alpha > 0 \) so that the cost of disclosure is always strictly less than the non-disclosure discount.\(^3\) The manager’s utility at time \( t \geq \tau \) is given by:

\[
W(g_t, f_t) = \frac{\bar{f} - c}{r\delta} + \frac{f_t - \bar{f}}{(r + \lambda)(\delta + \rho)}. \tag{21}
\]

Note that in the disclosure region, the manager’s utility is independent of the difference between the fundamental value and the investors’ belief. The manager would choose the optimal threshold level such that, at the disclosure boundary, her utility is continuous and differentiable. Thus, we have following value matching and smooth pasting conditions, respectively:

\[
\beta h(\bar{g}) = \frac{\bar{g}}{(r + \lambda)(\delta + \rho)} + \frac{\alpha}{r\delta}, \tag{22}
\]

\[
\beta h'(\bar{g}) = \frac{1}{(r + \lambda)(\delta + \rho)}. \tag{23}
\]

The value matching condition in (22) requires that, at the time of disclosure, there should be no jump in the manager’s utility. Also, the smooth pasting condition in (23) makes the manager worse off if she exercise the disclosure option at other boundary. The following result shows that there exists a unique pair \((\beta, \bar{g})\) that solves (22) and (23).

**Theorem 1** There exists a unique \( \beta \) and \( \bar{g} > -\frac{\alpha(r + \lambda)(\delta + \rho)}{r\delta} \) that solves (22) and (23).

\(^3\) We restrict \( \alpha \) to be positive so that the result is not driven by disclosure costs but rather from the option value of waiting. Allowing \( \alpha < 0 \) would not qualitatively affect our results since it only strengthens the benefit of delaying disclosure.
Note that there is a lower bound of the disclosure threshold. If the manager sets the disclosure boundary too low, the resulting non-disclosure discount is large so that the net benefit of overestimation and the option value is less than the disclosure benefit, $\alpha$. Moreover, we see that the lower bound is negative, which implies that the optimal disclosure boundary may also be negative, i.e. the manager begins disclosure even if the firm’s fundamental is overestimated. This is shown in the following corollary:

**Corollary 1** The manager’s optimal disclosure threshold is negative when the cost of disclosure satisfies the following condition:

$$\frac{r\delta}{(r + \lambda)(\delta + \rho)} \frac{h(0)}{h'(0)} < \alpha.$$  \hspace{1cm} (24)

where $h(\cdot)$ is defined in Proposition 4.

This result implies that when the benefit of disclosure is sufficiently high, the manager would decide to start disclosure even though the firm’s true fundamental is below the market belief. As shown by (24), this occurs when the cost of disclosure is sufficiently low. Intuitively, we find that this occurs because the manager’s gain in disclosing, avoiding the non-disclosure discount, outweighs the benefit of non-disclosure when the boundary is reached. This is a seemingly paradoxical result—we would not expect the manager to disclose a lower value than the estimate. However, this is possible because, as shown in the following section, disclosure during overestimation results in a price increase, thus making disclosure profitable for the manager, even in times of overestimation. This is not to say that unraveling occurs when (24) is satisfied. Rather, when the optimal threshold is negative, the manager continues to remain quiet as long as investors sufficiently overestimate the fundamental of the firm. Once this overestimation lessens to the point where the threshold is reached—even though this is still above the true fundamental value—the manager begins disclosure.

***Figure 2 Here***

Figure 2 plots the value of the disclosure option for two different values of $\alpha$. We use high $\alpha$ for Panel A. This value is greater than the left hand side in (24), i.e. we have a negative optimal disclosure boundary, $\bar{g} < 0$. We can see that the option value of waiting is decreasing in the difference between the true value and the investors’ belief. The manager waits until this option value is equal to the net benefit of disclosure, $\alpha/(r\delta)$. The benefit of delaying disclosure from the option value is greater than the cost of delaying from the downward adjustment in the
investors’ valuation when $g < \bar{g}$. At the disclosure boundary, the benefit and cost are identical and so the manager discloses. Investors adjust their estimate of true dividend growth by $q(\bar{g}) = 6.69\%$. In Panel B, we use low $\alpha$, which is less than the left hand side in (24). The net benefit of disclosure is not high enough to make the manager begin disclosure in overestimation. The manager thus bears some underestimation in order to allow for possible future overestimation. We find that the optimal disclosure boundary is positive $\bar{g} > 0$, and the non-disclosure penalty is $q(\bar{g}) = 4.26\%$.

***Figure 3 Here***

Figure 3 plots the simulated path of the true fundamental, the investors’ belief, and the threshold for disclosure. The manager’s strategy is to disclose when $g = \hat{f} - \hat{\bar{f}} \geq \bar{g}$, and thus we plot $\hat{f} + \bar{g}$ for the threshold for disclosure. We use a high $\alpha$ for Panel A and a low $\alpha$ for Panel B in order to generate disclosure boundaries during times of overestimation and underestimation, respectively. In this example, we assume that the initial belief is at the long run mean, $\bar{f}$, and that the initial fundamental is sufficiently below the investors’ belief so that there is not immediate disclosure. We see that the manager in Panel A waits until the true value crosses the disclosure boundary, which is below $\hat{f}$ since $\bar{g} < 0$. The manager discloses even if the investors’ belief is higher than the true value since the increase in the manager’s utility from removal of non-disclosure discount outweighs the option value from the investors’ overestimation and downward adjustment of beliefs. This is discussed further in the following section. On the other hand, in Panel B the disclosure boundary is above the investors’ belief, as $\bar{g} > 0$. Thus, the manager waits until the investors’ belief is sufficiently below the true value. The manager finds it optimal to disclose since the option value is dominated by the investors’ upward adjustment of beliefs and removal of the non-disclosure penalty.

3.3 Jump Size in Firm Value at Disclosure

The manager has a right to choose the optimal disclosure boundary so that her utility is smooth at the disclosure point. However, there will be a jump in the firm value at the time of disclosure decision. We can compute the jump size in firm value at the disclosure time:

$$V_{r+} - V_{r-} = \frac{\alpha}{r} + \frac{\bar{g}}{r + \lambda}.$$  

The first term in the right hand side is the present value of the disclosure benefit due to the elimination of non-disclosure discount. The second term in the right hand side is due to the
investors’ reaction to the disclosed true value. When $\bar{g} > 0$, i.e. disclosure at underestimation, the price jump is obviously positive. However, when $\bar{g} < 0$ it is questionable. The following proposition shows that the jump in the market valuation upon disclosure is always positive regardless of $\bar{g}$.

**Proposition 5** The jump size of firm value at the time of disclosure is always positive.

If the manager discloses during a period of underestimation ($\bar{g} > 0$), the increase in firm value is straightforward, since investors adjust their belief upward to the higher true fundamental value. Moreover, we also have a positive jump in the firm value even if the disclosure is made when investors are overestimating the firm’s fundamental ($\bar{g} < 0$). When the disclosure is made during overestimation, the investors would initially adjust their estimate to the lower value of the true fundamental. However, this downward adjustment is not enough to cancel out the net benefit of disclosure, since the non-disclosure discount is sufficiently greater than the disclosure cost (see (24).) Consequently, the market valuation increases following disclosure, even if the disclosed value was less than the market belief at the time of disclosure. Intuitively, because the cost of disclosure is such that the net benefit of disclosure is constant and independent of the disclosure boundary, the manager expects that there is always a constant net benefit of disclosure, $\alpha$, no matter what boundary he chooses. Hence, at the optimal boundary, the manager’s option value is equal to the net benefit of disclosure plus the negative downward adjustment of investors. The net result is positive.

### 3.4 Expected Duration of Delaying

We now investigate the frequency of disclosure, which can be thought of as the total expected delay before disclosure is made. We can derive the ex-ante duration of the delay period by using a transform analysis. Let $k(g_t, \delta) = E_t^M \left[ e^{-\delta(\tau-t)} \right]$, given that $g_t < \bar{g}$. Then, $k(g, \delta)$ is the manager’s present value of one dollar at the first time that the belief difference hits the disclosure boundary, given that the current belief difference is below the boundary. We can derive the ODE that $k(g, \delta)$ satisfies, given by:

$$\delta k = -\rho g k_g + \frac{1}{2} \sigma_g^2 k_{gg},$$

with $k(g, \delta) = 1$. Since $g_t = \bar{g}$, the solution is $k(g, r) = \frac{k(g)}{k(\bar{g})}$. Then, the expected duration of delaying is

$$E_0^M [\tau] = - \frac{\partial k(g_0, \delta)}{\partial \delta} \bigg|_{\delta=0}. \quad (27)$$
We can numerically compute the above value.

Figure 4 plots the optimal disclosure boundary and expected duration of delaying as functions of the true value’s rate of mean-reversion. We fix $\alpha$ and vary $\lambda$ from 0.2 to 1.8. These mean-reversion speeds correspond to autocorrelation values of 0.17 to 0.82. Also, we assume that the initial belief difference equal to zero (i.e., unbiased), which implies that the manager discloses immediately when $\bar{g}$ is negative. We see that the optimal disclosure boundary and expected duration of delaying disclosure are both decreasing in the rate of mean-reversion. Intuitively, the investors place a greater weight on the long-run mean of the fundamental value, $\bar{f}$, in their estimate when the mean-reversion speed is fast. Correspondingly, the option value of waiting decreases as the market estimation is tied more closely to $\bar{f}$ and less likely to result in significant overestimation. Hence, the manager’s optimal threshold for disclosure decreases as the option value of waiting decreases, as shown in Panel A. Similarly, because the optimal threshold is decreasing in the rate of mean-reversion, so will the expected duration of the delay period. As the optimal threshold for disclosure shrinks, it becomes more likely that investors will reach this boundary, and hence the duration of delay is decreasing in the rate of mean-reversion as well, as illustrated in Panel B.

4 Extended Model

The parsimonious model in the previous section captures the primary results of this paper. In this section, we extend the model by allowing for unlimited disclosure opportunities. The manager may now disclose information at any time and for however long she wishes. During the disclosure period, the manager may discontinue disclosure at any time, and begin disclosure at any time during a quiet period. For tractability, we assume that there are now two types of investors— informed and uninformed investors. Informed investors are able to observe when the manager discloses and the information that is disclosed. This can be thought of as investors who have access to analyst services that provide interpretation of the information that is disclosed. Thus, when the manager does not disclose, the informed investors use their filtered value and impose the non-disclosure discount as in the baseline model. When the manager discloses, the informed investors use the disclosed true value they observe. On the other hand, uninformed investors, do not observe the manager’s disclosure decision or the information disclosed. Uninformed investors consequently set their beliefs equal to the filtered value and do not impose the
We assume there is a unit mass of investors, where the stationary mass of informed investors at a given time is denoted as \( w \in (0, 1) \). Moreover, we assume that any given investors’ type is continually refreshed. Finally, we relax the assumption that the cost of disclosure, \( c = q(\bar{g}) - \alpha \), is always strictly less than non-disclosure discount. We allow \( \alpha \) to be negative so that the manager does not disclose unless the disparity between the true value and the market’s belief is high enough to induce the manager to bear the additional cost of disclosure.

As in the baseline model, the manager uses a threshold strategy for disclosure. When the difference between the fundamental value and the market’s belief exceeds this threshold, the manager begins disclosure. However, the manager now has a “shadow” threshold for when to discontinue disclosure. That is, when the noisy signal improves to the point where, if the manager had stopped disclosing, the belief difference is below the threshold value, then she discontinues disclosure. The manager discloses only for as long as the public signal is unfavorable, however, when the public signal improves sufficiently, she chooses to again keep her information private and incur the non-disclosure discount from the informed investors.

---

4 Uninformed investors do not know ex-ante whether they are in a disclosure region. They may infer that they are in a disclosure region ex-post if the resulting price is above the public signal. However, because they do not observe the true \( f \) during disclosure and because shocks are i.i.d., the probability that they will be in a disclosure region in the following instant is not affected by knowledge of having just been in a disclosure region. Consequently, ex-post knowledge of being in a disclosure or quiet region does not affect the uninformed investors’ ex-ante belief in the following instant.

5 This assumption is included for tractability. Let \( w \in (0, 1) \) be the stationary mass of informed investors. At every instant, we have the following transition matrix: \( p = \Pr(\text{remains informed}) \) and \( q = \Pr(\text{remains uninformed}) \). Then, we have a stationary mass of informed investors \( w \) given by

\[
\begin{bmatrix}
  w \\
  1 - w
\end{bmatrix} = \begin{bmatrix}
  p & 1 - p \\
  1 - q & q
\end{bmatrix} \begin{bmatrix}
  w \\
  1 - w
\end{bmatrix}.
\]

In the steady state, the mass of informed investors is fixed at \( w \), but the composition is time-varying; i.e. some investors become informed and some become uninformed, and this happens continuously. This assumption is common in the search literature, see Duffie, Grleau, and Pedersen (2005), among others.

6 The non-disclosure discount is only imposed by informed investors and the cost of disclosure is independent of the composition of investors. Hence, if the mass of informed investors is sufficiently low, the cost of disclosure may be higher than the non-disclosure discount.
4.1 Quiet Period

During a quiet period, the manager hides her private information. The fraction \( w \) of investors take into account the fact that the manager is not disclosing and value the firm using the truncated distribution according to the endogenously determined threshold. As in the baseline model, they again impose the non-disclosure discount, \( q(\bar{g}) \). However, the remaining \( 1 - w \) investors do not impose the non-disclosure discount. We can easily derive the manager’s utility as in the benchmark model:

\[
W(g, f) = \frac{\bar{f}}{r\delta} + \frac{f - \bar{f}}{(r + \lambda)(\delta + \lambda)} - \frac{w q(\bar{g})}{r\delta} + u(g),
\]  

(28)

where \( u(\cdot) \) solves the same ODE as (17). Note that the manager’s utility is decreased by \( w \) times the non-disclosure discount, \( q(\bar{g})/(r\delta) \).

4.2 Disclosure Period

In a disclosure period, the manager continuously discloses the firm’s true value, \( f \), but only informed investors react to the revealed information. This implies that even in a disclosure period, the manager has an option of discontinuing the disclosure and going back to a quiet period. The firm value in a disclosure period is given by:

\[
V_t = \frac{\bar{f} - c}{r} + \frac{wf_t + (1 - w)\bar{f}_t - \bar{f}}{r + \lambda}.
\]  

(29)

Note that informed investors no longer impose a punishment and the valuation is based on the weighted average of the true value, \( f \), and filtered value, \( \hat{f} \). There is also the cost of disclosure, \( c \). To be consistent with the baseline model, we assume that the cost of disclosure is \( c = w q(\bar{g}) - \alpha \), i.e. the wedge between keeping quiet and disclosure is again \( \alpha \). The manager’s utility is given by:

\[
W(g, f) = \frac{\bar{f}}{r\delta} + \frac{f - \bar{f}}{(r + \lambda)(\delta + \lambda)} - \frac{w q(\bar{g}) - \alpha}{r\delta} + \tilde{u}(g),
\]  

(30)

where \( \tilde{u}(\cdot) \) solves the following ODE:

\[
\delta \tilde{u}(g) = -\frac{(1 - w)g}{r + \lambda} - \rho g \tilde{u}(g)' + \frac{\sigma^2}{2} \tilde{u}(g)''.
\]  

(31)

We use this to solve for the unique belief threshold.
4.3 Equilibrium Threshold

We conjecture that, in equilibrium, whenever the belief difference, $g$, is less than the equilibrium threshold, $\bar{g}$, the manager remains quiet, otherwise she would begin disclosure. Thus, we have the following inequalities:

$$u(g) \begin{cases} > \bar{u}(g) + \frac{\alpha}{r\delta} & g < \bar{g}, \\ < \bar{u}(g) + \frac{\alpha}{r\delta} & g > \bar{g}. \end{cases}$$

(32)

Also, at the equilibrium threshold, the manager’s utility will be continuous and differentiable. This implies that

$$u(\bar{g}) = \bar{u}(\bar{g}) + \frac{\alpha}{r\delta},$$

(33)

$$u'(\bar{g}) = \bar{u}'(\bar{g}).$$

(34)

We conjecture that the solution of $u$ is same as (19) and the solution of $\bar{u}$ is given by:

$$\bar{u}(g) = \beta h(-g) - \frac{(1 - w)g}{(r + \lambda)(\delta + \rho)}.$$  

(35)

The following theorem shows that the functions $u$ and $\bar{u}$ established above satisfy (17), (31), and (32), and that there exists a unique pair $(\beta, \bar{g})$ that solves (33) and (34).

**Theorem 2** There exists a unique

$$\bar{g} \begin{cases} < -\frac{\alpha}{w}(r+\lambda)(\delta+\rho) & \text{if } \alpha > 0, \\ > -\frac{\alpha}{w}(r+\lambda)(\delta+\rho) & \text{if } \alpha < 0. \end{cases}$$

(36)

and $\beta$ that solve (33) and (34). The function $u$ and $\bar{u}$ constructed above are equilibrium option value functions. The optimal policy consists of hiding the information if $g < \bar{g}$; otherwise disclose the true $f$.

When $\alpha > 0$, we see that the optimal disclosure threshold has an upper boundary which is always negative and depends on the mass of informed investors. This implies that the manager would disclose optimally even when the market belief is greater than the true fundamental value. This occurs because of the unlimited disclosure opportunities. In the baseline model, the manager has only one irreversible disclosure decision, which results in a much greater option value, pushing the optimal disclosure threshold, in some cases, to be positive, $\bar{g} > 0$ (i.e. delay disclosure as much as the manager can and disclose only in times of sufficient underestimation.) However, in the extended model, the manager can disclose and remain quiet infinitely often.
Suppose that the manager is disclosing must decide whether to continue or stop disclosure. By entering a quiet region, the informed investors no longer observe the true value and must use the filtered value. Hence, the increase in the manager’s utility from the informed investors’ ignorance of the true fundamental should outweigh the decrease in the manager’s utility from the non-disclosure discount imposed by informed investors in the quiet region. The optimal threshold will be determined after the manager takes into account the option values in both regions. That is, the option value of continuing disclosure (paying a disclosure cost less than the non-disclosure discount) outweighs the option value of delaying disclosure (possibility of overestimation) at the upper boundary so the manager continues disclosure. This is not to say unraveling occurs; on the contrary, in equilibrium we have quiet regions and disclosure regions. Also, we see that the upper boundary for the optimal disclosure threshold is increasing in the mass of informed investors. The greater the mass of informed investors, the sooner the manager discontinues disclosure as the benefit from the ignorance of the informed investors is higher given the negative belief difference.

Next, consider the case where the disclosure cost exceeds the non-disclosure discount (i.e. when $\alpha < 0$). The manager is consequently further incentivized to remain quiet. We see that disclosure is only optimal when the market sufficiently undervalues the firm. The manager’s benefit from disclosure through the informed investors’ exact valuation outweighs the decrease in the manager’s utility due to the additional cost of disclosure over remaining quiet. The benefit is increasing in the mass of informed investors given the level of undervaluation, and thus the lower boundary for $\bar{g}$ is decreasing in $w$.

***Figure 5 Here***

Figure 5 plots the simulated path of the true fundamental, the investors’ belief, and the disclosure boundary ($\hat{f} + \bar{g}$). We assume for this example that the initial belief is at the long-run mean, $\bar{f}$, and that the initial fundamental is sufficiently below the investors’ belief to ensure that the manager begins in a quiet region. Also, we assume positive $\alpha$ which induces negative $\bar{g}$. We can see that the manager remains quiet until the true value crosses the threshold, which is below $\hat{f}$ since $\bar{g} < 0$. Hence, if the investors’ overestimation is not sufficiently high to compensate for the non-disclosure penalty imposed by the informed investors during a quiet period, the manager begins disclose. The manager may discontinue disclosure at any time while she is in a disclosure period. As we see in the example, once the true fundamental again becomes sufficiently low relative to investors’ belief and crosses the disclosure boundary (i.e. the shadow threshold), $\hat{f} + \bar{g}$, the manager discontinues disclosure and reverts to a quiet period.
4.4 Jump Size in Firm Value

As in the baseline model, we can compute the jump size in firm value when the manager switches from a quiet to a disclosure region, or vice versa. Let $\tau$ be the time when the manager starts to disclose, and $\hat{\tau}$ be the time when the manager discontinues disclosure. Then, we have

\[ V_{\tau^+} - V_{\tau^-} = \frac{\alpha}{r} + \frac{w\bar{g}}{r + \lambda}, \]  

(37)

\[ V_{\hat{\tau}^+} - V_{\hat{\tau}^-} = -\frac{\alpha}{r} - \frac{w\bar{g}}{r + \lambda}. \]  

(38)

In (37), the first term in the right hand side is the present value of the net disclosure benefit due to the elimination of the non-disclosure discount. The second term in the right hand side is due to the informed investors’ reaction to the revealed information at the time of disclosure. Theorem 2 tells that the optimal $\bar{g}$ has the opposite sign of $\alpha$. Thus, the price jump can be positive or negative. We also see that, upon discontinuing disclosure, there is a jump in the firm value in the opposite direction from a quiet to a disclosure region. The following proposition characterizes the sign of the price jump in the market valuation upon disclosure.

**Proposition 6** The jump size of firm value at the time of disclosure is negative when $\alpha > 0$, and positive when $\alpha < 0$.

As we discuss above, when disclosure is more costly than non-disclosure, i.e. $\alpha < 0$, the manager makes a decision of entering a disclosure region such that the positive price adjustment by informed investors is at least as large as the extra cost of disclosure. This implies that the price would jump positively when entering a disclosure region. Similarly, when disclosure is less costly than non-disclosure, the manager would enter a quiet region only if the upward price adjustment by informed investors due to no longer valuing the firm with the true fundamental exceeds the cost of the non-disclosure discount. Thus, the price jumps positively when entering a quiet region. Proposition 6 is very intuitive in the sense that releasing good (bad) news actually leads a price increase (decrease) at the time of disclosure. Also, if this price adjustment by informed investors is not enough, we would observe a continuation of price drift. We characterize condition under which this occurs in the following section.

4.5 Implications for Post-Earnings Announcement Drift

This framework can be used to explain a host of puzzles in disclosure. A well-documented phenomenon in accounting and finance that has few theoretical underpinnings outside of bounded
rationality models is post-earnings announcement drift. Investors tend to underreact to the information content of disclosure, thereby generating return continuation–negative abnormal returns occur in the days following a negative announcement while positive abnormal returns follow in the days after a positive announcement. This is known as the post-earnings announcement drift (PEAD) anomaly. We see that our extended model predicts PEAD. Since the extended model generates a positive or negative disclosure threshold, i.e. the “good news” or “bad news” announcement depending on the sign of \( \alpha \), the condition for PEAD can be stated as when the ex-ante firm value at the disclosure decision is lower or greater than the stationary firm value in a disclosure region. We can compute the ex-ante firm value at the time of disclosure (post-disclosure). This is given by:

\[
E_0^M [V_{\tau^+}] = \frac{\bar{f} - c}{r} + \frac{w\bar{g} - \bar{f}}{r + \lambda} + \frac{1}{r + \lambda} E_0^M \left[ \hat{g}_{\tau} \right],
\]

since at the time of disclosure we have \( f_{\tau} = \hat{f}_{\tau} + \bar{g} \). The ex-ante firm value depends on the investors’ belief at the time of disclosure. We have that the investors’ belief and the difference between the true value and beliefs both follow a multivariate mean-reverting process:

\[
d\hat{f}_t = \left\{ \lambda \left( \bar{f} - \hat{f}_t \right) - (\lambda - \rho)g_t \right\} dt + (\rho - \lambda) \sigma_D dZ^D_t,
\]

\[
dg_t = -\rho g_t dt + \sigma_g dZ^g_t.
\]

Without loss of generality, suppose that at time zero, the investors’ belief is \( \bar{f} \), i.e. \( \hat{f}_0 = \bar{f} \). Then, we can derive the expectation of \( \hat{f}_{\tau} \) by integrating the above equations and setting \( g_{\tau} = \bar{g} \):

\[
E_0^M \left[ \hat{f}_{\tau} \right] = \bar{f} + \bar{g} \left( E_0^M \left[ e^{(\rho - \lambda)\tau} \right] - 1 \right). \tag{42}
\]

Thus, the ex-ante firm value at the time of disclosure is

\[
E_0^M [V_{\tau}] = \frac{\bar{f} - c}{r} + \frac{\bar{g}}{r + \lambda} \left( E_0^M \left[ e^{(\rho - \lambda)\tau} \right] - (1 - w) \right). \tag{43}
\]

The stationary firm value in disclosure region is \( \overline{V} = \frac{\bar{f} - c}{r} \). The following proposition provides the condition for the ex-ante firm value at the time of disclosure to be lower or greater than \( \overline{V} \).

**Proposition 7** If the mass of informed investors satisfy

\[
w < 1 - E_0^M \left[ e^{(\rho - \lambda)\tau} \right], \tag{44}
\]

the firm value continues to decrease (increase) after the manager switches from a quiet region to a disclosure region when \( \alpha \) is positive (negative).

---

7 See Ball and Brown (1968) and Bernard and Thomas (1989, 1990), among several others.
Given that \( w \in (0, 1) \), if \( \rho > \lambda \) we will not see PEAD. Hence, a sufficient condition is \( \rho < \lambda \), i.e. the speed of mean-reversion for the difference in beliefs is slower than that of the true process. This happens when the correlation between shocks to the noisy signal and the true fundamental are sufficiently negative. This causes there to be a greater difference between the true value and the investors’ filtered value. Moreover, since the equilibrium belief-difference threshold is negative (positive) when \( \alpha > \left( < \right) 0 \), as shown in Theorem 2, this implies that the ex-ante filtered value at the time of disclosure is always greater (lower) than the long-run mean, \( \bar{f} \). Consequently, if there exists a sufficiently small mass of informed investors, the misestimation at the time of disclosure is not reflected into the firm value enough, which makes the continuation of price drift following disclosure.

4.6 Disclosure Frequency

We can also examine the frequency of disclosures in the extended model. We can define disclosure frequency as the likelihood of being in a disclosure period at any given time. We see from Theorem 2 that the disclosure frequency is decreasing in \( g \):

**Corollary 2** The manager’s frequency of disclosure is strictly decreasing in the optimal threshold \( g \).

This is intuitive as the greater the difference required between the market belief and the true fundamental for disclosure leads to lower likelihood of reaching this difference.

5 Conclusion

In this study, we have developed a framework for continuous-time disclosure that shows a number of interesting and hitherto uncaptured results. We have shown that, in the face of evolving news, a real option is borne to the manager, which results in delay even though the real costs of disclosure are assumed to be strictly less than the non-disclosure discount. We see that this results in a paradoxical punishment scheme by investors, where manager who disclose more frequently are punished more severely. Moreover, we characterize cases where the manager discloses even though the firm’s fundamental value is being overstated by investors. In the extended setting, we have shown that the manager may engage in quiet periods following disclosure.
This framework can be applied to investigate a number of questions in accounting and finance. For example, this model can be extended to investigate portfolio choice with disclosure, and clustering of announcements if mandatory disclosure is present. This study presents a number of interesting results and develops a framework that can be applied to investigate further questions in continuous time disclosure.
Appendix

A Proofs

Proof of Proposition 1. We have
\[ \frac{\partial \gamma}{\partial \lambda} \sim \lambda + \phi \sigma f / \sigma D - \rho \leq 0, \]  
(A.1)
and
\[ \frac{\partial \gamma}{\partial \phi} \sim \lambda + \phi \sigma f / \sigma D - \phi - \rho \leq 0. \]  
(A.2)

Proof of Proposition 2. We have
\[ \frac{\partial \rho}{\partial \phi} \sim \lambda \geq 0, \]  
(A.3)

Proof of Proposition 3. Clearly, \( q(\bar{g}) \) is positive and we have
\[ \frac{\partial q(\bar{g})}{\partial \bar{g}} \sim -\frac{\bar{g}}{\sqrt{\gamma}} - n \left( -\frac{\bar{g}}{\sqrt{\gamma}} \right) \left( 1 - N \left( -\frac{\bar{g}}{\sqrt{\gamma}} \right) \right)^{-1}. \]  
(A.4)

For positive \( \bar{g} \), clearly the above is negative and thus \( q(\bar{g}) \) is decreasing in \( \bar{g} \). For negative \( \bar{g} \), let \( R(x) = \frac{1 - N(x)}{n(x)} > 0 \). Then, we can express the above equation as
\[ x - \frac{1}{R(x)}, \]  
(A.5)

where \( x = -\bar{g}/\sqrt{\gamma} > 0 \). We want to show that \( R(x) \leq \frac{1}{x} \). We have followings relations
\[ R(x)' = xR(x) - 1 \]  
(A.6)
\[ \lim_{x \to \infty} xR(x) = 1, \]  
(A.7)

Suppose that at any point \( x_1, x_1 R(x_1) > 1 \) by contradiction. Then, by (A.6) \( R(x) \) would continue to increase and \( xR(x) \) also continue to increase. Hence, we have \( xR(x) > 1 \) for \( x \geq x_1 \), which contradicts (A.7). ■

Proof of Proposition 4. Two Kummer functions are defined as
\[ M(a, b, y) = 1 + \frac{ay}{b} + \frac{(a)2y^2}{(b)2!} + \cdots, \]  
(A.8)

with \((a)_{n} = a(a + 1)(a + 2) \cdots (a + n - 1)\) and \((a)_{0} = 1\), and
\[ U(a, b, y) = \frac{\pi}{\sin(\pi b)} \left[ \frac{M(a, b, y)}{\Gamma(1 + a - b)\Gamma(b)} - y^{1-b}M(1 + a - b, 2 - b, y) \right]. \]  
(A.9)

Consider the following differential equation
\[ yv''(y) + \left( \frac{1}{2} - y \right) v'(y) - \frac{r}{2\rho} v(y) = 0. \]  
(A.10)
It is straightforward to verify that \( u(g) = v (\rho g^2/\sigma_g^2) - g/((r + \lambda)(r + \rho)) \) satisfies (17). Then, a general solution to (A.10) is (see Abramowitz and Stegun 1964, Scheinkmen and Xiong 2003)

\[
v(y) = \alpha M \left( \frac{r}{2 \rho}, \frac{1}{2}, y \right) + \beta U \left( \frac{r}{2 \rho}, \frac{1}{2}, y \right).
\]  

(A.11)

We can construct two solutions \( u(g) \) for \( g < 0 \) and for \( g > 0 \). This gives us four unknowns. As \( g \to -\infty \), the disclosure option value goes away, i.e. \( u(g) = -g/((r + \lambda)(r + \rho)) \). Also at \( g = 0 \) two solutions should have same values and first-order derivatives. These three conditions make us left with one unknown, which is \( \beta \) in (19). For detail procedures, please refer to Scheinkmen and Xiong (2003).

**Lemma 1** Consider a function \( h(x) \) defined in equation (18). Then, \( h(x) > 0 \), \( \lim_{x \to -\infty} h(x) = 0 \), \( h'(x) > 0 \), \( h''(x) > 0 \), and \( h'''(x) > 0 \).

**Proof.** We omit proof here. Please refer to Scheinkmen and Xiong (2003) for details.

**Proof of Theorem 1.** Using equation (22) and (23), we can define the following function:

\[
F(g) = h'(g) \left( g + \alpha \frac{(r + \lambda)(\delta + \rho)}{r \delta} \right) - h(g).
\]  

(A.12)

We want to show that there is a unique \( \bar{g} \) such that \( F(\bar{g}) = 0 \). Then, \( \beta = 1/(h'(\bar{g})(r + \lambda)(\delta + \rho)) \). For all \( g < -\frac{\alpha(r + \lambda)(\delta + \rho)}{r \delta} \), \( F(g) < 0 \) by Lemma 1. Also, for all \( g > -\frac{\alpha(r + \lambda)(\delta + \rho)}{r \delta} \),

\[
F'(g) = h''(g) \left( g + \alpha \frac{(r + \lambda)(\delta + \rho)}{r \delta} \right) > 0
\]  

(A.13)

\[
F''(g) = h''(g) + h'''(g) \left( g + \alpha \frac{(r + \lambda)(\delta + \rho)}{r \delta} \right) > 0,
\]  

(A.14)

also by Lemma 1. Therefore \( F(g) = 0 \) has a unique solution \( \bar{g} > -\frac{\alpha(r + \lambda)(\delta + \rho)}{r \delta} \).

**Proof of Corollary 1.** In Theorem 1, we prove that for \( g > -\frac{\alpha(r + \lambda)(\delta + \rho)}{r \delta} \), \( F(g) \) is a increasing and convex function. Therefore, if \( F(0) \) is positive, we have negative \( \bar{g} \) solving \( F(\bar{g}) = 0 \). The condition of \( F(0) > 0 \) can be translated into equation (24).

**Proof of Proposition 5.** The optimal boundary solves \( F(\bar{g}) = 0 \). Hence, we can have

\[
V_{\tau^+} - V_{\tau^-} = \frac{\alpha}{r} + \frac{\bar{g}}{r + \lambda}
\]  

(A.15)

\[
= \frac{1}{h'(\bar{g})(r + \lambda)(\delta + \rho)} [h'(\bar{g})\rho\bar{g} + \delta h(\bar{g})]
\]  

(A.16)

\[
= \frac{\sigma_h^2 h''(\bar{g})}{2h'(\bar{g})(r + \lambda)(\delta + \rho)}
\]  

(A.17)

\[
> 0
\]  

(A.18)

Note that we use the property that \( h(x) \) solves \( \delta h(x) = -\rho x h'(x) + (1/2)\sigma_h^2 h(x)'' \) and Lemma 1.

**Proof of Theorem 2.** First, we prove the existence of a pair \( (\beta, \bar{g}) \) that solves (33) and (34). Substitute (19) and (35) in (34), then we have

\[
\beta = \frac{w}{(h'(\bar{g}) + h'(-\bar{g}))(r + \lambda)(\delta + \rho)}.
\]  

(A.19)
Plug this in (33), then we get the equation that $\bar{g}$ satisfies:

$$
(h'(\bar{g}) + h'(-\bar{g})) \left( \bar{g} + \frac{(r + \lambda)(\delta + \rho)}{r\delta} \frac{\alpha}{w} \right) - h(\bar{g}) + h(-\bar{g}) = 0.
$$

We can define the following function:

$$
K(g) = (h'(g) + h'(-g)) \left( g + \frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta} \right) - h(g) + h(-g).
$$

Then, we want to show that there is a unique $\bar{g}$ such that $K(\bar{g}) = 0$. The first and second derivatives of $K$ are

$$
K'(g) = (h''(g) - h''(-g)) \left( g + \frac{(r + \lambda)(\delta + \rho)}{r\delta} \right),
$$

$$
K''(g) = h''(g) - h''(-g) + (h''(g) + h''(-g)) \left( g + \frac{(r + \lambda)(\delta + \rho)}{r\delta} \right).
$$

First consider a case of $\alpha > 0$. For all $-\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta} \leq g \leq 0$, $K(g) > 0$ since $K \left( -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta} \right) > 0$, $K(0) > 0$, and $K'(g) < 0$. For all $g > 0$, $K(g) > 0$ since $K(0) > 0$ and $K'(g) > 0$. For all $g < -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta}$, $K'(g) > 0$ and $K''(g) < 0$. Therefore, $K(\bar{g}) = 0$ has a unique solution $\bar{g} < -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta}$ if $\alpha > 0$.

Next, consider a case of $\alpha < 0$. Then, for all $0 \leq g \leq -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta}$, $K(g) < 0$ since $K \left( -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta} \right) > 0$, $K(0) < 0$, and $K'(g) < 0$. For all $g < 0$, $K(g) < 0$ since $K(0) < 0$ and $K'(g) > 0$. For all $g > -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta}$, $K'(g) > 0$ and $K''(g) > 0$. Therefore, $K(\bar{g}) = 0$ has a unique solution $\bar{g} > -\frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta}$ if $\alpha < 0$.

Finally, we can easily verify that $u$ solves (17) and $\tilde{u}$ solves (31). Finally, the condition (32) can be translated into $K(g) < 0$ for $g < \bar{g}$ and $K(g) > 0$ otherwise. This is exactly what we have. ■

**Proof of Proposition 6.** Since the optimal threshold $\bar{g}$ solves $K(\bar{g}) = 0$, from (A.21) we have

$$
\bar{g} + \frac{\alpha (r + \lambda)(\delta + \rho)}{r\delta} = \frac{h(\bar{g}) - h(-\bar{g})}{h'(\bar{g}) + h'(-\bar{g})}
$$

Hence, the price jump upon disclosing can be expressed as

$$
V_{r+} - V_{r-} = \alpha + \frac{w\bar{g}}{r + \lambda}
$$

$$
= \frac{w [\delta h(\bar{g}) + \rho \bar{g} h'(\bar{g}) - \delta h(-\bar{g}) + \rho \bar{g} h'(-\bar{g})]}{(r + \lambda)(\delta + \rho) (h'(\bar{g}) + h'(-\bar{g}))}
$$

$$
= \frac{w \sigma^2 g [h''(\bar{g}) - h''(-\bar{g})]}{2(r + \lambda)(\delta + \rho) (h'(\bar{g}) + h'(-\bar{g}))}
$$

Note that we use that $h(x)$ solves $\delta h(x) = -\rho x h'(x) + \frac{1}{2} \sigma^2 h''(x)$. When $\alpha > (>)0$, $\bar{g} < (>)0$ and $h''(\bar{g}) - h''(-\bar{g}) < (>)0$. Hence, the price jump upon disclosing is negative (positive) when $\alpha > (>)0$.

■

**Proof of Proposition 7.** We want to show that the ex-ante firm value at the time of disclosure is greater (less) than the stationary firm value in a disclosure region when $\bar{g} < 0$ ($\bar{g} > 0$). This is satisfied when

$$
E_0^M [e^{(\rho - \lambda)\tau}] - (1 - w) < 0,
$$

which is (44). ■
**Proof of Corollary 2.** The stationary distribution of the difference between the true value of $f$ and the filtered value is a normal distribution with zero mean and variance of $\frac{\sigma^2}{\sigma^2}$. Hence, in a stationary state the probability that the manager is in a disclosure region is $N\left(-\frac{\tilde{g}}{\sigma^2}\right)$, which is strictly decreasing in the optimal threshold $\tilde{g}$. ■
References


We plot investor’s subjective distribution of true fundamental, $f$ when $\hat{f} = \bar{f}$. We normalize probability density using long-run mean $\bar{f}$ and stationary variance $\gamma$. Shaded area indicates that non-disclosure information truncates the distribution. Panel A is for a negative disclosure boundary, and Panel B is for a positive disclosure boundary.
Figure 2: The Value of Disclosure Option

Panel A: Disclosure at Overvaluation

We plot the value of disclosure option as a function of difference in belief for different values of $\alpha$. We use high $\alpha$ for Panel A and low $\alpha$ for Panel B.
Figure 3: Simulated Path of Fundamental and Belief

Panel A: Disclosure at Overvaluation

Panel B: Disclosure at Undervaluation

We plot the simulated path of fundamental and investor’s belief. We use high $\alpha$ for Panel A and low $\alpha$ for Panel B. The initial belief is at the long run mean.
We plot the optimal disclosure boundary in Panel A and expected duration of delaying in Panel B for various mean-reversion speed. We assume that the initial belief is unbiased.
Figure 5: Simulated Path of Fundamental and Belief in the Extended Model

We plot the simulated path of fundamental and investor’s belief in the extended model. The initial belief is at the long run mean.