Adverse Selection and Intermediation Chains *

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We propose a parsimonious model of over-the-counter trading with asymmetric information to explain the existence of intermediation chains that stand between buyers and sellers of assets. Trading an asset through multiple intermediaries can preserve the efficiency of trade by reallocating an information asymmetry over many sequential transactions. An intermediation chain that involves heterogeneously informed agents helps to ensure that the adverse selection problems counterparties face in each transaction are small enough to allow for socially efficient trading strategies by all parties involved. Our model makes novel predictions about network formation and rent extraction when adverse selection problems impede the efficiency of trade.

Keywords: Intermediation Chains, Asymmetric Information, OTC Trading Networks, Information Percolation, Payment for Order Flow

JEL Codes: G20, D82, D85
1 Introduction

Transactions in decentralized markets often feature the successive participation of several intermediaries. For example, Viswanathan and Wang (2004, p.2) note that in foreign exchange markets “much of the inter-dealer trading via direct negotiation is sequential (an outside customer trades with dealer 1 who trades with dealer 2 who trades with dealer 3 and so on) and involves very quick interactions”. Adrian and Shin (2010, p.604) argue that, more broadly, the whole U.S. financial system has shifted in recent decades from its traditional, centralized model of financial intermediation to a more complex, market-based model characterized by “the long chain of financial intermediaries involved in channeling funds” (see also Kroszner and Melick 2009, Cetorelli, Mandel, and Molineaux 2012, Pozsar et al. 2013, for similar characterizations).

In this paper, we propose a parsimonious model that rationalizes the existence of intermediation chains. We show that chains of heterogeneously informed agents can fulfill an important economic role in intermediating trade by reallocating information asymmetries over multiple sequential transactions. Our model considers two asymmetrically informed agents who wish to trade an asset over the counter (OTC) in order to realize exogenous gains to trade (for example, for liquidity reasons). One agent is assumed to be an expert who is well informed about the value of the asset, whereas the other agent is uninformed. A standard result in models like ours is that trade breaks down between agents when the potential gains to trade are small relative to the degree of information asymmetry about the asset’s value. In that case, we show that involving heterogeneously informed agents — whose information quality ranks between that of the ultimate buyer and that of the seller — to intermediate trade can improve trade efficiency. In contrast to other intermediation theories in which one intermediary suffices to eliminate inefficient behavior, our simple mechanism can explain why trading often goes through chains of intermediaries rather than through simpler trading networks centered around one dominant broker. We show that trade efficiency can be improved by reallocating the adverse selection problem over a large number of sequential transactions as long as the difference in information quality is small between the two counterparties involved in each transaction.

The original adverse selection problem between a buyer and a seller is reallocated in a non-linear

1We will refer, later in the introduction, to other papers that document the existence of intermediation chains in various markets.
fashion when several heterogeneously informed intermediaries are involved. Each pair of sequential traders bargains based on conditional distributions for the value of the asset that are different than the distribution that characterizes the original information asymmetry without intermediaries. It is then crucial to have intermediaries located within the trading network such that each trader’s information set is similar, although not identical, to that of nearby traders (i.e., his counterparties). For large adverse selection problems a high number of intermediaries is therefore needed to sufficiently reduce the information asymmetry that each agent faces when it is his turn to trade the asset. Greater information asymmetries require longer intermediation chains and, overall, more trading across agents, which contrasts with the conventional wisdom that asymmetric information should be associated with low trading volume (as was the case in the seminal model of Akerlof 1970).

However, each trader involved in such a network needs to be privately incentivized to sustain trade and preserve the surplus. The conditional distributions for the value of the asset that each heterogeneously informed trader and his counterparty face determine their incentives to trade efficiently. Our model thus speaks to how trading networks impact the ability of all involved parties to extract rents and their willingness to sustain socially efficient trade in equilibrium. In some cases, the intermediaries extract more rents through informed trading than the additional surplus they create by increasing liquidity. In those cases, intermediaries are willing to compensate other traders to secure a place in the socially optimal trading network. We characterize order-flow agreements that guarantee that every agent involved benefits from the implementation of a socially efficient network. These agreements, which allow to implement intermediated trade in equilibrium, are consistent with the practice by financial intermediaries of offering cash payments, or subsidized services, to traders in exchange for their order flow. The socially beneficial role that order-flow agreements can play in our model challenges recent proposals by regulatory agency and stock exchange officials to ban related practices.

Intermediate has been known to facilitate trade, either by minimizing transaction costs

\cite{Blume93, Chordia95, Reuter06, Nimalendran07} for empirical evidence.

\cite{Sprecher14, FSA12, Pitt04}
(Townsend 1978), by concentrating monitoring incentives (Diamond 1984), or by alleviating search frictions (Rubinstein and Wolinsky 1987, Yavaş 1994, Duffie, Gărleanu, and Pedersen 2005, Neklyudov 2013). Our paper, however, specifically speaks to how intermediaries may solve asymmetric information problems. We already know from Myerson and Satterthwaite (1983) that an uninformed third party who subsidizes transactions can help to eliminate these problems in bilateral trade. Trade efficiency can also be improved by the involvement of fully informed middlemen who care about their reputation (Biglaiser 1993) or who worry that informed buyers could force them to hold on to low-quality goods (Li 1998). Contrary to these models, our model considers the possibility that an intermediary’s information set differs from that of the agents initially involved in a transaction. In fact, in our static model without subsidies, warranties, or reputational concerns the involvement of an intermediary who is either fully informed or totally uninformed does not improve trade efficiency. Thus, the insight that moderately informed intermediaries can reduce trade inefficiencies simply by layering an information asymmetry over many sequential transactions fundamentally differentiates our paper from these earlier papers.

Rationalizing intermediation chains, which are observed in many financial markets, also distinguishes our paper from many market microstructure models with heterogeneously informed traders but where trading among intermediaries plays no role. Examples of those models include Kyle (1985) and Glosten and Milgrom (1985), where competitive market makers learn from order flow data and intermediate trade between liquidity traders and informed traders, and Jovanovic and Menkveld (2012), where high frequency traders learn quickly about the arrival of news and intermediate trade between early traders who post a limit order and late traders who react to the limit order using information that became available since its posting. The optimal involvement of multiple intermediaries also distinguishes our paper from Babus (2012) who endogenizes OTC trading networks when agents meet sporadically and have incomplete information about other traders’ past behaviors. In equilibrium, a central intermediary becomes involved in all trades and heavily penalizes anyone defaulting on prior obligations.4

On the other hand, Gofman (2011) allows for non-informational bargaining frictions in an OTC network and shows that socially optimal trading outcomes are easier to achieve if the network is

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4See also Farboodi (2014) who shows that a centralized trading network is socially optimal when banks must establish credit relationships prior to learning about the allocation of investment projects in the economy.
sufficiently dense (although the relationship is not necessarily monotonic). In our model, a trading network needs to be sparse enough to sustain efficient trade; otherwise, uninformed traders might be tempted to contact socially inefficient counterparties, in an attempt to reduce the number of strategic, informed intermediaries trying to extract surplus away from them. (We analyze the role that order-flow agreements can play in alleviating this problem.) Our paper also relates to Malamud and Rostek (2013) who study the concurrent existence of multiple exchanges in decentralized markets. Creating a new private exchange may improve the liquidity in incumbent exchanges by reducing the price impact that strategic traders impart when simultaneously trading the same asset at different prices on multiple exchanges. This particular mechanism plays no role in our model as trading is bilateral, occurs sequentially among intermediaries, and entails a fixed transaction size.

Although our framework could be used to shed light on the existence of intermediation chains in many contexts, we rely on the empirical literature on financial markets to contextualize our theory. In addition to the discussions in Viswanathan and Wang (2004) and Adrian and Shin (2010) mentioned earlier, many papers document the importance of transactions among intermediaries, a key prediction of our model, in centralized and decentralized financial markets. For example, in foreign exchange markets inter-dealer transaction volume averages $2.1 trillion per day, according to a 2013 report by the Bank of International Settlements. For metals futures contracts, Weller (2013) shows that a median number of 2 intermediaries are involved in round-trip transactions and up to 10% of transactions involve 5 or more intermediaries. In municipal bond markets, Li and Schürhoff (2014) show that 13% of intermediated trades involve a chain of 2 intermediaries and an additional 10% involve 3 or more intermediaries. Hollifield, Neklyudov, and Spatt (2014) also find evidence of intermediation chains for many securitized products: for example, intermediated trades of non-agency collateralized mortgage obligations involve 1.76 dealers on average and in some instances the chain includes up to 10 dealers.

Viswanathan and Wang (2004) show that the issuer of a security may prefer to have a set of dealers, heterogeneous only in their inventory levels, sequentially trading the security over having those same dealers participating in a centralized auction where the supply of the security is split

among them. The arguments highlighted in their model are based on differences in how strategic dealers can behave when trading bilaterally versus participating in a centralized auction — sequential bilateral trading allows a dealer who just bought a security to act as a monopolist and control the price and inventory levels in later periods. A few empirical papers such as Hansch, Naik, and Viswanathan (1998) document that inventory management motives explain part of the trading among intermediaries in financial markets, but key features of inter-dealer trading still remain unexplained. For example, Manaster and Mann (1996) show that the positive relationship between trader inventories and transaction prices in futures trading data violates the predictions of inventory control models such as Ho and Stoll (1983). Manaster and Mann (1996, p.973) conclude that the intermediaries they study are “active profit-seeking individuals with heterogeneous levels of information and/or trading skill”, elements that are usually absent from inventory control theories. Our model proposes an information-based explanation for intermediation chains, by combining asymmetric information with inventory management motives. The intermediaries in our model are effectively averse to holding inventories (i.e., non-zero positions) since they are not the efficient holders of the asset, that is, those who realize the gains to trade. Yet, information asymmetries may prevent them from offloading the asset to potential buyers and creating a surplus.

Recent empirical evidence appears to lend support to the main predictions from our model. In particular, Li and Schürhoff (2014) show that municipal bonds without a credit rating are more likely to be traded through long intermediation chains than municipal bonds with a credit rating (which arguably are less likely to be associated with large adverse selection problems). They also show that the average round-trip spread paid to dealers increases with the length of the chain. Hollifield, Neklyudov, and Spatt (2014) also show that securitized products such as non-agency collateralized mortgage obligations that can be traded by unsophisticated and sophisticated investors (i.e., “registered” instruments) are usually associated with longer chains of transactions and higher spreads paid to dealers (often viewed as a measure of adverse selection) than comparable instruments that can only be traded by sophisticated investors (i.e., “rule 144a” instruments). These

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7Madhavan and Smidt (1993) also combine asymmetric information and inventory management motives, but their model remains silent about the empirical phenomenon of intermediation chains; their model features centralized trading, rather than OTC trading, and does not allow for multiple intermediaries.
findings are all consistent with our model’s predictions that larger information asymmetries require longer intermediation chains, more inter-dealer trade, and are associated with larger rents being captured by intermediaries.

In the next section, we model a simple, and fairly standard, adverse selection problem between two asymmetrically informed traders. We show in Section 3 how adding moderately informed intermediaries can eliminate the trading inefficiencies that arose in the original setting. We study in Section 4 how order-flow agreements can be used to ensure that a socially optimal intermediation chain becomes privately optimal to implement for all traders involved. In Section 5 we show how our results can be extended to various information structures, and the last section concludes.

2 The Adverse Selection Problem

We start by assuming two risk-neutral agents who consider trading one unit of an asset over the counter: the current owner who values the asset at $v$ and a potential buyer who values it at $v + \Delta$. A potential interpretation for this interaction is that of a firm that wishes to offload a risk exposure (e.g., to interest rates, foreign exchange rates, or commodity prices) and meets an expert able to hold the risk exposure more efficiently (e.g., by means of pooling or diversification). The firm, also referred to as the seller, is thus trying to sell a risky asset to the expert, also referred to as the buyer, because the expert values the asset more than the firm does. Trade is then labeled as efficient only if the asset ends up in the hands of the expert with probability 1 and the gains to trade $\Delta$ are always realized.

We assume the gains to trade $\Delta$ are constant and known to all agents, but the common value $v$ is uncertain and takes the form:

$$v = \sum_{n=1}^{N} \phi_n \sigma,$$

where the $N$ factors $\phi_n \in \{0, 1\}$ are drawn independently from a Bernoulli distribution with $\Pr[\phi_n = 1] = 1/2$. The common value is thus binomial distributed with $v \sim B\left(N, \frac{1}{2}\right)$. We denote by $\Phi$ the full set of factor realizations $\{\phi_1, \phi_2, ..., \phi_N\}$.

Although the role that intermediation will play in our model is relatively simple, multi-layered

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Footnote 8: Shin (2003) also assumes a Binomial distribution, albeit a multiplicative one, to model uncertain asset values. His paper’s focus, however, differs from ours and pertains to the optimal disclosure of information by a manager and its effects on asset prices.
bargaining problems with asymmetric information are usually complex to analyze given the potential for multiple equilibria arising from the various types of off-equilibrium beliefs. We therefore make a few stylized assumptions that will allow us to keep the model sufficiently tractable, even when we consider in Section 3 multiple sequential transactions occurring among a large number of heterogeneously informed traders.

First, we assume that, in any transaction, the current holder of the asset makes an ultimatum offer (i.e., quotes an asking price) to his counterparty. Focusing on ultimatum offers simplifies the analysis of equilibrium bidding strategies and is consistent with the characterization of sequential inter-dealer trading by Viswanathan and Wang (2004, p.3) as “very quick interactions”. Ultimatum offers are also consistent with how Duffie (2012, p.2) describes the typical negotiation process in OTC markets and the notion that each OTC dealer tries to maintain “a reputation for standing firm on its original quotes.” Here, it is the seller who quotes a price rather than the buyer who makes an offer, but we show later how our results also apply to alternative settings, including one in which the buyer makes the ultimatum offer (see Subsection 5.3).

Second, we assume that prior to trading the seller is uninformed about the realizations of \( \phi_n \) that determine the common value \( v \), whereas the expert observes the full set \( \Phi \) of factor realizations. Note that for many financial products endowing a “buyer” with the informational advantage rather than the “seller” is an unrestrictive assumption; for example, a firm could be viewed as the buyer of an insurance policy, or, alternatively, as the seller of a risk exposure. In Section 5 we consider alternative information structures, including a case with an expert seller and a case with two-sided asymmetric information.

Third, agents know how well informed their counterparties are, that is, the set of factors that each agent observes is common knowledge. Although traders in our setting are asymmetrically informed about the common value component \( v \), all traders know the quality of the information available to their counterparties. Seppi (1990) lends support to this assumption arguing that agents knowing the identity of their trading counterparties is an important distinction between OTC trading and centralized/exchange trading.

Together, these three assumptions eliminate signaling concerns from our model and guarantee

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9Morris and Shin (2012) relax the common-knowledge assumption in a bilateral trading setup similar to the one in this section and show how the resulting coordination problems can magnify the effect of adverse selection on trade efficiency.
the uniqueness of our equilibrium without the need for equilibrium refinements. We are, thus, able to derive closed-form solutions for many objects of interest that would otherwise be hard to uniquely pin down. For example, the following lemma characterizes a limited set of price quotes the seller chooses from when trading directly with the expert buyer.

**Lemma 1 (Price candidates under direct trade)** If the seller and the expert buyer trade directly, the seller optimally chooses to quote one of $(N + 1)$ price candidates $p_i$, where $p_i$ is defined as:

$$p_i = i\sigma + \Delta, \quad i \in \{0, ..., N\}.$$

The unconditional probability with which the expert buyer accepts a price quote $p_i$ is given by:

$$\pi_i = \sum_{k=0}^{N-i} \binom{N}{k} \left(\frac{1}{2}\right)^N.$$

**Proof.** The expert buyer optimally accepts to pay a given price $\tilde{p}$ if and only if $\tilde{p} \leq v + \Delta$. Given the Binomial distribution for $v$, the price candidates $p_i = i\sigma + \Delta$ for $i \in \{0, ..., N\}$ represent the maximum prices the seller can charge conditional on ensuring any given feasible acceptance probability. Further, the seller strictly prefers the price quote $p_N$ to non-participation, since quoting $p_N$ increases his average payoff by $\frac{1}{2N}\Delta$. ■

For trade to be efficient and occur with probability one, the seller must find it optimal to quote $p_0$ in equilibrium rather than any other price candidate $p_i$ for which $i \in \{1, ..., N\}$. The following proposition provides a necessary and sufficient condition on the fundamentals of the asset $(\sigma, \Delta, N)$ to ensure efficient trade when the seller and the buyer trade directly with each other.

**Proposition 1 (Efficient direct trade)** Direct trade between the seller and the expert buyer is efficient if and only if:

$$\frac{\sigma}{\Delta} \leq \frac{1}{2^N - 1}. \quad (1)$$

Under efficient trade, the expected surplus from trade is split between the seller who obtains $\Delta - \frac{N}{2} \sigma$ and the buyer who obtains $\frac{N}{2} \sigma$.

**Proof.** Lemma 3, which is provided in Appendix A, shows that the incentive to increase the price
quote from $p_i$ to $p_{i+1}$ is strongest at $i = 0$ and the condition for the seller to prefer a price quote $p_0$ over $p_1$ also implies that he prefers quoting $p_0$ over any $p_i$ for which $i \in \{1, ..., N\}$. A seller who decides to quote $p_1$ rather than $p_0$ receives a higher price ($p_1 - p_0 = \sigma$) with probability $1 - \left(\frac{1}{2}\right)^N$, but forgoes extracting the gains to trade $\Delta$ with probability $\left(\frac{1}{2}\right)^N$. The seller thus chooses to quote $p_0$ among all prices if and only if doing so generates a weakly higher expected payoff than quoting $p_1$:

$$
\pi_0 p_0 \geq \pi_1 p_1 + (1 - \pi_1) \cdot 0
$$

$$
\Leftrightarrow \Delta \geq \left( 1 - \left(\frac{1}{2}\right)^N \right) (\sigma + \Delta).
$$

Eqn. (1) follows directly from the last inequality. Under efficient trade, the seller collects a surplus of:

$$
p_0 - E[v] = \Delta - \frac{N}{2} \sigma
$$

and the buyer collects an expected surplus of:

$$
E[v] + \Delta - p_0 = \frac{N}{2} \sigma.
$$

Efficient trade is thus only possible for small enough values of $\frac{\sigma}{\Delta}$, which quantifies the price concession made by the seller when quoting the lowest price $p_0$ relative to the gains to trade he extracts from sustaining trade. When $\frac{\sigma}{\Delta}$ is high and the information asymmetry is large relative to the surplus created by trade, trade breaks down with probability $\left(\frac{1}{2}\right)^N$ or greater and at least $\frac{\Delta}{2N}$ in surplus from trade is destroyed.\(^{10}\)

Next, we will show that a trading network that splits the information asymmetry over a sequence of transactions can induce fully efficient behavior on the part of heterogeneously informed traders. Although other mechanisms have been proposed to solve adverse selection problems (see,  

\(^{10}\) Asymmetric information could also affect the gains to trade $\Delta$ rather than only affecting the common value $v$ as is the case in our model. For example, gains to trade could be influenced by private information about a dealer’s order flow. We know, however, from Myerson and Satterthwaite (1983) that inefficient trading being a consequence of asymmetric information is a common result in bilateral bargaining, and we simplify the analysis by focusing on only one type of information asymmetry.
for example, the literature on optimal security design which includes: DeMarzo 2005, Chakraborty and Yilmaz 2011, Yang 2013), the idea that intermediation chains can by themselves fully alleviate these problems is novel and may shed light on the fact that chains are frequently observed in decentralized markets.

3 Intermediation Chains

In this section, we consider the involvement of $M$ intermediaries who observe different subsets of $\Phi$, the full set of factor realizations $\phi_n$. Like the seller, these intermediaries privately value the asset at $v$ and thus cannot help realize gains to trade unless they resell the asset and thereby facilitate a more efficient allocation. Moreover, these intermediaries do not bring new information to the table, as their information sets are nested by that of the expert buyer. However, as we show below, an intermediation chain that involves heterogeneously informed traders can improve the efficiency of trade by reallocating an information asymmetry over several sequential transactions.

Consider a simple trading network in which the uninformed firm offers to sell the asset to intermediary 1. If trade occurs, intermediary 1 offers to sell the asset to the next trader in the network, intermediary 2. Conditional on trade occurring, these bilateral interactions are repeated up until we reach the end of the chain, where intermediary $M$ offers to sell the asset to the expert buyer. (To simplify the notation, we label the firm/seller as trader 0 and the expert buyer as trader $M + 1$.) Traders are not allowed to deviate from the trading network by bypassing the trader who is next in line in the intermediation chain (we further discuss this assumption and its link to order-flow agreements in Section 4). Further, consistent with how we modeled trading without intermediaries, we assume that whoever owns the asset and tries to sell it quotes an ultimatum price to his counterparty.

The $M$ intermediaries are assumed to be heterogeneously informed, as it is often the case in OTC markets. In fact, the main mechanism that makes intermediation valuable in our model can be highlighted best by assuming that the subset of factor realizations that intermediary $m$ observes before trading is nested by the subset of factor realizations that intermediary $m + 1$ observes before trading: $\Phi_m \subseteq \Phi_{m+1} \subseteq \Phi$, for $m \in \{0, 1, ..., M\}$. Trader 1 is thus assumed to be the intermediary with the least expertise, as he only observes realizations from $N_1$ factors, say $\{\phi_1, \phi_2, ..., \phi_{N_1}\}$,
which can be interpreted as information that is relatively cheap to acquire and easy to interpret. Trader 2 observes the same $N_1$ factors $\{\phi_1, \phi_2, \ldots, \phi_{N_1}\}$ as well as $(N_2 - N_1)$ extra factors that are a little bit harder or more expensive to gather. The same logic applies for the remaining traders in the chain up until the expert (i.e., trader $M + 1$) is reached who observes all factors in the set $\Phi$. This simple network with increasingly informed traders implies that the information set of the proposer of a price quote is always weakly dominated by the responder’s information set. Figure 1 shows an example of information sets in a trading network with two intermediaries.

Figure 1: **Example of information sets in trading network.** The figure illustrates our informational structure when two intermediaries are involved ($M = 2$) in trading an asset whose common value $v$ depends on seven factors $\phi_i$ ($N = 7$). The dotted rectangles indicate the set of factor realizations that are observable to the two intermediaries and to the expert buyer (remember: the firm/seller observes none of these factors). Factor realizations $\phi_i \in \{0, 1\}$ are indicated by the circles that are either unfilled (for $\phi_i = 0$) or filled (for $\phi_i = 1$). The seller is uninformed and thus does not observe any of the factor realizations.

Nesting traders’ information sets eliminates signaling concerns and ensures a unique equilibrium despite the fact that we consider $(M + 1)$ bargaining problems among $(M + 2)$ heterogeneously informed agents. Moreover, the recursive nature of our model yields a clean and transparent analytical proof of our main result: an intermediation chain can preserve the efficiency of trade in situations in which surplus would be destroyed if trade were to occur through fewer intermediaries. As will become clear soon, what ultimately contributes to sustaining efficient trade is that the chain reduces the distance in counterparties’ information sets, although information sets do not necessarily have to be nested for our mechanism to work. In Section 5, we will show that if information sets are non-nested initially but information percolates through trade as in Duffie,
Malamud, and Manso (2009, 2013), a similar mechanism arises, as the asset is held by increasingly better informed agents through the chain. In addition, we will relax the assumption of one-sided asymmetric information and show how the proposed mechanism survives if both the buyer and the seller have private information about the value of the asset.

The proposition below formalizes our main result and is followed by the analysis of two special cases that help to illustrate the intuition behind our result.

**Proposition 2 (Efficient trade in an intermediation chain)** Trade is efficient throughout the trading network if and only if:

\[
\frac{\sigma}{\Delta} \leq \min_{m \in \{0, 1, \ldots, M\}} \frac{1}{2(N_{m+1} - N_m)} + \frac{N - N_{m+1}}{2} - 1.
\]

Under efficient trade, the expected surplus from trade is split between the original seller who obtains \(\Delta - \frac{N}{2} \sigma\) and each trader \(m \in \{1, \ldots, M + 1\}\) who obtains \(\left(\frac{N_m - N_{m+1}}{2}\right) \sigma\).

**Proof.** Consider a situation in which trader \(m\) currently holds the asset and tries to sell it to trader \(m + 1\). Trader \(m\) knows that \(G_m\) of the \(N_m\) factor realizations he observes have a value of 1. Similarly, trader \(m + 1\) knows that \(G_{m+1}\) of the \(N_{m+1}\) factor realizations he observes have a value of 1. The condition that information sets satisfy \(\Phi_m \subseteq \Phi_{m+1} \subseteq \Phi\) implies that \(0 \leq N_m \leq N_{m+1} \leq N\) and \(0 \leq G_m \leq G_{m+1} \leq N\). Assume for now that whenever trader \(m + 1\) acquires the asset, subsequent trading is efficient, which requires that traders \(k \in \{m + 1, m + 2, \ldots, M\}\) each charge a price:

\[
p_k^0 = G_k \sigma + \Delta,
\]

and maximize subsequent trade probability. Trader \(m\) then chooses to quote one of \((N_{m+1} - N_m + 1)\) price candidates defined as:

\[
p_i^m = (G_m + i) \sigma + \Delta, \quad i \in \{0, \ldots, N_{m+1} - N_m\}.
\]

The weakly better informed trader \(m + 1\) only accepts to pay a price \(p_i^m\) if it is weakly lower than the price he plans to quote to trader \(m + 2\), that is, \(p_0^{m+1} = G_{m+1} \sigma + \Delta\). For trade to be efficient between traders \(m\) and \(m + 1\), trader \(m\) must find it optimal to quote \(p_0^m\) in equilibrium rather
than any other price candidate $p''_i$. Lemma 3 in Appendix A shows that trader $m$ finds optimal to quote $p''_m$ rather than any other $p''_i$ if and only if quoting $p''_m$ makes him wealthier in expectation than quoting $p''_i$:

\[
G_m \sigma + \Delta \geq \left(1 - \left(\frac{1}{2}\right)^{(N_{m+1}-N_m)}\right) [G_m + 1] \sigma + \Delta + \left(\frac{1}{2}\right)^{(N_{m+1}-N_m)} (G_m + \frac{N - N_{m+1}}{2}) \sigma
\]

\[
\Leftrightarrow \frac{\sigma}{\Delta} \leq \frac{1}{2(N_{m+1}-N_m) + (\frac{N - N_{m+1}}{2}) - 1}.
\]

Recursively applying this condition to each trading stage yields the following condition for efficient trade throughout the trading network:

\[
\frac{\sigma}{\Delta} \leq \min_{m \in \{0, 1, \ldots, M\}} \frac{1}{2(N_{m+1}-N_m) + (\frac{N - N_{m+1}}{2}) - 1}.
\]

Under efficient trade, each trader $m \in \{1, \ldots, M\}$ collects an expected surplus of:

\[
E[p''_m|\Phi_{m-1}] - p''_0 = G_{m-1} \sigma + \left(\frac{N_m - N_{m-1}}{2}\right) \sigma + \Delta - [G_{m-1} \sigma + \Delta]
\]

\[
= \left(\frac{N_m - N_{m-1}}{2}\right) \sigma,
\]

the final buyer (trader $M + 1$) collects an expected surplus of:

\[
E[v|\Phi_M] + \Delta - p''_0 = G_M \sigma + \left(\frac{N - N_M}{2}\right) \sigma + \Delta - [G_M \sigma + \Delta]
\]

\[
= \left(\frac{N - N_M}{2}\right) \sigma,
\]

and the initial seller (trader 0) collects an expected surplus of:

\[
\Delta - E[v] = \Delta - \frac{N}{2} \sigma.
\]

The proposition formalizes the intuition that an asset characterized by $(\sigma, \Delta, N)$ is more likely to be traded efficiently within a network if informational advantages between sequential trading
partners \((N_{m+1} - N_m)\) are small. By focusing on traders’ behavior along the efficient trading path, we are able to exploit the recursivity of the sequence of transactions and show in a tractable way how intermediation chains can help to solve an adverse selection problem. Formally, the condition in eqn. (2) for efficient trade is weakly less restrictive than the corresponding condition in eqn. (1) for the case without intermediaries. (In fact, due to the recursive nature of our model, eqn. (2) corresponds to eqn. (1) when we set \(M = 0\).)

The holder of an asset faces the following trade-off when choosing the price he quotes to his counterparty. If the conditions for efficient trade are satisfied for all subsequent transactions in the chain, the prospective seller recognizes that subsequent trading will preserve the whole gains to trade \(\Delta\). Hence, he compares the benefit of extracting the full \(\Delta\) with the cost of quoting a price that is low enough to be accepted by a counterparty who possesses an informational advantage of \((N_{m+1} - N_m)\) factors. When a trader faces a counterparty who is significantly better informed than him, he might find optimal to quote a high price, in case the informed counterparty receives good signals and accepts to pay the high price. However, this strategy also comes at a cost since the asking price may sometimes exceed the counterparty’s valuation of the asset. Although such trading strategies may be privately optimal for less informed traders, they are socially inefficient since the surplus from trade is destroyed with positive probability. Transactions between more homogenously informed agents give asset holders lower incentives to quote inefficiently high prices as marginally better informed counterparties are less likely to accept such high offers. Intermediation chains can thus preserve efficient trade in situations in which trade would otherwise break down with positive probability.

Moreover, as the ratio \(\frac{\sigma}{\Delta}\) increases and the adverse selection problem worsens, a higher number of intermediaries \(M\) are needed to sufficiently bound the information asymmetries that each trading counterparty faces, consistent for example with Li and Schürhoff (2014) who show that municipal bonds with no credit rating are typically traded through longer intermediation chains than municipal bonds with a credit rating (which arguably are less likely to be associated with large adverse selection problems). Specifically, it is easy to show that adding intermediaries helps to relax the restriction imposed on \(\frac{\sigma}{\Delta}\) in Proposition \(2\). Suppose an intermediary \(m'\) is added between traders \(m\) and \(m + 1\). If the expertise of intermediary \(m'\) differs from that of those already involved in the chain, in particular if \(N_m < N_{m'} < N_{m+1}\), the terms on the right-hand side of eqn. \(2\) should
weakly increase for all layers of transactions. First, all terms on the right-hand side of (2) that do not involve trader \( m' \) remain the same as before. Second, both of the terms that involve trader \( m' \) are strictly greater than the old term they replace:

\[ \frac{1}{2(N_{m+1} - N_{m'}) + \frac{N - N_{m+1}}{2} - 1} > \frac{1}{2(N_{m+1} - N_{m}) + \frac{N - N_{m+1}}{2} - 1}, \]

and

\[ \frac{1}{2(N_{m'} - N_{m}) + \frac{N - N_{m'}}{2} - 1} > \frac{1}{2(N_{m+1} - N_{m}) + \frac{N - N_{m+1}}{2} - 1}. \]

The socially optimal response to greater information asymmetries is thus longer intermediation chains and more trading among all agents involved.

The proposition also shows that, given equal informational distances between bilateral counterparties (i.e., the same \((N_{m+1} - N_{m})\) for all \( m \)), efficient trade is hardest to sustain at the beginning of the chain where less is known about the overall value of the asset. Early in the chain, the expected value of the asset linked to the factors that are unknown to trading counterparties is greater, which makes the possibility of charging a high price and being stuck with the asset less costly than it is late in the chain.

Conditional on efficient trade throughout the network, each informed trader collects rents that increase with the uncertainty in asset value, \( \sigma \), as well as with his informational advantage over the trader that sells him the asset, \((N_{m} - N_{m-1})\). These rents come from the optimality for trader \( m-1 \) to charge a low price to trader \( m \) in order to ensure his full participation in the trade and preserve the whole gains to trade \( \Delta \). Trader \( m \) only pays \( G_{m-1}\sigma + \Delta \) and expects to collect \( G_{m}\sigma + \Delta \).

The intermediary sector as a whole is therefore able to extract rents of \( \frac{N \Delta}{2} \sigma \) in total. Among the networks that sustain efficient trade, networks with fewer, more distanced, intermediaries increase the rents that accrue to the expert as well as the average rent a moderately informed intermediary extracts. Our model thus makes predictions about how surplus from trade should be distributed among heterogeneously informed OTC market participants and contributes to the literature on rent-extraction in finance (Murphy, Shleifer, and Vishny 1991, Philippon 2010, Bolton, Santos, and Scheinkman 2012, Glode, Green, and Lowery 2012, Biais and Landier 2013, Glode and Lowery 2013).

To further illustrate how moderately informed intermediaries can help to solve an adverse
selection problem between two asymmetrically informed traders, we now analyze two special cases of our model (with \( N = 2 \) and \( N = 3 \), respectively).

**Two-Factor Case:** Suppose an asset is worth \( v = \phi_1 \sigma + \phi_2 \sigma \) to the seller and \( v + \Delta \) to the buyer. Without an intermediary, the seller chooses to quote one of three price candidates: (i) \( \Delta \), which is accepted by the buyer with probability 1; (ii) \( \sigma + \Delta \), which is accepted with probability \( 3/4 \); (iii) \( 2\sigma + \Delta \), which is accepted with probability \( 1/4 \).

The first price candidate \( \Delta \) splits the surplus from trade such that the seller collects \( \Delta - \sigma \) and the buyer collects \( \sigma \). The second price candidate \( \sigma + \Delta \) produces an expected surplus of \( \frac{3}{4} \Delta - \frac{1}{4} \sigma \) for the seller and \( \frac{1}{4} \sigma \) for the buyer. The third price candidate produces an expected surplus of \( \frac{1}{4} \Delta \) for the seller and no surplus for the buyer. Quoting the low price \( \Delta \) is thus optimal for the seller, making trade efficient, if and only if \( \frac{\sigma}{\Delta} \leq \frac{1}{3} \).

However, when an agent observes \( \phi_1 \) and intermediates trade between the seller and the buyer, trade can be efficient even though \( \frac{\sigma}{\Delta} > \frac{1}{3} \). Specifically, when holding the asset the intermediary is in expectation wealthier from quoting \( \phi_1 \sigma + \Delta \) rather than \( \phi_1 \sigma + \sigma + \Delta \) if and only if:

\[
\phi_1 \sigma + \Delta \geq \frac{1}{2}(\phi_1 \sigma + \sigma + \Delta) + \frac{1}{2} \phi_1 \sigma,
\]

which simplifies to \( \frac{\sigma}{\Delta} \leq 1 \). Given that, the seller chooses between a price candidate \( \Delta \), which is accepted by the intermediary with probability 1, and a price candidate \( \sigma + \Delta \), which is accepted by the intermediary with probability \( 1/2 \). The seller is in expectation wealthier when quoting \( \Delta \) rather than \( \sigma + \Delta \) if and only if:

\[
\Delta \geq \frac{1}{2}(\sigma + \Delta) + \frac{1}{2} \left( \frac{\sigma}{\Delta} \right),
\]

which simplifies to \( \frac{\sigma}{\Delta} \leq \frac{2}{3} \).

Hence, in the region where \( \frac{1}{3} < \frac{\sigma}{\Delta} \leq \frac{2}{3} \), trade is efficient if an intermediary who observes only one of the two factors is involved, but inefficient without an intermediary. The total surplus generated by trade in equilibrium increases from \( \frac{3}{4} \Delta \) without an intermediary to \( \Delta \) with an intermediary. The buyer extracts \( \sigma/2 \) with an intermediary, which is twice as much as what he
would get without an intermediary. Because trade occurs at a low price between the seller and the intermediary, the intermediary is also able to extract a surplus $\sigma/2$.

The seller extracts $\Delta - \sigma$ with the intermediary, but is worse off than without an intermediary when $\frac{\sigma}{\Delta} > 1/3$. When an intermediary is involved, the difference in information quality between counterparties is small enough in both transactions to allow for efficient trade throughout the network. However, this comes at the cost of adding a strategic agent, the intermediary, who captures a share of the surplus and makes the uninformed seller worse off. When trading directly with the expert, the seller has the (socially inefficient) option of selling the asset at a price $\sigma + \Delta$, which the expert accepts to pay with probability $3/4$. With the intermediary, the seller can still sell the asset at a price $\sigma + \Delta$, this time to the intermediary, but the intermediary only accepts to pay this price with probability $1/2$. By making the socially inefficient price quote $\sigma + \Delta$ less attractive to the seller, the intermediary makes him worse off in the region where $1/3 < \frac{\sigma}{\Delta} \leq 2/3$, thus he makes trade more efficient. As a consequence, if allowed the seller would prefer to bypass the intermediary and make an ultimatum offer to the buyer. This deviation would lead to a lower social surplus than if trade goes through the intermediary. The socially efficient trading network therefore centers around a moderately informed intermediary, and it is also sparse, in the sense that the seller cannot contact the buyer himself. Alternatively, the expert buyer could commit to ignore any offer coming directly from the uninformed seller, since the buyer is better off when trade goes through a moderately informed intermediary; the expert buyer collects a surplus of $\sigma/2$ when trade goes through the intermediary and is efficient compared to $\sigma/4$ when trade breaks down because no intermediary is involved. The fact that, in practice, it is nearly impossible for retail investors and unsophisticated firms to contact the most sophisticated trading desks directly and bypass the usual middlemen suggests that sparse intermediated networks, or equivalent commitments by sophisticated trading desks, are sensible outcomes of our theory. We discuss in Section 4 the role that ex ante transfers such as payments for order flow can play in ensuring that the socially efficient trading network is Pareto dominant.

Note also that in the region where $1/3 < \frac{\sigma}{\Delta} \leq 2/3$ the surplus the moderately informed agent collects from intermediating trade is greater than the surplus he could collect if he stayed outside the trading network and (credibly) offered to sell his signal to the uninformed agent, in the spirit of Admati and Pfleiderer (1988, 1990). The reason for this result is that a moderately
informed intermediary is rewarded for improving trade efficiency, but he also extracts rents from the uninformed agent.

Moreover, replacing the intermediary with a different one who instead observes zero or two factors would eliminate any benefit of intermediation here. Hence, if offered the opportunity to choose his own information set, an intermediary should opt for acquiring more information than the least informed trader and less information than the most informed trader, as it is the only way to extract rents in the intermediation chain.

Finally, note that if trade breaks down despite the involvement of an intermediary, the total surplus that is generated from trade is weakly greater without an intermediary than with one. The intermediary’s strategic behavior aimed at appropriating a share of the surplus then becomes an impediment to trade that overpowers the benefits of his involvement that we highlighted so far. This result might help to formalize the role that intermediation chains have played in the recent crisis (i.e., times of high uncertainty), as suggested by Adrian and Shin (2010)

The next special case we consider serves to illustrate that, as the adverse selection problem between the ultimate buyer and seller worsens, more intermediaries may be needed to preserve efficient trade.

**Three-Factor Case:** Suppose that the asset is worth \( v = \phi_1 \sigma + \phi_2 \sigma + \phi_3 \sigma \) to the seller and \( v + \Delta \) to the buyer. Without the involvement of intermediaries, we know from eqn. (2) that the seller chooses to quote the efficient price \( \Delta \) if and only if \( \frac{\sigma}{\Delta} \leq \frac{1}{7} \). Proposition 2 also implies that an intermediary who observes one factor realization allows for efficient trade if and only if:

\[
\frac{\sigma}{\Delta} \leq \min \left\{ \frac{1}{2^2 - 1}, \frac{1}{2 + \left( \frac{7}{2} \right) - 1} \right\} = 1/3,
\]

whereas an intermediary who observes two factor realizations allows for efficient trade if and only if:

\[
\frac{\sigma}{\Delta} \leq \min \left\{ \frac{1}{2 - 1}, \frac{1}{2^2 + \left( \frac{7}{2} \right) - 1} \right\} = 2/7.
\]

Thus, as in the two-factor case, adding a second layer of transactions to reduce the distance between counterparties’ information sets can eliminate entirely the trading inefficiencies that adverse
selection imposes. Overall, in the region where $1/7 < \frac{\sigma}{\Delta} \leq 1/3$, trade is efficient if a moderately informed intermediary is involved, but is inefficient without him.

Moreover, involving a second intermediary further extends the region of efficient trade. An intermediation chain in which the seller trades with a first intermediary who observes one factor realization before trading with a second intermediary who observes two factor realizations (including the one the first intermediary observes) before trading with the expert buyer allows for efficient trade if and only if:

$$\frac{\sigma}{\Delta} \leq \min \left\{ \frac{1}{2} - 1, \frac{1}{2} - 1, \frac{1}{2} + \left( \frac{1}{2} \right) - 1 \right\} = \frac{1}{2}. $$

In the region where $1/3 < \frac{\sigma}{\Delta} \leq 1/2$, trade is thus efficient if two heterogeneously informed intermediaries are involved, but is inefficient with zero or one intermediary.

An important implication of our analysis is that intermediaries should be located within the trading network such that each trader’s information set is similar, but not identical, to that of nearby traders. It is socially optimal to have, for example, the least sophisticated intermediaries trading directly with the least informed end-traders, in this case the firm, and the most sophisticated intermediaries trading directly with the most informed end-traders, in this case the expert.

Our paper also highlights that the optimality of specific trading networks greatly depends on the trading frictions that are most relevant in a given context. If efficient trade is impeded by a large information asymmetry related to the value of the asset being traded, our model shows that multiple heterogeneously informed intermediaries may be needed to sustain the social efficiency of trade. If the information asymmetry instead relates to traders’ past behavior, Babus (2012) shows that the optimal trading network is centered around a single intermediary who heavily penalizes anyone defaulting on his prior obligations. In Gofman (2011), traders face non-informational bargaining frictions that imply that socially efficient outcomes are easier to achieve when the network is sufficiently dense (although the relationship is not always monotonic). On the other hand, in our model a trading network needs to be sufficiently sparse to sustain efficient trade; otherwise, uninformed parties might privately benefit from trading relationships that reduce social efficiency. (We analyze in the next section the role that order-flow agreements can play in alleviating this
Given that different trading frictions are more relevant in some situations than others, our results and those derived in the papers cited above can help us to understand which type of networks we observe in various contexts.

4 Order-Flow Agreements

We showed in Section 3 that if a social planner wants to maximize the social surplus generated by trade between an uninformed seller and an expert buyer, he may have the uninformed seller trade the asset to a slightly better informed intermediary, who then trades it to another slightly better informed intermediary, and so on until the asset reaches the expert buyer. This intermediation chain will allow trade to occur efficiently, preserving all gains to trade, even in situations where direct trading between the buyer and the seller would be inefficient. We, however, also learned that when an intermediation chain helps to preserve more social surplus than direct trade, the seller may have private incentives to bypass the intermediation chain if allowed. In this section, we characterize order-flow agreements that traders commit to ex ante (i.e., before trading takes place) and ensure that no trader involved in an intermediation chain that sustains efficient trade will be tempted to form an alternative trading network. These order-flow agreements will render socially optimal trading networks privately optimal for all traders involved.

Definition 1 (Order-flow agreement) Consider an economy with a set of traders $\mathbb{T}$. An order-flow agreement $\Sigma$ between a subset of traders $C \subseteq \mathbb{T}$ specifies the following objects:

1. A collection of directed network links: each trader $i$ in the set $C$ is exclusively connected to a unique counterparty $j \in \{C \setminus i\}$ to which trader $i$ quotes an ultimatum price whenever he wishes to sell.

2. A collection of ex ante transfers between the traders in the set $C$.

A key component of these order-flow agreements are the ex ante transfers that incentivize traders to transact with specific counterparties. In financial markets, these transfers may come in the form of explicit agreements involving cash payments for order flow or soft dollars, or they may be implicit arrangements involving profitable IPO allocations or subsidies on the various other services that intermediaries provide. In fact, there is ample empirical evidence that such
“perks” are commonly used by financial intermediaries to compensate traders for their business (see, e.g., Blume 1993, Chordia and Subrahmanyam 1995, Reuter 2006, Nimalendran, Ritter, and Zhang 2007).

**Definition 2 (Equilibrium)** An order-flow agreement $\Sigma$ between a set of traders $C \subseteq T$ constitutes an equilibrium if there is no coalition of traders $C' \subseteq T$ that can block the agreement, that is, there does not exist an order-flow agreement $\Sigma'$ that only includes traders in coalition $C'$ and that makes every trader in $C'$ weakly better off and at least one trader in $C'$ strictly better off.

Consistent with our previous analysis, we are interested in the parameter region in which intermediation chains help to sustain efficient trade. For expositional convenience, we summarize the corresponding conditions as follows.

**Condition 1 (Efficient intermediation chain)** The set $T$ contains traders who are endowed with information sets as described in Section 3, and inequality 2 in Proposition 2 is satisfied.

We now formally characterize the existence of equilibrium order-flow agreements that support the types of intermediation chains that we introduced in Section 3.

**Proposition 3 (Equilibrium order-flow agreements)** If Condition 1 is satisfied, the following results obtain:

1. Any order-flow agreement that does not lead to efficient trade is not an equilibrium.

2. For any intermediation chain that allows for efficient trade there exists a corresponding order-flow agreement that constitutes an equilibrium.

**Proof.** [Part 1] Suppose there exists a set of traders $C \subseteq T$ and an order-flow agreement $\Sigma$ for which trade breaks down with strictly positive probability so that the total surplus across all traders in $C$ is less than $\Delta$. Further, assume that every trader in $C$ obtains an ex ante surplus, net of transfers, that is weakly positive (otherwise equilibrium conditions are immediately violated, as

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11 Note that arrangements on cash payments for order flow in equity and option markets are required to be disclosed in advance in Rule 606 reports. Thus, just like in our model, transfers of this type do not vary based on transaction-specific information (i.e., a particular realization of $v$), although they vary based on the expertise of the traders involved (Easley, Kiefer, and O’Hara 1996). This characterization distinguishes these ex ante transfers from the transfers that occur later as part of the trading process (i.e., the transaction prices $p^m_i$).
every trader with negative surplus strictly prefers to exit the agreement). Order-flow agreement \( \Sigma \) can be blocked by a coalition of traders \( C' \subseteq \mathcal{T} \): since Condition I is satisfied, there exists an order-flow agreement \( \Sigma' \) associated with an intermediation chain that generates efficient trade and preserves a total surplus of \( \Delta \). Since the total surplus is greater under agreement \( \Sigma' \) and any trader not involved in \( \Sigma \) collects zero surplus, ex ante transfers can be chosen such that every trader in \( C' \) is strictly better off.

[Part 2] An intermediation chain that allows for efficient trade yields a total ex ante surplus of \( \Delta \) across all traders. To prove the existence of an order flow agreement that constitutes an equilibrium and supports the efficient intermediation chain consider an order-flow agreement \( \Sigma \) that specifies a set of transfers that imply that all intermediaries involved in agreement \( \Sigma \) obtain zero ex ante surplus (net of transfers), and the ultimate buyer and seller split the total surplus of \( \Delta \). Any coalition of traders \( C' \) that attempts to block this order flow agreement would need to include the ultimate buyer and seller, since they are needed to generate a positive surplus from trade. A blocking order-flow agreement \( \Sigma' \) would thus need to make these end traders weakly better off and at least one agent in coalition \( C' \) strictly better off, which is impossible since the ultimate buyer and seller already split the maximum surplus of \( \Delta \) under agreement \( \Sigma \) and no intermediary would be willing to participate in the blocking order-flow agreement if promised a negative expected surplus.

In our model, deal-flow is valuable to any intermediary included in an efficient trading network, since his informational advantage over his counterparty allows him to extract a fraction of the gains to trade \( \Delta \). Hence, intermediaries are willing to offer cash payments, or subsidized services, to the ultimate buyer and seller of the asset if these are required concessions for being involved in the trading network. In the proof of Proposition 3 we have focused on order-flow agreements that set the profits of intermediaries, net of these transfers, equal to zero. There, however, may also exist order-flow agreements that provide some intermediaries with strictly positive ex ante surplus. In cases where full efficiency can only be achieved with the involvement of a particular intermediary, equilibrium order-flow agreements will exist such that this important intermediary extracts strictly positive surplus.
5 Other Information Structures

Our main result that chains of intermediaries can facilitate efficient trade was made tractable in our baseline setting thanks to a few stylized assumptions about traders’ information structures. In this section, we revisit the special cases analyzed in Section 3 and show how the intuition developed so far can be extended to more complex informational settings.

Since in these more complex settings some transactions will involve bargaining games in which a proposer (seller) has private information not known to a responder (buyer), the model will no longer have a unique equilibrium. The goal of the analysis below is to show, under various circumstances, the existence of at least one type of equilibria in which intermediation chains expand the parameter region in which efficient trade is attainable. To ensure that our results are not driven by the multiplicity of equilibria that off-equilibrium beliefs trigger in signaling games, we will first fix off-equilibrium beliefs and then compare the efficiency of trade across various trading networks given those beliefs. We will show that, for a class of beliefs that we argue is reasonable, our baseline result that intermediation chains facilitate efficient trade is robust to variations in information structures.

Throughout, we will assume that transaction prices quoted in earlier rounds of trade are not observable to traders that were not involved in those rounds. This assumption will streamline our analysis, since an off-equilibrium price quote in one round of trade will trigger belief adjustments for only one trader (that is, the responder in that round of trade). In the context of decentralized markets price opacity appears more suitable than price transparency (see, e.g., Green, Hollifield, and Schürhoff 2007, Duffie 2012), but our results would survive if all traders were to observe the prices quoted in earlier rounds and their beliefs would adjust following a deviation by any informed proposer.

5.1 Two-Sided Asymmetric Information

In Section 2 we introduced an information asymmetry between a buyer and a seller that was one sided. We now show that the intuition developed in our baseline model extends to situations in which both end-traders have private information about the value of the asset. We revisit the two- and three-factor cases analyzed in Section 3 and prove the existence of perfect Bayesian equilibria in which intermediation chains improve trade efficiency just as they do in our baseline setting.
Before solving for the conditions for efficient trade throughout a given trading network, we introduce the following Lemma:

**Lemma 2 (Efficient trade and pooling equilibria)** The only equilibria in which efficient trade occurs are pooling equilibria in which the proposer does not alter his price quote based on his private information, and this price quote is always accepted by the responder.

**Proof.** Suppose there is an equilibrium in which the proposer alters his price quote based on his private information. In such an equilibrium, for trade to be efficient the responder needs to accept all of the proposer’s offers. If the proposer anticipates such a response, then he should quote the highest equilibrium price, regardless of his information, contradicting the initial claim. ■

**Two-Factor Case, revisited:** Recall that in Section 3 we showed that, if the seller is uninformed about \( v \) but the buyer observes \( \{ \phi_1, \phi_2 \} \), involving an intermediary who observes one factor allows for efficient trade as long as: \( \frac{\sigma}{\Delta} \leq \frac{2}{3} \). Trade is, however, inefficient without the intermediary whenever \( \frac{\sigma}{\Delta} > \frac{1}{3} \).

Here, we consider instead the case where asymmetric information is two sided, that is, the seller only observes \( \phi_1 \) and the buyer only observes \( \phi_2 \). Both end traders are thus partially informed about \( v \) and the trader who makes the ultimatum offer now possesses information his counterparty does not possess. It will greatly simplify our analysis to restrict our attention to off-equilibrium beliefs that have the responder updating the probability that \( \phi_1 = 1 \) from \( \frac{1}{2} \) to \( \mu \) when quoted by the seller any price higher than the equilibrium price quote. Since efficient trade cannot be sustained, with or without intermediaries, whenever \( \mu > \frac{1}{2} \), we restrict our attention to situations for which \( \mu \in [0, \frac{1}{2}] \) and compare the parameter regions that allows for efficient trade in different networks, just as we did when analyzing the baseline model\(^{12}\). This class of beliefs allows our equilibrium to satisfy the Intuitive Criterion of Cho and Kreps (1987). In our context, the Intuitive Criterion requires that a buyer ascribes zero probability to any seller type who would be worse off by quoting a higher price regardless of the buyer’s actions. Clearly, both seller types would be better off with a higher price should the buyer accept. A natural example for these off-equilibrium beliefs sets \( \mu = 1/2 \), meaning that a deviation to a higher price quote is uninformative about the proposer’s

\(^{12}\)When \( \mu > 1/2 \), a seller always finds profitable to quote an infinitesimally higher price than the pooling equilibrium price because it is accepted by the buyer given his beliefs. This profitable deviation implies that no pooling, perfect Bayesian equilibrium exists and trade cannot be efficient according to Lemma 2.
private information. Such off-equilibrium beliefs are particularly reasonable given that any seller would strictly prefer to collect more than the equilibrium price, whenever possible. As we will show though, many other off-equilibrium beliefs $\mu$ allow our results to survive qualitatively, but the region over which intermediation chains sustain efficient trade differs.

We know from Lemma 2 that without an intermediary efficient trade is possible if and only if there exists a pooling price that is always accepted by the buyer. We denote the highest pooling price that a buyer always accepts by $\bar{p} = \frac{\sigma}{2} + \Delta$. This price is also the pooling price best able to sustain efficient trade. The buyer believes that any higher price quote coming from the seller implies that $\phi_1 = 1$ with probability $\mu \leq 1/2$. All that is left to check then is that the seller prefers to quote the buyer $\bar{p}$, which is always accepted, rather than $\mu \sigma + \sigma + \Delta$, which is only accepted half the time:

$$\frac{\sigma}{2} + \Delta \geq \frac{1}{2} (\mu \sigma + \sigma + \Delta) + \frac{1}{2} \phi_1 \sigma.$$

This condition is always satisfied as long as: $\frac{\sigma}{\Delta} \leq \frac{1}{1+\mu}$. Trade is inefficient if no intermediary is involved and this inequality is violated.

Now, the counterpart for the two-sided asymmetric information case of the moderately informed intermediary we had in the baseline model is an uninformed intermediary: his involvement splits an information asymmetry of two factors into two transactions that each involve a one-factor informational advantage. Conjecturing that efficient trade occurred in the first transaction, the uninformed intermediary prefers to quote the buyer $\bar{p}$ rather than $\bar{p} + \sigma$ if and only if:

$$\frac{\sigma}{2} + \Delta \geq \frac{1}{2} (\sigma + \sigma + \Delta) + \frac{1}{2} \phi_1 \sigma,$$

which simplifies to $\frac{\sigma}{\Delta} \leq 1$. Given this, the highest pooling price the uninformed intermediary accepts to pay to the seller is also $\bar{p}$. Any higher price quote would be rejected by the intermediary, given his off-equilibrium beliefs. The seller then prefers to quote $\bar{p}$ rather than holding on to the asset if

$$\frac{\sigma}{2} + \Delta \geq \phi_1 \sigma + \frac{\sigma}{2}.$$

This condition is always satisfied as long as: $\frac{\sigma}{\Delta} \leq 1$. Hence, similarly to what happens in the baseline model, as long as $\mu \in (0, \frac{1}{2}]$ there exists a region, i.e., $\frac{1}{1+\mu} < \frac{\sigma}{\Delta} \leq 1$, in which trade is
efficient if an intermediary is involved and is inefficient otherwise.

As in Section 3, the two-factor case helped to illustrate how an intermediary can facilitate efficient trade. It, however, takes an environment with at least three factors to observe an intermediation chain that sustains efficient trade.

**Three-Factor Case, revisited:** Instead of the seller being uninformed about \( v \) and the buyer observing \( \{\phi_1, \phi_2, \phi_3\} \) as in Section 3, we now assume that the seller observes \( \phi_1 \) and the buyer observes \( \{\phi_2, \phi_3\} \). The highest pooling price quoted by the seller that is always accepted by the buyer is still \( \bar{p} \). The buyer believes that any higher price quote from the seller implies that \( \phi_1 = 1 \) with probability \( \mu \leq 1/2 \). We show in Appendix B that efficient trade in this case occurs without intermediaries only if \( \frac{\sigma}{\Delta} \leq \frac{1}{2+3\mu} \).

Next, we consider a trading network in which the seller, who observes \( \phi_1 \), trades with an uninformed intermediary who then trades with a second intermediary who observes \( \phi_2 \) and then trades with the buyer, who observes \( \{\phi_2, \phi_3\} \). We show in Appendix B that intermediated trade can be efficient as long as: \( \frac{\sigma}{\Delta} \leq 2/3 \). Thus, for any \( \mu \in [0, \frac{1}{3}] \) there exists a region, i.e., \( \frac{1}{2+3\mu} < \frac{\sigma}{\Delta} \leq 2/3 \), in which trade is inefficient without intermediaries but efficient with a chain of two intermediaries.

### 5.2 Information Percolation

We now analyze how the intuition developed in our baseline model extends to situations in which traders’ information sets are non-nested initially, but information percolates through trade as in Duffie, Malamud, and Manso (2009, 2013). We revisit the three-factor case from Section 3 and prove the existence of perfect Bayesian equilibria in which intermediation chains improve trade efficiency just as they do in our baseline setting.

**Three-Factor Case, revisited:** Recall that in Section 3 we showed that involving two intermediaries, who respectively observe the information sets \( \{\phi_1\} \) and \( \{\phi_1, \phi_2\} \), between an uninformed seller and a fully informed buyer allows for efficient trade as long as: \( \frac{\sigma}{\Delta} \leq 1/2 \). Trade is, however, inefficient with one or no intermediary if: \( \frac{\sigma}{\Delta} > 1/3 \).

In this section, the structure of information sets deviates from the one in our baseline setting in
that there are two intermediaries who observe disjoint sets of factors before trading occurs. Traders, however, learn the information of their respective counterparty after trading has occurred, which is analogous to the notion of information percolation analyzed by Duffie, Malamud, and Manso (2009, 2013).\footnote{As in Duffie, Malamud, and Manso (2009, 2013) the traders in our model do not have any reason to refrain from sharing their information with their counterparty once a transaction has been finalized. Sharing information prior to the transaction occurring would, however, not be optimal for informed traders. A prospective seller does not want to share a bad private signal about \( v \) prior to the transaction, but he has no reason not to do so once the transaction has been finalized. Unlike Duffie, Malamud, and Manso (2009, 2013), we abstract away from the specific process through which information sharing occurs. Instead, we focus on showing that the efficiency gains that intermediation chains produce with nested information sets survive in an environment with information percolation.}

Specifically, we consider a trading network in which the uninformed seller trades with a first intermediary, who initially observes only \( \phi_1 \), and then trades with a second intermediary, who initially observes only \( \phi_2 \). Finally, the second intermediary trades with the expert buyer. Because information percolates once the two intermediaries have finalized their joint transaction, the second intermediary knows the realizations of factors \( \{\phi_1, \phi_2\} \) by the time he quotes a price to the expert. As in the scenario above with two-sided asymmetric information, the bargaining game now involves a proposer (the first intermediary) with private information not known to a responder (the second intermediary) so that the model no longer has a unique equilibrium. The purpose of the current analysis is to show the existence of at least one type of equilibria in which intermediation chains facilitate efficient trade. We conjecture an equilibrium that sustains efficient trade and satisfies the following properties:

- The uninformed seller quotes a price \( \bar{p} \) to the first intermediary, who always accepts.

- Regardless of the realization of \( \phi_1 \) he observes, the first intermediary quotes the highest price at which the second intermediary, knowing nothing about the first intermediary’s information, always accepts: \( \bar{p} = \frac{\sigma}{2} + \Delta \).

- The second intermediary updates the probability that \( \phi_1 = 1 \) from \( 1/2 \) to \( \mu \in [0, \frac{1}{2}] \) when quoted any price higher than \( \bar{p} \) by the first intermediary.

- The second intermediary quotes a price \( \phi_1 \sigma + \phi_2 \sigma + \Delta \) to the expert buyer, who always accepts.

In Appendix C, we prove the existence of a perfect Bayesian equilibrium as defined above as long...
as: $\frac{\sigma}{\Delta} \leq \frac{2}{3+2\mu}$. Nested information sets are thus not necessary for intermediation chains to facilitate efficient trade under asymmetric information. As before, the class of beliefs we assume ensures that our equilibrium satisfies the Intuitive Criterion from Cho and Kreps (1987). However, what is special here is that reasonable off-equilibrium beliefs for which $\mu = 1/2$ also produce a condition for efficient intermediated trade that is identical to the condition we derived in Section 3 when information sets were nested. When $\mu = 1/2$, trade is efficient in the region where $1/3 < \frac{\sigma}{\Delta} \leq 1/2$ if two heterogeneously informed intermediaries are involved, but is inefficient with zero or one intermediary. This example shows that in the presence of information percolation, replacing an intermediation chain with nested information sets by a chain with non-nested information sets may sustain efficient trade in a very similar manner.

5.3 Expert Sellers

The last point we want to highlight in this section is that results similar to those derived in Section 3 can arise when the seller is the expert and the buyer is uninformed. If we allow intermediaries to short sell the asset, those results can be obtained without the complications that arise in signaling games. We revisit the two-factor case to show that conditions on $\frac{\sigma}{\Delta}$ for efficient trade are identical to the conditions we derived earlier. Extending this comparison to an $N$-factor case with an $M$-intermediary chain would be straightforward, yet redundant.

**Two-Factor Case, revisited:** As before, suppose the asset is worth $v = \phi_1\sigma + \phi_2\sigma$ to the seller and $v + \Delta$ to the buyer. However, the seller now observes $\{\phi_1, \phi_2\}$ while the buyer is uninformed about these factors. To eliminate signalling concerns and remain consistent with the analysis in Section 3, the uninformed buyer is assumed to be making an ultimatum offer to the seller. Without an intermediary, the buyer chooses to offer one of three price candidates: (i) $2\sigma$, which is accepted by the seller with probability 1; (ii) $\sigma$, which is accepted with probability $3/4$; (iii) 0, which is accepted with probability $1/4$.

The first price candidate $2\sigma$ splits the surplus from trade such that the buyer collects $\Delta - \sigma$ and the seller collects $\sigma$. The second price candidate $\sigma$ produces an expected surplus of $\frac{3}{4}\Delta - \frac{1}{4}\sigma$ for the buyer and $\frac{1}{4}\sigma$ for the seller. The third price candidate produces an expected surplus of $\frac{1}{4}\Delta$.
for the buyer and no surplus for the seller. Offering the high price $2\sigma$ is thus optimal for the buyer, making trade efficient, if and only if $\frac{\sigma}{\Delta} \leq 1/3$.

However, when an agent observes $\phi_1$ and intermediates trade between the seller and the buyer, trade can be efficient even though $\frac{\sigma}{\Delta} > 1/3$. Remember that here we allow the intermediary to sell the asset short, that is, he can accept to sell the asset to the buyer as long as he also buys the asset from the seller. Consistent with the nested information sets assumed in Section 3, the uninformed buyer first makes an offer to purchase the asset from the intermediary who then makes an offer to the seller.

In order to buy the asset from the seller, the intermediary can offer a price $\phi_1\sigma + \sigma$ to the seller, which is always accepted, or a price $\phi_1\sigma$, which is only accepted half the time. Since the buyer makes an ultimatum offer to the intermediary and the intermediary can only accept it if he commits to buy the asset from the seller, trade is efficient as long as the buyer prefers to quote the buyer $2\sigma$, which is always accepted by the intermediary, rather than $\sigma$, which is only accepted half the time:

$$\sigma + \Delta - 2\sigma \geq \frac{1}{2} \left( \frac{\sigma}{2} + \Delta - \sigma \right),$$

which simplifies to $\frac{\sigma}{\Delta} \leq 2/3$.

As we can see, although the mechanics of intermediation with short selling are different, the region $1/3 < \frac{\sigma}{\Delta} \leq 2/3$ for which trade can only be efficient if a moderately informed intermediary is involved is identical to the corresponding region derived in Section 3 when the expert was a buyer instead of a seller.

6 Conclusion

This paper shows how chains of heterogeneously informed intermediaries can help to alleviate adverse selection problems that impede efficient trading between asymmetrically informed agents. Complex trading networks that sequentially involve several intermediaries may be the socially optimal response to information asymmetries as reallocating a large adverse selection problem over multiple transactions reduces agents’ incentives to inefficiently limit trade when facing their better informed counterparties. Thus, greater information asymmetries require longer intermediation chains to sustain efficient trade, consistent for example with Li and Schürhoff (2014) who show that
unrated municipal bonds are typically traded through longer intermediation chains than rated municipal bonds (which arguably are less likely to be associated with large adverse selection problems). Moreover, if market participants implement efficient networks, our theory predicts that larger information asymmetries will be associated with more trading being observed, which contrasts with the conventional wisdom that empirically, large information asymmetries should be associated with low trading volume (as in Akerlof 1970). Finally, because informed intermediaries extract rents in a socially optimal trading network, they are willing to offer transfers such as cash payments, or subsidies on services they perform, to other traders in exchange for their order flow. This result might help to inform the current policy debate on the use of order-flow agreements in financial markets.
Appendix A: Proofs

Lemma 3 (Necessary and sufficient condition for efficient trade)  Given that trade is efficient in all subsequent transactions, trader \( m \) finds optimal to quote \( p^m_0 \) rather than any other price if and only if he prefers to quote \( p^m_0 \) over \( p^m_1 \).

Proof. Consider a situation in which trader \( m \) currently holds the asset and wants to sell it to trader \( m+1 \). Trader \( m \) knows that out of the \( N_m \) factors \( \phi_n \) he observes, \( G_m \) realizations have a value of 1. Similarly, trader \( m+1 \) knows that out of the \( N_{m+1} \) factors \( \phi_n \) he observes, \( G_{m+1} \) realizations have a value of 1. Assume that whenever trader \( m+1 \) acquires the asset, subsequent trading is efficient, which requires that all subsequent traders \( k \in \{m+1, m+2, \ldots, M\} \) charge prices:

\[
p^k_0 = G_k \sigma + \Delta,
\]

which maximize trade probability. Trader \( m \) then chooses to quote one of \((N_{m+1} - N_m + 1)\) price candidates, defined as:

\[
p^m_i = (G_m + i) \sigma + \Delta, \quad i \in \{0, \ldots, N_{m+1} - N_m\}.
\]

The weakly better informed trader \( m+1 \) only accepts to pay a price \( p^m_i \) if \( G_{m+1} \sigma + \Delta \geq p^m_i \), which occurs with probability \( \pi^m_i \).

Trader \( m \) prefers quoting \( p^m_i \) over \( p^m_{i+1} \) if and only if:

\[
\pi^m_i p^m_i + (1 - \pi^m_i) E[v|G_{m+1} < G_m + i] \geq \pi^m_{i+1} p^m_{i+1} + (1 - \pi^m_{i+1}) E[v|G_{m+1} < G_m + i + 1]
\]

\[
\Leftrightarrow \pi^m_i p^m_i - \pi^m_{i+1} p^m_{i+1} \geq (1 - \pi^m_i) E[v|G_{m+1} < G_m + i + 1] - (1 - \pi^m_{i+1}) E[v|G_{m+1} < G_m + i]
\]

\[
\Leftrightarrow (\pi^m_i - \pi^m_{i+1}) [(G_m + i) \sigma + \Delta] - \pi^m_{i+1} \sigma \geq (\pi^m_i - \pi^m_{i+1}) (G_m + i) \sigma
\]

\[
\Leftrightarrow \frac{\sigma}{\Delta} \leq \frac{\pi^m_i - \pi^m_{i+1}}{\pi^m_{i+1}}.
\]

When the probability distribution that characterizes the information asymmetry between traders \( m \) and \( m+1 \) is such that the (discrete) hazard rate (i.e., the RHS in (3)) reaches its global mini-
mum at \( i = 0 \), trader \( m \) quotes \( p_0^m \) if and only if he prefers to quote \( p_0^m \) over \( p_1^m \). The binomial distribution has a (discrete) hazard rate that reaches its global minimum at \( i = 0 \): the probability mass function \( \pi_i^m - \pi_{i+1}^m \) is minimized at the two extremes of the distribution, that is, at \( i = 0 \) and \( i = N_m + 1 - N_m - 1 \) and the complementary cumulative distribution function \( \pi_{i+1}^m \) is decreasing in \( i \). ■

### Appendix B: Efficient Trade with Two-Sided Asymmetric Information

In this scenario, we assume that the seller observes \( \phi_1 \) and the buyer observes \( \{\phi_2, \phi_3\} \). The highest pooling price quoted by the seller that is always accepted by the buyer is still \( \bar{p} \). Given his off-equilibrium beliefs, any higher price quote would be perceived by the buyer as meaning that \( \phi_1 = 1 \) with probability \( \mu \leq 1/2 \). Hence, the two conditions that need to be satisfied for efficient trade to occur without intermediaries are:

\[
\frac{\sigma}{2} + \Delta \geq \frac{3}{4} (\mu \sigma + \sigma + \Delta) + \frac{1}{4} \sigma,
\]

which simplifies to \( \frac{\sigma}{\Delta} \leq \frac{1}{2 + 3 \mu} \), and

\[
\frac{\sigma}{2} + \Delta \geq \frac{1}{4} (\mu \sigma + 2 \sigma + \Delta) + \frac{1}{2} 2 \sigma + \frac{1}{4} \sigma,
\]

which simplifies to \( \frac{\sigma}{\Delta} \leq \frac{3}{5 + \mu} \). When \( \mu \geq 0 \), the first condition is more restrictive than the second one and efficient trade is thus possible without intermediaries only if \( \frac{\sigma}{\Delta} \leq \frac{1}{2 + 3 \mu} \).

Next, we consider a trading network in which the seller, who observes \( \phi_1 \), trades with an uninformed intermediary, who then trades with a second intermediary who observes \( \phi_2 \) and then trades with the buyer, who observes \( \{\phi_2, \phi_3\} \). Conjecturing that efficient trade occurred in the first two transactions, the second intermediary prefers to quote the buyer \( \bar{p} + \phi_2 \sigma \) rather than \( \bar{p} + \phi_2 \sigma + \sigma \) if and only if:

\[
\frac{\sigma}{2} + \phi_2 \sigma + \Delta \geq \frac{1}{2} (\frac{\sigma}{2} + \phi_2 \sigma + \sigma + \Delta) + \frac{1}{2} (\frac{\sigma}{2} + \phi_2 \sigma),
\]

14More generally, a hazard rate function is defined as \( pmf(x) \), where \( pmf \) and \( cdf \) respectively denote the probability mass function and the cumulative distribution function.
which simplifies to \( \frac{\sigma}{\Delta} \leq 1 \). Given that, the first intermediary prefers to quote \( \bar{p} \) rather than \( \bar{p} + \sigma \) to the second intermediary if and only if:
\[
\frac{\sigma}{2} + \Delta \geq \frac{1}{2} \left( \frac{\sigma}{2} + \sigma + \Delta \right) + \frac{1}{2} \sigma,
\]
which simplifies to \( \frac{\sigma}{\Delta} \leq 2/3 \). Given that, the highest pooling price that the uninformed intermediary will accept to pay to the seller is also \( \bar{p} \). Any higher price quote would be rejected by the uninformed intermediary, given his off-equilibrium beliefs. All that is left to check then is that the seller prefers to quote \( \bar{p} \) rather than holding on to the asset, even if \( \phi_1 = 1 \):
\[
\frac{\sigma}{2} + \Delta \geq 2\sigma,
\]
which simplifies to \( \frac{\sigma}{\Delta} \leq 2/3 \).

**Appendix C: Efficient Trade with Information Percolation**

In this scenario, we consider a trading network in which the uninformed seller trades with a first intermediary who observes \( \phi_1 \) and then trades with a second intermediary, who observes \( \phi_2 \). Finally, the second intermediary trades with the expert buyer. We conjecture a perfect Bayesian equilibrium in which trade is efficient and the following properties apply:

- The uninformed seller quotes a price \( \bar{p} \) to the first intermediary, who always accepts.

- Regardless of the realization of \( \phi_1 \) he observes, the first intermediary quotes the highest price at which the second intermediary, knowing nothing about the first intermediary’s information, always accepts: \( \bar{p} = \frac{\sigma}{2} + \Delta \).

- The second intermediary updates the probability that \( \phi_1 = 1 \) from 1/2 to \( \mu \in [0, \frac{1}{2}] \) when quoted any price higher than \( \bar{p} \) by the first intermediary.

- The second intermediary quotes a price \( \phi_1 \sigma + \phi_2 \sigma + \Delta \) to the expert buyer, who always accepts.
To prove the existence of such equilibrium, we first need to analyze the last stage of trading between the second intermediary and the expert, which is identical to the last stage of trading in the two-factor and three-factor cases from Section 3. If $\frac{\sigma}{\Delta} \leq 1$, the second intermediary finds optimal to quote a price $\phi_1\sigma + \phi_2\sigma + \Delta$ to the expert, which he always accepts.

Trading between the two intermediaries is slightly more complex to analyze, since information sets are non-nested. The first intermediary quotes a price, after observing $\phi_1$, to the second intermediary who only observes $\phi_2$. Given the beliefs assumed, the most attractive deviation by the first intermediary from the conjectured equilibrium action is to quote a price $\mu\sigma + \sigma + \Delta$, which is accepted by the second intermediary only if $\phi_2 = 1$. Such deviation is dominated by the equilibrium strategy of quoting $\bar{p}$, even if $\phi_1 = 1$, as long as:

$$\frac{\sigma}{2} + \frac{\Delta}{2} \geq \frac{1}{2} (\mu\sigma + \sigma + \Delta) + \frac{1}{2} \left( \sigma + \frac{\sigma}{2} \right)$$

$$\Leftrightarrow \frac{\sigma}{\Delta} \leq \frac{2}{3 + 2\mu}.$$ 

Further, collecting $\bar{p}$ also dominates non-participation for the first intermediary, as long as $\frac{\sigma}{\Delta} \leq \frac{1}{\frac{1}{2} + \phi_1}$. When $\frac{\sigma}{\Delta} \leq \frac{2}{3 + 2\mu}$, quoting $\bar{p}$ is thus always the equilibrium strategy for the first intermediary in this stage.

Finally, the seller can quote $\bar{p}$ to the first intermediary, which is accepted with probability 1, but he might also consider quoting a higher price. Since the first intermediary plans on subsequently quoting $\bar{p}$, regardless of his information, no such higher price quote by the seller can sustain trade with positive probability. The seller thus chooses to quote $\bar{p}$ as long as it dominates non-participation:

$$\frac{\sigma}{2} + \frac{\Delta}{2} \geq \frac{3}{2} \sigma$$

$$\Leftrightarrow \frac{\sigma}{\Delta} \leq 1.$$ 

Overall, all the conditions required for the conjectured perfect Bayesian equilibrium to exist are verified if and only if $\frac{\sigma}{\Delta} \leq \frac{2}{3 + 2\mu}$.
References


