DIVISION OF LABOR IN MULTI-BUSINESS FIRMS:

HUMAN CAPITAL, JOB DESIGN, AND LABOR CONTRACTS

by

Birger Wernerfelt*

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*bwerner@mit.edu

ABSTRACT

We ask how division of labor is enhanced in multi-business firms. What human capital do the workers acquire, how are their jobs designed, and under what kinds of contracts do they work? To answer the questions in a way that is consistent with the existence of multi-business firms, we develop a simple model with endogenous human capital acquisition, job design, labor contracts, and firm size. We find conditions under which several different sets of human capital and job designs will be observed and characterize the mechanisms in which each is traded. The mechanisms used in equilibria include markets of different sizes, employment in single- and multi-business firms, and bilateral non-employment relationships. When they exist, multi-business firms allow workers to acquire human capital in, and work on, a narrow set of services, and they can therefore use dedicated employees to perform services that smaller firms buy in the market or get from employees with broader job descriptions.

JEL Codes: D02, D23

I. Introduction

The efficiency gains from division of labor, and the role of markets in making them feasible, is a fundamental premise of economics. We here identify multi-business firms as another enabler of specialization and show how they may owe their existence to that very role. The analysis will, for example, explain when a worker will be a handy man, a superintendent, a plumber who serves a number of customers as an independent contractor, or a plumber who is an employee of a large firm. Our main result is that multi-business firms allow workers to acquire human capital in, and work on, a narrow set of services and that they can therefore use dedicated employees to perform services that smaller firms buy in the market or get from employees with broader job descriptions.

Along the way to developing a rationale for multi-business firms, we identify several determinants of how narrow human capital a worker acquires, how many services his job description includes, how many businesses he serves, the nature of the contracts that govern his work, and the kinds of firms he contracts with.

Intuition

A worker's cost of performing a particular service for a particular business, say fixing a bannister in an apartment building, depends on the narrowness of his human capital and the match between it and the (service, business) pair in question. His costs are lower the more narrowly his human capital is focused on the service, but they are higher if he has to perform a service outside his area of expertise. Analogously with the business; his costs are lower the more narrowly his human capital is focused on the business, but higher if he has to work for someone else.

Everything else being equal, a worker would prefer his job description to be as narrow as possible in both the service and business domains. If he can perform the same service for the same business in every period, his human capital investments can exhibit the identical dual specialization and he will be maximally efficient. However, since the services needed by an individual business cannot be perfectly predicted, workers often select mixed mode human capital investments to protect themselves from the risk of having to work outside their areas of expertise. For example, a worker who is an expert in a single service but has general business skills, can take a job with matching description and follow the demand for that service from business to business, like e.g. an independent plumber. Conversely, if a worker is an expert on a single business but has general service skills, he can take a job performing odd services for the business in question, like e.g. a building superintendent.

Workers can, however, approximate dual specialization by taking advantage of business "neighborhoods". This concept captures the observation that some pairs of businesses are more "similar" than others. There are two aspects of similarity: Human capital and services needed.[1] First, the human capital used to work in some pairs of businesses are more similar to each other than to the those required to do the same work in other businesses. Second, some pairs of businesses tend to need more or less the same services, while others need very different services. Think, for example, about maintaining two 1920's brownstones vs. one brownstone and a modern high-rise. Both the building specific information necessary and the typical problems are likely to be more similar in the two brownstones.

To see the role of neighborhoods in efficient production, consider a worker whose human capital is focused on a business neighborhood and an individual service. This worker may be able to spend most or all of his time providing the chosen service to businesses in the neighborhood. When these businesses are independently operated, they could be seen as a local market. More importantly, if all the neighborhood businesses are operated by a single entrepreneur *and* the workers are employees, they would constitute a multi-business firm.

Finding conditions under which workers are employees as opposed to independent contractors, requires a theory of employment. We take this component of the model from Wernerfelt (1997, 2015) and model distortions brought about by bilateral price-determination by the reduced form assumption that a worker incurs bargaining costs, if his per-service payment is negotiated with a single entrepreneur. We assume that these costs are sub-additive in the breadth of the job description, the number of possible services covered by the negotiation. That is, if the contract gives the entrepreneur the right to ask for any one of S services, then the cost of negotiating it are less than S times the cost of negotiating a price for a single specific service.

The sub-additivity is key because it is that which makes it meaningful to compare several alternative contracts: The parties may bargain over individual prices on an as-needed basis, they may eliminate all later bargaining by once-and-for-all agreeing on a price for which they would

[1] As shown in the Corollary in Section III, it turns out that we need both properties to make our argument.

perform any service in any business the entrepreneur asks for, or they may negotiate over a price for any element of a set of service/business pairs with intermediate cardinality - hoping to avoid additional negotiations later on. We use the term "employee" to describe a worker who has agreed to follow orders in the sense that he will perform any of several service/business pairs "on demand" with no further negotiation, thus adapting costlessly.

While it is non-standard to rely on the existence of bargaining costs, we assume that they are so "small" that they only influence choices that cannot be made on the basis of production costs.

We can now see how the model can explain the existence of multi-business firms. Think of a business neighborhood consisting of several apartment buildings and a number of workers who have human capital focused on the business neighborhood and individual services, such plumbing, carpentry, etc. No single building will need a plumber on any given day, but on many days, at least one of them will. For simplicity, assume exactly one. If the plumber is an independent contractor, he will negotiate a [one business, plumbing] contract every day. But if all the businesses are managed by the same entrepreneur, our plumber can negotiate an employment contract of the form [business neighborhood, plumbing]. Because of the sub-additivity, this may well be cheaper. So the model allows us to characterize at least some determinants of the scope of the firm. In particular, it suggests that multi-business firms are comparatively more efficient if neighboring businesses are more similar in terms of human capital required and services needed.

Literature

It is useful to organize the literature review by going through each of the model components.

As mentioned above, the argument relies on the assumption that human capital has two dimensions, such that workers invest in more or less narrow sets of services and businesses. The first dimension has been never been controversial: Economists have, at least since Adam Smith (1776), accepted that individuals can learn how to perform a service (a craft) and thereby become more efficient at it. In contrast, the claim that there is a business-specific dimension of human capital was only comparatively recently made by Becker (1962), and has been the subject of considerable debate.

Another aspect of model, the idea that human capital can be more or less narrow, has perhaps been most explicitly made by Lazear (2009), but would not seem to be controversial. However, an unusual aspect of the model is that we look exclusively at how narrow human capital investments are, while holding their levels constant.

On the efficiency advantages associated with division of labor (job descriptions that cover a small set of services and businesses), we follow a very old line originally introduced by Adam Smith (1776) and developed further by Stigler (1951). Where they portray division of labor as limited by market size, we here make the point that a multi-business firm could sustain specialization just as well as a local market. If a set of neighboring firms between them has enough demand for a particular service, a worker can specialize in it and serve the firms either though a local market or as an employee of a multi-business firm.

As mentioned earlier, the theory of employment is taken from Wernerfelt (1997, 2015). Beyond that, the most closely related literature are the theory of the firm as proposed by Coase (1937) and the view of employment pioneered by Simon (1951). Coase talks about the costs of determining prices and Simon views the essence of employment as order-taking and introduces the idea that workers might be willing to comply if they are more or less indifferent between the possible orders.

The assumptions that employment only appears in bilateral situations and that these are created by specific investments, are made by both Williamson (1979) and Grossman and Hart (1986).[2] Ours is, however, a model of ex post adaptation rather than ex ante investment distortions and in contrast to Hart and Moore (2008), we do not rely on deviations from rationality (beyond the postulated impossibility of costless complete contracting). The focus on ex post adaptation is also found in, among others, Bajari and Tadelis (2001), Bolton and Rajan (2001), Dessein, Galeotti, and Santos (2016), Matouschek (2004), and Wernerfelt (1997, 2015, 2016). The latter papers also use the sub-additivity assumption.

[2] On the other hand, the present paper is also part of an emerging literature on organizations that more explicitly considers multi-lateral alternatives (e. g. Legros and Newman, 2013).

The results on multi-business firms are consistent with much literature in management, suggesting that such firms will enter industries that require human capital similar to that used in their existing industries, thereby allowing them to leverage excess capacity of a specialized workforce (Penrose, 1959).

Plan of the paper

The model is formulated in Section II and we consider the special case in which all firms operate a single business in Section III. Another special case, in which pairs of businesses are operated together, in analyzed in Section IV. This then allows us to characterize the optimal scope of the firm in Section V. The paper concludes with a discussion in Section VI.

II. Base Model

The model covers two time "periods", $t = 1, 2$, and $\delta > 0$ is the weight on second period payoffs, representing the long run and the rate at which things change. (So it is possible, and perhaps even natural, to think of $\delta > 1$.) There is a set Ω_B of businesses with $|\Omega_B| = B$, a set Ω_S of services with $|\Omega_S| = S$, a set Ω_W of risk-neutral workers with $|\Omega_W| = W$, and a set Ω_E of risk-neutral entrepreneurs with $|\Omega_E| = E$. Workers and entrepreneurs are ex ante identical and we use w and e to denote generic elements of Ω_W and Ω_E. In contrast, we assume that all businesses and services are unique. The characteristics of specific businesses and services are summarized by their addresses b and s, respectively. Businesses come in pairs such that each has exactly one neighbor. (So B is an even number.) We use $N(b)$ to denote the two element business "neighborhood" and note that $b' \in N(b) \Leftrightarrow b \in N(b')$.[3]

Businesses, operated by entrepreneurs, produce by using workers to perform services. Each business b *needs* one service, $s^t_b \in \Omega_S$, in each period t, and if a needed service is performed by a worker, it results in one unit of output. Any worker can perform any service, but only one per period, and output cannot be expanded by performing a needed service more than once, or by performing an unneeded service. We make the convenient assumptions that it is efficient to

[3] To fix ideas, one might think of pairs of businesses as located on the tips of a star with $B/2$ arms.

perform any needed service regardless of the mechanism used and that $B = W$ (such that the aggregate labor market can clear).

The allocation of businesses to entrepreneurs is a partition of Ω_B denoted by the 0-1 $B\times E$ matrix $\pi \equiv (\pi_1, .. \pi_e, .. \pi_E)$, where the column vector π_e describes the set of businesses operated by e, and Π is the set of feasible partitions (those with exactly one 1-entry in each row). Throughout the paper, π is chosen by a social planner prior to period 1, but in Section V, we allow the entrepreneurs to adjust the initial allocation by trading the rights to operate individual businesses.

The needs of businesses are determined in a publicly observed randomization at the start of each period. The distribution of needs is such that B/S businesses will need each service in each period.[4] Other than that, the vector of needs in the first period, $s^1 \equiv (s^1{}_1, .. s^1{}_b, .. s^1{}_B)$, is random. The vector of needs in the second period, $s^2 \equiv (s^2{}_1, .. s^2{}_b, .. s^2{}_B)$, depends on s^1. To capture the idea that neighboring businesses have similar needs, we assume that

- iff $\{b, b'\} = N(b)$, $s^2{}_{b'} = s^1{}_b$ with probability $p \geq 1/S$.

To keep some of the formulas uncluttered, we will assume that the probability of business needing the same service in both periods, $s^2{}_b = s^1{}_b$, is zero, (or alternatively that S is large). We use S^1 and S^2 to denote the sets of feasible need distributions in periods 1 and 2, respectively.

After observing π and s^1, workers choose their *human capital profiles* by costlessly acquiring more or less narrow business and service skills.[5] In the business domain, the skills may be focused on an individual business, a neighborhood, or businesses in general.[6] In the service domain, the skills may be focused on an individual service or services in general. Because no misunderstanding should be possible, we use the subset of $\Omega_B \times \Omega_S$ in which a worker is invested as shorthand for his human capital profile. So w's human capital is summarized in his profile $(h_{wB}, h_{wS}) \in \{\{b\}_{b\in\Omega_B}, \{N(b)\}_{b\in\Omega_B}, \Omega_B\} \times \{\{s\}_{s\in\Omega_B}, \Omega_S\} \equiv H_B \times H_S$.

[4] For convenience, we abstract from integer problems (effectively assuming that B is a multiple of S).
[5] The first period does not play a big role in the model other than allowing workers to choose their human capital in light of demand.
[6] The results change little if we portray the human capital profiles as results of learning from first period assignments rather than driven by education.

A worker's production costs are lower the more narrowly invested he is and higher as he works further from his area of expertise. Specifically, w's production costs of performing s for b is $c_{wbs} = c_{wb} + c_{ws}$ where c_{wb} depends on h_{wB} and the match between it and b, while c_{ws} depends on h_{wS} the match between it and s. Narrower specialization yields lower costs, but it is costly to work outside one's are of specialization. We define the costs as follows: When $h_{wB} = b'$, $c_{wb} = 0$ if $b = b'$ and $c_{wb} = c_B^+$ if not. When $h_{wB} = N(b')$, $c_{wb} = c_B^-$ if $b \in N(b')$, and $c_{wb} = c_B^+$ if not. Finally, $c_{wb} = c_B$ when $h_{wB} = \Omega_B$. Similarly for the service component: When $h_{wS} = s'$, $c_{ws} = o$ if $s = s'$ and $c_{ws} = c_S^+$ if not, while $c_{ws} = c_S$ when $h_{wS} = \Omega_S$. As suggested by the notation and the idea of gains from focus and match, we posit that $0 < c_B^- < c_B < c_B^+$ and that $0 < c_S < c_S^+$

The notation is summarized in Tables 1 and 2 below.

Table 1

Business Component of Production Costs (c_{wB})

h_{wB} \ *Business*	$b = b'$	$b \in N(b')\backslash b'$	$b \in \Omega_B\backslash N(b')$.
b'	0	c_B^+	c_B^+
$N(b')$	c_B^-	c_B^-	c_B^+
Ω_B	c_B	c_B	c_B

Table 2

Service Component of Production Costs (c_{wS})

h_{wS} \ *Service*	$s = s'$	$s \in \Omega_S\backslash s'$
s'	0	c_S^+
Ω_S	c_S	c_S

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To eliminate equilibria in which the worker simply minimizes first period costs, we assume that the importance of the second period and a worker's cost of working outside his area of expertise are sufficiently large. Formally:

Assumption Δ: $\delta > Max\ \{c_B^+/(c_B^+ - c_B), c_S^{\ +}/(c_S^+ - c_S)\}$

The distribution of s^2 and the costs described in Tables 1 and 2 mean that business neighborhoods are defined by two advantages: Needs are correlated and human capital degrades less between neighboring businesses. The parameters measuring these effects are p and $c_B - c_B^-$. We will later show (in the Corollary after Proposition 1) that both properties are necessary to explain human capital investments in business neighborhoods and ultimately the existence of multi-business firms.

Trades are governed by *mechanisms.* Each worker and entrepreneur can enter one mechanism per period. A mechanism in period t specifies two sets: $(m^t_B, m^t_S) \subseteq \Omega_B \times \Omega_S$, and by entering, a worker agrees to perform any $s \in m^t_S$ for any $b \in m^t_B$ in exchange for a single price determined by the number of workers and businesses who enter the mechanism.[7] Similarly, the entrepreneurs agree to choose one $s \in m^t_S$ for each $b \in m^t_B$ they operate and pay the price to any worker who performs it. The agreement is a long term contract in the sense that it is valid for both periods. No trade takes place if an entrepreneur asks for $s' \notin m^t_S$. The set of mechanisms is thus $\{0, 1\}^B \times \{0, 1\}^S$ and workers and businesses can enter any one of them, (though it only makes sense to enter a business b in one of the $|\{0, 1\}^{B-1} \times \{0, 1\}^{S-1}|$ mechanisms in which $b \in m^t_B$ and $s^t_b \in m^t_S$). A mechanism *clears* iff it is entered by the same number of workers and businesses, and in that case the price is such that all participants (workers and entrepreneurs) get identical net payoffs. If a mechanism does not clear, players on the long side get negative payoffs. In either case, businesses and workers are paired up randomly.

[7] It is an important assumption that no mechanism can produce agreement on more than one price per period. The price can cover any number of services, but cannot differ between services. This is clearly an unusual assumption, but we are not aware of any fully satisfactory ways to rule out complete contracting.

A worker is an *employee* if $m^t_B = b$ and $|m^1_S| > 1$, a *contractor* if $(m^t_B, m^t_S) = (b, s^t_b)$, and a *market worker* if $(m^t_B, m^t_S) = (\Omega_B, s)$.

We make the non-standard assumption that the parties face costs of bilateral bargaining and that these are sub-additive in the number of items bargained over.[8] To fix ideas, we assume that bargaining costs are symmetric. So iff a mechanism is entered by a single entrepreneur and one or more workers, each worker incurs bargaining costs $K(|m^t_S|)/2$ and the entrepreneur incurs the same costs per worker. The key assumption is that $K(\ )$ is weakly increasing and sub-additive, but to keep the number of parameters down, we also posit that $K(1) < K(2) < K(3) = K(S)$.

While bargaining costs will be important for the analysis, we will state our results under the assumption that they are so small that it never pays off to take on extra production costs in order to save on bargaining costs. Formally,

Assumption K: $\text{Min}\{K(S), (1 + \delta)K(1)\} < (1 + \delta)c_B^-$.

This assumption also allows us to eliminate equilibria in which parties engage in inefficient production for no reason other than to reduce bargaining costs.

In Sections III and IV, the sequence of events is as follows:

1. A social planner distributes the businesses in Ω_B between the entrepreneurs in Ω_E according to the publicly observed partition π.
2. Business needs for period 1, s^1, are realized and publicly observed.
3. Workers choose their human capital profiles $\Pi \times S^1 \rightarrow H_B \times H_S$ and these are publicly observed.
4. Workers and businesses select mechanisms for period 1: $\Pi \times (H_B \times H_S)^W \times S^1 \rightarrow \{0, 1\}^B \times \{0, 1\}^S$, and are randomly matched within each. All workers perform the service needed by the business with which they is matched.
5. Business needs for period 2, s^2, are realized and publicly observed.

[8] While this is an unusual premise, it is not unreasonable: Most people prefer not to bargain, but if they have to, would rather bargain once over a \$300 pie than 30 times over \$10 pies. Consistent with this, Maciejovsky and Wernerfelt (2011) report on a laboratory experiment in which bargaining costs are found to be positive and sub-additive.

6. Workers and businesses select mechanisms for period $2: \Pi \times (H_B \times H_S)^W \times S^2 \rightarrow \{0, 1\}^B \times \{0, 1\}^S$, and are randomly matched within each. All workers perform the service needed by the business with which they is matched.
7. All payoffs, net of any bargaining and production costs, are distributed.

We will be looking for the *most efficient symmetric subgame perfect equilibria*. [9]

Since mechanisms give both sides non-negative payoff iff they clear, we immediately get:

LEMMA: Stage games *4* and *6* are in equilibrium iff all mechanisms clear.

The model is unusual in the sense that some elements that normally are given a lot of attention (such as price determination and small numbers inefficiencies) are entered in reduced form, while things that normally are treated as exogenous (such as the size of markets and the allocation of workers to entrepreneurs) are endogenous. We first look at an economy in which each entrepreneur operates a single business and later compare it to one in which each operates two.

III. One Business per Entrepreneur.

Since entrepreneurs who did not get a business assigned to them are irrelevant, we effectively have that $B = W = E$ (recall that the mechanism clears). The matrix π is thus square and has a single *1*'s in each column (and *0*'s everywhere else).

PROPOSITION 1: One of the following is the most efficient symmetric subgame perfect equilibrium when each entrepreneur operates exactly one business:

Employment: $(h_{wB}, h_{wS}) = (b, \Omega_S)$, $(m^1{}_B, m^1{}_S) = (b, \Omega_S)$, and $(m^2{}_B, m^2{}_S) = (\varnothing, \varnothing)$, total (two period) cost is $(1 + \delta)c_S + K(S)$

[9] It will turn out that the restriction to symmetric equilibria is unimportant

Sequential Contracting: $(h_{wB}, h_{wS}) = (b, \Omega_S)$, $(m^1_B, m^1_S) = (b, s^1_b)$ and $(m^2_B, m^2_S) = (b, s^2_b)$, with cost $(1 + \delta)c_S + (1 + \delta)K(1)$.

Global Market: $(h_{wB}, h_{wS}) = (m^t_B, m^t_S) = (\Omega_B, s)$, with cost $(1 + \delta)c_B$

Local Market: Two workers set $(h_{wB}, h_{wS}) = (N(b), s^1_b)$, $(m^1_B, m^1_S) = (b, s^1_b)$, $(m^2_B, m^2_S) = (b', s^2_{b'})$ and $(h_{wB}, h_{wS}) = (N(b), s^1_{b'})$, $(m^1_B, m^1_S) = (b', s^1_{b'})$, $(m^2_B, m^2_S) = (b, s^2_b)$, respectively, where $N(b) = \{b, b'\}$ and expected cost is $(1 + \delta)c_B^- + \delta(1 - p)c_S^+ + (1 + \delta)K(1)$.

Proof: Since we are looking for the most efficient of many equilibria, it is not convenient to solve the game from the back. Instead, we will look for the most efficient equilibria contingent on each of the six possible human capital profiles. We will then compare the payoffs from each of these at the end.

(1) Suppose first that $(h_{wB}, h_{wS}) = (b, \Omega_S)$. In this case the worker investing in b and the business (entrepreneur) b will enter the same mechanism in all equilibria.

   (i) If the individual worker-entrepreneur pair negotiates a price for any possible service, $(m^1_B, m^1_S) = (b, \Omega_S)$, $(m^2_B, m^2_S) = (\varnothing, \varnothing)$, and total (two period) cost is $(1 + \delta)c_S + K(S)$.

   (ii) If they only negotiate for the currently needed service, $(m^1_B, m^1_S) = (b, s^1_b)$ and $(m^2_B, m^2_S) = (b, s^2_b)$, this gives cost $(1 + \delta)c_S + (1 + \delta)K(1)$.

   (iii) Differently, if two pairs enter the same mechanism in order to save on bargaining costs, $(m^t_B, m^t_S) = (N(b), \Omega_S)$, cost is $(1 + \delta)(c_B^- + c_S)$.

   (iv) Mechanisms in which m^t_B, has more than two elements are less efficient since there the worker is more likely to work outside his area of expertise. Finally, if $m^t_S \subset \Omega_S$ there is a chance he will be asked to perform services for which no price has been negotiated (outside his job description).

(2) Suppose next that $(h_{wB}, h_{wS}) = (N(b), \Omega_S)$ with two workers investing in $N(b)$. In this case it is weakly efficient if the two businesses in $N(b)$ enter the same mechanism as the two workers. If $(m^t_B, m^t_S) = (N(b), \Omega_S)$, cost is $(1 + \delta)(c_B^- + c_S)$. Mechanisms in which $m^t_B \subset N(b)$ perform identically if $|m^t_B| = 1$, but the parties then have to incur bargaining costs. It is less efficient to use $m^t_B \supset N(b)$ because the worker risks having to work outside his area

of expertise. Finally, if $m^t_S \subset \Omega_S$ there is a chance he will be asked to perform services outside his job description.

(3) Suppose that $(h_{wB}, h_{wS}) = (\Omega_B, \Omega_S)$. In this case it is weakly efficient for two or more worker-business pairs to enter the same mechanism. If $(m^t_B, m^t_S) = (\Omega_B, \Omega_S)$, per-period cost is $(1 + \delta)(c_B + c_S)$. There is no gain from using $m^t_B \subset \Omega_B$ and it is less efficient to use $m^t_S \subset \Omega_S$, since there is a chance a worker will be asked to perform services outside his job description.

(4) Suppose that $(h_{wB}, h_{wS}) = (b, s^1_b)$ in which case the worker investing in b and the business (entrepreneur) b will enter the same mechanism in all equilibria. If $(m^t_B, m^t_S) = (\Omega_B, s^1_b)$ the costs are δc_B^+ and if $m^t_B = b$, they are δc_S^+ plus bargaining costs. For the reasons identified above, it is less efficient to have $m^t_B \subset \Omega_B$ and $m^t_S \supset s^1_b$.

(5) Suppose that two workers invest in $(h_{wB}, h_{wS}) = (N(b), s^1_b)$ and $(N(b), s^1_{b'})$ where $N(b) = \{b, b'\}$. In the first period, the worker investing in s^1_b and the business b will enter $(m^1_B, m^1_S) = (b, s^1_b)$, and the worker investing in $s^1_{b'}$ and the business b' will enter $(b', s^1_{b'})$. In the second period, they will swap partners and enter $(b', s^2_{b'})$ and (b, s^2_b), respectively. In both periods, only a single price will be negotiated and cost is $(1 + \delta)c_B^- + \delta(1 - p)c_S^+ + (1 + \delta)K(1)$.

(6) Suppose that $(h_{wB}, h_{wS}) = (\Omega_B, s)$ in which case it is weakly efficient for all (W/S) workers investing in s and all (W/S) businesses needing s to enter the same mechanism. If $(m^t_B, m^t_S) = (\Omega_B, s)$, cost is $(1 + \delta)c_B$. Mechanisms with $m^t_S \supset s$ and/or $m^t_B \subset \Omega_B$ are less efficient for the reasons identified above.

(1, iii) and (2), are eliminated by Assumption K, (3) is dominated by (6), and (4) is dominated by (1) or (6) by Assumption Δ. This leaves us with (1, i), (1, ii), (5), and (6).

QED

Comparing the different equilibria gives the following results for the case in which no entrepreneur operates more than one business:

-Human capital in the business domain is more likely to be narrow when it is more (less) costly to be a business generalist and less (more) costly to be a service generalist, when neighborhood effects in businesses are large (small), and when neighborhood effects in services are small (large).

-Employment is more efficient if needs change frequently and the horizon is long.

-Employee job descriptions are more likely to be broad when bargaining costs are more sub-additive in the sense that $K(2) + \delta K(1) – K(S)$ is larger.

-Markets are more efficient if businesses differ less than services.

-Local markets are more efficient when business neighborhood effects are large.

We now also see why it is necessary to define neighborhoods by the two properties summarized in Table 2.

COROLLARY: Only if $p > 0$ and $c_B > c_B^-$, can the production costs of $(h_{wB}, h_{wS}) = (N(b), s), (m^1_B, m^1_S) = (b, s^1_b)$ be lower than the lowest of $(h_{wB}, h_{wS}) = (b, \Omega_S), (m^t_B, m^t_S) = (b, \Omega_S)$, and $(h_{wB}, h_{wS}) = (\Omega_B, s), (m^t_B, m^t_S) = (\Omega_B, s)$.

Proof: The production costs of $(h_{wB}, h_{wS}) = (N(b), s)$ and $(m^1_B, m^1_S) = (b, s^1_b)$ are $(1 + \delta)c_B^- + \delta(1 – p)C_S$, while the two alternatives in the Corollary have costs of $(1 + \delta)c_S$ and $(1 + \delta)c_B$, respectively. So we need $(1 + \delta)c_B^- + \delta(1 – p)c_S^+ < \mathrm{Min}\{(1 + \delta)c_S, (1 + \delta)c_B\}$. If $p = 0$, Assumption Δ implies that the condition would require that $c_B^- < 0$, and if $c_B^- = c_B$, it would require that $c_S^+ < 0$. Since these cost values are ruled out by assumption, we are done.

QED

IV. Each Entrepreneur Operates Two Businesses

Since once again the entrepreneurs with no businesses are irrelevant, we can assume that $B = W = 2E$. If we order the businesses such that neighbors are adjacent, the $(B/2)$ columns of π have two adjacent 1's and zeros everywhere else.

In this multi-businesses setting, all the equilibria listed in Proposition 1 continue to exist. However, the new allocation supports some possibly more efficient equilibria as well, equilibria in which the multi-business property plays an essential role. To facilitate comparison with the single business case, we express cost on a two-period per-business basis.

PROPOSITION 2: When each entrepreneur operates a pair of businesses, two equilibria exist that are not replications of equilibria found in the single business case and can be more efficient than any that exists there. In both of these, the two businesses are neighbors and workers select human capital profiles to fit the likely needs of these businesses, such that a pair of workers will choose $(h_{wB}, h_{wS}) = (N(b), s^1_b)$ and $(N(b), s^1_{b'})$ where $N(b) = \{b, b'\}$. The two new equilibria are similar to local markets, only with employees:

- Multi-business Employment 1 *(broad job description):* $(m^1_B, m^1_S) = (N(b), \Omega_S)$, $(m^2_B, m^2_S) = (\emptyset, \emptyset)$ and expected cost is $(1 + \delta)c_B^- + \delta(1 - p)c_S^+ + K(S)$.

- Multi-business Employment 2 *(narrow job description):* $(m^1_B, m^1_S) = (N(b), \{s^1_b, s^1_{b'}\})$, $(m^2_B, m^2_S) = (\emptyset, \emptyset)$ if $\{s^2_b, s^2_{b'}\} = \{s^1_{b'}, s^1_b\}$, and $(m^2_B, m^2_S) = (b, s^2_b)$ if $s^2_b \neq s^1_{b'}$, and/or $(m^2_B, m^2_S) = (b', s^2_{b'})$ if $s^2_{b'} \neq s^1_b$. In this case, expected cost is $(1 + \delta)c_B^- + \delta(1 - p)c_S^+ + K(2) + \delta(1 - p)K(1)$.

Proof: We again go through all possible human capital profiles.

(1) Suppose first that workers divide into sets of two and select human capital profiles to fit the two firms in a neighborhood operated by a single entrepreneur. So the human capital profiles of a pair of such workers are $(N(b), s^1_b)$ and $(N(b), s^1_{b'})$ where $N(b) = \{b, b'\}$. Both of them enter the same mechanism as the entrepreneur who operates the two firms in $N(b)$.

(i) If the parties negotiate a price for any service, $(m^1_B, m^1_S) = (N(b), \Omega_S)$, $(m^2_B, m^2_S) = (\emptyset, \emptyset)$ and cost is $(1 + \delta)c_B^- + \delta(1 - p)c_S^+ + K(S)$.

(ii) If the entrepreneur and her workers negotiate a price for both first period services only, $(m^1_B, m^1_S) = (N(b), \{s^1_b, s^1_{b'}\})$, $(m^2_B, m^2_S) = (\emptyset, \emptyset)$ if $\{s^2_b, s^2_{b'}\} = \{s^1_b, s^1_{b'}\}$, and $(m^2_B, m^2_S) = (b, s^2_b)$ if $s^2_b \notin \{s^1_b, s^1_{b'}\}$, and/or $(m^2_B, m^2_S) = (b', s^2_{b'})$ if $s^2_{b'} \notin \{s^1_b, s^1_{b'}\}$. In this case, cost is $(1 + \delta)c_B^- + \delta(1 - p)c_S^+ + K(2) + \delta K(1)(1- p)$.

(iii) Since we are assuming that the entrepreneur cannot go to more than one mechanism per period, it is not possible for the workers to negotiate a price for one service each. If they could, $(m^1{}_B, m^1{}_S) = (N(b), s^1{}_b), (N(b), s^1{}_{b'})$ and $(m^2{}_B, m^2{}_S) = (\varnothing, \varnothing)$ with probability p, while $(m^2{}_B, m^2{}_S) = (b, s^2{}_b)$ if $s^2{}_b \neq s^1{}_{b'}$ and $(m^2{}_B, m^2{}_S) = (b', s^2{}_{b'})$ if $s^2{}_{b'} \neq s^1{}_b$. This would dominate the "Local market" in the single business case since cost is $(1 + \delta)c_B{}^- + \delta(1 - p)c_S{}^+ + (1 + [1 - p]\delta)K(1)$.

(iv) Differently, if two triples enter the same mechanism in order to save on bargaining costs, $(m^t{}_B, m^t{}_S) = (N(b) \cup N(b''), \Omega_S)$, cost is $(1 + \delta)(c_B{}^- + c_B{}^+)/2 + \delta(1 - p)c_S{}^+$. (Mechanisms in which $m^t{}_B$, has more than four elements are less efficient since there the workers are more likely to work outside their areas of expertise.)

(2) Equilibria in which the human capital profiles are confined to a single business, such as $(h_{wB}, h_{wS}) = (b, s^1{}_b)$ or (b, Ω_S), are no more efficient than the corresponding equilibria in the single business case.

(3) Similarly with human capital profiles covering all businesses, for example $(h_{wB}, h_{wS}) = (\Omega_B, s^1{}_b)$ or (Ω_B, Ω_S),

Since Assumption K eliminates case (1, iv), we are left with (1, i) and (1, ii).

QED

By combining Propositions 1 and 2:

-Workers can acquire narrower human capital if they work in multi-business firms (multi-business firms can hire workers with narrower human capital)

-Multi-business firms can hire dedicated employees to perform services that single business firms have to get from the market or employees with broader job descriptions.

V. Entrepreneurs Choose Between Operating One or Two Businesses

We here assume that the social planner initially allocates one business to each of B entrepreneurs, exactly as in Section III. However, before s^1 is realized, each of the entrepreneurs has the option of buying (the right to operate) one more business from another entrepreneur.

So the sequence of events is as follows:

1. A social planner distributes one businesses to each of B entrepreneurs.
2. Each entrepreneur may buy a business from any other entrepreneur. Prices are such that trades will take place iff they are efficient in terms of net surplus (value less production and bargaining costs). The trades are publicly observed and the set of possible post-trade distributions is Π'.
3. Business needs for period 1, s^1, are realized and publicly observed.
4. Workers choose their human capital profiles $\Pi' \times S^1 \rightarrow H_B \times H_S$ and these are publicly observed.
5. Workers and businesses select mechanisms for period 1: $\Pi' \times (H_B \times H_S)^W \times S^1 \rightarrow \{0, 1\}^B \times \{0, 1\}^S$, and are randomly matched within each. All workers perform the service needed by the business with which they is matched.
6. Business needs for period 2, s^2, are realized and publicly observed.
7. Workers and businesses select mechanisms for period 2: $\Pi' \times (H_B \times H_S)^W \times S^2 \rightarrow \{0, 1\}^B \times \{0, 1\}^S$, and are randomly matched within each. All workers perform the service needed by the business with which they is matched.
8. All payoffs, net of any bargaining and production costs, are distributed.

PROPOSITION 3: In the most efficient equilibria, any jointly operated businesses will be neighbors. Furthermore, entrepreneurs will operate two businesses in equilibrium iff

$$(1 + \delta)c_B^- + \delta(1 - p)c_S^+ < \text{Min}\{(1 + \delta)c_B, (1 + \delta)c_S\} \text{ and}$$

$$\text{Min}\{K(2) + \delta(1 - p)K(1), K(S)\} < (1 + \delta)K(1).$$

Proof: Any advantages to joint operation of two non-neighboring businesses also apply to two neighboring businesses, but there are extra advantages that only apply to neighbors. So the jointly operated pairs of businesses will be neighbors in all the most efficient equilibria.

The inequalities follows from Propositions 1 and 2.

QED

VI. Conclusion

The key implications of the model are consistent with several stylized facts and many findings in the empirical literature. We will here look at the two most important predictions:

1. The theory predicts that *firms will combine businesses with three simultaneous properties: (i) Their workers use similar business skills, (ii) they need the same services, and (iii) these needs change frequently.*

To relate this to real data, we make the natural assumption that c_B^-, c_B, δ, and p vary with the neighborhood and thus write them as c_b^-, c_b, δ_b and p_b, respectively. From Proposition 3, the attractiveness of multi-business firms vs. market specialists and generalist employees, is increasing in $(1 + \delta_b)(c_b - c_b^-) - \delta_b(1 - p_b)c_S^+$ and $(1 + \delta)(c_S - c_b^-) - \delta_b(1 - p_b)c_S^+$, respectively. Furthermore, the two businesses are more likely to merge or separate if c_b^-, δ_b, and p_b change. (Note also, from Table 2, that *b* and *b'* may start or stop being neighbors depending on the size of such a shift.)

It is easy to find dramatic examples of cross-industry combinations or separations that could be interpreted as being consistent with the theory: Think of the current convergence between the car- and tech industries brought about by the possibility of self-driving cars, or the break-up of the 1960s conglomerates made feasible by the growing specialization of management talent.

The literature contains at least two large sample studies reporting results that are consistent with the hypothesis. Atalay, Hortacsu, and Syverson (2014) find that manufacturing plants, after being acquired, change their ship-to locations and product mix to more closely resemble those of their new parent firm. Perhaps more directly, Montgomery and Hariharan (1991) show that firms are more likely to merge if they have similar advertising and R&D intensities.[10] Both sets of authors explicitly interpret their findings as being consistent with the view that mergers are driven by excess capacity of intangible assets ("skills").

2. The theory predicts that *larger firms will hire dedicated employees to perform services that smaller firms get from the market or employees with broader job descriptions.*

The alternative with market specialists is consistent with many stylized facts. At the level of individual workers: Only bigger firms hire corporate counsels, landlords only hire their own maintenance crews (plumbers, electricians, etc.) once they have a large number of properties, and firms transition from independent sales reps to their own sales force when sales are sufficiently strong.

Many of the same examples also illustrate the alternative with generalist employees: Corporate counsel working alone will perform many different types of legal tasks (a common rule of thumb is that firms should add one more in-house lawyer for every $200M in sales), small landlords hire super intendents to take care of all minor maintenance issues, and the marketing and sales functions are one and the same in the typical start-up.

It should, however, be clear, that none of these papers were written to test the specific theory put forward here. Producing such a paper is an important goal for future research.[11]

[10] A number of other studies provide less direct support. For example, consistent with the hypothesis that multi-business firms do better when the underlying business neighborhoods are more homogeneous, Montgomery and Wernerfelt (1988) and Wernerfelt and Montgomery (1988) find that margins are smaller in firms with more interindustry diversification.

[11] To perform these tests on anything but the smallest firms, it is probably best to think of the workers in the model as managers in charge of departments with a minimum efficient size.

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