Implications of Breach Remedy and Renegotiation Design for Innovation and Capacity

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Implications of Breach Remedy and Renegotiation Design for Innovation and Capacity

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Abstract

A manufacturer writes supply contracts with $N$ buyers. Then, the buyers invest in innovation, and the manufacturer builds capacity. Finally, demand is realized, and the firms renegotiate the supply contracts to achieve an efficient allocation of capacity among the buyers. The court remedy for breach of contract (specific performance versus expectation damages) affects how the firms share the gain from renegotiation, and hence how the firms make investments ex ante. The firms may also engage in renegotiation design, inserting simple clauses into the supply contract to shape the outcome of renegotiation. For example, when a buyer grants a financial “hostage” to the manufacturer or is charged a per diem penalty for delay in bargaining, the manufacturer captures the gain from renegotiation. “Tradable options,” which grant buyers the right to trade capacity without intervention from the manufacturer, return the gain from renegotiation to the buyers. This paper proves that, under surprisingly general conditions, the firms can coordinate their investments with the simplest of supply contracts (fixed-quantity contracts). This may require renegotiation design, and certainly requires that the firms understand the breach remedy and set their contract parameters accordingly.

Subject Classifications: renegotiation, bargaining, contract manufacturing, capacity pooling and allocation, renegotiation design

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1 Introduction

Firms often renegotiate supply contracts to adapt to their business environment, and rarely go to court to enforce the contracts (Cahn 2000, Serant and Ojo 2001). Nevertheless, the remedy that a court would impose for unilateral breach of contract influences the financial outcome of any renegotiation. In designing supply contracts and making investments, managers need to anticipate renegotiation, and understand recent trends in how courts enforce supply contracts. Proactive managers may employ various instruments (e.g., tradable options, financial hostages, penalties for delay in bargaining) to shape the renegotiation process. This paper proves that simple fixed-quantity contracts with renegotiation design can coordinate multiple firms’ investments in innovation and capacity.

Contract law stipulates payments and actions in the event that one party fails to execute the terms of a contract. Two standard remedies for breach of contract are expectation damages and specific performance. The expectation damages remedy is that a firm that breaches a contract must pay the injured firm the money required to achieve the same profit as if the contract were executed. The alternative is that the court enforces performance of the specific terms of the contract. The academic literature on supply contracts typically assumes specific performance. However, expectation damages has been routine in practice in the U.S. since the late 19th century, motivated by the argument of Oliver Wendell Holmes that the remedy of expectation damages is socially efficient, because a promisor will breach if and only if his gain in profit from breach exceeds the loss to the promisee. Specific performance may be enforced at the discretion of the court for unique goods, for which no substitute exists. Recently, legal scholars have argued for routine availability of the specific performance remedy because (1) expectation damages are difficult for the court to measure and therefore often undercompensatory (e.g., when a supplier breaches a contract to deliver goods, the buyer is not compensated for managerial effort expended in searching for an alternative); (2) efficient breach would occur anyway under specific performance through renegotiation; and (3) with multiple potential buyers, the interference tort is used to indirectly (and inefficiently) enforce specific performance (Varadarajan 2001).

Economists have demonstrated that simple contracts, with the potential for renegotiation, can induce efficient investments in a bilateral trading relationship. Tirole (1999) provides an excellent survey of this literature (not including the first papers to incorporate capacity investment (Wielenberg 2000, Golovachkina and Bradley 2003)). How does breach remedy (expectation dam-
ages vs. specific performance) influence relationship-specific investment under simple contracts? In Shavel (1980) a manufacturer and buyer contract to trade a single unit at a fixed price, and cannot subsequently renegotiate the contract. The manufacturer’s production cost is a random variable and the buyer makes an investment that increases her value for the item. Under expectation damages, the manufacturer breaches the contract when the realized production cost exceeds the buyer’s value for the item, and pays the buyer the value of the item. However, although expectation damages results in efficient breach, it causes inefficiency in investment. Specifically, the buyer spends too much to increase the value of the item, because she is guaranteed to receive the value of the item, even when the manufacturer does not produce and deliver the item. Rogerson (1984) incorporates renegotiation into the model, and finds that expectation damages results in overinvestment, and that specific performance Pareto-dominates expectation damages. Edlin and Reichelstein (1996) incorporate a variable production quantity and find that contracting for a fixed price and quantity induces optimal investment by the buyer, under expectation damages or specific performance. However, when the manufacturer also makes an investment that stochastically reduces variable production cost, the optimal investments can never be induced under expectation damages, but can often be induced under specific performance. In a similar model, Aghion et al. (1994) prove that the first best investments are achieved with specific performance and renegotiation design that shifts 100% of the gain from renegotiation to one party. Alessi and Staaf (1994) observe that many firms use relationships and reputation to enforce specific performance where courts will not. In summary, the economics literature suggests that the specific performance remedy is more effective than expectation damages for inducing optimal investment.¹

Partially in response to these academic insights, the specific performance remedy is becoming increasingly common in practice. The Official Comment to the U.S. Uniform Commercial Code in Section 2-716 indicates that “specific performance is increasingly applied...for output and requirements contracts involving a particular or peculiarly available source of supply.” For example, specific performance of a contract for supply of commercial steel will be enforced if the buyer is unable to buy steel on the market within a reasonable leadtime (Murray 1996).

In a model motivated by contract manufacturing practices in the biopharmaceutical and semiconductor industries, we address the following questions: Can simple fixed-quantity contracts induce optimal investments in innovation and capacity? How should firms design the renegotiation

¹An exception to this point of view is Edlin (1996) who shows that expectation damages is preferred when the investment is made by a single firm and is multidimensional.
process (jointly with the initial contracts) to induce the optimal investments? How do the answers to these questions change depending on the remedy that a court would impose for breach of contract? Our model is differentiated from the existing literature on renegotiation by the interaction of multiple buyers.

Currently, biopharmaceutical manufacturing capacity is tightly constrained and the lead time for building this capacity is three to five years (Molowa 2001). Therefore, before making important investments in drug development and clinical trials, pharmaceutical giants like Eli Lilly and small biotechnology firms are contracting for capacity with the manufacturer Lonza, typically three years in advance of production. Before signing a contract, for each drug that a prospective buyer has under development, Lonza estimates the likelihood of success in clinical trials, the market value, dosage requirements, manufacturing yield, and hence capacity requirements. Lonza employs various scientists to understand these prospects as well as the buyer does. At the time of contracting, the buyer’s capacity requirements are highly uncertain. Fortunately, Lonza’s manufacturing facilities can readily be adapted to produce various proteins. When one buyer has greater demand than anticipated and another buyer has low demand (or zero demand, in the case that a drug fails its clinical trials), Lonza renegotiates the contracts with the buyers to achieve a more profitable allocation of its fixed capacity (Thomas 2001). Lonza is the sole source of supply for many biotechnology firms, and developing an alternative supplier would require FDA approval, a process which takes at least two years. Hence the recent trend to enforce specific performance for a unique source of supply affects Lonza and its customers.

Similar renegotiation of supply contracts occurs in the semiconductor industry, where capacity lead times are also lengthy. In the late 1990s, manufacturer AMD contracted with Original Equipment Manufacturers in diverse market segments (cell phones, telecommunications equipment, appliances, personal computers) to provide capacity with a two-year lead time. By the time the capacity came on-line, market demand had shifted, and AMD and its customers renegotiated the contracts to achieve the most profitable allocation of scarce capacity (Doran 2001). According to a Dell executive (Painter 2004), successful buyers anticipate the potential for renegotiation and, before signing a long-term supply contract, identify the manufacturer’s other customers, the likely demand for capacity from each of these customers, and the total capacity that will be available.

In the biopharmaceutical and semiconductor industries, firms should contract before investing in innovation and capacity. If they wait to contract for production until capacity is built and demand realized, the buyer will benefit from the manufacturer’s sunk capacity investment and the
Specific Performance

Expectation Damages | Specific Performance
---|---
first best with simple fixed-quantity contracts | excess capacity, too little innovation

too little capacity, excess innovation | first best with simple fixed-quantity contracts
(assumes separable revenue functions & a linear bargaining outcome)

Figure 1: Efficacy of simple fixed-quantity contracts depends on breach remedy and the relative bargaining power of the firms. Penalizing buyers for delay in bargaining shifts bargaining power to the manufacturer. Tradable options shift bargaining power from the manufacturer to the buyers.

The manufacturer will benefit from the buyer’s sunk investment in innovation. This reduces incentives for investment and hence profit (Plambeck and Taylor 2005).

In our model, a manufacturer contracts to supply quantity $Q_i$ to each of $i = 1..N$ buyers. Then the manufacturer builds capacity, and each buyer invests in innovation (e.g., research and development, clinical trials, marketing) to influence its demand distribution. After demand is realized, the efficient allocation of capacity may differ from the contracted quantities. Then, the buyers and manufacturer renegotiate their contracts to achieve the efficient capacity allocation, and bargain over how to share the resulting gain in total profit.

The financial outcome of the renegotiation, and hence the firms’ incentives for investment, depend upon the breach remedy. Under the expectation damages remedy, the manufacturer can unilaterally choose to supply less than $Q_i$ to buyer $i$ and pay enough money to provide the same profit as buyer $i$ would have had with the full quantity $Q_i$. However, under the specific performance remedy, buyer $i$ can petition the court to compel the manufacturer to provide the full quantity $Q_i$ or pay an extreme penalty. Hence, under specific performance, the manufacturer is effectively constrained to build sufficient capacity to meet his obligations to all of the buyers.

The problem is to set the contractual quantities $\{Q_i\}_{i=1..N}$ so that, in Nash equilibrium, the manufacturer and buyers choose the investments that maximize total expected profit (first best investments). Implicitly, this expected profit is allocated through transfer payments at the time of contracting. For tractability, we assume the firms have common information; this assumption is not unreasonable for the biopharmaceutical and semiconductor examples described above. Figure 1 summarizes the conditions under which simple fixed-quantity contracts with renegotiation coordinate the system. If the manufacturer has the power to extract all the gain from renegotiation
and the remedy for breach is expectation damages, then the first best investments are implemented as a Nash equilibrium with properly designed fixed-quantity contracts. However, switching to the breach remedy of specific performance, we find that the first best is not implementable when innovation creates positive externalities and/or the cost of capacity is high. Then, under the best fixed-quantity contracts and associated Nash equilibrium, the firms invest in excess capacity but too little innovation.

For the case that the buyers extract some gain from renegotiation, we obtain symmetric but opposite results. Under expectation damages, the first best can never be implemented with fixed-quantity contracts. The firms invest too little in capacity and too much in innovation, given the limited capacity. In contrast, under specific performance, the first best can be implemented. Indeed, for a class of revenue functions and bargaining outcomes (described in §3.2), the first best can always be implemented by simply setting $Q_i$ equal to the expected value of the optimal allocation of capacity to buyer $i$ conditioned on the first best investments in capacity and innovation.

Through renegotiation design, the firms may select the top row or the bottom row of Figure 1. In the seminal paper on renegotiation design in a bilateral trading relationship, Aghion et al. (1994) model renegotiation as a noncooperative bargaining game with alternating offers. They impose a penalty on the buyer for delay in bargaining, and prove that when the penalty is sufficiently large, the manufacturer has all the bargaining power. That is, in the unique perfect equilibrium, the manufacturer makes a “take it or leave it” offer to the buyer, the buyer accepts immediately, and thus the manufacturer captures all the gain from renegotiation. Aghion et al. (1994) also provide examples of how such delay penalties are implemented in practice, in various industries. One means to implement the delay penalty is a financial hostage: the buyer grants a sum of money to the manufacturer to hold, without paying interest, until trade takes place. Alternatively, the contract may specify a per diem penalty when a specified deadline is not met. Either of these instruments enable the buyer and manufacturer to precommit themselves to give all the gain from renegotiation to the manufacturer. In the semiconductor industry, we have observed an alternative approach to renegotiation design that shifts bargaining power from a manufacturer to his customers. So-called “tradable capacity options” give buyers the legal right to trade capacity among themselves without interference from the manufacturer. (Semiconductor contract manufacturer TSMC sells tradable options.) In the setting with specific performance, we prove that tradable options increase the incentive for innovation and yield greater expected profit than simple fixed-quantity contracts, but not necessarily the first best.
The default remedy for breach is set exogenously in the general rules of contract within the jurisdiction that the firms operate. However, subject to jurisdictional limits, the firms may stipulate damages for breach in their contract, as an instrument for renegotiation design. Typically, in the United States, a contractual provision which specifies an amount of damages upon breach is enforceable if and only if it constitutes a reasonable estimate of the injury caused by breach (Goetz and Scott 1977, Schwartz 1990). In effect, the firms may specify expectation damages, but not specific performance. Hence the firms may select the upper left box in Figure 1, in order to induce the first best investments with fixed-quantity contracts. Specifying damages would, unfortunately, make the contract very complex. Therefore, adopting the specific performance breach remedy, as favored by the papers with bilateral investment surveyed above, is not necessarily beneficial.

The supply chain literature typically assumes that if contractual quantity commitments are enforceable, the remedy for breach is specific performance; Cachon and Lariviere (2001) label this “forced compliance” and use the term “voluntary compliance” to describe contracts where quantity commitments are not enforceable. In contrast, Tomlin (2003) and Erkoc and Wu (2005) consider a linear breach remedy. They analyze a model very similar to ours, but with \( N = 1 \) buyer and an exogenous demand distribution. The buyer and manufacturer contract for capacity \( Q \) plus a positive exercise price per unit ordered and fixed penalty per unit that the manufacturer fails to provide. Tomlin (2003) shows that the penalty leads the supplier to invest in greater capacity, and Erkoc and Wu (2005) show that a properly designed contract induces the first best capacity investment.

Several papers examine how firms invest in capacity/inventory and subsequently bargain cooperatively over its use. Anupindi et al. (2001) and Granot and Sosic (2003) consider a network of retailers with stochastic demands: Each chooses her inventory level; then demand is realized; and the retailers bargain cooperatively over the transshipment of excess inventory to meet excess demand. Van Mieghem (1999) and Chod and Rudi (2003) consider settings in which two firms trade capacity after receiving demand information. Anupindi and Bassok (1999) examine the effect of inventory pooling on the quantities ordered by two retailers. This research demonstrates that capacity trading can increase or decrease capacity investment. In contrast, Hartman et al. (2000), Müller et al. (2002), and Slikker et al. (2003) consider retailers that negotiate over pooling inventory ownership and the sharing of the expected profits. In these papers and ours, the firms have common information so the bargaining leads to an efficient allocation. Our paper is differentiated by the assumption that the demand distribution depends on the buyers’ investments in innovation.
An alternative is to allocate capacity through market mechanisms, which may be particularly relevant when the number of parties is large. Porteus and Whang (1991) and Kouvelis and Lariviere (2000) develop internal market mechanisms to allocate capacity efficiently within a single firm. These mechanisms depend on the existence of an independent “headquarters” that is willing to incur losses for the sake of maximizing system profit. Nonetheless, these papers are relevant in that they allow for investments in both capacity and demand stimulation. In contrast, Lee and Whang (2002), Wu et al. (2002), Dong and Durbin (2003) and Tunca and Mendelson (2004a, b) consider market mechanisms for allocating capacity across firms when the demand function is exogenous.

The paper is organized as follows. §2 presents the model, §3 presents the results, and §4 provides concluding remarks.

2 Model Formulation

Consider a contract manufacturer with $N$ buyers. Figure 2 depicts the sequence of events. Each buyer $i$ makes a transfer payment to the manufacturer for $Q_i$ units of capacity. (Our analysis will characterize the optimal $\{Q_i\}_{i=1..N}$ but not the transfer payments, which simply serve to divide up the total expected profit.) Then buyer $i$ chooses innovation $e_i \in [0,1]$ and incurs cost $g_i(e_i)$. We assume that the function $g_i(\cdot)$ is differentiable, increasing and strictly convex, with $g_i(0) = 0$, $g'_i(0) = 0$ and $g'_i(e_i) \to \infty$ as $e_i \to 1$. All functions described as increasing, convex or concave are weakly so, unless specified otherwise. Associated with each market state $\omega \in \Omega$ is a set of revenue functions $\{R_i(q;\omega) : q \geq 0\}_{i=1..N}$ (the maximum revenue that buyer $i$ can generate with $q$ units of capacity is $R_i(q;\omega)$). Because the buyer need not bring every unit of production to market, having more units of capacity available increases the revenue that can be generated: $R_i(q;\omega)$ is increasing in $q$. Further, we assume the functions $R_i(q;\omega)$ are differentiable and concave with $R_i(0;\omega) = 0$. The innovation vector $e = \{e_i\}_{i=1..N} \in [0,1]^N$ induces a probability measure $P_e$ on $\Omega$. $E_e[\cdot]$ denotes expectation with respect to the probability measure $P_e$. We assume that (the distribution of) each buyer’s revenue function is independent of the innovation investments of the other buyers. However, the results in the first half of the paper (through Theorem 3a) continue to hold when each revenue function depends on the entire innovation vector $e$; the proofs of these results extend without modification.

While the buyers are investing in innovation, the manufacturer invests in capacity $c$ at a cost of $k$ per unit; thus, in contrast to the buyers’ innovation investments, the outcome of the manufacturer’s investment is deterministic. After the firms invest in innovation and capacity,
Buyers invest in innovation e Production

Market state \( \omega \) realized

Renegotiation

Production

<table>
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<tr>
<th>Contracting</th>
<th>Buyers invest in innovation e</th>
<th>Market state ( \omega ) realized</th>
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<tr>
<td>Manufacturer invests in capacity ( c )</td>
<td>All firms observe ((c, \omega))</td>
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Figure 2: Sequence of events.

The market state \( \omega \) and the consequent revenue functions \( \{R_i(\cdot;\omega)\}_{i=1..N} \) are realized. All firms observe the buyers’ revenue functions and the manufacturer’s capacity. Let \( \{q^*_i(c;\omega)\}_{i=1..N} \) denote an efficient allocation of capacity among the buyers, obtained by solving

\[
\max \sum_{i=1}^N R_i(q_i;\omega) \\
\text{s.t.} \sum_{i=1}^N q_i \leq c \text{ and } q_i \geq 0.
\]

If \( q^*_i(c;\omega) \neq Q_i \) for some buyer \( i \), the firms have an opportunity to cooperate to reallocate capacity and thus increase revenue. The optimal capacity allocation depends on the realized market state \( \omega \), so this occurs with positive probability:

\[
P_e(q^*_i(c;\omega) \neq Q_i \text{ for some } i \in 1..N) > 0. \tag{1}
\]

The optimal quantity \( q^*_i(c;\omega) \) to deliver to buyer \( i \) may be less or more than her contractual quantity \( Q_i \). If buyer \( i \) refuses to cooperate, then under specific performance the manufacturer must deliver \( Q_i \), but under expectation damages the manufacturer may deliver strictly less than \( Q_i \) and pay the buyer for lost revenue. Hence buyer \( i \) is guaranteed revenue of \( R_i(Q_i;\omega) \), but may achieve more by cooperating. Because the firms have common information, they will renegotiate the fixed-quantity contracts to achieve the maximum total revenue, and every firm will benefit (weakly) from renegotiation.

The bargaining outcome \( \{x_i(Q_1,..Q_N,c;\omega)\}_{i=1..N} \) specifies the share of the gain from renegotiation for each buyer, so total revenue for buyer \( i \) after renegotiation is \( R_i(Q_i;\omega) + x_i(Q_1,..Q_N,c;\omega) \). The difference in total revenue from allocating capacity optimally, rather than according to the contracts \( \{Q_i\}_{i=1..N} \), is

\[
\sum_{i=1}^N R_i(q^*_i(c;\omega);\omega) - \sum_{i=1}^N R_i(Q_i;\omega). \tag{2}
\]
Because the bargaining leads to the optimal capacity allocation, the manufacturer receives

$$\sum_{i=1}^{N} R_i(q^*_i(c; \omega); \omega) - \sum_{i=1}^{N} R_i(Q_i; \omega) - \sum_{i=1}^{N} x_i(Q_1,..Q_N, c; \omega)$$

in the renegotiation.  \(\{x_i(Q_1,..Q_N, c; \omega)\}_{i=1..N}\) is implemented through a second set of transfer payments between the manufacturer and buyers in the renegotiation stage. The second transfer payments serve to divide up the increase in total profit from efficient capacity allocation, relative to the original contract.)

We assume that when the firms contract and invest, they know \(\{x_i(Q_1,..Q_N, c; \omega)\}_{i=1..N}\) but not \(\omega\). This formulation allows for uncertainty regarding the bargaining outcome, as two states \(\omega_1\) and \(\omega_2\) may result in identical revenue functions and differ only in terms of how the bargaining will proceed. Given a set of supply contracts \(\{Q_i\}_{i=1..N}\), if the manufacturer anticipates innovation \(e\), he chooses capacity to

$$\max_{c \geq 0} \left\{ E_e[\sum_{i=1}^{N} R_i(q^*_i(c))] - \sum_{i=1}^{N} R_i(Q_i) - \sum_{i=1}^{N} x_i(Q_1,..Q_N, c) \right\} - kc. \quad (4)$$

If buyer \(i\) anticipates capacity \(c\) and innovation by the other buyers of \(e_i\), he chooses his investment in innovation to

$$\max_{e_i \in [0,1]} \left\{ E_{e_i,e_{-i}}[R_i(Q_i) + x_i(Q_1,..Q_N, c)] - g_i(e_i) \right\}.$$

We assume \(E_e[\sum_{i=1}^{N} R_i(q^*_i(c))]\) is strictly jointly concave and continuously differentiable in \(e\) and \(c\), so there exists a unique optimal capacity and innovation vector that maximize total expected profit:

$$(e^*, c^*) = \arg \max_{e \in [0,1]^N, c \geq 0} \left\{ E_e[\sum_{i=1}^{N} R_i(q^*_i(c))] - \sum_{i=1}^{N} g_i(e_i) - kc \right\}.$$

We assume the optimal capacity and innovation are strictly positive: \(c^* > 0\) and \(e_i^* > 0\) for \(i = 1..N\). For each buyer in isolation, \(E_e[R_i(Q)]\) is differentiable and concave as a function of \(e_i\). It is natural to assume that capacity and innovation are complements: for any two random variables \(q'(\omega)\) and \(q(\omega)\) defined on \(\Omega\) that satisfy \(P_e(q'(\omega) \geq q(\omega)) = 1\) and \(P_e(q'(\omega) > q(\omega)) > 0\),

$$\frac{\partial}{\partial e_i} E_e[R_i(q'(\omega); \omega)] > \frac{\partial}{\partial e_i} E_e[R_i(q(\omega); \omega)] \text{ for } i \in 1..N. \quad (5)$$
Because the buyers compete for capacity, it is natural to assume

$$\frac{\partial}{\partial e_i} E_e[\sum_{j \neq i} R_j(q_j^*(c))] \leq 0 \text{ for } c \leq c^* \text{ and } i \in 1..N. \quad (6)$$

Assumption (6) and strict inequality in (5) are needed only for Theorem 1. We also require notation for the system-wide optimal capacity contingent on the buyers’ innovation:

$$c^*(e) = \arg \max_{c \geq 0} \left\{ E_e \left[ \sum_{i=1}^{N} R_i(q_i^*(c)) \right] - \sum_{i=1}^{N} g_i(e_i) - kc \right\},$$

and the system-wide optimal innovation for buyer $i$ contingent on the capacity and other buyers’ innovation:

$$e_i^*(c, e_{-i}) = \arg \max_{e_i \in [0,1]} \left\{ E_{e, e_{-i}} \left[ \sum_{i=1}^{N} R_i(q_i^*(c)) \right] - \sum_{i=1}^{N} g_i(e_i) - kc \right\}.$$

The problem for the manufacturer and buyers is to choose quantities $\{Q_i\}_{i=1..N}$ that implement the first best investments $(e^*, c^*)$ as a Nash equilibrium, if possible. Otherwise, the firms should choose the second best quantities $\{\tilde{Q}_i\}_{i=1..N}$ and associated Nash equilibrium $(\tilde{e}, \tilde{c})$ that maximize total expected profit. The firms can achieve any division of this expected profit through the transfer payments at the time of contracting; our results do not depend on how the firms divide up the expected profit.

Although the firms do not go to court, the remedy the court would impose for breach of contract shapes the bargaining outcome (and hence the Nash equilibrium investments and optimal quantities). For each firm, the bargaining outcome must yield weakly greater profit than the firm can obtain by going to court. In the specific performance remedy, the court would enforce immediate delivery of $Q_i$ to each buyer $i$, or, if the manufacturer could not deliver, would impose a very large financial penalty. This penalty is sufficiently large to force a manufacturer facing the specific performance remedy to build sufficient capacity to meet his obligations:

$$c \geq \sum_{i=1}^{N} Q_i \quad (7)$$

is added to (4). In contrast, under the expectation damages remedy, the manufacturer is not constrained to choose capacity $c \geq \sum_{i=1}^{N} Q_i$. The manufacturer can unilaterally choose to deliver $q < Q_i$ to buyer $i$ and pay

$$R_i(Q_i; \omega) - R_i(q; \omega). \quad (8)$$
A court applying the specific performance remedy would not evaluate (8), meaning that buyer \( i \) could force the manufacturer to deliver \( Q_i \) even if he had no use for it: \( R_i(Q_i; \omega) = 0 \). Under specific performance or expectation damages, each buyer \( i \) is guaranteed revenue of \( R_i(Q_i) \), so the bargaining outcome must satisfy

\[
x_i(Q_1,..Q_N, c; \omega) \geq 0 \text{ for } i = 1..N.
\] (9)

The manufacturer must also benefit from the renegotiation, so under specific performance

\[
\sum_{i=1}^{N} x_i(Q_1,..Q_N, c; \omega) \leq \sum_{i=1}^{N} R_i(q^*_i(c; \omega); \omega) - \sum_{i=1}^{N} R_i(Q_i; \omega).
\]

Under expectation damages, the manufacturer can unilaterally achieve revenue of

\[
\max_{q_i \leq Q_i, \sum q_i \leq c} \sum_{i=1}^{N} R_i(q_i; \omega) - \sum_{i=1}^{N} R_i(Q_i; \omega).
\] (10)

(Note that (10) is zero if \( c \geq \sum_{i=1}^{N} Q_i \). However, (10), (2) and (3) may be negative if \( c < \sum_{i=1}^{N} Q_i \), meaning that the manufacturer pays a penalty for building insufficient capacity to meet his obligations.) Therefore, under expectation damages, the manufacturer will accept the bargaining outcome only if

\[
\sum_{i=1}^{N} x_i(Q_1,..Q_N, c; \omega) \leq \sum_{i=1}^{N} R_i(q^*_i(c; \omega); \omega) - \max_{q_i \leq Q_i, \sum q_i \leq c} \sum_{i=1}^{N} R_i(q_i; \omega).
\] (11)

Owen (1995) reviews the various established methods for computing a specific bargaining outcome (e.g., Shapley value, kernel). We impose minimal assumptions about the bargaining outcome, rather than adopt one of these methods, in order to generate robust insights.

The following two technical definitions are needed for Theorems 1 and 4. First, as defined in Edlin and Reichelstein (1996), with a slight abuse of our notation, the revenue functions are separable if

\[
R_i(q, e_i, \theta) = R^{a}_i(e_i)q + R^{b}_i(q, \theta) + R^{c}_i(e_i, \theta) \text{ for } i = 1..N,
\] (12)

where \( \theta \) is a random variable with continuous, bounded support, the functions \( R^{a}_i, R^{b}_i \) and \( R^{c}_i \) are twice-differentiable, \( R^{a}_i(e_i) > 0 \), and \( \frac{\partial}{\partial q} R^{b}_i(q, \theta) > 0 \). One may interpret a separable revenue
function (12) as an approximation to the “true” revenue function. Second, the probability measures $P_e$ are absolutely continuous if $P_e(A) = 0$ for some $e \in [0, 1]^N$ and $A \in \Omega$ implies that $P_{e'}(A) = 0$ for all $e' \in [0, 1]^N$.

For ease of exposition, we focus on capacity cost and suppress any ex post production cost. In the biopharmaceutical industry, the buyer incurs a significant cost for procurement and delivery of raw materials to the manufacturer (Thomas 2001). In general, the firms may specify in their contract that the buyer bears any ex post cost of production. Our results extend to this setting as follows. Let $p_i(q)$ denote the convex total production cost as a function of the production quantity $q$ for buyer $i$. The contribution for buyer $i$ from capacity $Q$ is

$$C_i(Q; \omega) = \max_{q \in [0, Q]} \{R_i(q; \omega) - p_i(q)\}.$$

All propositions and theorems hold with substitution of contribution $C_i(\cdot; \omega)$ for revenue $R_i(\cdot; \omega)$.

For brevity, in the remainder of the paper we will suppress the notation $\omega$ as much as possible, and when the set of contracts is fixed, we will write $x_i(c)$ for $x_i(Q_1, \ldots, Q_N, c; \omega)$.

3 Results

With only one buyer, under the specific performance or expectation damages remedy, the contract design problem is straightforward. With $Q_1 = c^*$, the first best investments $(e_1^*, c^*)$ constitute a Nash equilibrium. The remainder of the paper assumes $N \geq 2$ buyers. §3.1 considers the expectation damages breach remedy, and §3.2 considers specific performance. §3.3 investigates renegotiation design through tradable options.

3.1 Expectation Damages

This section establishes that the first best cannot be achieved under expectation damages, except in the extreme case that the manufacturer captures all the gain from renegotiation. The firms can implement this extreme case by the renegotiation design described in §1. We first argue that, under expectation damages, it is reasonable to focus on bargaining outcomes in which each buyer’s share increases with the manufacturer’s capacity investment. Furthermore, commonly-used bargaining outcomes exhibit this monotonicity. Then, we show that this monotonicity causes underinvestment in capacity and/or overinvestment in innovation.

It is reasonable to assume that a player’s bargaining outcome is some increasing function of the value that he can add by cooperating with each coalition of the other players. Under expectation
damages, the value added by buyer $j$ to a coalition consisting of the manufacturer and a subset $S \subset \{1..N\}\backslash j$ of the other buyers is:

$$v_j^S(Q_1..Q_N, c; \omega) = \max \sum_{i=1}^N R_i(q_i; \omega) - \max \sum_{i=1}^N R_i(q_i; \omega) \quad (13)$$

$$q_i \leq Q_i \text{ for } i \notin S \cup j \quad \sum_{i=1}^N q_i \leq c \quad q_i \geq 0 \text{ for } i = 1..N$$

we refer to the value buyer $j$ adds to the coalition of all the other players $v_j^{\{1..N\}\backslash j}$ as his added value, and focus on coalitions that include the manufacturer because buyers cannot trade capacity without the manufacturer’s cooperation. That $x_i$ increases with $v_i^S$ is a common assumption in cooperative game theory. Indeed, the best-known normative bargaining outcome in cooperative game theory, the Shapley value, is a weighted average of the value that a buyer adds to every coalition:

$$x_i = \sum_{S \subset \{1..N\}\backslash i} \left[ \frac{(N - |S| - 1)(|S| + 1)!}{(N + 1)!} v_i^S \right] \quad \text{for } i = 1..N \quad (14)$$

(Shapley 1953). The Shapley value is used to analyze inventory pooling among two retailers and their common supplier in Bartholdi and Kemahlıoğlu-Ziya (2003) and inventory cost sharing in Hartman and Dror (1996). Other researchers assume that each buyer $i$ receives a fraction $\alpha_i \in [0, 1]$ of his added value:

$$x_i = \alpha_i v_i^{\{1..N\}\backslash i}$$

see Plambeck and Taylor (2004a) and references therein; buyer $i$ cannot not take more than $v_i^{\{1..N\}\backslash i}$ or the manufacturer and other buyers would benefit from excluding him (Brandenburger and Nalebuff 1996). A useful observation is that under expectation damages, the value added by any buyer $j$ to any coalition increases in the manufacturer’s capacity.

**Lemma 1** Suppose that the buyers’ revenue functions are strictly concave. Under expectation damages, for any $S \subset \{1..N\}\backslash j$

$$\frac{\partial v_j^S(Q_1..Q_N, c; \omega)}{\partial c} \geq 0 \quad (15)$$
and if $q^*_j(c; \omega) > Q_j$ then

$$\frac{\partial v^x_j(Q_1..Q_N, c; \omega)}{\partial c} > 0. \quad (16)$$

All proofs are in the appendix.

According to Young (1985) the principle of fair division implies strong monotonicity: if the value added by a player to every possible coalition increases, then the bargaining outcome for that player increases. We add that if the increase in value added is strict, then the increase in bargaining outcome is also strict. Then, Lemma 1 guarantees that under expectation damages

$$\frac{\partial x_i(Q_1..Q_N, c; \omega)}{\partial c} \geq 0, \text{ where the inequality is strict if } q^*_i(c; \omega) > Q_i. \quad (17)$$

In particular, Lemma 1 guarantees that the Shapley value and the Nash bargaining solution satisfy (17). We believe that (17) is a plausible condition. Our first theorem establishes that this strict monotonicity results in underinvestment in capacity or overinvestment in innovation.

**Theorem 1** Suppose that $\{R_i\}_{i=1..N}$ are separable or that $P_e$ is absolutely continuous. Also assume the bargaining outcome satisfies (17). Then the first best investments cannot be implemented as a Nash equilibrium under simple fixed-quantity contracts. Any contracts $\{\tilde{Q}_i\}_{i=1..N}$ and corresponding Nash equilibrium $(\tilde{e}, \tilde{c})$ exhibit underinvestment in capacity or overinvestment in innovation, i.e., at least one of the following conditions must be satisfied: (i) $\tilde{c} = 0$, (ii) $\tilde{c} < c^*(\tilde{e})$, (iii) $e_i > e^*_i(\tilde{e}, \tilde{e}_{-i})$ for some $i \in 1..N$.

The intuition behind the proof of Theorem 1 is that when the manufacturer builds a marginal unit of capacity, the value added by each player to every possible coalition is increased, which means that the buyers obtain more revenue in the renegotiation. Anticipating that the buyers will share the revenue generated by marginal capacity, the manufacturer underinvests. In general, $\tilde{c}$ might be larger or smaller than $c^*$ and $\tilde{e}$ might have components larger and smaller than $e^*$, but we can rule out the case that both $\tilde{c} > c^*$ and $\tilde{e} > e^*$.

Now suppose that, by renegotiation design, the manufacturer will capture all the gain from renegotiation. Then the manufacturer has optimal incentives for capacity investment. Each buyer, anticipating zero gain from renegotiation, chooses innovation to maximize her own expected profit given fixed quantity $Q_i$. The first best is implemented by tuning the quantity parameters $\{Q_i\}_{i=1..N}$
to give the buyers optimal incentives for innovation. Let $\hat{Q}_i$ denote the unique solution to

$$
\frac{\partial}{\partial e_i} E_{e^*}[R_i(\hat{Q}_i)] = \frac{\partial}{\partial e_i} E_{e^*}[\sum_{i=1}^N R_i(q_i^*(c^*))].
$$

(18)

**Theorem 2** Suppose that the manufacturer captures all the gain from renegotiation: $x_i = 0$ for $i = 1..N$. Then the first best investments are implemented as a Nash equilibrium with contracts $\{\hat{Q}_i\}_{i=1..N}$.

A manufacturer may be dominant in bargaining due to exogenous conditions in the business environment, and capture most of the gain from renegotiation. Then, even without explicit renegotiation design, $x_i$ will be small, and Theorem 2 suggests that fixed-quantity contracts will be effective if not precisely optimal.

### 3.2 Specific Performance

Our results for the setting with specific performance are opposite to those for expectation damages. Our first result is that fixed-quantity contracts may fail to achieve the first best when the manufacturer is dominant. This follows from the requirement that under specific performance the manufacturer build sufficient capacity to meet his obligations (7).

**Theorem 3** (a) Suppose that the manufacturer captures all the gain from renegotiation: $x_i = 0$ for $i = 1..N$. The first best investments are implemented as a Nash equilibrium with the contracts $\{\hat{Q}_i\}_{i=1..N}$ defined in (18) if and only if

$$
\sum_{i=1}^N \hat{Q}_i \leq c^*.
$$

(19)

If (19) is not satisfied, then under specific performance, the first best cannot be achieved with fixed-quantity contracts. (b) Any second best set of contracts $\{\tilde{Q}_i\}_{i=1..N}$ and associated Nash equilibrium $(\tilde{e}, \tilde{c})$ is characterized by overinvestment in capacity: $\tilde{c} = \sum_{i=1}^N \tilde{Q}_i \geq c^*(\tilde{e})$ and underinvestment in innovation: $\tilde{e}_i \leq e_i^*(\tilde{c}, \tilde{e}_{-i})$ for $i = 1..N$.

Although we have assumed that each buyer’s revenue function is independent of the innovation investments of the other buyers, all of our preceding results through Theorem 3a continue to hold when each revenue function depends on the entire innovation vector $e$. When such dependency is allowed, the critical condition (19) is violated (fixed-quantity contracts fail to implement the first best) when investment in innovation generates sufficient positive externalities for other buyers.
Recall that capacity and innovation are assumed to be complements for each buyer $i$ (5), so in a setting with positive externalities $\hat{Q}_i$ must be relatively large to induce the first best innovation investment from buyer $i$. Even without positive externalities, (19) is violated when capacity is expensive, so that $c^*$ is small and pooling increases the marginal value of innovation.

When (19) is violated, the firms face a trade-off: On the one hand, providing incentives for innovation requires relatively large quantity commitments. On the other hand, large quantity commitments, enforced by the coercive specific performance remedy, compel the manufacturer to overinvest in capacity. The second best contract reflects this trade-off: the contract leads to overinvestment in capacity and underinvestment in innovation. When (19) is satisfied (fixed-quantity contracts implement the first best) the optimal contractual quantities are the same under specific performance and expectation damages. Finally, note that $\tilde{c}$ might be larger or smaller than $c^*$ and $\tilde{e}$ might have components larger and smaller than $e^*$, but we can rule out the case that both $\tilde{c} < c^*$ and $\tilde{e} > e^*$.

Now suppose the buyers have some bargaining power vis-à-vis the manufacturer. In contrast to Theorem 1 for the previous case with expectation damages, the first best can be achieved with specific performance for some plausible bargaining outcomes. The problem in the setting with expectation damages is that the manufacturer underinvests in capacity, because the buyers will capture some gain from each marginal unit of capacity that he builds. Specific performance can eliminate this problem by either of the following two mechanisms. First, specific performance imposes a lower bound on the manufacturer's capacity investment: $c \geq \sum_{i=1}^{N} Q_i$. Second, specific performance breaks Lemma 1. Under specific performance, the value added by buyer $j$ to a coalition consisting of the manufacturer and a subset $S \subset \{1..N\}\setminus j$ of the other buyers is:

$$v_j^S(Q_1..Q_N, c; \omega) = \max \sum_{i=1}^{N} R_i(q_i; \omega) - \max \sum_{i=1}^{N} R_i(q_i; \omega)$$

subject to:

- $q_i = Q_i$ for $i \notin S \cup j$ and $q_i = Q_i$ for $i \notin S$
- $\sum_{i=1}^{N} q_i \leq c$
- $\sum_{i=1}^{N} q_i \leq c$
- $q_i \geq 0$ for $i = 1..N$
- $q_i \geq 0$ for $i = 1..N$.

(The reader is advised to compare (20) to (13); $q_i = Q_i$ replaces $q_i \leq Q_i$.) Thus Lemma 1 is violated because by adding capacity, the manufacturer decreases $v_j^S$ when buyer $j$ is a donor of capacity to the coalition $S$. To see this, suppose that $N = 2$ and the buyers have independent and identical
revenue functions:

\[ R_i(q) = \begin{cases} 
3\ln(1 + q) & \text{with probability } e_i \\
\ln(1 + q) & \text{with probability } 1 - e_i
\end{cases} \]

for \( i = 1, 2 \). Let \( c = 2 \) and \( (Q_1, Q_2) = (1, 1) \). Then with probability \((1 - e_1) \cdot e_2\) an efficient capacity allocation is \( q_1^*(2) = 0, q_2^*(2) = 2 \), and from (20), the value that buyer 1 adds to the coalition of the manufacturer and buyer 2 satisfies \( \frac{\partial v}{\partial c} = -\frac{1}{2} \). With specific performance, in contrast to expectation damages, the manufacturer might overinvest in capacity in order to reduce the buyers’ gain from renegotiation.

We say that the bargaining outcome for buyer \( i \) is \textit{linear in the total gain from reallocating capacity} if

\[ x_i(Q_1, ..., Q_N, c; \theta) = \alpha_i \left[ \max_{\sum_{i=1}^N q_i \leq c} \sum_{i=1}^N R_i(q_i, e_i, \theta) - \sum_{i=1}^N R_i(Q_i, e_i, \theta) \right] \text{ for some } \alpha_i \in [0, 1]. \]  

(21)

The Nash bargaining solution is (21) with \( \alpha_i = \frac{1}{N+1} \) for \( i = 1..N \). However, this bargaining outcome may give a buyer more than his added value. Therefore a more realistic alternative is that a buyer receives a fraction of his added value. If

\[ x_i(Q_1, ..., Q_N, c; \theta) = \alpha_i v_i^{[1..N] \setminus i} \text{ for some } \alpha_i \in [0, 1] \]

(22)

we say that the bargaining outcome for buyer \( i \) is \textit{linear in his added value}.

**Theorem 4** Suppose that the revenue functions are separable and strictly concave in \( e \). If the bargaining outcome for each buyer is linear in the total gain from reallocating capacity (21) then the first best investments are implemented with fixed-quantity contracts

\[ Q_i^* = E_{e^*}[q_i^*(c^*)] \text{ for } i = 1..N. \]  

(23)

If the bargaining outcome for each buyer is linear in his added value (22) or, for \( N = 2 \), the Shapley value (14), and

\[ c^* \in \arg \max_{c \geq c^*} \left\{ E_{e^*}[\sum_{i=1}^N R_i(q_i^*(c)) - \sum_{i=1}^N x_i(Q_i^*, ..Q_N^*, c)] - kc \right\}, \]

(24)

then the first best investments are implemented with fixed-quantity contracts (23).

The class of bargaining outcomes assumed in Theorem 4 is broad. If the firms interact repeatedly, they may engage in renegotiation design by adopting a relational contract to coordinate
Edlin and Reichelstein (1996) point out that for any revenue function, one may construct a separable revenue function (12) by a second-order approximation to the “true” revenue function. In this sense, Theorem 4 can be thought of as providing an approximate solution to the problem of designing fixed-quantity contracts that will induce the maximum total expected profit. The solution is remarkably simple: each buyer commits to purchase a quantity equal to her expected optimal quantity allocation under the first best innovation and capacity. The essential insight for proving Theorem 4 is that with separable revenue functions (12), when buyer $i$ has purchased capacity $Q_i = E[q_i^*(c^*, e^*, \theta)]$, 

$$
\frac{\partial}{\partial e_i} E[R_i(Q_i, e_i, \theta)] = \frac{\partial}{\partial e_i} E \left[ \pi((e_i, e_i^*), \theta, c^*) \right],
$$

where $\pi$ denotes total system expected profit with the optimal capacity allocation. That is, buyer $i$’s marginal revenue from innovation in the absence of renegotiation equals the marginal revenue for the system with renegotiation. Then linear structure in the bargaining outcome, which is present in (14), (21) and (22), guarantees that buyer $i$ has optimal incentives for innovation.

Under specific performance, the manufacturer is forced to build at least the optimal capacity $c \geq \sum_{i=1}^{N} E[q_i^*(c^*, e^*, \theta)] = c^*$. We were able to construct a pathological example, with a bargaining outcome satisfying (22) and contracts (23), in which the manufacturer builds strictly more capacity than $c^*$ in order to reduce the buyer’s gain from renegotiation. In most cases, (24) is satisfied, which means that the manufacturer will not speculatively build more capacity than is optimal for the system as a whole.

Of our four theorems, Theorem 4 requires the strongest assumptions, and hence should be interpreted with the most caution. In particular, judicial policy-makers should not conclude from Theorem 4 that routinizing the specific performance remedy will necessarily yield efficient investments in practice. Separability is a strong assumption, and the results are sensitive to this assumption. Plambeck and Taylor (2004a) show, in a setting where the bargaining outcome satisfies the conditions of Theorem 4 but the revenue functions are not separable, that the first best need not be achieved and that the loss in profit relative to the first best can be substantial. That is, the quality of the “approximate solution” (23) can be poor. A second concern is that some members of the class of bargaining outcomes assumed in Theorem 4 are not in the core, and therefore will not
occur in practice. (A bargaining outcome is in the core if no subset of firms, working together and excluding the other firms, can obtain a payoff that exceeds the sum of its members’ current payoffs.) With the linear bargaining outcomes (21) and (22), a sufficient condition for the bargaining outcome to be in the core is that the $\alpha_i$’s be sufficiently small. However, with large $\alpha_i$’s or the Shapley value, the bargaining outcome may lie outside the core.

3.3 Tradable Options

Renegotiation design can be employed to transfer the bargaining power to the manufacturer via financial hostages or delay penalties. When the remedy for breach is expectation damages, this is a powerful tool: it allows simple fixed-quantity contracts to achieve the first best (Theorem 2). However, allocating all the bargaining power to the manufacturer may not be attractive under specific performance (Theorem 3). This section demonstrates that when the manufacturer is exogenously powerful, it may be attractive to endogenously transfer bargaining power to the buyers by modifying the fixed-quantity contracts so as to give buyers the right to trade capacity and retain all the gains from trade. Such contracts are commonly called “tradable options,” and are used by the giant semiconductor contract manufacturer TSMC (Economist 1996). We focus on the case where the manufacturer is dominant because this lends clarity as to how tradable options transfer bargaining power to the buyers and because in this case transferring bargaining power to the buyers is attractive.

To demonstrate the value of tradable options, we will consider a simplified example with two buyers. When there are two firms, a common view is that the firms will split the gains from cooperation 50:50 (Nash 1953, Rubinstein 1982, Kagel and Roth 1995). Thus, assuming $N = 2$ facilitates identifying a natural bargaining outcome.

The manufacturer sells tradable options $\{Q^t_i\}_{i=1,2}$ to the buyers. These differ from the fixed-quantity contracts discussed above in that the buyers are given the explicit right to trade their capacity without intervention of the manufacturer and without additional compensation to the manufacturer. The market state $\omega$ is realized, and then the buyers trade. They are guaranteed to split the gains from trading their capacity options 50:50, according to the Nash bargaining solution. Finally, if the manufacturer has built excess capacity $c > \sum_{i=1,2} Q^t_i$, he can sell the excess $c - \sum_{i=1,2} Q^t_i$ to the buyers and keep the associated gains from trade. Under expectation damages, the manufacturer may choose to build $c < \sum_{i=1,2} Q^t_i$ and compensate the buyers financially for the
shortage. The bargaining outcome is symmetric for the buyers:

\[ x_i = \frac{1}{2} \left\{ \sum_{i=1,2} R_i \left( q_i^* \left( \sum_{i=1,2} Q_i^f \right) \right) - \sum_{i=1,2} R_i(Q_i^f) \right\} \text{ for } i = 1, 2, \]

and the manufacturer’s share is

\[ \sum_{i=1,2} R_i(q_i^*(c)) - \sum_{i=1,2} R_i \left( q_i^* \left( \sum_{i=1,2} Q_i^f \right) \right). \]

Note that this is negative if, under expectation damages, \( c < \sum_{i=1,2} Q_i^f \).

Therefore, given innovation \( e \), the manufacturer will choose capacity to

\[ \max_{c \geq \zeta} \left\{ E_e \left[ \sum_{i=1,2} R_i(q_i^*(c)) - \sum_{i=1,2} R_i \left( q_i^* \left( \sum_{i=1,2} Q_i^f \right) \right) \right] - kc \right\}, \]

where \( \zeta = \Sigma Q_i^f \) under specific performance and \( \zeta = 0 \) under expectation damages. Buyer 1 chooses innovation to

\[ \max_{e_1} \left\{ E_{e_1} \left[ R_1(Q_1^f) + \frac{1}{2} \left\{ \sum_{i=1,2} R_i \left( q_i^* \left( \sum_{i=1,2} Q_i^f \right) \right) - \sum_{i=1,2} R_i(Q_i^f) \right\} \right] - g_1(e_1) \right\}, \]

and buyer 2 faces a symmetric optimization problem.

The following proposition establishes that the first best is achieved with tradable options if and only if the first best is achieved under simple fixed-quantity contracts when the manufacturer is dominant \( (x_i = 0) \). When the first best is not achieved with fixed-quantity contracts, tradable options result in greater expected profit.

**Proposition 1** Under expectation damages, the first best can be implemented with tradable options. Under specific performance, the first best can be implemented with tradable options if and only if (19) is satisfied; the second best expected profit with tradable options is greater than with simple fixed-quantity contracts when the manufacturer is dominant.

The intuition is that tradable options, by increasing each buyer’s gain from renegotiation, increase each buyer’s incentive for innovation. In comparison to the setting with a dominant manufacturer, tradable options enable the firms to reduce the contractual quantity \( Q_i \) for each buyer while maintaining the same innovation investments. Theorem 2 (for the setting with expectation damages and a dominant manufacturer) and Theorem 3 (for the setting with specific performance, a dominant manufacturer and (19)) establish that the contractual quantities (18) implement the first best; to implement the first best with tradable options, the firms simply reduce the contractual
quantities. Theorem 3 (for the setting with specific performance, a dominant manufacturer and (19) violated) establishes that with the second best contracts and associated equilibria, the buyers underinvest in innovation and the manufacturer is forced to build excess capacity. Introducing tradable options increases the buyers’ investment in innovation and enables the firms to reduce the contractual quantities and hence the manufacturer’s capacity investment. This strictly increases the expected discounted profit, though not to the first best because the buyers must share the gain from renegotiation and therefore have imperfect incentives for innovation when the total contracted quantity equals the capacity. In a numerical study, Plambeck and Taylor (2004a) demonstrate that renegotiation design through tradable options can yield substantially greater expected profit than fixed-quantity contracts, and that relative gain is most pronounced when capacity is expensive and buyers are exogenously weak.

4 Discussion

We have proven that simple fixed-quantity contracts, with renegotiation design, can coordinate investment in innovation and capacity. We have considered three alternatives for renegotiation design. The first shifts the gain from renegotiation to the manufacturer by penalizing the buyer for delay in bargaining, as described in Aghion et al. (1994). The second shifts the gain from renegotiation to the buyers, by giving them the right to trade capacity without interference from the manufacturer (tradable options). Either of these guarantees that the firms can induce the first best investments with fixed-quantity contracts—assuming the breach remedy is expectation damages. However, the trend in the U.S. is for courts to enforce specific performance when the source of supply is unique. Under specific performance, the firms might not be able to induce the first best investments with fixed-quantity contracts. The third renegotiation-design alternative is for the firms to stipulate, in the contract, the penalty for noncompliance. Most jurisdictions limit the stipulated damages, so in effect the firms can opt for expectation damages, but cannot opt for specific performance when the standard remedy is expectation damages (Schwartz 1990). Unfortunately, to stipulate expectation damages will make the supply contract complex.

Under specific performance, the firms may benefit from using flexible supply contracts rather than simple fixed-quantity contracts with renegotiation design. For \( N = 2 \) buyers with independent demand and the added-value bargaining outcome (22), Plambeck and Taylor (2004a) characterize the conditions under which flexibility is necessary and induces the first best investments. The role of flexibility in the contract is to fine-tune the firms’ incentives for investment rather than to
eliminate the need for renegotiation.

Renegotiation is a “hassle” for managers (Doran 2001), and the potential for renegotiation might weaken incentives for investment. Therefore, firms might benefit from developing and maintaining a reputation for never renegotiating supply contracts. Plambeck and Taylor (2004a) evaluate how commitment to never renegotiate changes optimal contract structure, investments and expected profit.

One shortcoming of our analysis is that it is essentially static: firms make investments once, learn about the state of the world and then renegotiate. In practice, firms’ investments in innovation, capacity and production are made progressively over time. Similarly, firms may repudiate or repeatedly renegotiate contracts over time as they obtain new information and demand conditions evolve (Triantis and Triantis 1998). Future research in this area may build on recent work on progressive learning and investment in product development or capacity for a single firm (e.g., Roberts and Weitzman 1981, Granot and Zuckerman 1991, Burnetas and Gilbert 2001, Ryan 2003, Bassamboo and Zenios 2003). Another shortcoming is that we assume the firms have common information. Information asymmetry will typically cause inefficiency in investment and trade, despite the potential for renegotiation (see, for example, Tirole 1986, Laffont and Tirole 1990, Beaudry and Poitevin 1993). Economists are making progress on bargaining with information asymmetry between two parties (see Feinberg and Skrzypacz 2003 and references therein). Extending these results to capacity allocation with multiple buyers will require additional assumptions regarding the renegotiation process; the outcome of the renegotiation is often quite sensitive to the details of the assumed process. Although considering such issues is beyond the scope of this paper, information asymmetry is an important aspect of many applications, and we hope that future work will follow.

References


Serant, C., B. Ojo. 2001. OEMs walking away from contracts–AMD sues Alcatel as suppliers feel full force of inventory wave. Electronics Buyers News April 9 1257.


Appendix

Proof of Lemma 1: For \( S \subset \{1..N\} \) define
\[
\Pi^S = \max \left\{ \sum_{i=1}^{N} R_i(q_i) \middle| \begin{array}{l}
q_i \leq Q_i \text{ for } i \notin S \\
q_i \geq 0 \text{ for } i = 1..N \\
\sum_{i=1}^{N} q_i \leq c
\end{array} \right\}
\]
and let \( \{q^S_i\}_{i=1..N} \) denote the unique optimal solution and \( \lambda^S \) denote the Lagrange multiplier associated with the constraint \( \sum_{i=1}^{N} q_i \leq c \). Then \( v_j^S = \Pi^{S \cup j} - \Pi^S \). Observe that
\[
\frac{\partial \Pi^S}{\partial c} = \lambda^S \left\{ \begin{array}{l}
\geq R'_i(q^S_i) \text{ if } q^S_i = 0 \\
= R'_i(q^S_i) \text{ if } q^S_i > 0 \text{ and } i \in S \\
= R'_i(q^S_i) \text{ if } q^S_i \in (0, Q_i) \\
\leq R'_i(q^S_i) \text{ if } q^S_i = Q_i > 0 \text{ and } i \notin S.
\end{array} \right.
\] (26)
The last line is by application of the envelope theorem when constraint \( q_i \leq Q_i \) binds. In the case that \( q^S_j \leq Q_j \), we have \( q^S_i = q^S_{i \cup j} \) for \( i = 1..N \), and
\[
\frac{\partial \Pi^{S \cup j}}{\partial c} = \frac{\partial \Pi^S}{\partial c}.
\]
In the case that \( q^S_j > Q_j \), there exists \( k \neq j \) such that \( q^S_{k \cup j} < q^S_k \), and by strict concavity of \( R_k \) and (26)
\[
\frac{\partial \Pi^{S \cup j}}{\partial c} = R'_j(q^S_j) \geq R'_k(q^S_{k \cup j}) > R'_k(q^S_k) \geq \frac{\partial \Pi^S}{\partial c}.
\] (27)
This establishes (15). Furthermore, for any \( k \subset \{1..N\} \backslash (S \cup j) \)
\[
q^S_{j \cup j} \geq q^S_{j \cup j \cup k},
\]
and by repeated application of (28), \( q^S_{j \cup j} \geq q^*_j(c) \). Therefore, \( q^*_j(c) > Q_j \) implies that \( q^S_{j \cup j} > Q_j \). With (27) this establishes (16).

**Proof of Theorem 1:** We will first prove that if \( \{R_i\}_{i=1..N} \) are separable or \( P_e \) is absolutely continuous, then
\[
P_e(q^*_i(c) \leq Q_i \text{ for } i = 1..N) = 1 \text{ implies that } \frac{\partial}{\partial e_i} E_e[x_i] = 0 \text{ for } i = 1..N.
\]
If \( P_e(q^*_i(c) \leq Q_i \text{ for } i = 1..N) = 1 \) then (9) and (11) ensure that the manufacturer expropriates all the gain from reallocating capacity
\[
P_e(x_i = 0 \text{ for } i = 1..N) = 1.
\]
If the probability measures induced by \( e \in [0,1]^N \) are absolutely continuous, (30) holds for all \( e \in [0,1]^N \), hence
\[
\frac{\partial}{\partial e_i} E_e[x_i] = 0 \text{ for } i = 1..N.
\]
Alternatively, if the revenue functions are separable, then (31) follows immediately from (3), (8), (11), and the envelope theorem. This completes the proof of (29).

By definition, \( c^*(\tilde{e}) \) is the optimal capacity given innovation \( \tilde{e} \), and satisfies the first order condition
\[
\left. \frac{\partial}{\partial c} E_e[\sum_{i=1}^N R_i(q^*_i(c))] \right|_{c=c^*(\tilde{e})} - k = 0.
\]
Because the total expected revenue \( E_e[\sum_{i=1}^N R_i(q^*_i(c))] \) is strictly concave as a function of \( c \), for any \( c > c^*(\tilde{e}) \),
\[
\left. \frac{\partial}{\partial c} E_e[\sum_{i=1}^N R_i(q^*_i(c))] \right|_{c=c^*(\tilde{e})} - k < 0.
\]
The manufacturer’s best response \( \tilde{c} \) must satisfy the first order condition: either \( \tilde{c} = 0 \) or
\[
\left. \frac{\partial}{\partial c} E_e[\sum_{i=1}^N R_i(q^*_i(c)) - \sum_{i=1}^N x_i(\tilde{Q}_1,..\tilde{Q}_N, c)] \right|_{c=\tilde{c}} - k = 0.
\]
Together, (17) and (32) imply that for \( c > c^*(\tilde{e}) \),
\[
\left. \frac{\partial}{\partial c} E_e[\sum_{i=1}^N R_i(q^*_i(c)) - \sum_{i=1}^N x_i(\tilde{Q}_1,..\tilde{Q}_N, c)] \right|_{c=\tilde{c}} - k < 0.
\]
This establishes that the manufacturer weakly underinvests in capacity

\[ \tilde{c} \leq c^*(\tilde{e}). \]

Now let us assume that

\[ \tilde{c} = c^*(\tilde{e}) > 0. \]

From (1), we know that for some \( j \in 1..N \)

\[ P_{\tilde{e}}(q_j^*(\tilde{c}) = \tilde{Q}_j) < 1. \]

To complete the proof, it is sufficient to show that \( \tilde{e}_j > e_j^*(\tilde{c}, \tilde{e}_{-j}) \). Because \( \tilde{c} \) is optimal for the whole system and for the manufacturer,

\[ \frac{\partial}{\partial c} E_{\tilde{e}}[\Sigma_{i=1}^N R_i(q_i^*(c))] \bigg|_{c=\tilde{c}} - k = 0, \]

\[ \frac{\partial}{\partial c} E_{\tilde{e}}[\Sigma_{i=1}^N R_i(q_i^*(c)) - \Sigma_{i=1}^N x_i(\tilde{Q}_1,..\tilde{Q}_N, c)] \bigg|_{c=\tilde{c}} - k = 0, \]

so that

\[ \frac{\partial}{\partial c} E_{\tilde{e}}[\Sigma_{i=1}^N x_i(\tilde{Q}_1,..\tilde{Q}_N, c)] \bigg|_{c=\tilde{c}} = 0. \]

Then (17) implies that

\[ P_{\tilde{e}}(q_i^*(\tilde{c}) \leq \tilde{Q}_i) = 1 \text{ for } i = 1..N. \]

Then from (29),

\[ \frac{\partial}{\partial e_i} E_{\tilde{e}}[x_i] = 0 \text{ for } i = 1..N, \]

and from conditions (5) and (6),

\[ \frac{\partial}{\partial e_j} E_{\tilde{e}}[R_j(\tilde{Q}_j)] > \frac{\partial}{\partial e_j} E_{\tilde{e}}[\Sigma_{i=1}^N R_i(q_i^*(c))]. \]

Because \( \tilde{e}_j \) satisfies the buyer’s first order condition

\[ \frac{\partial}{\partial e_j} E_{\tilde{e}}[R_j(\tilde{Q}_j) + x_j] - g_j'(\tilde{e}_j) = 0, \]

and \( e_j^*(\tilde{c}, \tilde{e}_{-j}) \) satisfies the first order optimality condition for the whole system

\[ \frac{\partial}{\partial e_j} E_{e_j, \tilde{e}_{-j}}[\Sigma_{i=1}^N R_i(q_i^*(c))] - g_j^*(e_j^*(\tilde{c}, \tilde{e}_{-j})) = 0, \]
we conclude that
\[ \tilde{e}_j > e_j^*(\tilde{c}, \tilde{e}_{-j}). \]

Therefore, the first best investments cannot be implemented as a Nash equilibrium under simple fixed-quantity contracts. 

**Proof of Theorem 2:** Trivial and similar to the proof of Theorem 3; hence omitted.

**Proof of Theorem 3:** Assuming innovation \( \tilde{e} \), the manufacturer’s optimization problem is

\[
\max_{c \geq \Sigma Q_i} \left\{ E_{\tilde{e}} \left[ \Sigma_{i=1}^{N} R_i(q_i^*(e)) \right] - kc \right\};
\]

he will choose the optimal capacity if the contracted capacity is not too large, that is,

\[ \tilde{c} = (\Sigma_{i=1}^{N} Q_i) \lor c^*(\tilde{e}), \]

where \( a \lor b \) denotes the maximum of \( a \) and \( b \). Furthermore, given contract \( Q_i \) and assuming optimal innovation from the other buyers, buyer \( i \)’s optimization problem is

\[
\max_{e_i \in [0,1]} \left\{ E_{e_i, e_{-i}} \left[ R_i(Q_i) \right] - g_i(e_i) \right\}.
\]

The objective is strictly concave in \( e_i \). Hence buyer \( i \)’s best response is \( e_i^* \) if and only if \( Q_i = \hat{Q}_i \). With contracts \( \{\hat{Q}_i\}_{i=1..N} \) the manufacturer chooses the optimal capacity \( c^* \) if and only if (19) is satisfied.

Now let us suppose that (19) is not satisfied, so the first best cannot be achieved. We will prove that the second best contracts and associated Nash equilibrium require underinvestment by the buyers and overinvestment by the manufacturer. The argument is by contradiction. Suppose that \( \tilde{e}_i > e^*(\tilde{c}, \tilde{e}_{-i}) \) for some \( i \). Then there exists \( Q'_i \in [0, \hat{Q}_i) \) such that

\[
e_i^*(\tilde{c}, \tilde{e}_{-i}) = \arg \max_{e_i} \left\{ E_{e_i, \tilde{e}_{-i}} \left[ R_i(Q'_i) \right] - g_i(e_i) \right\}.
\]

That is, reducing the capacity allocated to buyer \( i \) reduces her innovation to the optimal level \( e_i^*(\tilde{c}, \tilde{e}_{-i}) \). The other buyers’ innovation levels \( \tilde{e}_{-i} \) are unaffected by this change (here we use, for the first time, the assumption that the distribution of each buyer’s revenue function is independent of the innovation investments of the other buyers). If the manufacturer’s capacity were fixed at \( \tilde{c} \), reducing buyer \( i \)’s contracted quantity to \( Q'_i \) to reduce the innovation for buyer \( i \) to \( e_i^*(\tilde{c}, \tilde{e}_{-i}) \) would yield strictly greater expected system profits. Better yet, as the contractual lower bound on capacity investment is relaxed \( (Q'_i + \Sigma_{j \neq i} \hat{Q}_j < \Sigma_{j=1}^{N} \hat{Q}_j) \) the manufacturer will choose a capacity
that yields weakly greater expected system profit than would \( \tilde{c} \) with the innovation investments \((e_i^*(\tilde{c}, \tilde{e}_{-i}), \tilde{e}_{-i})\). (Recall that the manufacturer’s objective function in choosing capacity investment (33) is the expected system profit less the cost of innovation.) We conclude that by reducing contracted quantity for buyer \( i \), the firms can construct a set of fixed-quantity contracts and associated Nash equilibrium investments that yield strictly greater expected system profit than \((\tilde{e}, \tilde{c})\). This contradicts our assumption that \( \{\tilde{Q}_i\}_{i=1..N} \) and \((\tilde{e}, \tilde{c})\) with \( \tilde{e}_i > e^*(\tilde{c}, \tilde{e}_{-i}) \) are second best.

Specific performance implies that \( \tilde{c} = \sum_{i=1}^{N} \tilde{Q}_i \vee c^*(\tilde{e}) \), so it remains to show that \( \tilde{c} = \sum_{i=1}^{N} \tilde{Q}_i \).

The argument proceeds by contradiction. Suppose that \( \tilde{c} = c^*(\tilde{e}) > \sum_{i=1}^{N} \tilde{Q}_i \). Then it must be true that

\[
\tilde{e}_i = e_i^*(\tilde{c}, \tilde{e}_{-i}) \quad \text{for all } i.
\]

(We established above that \( \tilde{e}_i \leq e_i^*(\tilde{c}, \tilde{e}_{-i}) \); if \( \tilde{e}_i < e_i^*(\tilde{c}, \tilde{e}_{-i}) \) we could increase \( \tilde{Q}_i \), and thus increase the innovation from buyer \( i \) to achieve strictly greater expected profit.) The first best is not achieved, so we must have either \( \tilde{c} > c^* \) or \( \tilde{c} < c^* \). Capacity and innovation are complements, so \( \tilde{c} > c^* \) would imply that \( \tilde{e}_i > e_i^* \) for some buyer \( i \). But then, as above, we could reduce the capacity allocated buyer \( i \) to achieve strictly greater expected profit. Similarly, \( \tilde{c} < c^* \) would imply that \( \tilde{e}_i < e_i^* \) for some buyer \( i \), and that we could increase the capacity allocated buyer \( i \) to achieve strictly greater expected profit. This contradicts our assumption that \( \{\tilde{Q}_i\}_{i=1..N} \) and \((\tilde{e}, \tilde{c})\) with \( \tilde{e}_i > e_i^*(\tilde{c}, \tilde{e}_{-i}) \) are second best. So it must be true that \( \tilde{c} = \sum_{i=1}^{N} \tilde{Q}_i \). □

**Proof of Theorem 4:** Define

\[
\pi(e, \theta) = \max \left\{ \sum_{i=1}^{N} R_i^0(e_i) q_i + R_i^b(q_i, \theta) + R_i^c(e_i, \theta) \right\}
\]

\text{s.t.} \( \sum_{i=1}^{N} q_i \leq c^* \) and \( q_i \geq 0 \)

and let \( q_i^*(c^*, e, \theta) \) denote the optimal solution. Because \( R_i^0(e_i) \) is strictly positive and \( R_i^b(q, \theta) \) increases with \( q \),

\[
\sum_{i=1}^{N} q_i^*(c^*, e, \theta) = c^*, \tag{34}
\]

and therefore

\[
\sum_{i=1}^{N} Q_i^* = \sum_{i=1}^{N} E[q_i^*(c^*, e^*, \theta)] = c^*. \tag{35}
\]

By the envelope theorem,

\[
\frac{\partial}{\partial e_i} \left[ \pi(e, \theta) \right] = \frac{\partial}{\partial e_i} \left[ R_i^0(e_i) \right] \cdot q_i^*(c^*, e, \theta) + \frac{\partial}{\partial e_i} \left[ R_i^c(e_i, \theta) \right].
\]

Given investment by the other buyers of \( e_{-i}^* \) and manufacturer capacity of \( c^* \), and fixed-quantity
quantity \(Q_i^*\), buyer \(i\) chooses her investment in innovation \(e_i\) according to

\[
\max_{e_i \in [0,1]} \left\{ E \left[ R_i(Q_i^*, (e_i, e_{-i}^*), \theta) + \alpha_i \left[ \pi((e_i, e_{-i}^*), \theta) - R_i(Q_i^*, e_i, \theta) - F(\{Q_i^*\}_{i=1..N}, e_{-i}^*, \theta) \right] \right] - g_i(e_i) \right\}.
\]

(36)

where, when the bargaining outcome is linear in the total gain from renegotiation:

\(F(\{Q_i^*\}_{i=1..N}, e_{-i}^*, \theta) = \sum_{j \neq i} R_j(Q_j^*, e_j, \theta);\) when the bargaining outcome is linear in the added value:

\(F(\{Q_i^*\}_{i=1..N}, e_{-i}^*, \theta) = \max\{ \sum_{j \neq i} R_j(q_j, e_j, \theta) : q_j \geq 0, \sum q_j \leq c^* - Q_i^* \};\) when the bargaining outcome is the Shapley value and \(N = 2: \alpha_i = \frac{1}{3}\) and \(F(\{Q_i^*\}_{i=1..N}, e_{-i}^*, \theta) = R_j(Q_j^*, e_j, \theta)\) for \(j \in \{1, 2\}\). (The Shapley value for buyer \(i\) is:

\[
(\pi((e_i, e_{-i}^*), \theta) - R_i(Q_i^*, e_i, \theta) - R_j(c^* - Q_j^*, e_j, \theta))/3 + (R_i(c^* - Q_j^*, e_i, \theta) - R_i(Q_i^*, e_i, \theta))/3,
\]

because \(Q_1^* + Q_2^* = c^*\). Because \(F(\{Q_i^*\}_{i=1..N}, e_{-i}^*, \theta)\) is invariant with respect to \(e_i\), (36) reduces to

\[
\max_{e_i \in [0,1]} \left\{ (1 - \alpha_i)E[R_i(Q_i^*, e_i, \theta) + \alpha_i E[\pi((e_i, e_{-i}^*), \theta)] - g_i(e_i) \right\}.
\]

Because \(R_i^c(e_i, \theta)\) is strictly concave in \(e_i\), the unique optimal investment by buyer \(i\) is the value of \(e_i\) that satisfies

\[
\frac{\partial}{\partial e_i} \left[ (1 - \alpha_i)E[R_i(Q_i^*, e_i, \theta) + \alpha_i E[\pi((e_i, e_{-i}^*), \theta)] - g_i'(e_i) \right] = 0.
\]

Because \(R_i^c\) is twice-differentiable and the support of \(\theta\) is continuous and bounded, \(\frac{\partial}{\partial e_i} R_i^c(e_i, \theta)\) is uniformly bounded on the support of \(\theta\). This justifies interchange of expectation and differentiation:

\[
\frac{\partial}{\partial e_i} E[R_i(Q_i^*, e_i, \theta)] = \frac{d}{de_i} E[R_i^c(e_i)] \cdot Q_i^* + \frac{d}{de_i} E[R_i^c(e_i, \theta)]
\]

\[
= \frac{d}{de_i} E[R_i^c(e_i)] \cdot E[q_i^c(e^*, e^*, \theta)] + \frac{d}{de_i} E[R_i^c(e_i, \theta)]
\]

\[
= E \left[ \frac{\partial}{\partial e_i} \pi((e_i, e_{-i}^*), \theta) \right]
\]

\[
= \frac{\partial}{\partial e_i} E \left[ \pi((e_i, e_{-i}^*), \theta) \right].
\]

Therefore buyer \(i\) will choose the first best investment \(e_i^*\).

Under specific performance, from (35), the manufacturer must build capacity of at least \(c^* = \Sigma_{i=1}^N Q_i^*\). Assumption (24) ensures that the manufacturer will not build more than \(c^*\). Assumption (24) is trivially satisfied in the case that the bargaining outcome is linear in the total gain from
renegotiation. Therefore \((e^*, c^*)\) is a Nash equilibrium. ■

**Proof of Proposition 1:** The buyers have independent revenue functions, so buyer \(j\)'s objective function reduces to

\[
\max_{e_j} \left\{ E_e \left[ \frac{1}{2} R_j(Q_j^1) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(Q_{i1})) \right] - g_j(e_j) \right\}.
\]

Each buyer \(j\)'s objective function is strictly concave so first order conditions are sufficient for optimality. We will construct a set of tradable options \(\{Q_{ti}^t\}_{i=1,2}\) that satisfy

\[
\frac{\partial}{\partial e_1} E_e^* \left[ \frac{1}{2} R_1(Q_1^t) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(Q_{i1})) \right] = g_1'(e_1^*) \tag{37}
\]

\[
\frac{\partial}{\partial e_2} E_e^* \left[ \frac{1}{2} R_2(Q_2^t) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(Q_{i1})) \right] = g_2'(e_2^*), \tag{38}
\]

and, if (19) is satisfied,

\[
\sum_{i=1,2} Q_i^t \leq c^*. \tag{39}
\]

Then it follows that under expectation damages and also, if (19) is satisfied, under specific performance, that \((e^*, c^*)\) is implemented as a Nash equilibrium with tradable options \(\{Q_{ti}^t\}_{i=1,2}\). For \(\tau \in [0, c^* - \hat{Q}_1]\) define

\[
Q_2^t(\tau) = c^* - \hat{Q}_1 - \tau
\]

\[
Q_1^t(\tau) = \min \left\{ Q : \frac{\partial}{\partial e_1} E_e^* \left[ \frac{1}{2} R_1(Q) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(Q + Q_2^t(\tau))) \right] = g_1'(e_1^*) \right\}.
\]

Note that \(Q_1^t(0) = \hat{Q}_1\). If \(\sum_{i=1,2} \hat{Q}_i = c^*\), then we also have \(Q_2^t(0) = \hat{Q}_2\) so that \(\{Q_{ti}^t(0)\}_{i=1,2}\) satisfies (37)-(39). To see that \(Q_1^t(\tau)\) exists and satisfies \(\sum_{i=1,2} Q_i^t(\tau) \leq c^*\) for all \(\tau \in [0, c^* - \hat{Q}_1]\) observe that

\[
\frac{\partial}{\partial e_1} E_e^* \left[ \frac{1}{2} R_1(c^* - Q_2^t(\tau)) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(c^*)) \right] \geq g_1'(e_1^*)
\]

because \(c^* - Q_2^t(\tau) \geq \hat{Q}_1\). Also observe that at \(Q = 0\),

\[
\frac{\partial}{\partial e_1} E_e^* \left[ \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(Q_2^t(\tau))) \right] \leq \frac{\partial}{\partial e_1} E_e^* \left[ \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(c^*)) \right] < g_1'(e_1^*)
\]

because \(Q_2^t(\tau) \leq c^*\). Because \(\frac{\partial}{\partial e_1} E_e^* \left[ \frac{1}{2} R_1(Q) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(Q + Q_2^t(\tau))) \right]\) increases continuously with \(Q\), we conclude that \(Q_1^t(\tau)\) exists and satisfies \(\sum_{i=1,2} Q_i^t(\tau) \leq c^*\). Similarly, at \(\tau = 0\) we have...
\(Q_2'(0) = c^* - \hat{Q}_1 \geq \hat{Q}_2\) and \(Q'_1(0) + Q'_2(0) = c^*\), so that
\[
\frac{\partial}{\partial e} E_{\bar{e}} \left[ \frac{1}{2} R_2(Q_2'(\tau)) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(\sum_{i=1,2} Q'_i(\tau))) \right] \bigg|_{\tau=0} \geq \frac{\partial}{\partial e} E_{\bar{e}} \left[ \frac{1}{2} R_2(\hat{Q}_2) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(c^*)) \right]
= g'_2(c_2^*).
\]

At the other extreme of \(\tau = c^* - \hat{Q}_1\), \(Q'_2(c^* - \hat{Q}_1) = 0\) and
\[
\frac{\partial}{\partial e} E_{\bar{e}} \left[ \frac{1}{2} R_2(Q_2'(\tau)) + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(\sum_{i=1,2} Q'_i(\tau))) \right] \bigg|_{\tau=c^* - \hat{Q}_1} \leq \frac{\partial}{\partial e} E_{\bar{e}} \left[ \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(c^*)) \right] < g'_2(c_2^*).
\]

We conclude that there exists \(\tau^* \in [0, c^* - \hat{Q}_1]\) with \(\{Q_i^{\tau^*}\}_{i=1,2} = \{Q_i'(\tau^*)\}_{i=1,2}\) satisfying (37)-(39).

It remains to consider the case that (19) is not satisfied (i.e., \(\sum_{i=1,2} \hat{Q}_i > c^*\)) under specific performance. From Theorem 3, under the second best optimal fixed-quantity contracts \(\{\hat{Q}_i\}_{i=1,2}\) and corresponding equilibrium \((\hat{e}, \hat{c})\), we know that \(\hat{e}_i \leq e_i^*(\hat{c}, \hat{e}_{-i})\) for \(i = 1, 2\), which implies
\[
\frac{\partial}{\partial \hat{e}_i} E_{\hat{e}} \left[ R_i(\hat{Q}_i) \right] \leq \frac{\partial}{\partial \hat{e}_i} E_{\hat{e}} \left[ \sum_{i=1,2} R_i(q_i^*(\hat{c})) \right].
\]

We also know that
\[
\hat{Q}_1 + \hat{Q}_2 = \hat{c}.
\]

Therefore, keeping the same capacity \(\hat{Q}_i\) for each buyer but making it tradable, i.e. selling \(Q_i' = \hat{Q}_i\) tradable options to each buyer, will increase each buyer’s incentive for innovation:
\[
\frac{\partial}{\partial e_i} E_{\bar{e}} \left[ \frac{1}{2} R_i(Q_i') + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(\sum_{i=1,2} Q_i')) \right] \geq g'_i(\hat{e}_i).
\]

If (40) is satisfied with equality for \(i = 1, 2\) then \((\hat{e}, \hat{c})\) remains an equilibrium with tradable options \(Q_i' = \hat{Q}_i\). Otherwise we can reduce the quantity for each buyer to \(Q_i'' < \hat{Q}_i\) to satisfy
\[
\frac{\partial}{\partial e_i} E_{\bar{e}} \left[ \frac{1}{2} R_i(Q_i'') + \frac{1}{2} \sum_{i=1,2} R_i(q_i^*(\sum_{i=1,2} Q_i'')) \right] = g'_i(\hat{e}_i) \text{ for } i = 1, 2.
\]

If \(c^*(\hat{e}) < \hat{c} = \sum_{i=1,2} \hat{Q}_i\) this enables the manufacturer to reduce his capacity, strictly increasing the total expected profit, with equilibrium \((\hat{e}, c^*(\hat{e}) \lor \sum Q_i'')\). Otherwise if \(c^*(\hat{e}) = \hat{c}\) then \((\hat{e}, \hat{c})\) is an equilibrium with tradable options \(\{Q_i''\}_{i=1,2}\). ■