

# Public Disclosures, Price Efficiency, and Information Asymmetries: A Theory of the Mosaic

Edwige Cheynel  
*Graduate School of Business*  
*Columbia University*

Carolyn Levine  
*Rutgers Business School*  
*Rutgers University*

## **Abstract**

We develop a model to investigate the impact of public disclosure on price efficiency and information asymmetry. Public disclosure policies can enhance the mosaic of information, allowing informed traders to improve the precision of their private information while simultaneously increasing the amount of information impounded in price. Managers who care about both share price and private benefits may optimally choose no disclosure, partial disclosure, or full disclosure. When managers can selectively disclose, they will selectively disclose whenever they choose no public disclosure, consistent with concerns that prompted Reg FD. However, if there is partial disclosure, the manager will not always selectively disclose. Finally, when managers have a duty to disclose negative news, overall disclosure may either increase or decrease. Mandating disclosure of one state may have the unintended consequence of enabling the manager to withhold information on other states.

# 1 Introduction

Standard arguments in favor of disclosure regulation assert that increasing disclosure decreases information asymmetry. As such, one might expect privately informed traders to dislike all subsequent public disclosures as they would diminish opportunities to exploit their information advantage. However, considerable empirical evidence exists suggesting that information asymmetry *increases* following both earnings announcements and management forecasts (e.g., Lee, Mucklow, and Ready (1993); Coller and Yohn (1997)). By assuming that information gathering can occur only after earnings announcements, Kim and Verrecchia (1994) provide a model consistent with that empirical finding. However, their model seems descriptive of settings with few market participants, conducting fundamental analyses based on publicly disclosed financial information. When markets are highly liquid and the number of sophisticated investors is large, it seems that an information advantage gained by superior interpretation of an earnings announcement would be quickly dissipated. When private information can be gathered even in the absence of public disclosures, the value of public disclosure to different market participants is less clear.

In this paper, we model an informed trader, who can gather noisy private information, and a manager that makes public disclosure decisions. More public disclosures help reduce the uncertainty about the cash flows and thus increase price efficiency. One might wrongly conclude that because prices are more informative, informed traders are worse off. Partial public disclosures will indeed eliminate the ability to trade on the “disclosed” information. However, the disclosure policy can make the undisclosed information increase in value. That is, a skilled investor, may piece together less informative data with public information into a mosaic, generating a more precise signal and avoiding some unprofitable trades they would have incurred in the absence of disclosure. Therefore, price efficiency can increase at the same time that information asymmetry and informed traders’ expected profits increase. This is in sharp contrast to conventional wisdom which suggests that disclosure lowers information asymmetry which in turn increases price efficiency. Overall, since trading profits would be higher under partial disclosure, it is consistent with the empirical evidence of an average increase in information asymmetry in disclosure periods.

We begin by evaluating the conditions under which informed traders benefit from public disclosures in a simple model. The distribution of the firm’s cash flows is symmetric with three outcomes, high, medium and low. Partial public disclosures can increase information asymmetries by removing any uncertainty about one state of the world and thus preventing the informed trader from incurring some losses. We show that partial disclosure policies can increase the informed trader’s overall expected profits when the informed trader does not have “decisive” directional information, i.e., when the distribution of his signal violates the Monotone Likelihood Ratio Property, MLRP. With this in-

formation structure, the informed trader buys if his signal indicates that the high state is more likely than the low state and the medium state is more likely than the high state. The informed trader loses when he buys but the medium state occurs absent any public disclosures. Partial disclosure of the high state will make the informed trader better off if the medium state occurs as his private information combined with the public information more closely reflects the true cash flows of the firm. If the ex ante probability of the high state is low, these gains outweigh the forgone profits from elimination of trading opportunities on the high state.

After identifying the conditions under which partial public disclosure increases information asymmetries, we discuss the endogenous choice of disclosure by incorporating the preferences of the manager. We model the manager's utility as an increasing function of stock price and private benefits (which are increasing in trading profits for informed traders).<sup>1</sup> The manager, who will subsequently learn the realization of firm value selects, but cannot credibly commit to, a disclosure policy. Since the actual disclosure is made after the manager becomes informed, we consider only those disclosure policies that are ex post incentive compatible. To maximize stock price, managers would disclose whenever price is less than the realized state, consistent with the standard unraveling argument Milgrom (1981). Information asymmetry can be highest when the manager partially discloses information. Although a partial disclosure strategy could increase trading profits (relative to no disclosure) in expectation, a manager that cares only about private benefits would not stick to his partial disclosure policy when the true state is the state he planned to disclose. The manager would renege, allowing price to drop by getting private benefits, making partial disclosure infeasible. Therefore, when the manager cares predominantly about private benefits, he follows a no disclosure policy. When the manager cares predominantly about stock price, the manager fully discloses. When his preferences over stock price and private benefits are more evenly balanced, the manager follows a partial disclosure policy where he partially discloses good news, whenever possible. If he reneges on the partial disclosure policy, he will be forced to fully disclose.

RegFD was implemented to prevent increases in information asymmetry resulting from private disclosure to select traders. To understand the impact of RegFD in our setting, we relax the assumption that managers cannot selectively disclose and allow the manager full discretion over public and private disclosures. If the manager mainly cares about private benefits, selective disclosure will decrease the amount of public disclosure and increase the information asymmetry. Specifically, the ability to privately disclose crowds out partial disclosure and within the expanded no disclosure region, the manager selectively discloses. However, because the informed trader will get more precise

---

<sup>1</sup>A similar approach is taken in Zou (1994) and dating back as far as Kurz (1968), where an individual's utility is modeled as a weighted average of consumption and wealth accumulation. The emphasis on these two opposing interests lead to differences in savings rates and growth rates.

information, selective disclosure also increases price efficiency as information gets impounded into price through trading. More surprisingly, when the manager's objective is more balanced between protecting his private benefits and maximizing the market price, he cannot credibly commit to selective disclosures and the optimal disclosure policy remains unchanged.

Finally, we explore the consequences of mandatory regulation (or the duty to disclose) on managers' optimal voluntary disclosures, when low outcomes are required to be disclosed. In absence of mandatory disclosures, the manager always has the incentive to hide the low state if it occurs and disclosure of the low state is never credible. This lack of credibility forces the manager to fully disclose his information when his private benefits are high and he wants to defect to retaining the information he was supposed to disclose. The duty to disclose low outcomes prevents the manager from withholding information about the low state, if it occurs and will help him to avoid full disclosure, in part or even entirely. While mandating disclosure can lead to more disclosure in some regions, it leads to less disclosure in others, and "different" but not better disclosure in still others. An unintended consequence of mandatory regulation is a decrease in price efficiency or an increase in information asymmetry relative to that which would result from unconstrained, optimal voluntary disclosure when the manager mainly cares about private benefits.

In auction theory, the linkage principle is a strong result that shows that a seller possessing private information about an appraisal of an asset to be auctioned should disclose this information to bidders to reduce information asymmetry and thus improve the selling price. Specifically, Milgrom and Weber (1982) find that among policies for revealing information (including censored, noisy, or partial revelation), fully and publicly announcing all information a seller possesses is an expected-revenue-maximizing policy.<sup>2</sup> More public information reduces information asymmetries, but the result heavily relies on a notion of affiliation, which requires a positive correlation between the bidder's valuations and in a simplified setting that the distribution of the bidder's signals satisfy the monotone likelihood ratio property MLRP property, that implies a positive correlation between signals and the value of the asset.

However, the opposite relation, that increased disclosure leads to increased information asymmetry has been documented by a large body of empirical papers (e.g., Lee, Mucklow, and Ready (1993), Yohn (1998), Krinsky and Lee (1996), Affleck-Graves, Callahan, and Chipalkatti (2002), Barron, Byard, and Kim (2002) and Barron, Harris, and Stanford (2005)). Without assuming that the disclosures themselves lead to increased opportunities for information gathering, we believe this is the first paper to provide a theoretical explanation for the simultaneous increase in information asymmetry and decrease of price volatility resulting from voluntary disclosure. Our paper relaxes the MLRP assumption and explores the opportunities for public disclosure to exacerbate information asymmetries

---

<sup>2</sup>This central result is then recast in Milgrom (2004) as the "publicity effect."

in a microstructure model.

The theoretical literature varies on whether public and private information are substitutes, complements or both. Naturally, if public information substitutes for private information (Diamond and Verrecchia (1991)), information asymmetries will decrease after the public information is released. Conversely, if public and private information are complements, increased earnings precision will increase information asymmetry.<sup>3</sup> Alternatively, more precise public disclosure may induce increased information acquisition by some investors (Kim and Verrecchia (1991) and McNichols and Trueman (1994)). This paper differs from these studies by considering a setting where informed traders possess private information before any public disclosures and the public disclosures (lack of disclosures) either subsume (complement) their information advantage. We connect the cost and benefit analysis of public disclosures to the information mosaic of the informed trader, where he may interpret the public information differently and in a more efficient way than the market maker. In a different context, the “divergence” between traders on public information is also analyzed in Varian (1988), Harris and Raviv (1993) and Kandel and Pearson (1995), where different perceptions of the same public information produce disagreement and volume.

Our paper extends the voluntary disclosure literature in several ways. First, the tension in our paper that prevents unravelling is related to the objective function of the manager that consists in trading off maximization of the market price and private benefits, generated through receiving an implicit payout related to the magnitude of informed trading profits. Bertomeu, Beyer, and Dye (2011) provide conditions where more public disclosures can increase information asymmetries between the uninformed traders and a trader who does not know the value of the firm but knows whether the manager has received information. In contrast, our trader is directly informed (albeit imperfectly) about firm value. When the manager optimally chooses his disclosure policy, he weighs whether his choice can enhance informed trading profits and if those potential gains are sufficiently high relative to his other objective of maximizing the market price. In a setting where liquidity traders have some discretion over their trades, Levine and Smith (2003) show that an insider-manager may need to disclose part of his information to reduce the bid-ask spreads. Without disclosure, the information asymmetries can be too large causing discretionary liquidity traders to flee the market, eliminating all opportunities for profitable informed trading. In contrast, ex post optimal disclosures in our model may diminish the informed traders’ profits but the offsetting benefit is an increase in stock price.

This paper contributes to the literature that evaluates the benefits of selective disclosure rules and mandatory disclosure regulation. In particular, Dutta (1996) shows that when analysts’ relative infor-

---

<sup>3</sup>Lundholm (1988) focuses on how the correlation between public and private signals relates to the signal’s impact on stock prices, Indjejikian (1991) models some heterogeneity in the cost of processing information, while Fischer and Verrecchia (1999) rely on heterogeneity in information-processing biases.

mation advantage is not too great, some private disclosures can increase price efficiency, a common metric of Pareto efficiency (Grossman (1995)). We, instead, highlight the often disconnection between information asymmetries and price efficiency and more interestingly, we show that selective disclosures do not necessarily worsen information asymmetries because the ex-ante disclosure policy is never ex-post optimal if the manager’s private benefits are significant. With regards to mandatory regulation requiring disclosure of bad news, we show that the “duty to disclose” admits partial disclosures when private benefits are high, but partial disclosure was not credible without it. Trueman (1997) justifies the strategic choice of releasing disclosures of low outcomes to avoid litigation risk. Dye (2013) embeds a voluntary disclosure model with uncertainty about information endowment with a materiality threshold that will capture when a piece of information is deemed material and should be disclosed. He shows that the voluntary disclosure threshold determining when firms disclose is not monotonic in the materiality threshold. Although the duty to disclose increases disclosure for some values of  $a$ , they reduce the amount of disclosure in others.

## 2 Model

In the model, there are four risk-neutral players: a manager, an informed trader, liquidity traders and a market maker. There is a single risky asset (the “firm”) which generates cash flows  $\theta_\tau$  at the end of period  $\tau$ . Within each period, there are four relevant dates. At date 1, the manager of the firm chooses a public disclosure policy. At date 2, the informed trader receives a private signal informative about the distribution of the cash flows of the firm. At date 3, the manager observes true cash flows and discloses or withholds his information according to the pre-specified disclosure policy. At date 4, traders (informed and liquidity) submit their orders, the market maker observes net order flow, and the market maker sets price. For details, see the timeline given in Figure 1.

The cash flows  $\theta_t$  can take on three possible values,  $\{L, M, H\}$ , where

$$L = \mu - e; \quad M = \mu; \quad H = \mu + e;$$

with  $0 < e < \mu$ . State  $M$  occurs with probability  $1 - n$  and states  $L$  and  $H$  each occur with probability  $n/2$ , giving us a symmetric distribution. Further, we assume that  $0 < n < 2/3$  to ensure the probability distribution is single peaked and that extreme outcomes are less likely than moderate ones.

The manager’s disclosure policy specifies the states of the world he will disclose publicly at date 3. We denote the disclosure policy by  $d_P$ , where  $d_N$  indicates no disclosure,  $d_F$  indicates full disclosure, and  $d_\theta$  indicates disclosure of state  $\theta$  only. Specifically, if the manager’s policy is  $d_H$ , whenever he observes  $\theta = H$  he discloses and whenever he observes  $\theta \in \{M, L\}$  he stays silent. We

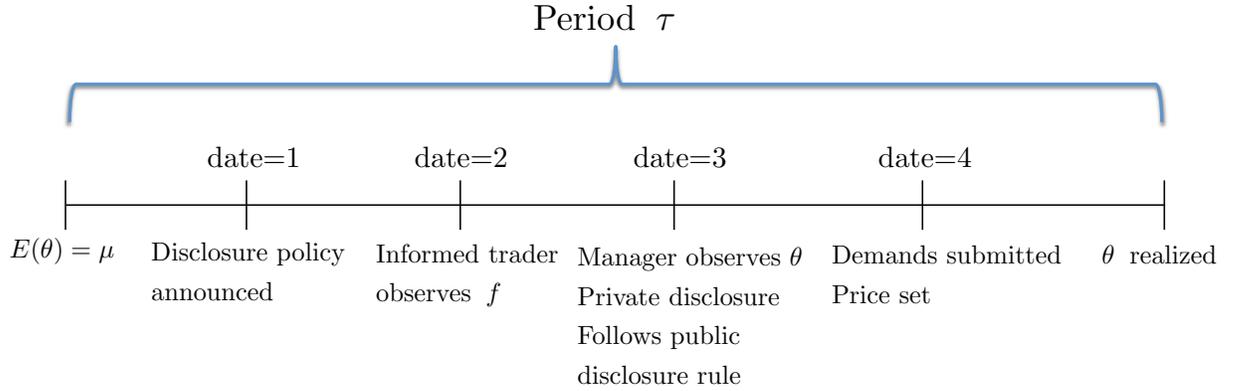


Figure 1: Timing of events within period  $\tau$

refer to disclosure policies in which only one state is disclosed as partial disclosure.

The manager does not trade on his private information directly and does not disclose his private information selectively.<sup>4</sup> However, he cares about both shareholders and informed traders. We model his utility as an increasing function of private benefits and his firm's stock price, computed prior to the realization of cash flows. We assume that private benefits are provided by the informed trader and are therefore increasing in the informed trader's expected profits. For example, the informed trader might give the manager (or one of the manager's family members) a position on the board of his company, provide some favorable trading terms on corporate or personal goods, or other perquisites. If the informed trader is an institutional investor, the informed trader might approve a greater compensation package or demand less monitoring of a manager whose disclosure policies enhance his trading advantage.<sup>5</sup> Since stock price is part of the manager's utility, absent monitoring or a duty to disclose low valued information, partial disclosure of the low state is never ex post optimal.<sup>6</sup>

At date 2, the informed trader observes a binary signal  $f \in \{0, 1\}$ . When  $\theta = i$ , the informed trader observes signal  $f = 1$  with probability  $t_i$  and  $f = 0$  with probability  $(1 - t_i)$ . The signal  $f$  is informative in that trades based on  $f$  earn positive expected profits. We assume that  $t_L < t_H$ . That way, in the absence of additional public disclosure, the action associated with  $f = 1$  is a purchase and with  $f = 0$  is a sale.<sup>7</sup>

<sup>4</sup>In section 5, we suppress this latter assumption and allow the manager to engage in some selective disclosure.

<sup>5</sup>There is an extensive literature on private benefits of control that supports the argument that managers take advantage of their position within the company for their own benefit. See Barclay and Holderness (1989), Kandel and Pearson (1995), Affleck-Graves, Callahan, and Chipalkatti (2002), Eckbo and Thorburn (2003), Belen and Raphael (2006) and Doidge, Karolyi, Lins, and Stulz (2009).

<sup>6</sup>We consider the implications of a duty to disclose in Section 3.3.

<sup>7</sup>The ordering of  $t_H$  and  $t_L$  is without loss of generality. If we made the opposite assumption, the informed trader would simply sell whenever  $f = 1$  and buy when  $f = 0$ . The key is that the signal is informative, not which signal has which

At date 3, the manager learns (privately) the realization of the firm's cash flows. The manager either discloses his information,  $\theta$  or withholds his information. We assume that when he discloses his information, he is required to report it truthfully.<sup>8</sup> We require the manager's disclosure decision to be ex post incentive compatible; in other words, his disclosure must be optimal after he has observed the true  $\theta$ . If the manager were interested in maximizing stock price alone, we would have the standard unraveling result and full disclosure. The joint interest in maximizing stock price and informed traders' profits prevents from unraveling.

At  $t = 4$ , liquidity traders post a demand  $\tilde{\epsilon}$  equal to either 1 or  $-1$ , each with probability  $\frac{1}{2}$ . The informed trader chooses a demand  $X$  conditional on his public information and the disclosure. In order to use liquidity trades as disguise, the informed trader will only buy or sell a single unit. The market clearing price of the firm is then established via a Kyle (1985)-type market maker, who sets the market price  $P$  equal to expected value based on observing the aggregate order flow  $\tilde{Y} = \tilde{X} + \tilde{\epsilon}$  and public disclosures.

## 2.1 Informed Trading Profits

In this section we take disclosure policy as fixed. We endogenize disclosure policies in a later section. Under a full disclosure policy, the firm value is disclosed in every state. The market maker sets price equal to value. There are no opportunities for the informed trader to benefit from his information, as it is wholly subsumed by the public disclosure. Under a no disclosure policy, market price can only update price based on order flow. Under a partial disclosure policy, the market maker sets price equal to value when the state is disclosed, and sets it to the conditional expectation based over the remaining undisclosed states and the order flow when the state is not revealed. We then measure the price efficiency and information asymmetries of each possible disclosure policy.

There are only three possible levels of order flow,  $Y \in \{2, 0, -2\}$ . In the case of no disclosure, the informed trader always trades since his information is noisy. An order flow of  $Y = 2$  ( $Y = -2$ ) indicates to the marker maker that the informed trader submitted a buy (sell) order. An order flow  $Y = 0$  provides no information about the direction of the informed trader's trade as it is completely disguised by the offsetting liquidity trade.

---

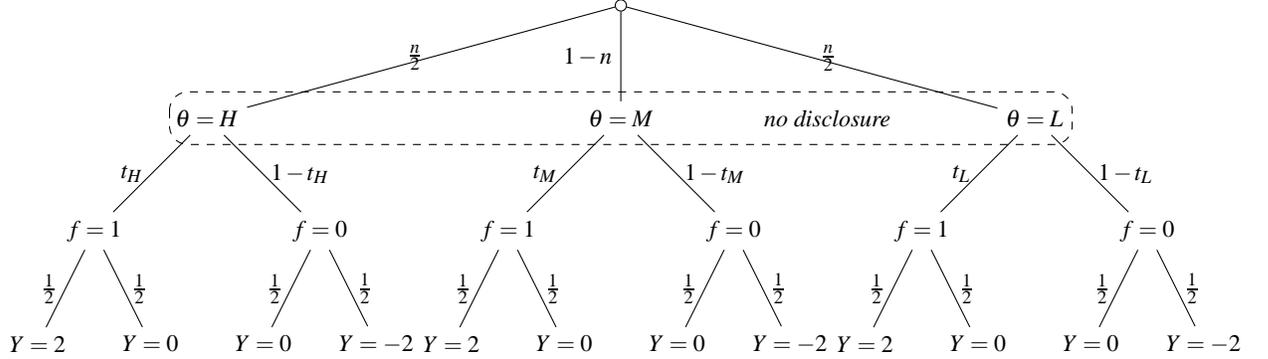
designation.

<sup>8</sup>Truthfulness is a standard assumption in the voluntary disclosure literature (e.g., Verrecchia (1983), Dye (1985)). Stocken (2000) provides a setting where voluntary disclosures are truthful if we take into account that in a multi-period game, the cost of lying through severe punishment disciplines the manager's behavior. In the context of mandatory disclosures, Goex and Wagenhofer (2009) also assume that the manager reports truthfully the information generated by an information system. Beyer, Cohen, Lys, and Walther (2010) offers a nice survey and arguments to support truthful disclosures.

### 2.1.1 No Disclosure

The probability tree is presented in Figure 2

Figure 2: No disclosure



The market maker sets price equal to expected value given the disclosure policy, disclosure and order flow. In this case, the disclosure policy is  $d_N$ , or no disclosure, and therefore the disclosure itself is  $\emptyset$ .

$$\begin{aligned}
 P(2, \emptyset, d_N) &= E(\theta|Y = 2) = \mu + \frac{en(t_H - t_L)}{n(t_H + t_L - 2t_M) + 2t_M} \\
 P(0, \emptyset, d_N) &= E(\theta|Y = 0) = \mu \\
 P(-2, \emptyset, d_N) &= E(\theta|Y = -2) = \mu + \frac{en(t_H - t_L)}{n(t_H + t_L - 2t_M) - 2(1 - t_M)}
 \end{aligned}$$

By inspection, we can see that  $H > P(2, \emptyset) > P(0, \emptyset) = M > P(-2, \emptyset) > L$ , or price is increasing in order flow, as it should. The informed trader earns profits, in expectation, because price does not fully adjust to order flow, since his information is noisy and when the market maker observes an order flow  $Y = 0$ , he does not know the trading decision of the informed trader.

Calculating the informed trader's expected profits, when  $f = 1$  and the insider has purchased or  $f = 0$  and sold gives:

$$\Pi(f = 1, \emptyset, d_N) = \frac{en(t_H - t_L)}{2(n(t_H + t_L - 2t_M) + 2t_M)} \quad (1)$$

$$\Pi(f = 0, \emptyset, d_N) = \frac{en(t_H - t_L)}{2(2(1 - t_M) - n(t_H + t_L - 2t_M))} \quad (2)$$

The expected profits are positive if the insider buys when  $f = 1$  and sells when  $f = 0$ . Expected profits of trading (buying or selling) are increasing in  $t_H$ ,  $e$  and  $n$  and decreasing in  $t_L$  and  $t_M$ . Trading profits increase if the informed trader buys and the information is more precise about the high event

relative to the low event. Trading profits increase if the informed trader sells and the information is more precise about the low event relative to the high event.

Now, consider the *ex ante* profits for the no-disclosure case, the probability that the informed trader gets signals  $f = 1$  and  $f = 0$  are

$$Pr(f = 1, \emptyset) = \frac{n}{2}(t_H + t_L) + (1 - n)t_M \quad (3)$$

$$Pr(f = 0, \emptyset) = \frac{n}{2}(2 - t_H - t_L) + (1 - n)(1 - t_M) \quad (4)$$

Applying these probabilities to the expected profit in each case (equations (1)-(2) respectively) yields

$$\Pi(d_N) = \frac{en(t_H - t_L)}{2} \quad (5)$$

We compute price informativeness as the conditional variance of value given prices, or

$$V(d_N) = \frac{e^2n(2n^2(t_H + t_L - 2t_M)^2 + n(t_H^2 - 2t_H(t_L - 4t_M + 2) + 8(t_L + 1)t_M + (t_L - 4)t_L - 16t_M^2) + 8(t_M - 1)t_M)}{2(n(t_H + t_L - 2t_M) + 2(t_M - 1))(n(t_H + t_L - 2t_M) + 2t_M)} \quad (6)$$

The variance is increasing in  $e$ ,  $n$  and  $t_L$ , decreasing in  $t_H$  and inversely U-shaped in  $t_M$ . In other words, price becomes more informative if the informed trader's purchase decision is driven by a higher probability that the high event will occur or a lower probability of the low event. Price efficiency is ambiguously affected by  $t_M$  because it simply adds noise to orders; if the informed trader knew  $M$  perfectly, he would prefer not to trade.

### 2.1.2 Partial Disclosure

#### Disclosure of $M$ only ( $d_M$ )

First, we show that the informed trader is indifferent between no disclosure and partial disclosure of  $M$ . While partial disclosure eliminates the unprofitable trades when the signal indicates buy or sell and the true value is  $M$ , the price sensitivity increases commensurately to reflect the greater precision of information. To see this,

$$\begin{aligned} P(2, \emptyset, d_M) &= \mu + \frac{e(t_H - t_L)}{(t_H + t_L)} \\ P(0, \emptyset, d_M) &= \mu \\ P(2, \emptyset, d_M) &= \mu - \frac{e(t_H - t_L)}{(2 - t_H - t_L)}, \end{aligned}$$

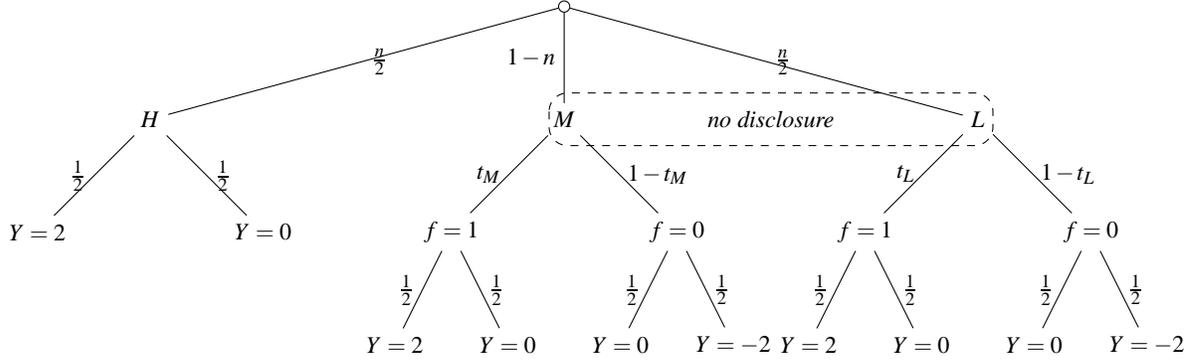
and expected profits,  $\Pi(d_M)$  are  $en(t_H - t_L)/2$  which is identical to the expected profits under no

disclosure (Equation (11)).

Disclosure of H only ( $d_H$ )

When the disclosure policy is  $d_H$ , the market perfectly knows the value of the firm when  $H$  is disclosed and knows the value is either  $L$  or  $M$  when there is no disclosure. To simplify the exposition, we present here the case where  $t_M > t_L$ , or  $f = 1$  is “good” news.

Figure 3: Partial disclosure,  $d_H$



The market maker then sets prices conditional on the observed disclosure (or lack of disclosure), given  $H$  will be disclosed whenever the manager has it, as:

$$\begin{aligned}
 P(\cdot, H, d_H) &= \mu + e \\
 P(2, \emptyset, d_H) &= \mu - \frac{ent_L}{2t_M(1-n) + nt_L} \\
 P(0, \emptyset, d_H) &= \mu - \frac{en}{2-n} \\
 P(-2, \emptyset, d_H) &= \mu - \frac{en(1-t_L)}{2(1-t_M) - n(1+t_L - 2t_M)}
 \end{aligned}$$

When  $H$  is disclosed, price is set equal to  $H = \mu + e$  for all possible order flows.<sup>9</sup> Otherwise, price is in the range  $(\mu - e, \mu)$ . If  $t_M > t_L$  then when the informed trader observes  $f = 1$ , he knows that the state is more likely to be  $M$ . Since price will always be below  $\mu$ , he prefers to buy. Similarly, when the informed trader observes  $f = 0$ , he prefers to sell since the state is more likely to be  $L$  and price is always above  $\mu - e$ .

<sup>9</sup>The informed trader is indifferent between trading and not trading when the state is fully disclosed. In both cases, he earns zero profits

Expected profits are then computed as:

$$\begin{aligned}\Pi(H, d_H) &= 0 \\ \Pi(\emptyset, d_H) &= \frac{2en(1-n)(t_M - t_L)}{(2-n)^2}\end{aligned}$$

Thus, the total *ex ante* profits of the partial disclosure regime,  $d_H$  is

$$\Pi(d_H) = \frac{n}{2}(0) + \left(1 - \frac{n}{2}\right) \frac{2en(1-n)(t_M - t_L)}{(2-n)^2} = \frac{en(1-n)(t_M - t_L)}{(2-n)} \quad (7)$$

Ex ante profits are increasing in  $t_M$  and  $e$ , and decreasing in  $t_L$ . The informed trading with partial public disclosure is valuable because the informed trader can better assess the likelihood of the medium state relative to the low state. However, the ex ante profits are maximized at  $n^* = 2 - \sqrt{2}$ . On one hand, if the spread between the occurrence of the medium and the low is large, the signal  $f$  is quite informative above to the remaining undisclosed two states. On the other hand, more weight on the extreme events is associated with larger foregone profits (for the often realized high state).

Measuring price informativeness under  $d_H$  using the volatility of prices gives

$$V(d_H) = \frac{e^2(n-1)n(-2(n-1)t_M(nt_L + n - 2) + nt_L(n + t_L - 2) + (n-1)(3n-4)t_M^2)}{(n-2)(n(t_L - 2t_M) + 2t_M)(nt_L - 2nt_M + n + 2t_M - 2)} \quad (8)$$

The volatility is decreasing in  $t_M$  and increasing in  $t_L$  and  $e$ , and inversely U-shaped in  $n$ . When  $t_M > t_L$ , the informed trader submits a buy order if he does not observe any public disclosures and thus, when  $t_M$  is large or  $t_L$  is low, the informed trader's information is closer to the true cash flows of the firm and this is impounded in the price. The higher the accuracy of the informed trader's information, the lower the volatility in prices. The unconditional probabilities play an important role in the expected volatility. Public disclosures of the high state removes any volatility in the high state but if the unconditional probability on the extremes is too high, the price is less informative in the remaining non disclosed events.

#### Disclosure of L only ( $d_L$ )

When the disclosure policy is  $d_L$ , the market perfectly knows the value of the firm when  $L$  is disclosed and knows the value is either  $M$  or  $H$  when there is no disclosure. The analysis is parallel to the case with partial disclosure of  $H$ . Therefore, we can suppress the details of the analysis and directly derive the ex-ante expected profits and the price informativeness under  $d_L$  using the volatility of prices when  $t_H > t_M$ .<sup>10</sup>

<sup>10</sup>If  $t_H < t_L$ , the ex-ante profits and the volatility of prices are symmetric .

$$\Pi(d_L) = \frac{n}{2}(0) + \left(1 - \frac{n}{2}\right) \frac{2en(1-n)(t_H - t_M)}{(2-n)^2} = \frac{en(1-n)(t_H - t_M)}{(2-n)} \quad (9)$$

$$V(d_L) = \frac{e^2(n-1)n(-2(n-1)t_M(nt_H + n - 2) + nt_H(n + t_H - 2) + (n-1)(3n-4)t_M^2)}{(n-2)(n(t_H - 2t_M) + 2t_M)(nt_H - 2nt_M + n + 2t_M - 2)} \quad (10)$$

Again, the informed trader benefits from a larger spread between  $t_H$  and  $t_M$  and simultaneously more information is impounded in the price reducing the volatility of prices.

## 2.2 Comparison of no-disclosure and partial disclosure: Informed trader's perspective

In a setting where the informed trader's information is perfect, public disclosure is bad for the informed trader because it reduces his opportunities for advantaged trade. Even without perfect information, when the signal satisfies the monotone likelihood ratio property (such that the probability of the good signal is increasing in the outcome), informed traders are made worse off by any public disclosures.

**Proposition 1** *Suppose the probabilities of the informed trader's signal preserve the monotone likelihood ratio property, or  $t_H > t_M > t_L$ . Information asymmetry is highest and price efficiency is lowest under no disclosure when compared to partial and full disclosure.*

With MLRP met, the informed trader 'owns' directional information about the firm's cash flows. Partial disclosure policies  $d_L$  and  $d_H$  provide information about the extremes that overlap with the informed trader's directional information. Therefore, such disclosures decrease the informed trader's information advantage (i.e., information asymmetry is lower under partial disclosure than no-disclosure). At the same time, price efficiency is the lowest (or prices are most volatile) if no public disclosures are released. These results supports the conventional wisdom that more public disclosures decrease information asymmetries and improves price informativeness. Thus, a necessary condition to exacerbate information asymmetries with more public disclosures is to violate MLRP.

**Proposition 2** *Suppose the probabilities of the informed trader's signal violate the monotone likelihood ratio property. Let*

$$\begin{aligned} t_1 &= \frac{2(1-n)t_M + nt_L}{2-n} & t_2 &= \frac{(2-n)t_L - 2(1-n)t_M}{n}, \\ t_3 &= \frac{2(1-n)t_M + (2-n)t_L}{4-3n} & t_4 &= \frac{(4-3n)t_L - 2(1-n)t_M}{2-n}. \end{aligned}$$

*Then, the following orderings hold for information asymmetry and price efficiency.*

<u>Parameters</u>	<u>Information asymmetry</u>	<u>Price efficiency</u>
$t_M > t_H > t_1 > t_L$	$IA(d_N) > IA(d_H) > IA(d_L)$	$PE(d_H) > PE(d_L) > PE(d_N)$
$t_M > t_1 > t_H > t_3 > t_L$	$IA(d_H) > IA(d_N) > IA(d_L)$	$PE(d_H) > PE(d_L) > PE(d_N)$
$t_M > t_3 > t_H > t_L$	$IA(d_H) > IA(d_L) > IA(d_N)$	$PE(d_H) > PE(d_L) > PE(d_N)$
$t_H > t_2 > t_L > t_M$	$IA(d_N) > IA(d_L) > IA(d_H)$	$PE(d_L) > PE(d_H) > PE(d_N)$
$t_4 > t_H > t_L > t_M$	$IA(d_L) > IA(d_H) > IA(d_N)$	$PE(d_L) > PE(d_H) > PE(d_N)$
$t_2 > t_H > t_4 > t_L > t_M$	$IA(d_L) > IA(d_N) > IA(d_H)$	$PE(d_L) > PE(d_H) > PE(d_N)$

Partial disclosure policies can be beneficial because they enhance the quality of the informed trader's signal in the undisclosed states. Effectively, the disclosure can exacerbate the information asymmetry by eliminating some of the noise in the informed trader's signal. To understand why, note that when the information structure violates MLRP (i.e.,  $t_M \geq t_H \geq t_L$  or  $t_H \geq t_L \geq t_M$ ), the informed trader does not have "decisive" directional information, although trading based on his signal is profitable in expectation. If the manager discloses the high state only or the low state, the informed trader can now order the remaining states more reliably using  $f$ . The disclosure comes at a cost since the informed trader forgoes his potential profits whenever the state disclosed occurs. It follows that disclosure is more valuable for the informed trader if the unconditional probabilities on the extreme cash flows are low. When  $t_M \geq t_L$ , the most desirable disclosure for the informed trader is partial disclosure of  $H$  while when  $t_L \geq t_M$ , partial disclosure of  $L$  is the optimal policy for the informed trader. The most desirable outcome for the informed trader when MLRP is violated is associated with more price informativeness. This positive relation between information asymmetries and price informativeness suggests some caution in confounding information asymmetries and price informativeness. These two measures capture the impact of public disclosures but with a different angle.

Figure 4 illustrates the information asymmetry results of Propositions 1 and 2. The region between the dotted line and the dashed line are the parameters for which MLRP is satisfied. There, no disclosure is always better for the informed trader than partial disclosure of  $L$  or  $H$ . A higher spread between the probability to receive a buy signal given that the true state is  $H$ , i.e.,  $t_H$  and the probability to receive a buy signal given that the true state is  $t_L$ , leads to higher profits and with MLRP satisfied, the informed trader is more likely to have correct information about the extreme values. The section of the graph below the dashed line represents parameters for which  $t_M < t_L < t_H$ . The signal is still informative, but there is a high probability of trading losses stemming from incorrect trades when the signal indicates to sell and the true value is actually  $M$ . In that region, if these trading losses are too big if  $M$  occurs, information asymmetry is highest under a policy of  $d_L$  which allows the informed trader to more precisely order the states  $M$  and  $H$ , when  $L$  is disclosed. The region to the northwest of the dotted line is the MLRP violation where  $t_L < t_H < t_M$ . A policy of  $d_H$  reduces the informed

traders profits when  $H$  is realized, but increases the profitability of trades on  $L$  and  $M$  since disclosure of  $H$  effectively recovers MLRP for the remaining (undisclosed) states. In all cases, partial disclosure leads to greater price efficiency than no disclosure; above the dashed line, the price efficiency is highest when  $H$  is disclosed and below it, price efficiency is highest when  $L$  is disclosed. In addition to getting perfect pricing for the disclosed state, by disclosing the state with the “middling” probability, it increases the precision of the remaining information the most.

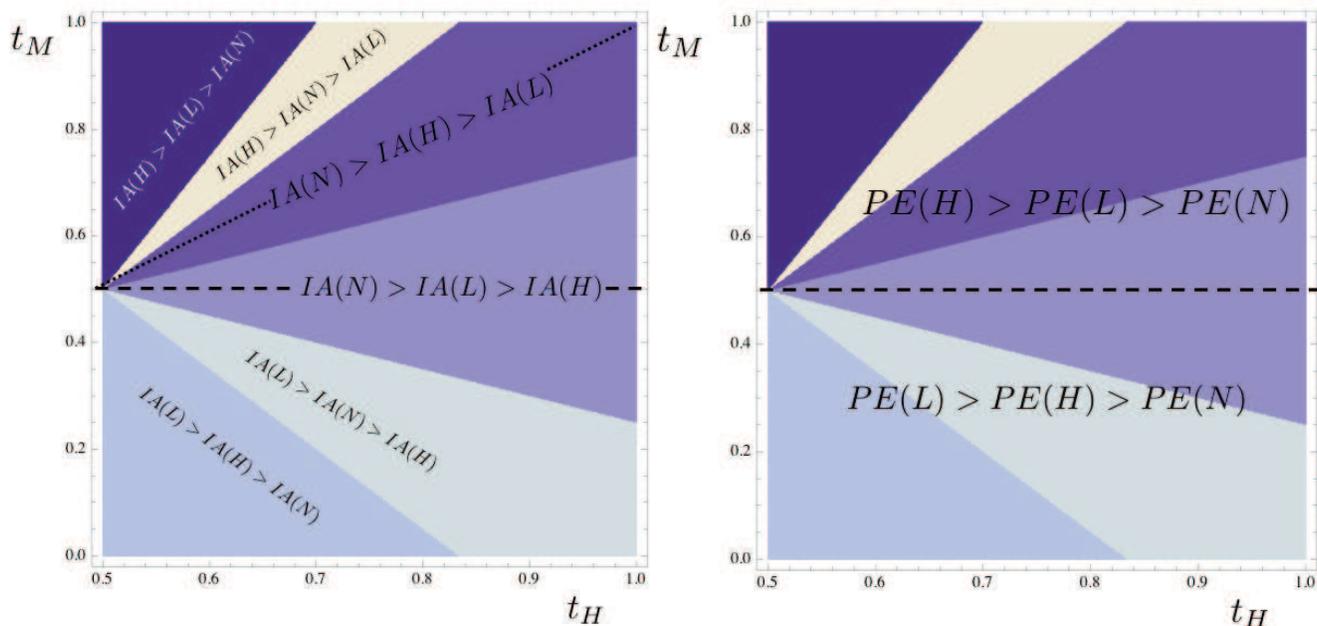


Figure 4: Information Asymmetry Orderings with  $t_L = \frac{1}{2}$  and  $n = \frac{1}{2}$

**Corollary 1** *Partial disclosure policies always increase price efficiency. The benefits to the insider of partial disclosure (i.e., increases to information asymmetry) are decreasing in the difference between  $t_H$  and  $t_L$ .*

When the expected losses avoided by partial disclosures outweigh the expected profits forgone, information asymmetries increase under disclosure. Still, price is more informative because prices benefit from both the disclosure and the more precise information implicit in demand. Therefore, it is possible through public disclosure to simultaneously improve the welfare of the informed trader and increase the amount of information impounded into price. This contradicts the conventional wisdom that more public disclosure increases price efficiency and decreases information asymmetries. In our setting, the informed trader has information which when combined with a disclosure policy forms a “mosaic,” resulting in information that can better direct his trades. Absent any public disclosure and if

true outcome is  $M$ , the informed trader is suffering larger losses with a buy decision if the distribution of his signal violates MLRP than if the distribution of his signal satisfies MLRP. Partial disclosure eliminates any of his profits if the state disclosed occurs but will now turn his losses into substantial gains when the true state is  $M$ . In the extreme case where  $t_H = t_L$ , partial disclosure always increases information asymmetries because in absence of public disclosures, the informed trader cannot profitably trade. His otherwise non-material private information becomes valuable when he is able to piece together his information and material public information into a mosaic.

### 3 Implementation of Public Disclosures

For simplicity, we assume that  $t_L = 0$  for the remainder of the paper. This assumption means that the informed trader never observes a good signal if true cash flows are low. This assumption allows us to focus on only one possible violation of MLRP ( $t_M > t_H > t_L$ ) and seems consistent with the general conservatism in reporting that would understate good outcomes but not overstate bad ones. Additionally, the expressions are more tractable yet results are qualitatively similar to settings in which  $0 < t_L < t_H$ .

We endogenize the disclosure policy, allowing the manager to select the policy that maximizes his personal utility, after observing the true realization  $\theta$ . Most compensation packages provide CEOs with incentives to increase the firm's stock price. Were the manager to maximize market price alone, he would disclose all of his information.<sup>11</sup> This is the standard unraveling result, where the manager, when faced with a price below his signal, will always disclose his signal to increase the price, leaving only the lowest possible value undisclosed. In our model, the manager has a second component to his utility – the incentive to maximize informed trading profits. The benefits from trading profits need not be associated with illegal insider trading by the manager, but may arise instead from other informed traders (e.g., analysts, institutional investors, other influential investors) that provide the manager with private benefits (i.e., favorable recommendations, board memberships, exclusive memberships and other perks) based on their perceived level of investing success. Regulation FD was designed to ‘level the playing field’ for all investors by prohibiting selective disclosures from managers to analysts and institutional investors, thereby requiring firms to make public, within 24 hours, all disclosures of material information. Our modeling of the manager's utility supports the idea that managers would, if they could, share valuable information with specific traders. Specifically, the CEO's utility function is defined over both informed traders' benefits and stock price, as

$$U(\Phi, d|\theta) = a\Pi(\Phi, d) + (1 - a)P(\Phi, d|\theta)$$

---

<sup>11</sup>There are no exogenous disclosure costs in this model.

where  $\Phi \in \{\emptyset, \theta\}$  is the public information set, where the outcome  $\Phi = \theta$  of the cash flows is disclosed or the information is withheld, i.e.,  $\Phi = \emptyset$ . The manager's decision to disclose depends on his beliefs about the stock price, given his information, and his expected private benefits. The conditional stock price is not the same as the unconditional stock price since the manager has better information than the market maker about which order flow to expect. We assume that the informed trader offers the private benefits to the manager prior to the cash flow realization but after observing the manager's disclosure, and therefore  $\Pi(\Phi, d)$  are based on the informed trader's expected profits given the disclosure policy and disclosure, not realized profits. Based on the results of the previous section, expected informed trading can be greater with disclosure than without it. Therefore, a manager who cares only about informed trading profits ( $a = 1$ ) might have incentives to voluntarily disclose part of his information for some parameters. On the other hand, a manager who cares only about stock price only ( $a = 0$ ) would have a policy of full disclosure.

**Definition 1** *A value maximizing disclosure equilibrium is one in which*

$$d = \operatorname{argmax}_D U(d|\theta) > U(d'|\theta) \forall \theta \quad \text{where } d, d' \in D = \{d_N, d_\theta, d_F\},$$

$$\text{subject to } U(\Phi, d|\theta) > U(\Phi', d|\theta) \quad \text{where } \Phi, \Phi' \in \{\emptyset, \theta\}.$$

The first part of the definition requires that disclosure policy  $d$  maximizes the manager's ex ante expected profits relative to  $d$ , under beliefs consistent with  $d$  and  $d'$  respectively. The second part of the definition requires that the disclosure policy is ex post incentive compatible. In other words, the manager follows his stated disclosure policy for all realizations of  $\theta$  and has no incentives to disclose more (or less) than he committed to with his disclosure policy.<sup>12,13</sup>

Proposition 2 shows that there are net benefits to partial public disclosure when the distribution of the private signal violates MLRP and either  $t_H < t_1 < t_M$  or  $t_1 < t_H < t_3 < t_M$ . The gains from improved information in the undisclosed states compensate for the lost profits of the disclosed state. However, when the manager cares about maximizing private benefits, he may renege on his stated disclosure policy to avoid the foregone profits if the true state is the one he agreed to disclose.

### 3.1 Manager's preferred disclosure policy

First, we compute the expected price under no disclosure, conditional on  $\theta$  as

$$P_\theta(d_N) = \frac{1}{2} (t_\theta(P(2, \emptyset, d_N) + P(0, \emptyset, d_N)) + (1 - t_\theta)(P(0, \emptyset, d_N) + P(-2, \emptyset, d_N)))$$

The two incentive compatibility constraints to guarantee no disclosure is preferred to disclosing

---

<sup>12</sup>We do not allow the manager to falsely disclose.

<sup>13</sup>Kamenica and Gentzkow (2011) also focus on equilibria candidates that maximizes the sender's profits.

either  $H$  or  $M$  when the manager observes those states are:

$$a\Pi(d_0) + (1-a)P_H(d_N) \geq (1-a)(\mu + e) \quad (IC_{N,H})$$

$$a\Pi(d_0) + (1-a)P_M(d_N) \geq (1-a)\mu \quad (IC_{N,M})$$

Rewriting and solving for  $a$ , we have

$$a \geq \frac{2 + Gnt_H}{(G+1)nt_H + 2} \equiv a_{N,H}$$

$$\text{where } G = \frac{1 - t_H}{n(2t_M - t_H) + 2(1 - t_M)} - \frac{t_H}{n(t_H - 2t_M) + 2t_M}$$

$$a \geq \frac{n(2t_M - t_H)}{n^2(2t_M - t_H)^2 - 4(1 - t_M)t_M + n(2t_m - t_H)(3 - 4t_M)} \equiv a_{N,M}$$

When there is uncertainty about the state, price is always below  $\mu + e$ . Therefore, to satisfy  $IC_{N,H}$ , the weight on private benefits,  $a$ , needs to be sufficiently large (above  $a_{N,H}$ ) so that the private benefits exceed the cost of a lower stock price when the true state is  $H$ . If the expected price, conditional on observing  $M$  is above  $\mu$  then there is no benefit to disclosing  $M$ . If the expected price, conditional on observing  $M$  is below  $\mu$ , then, similar to the condition for disclosing  $H$ ,  $a$  has to be sufficiently large (larger than  $a_{N,M}$ ) so that the cost of a lower stock price is outweighed by the private benefits.

The threshold  $a_{N,H}$  is decreasing in  $n$  and  $t_M$  and increasing in  $t_H$ . The threshold  $a_{N,M}$  is decreasing in  $t_H$  and increasing in  $t_M$ ; if MLRP is violated, it is also increasing in  $n$ . When  $n$  is small,  $a_{N,H} > a_{N,M}$  because the price (with no disclosure) is largely driven by the moderate outcome (since the probability  $M$  is realized is large). A disclosure of  $H$  will have a large impact on the price and therefore the value of the private benefits must be quite large to surpass the utility gains of increasing stock price. When  $n$  is large, trades on  $H$  are very profitable, but trades on  $M$  are unprofitable. By retaining  $M$ , the private benefits decrease, and thus for large  $n$ , it is possible that  $a_{N,H} < a_{N,M}$ . The lower the value of  $t_M$ , the more informative the private signal. By the previous analysis, we know that if  $t_M < t_H$ , disclosure reduces the information asymmetry, and therefore reduces the expected private benefits, pushing down the threshold above which the manager prefers no disclosure to  $H$  disclosure. If  $t_M \gg t_H$ , the lost private benefits when  $H$  is realized are offset by the significantly higher price, because price was not particularly sensitive to order flow when the signal was very noisy.

Revealing  $H$  when  $a < a_{N,H}$  is preferred to following a no disclosure policy. However, that does not ensure that  $d_H$  is ex post incentive compatible. First, we need to confirm that a manager would indeed disclose  $H$  when it occurs, given beliefs market participants believe the manager is following the disclosure policy  $d_H$ . The informed trader earns profits (and provides private benefits to the manager) only when there is no disclosure. Therefore, if the manager's compensation heavily

weights private benefits, the manager will have incentives to renege on his stated policy and mislead the market. Although the stock price will be lower, if private benefits are highly valued, he might prefer to withhold his information when  $\theta = H$ . Comparing disclosure with withholding when  $\theta = H$  and non-disclosure is interpreted as  $\theta \in \{L, M\}$  yields:

$$(1-a)(\mu + e) > a\Pi(\emptyset, d_H) + (1-a)P_H(\emptyset, d_H)$$

$$\rightarrow a < \frac{(n-2)((n-2)(nt_H - 4) - 2(n-4)(n-1)t_M)}{(n-2)^2(nt_H - 4) + 8(n-1)^2nt_M^2 - 2(n-1)(3n-4)(n-2)t_M} \equiv a_{H,N}$$

Additionally, to guarantee a preference for partial disclosure over full disclosure, we must consider the manager's utility when the true state is  $M$ . The price of the firm when nothing is disclosed under  $d_H$  is always below  $\mu$ . The manager prefers to (disclose  $M$ ) withhold  $M$  whenever his private benefits from trading (do not) offset the lower price. The ex post incentive compatibility condition such that the manager prefers partial disclosure of  $H$  to full disclosure is

$$a\Pi(\emptyset, d_H) + (1-a)P_M(\emptyset, d_H) \geq (1-a)\mu \quad \rightarrow \quad a > \frac{(n-2)(n(3t_M - 2) - 4t_M + 4)}{8(n-1)^2t_M^2 - n(n-2)t_M - 2(n-2)^2} \equiv a_{H,M}$$

Suppose  $d_M$  is planned. Then, whenever  $M$  is realized, the stock price is  $M$  and the private benefits are zero. Since stock price would be  $H$  for positive order flow and  $M$  when order flow is zero, the manager has incentives to renege and mislead the market. Therefore,  $d_M$  cannot be part of any equilibrium strategy. partial disclosure of  $L$  cannot be part of an equilibrium disclosure strategy. If the manager were following  $d_L$ , the informed traders and market maker would have to believe that the true state is either  $M$  or  $H$  when nothing is disclosed. Under those beliefs, the manager would always withhold his information upon observing  $L$  since he can increase both his private benefits and the market price. Consequently, if there is partial disclosure, it will be partial disclosure of  $H$  only.

Full disclosure eliminates all private benefits. The more weight the manager places on stock price, the less costly is full disclosure. Moreover, if the market expects full disclosure, there is no advantage to withholding information. Beliefs when nothing is revealed would be that value is  $L$ . Price would be set accordingly and the informed traders would expect to earn zero profits (and therefore pass along no private benefits) because price is equal to value. When there is no other ex post incentive compatible disclosure policy, the manager will have to resort to full disclosure.

### 3.2 Optimal disclosure policies

Recall from Proposition 2, partial disclosure of increases information asymmetry (and the informed traders expected profits) whenever MLRP is violated and either  $t_M > t_1 > t_H$  or  $t_M > t_3 > t_H$ . However, even under those conditions, the further condition that  $a < a_{H,N}$  is required for partial disclosure to

be incentive compatible. Therefore, when the manager cares predominantly about private benefits ( $a$  large), he will implement a policy of no disclosure. Although he might prefer partial disclosure, ex ante, he cannot sustain it ex post (i.e., in the absence of forced compliance with the stated disclosure policy, he would renege). If partial disclosure is ex post incentive compatible, the manager will select partial disclosure. In all other regions, the manager chooses full disclosure. When  $a$  is low, it is his preference to prioritize stock price over private benefits. When  $a$  is moderately high, he may not be able to convincingly follow a policy of partial disclosure.

**Proposition 3**

*I: If  $t_M > t_H > t_1 > t_L \rightarrow a_{H,N} > a_{N,H} > a_{N,M}$ . The manager's optimal disclosure policy is*

$$\text{If } a > a_{N,H} \rightarrow d_N; \quad \text{If } a_{H,M} < a \leq a_{N,H} \rightarrow d_H; \quad \text{If } a \leq a_{H,M} \rightarrow d_F.$$

*II: Otherwise,  $a_{H,N} < \max\{a_{N,H}, a_{N,M}\}$ . The manager's optimal disclosure policy is*

$$\begin{aligned} \text{If } a > \max(a_{N,H}, a_{N,M}) \rightarrow d_N; \quad \text{If } a_{H,N} < a \leq \max(a_{N,H}, a_{N,M}) \rightarrow d_F; \\ \text{If } a_{H,M} < a \leq a_{H,N} \rightarrow d_H; \quad \text{If } a \leq a_{H,M} \rightarrow d_F \end{aligned}$$

No disclosure is credible and optimal for large  $a$  as the manager's interests are tied to the informed trader's profits and he would not deviate to any other disclosure to increase the market price. The reason that no disclosure is chosen over partial disclosure is that partial disclosure may enhance ex ante profits for the informed trader, but the manager is considering the impact on ex post profits. If the manager discloses  $H$  when he has it, the informed traders lose their full trading advantage, and the manager will receive no private benefits. When the manager's incentives place more emphasis on market price, he switches to partial disclosure of  $H$ . Finally, when partial disclosure of  $H$  is no longer credible, he fully discloses his information. The additional disclosure when moving from partial to full always increases price efficiency and decreases information asymmetry.

This provides an interesting and somewhat surprising conclusion. Informed traders' expected profits are highest when managers care (relatively) less about them, but not so little that there is full disclosure. As we have discussed, when  $a$  is very large (and managers care primarily about informed trading profits, they cannot credibly commit to a disclosure policy of  $d_H$ , although that is the policy preferred by informed traders, ex ante). This may explain why Institutional investors, who provide private benefits, may also monitor the manager by tying his compensation to current share price. Liquidity traders' expected losses are non monotonic in disclosure; they are lowest for full disclosure but highest for partial disclosure of  $H$ . The optimal disclosure as a function of  $a$  when MLRP is

satisfied follows the same pattern although no disclosure is the informed traders preferred ex ante disclosure.

To gain more intuition on the different disclosure regions, consider the extreme case where  $t_L = t_H = 0$ , implying  $a_{N,H} = 1$ . No disclosure is never optimal as the informed trader cannot profitably buy or sell, since it is equally likely that the outcome is  $H$  or  $L$  for each signal. Full disclosure is chosen either if the manager is predominantly concerned with maximizing the market price or if his private benefits are so large that he cannot credibly commit to partial disclosure of  $H$ . This result suggests that in economies or industries where private information is extremely noisy and therefore would not materially affect beliefs about value, we would observe more public disclosures than in environment where posteriors based on private information are materially updated.

### 3.3 The duty to disclose: mandatory disclosure of $L$ state

In the previous sections, disclosure of the low state was never part of an equilibrium strategy. Recognizing this to be generally true, regulators have imposed rules specifically to illicit bad news (e.g., impairment rules, duty to disclose rules). Therefore, we consider a disclosure environment where public disclosure of  $L$  is mandatory (and enforceable). This imposes a minimum disclosure requirement, but not a maximum disclosure requirement on the manager. That is, the manager can effectively choose full disclosure if disclosing  $H$ , when it is observed, is preferred to staying silent. The manager compares disclosing  $L$  only over revealing  $H$  when he has it, generating a threshold above which he partially discloses, and below which he fully discloses.

$$a > \frac{\phi}{\phi + \frac{4(1-n)n(t_M-t_H)}{(2-n)^2}} \equiv a_{L,F}$$

where 
$$\phi = n \left( -\frac{t_H^2}{nt_H - 2nt_M + 2t_M} + \frac{(1-t_H)^2}{nt_H - 2nt_M + n + 2t_M - 2} + \frac{1}{n-2} \right) + 2$$

**Proposition 4** *When a duty to disclose  $L$  exists, if  $a > a_{L,F}$ , the manager follows policy  $d_L$ . If instead  $a < a_{L,F}$ , the manager chooses a full disclosure policy, or  $d_F$ .*

Depending on the location of  $a_{L,F}$  and the other parameters, the duty to disclose can increase disclosures, decrease disclosures or simply change the disclosure policy from  $d_H$  to  $d_L$ . There are several cases, but they can be easily illustrated in Figure 5. We plot Cases I and II from Proposition 3, below the top and bottom lines segmented by thresholds on  $a$  respectively. In Case I and no duty to disclose, there was a region of full disclosure (low  $a$ ), a region of partial disclosure (moderate  $a$ ) and a region of no disclosure (high  $a$ ). The duty to disclose pushes up the full disclosure threshold and replaces no disclosure with partial disclosure, creating overall more disclosure in those regions. If  $a_{L,F}$  is below

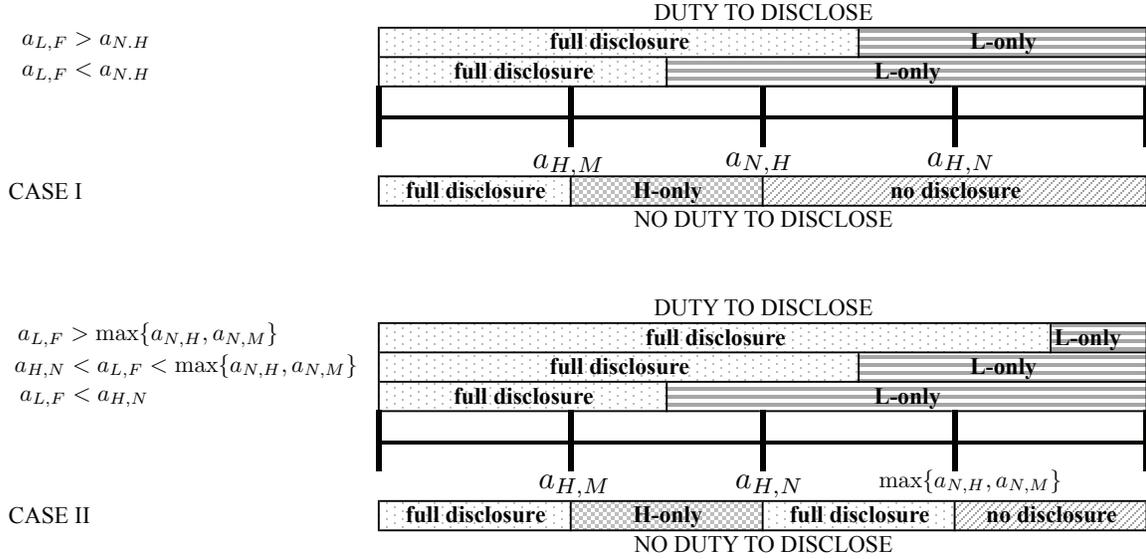


Figure 5: Cases with Duty to Disclose

$a_{N,H}$ , there is a region in which partial disclosure of  $L$  replaces partial disclosure of  $H$ . Case II has a surprising twist. When  $a_{L,F} < \max\{a_{N,H}, a_{N,M}\}$ , mandatory disclosure of  $L$  will lead to less disclosure if MLRP is violated. There was a region of full disclosure above  $a_{H,N}$  because of the inability of the manager to credibly commit to a partial disclosure policy. Mandatory disclosure introduces an effective commitment to partial disclosure, allowing the manager to avoid full disclosure. The other characteristics of mandatory disclosure are similar to Case I; the full disclosure region expands and the no disclosure region is eliminated and replaced by partial disclosure. As before,  $L$  may replace  $H$ , but only for some parameters. To summarize, when the manager places a relatively high weight on private benefits, the duty to disclose  $L$  grants him the ability to credibly commit to partial disclosure, increasing his private benefits and increasing the information asymmetry in the market, an unintended consequence of the duty to disclose.

The value of  $a_{L,F}$ , which partitions the line into  $L$ -only and full disclosure is increasing in the probability of a good signal when the true state is high,  $t_H$ , and decreasing in the probability of a good signal when the true state is medium,  $t_M$ . When MLRP is satisfied,  $a_{L,F}$  is increasing in the prior probability of the middle state,  $n$ , and when violated, it is decreasing in  $n$ . Finally,  $a_{N,H}$  is decreasing in  $t_H$ ; therefore if  $t_H$  is large (small), it is more likely that  $a_{L,F}$  is greater than (less than)  $a_{N,H}$ .

## 4 Selective Disclosure

The practice of selective disclosure, providing material, non-public information about a company to an analyst or other investor before disclosing it to the general public, is prohibited by RegFD. Specifically, Reg FD requires that when a manager discloses material non-public information to certain individuals, it must publicly report that information promptly (or simultaneously for intentional disclosure). In this section, we look at the impact of (now prohibited) selective disclosure on information asymmetries and price efficiency. We allow the manager full discretion over private (selective) and public disclosure and the manager is not bound to public disclosure if he has provided information privately to a subset of investors.

### 4.1 Full Private Disclosure

If the manager decides to engage in selective disclosure, he will (effectively) disclose all of his information privately.<sup>14</sup> When private disclosures are possible, the market maker must anticipate the selective disclosure and price securities under the belief that the informed trader is perfectly informed. We refer to the disclosure policy of selective disclosure and no public disclosure as  $d_N^S$ . The informed trader trades if and only he knows that the state of the world is  $H$  or  $L$ . He does not trade on the state  $M$  because relative to ex ante prices of  $\mu$ , he cannot earn profits. Therefore the market maker's pricing schedule is as follows when the informed trader submits a demand:

$$\begin{aligned}
 P(2, \emptyset, d_N^S) &= E(\theta|Y = 2) = \mu + e \\
 P(1, \emptyset, d_N^S) &= E(\theta|Y = 1) = \mu \\
 P(0, \emptyset, d_N^S) &= E(\theta|Y = 0) = \mu \\
 P(-1, \emptyset, d_N^S) &= E(\theta|Y = -1) = \mu \\
 P(-2, \emptyset, d_N^S) &= E(\theta|Y = -2) = \mu - e
 \end{aligned}$$

When order flow is  $Y = 2$ , the market maker knows the informed trader has issued a purchase order and can infer that the true state is  $H$ . However, when  $Y = 0$ , either the informed trader has bought and the liquidity trader sold or the informed trader has sold and the liquidity trader has bought. Since  $H$  and  $L$  are equally likely, the market maker cannot update his inferences about the state, and price is equal to the prior. The ex-ante profits of the informed trader with selective are equal to  $\frac{en}{2}$ . Not surprisingly, the informed trader is always better off under selective disclosure relative to public disclosure because he has perfect information. The informed trading will impact the price efficiency since trades are more informative, and the price volatility is equal to  $\frac{e^2n}{2}$ . Therefore, if the manager

---

<sup>14</sup>The manager would always choose to fully disclose privately his information if he can. When he cannot, the manager cannot partially disclose his information either.

does not have a preference to increase stock price, it is easy to see why regulation to eliminate selective disclosure would protect uninformed traders.

**Proposition 5**

- Suppose the probabilities of the informed trader's signal satisfy the monotone likelihood ratio property. The following orderings hold for information asymmetry and price efficiency.

<u>Parameters</u>	<u>Information asymmetry</u>	<u>Price efficiency</u>
$t_H < t_5$	$IA(d_N^S) > IA(d_H^S) > IA(d_N) > IA(d_H)$	$PE(d_H^S) > PE(d_H) > PE(d_N^S) > PE(d_N)$
$t_H > t_5$	$IA(d_N^S) > IA(d_N) > IA(d_H^S) > IA(d_H)$	$PE(d_H^S) > PE(d_H) > PE(d_N^S) > PE(d_N)$

- Suppose the probabilities of the informed trader's signal violate the monotone likelihood ratio property and let  $t_5 = 2 - \frac{2}{2-n}$  and  $t_6 = \frac{2(1-n)t_M}{2-n}$ . The following orderings hold for information asymmetry and price efficiency.

<u>Parameters</u>	<u>Information asymmetry</u>	<u>Price efficiency</u>
$\min(t_M, t_5) > t_H > t_6$	$IA(d_N^S) > IA(d_H^S) > IA(d_N) > IA(d_H)$	$PE(d_H^S) > PE(d_H) > PE(d_N^S) > PE(d_N)$
$\min(t_M, t_5) > t_6 > t_H$	$IA(d_N^S) > IA(d_H^S) > IA(d_H) > IA(d_N)$	$PE(d_H^S) > PE(d_H) > PE(d_N^S) > PE(d_N)$
$t_M > t_H > t_5 > t_6$	$IA(d_N^S) > IA(d_N) > IA(d_H^S) > IA(d_H)$	$PE(d_H^S) > PE(d_H) > PE(d_N^S) > PE(d_N)$

Price efficiency is higher with selective disclosure than without it, holding the disclosure policy fixed. Holding selective disclosure fixed, price efficiency is higher with partial disclosure than with no disclosure. In contrast, information asymmetry is highest with selective disclosure and no public disclosure. The informed trader has the most to earn when his information is perfect and there are no public disclosures (i.e., any advantages of public disclosure are absent when the informed trader's signal is perfect). Under partial public disclosure, information asymmetry is higher with selective disclosure than without it. Thus, we note again that the ordering of disclosure regimes based on price efficiency is not the same as the ordering based on information asymmetry. Price efficiency is higher with selective disclosure because the informed trader's information is fully revealed for some order flows. As a consequence, prices reflect better the true cash flows of the firm. At the same time, when the market maker cannot infer the signal (net order flow is zero), the informed trader's gains are much more valuable since the informed trader only makes profitable trades, based on perfect information. Public disclosure of  $H$  works against the informed trader and in favor of market price informativeness; it reduces the volatility in prices and eliminates trading gains whenever  $H$  is realized.

## 4.2 Manager's Optimal Public and Private Disclosure

First, suppose the manager considers no public disclosure. No public disclosure and selective disclosure to the informed trader ( $d_N^S$ ) is the most desirable outcome for the manager if he can implement it. We determine whether  $d_N^S$  is ex post incentive compatible. Expected profits for the informed trader are increasing in the precision of his information and therefore highest with perfect information. Price will be fully revealing when the liquidity trade is in the same direction as the informed trade, or uninformative when the liquidity trade offsets the informed trade. Therefore, from the manager's perspective, he could increase price half of the time by revealing  $H$ . He will forego the price increase for the higher trading profits whenever

$$a\Pi(H, d_N^S) + (1-a)P_H(\emptyset, d_N^S) \geq (1-a)(\mu + e) \quad \rightarrow \quad a > 1/2 \equiv a_{N,H}^S$$

If no public disclosure is not incentive compatible, will the manager engage in partial public disclosures by disclosing  $H$ ? We determine the threshold  $a_{H,M}^S$  to guarantee that the manager does not want to deviate to disclosing  $M$ , if it occurs.

$$a\Pi(M, d_H^S) + (1-a)P_M(\emptyset, d_H^S) \geq (1-a)\mu \quad \rightarrow \quad a > 1/2 \equiv a_{H,M}^S$$

Selective disclosure and partial public disclosure of  $H$  is therefore never optimal, as the manager prefers no disclosure when  $a > 1/2$ . When  $a < 1/2$ , no disclosure (with or without private disclosure) can be ruled out with the manager's preference to deviate to disclosure of  $H$ . Can the manager credibly follow the policy  $d_H$  without private disclosure? If  $a_{H,M} < 1/2$ , there is a region in which the manager prefers partial public disclosure without selectively sharing his information privately.

### Proposition 6

- (i) If  $a_{H,M} \leq 1/2$ , the manager chooses no public disclosure with selective disclosure when  $a > 1/2$ , partial public disclosure of  $H$  with no selective disclosure ( $d_H$ ) for  $a_{H,M} < a < 1/2$  and full disclosure for  $a < a_{H,M}$ .
- (ii) If  $a_{H,M} > 1/2$ , the manager chooses no public disclosure with selective disclosure when  $a > 1/2$  and full disclosure when  $a < 1/2$ .

Figure 6 illustrates Proposition 6. Part (i) is the top portion of the Figure and Part (ii) is the bottom portion. If the threshold at which the manager prefers full over partial disclosure (without selective disclosure)  $a_{H,M}$  is low, there is a region of partial disclosure with no selective disclosure. Otherwise

there is full disclosure (low  $a$ ) or no disclosure with selective disclosure (high  $a$ ). Partial public disclosure with selective disclosure is not part of any equilibrium disclosure strategy.

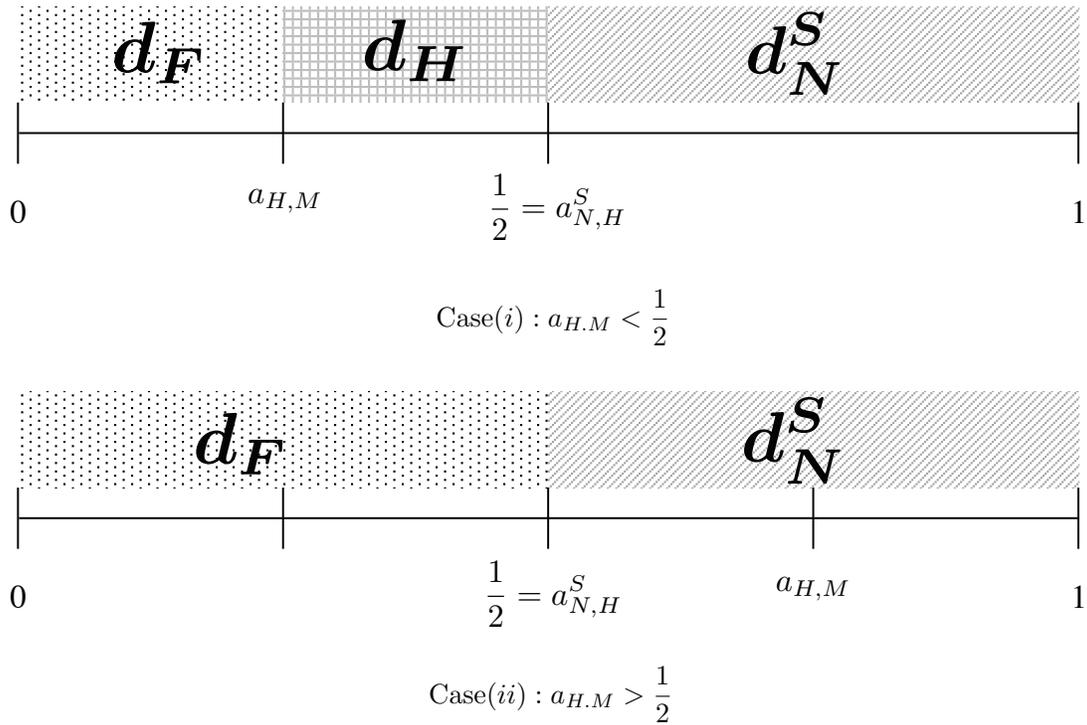


Figure 6: Selective Disclosure

Proposition 6 highlights the costs and benefits of selective disclosure. If the manager’s interests are closely tied to the informed trader (high  $a$ ), the manager prefers no public disclosure along with selective disclosure. Because selective disclosure improves price efficiency, it provides some ammunition for opponents of RegFD. However, price efficiency is not necessarily the measure of interest. A clear goal of Reg FD is to eliminate those cases in which the disclosure of important information allowed “those who were privy to such information to make a profit or avoid a loss at the expense of those kept in the dark.”<sup>15</sup> Based on part (i) of Prop 6, it is clear that selective disclosure with no public disclosure indeed increases the expected profits for the informed trader at the expense of the uninformed. If the CEO’s utility function places more weight on market price, he may replace  $d_N^S$  with  $d_H$ , meaning he provides partial public disclosure and no private disclosures.

The ability to selectively disclose increases the region of no disclosure relative to its size when public disclosure was not permitted. Therefore Reg FD (or the prohibition of selective disclosure)

<sup>15</sup>See <http://www.sec.gov/rules/final/33-7881.htm>

indeed increases the amount of disclosure. However, in some cases, even if selective disclosure were permitted, the manager *cannot* necessarily choose it. As well, if  $a_{H,M} < 1/2$ , the full disclosure region is the same “size” with or without Reg FD. Because partial disclosure of  $H$  is never accompanied by selective disclosure, the threshold  $a_{H,M}$  is exactly the same as in the previous analysis.

If we restrict our attention to  $t_H = 0$ , we would still get the two scenarios described above. However, this specific case allows us to gain intuition on what would happen if one might subvert the regulation by privately disclosing  $M$ . Knowing  $M$  would not affect beliefs about the firm relative to priors for the informed trader, but since prices are set after the the market maker observes public disclosures, his beliefs might be updated allowing the informed trader to profit. Reg FD that specifically relates to *material* non-public information. Material is (somewhat vaguely) defined as “there is a substantial likelihood that a reasonable shareholder would consider it important.” Managers could claim that it was not material information at the time they shared this information with the informed trader. Even if the manager had still the option to disclose privately  $M$ , he cannot implement no disclosure (private or public) or no public disclosure with private disclosure of  $M$ . However, he can sometimes credibly implement public disclosure of  $H$  with private disclosure of  $M$  and achieve his most desirable outcome (as the informed trader has perfect information). He optimally chooses to disclose publicly  $H$  if  $1/2 < a < \frac{2-n}{4(1-n)}$ . If  $a > 1/2$ , the manager does not renege (i.e., disclose  $M$  publicly). If  $a < \frac{2-n}{4(1-n)}$ , the manager will not withhold his information if  $M$  occurs. For  $a > \frac{2-n}{4(1-n)}$ , the manager can either implement public disclosures of  $H$  without privately disclosing  $M$  or has to switch to full disclosure when no partial public disclosure is feasible. For  $a < 1/2$ , depending on whether  $a_{H,M}$  is greater or below  $1/2$ , he will switch to full disclosure or first implement disclosure of  $H$  publicly without privately disclosing  $M$  and when the latter disclosure is no longer feasible, he fully discloses his information. In this context, RegFD would also lead to more disclosure if the manager receives high private benefits but might not affect the optimal disclosure policy when the manager cares less about private benefits.

## 5 Conclusion

Our paper helps to put the empirical findings that information asymmetry increase around public disclosures into context, without requiring us to assume that there are greater information gathering opportunities generated by the public disclosure. The private information is available with or without the disclosure, but the disclosure can improve the precision of the information. Depending on the initial quality of the information, disclosure can increase or decrease expected informed trading profits.

We then expand our understanding of how public disclosures affect information asymmetry and

the price formation process in a model with optimal disclosure decisions selected by a manager who cares about both stock prices and informed trading profits. The informed traders in our model have a noisy signal about firm value and managers can use public disclosure policies to implicitly reveal information about outcomes. That is, if the manager follows a policy where he discloses good news, no disclosure can be interpreted as “not good news.” These partial disclosure policies can increase expected informed trading profits by improving the precision of the informed trader’s signal. Because the information, if disclosed, can decrease informed trading profits for a particular realization, the optimal ex ante and ex post disclosure policies are not necessarily the same. Since our manager chooses optimally based on his private information and his compensation, providing the manager with joint incentives to maximize stock price and informed trading profits, rather than focusing on only informed profits, allows informed traders to actually increase their expected profits.

When we add in the possibility of mandatory disclosure, through a duty to disclose bad news, we find that optimal disclosures change. There is more disclosure in regions in which there would have been no disclosure without the mandatory policy, but mandatory regulation allows for commitment to an ex ante disclosure policy which may actually improve the expected profits of the informed traders. Therefore, such regulation may be counterproductive as it can increase information asymmetry and increase expected uninformed losses.

Finally, we examine selective disclosure and evaluate the impact that Reg FD has on optimal disclosure decisions. Although there are some regions in which the manager would choose to privately share his information with a subset of market participants, there are other regions in which the manager would not selectively disclose even if it were possible.

## References

- AFFLECK-GRAVES, J., C. M. CALLAHAN, AND N. CHIPALKATTI (2002): “Earnings Predictability, Information Asymmetry, and Market Liquidity,” *Journal of Accounting Research*, 40(3), pp. 561–583.
- BARCLAY, M. J., AND C. G. HOLDERNESS (1989): “Private benefits from control of public corporations,” *Journal of Financial Economics*, 25(2), pp. 371–395.
- BARRON, O. E., D. BYARD, AND O. KIM (2002): “Changes in analysts’ information around earnings announcements,” *The Accounting Review*, 77(4), 821–846.
- BARRON, O. E., D. G. HARRIS, AND M. STANFORD (2005): “Evidence that investors trade on private event-period information around earnings announcements,” *The Accounting Review*, 80(2), 403–421.

- BELEN, V., AND A. RAPHAEL (2006): "How do family ownership, control and management affect firm value?," *Journal of Financial Economics*, 80(2), pp.385–417.
- BERTOMEU, J., A. BEYER, AND R. A. DYE (2011): "Capital Structure, Cost of Capital, and Voluntary Disclosures," *The Accounting Review*, 86(3), 857–886.
- BEYER, A., D. A. COHEN, T. Z. LYS, AND B. R. WALTHER (2010): "The financial reporting environment: Review of the recent literature," *Journal of Accounting and Economics*, 50(2-3), 296–343.
- COLLER, M., AND T. L. YOHN (1997): "Management forecasts and information asymmetry: An examination of bid-ask spreads," *Journal of Accounting Research*, 35(2), 181–191.
- DIAMOND, D. W., AND R. E. VERRECCHIA (1991): "Disclosure, Liquidity, and the Cost of Capital," *The Journal of Finance*, 46(4), 1325–1359.
- DOIDGE, C., G. A. KAROLYI, K. V. LINS, AND R. M. STULZ (2009): "Private Benefits of Control, Ownership, and the Cross-listing Decision," *The Journal of Finance*, 64(1), pp. 425–466.
- DUTTA, S. (1996): "Private and public disclosures and the efficiency of stock prices," *Review of Accounting Studies*, 1(4), 285–307.
- DYE, R. A. (1985): "Disclosure of Nonproprietary Information," *Journal of Accounting Research*, 23(1), 123–145.
- (2013): "Voluntary Disclosure and the Duty to Disclose," *Kellogg Working Paper Series*.
- ECKBO, B. E., AND K. S. THORBURN (2003): "Control benefits and CEO discipline in automatic bankruptcy auctions," *Journal of Financial Economics*, 69, 227–258.
- FISCHER, P. E., AND R. E. VERRECCHIA (1999): "Public information and heuristic trade," *Journal of Accounting and Economics*, 27(1), 89–124.
- GOEX, R. F., AND A. WAGENHOFER (2009): "Optimal impairment rules," *Journal of Accounting and Economics*, 48(1), 2–16.
- GROSSMAN, S. J. (1995): "Dynamic asset allocation and the informational efficiency of markets," *The Journal of Finance*, 50(3), 773–787.
- HARRIS, M., AND A. RAVIV (1993): "Differences of opinion make a horse race," *Review of Financial Studies*, 6(3), 473–506.

- INDJEKIAN, R. J. (1991): "The impact of costly information interpretation on firm disclosure decisions," *Journal of Accounting Research*, 29(2), 277–301.
- KAMENICA, E., AND M. GENTZKOW (2011): "Bayesian Persuasion," *American Economic Review*, 101(6), 2590–2615.
- KANDEL, E., AND N. D. PEARSON (1995): "Differential Interpretation of Public Signals and Trade in Speculative Markets," *Journal of Political Economy*, 103(4), pp. 831–872.
- KIM, O., AND R. E. VERRECCHIA (1991): "Trading volume and price reactions to public announcements," *Journal of accounting research*, 29(2), 302–321.
- (1994): "Market Liquidity and Volume around Earnings Announcements," *Journal of Accounting and Economics*, 17, 41–67.
- KRINSKY, I., AND J. LEE (1996): "Earnings Announcements and the Components of the Bid-Ask Spread," *The Journal of Finance*, 51(4), pp. 1523–1535.
- KURZ, M. (1968): "Optimal Economic Growth and Wealth Effects," *International Economic Review*, 9, 348–357.
- LEE, C., B. MUCKLOW, AND M. READY (1993): "Spreads, depths, and the impact of earnings information: an intraday analysis," *Review of Financial Studies*, 6(2), 345–374.
- LEVINE, C. B., AND M. J. SMITH (2003): "Ex Post Voluntary Disclosure by Insiders," *Contemporary Accounting Research*, (20), 719–746.
- LUNDHOLM, R. J. (1988): "Price-Signal Relations in the Presence of Correlated Public and Private Information," *Journal of Accounting Research*, 26(1), 107–118.
- MCNICHOLS, M., AND B. TRUEMAN (1994): "Public disclosure, private information collection, and short-term trading," *Journal of Accounting and Economics*, 17(1), 69–94.
- MILGROM, P. R. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12(2), 380–391.
- MILGROM, P. R. (2004): *Putting auction theory to work*. Cambridge University Press.
- MILGROM, P. R., AND R. J. WEBER (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50(5), 1089–1122.

- STOCKEN, P. (2000): “Credibility of Voluntary Disclosure,” *Rand Journal of Economics*, 31(2), 359–374.
- TRUEMAN, B. (1997): “Managerial disclosures and shareholder litigation,” *Review of Accounting Studies*, 2(2), 181–199.
- VARIAN, H. R. (1988): “Differences of opinion in financial markets,” in *financial Risk: Theory, Evidence and implications*, pp. 3–37. Springer.
- VERRECCHIA, R. E. (1983): “Discretionary Disclosure,” *Journal of Accounting and Economics*, 5, 179–194.
- YOHAN, T. (1998): “Information Asymmetry Around Earnings Announcements,” *Review of Quantitative Finance and Accounting*, 11(2), pp. 165–182.
- ZOU, H.-F. (1994): “The spirit of capitalism’ and long-run growth,” *European Journal of Political Economy*, 10(2), 279–293.

## Proofs

### Sketch of Proof of Proposition 1:

$$\begin{aligned}\Pi(d_N) &= \frac{en(t_H - t_L)}{2} \\ \Pi(d_H) &= \frac{en(1-n)(t_M - t_L)}{(2-n)} \\ \Pi(d_L) &= \frac{en(1-n)(t_H - t_M)}{(2-n)}\end{aligned}$$

The derivative of  $\frac{(1-n)}{(2-n)}$  in  $n$  is  $-\frac{1}{(2-n)^2} < 0$ . Thus,  $\frac{(1-n)}{(2-n)} < \frac{1}{2}$ . Further  $t_M - t_L \leq t_H - t_L$  and  $t_H - t_M \leq t_H - t_L$  because  $t_H \geq t_M \geq t_L$ . We conclude that  $\Pi(d_N) \geq \Pi(d_H)$  and  $\Pi(d_N) \geq \Pi(d_L)$ . Further,  $\Pi(d_H) \geq \Pi(d_L)$  if  $t_H \leq 2t_M - t_L$ . Otherwise  $\Pi(d_H) < \Pi(d_L)$ .

We turn to the analysis of the variance of value given prices:  $\forall i \in \{H, L, N\}, V(\theta - P|d_i) = V(d_i)$ .

$$\begin{aligned}
V(d_i) &= \frac{1}{2}nt_H \left( \frac{1}{2}(e - P(2, \emptyset, d_i) + \mu)^2 + \frac{1}{2}(e - P(0, \emptyset, d_i) + \mu)^2 \right) + \\
&\quad \frac{1}{2}n(1 - t_H) \left( \frac{1}{2}(e - P(-2, \emptyset, d_i) + \mu)^2 + \frac{1}{2}(e - P(0, \emptyset, d_i) + \mu)^2 \right) \\
&\quad + \frac{1}{2}nt_L \left( \frac{1}{2}(-e - P(2, \emptyset, d_i) + \mu)^2 + \frac{1}{2}(-e - P(0, \emptyset, d_i) + \mu)^2 \right) \\
&\quad + \frac{1}{2}n(1 - t_L) \left( \frac{1}{2}(-e - P(-2, \emptyset, d_i) + \mu)^2 + \frac{1}{2}(-e - P(0, \emptyset, d_i) + \mu)^2 \right) \\
&\quad + (1 - n)t_M \left( \frac{1}{2}(\mu - P(2, \emptyset, d_i))^2 + \frac{1}{2}(\mu - P(0, \emptyset, d_i))^2 \right) \\
&\quad + (1 - n)(1 - t_M) \left( \frac{1}{2}(\mu - P(-2, \emptyset, d_i))^2 + \frac{1}{2}(\mu - P(0, \emptyset, d_i))^2 \right)
\end{aligned}$$

After rearranging the terms, we obtain expressions 6, 8 and 10.

We want to show that  $\frac{\partial V(d_N)}{\partial t_H} = \frac{e^2 n^2 (t_H - t_L) (2n^2 (t_L - t_M) (t_H + t_L - 2t_M) + n(2t_M - 1) (t_H + 3t_L - 4t_M) - 4(1 - t_M)t_M)}{(n(t_H + t_L - 2t_M) + 2(t_M - 1))^2 (n(t_H + t_L - 2t_M) + 2t_M)^2} < 0$ . To see that  $V(d_N)$  is decreasing in  $t_H$ , we need to sign

$$\Gamma(n) = (2n^2(t_L - t_M)(t_H + t_L - 2t_M) + n(2t_M - 1)(t_H + 3t_L - 4t_M) - 4(1 - t_M)t_M),$$

which a quadratic polynomial expression in  $n$ . Let  $a_1$ ,  $b_1$  and  $c_1$  be the following coefficients of the quadratic polynomial expression:

$$\begin{aligned}
a_1 &= 2n^2(t_L - t_M)(t_H + t_L - 2t_M) \\
b_1 &= (2t_M - 1)(t_H + 3t_L - 4t_M) \\
c_1 &= -4(1 - t_M)t_M < 0
\end{aligned}$$

If  $\Gamma(n)$  admits two real roots then we have three scenarios to consider.

1. If  $a_1 > 0$ , then the two roots have opposite signs.
2. If  $a_1 < 0$  and  $b_1 > 0$  then the roots are positive.
3. If  $a_1 < 0$  and  $b_1 < 0$  both roots are negative.

For case 1., we know that  $\Gamma(0) = c_1 < 0$ . So if  $\Gamma(1) < 0, \forall n \in [0, 1], \Gamma(n) < 0$ . Let us show that  $\Gamma(1) < 0$ :

$$\Gamma(1) = 2t_L^2 + (2t_H - 3)t_L - t_H \tag{11}$$

This expression is a quadratic polynomial expression  $t_L$  where his coefficients are  $2 > 0$  and  $-t_H < 0$  and  $(2t_H - 3)$ , which sign is indeterminate. If expression 11 admits two real roots, they are of opposite signs. At  $t_L = 0$ , expression 11 is equal to  $-t_H < 0$  and at  $t_L = 1$ , it is equal to  $-(1 - t_H) < 0$ . Thus,  $\Gamma(1) < 0$ . Further notice that expression 11 cannot have a negative discriminant because at  $t_L = 0$ , the expression is negative.

For case 2., we further need to prove that the root of  $\Gamma(n) \frac{-b_1 + \sqrt{\Delta}}{2a_1} > 1$  to guarantee that  $\Gamma(n) < 0$ . Rearranging the terms, we need to show that  $a_1 + b_1 + c_1 < 0$ . Taking the coefficients  $a_1$ ,  $b_1$  and  $c_1$  of  $\Gamma(n)$ ,  $a_1 + b_1 + c_1 = 2t_L^2 + (2t_H - 3)t_L - t_H = \Gamma(1) < 0$ . Thus,  $\Gamma(n) < 0$ .

For case 3., it is immediate to see that  $\Gamma(n) < 0$ .

If  $\Gamma(n)$  has a negative discriminant and given that  $\Gamma(0) < 0$ ,  $a_1 < 0$ . Thus,  $\forall n \in [0, 1], \Gamma(n) < 0$ .

At  $t_H = 1$ , we need to show that:

$$V(d_N) - V(d_H) = \frac{e^2 n (2n^3 (t_L - t_M)^2 - n^2 (5t_L^2 - 16t_L t_M + t_L + 2t_M (6t_M - 1)) + 2nt_M (-6t_L + 9t_M - 1) - 8t_M^2)}{2(n-2)(n(t_L - 2t_M) + 2t_M)(n(t_L - 2t_M + 1) + 2t_M)} > 0 \quad (12)$$

Collecting the terms in  $t_M$  of the numerator of expression 12, it yields:

$$\underbrace{(-8 + 18n - 12n^2 + 2n^3)t_M^2}_{<0} + \underbrace{(-2n + 2n^2 - 12nt_L + 16n^2 t_L - 4n^3 t_L)t_M}_{<0} - \underbrace{n^2 t_L - 5n^2 t_L^2 + 2n^3 t_L^2}_{<0} < 0 \quad (13)$$

Collecting the terms in  $t_M$  of the denominator of expression 12, it yields:

$$\underbrace{-2(2-n)4(1-n(2-n))t_M^2}_{<0} - \underbrace{2(2-n)(2n(1-n) + 4nt_L(1-n))t_M}_{<0} - \underbrace{2(2-n)(n^2 t_L + n^2 t_L^2)}_{<0} < 0 \quad (14)$$

Therefore,  $V(d_N) > V(d_H)$ .

The reasoning is symmetric to prove that  $\frac{\partial V(d_N)}{\partial t_L} > 0$  and at  $t_L = 0$ ,  $V(d_N) - V(d_L) > 0$ . Therefore,  $V(d_N) > V(d_L)$ .

Moreover, at  $t_H = t_L$ ,  $V(d_H) = V(d_L)$ . If  $t_H > t_L$ ,  $V(d_H) < V(d_L)$  otherwise  $V(d_H) > V(d_L)$ .

**Sketch of Proof of Proposition 2:** If  $t_M \geq t_H \geq t_L$ ,  $\Pi(d_H) > \Pi(d_L)$ .

- i. At  $t_M = t_1 > t_L$ ,  $\Pi(d_H) = \Pi(d_N)$  and if  $t_M > t_1$ ,  $\Pi(d_H) < \Pi(d_N)$ .
- ii. At  $t_H = t_3 > t_L$ ,  $\Pi(d_L) = \Pi(d_N)$  and if  $t_H > t_3$ ,  $\Pi(d_L) < \Pi(d_N)$ .
- iii. By inspection,  $t_1 > t_3$ .

If  $t_M \geq t_L \geq t_H$ ,  $\Pi(d_L) > \Pi(d_H)$ .

- i. At  $t_H = t_2 > t_L$ ,  $\Pi(d_L) = \Pi(d_N)$  and if  $t_H > t_2$ ,  $\Pi(d_L) < \Pi(d_N)$ .
- ii. At  $t_H = t_4 > t_L$ ,  $\Pi(d_L) = \Pi(d_N)$  and if  $t_H > t_4$ ,  $\Pi(d_L) < \Pi(d_N)$ .
- iii. By inspection,  $t_2 > t_4$ .

**Sketch of Proof of Proposition 3:**

Conditions to implement no disclosure:

i. If  $H$  occurs, the profits by following the no disclosure policy are:

$$\begin{aligned} & \underbrace{a \frac{1}{2} ent_H}_{\text{private benefits}} + (1-a) \underbrace{\left( \frac{t_H(P(2, \emptyset, d_N) + P(0, \emptyset, d_N))}{2} + \frac{(1-t_H)(P(-2, \emptyset, d_N) + P(0, \emptyset, d_N))}{2} \right)}_{\text{price}} \\ = & a \frac{1}{2} ent_H + (1-a) \left( \frac{ent_H^2}{2n(t_H - 2t_M) + 4t_M} - \frac{en(t_H - 1)t_H}{2(n(t_H - 2t_M) + 2(t_M - 1))} + \mu \right) \end{aligned} \quad (15)$$

The profits in 15 needs to be greater than disclosing publicly  $H$  and getting  $(1-a)(\mu + e)$ . Simplifying and rearranging  $a \geq a_{N,H}$ .

ii. If  $M$  occurs. the profits by following the no disclosure policy are:

$$\begin{aligned} & \underbrace{a \frac{1}{2} ent_H}_{\text{private benefits}} + (1-a) \underbrace{\left( \frac{t_M(P(2, \emptyset, d_N) + P(0, \emptyset, d_N))}{2} + \frac{(1-t_M)(P(-2, \emptyset, d_N) + P(0, \emptyset, d_N))}{2} \right)}_{\text{price}} \\ = & a \frac{1}{2} ent_H + (1-a) \left( \frac{(1-a)(en^2 t_H(t_H - 2t_M) + 2\mu(n(t_H - 2t_M) + 2(t_M - 1))(n(t_H - 2t_M) + 2t_M))}{2(n(t_H - 2t_M) + 2(t_M - 1))(n(t_H - 2t_M) + 2t_M)} \right) \end{aligned} \quad (16)$$

The profits in 16 needs to be greater than disclosing publicly  $H$  and getting  $(1-a)\mu$ . Simplifying and rearranging  $a \geq a_{M,H}$ .

Therefore no disclosure is implementable if  $a \geq \max(a_{N,H}, a_{N,M})$ .

Conditions to implement partial disclosure of  $H$ :

i. If  $H$  occurs, the profits hiding  $H$  are:

$$\begin{aligned} & \underbrace{a \frac{2e(1-n)nt_M}{(n-2)^2}}_{\text{private benefits}} + (1-a) \underbrace{\left( \frac{th(P(2, \emptyset, d_H) + P(0, \emptyset, d_H))}{2} + \frac{(1-th)(P(-2, \emptyset, d_H) + P(0, \emptyset, d_H))}{2} \right)}_{\text{price}} \\ = & a \frac{2e(1-n)nt_M}{(n-2)^2} + (1-a) \left( en \left( \frac{1}{n-2} - \frac{t_H - 1}{-2nt_M + n + 2t_M - 2} \right) + 2\mu \right) \end{aligned} \quad (17)$$

The profits in 17 needs to be lower than disclosing publicly  $H$  and getting  $(1-a)(\mu + e)$ . Simplifying and rearranging  $a \leq a_{H,N}$ .

ii. If  $M$  occurs. the profits by following partial disclosure of  $H$  are:

$$\begin{aligned} & \underbrace{a \frac{2e(1-n)nt_M}{(n-2)^2}}_{\text{private benefits}} + (1-a) \underbrace{\left( \frac{tm(P(2, \emptyset, d_H) + P(0, \emptyset, d_H))}{2} + \frac{(1-tm)(P(-2, \emptyset, d_H) + P(0, \emptyset, d_H))}{2} \right)}_{\text{price}} \\ = & a \frac{2e(1-n)nt_M}{(n-2)^2} \\ & + (1-a) \left( \frac{en((n-1)(3n-4)t_M^2 - 2(n-2)(n-1)t_M) + \mu(n-2)(2t_M - 2nt_M)(-2nt_M + n + 2t_M - 2)}{(n-2)(2t_M - 2nt_M)(-2nt_M + n + 2t_M - 2)} \right) \end{aligned} \quad (18)$$

The profits in 18 needs to be greater than disclosing publicly  $H$  and getting  $(1-a)\mu$ . Simplifying and rearranging  $a \geq a_{H,M}$ .

Therefore, partial disclosure of  $H$  is implementable if  $a \geq a_{H,M}$  and  $a \leq a_{H,N}$ .

Conditions to implement partial disclosure of  $M$ :

i. If  $H$  occurs, the profits by following partial disclosure of  $M$  are:

$$\underbrace{\frac{1}{2}et_H}_{\text{private benefits}} + \underbrace{(1-a) \left( \frac{t_H(P(2, \emptyset, d_M) + P(0, \emptyset, d_M))}{2} + \frac{(1-t_H)(P(-2, \emptyset, d_M) + P(0, \emptyset, d_M))}{2} \right)}_{\text{price}} \quad (19)$$

$$= \frac{1}{2}et_H + (1-a) \left( \frac{(1-a)(2\mu(t_H-2)t_H - et_H^2)}{2(t_H-2)t_H} \right) \quad (20)$$

The profits in 20 needs to be greater than disclosing publicly  $H$  and getting  $(1-a)(\mu + e)$ . Simplifying and rearranging  $a \geq a_{M,H} \equiv \frac{3t_H-4}{t_H^2+t_H-4}$ .

ii. If  $M$  occurs, the profits hiding  $M$  are:

$$\underbrace{\frac{1}{2}et_H}_{\text{private benefits}} + \underbrace{(1-a) \left( \frac{t_M(P(2, \emptyset, d_M) + P(0, \emptyset, d_M))}{2} + \frac{(1-t_M)(P(-2, \emptyset, d_M) + P(0, \emptyset, d_M))}{2} \right)}_{\text{price}} \quad (21)$$

$$= \frac{1}{2}et_H + (1-a) \left( \frac{e(t_H-2t_M) + 2\mu(t_H-2)}{2(t_H-2)} \right)$$

The profits in 21 needs to be lower than disclosing publicly  $M$  and getting  $(1-a)\mu$ . Simplifying and rearranging  $a < a_{M,N} \equiv \frac{2t_M-t_H}{(t_H-3)t_H+2t_M}$ , which needs to be lower than 1.

Therefore, partial disclosure of  $M$  is implementable if  $a \geq a_{M,H}$  and  $a \leq a_{M,N}$ .

We show that partial disclosure of  $M$  is never implementable.

If  $t_H < \frac{1}{2}(3 - \sqrt{9 - 8t_M})$ ,  $a_{M,N} > 1$ . From expression 21, at  $a = 1$ , it is immediate to see that the manager would always withhold disclosure of  $M$  if it occurs. If  $t_H \geq \frac{1}{2}(3 - \sqrt{9 - 8t_M})$ ,  $a_{M,H} > a_{M,N}$ .

Ordering of the thresholds.

**Step 1.  $a_{H,N}$  and  $a_{H,M}$ :**

$$\frac{\partial a_{H,N}}{\partial t_H} = \frac{4(n-2)^2(1-n)n^2t_M(2(1-n)t_M+n-2)}{((n-2)^2(nt_H-4) + 8(n-1)^2nt_M^2 - 2(n-1)(3n-4)(n-2)t_M)^2} < 0 \quad (22)$$

The difference  $a_{H,N} - a_{H,M}$  is decreasing in  $t_H$ .

$$\text{At } t_H = 1, a_{H,N} - a_{H,M} = \frac{4(n-2)^2(1-n)t_M(n(5t_M-3) + 4(1-t_M))}{(8(n-1)^2t_M^2 - n(n-2)t_M - 2(n-2)^2)(n(4(n-1)t_M - n + 6) - 8)} > 0 \quad (23)$$

Thus,  $a_{H,M} < a_{H,N}$  and partial disclosure of  $H$  is always implementable.

**Step 2.**  $a_{N,H}$  and  $a_{H,N}$ :

$$\frac{\partial a_{N,H}}{\partial t_H} = \frac{n(n t_H^2 G'(t_H) - 2)}{(n t_H G(t_H) + n t_H + 2)^2}$$

where  $G'(t_H) = \frac{(n-2)n^2 t_H^2 - 4(n-1)t_M(n^2 t_H - 2) + 4(n-1)^2(n+2)t_M^2}{(n(t_H - 2t_M) + 2t_M)^2(n t_H - 2n t_M + 2t_M - 2)^2}$

We show that  $G'(t_H) < 0$ . To do that, we only need to sign the numerator, which is a quadratic polynomial expression in  $t_M$ .

$$\underbrace{-(2-n)n^2 t_H^2}_{<0} \underbrace{-4(1-n)(2-n^2 t_H)}_{<0} t_M + \underbrace{4(1-n)^2(n+2)}_{>0} t_M^2,$$

which admits only one positive root. At  $t_M = 0$ ,  $-(2-n)n^2 t_H^2 < 0$

and at  $t_M = 1$ ,  $n(n(2-t_H)(-n(2-t_H)+2t_H)-4) < 0$ . Thus,  $G'(t_H) < 0$  and  $a_{N,H}$  is decreasing in  $t_H$ .

$$\begin{aligned} \text{At } t_H = 0, a_{N,H} - a_{H,N} &= \frac{2(n-1)n t_M(n(2t_M-1)-2t_M+2)}{n^3 t_M(4t_M-3) + n^2(-8t_M^2+13t_M-2) + 2n(2t_M^2-9t_M+4) + 8(t_M-1)} > 0 \\ \text{At } t_H = t_1 < t_M, a_{N,H} - a_{H,N} &= 0 \end{aligned}$$

Thus, if  $t_M \geq t_H \geq t_1$ ,  $a_{N,H} < a_{H,N}$ . Otherwise  $a_{N,H} > a_{H,N}$ .

**Step 3.**  $a_{N,M}$  and  $a_{H,M}$ :

We only need to order them for  $t_M > t_H > t_1$ .

$$\frac{\partial a_{N,M}}{\partial t_H} = \frac{n(n^2(t_H - 2t_M)^2 + 4(1-t_M)t_M)}{(n^2(t_H - 2t_M)^2 + n(4t_M - 3)(t_H - 2t_M) + 4(t_M - 1)t_M)^2} > 0 \quad (24)$$

The difference  $a_{N,M} - a_{N,H}$  is increasing in  $t_H$ .

$$\text{At } t_H = t_M, a_{N,M} - a_{N,H} = -\frac{2(n-2)^2((n-2)t_M+2)^2}{(n^2 t_M - 4n t_M + 3n + 4t_M - 4)(n^3 t_M^2 + n^2(5-4t_M)t_M + 4n(t_M^2 - 3t_M + 1) + 8(t_M - 1))} < 0$$

Thus,  $a_{N,M} < a_{N,H}$ .

**Sketch of Proof of Proposition 4:** We take the derivative of  $a_{L,F}$  in  $t_H$  and show that it is positive.  $a_{L,F}$  is increasing in  $t_H$ .

- i. The difference  $a_{L,F} - a_{H,M}$  is increasing in  $t_H$ . At  $t_H = 0$ , we can show that  $a_{L,F} - a_{H,M} > 0$  and at  $t_H = t_M$ ,  $a_{L,F} - a_{H,M} > 0$ . Thus,  $a_{L,F} > a_{H,M}$ .
- ii. The difference  $a_{L,F} - a_{H,N}$  is increasing in  $t_H$ . At  $t_H = 0$ , we can show that  $a_{L,F} - a_{H,N} < 0$  and

at  $t_H = t_M$ ,  $a_{L,F} - a_{H,N} > 0$ . Thus,  $a_{L,F} > a_{H,N}$  if  $t_H > t_M$ .

- iii. The difference  $a_{L,F} - a_{N,H}$  is increasing in  $t_H$ . At  $t_H = 0$ , we can show that  $a_{L,F} - a_{N,H} < 0$  and at  $t_H = t_M$ ,  $a_{L,F} - a_{N,H} > 0$ . Thus,  $a_{L,F} > a_{N,H}$  if  $t_H > t_M$ .

**Sketch of Proof of Proposition 6:**

- i. We prove next that  $a_{N,H}$  is increasing in  $t_M$ .

$$\frac{\partial a_{N,H}}{\partial t_M} = \frac{n^2 t_H^2 G'(t_M)}{(n t_H G(t_M) + n t_H + 2)^2}$$

where  $G'(t_M) = 2(1-n) \left( \frac{1-t_H}{(n t_H - 2 n t_M + 2 t_M - 2)^2} + \frac{t_H}{(n(t_H - 2 t_M) + 2 t_M)^2} \right) > 0$

We already know that  $a_{N,H}$  is decreasing in  $t_H$ . At  $t_H = 1$  and  $t_M = 0$ ,  $a_{N,H}$  is minimized and equal to  $\frac{1}{1+n}$ . Thus,  $a_{N,H} > 1/2$ .

- ii.  $a_{H,N}$  is decreasing in  $t_H$ . At  $t_H = 1$ ,  $a_{H,N} = -\frac{(n-4)(n-2)}{n(4(n-1)t_M - n + 6) - 8}$ , which is increasing in  $t_M$  and greater than  $\frac{n^2 - 6n + 8}{-3n^2 - 2n + 8} > 1/2$ . Thus,  $a_{H,N} > 1/2$ .