Valuation Issues Related to Buy-in Payments in Cost-Sharing Agreements

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Abstract

This paper studies valuation issues related to the buy-in payments in cost-sharing agreements. Three issues are studied. First, the paper shows how to calculate depreciation rates using the so-called "residual profit split method" so that the resulting buy-in payment equals the market value of the pre-buy in intangibles. Second, the paper establishes that if a taxpayer elects to calculate the buy-in payment using a royalty rate, the taxpayer has an incentive to overestimate its forecast of the revenues of the subsidiary that makes use of the pre-buy in intangibles. Third, the paper shows how the "market cap" method can be implemented so as to calculate the buy-in payment correctly.

Introduction

Cost sharing agreements between a U.S.-based parent and a foreign-based subsidiary have become an increasingly popular method of allocating the costs, and determining the tax liabilities, associated with intangibles used by both the parent and subsidiary under Treasury regulations §1.482–7. (See, e.g., Boos [2003], Bose [2002], Brooks, Kwiat, Weissler [2004], Feinschreiber [2004], Femia, Kirmil [2005], Levey [2000], Levey [2001], Markham [2005], Schrotenboer [2003].) Under a cost-sharing agreement, the parent and subsidiary share the costs of intangible assets they develop after the agreement in proportion to the "pattern of benefits" the parent and subsidiary anticipate receiving from the intangibles they both contribute to the agreement.[1] The attractiveness to the taxpayer of cost-sharing agreements is that, in the typical case where the parent is responsible for generating most of the intangibles used by the sub, and the sub is located in a jurisdiction where the tax rate on corporate profits is lower than in the U.S., the sub only has to make a payment to the parent for the *cost* of (its share of the benefits from) the intangibles the parent develops, rather than a payment based on the *market value* of those intangibles.

The date the cost-sharing agreement becomes effective is referred to as the "buy-in date." The parent often produces intangibles used by the subsidiary prior the buy-in date. The (so-called) pre-buy in intangibles the parent contributes to the sub could be, for example, marketing intangibles (e.g., brand name, distribution channels, relationships with customers) or technology intangibles (e.g., licenses to allow the subsidiary to sell products made by the parent, methods of production the parent developed, etc.). A necessary component of implementing a cost-sharing agreement, when the parent has produced such pre-buy in intangibles, is that the subsidiary must make a payment to the parent, called the buy-in payment, that compensates the parent for the value of the

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[1]The determination of what constitutes the "pattern of benefits," whether it be the income, sales (in units or dollars), cost-savings, or something else, generated by the intangibles is also described in the -7 regulations, and is discussed further below.

(pre-buy in) intangibles the parent developed.[2] As these pre-buy in intangibles can be substantial, the buy-in payment – which constitutes taxable income to the parent – can be sizeable. Consequently, how this buy-in payment is calculated can be of considerable import to both the taxpayer and the Internal Revenue Service.

According to section §1.482-7, the buy-in payment is supposed to equal the market price, or arm's length value, of the pre-buy in intangibles.[3] Section §1.482 also requires that arms' length or market prices must be used to value *tangible* assets transferred between a parent and its sub. Unfortunately, the most common method of valuing tangible assets for tax purposes - by means of "comparable uncontrolled prices" (CUPS) - is seldom useful in valuing the transfer of intangible assets. The CUPS method entails identifying the price paid for (tangible) assets transferred between independent (uncontrolled) taxpayers that are similar to the (tangible) assets transferred between the (controlled) parent and sub, and adjusting this price, as necessary, for any differences between the controlled and uncontrolled transfers. The problem with applying the CUPS method to intangibles is that it is extremely rare to be able to find, for either product or marketing intangibles, exchanges of intangibles in uncontrolled transactions that are even remotely similar to the intangibles the parent transfers to the sub prior to the implementation of a cost sharing agreement.

This paper studies what is perhaps the most common method used to construct the buy-in payment for intangibles, the so-called residual profit split (RPS) method. According to this method, the buy-in payment is constructed by first estimating the projected profits of the sub for each period after the buy-in date, then subtracting a so-called "routine return" for that portion of the sub's profits that are (believed to be) generated by the routine activities the

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[2] In general, both the parent and sub may contribute pre-buy in intangibles. In that case, each party's pre-buy in intangibles must be valued on the buy-in date.

[3] Later in the paper, we cite the precise language for the buy-in payment contained in the -7 regulations.

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sub engages in. The projected profits that result from the subtraction of the routine profits from the sub's projected total profits are referred to as the sub's "residual profits." If the routine profits are calculated correctly, the residual profits are due entirely to intangibles supplied by one of: either the parent or the sub, either before or after the buy-in date. Implementing the RPS method requires identifying that portion of the sub's residual profits for each period after the buy-in date that is attributable exclusively to the parent's pre-buy in intangibles. Once this portion is identified for each post-buy in period, the buy-in payment is calculated as the present value of the sum of those portions, i.e., the present value of the sum of the portions of each post buy-in period's residual profits attributable to the parent's pre buy-in intangibles.

One of the questions this paper addresses is how to calculate these portions attributable to the parent's pre-buy in intangibles. In practice, they are often determined as follows: in each post buy-in period, calculate what remains of the parent's pre-buy in expenditures on intangibles as of that period - that is, calculate the undepreciated (or capitalized) value of the parent's pre-buy in expenditures as of that period. Also calculate, for each such post buy-in period, the total undepreciated value of all expenditures on intangibles, regardless of whether those expenditures were made by the parent or sub, and regardless of whether those expenditures were made before or after the buy-in date. Then, take the ratio of the capitalized value of the parent's pre-buy in expenditures on intangibles as of that period to the total capitalized value of all expenditures on intangibles as of that period as the portion of that period's residual profits attributable to the parent's pre-buy in intangibles.

Obviously, this procedure is sensitive to what depreciation rates are used to map the history of the parent's and sub's expenditures on intangibles into the capitalized values of intangibles in each post buy-in period. Unlike many settings in accounting, where there is no clear criterion for whether one depreciation procedure is better or worse than another, there is an unambiguous theoretical

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criterion by which to judge depreciation rates or schedules here: a depreciation schedule is "correct" if, when combined with the other elements of the RPS method described above, the present value of the sums of the portions of the sub's post-buy in residual profits assigned to the parent's pre buy-in intangibles equals the arm's length value of the parent's pre-buy in intangibles. A "correct" depreciation schedule can be described alternatively as capitalizing the observed pre buy-in and post buy-in expenditures on intangibles so as to construct (from the observed sequence of residual profits) the *counterfactual* or *"as if"* sequence of residual profits that would have resulted had the parent stopped contributing intangibles to the sub as of the buy-in date.[4]

We show how to calculate the "correct" depreciation schedule. The "correct" schedule depend on the specifics of the production process mapping expenditures on intangibles into the sub's residual profits. In a broad sense, the sensitivity of the "correct" depreciation schedule to the particular time series process generating the post buy-in date residual profits of the sub is reminiscent of the observation that the appropriate depreciation method for an asset depends upon the pattern of future benefits anticipated to be generated by the asset (Christensen and Demski [1990]). We show that, often, none of the common methods of accounting for depreciation - none of the straight-line methods, accelerated methods, or compound depreciation methods - constitutes a "correct" method. For many plausible time series processes generating the sub's residual profits, the "correct" depreciation method involves either no, or even negative, depreciation of the parent's pre-buy in expenditures on intangibles for several periods after the buy-in date. The reason traditional methods of depreciation generally do not work is a combination of (a). many traditional depreciation methods are designed solely as a vehicle for allocating the acquisition cost of an asset over

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[4]This is one interpretation of the -7 regulations specifying the buy-in payment. We discuss this, along with other, interpretations of the -7 regulations specifying the buy-in payment below.

the length of the asset's economic life without regard to any other objective,[5] and (b). none of the traditional depreciation methods was developed for the specific purpose of trying to recover the *"as if"* residual profit sequence of the sub from the sub's *observed* residual profit sequence.

In addition to determining "correct" depreciation schedules, another issue we study is the influence of alternative methods of making the buy-in payment allowed under §1.482–7 on the taxpayer's incentives to forecast accurately the sub's post buy-in sales.[6] The taxpayer has three options under -7: make a single lump sum payment, make a sequence of installment payments, or make a sequence of royalty payments. If the taxpayer opts for either the single lump sum payment or a sequence of installment payments, it is obvious that, in the typical case where the sub's post buy-in forecasted residual profits increase in the sub's post buy-in forecasted sales, the taxpayer has an incentive to understate the sub's future sales. In contrast, we show that if the taxpayer opts for the sequence of royalty payments, the taxpayer has an incentive to overstate the sub's future sales forecasts. A brief explanation for why this happens is as follows. (More complete enumeration may be found in the text below.) By definition, the sub's periodic royalty payment is the product of the royalty rate and the sub's actual sales in each post buy-in period. The royalty rate is calculated so that if the

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[5] For example, consider the following observation from the Committee on Accounting Terminology of the AIA (the predecessor of the AICPA) in Accounting Terminology Bulletin No. 1: "Depreciation accounting is a system of accounting which aims to distribute the cost or other basic value of tangible capital assets, less salvage value (if any), over the estimated useful life of the unit (which may be a group of assets) in a systematic and rational manner. It is a process of allocation, not of valuation. [paragraph 56]

.........a cost or other basic value is allocated to accounting periods in a rational and systematic method... and this method does not attempt to determine the sum allocated to an accounting period solely by relation to occurrences within that period which affect either the length of life or the monetary value of the property. Definitions are unacceptable which imply that depreciation for the year is a measurement, expressed in monetary terms, of the physical deterioration within the year, or the decline in monetary value within the year, or indeed, of anything that actually occurs within the year. [paragraph 54]."

As another example, consider the remark in Revsine, Collins, and Johnson [2002]

"We stress this *absence of correspondence* between accounting measures of depreciation and value decrement, because accounting depreciation, computed following GAAP, ia a process of cost allocation, not asset valuation." (emphasis in original), p. 478.

[6] See §1-482-7(g)(7).

sub's actual sales coincides with the sub's forecasted sales, then the present value of the sequence of royalty payments, and therefore, the parent's tax liability, will coincide with the single lump sum payment. This property of the royalty rate *requires* the royalty rate to be set as the ratio of the lump sum buy-in payment to the expected present value of the sub's post buy-in sales. While understating its forecast of its sub's future sales has the advantage of reducing the estimated residual profits attributable to pre-buy in intangibles - and therefore reducing the numerator of the royalty rate - overstating the taxpayer's forecast of the sub's future sales has the advantage of increasing the estimated present value of the sub's post buy-in sales - and therefore increasing the denominator of the royalty rate. Our calculations indicate that, when the sub's forecasted post buy-in sales and residual profits are expected to grow at constant rates, this "denominator effect" is always larger than the "numerator effect," and so the royalty rate declines in the sub's forecasted growth rate of sales. Thus, a rational (parent) taxpayer who elects to have its sub pay the buy-in payment based on a sequence of royalty payments can reduce the expected present value of its tax liability by overstating the sub's post buy-in forecasted sales.

A third issue addressed in the paper is the use of the so-called "market cap" method for assessing the buy-in payment. The essential idea of the market cap method is that the value of some particular intangible for a publicly traded company can be determined by subtracting estimates of the market values of all other assets of the company from the market value of the company to determine the market value of the particular intangible in question. The residual value obtained after this subtraction is the inferred value of the specific intangible whose value is to be estimated. Since the sub is not typically a publicly traded company, when the market cap method is applied to calculating the buy-in payment, the "market cap" refers to an imputed estimate of the sub's market value. After calculating this imputed market value, the value of the intangibles transferred from the parent can be estimated upon subtracting from the

sub's imputed market value the value of all assets other than those intangibles provided by the parent.

The market cap method has been criticized as being either difficult or inappropriate to apply on a variety of grounds: (1) if a company has more than one intangible asset, any attempt to determine the market value of any particular intangible asset of the company can proceed under the market cap method only by valuing the company's other intangibles. Since the market cap method is often resorted to only when direct valuation of the company's various intangibles is impossible, the market cap method contains an inherent circularity: any given intangible can be valued using this method only if all of the company's other intangibles can be valued directly. That is, the market cap method simply shifts the burden of valuing one particular intangible onto the problem of determining the value of the firm's other intangibles. (2) Even if the problem of separating the value of the company's, or its subsidiaries', current intangibles (discussed in (1)) can be resolved, the problem remains that the market cap of a company as of any particular date is the sum of not only the value of intangibles the company has already produced, but also includes the market's assessment of the value of tangible and intangible assets (net of their acquisition costs) the company intends to acquire in the future, some of which may have not even been thought of yet. So, in order to apply the market cap method to assess the value of pre-existing intangibles, there must be some method of dividing the company's market cap into that portion of the market cap attributable to intangibles in place versus that portion of the market cap attributable to intangibles not yet created.

Notwithstanding these difficulties, in Section 9 of the paper, we show how the market cap method can be combined with the RPS profit method to assign a value for the parent's pre-buy in intangibles. The method takes, as inputs, the profit margins and sales growth rates of both the parent and the sub, and then computes the sub's imputed market cap by combining these inputs with the

market cap of the (assumed publicly traded) parent. The method multiplies the sub's imputed market cap by the royalty rate (computed from the RPS method) to determine the component of the sub's market cap generated by the parent's pre-buy in intangibles. We confirm that when the market value of the pre-buy in intangibles the parent makes available to the sub is computed in this way, the value that results coincides with the value obtained by exclusive reliance on the RPS method.

This paper is linked to the professional literature on cost-sharing and also the literature on depreciating intangibles. The professional literature on cost-sharing was cited in the opening paragraph above. The literature dealing with the depreciation of intangibles is empirical. It attempts to estimate how fast advertising, research and development, and/or other expenditures on intangibles depreciate. Representative articles in this literature include Herschey [1982], Herschey and Weygandt [1985], Landes and Rosenfield [1994], Lev and Sougiannis [1996], Nadiri and Prucha [1996], Pakes and Griliches [1984], Peles [1970], Ravenscraft and Scherer [1982]. We are not aware of any literature concerned with the specific problem of developing depreciation methods for intangibles in the context of buy-in payments, or of any other academic literature on cost-sharing.

The paper proceeds as follows. Section 1 discusses certain preliminaries, e.g., the definition of intangibles, the determination of residual profits, notation for the parent and sub. Section 2 describes the typical way the sub's residual profits are allocated under the -7 regulations. Section 3 presents alternative interpretations of the -7 regulations of the buy-in payment. Section 4 rationalizes the conventional procedure for allocating the post buy-in residual profits to the parent's pre-buy in intangibles when the sub's time series of residual profits are determined by a lagged linear function of current and past expenditures on intangibles. Section 5 determines a "correct " depreciation schedule, in the following sense: if the sub's post buy-in residual profits are multiplied

by the ratio of the capitalized value of the parent's pre-buy in expenditures to the capitalized value of the parent's total expenditures on intangibles - where the capitalized values are calculated using this "correct" depreciation schedule - the result is an "as if" residual profit sequence that captures only the residual profits of the sub generated by the parent's pre-buy in intangibles. Section 6 considers an extension of the analysis where both the parent and sub contribute intangibles at all dates. Section 7 contains another extension based on an alternative specification of the mapping from investments in intangibles to the sub's residual profits. Section 8 discusses how the choice of payment method - lump sum vs. periodic royalty rates - influences the (U.S. parent) taxpayer's incentives to bias the (foreign) sub's forecasted future sales growth. Section 9 discusses how the "market cap" method can be used to assess the buy-in payment. Section 10 identifies some unresolved issues associated with cost-sharing agreements, including how to execute cost sharing agreements when the returns to developing intangibles are random and extend over multiple years, along with the problem of determining what constitutes the most beneficial time to execute a cost-sharing agreement.

1 Preliminaries

In the following, we sometimes refer to the parent as USP (for "U.S. parent") and the sub as FS (for "foreign sub"). Also, we refer repeatedly to investments in or expenditures on "intangibles." §1.482-4 defines intangible property as an asset that "derives its value not from its physical attributes but from its intellectual content or other intangible properties" and "has substantial value independent of the services of any individual." In practice, in buy-in calculations involving the residual profit split method, "expenditures on intangibles" are operationalized as those expenditures on marketing, product development, etc. that are expected to generate "above normal" returns. Also, "residual profits" of the sub are

calculated as the sub's profits net of its "routine returns." In practice, what constitutes "intangibles," "above normal" returns, and/or "residual profits" can be sources of disagreement between the taxpayer and the IRS. As the present paper is not focused on these definitional questions, we proceed by positing that all the expenditures under discussion are expenditures on intangibles, and all profits of the sub are residual profits. That is, the paper takes "intangibles" and "residual profits" to be primitives of the analysis – i.e., any issues regarding the classification of intangibles or residual profits have been resolved through some prior unspecified analysis.

2 The typical way the sub's residual profits are allocated to pre-buy in intangibles under the RPS method

As was noted in the Introduction, the typical way the residual profits of the sub in any post buy-in period t in practice are allocated to the pre-buy in intangibles of the parent under the RPS method is to multiply these residual profits by the ratio of the period t capitalized value of the pre-buy intangibles (supplied by the parent to the sub) to the period t capitalized value of all intangibles generated on behalf of the sub (whether pre- or post- buy in, whether by the parent or the sub). While this verbal description might seem complicated, the arithmetic underlying the calculation is actually simple. Let:

- $t = 0$ be the buy-in date;

- π_t be the post buy-in residual profits of the sub for any period $t > 0$;

- $K_t^{parentpre}$ be the period t capitalized value of all the parent's pre-buy in intangibles (that is, the undepreciated value as of period t of the expenditures on intangibles made by the parent in all periods $\tau \leq 0$); and

- K_t^{total} be the total period t capitalized value of all past intangibles on

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behalf of the sub (made by either the parent or the sub, either pre or post the buy-in date).

Then, in the typical procedure, the residual profits allocated to the parent's pre-buy in intangibles are simply:

$$\frac{K_t^{parentpre}}{K_t^{total}} \times \pi_t. \tag{1}$$

Then, the buy-in payment is the present value of this sum, i.e., with r being the applicable interest rate, the buy-in payment is:

$$\sum_{t=1}^{\infty} \frac{\left(\frac{K_t^{parentpre}}{K_t^{total}} \times \pi_t \right)}{(1+r)^t}. \tag{2}$$

This is consistent with what was described verbally above.

To understand more about whether the preceding procedure is appropriate, we start in the next section by considering the formal specification of the buy-in payment in the -7 regulations. We establish two things in that section. First, there are several distinct mathematical formalizations of a buy-in payment, all of which appear to be consistent with the -7 regulations. Second, regardless of which of these formalizations is chosen to calculate the buy-in payment, if one knows enough about the process mapping the expenditures on intangibles into residual profits, none of those definitions necessarily involves having a buy-in payment calculated according to (2). These two results lead us to search, in the following section, for relationships between expenditures on intangibles and residual profits that (a). are robust with respect to the alternative possible definitions of intangibles, and (b). in which ratios of the form (1) arise naturally in the calculation of the buy-in payment.

3   Alternative ways of defining the buy-in payment

The stated requirement for the buy-in payment is the following (from § 1-482-7(g)(2)).

"If a controlled participant makes pre-existing intangible property in which it owns an interest available to other controlled participants for purposes of research in the intangible development area under a qualified cost sharing arrangement, then each such other controlled participant must make a buy-in payment to the owner. The buy-in payment by each such other controlled participant is the arm's length charge for the use of the intangible under the rules of Secs. 1.482-1 and 1.482-4 through 1.482-6, multiplied by the controlled participant's share of reasonably anticipated benefits ..."

This section starts by proposing alternative interpretations of the preceding paragraph's reference to "an arm's length charge," in the context of buy-in payments.

Consider a general production process describing how intangibles generate residual profits. Let $\mathbf{E}_t^p = (E_t^p, E_{t-1}^p, E_{t-2}^p, ...)$ denote the history of expenditures by the parent on intangibles up through and including period t, with E_{t-k}^p being the expenditures on intangibles by the parent in period $t - k$. It is important to recognize that "expenditures" in a period are distinct from the "capitalized values" of intangibles in that period: for example, the capitalized value K_t^{total} referenced above constitutes a single measure of the cumulative stock of intangibles contributed by the parent as of period t; capitalized values such as K_t^{total} can be constructed only by knowing both the sequence of past expenditures on intangibles and the appropriate rate of depreciation of the value of those expenditures. In contrast, the history of expenditures, $\mathbf{E}_t^p$, requires no specification of any depreciation rates.

Similarly, we let $\mathbf{E}_t^s = (E_t^s, E_{t-1}^s, E_{t-2}^s, ...)$ denote the history of expenditures on intangibles up through period t by the subsidiary.

The investment function

$$\pi_t = f_t(\mathbf{E}_t^p, \mathbf{E}_t^s) \tag{3}$$

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maps the expenditures on intangibles into the sub's residual profits[7]. This function f_t will be the basis of providing alternative definitions of the buy-in payment.

We employ the following additional notation: for any period $t > 0$, define $\mathbf{0}_t$ be a sequence of t zeroes. Concatenate vectors in the usual way, i.e., let the vector

$$(\mathbf{0}_t, \mathbf{E}_0^p) \tag{4}$$

denotes zero expenditures on intangibles by the parent between periods 1 and t, and expenditures $\mathbf{E}_0^p$ from period 0 backwards in time. Define $f_t^{asif}(\mathbf{E}_0^p, \mathbf{E}_t^s)$ by:

$$f_t^{asif}(\mathbf{E}_0^p, \mathbf{E}_t^s) \equiv f_t((\mathbf{0}_t, \mathbf{E}_0^p), \mathbf{E}_t^s) \equiv \pi_t^{asif}. \tag{5}$$

This is the residual profits of the sub that would have resulted in period t were the parent to have stopped its expenditures on intangibles as of the buy-in date. Let $\mathbf{0}_\infty$ be an infinite sequence of zeroes. Given these definitions, one way of defining the buy-in payment is given by:

$$\Sigma_{t=1}^\infty \frac{f_t^{asif}(\mathbf{E}_0^p, \mathbf{E}_t^s = \mathbf{0}_\infty)}{(1+r)^t} = \Sigma_{t=1}^\infty \frac{\pi_t^{asif}}{(1+r)^t}. \tag{6}$$

This is the present value of residual profits solely attributable to the parent's pre-buy in intangibles. It is the present value of a counterfactual, or "as if", profit stream, since it is unlikely that the parent will stop making expenditures on intangibles after the buy–in date and that the sub will make no expenditures on intangibles whatsoever.

A second method of calculating the buy-in payment is quite different from this first method. Under the second method, one asks: how much did the value of the sub increase as a consequence of the parent having made the pre-buy in expenditures on intangibles? To be more precise, let $v^s(\mathbf{E}_0^p, \mathbf{E}_0^s)$ be the value

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[7]Here we have employed the assumption that residual profits are taken as a primitive. If we had not taken residual profits as a primitive, we would have had to starti with the sub's total profits and then subtracted the "routine" component of those profits to deduce residual profits.

of the sub as of date 0, given that, up to the buy-in date, the parent and sub respectively had made the sequence of past expenditures on intangibles given by $(\mathbf{E}_0^p, \mathbf{E}_t^s)$, the parent stops making expenditures on intangibles after the buy-in date, and the sub makes optimal investments in intangibles following the buy-in date. We allow there to be some restrictions on E_t^s (perhaps due to production technology constraints, capital rationing, etc.,) by confining E_t^s to some set $\mathbf{\Omega}$. Then, $v^s(\mathbf{E}_0^p, \mathbf{E}_0^s)$ is defined by:

$$v^s(\mathbf{E}_0^p, \mathbf{E}_0^s) = \max_{\{E_t^s\}_{t \geq 1}} \sum_{t=1}^{\infty} \frac{f_t^{asif}(\mathbf{E}_0^p, \mathbf{E}_t^s) - E_t^s}{(1+r)^t}.$$

$$\text{where } \mathbf{E}_t^s = (E_t^s, E_{t-1}^s, E_{t-2}^s, ..., E_1^s, \mathbf{E}_0^s), \ E_t^s \in \mathbf{\Omega}$$

Then, a second way of valuing the parent's pre-buy in intangibles is given by:

$$\Delta(\mathbf{E}_0^p, \mathbf{E}_0^s) = v^s(\mathbf{E}_0^p, \mathbf{E}_0^s) - v^s(\mathbf{0}_\infty, \mathbf{E}_0^s). \tag{7}$$

According to this definition, the value of the parent's pre buy in expenditures on intangibles is the amount by which the value of the sub increases as a consequence of the parent having made these expenditures, where the value of the sub is determined by selecting investments in the sub optimally after the buy-in date, taking as given whatever pre-buy in expenditures were made by both the parent and sub.

A potential criticism of the second method is that the parent's investment in intangibles is assumed to stop as of the buy-in date. Even if the parent were to sell the sub, it is unlikely that the parent will stop making expenditures on intangibles that benefit the sub, because even if the parent acts exclusively in its own interest, some of its expenditures on intangibles will either directly or indirectly benefit the sub. A third method of specifying the buy-in value is a variation on the second method: it takes into account the post buy-in expenditures on intangibles the parent makes.[8] Suppose $E_t^{*p}(\mathbf{E}_0^p)$ is the parent's preferred expen-

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[8] One could further refine this method by considering history-dependent stochastic investments in intangibles by the parent after the buy-in date. While notationally more cumbersome, such stochastic sequences introduce no new conceptual issues to the analysis that follows and, for that reason, are not developed in the following.

ditures in period $t > 0$, given that its history of expenditures on intangibles up to period 0 was $\mathbf{E}_0^p$; let $\mathbf{E}_t^{*p}(\mathbf{E}_0^p)$ be the total history of the parent's expenditures up to period t, i.e., $\mathbf{E}_t^{*p}(\mathbf{E}_0^p) = (E_t^{*p}(\mathbf{E}_0^p), E_{t-1}^{*p}(\mathbf{E}_0^p), E_{t-2}^{*p}(\mathbf{E}_0^p), ..., E_1^{*p}(\mathbf{E}_0^p), \mathbf{E}_0^p)$ and let the infinite (future) history of the parent's expenditures be denoted by $\mathbf{E}_\infty^{*p}(\mathbf{E}_0^p) \equiv \lim_{t \to \infty} \mathbf{E}_t^{*p}(\mathbf{E}_0^p)$. All this notation $\mathbf{E}_\infty^{*p}(\mathbf{E}_0^p)$ is intended to capture is that what constitutes a preferred expenditure on intangibles by the parent after the buy-in date depends upon what expenditures on intangibles it made prior to the buy-in date.

Now define

$$v^{*s}(\mathbf{E}_\infty^{*p}(\mathbf{E}_0^p), \mathbf{E}_0^s) = \max_{\{E_t^s\}_{t \geq 1}} \sum_{t=1}^{\infty} \frac{f_t(\mathbf{E}_t^{*p}(\mathbf{E}_0^p), \mathbf{E}_t^s) - E_t^s}{(1+r)^t}.$$

$$\text{where } \mathbf{E}_t^s = (E_t^s, E_{t-1}^s, E_{t-2}^s, ..., E_1^s, \mathbf{E}_0^s), \ E_t^s \in \mathbf{\Omega}$$

$v^{*s}(\mathbf{E}_\infty^{*p}(\mathbf{E}_0^p), \mathbf{E}_0^s)$ is the value of the subsidiary when the subsidiary takes as given the sequence of expenditures $\mathbf{E}_\infty^{*p}(\mathbf{E}_0^p)$ by the parent, and the subsidiary selects its own expenditures on intangibles to maximize its own value. Then, the value of the parent's pre-buy in intangibles according to this third definition is given by:

$$\Delta^{*s}(\mathbf{E}_0^p, \mathbf{E}_0^s) = v^{*s}(\mathbf{E}_\infty^{*p}(\mathbf{E}_0^p), \mathbf{E}_0^s) - v^{*s}(\mathbf{E}_\infty^{*p}(\mathbf{0}_\infty), \mathbf{E}_0^s). \tag{8}$$

This is the amount by which the value of the sub (presumably) increased, as a consequence of the parent having made pre-buy in intangibles to the benefit of the sub, had the parent continued to make expenditures on intangibles after the buy-in date, as compared to what the sub's value would have been had the parent made no expenditures on intangibles to the benefit of the sub prior to the buy-in date.

(6), (7), and (8) illustrate various possible constructions of the buy-in payment which may or may not be the same. They also illustrate that there need be no obvious role for ratios of the form $\frac{K_t^{parentpre}}{K_t^{total}}$ in calculating the buy-in payment. But that does not rule out the possibility that there are other math-

ematical descriptions of the buy-in payment that make use of these ratios that are algebraically equivalent to one or more of these constructions.

In the next section, we identify stochastic versions of (3) in which buy-in payments involving ratios of the form (1) arise naturally. In that section, we also show that, sometimes, the buy-in payment can be calculated even when the production functions f_t are not completely understood by those making the buy-in payment calculations.

4 Rationalizing the conventional procedure for allocating the post buy-in residual profits to the parent's pre-buy in intangibles

In several parts of this paper, we employ the following process for generating the sub's residual profits:

$$\tilde{\pi}_t = \tilde{\alpha}_t \times \sum_{i=0}^{n} a_i E_{t-i}, \text{ where } a_0 = 1, \ E_t = E_t^p \text{ for } t \leq 0 \text{ and } E_t = E_t^s \text{ for } t > 0. \tag{9}$$

Some comments on this specification follow.

- $\tilde{\alpha}_t$ is posited to be a random variable not known or observed by anyone. Initially, we put no structure whatsoever on the distribution of the random variable $\tilde{\alpha}_t$ over time: any and all possible time series interdependencies are allowed.

- The specification $a_0 = 1$ is simply a normalization; if it were not initially true, it could be made true by redefining the random variable $\tilde{\alpha}_t$.

- The specification $E_t = E_t^p$ for $t \leq 0$ and $E_t = E_t^s$ for $t > 0$ means that only the parent invests in intangibles before the buy-in date, and only the sub invests in intangibles after the buy-in date. This is a simplification that can, and subsequently will, be dispensed with.

- What this process assumes is the following: while the level of residual profits is not perfectly predictable in advance of making expenditures on intangibles, the relative contributions of intangibles of various vintages (a "vintage" referring to the time at which the expenditures on the intangibles are made), is known and stable across time. In stable economic environments, this seems to be a reasonable assumption.

- With this process, there is no difference among the three methods of defining the buy-in payment in the previous section, because the increase in the residual profits of the sub due to any post buy-in expenditures the sub makes is independent of the level or vintages of the parent's pre-buy in expenditures (because the profit function (9) is linear in these expenditures). Accordingly, with no loss of generality, we revert to definition (6) for the buy-in payment in the following.

If (9) characterizes the observed profit stream, the counterfactual or "as if" profit stream generated by assuming that no investments in intangibles takes place after the buy-in date - that is, (5) - is given by:

$$
\begin{aligned}
\pi_1^{asif} &= \tilde{\alpha}_1 \times \sum_{i=0}^{n-1} a_{i+1} E_{-i}; \\
\pi_2^{asif} &= \tilde{\alpha}_2 \times \sum_{i=0}^{n-2} a_{i+2} E_{-i}; \text{ and in general,} \\
\pi_t^{asif} &= \tilde{\alpha}_t \times \sum_{i=0}^{n-t} a_{i+t} E_{-i}.
\end{aligned}
\tag{10}
$$

The reason that this is the correct specification of the "as if" stream of residual profits is clear. In period 1, for example, the "as if" residual profits are $\pi_1^{asif} = \tilde{\alpha}_1 \times \sum_{i=0}^{n-1} a_{i+1} E_{-i}$, since the "as if" profits are computed by assuming E_1 had been set equal to zero and the age of expenditures made in period i, $i \leq 0$, is (in period 1) is $i + 1$. The justification for the other "as if" residual profit calculations is similar.

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The specialization of (6) to time series (9) is then given by:

$$\sum_{t=1}^{\infty} \frac{\tilde{\alpha}_t \times \sum_{i=0}^{n-t} a_{i+t} E_{-i}}{(1+r)^t}. \tag{11}$$

The problem in practice that has to be confronted in calculating the buy-in payment is that neither the "as if" profit stream (10) nor its present value (11) is directly observable. The *observable* profit stream (9) reflects both the contribution of the capital expenditures made by USP prior to the buy-in date as well as the capital expenditures made by FS after the buy-in date.[9] Since - clearly - the actual buy-in payment must be based on observables, the goal is to reconstruct the "as if" profit stream from the observed profit stream, i.e., to compute (6) based on observables.

We now proceed to construct a buy-in payment based on the capitalized values of past expenditures on intangibles. The goal is to use the ratios of these capitalized values, as in (1) above, to extract the "as if" residual profit stream from the observable residual profit stream. These capitalized values of intangibles used in this construction of a buy-in payment depend upon what specific depreciation schedule is used. We first calculate capitalized values for any depreciation schedule, and then we find a particular depreciation schedule that converts the observed sequence of residual profits into the "as if" sequence of residual profits.

For any $k \geq 1$, we let d_t^k denote the depreciation rate to be applied to expenditures made in period t over the time interval $t + k - 1$ to $t + k$. In this notation, $E_t(1 - d_t^1)$ is the capitalized, or undepreciated, value of the expenditures E_t made in period t as of the end of period $t + 1$; $E_t(1 - d_t^1)(1 - d_t^2)$ is the capitalized or undepreciated value of the expenditures E_t made in period t as of the end of period $t + 2$, etc.

The set $\{d_t^k | t, k \geq 1\}$ constitutes the depreciation policy for the firm.

---

[9] We consider the residual profit stream to be "observable" because, as was noted previously, we take the residual profit stream as a primitive in this analysis. In instances in which residual profits are not a primitive, then residual profits are themselves not observable: they have to be constructed from the sub's actual reported profits.

Given a depreciation policy, any history of expenditures $\mathbf{E}_t$ has associated with it a capitalized value of intangibles K_t defined by:

$$K_t = E_t + E_{t-1}(1-d^1_{t-1}) + E_{t-2}(1-d^1_{t-2})(1-d^2_{t-2}) + E_{t-3}(1-d^1_{t-3})(1-d^2_{t-3})(1-d^3_{t-3}) + ...$$

Here, K_t is the capital stock, or capitalized value, of intangibles as of date t. Sometimes, to be explicit about the dependence of the capital stock on the history of expenditures, we write $K_t(\mathbf{E}_t)$; for even more completeness, to indicate that the capital stock also depends on the depreciation schedule, we sometimes write $K_t(\mathbf{E}_t, \{d^k_t | t, k\})$.

For any two dates t' and $t > t'$, we let $K^{t'}_t$ be the date t value of the capital stock corresponding to expenditures made only up to date t'. In other words, using the concatenation convention (4),

$$K^{t'}_t(\mathbf{E}_t) \equiv K_t(\mathbf{0}_{t-t'}, \mathbf{E}_{t'}), \tag{12}$$

i.e., it is what the capitalized value of expenditures on intangibles would have been as of date t had there been no expenditures on intangibles after date t'. $K^{t'}_t(\mathbf{E}_t)$ is a counterfactual (or "as if") value, obtained by replacing the actual expenditures $E_{t'+1}, E_{t'+2}, ..., E_t$ on intangibles over the periods $t'+1, t'+2, ..., t$ by $\mathbf{0}_{t-t'}$. When the specification of t' is clear from context, we sometimes refer to $K^{t'}_t(\mathbf{E}_t)$ as the truncated capitalized value of the expenditures on intangibles.

We can rationalize the typical procedure used to calculate residual profits attributable to pre-buy in intangibles, as described initially in (1) above, if we can find depreciation schedules so that the capitalized values that appear in (1) using these depreciation schedules coincide with the "as if" residual profits in (6). The fraction of residual profits attributable to the pre-buy in intangibles would be given by:

$$\frac{K^0_t(\mathbf{E}_t, \{d^k_t | t, k\})}{K_t(\mathbf{E}_t, \{d^k_t | t, k\})}. \tag{13}$$

Formally, we say that a depreciation schedule is "correct" if we can recover the "as if" residual profit sequence (10) from the observed residual profit sequence.

Definition A depreciation policy is "correct" if, for all periods $t > 0$ after the buy-in date,

$$\pi_t^{asif} = \frac{K_t^0(\mathbf{E}_t, \{d_t^k|t,k\})}{K_t(\mathbf{E}_t, \{d_t^k|t,k\})} \times \pi_t.$$

If a depreciation schedule is correct, then the present value

$$\Sigma_{t=1}^{\infty} \frac{\frac{K_t^0(\mathbf{E}_t, \{d_t^k|t,k\})}{K_t(\mathbf{E}_t, \{d_t^k|t,k\})} \times \pi_t}{(1+r)^t}$$

will equal (6). So, an economically accurate depreciation schedule correctly estimates the buy-in payment.

We noted in the Introduction that the conventional depreciation procedures used in financial reporting and tax reporting are often not "correct." We now expand on the observations made there. It is widely acknowledged that the tax depreciation rates specified by the Modified Accelerated Cost Recovery System (MACRS) bear little relationship to economic depreciation rates for the assets that are covered by the MACRS schedules.[10] In financial accounting, there is a limited attempt to select depreciation schedules for assets to match the pattern of future benefits associated with the assets - in so far as if a firm becomes aware of information that leads it to believe that the future revenues generated by some of its assets are either longer or shorter than was anticipated at the time the depreciation policy for those assets was chosen, the depreciation policy must be revised prospectively so as to ensure that the remaining depreciation of the assets is recorded over the periods during which the assets' remaining future benefits are anticipated to be realized.[11] But, the extent to which the depreciation policies firms in practice choose for financial reporting purposes reflect the pattern of the future benefits associated with those assets is limited. For example, to the extent that an asset is known to generate a relatively constant profit or revenue stream, the decline in the asset's value is most accurately captured by compound depreciation methods. But, compound depreciation methods are

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[10] See, e.g., Stickney and Weil [2007], Chapter 8.
[11] Ibid.

decelerated depreciation methods, and decelerated depreciation methods are forbidden from being used under U.S. GAAP.[12]

But whether the depreciation rates properly reflect the decline in the value of the assets being depreciated is not the point of "correct" depreciation schedules. Whether a depreciation schedule is "correct" can only be assessed based on whether it permits the recovery/identification of the "as if" residual profit stream from the observed residual profit stream.

5   "Correct " depreciation schedules

The problem to be solved is the construction of the "as if" residual profit stream from the observed profit stream. Given that the relation in (9) is known, and that the sequence of past expenditures on intangibles is observable, it is clear how this can be done:

$$\pi_1^{asif} = \frac{\sum_{i=0}^{n-1} a_{i+1} E_{-i}}{\sum_{i=0}^{n} a_i E_{1-i}} \times \pi_1; \tag{14}$$

$$\pi_2^{asif} = \frac{\sum_{i=0}^{n-2} a_{i+2} E_{-i}}{\sum_{i=0}^{n} a_i E_{2-i}} \times \pi_2; \text{ and in general,} \tag{15}$$

$$\pi_t^{asif} = \frac{\sum_{i=0}^{n-t} a_{i+t} E_{-i}}{\sum_{i=0}^{n} a_i E_{t-i}} \times \pi_t. \tag{16}$$

(16) shows how the "as if" residual profit stream can be constructed from the observed residual profit stream with the aid of the history of expenditures on intangibles. What the following theorem does is show how one can construct depreciation schedules so that ratios involving the capitalized expenditures on intangibles can accomplish the same thing.

Theorem 1 *When residual profit streams can be represented by the linear time series (9), the "correct" depreciation schedule is the following:*

---

[12] Ibid.

$$
\begin{aligned}
d_t^1 &\equiv d^1 = 1 - a_1; \\
d_t^2 &\equiv d^2 = 1 - \frac{a_2}{a_1}; \\
d_t^3 &\equiv d^3 = 1 - \frac{a_3}{a_2}; \text{ and in general for any } k \leq n \\
d_t^k &\equiv d^k = 1 - \frac{a_k}{a_{k-1}}; \text{ and} \\
d_t^{n+1} &\equiv d^{n+1} = 1.
\end{aligned}
$$

Proof Notice that, using this depreciation schedule:

- the expenditure E_{t-1} made in period $t-1$ depreciates in value to $E_{t-1}(1 - d^1) = a_1 E_{t-1}$ in period t;

- the expenditure E_{t-2} made in period $t-2$ depreciates in value to $E_{t-2}(1 - d^1)(1 - d^2) = a_2 E_{t-2}$ in period t;

- in general, for $k \leq n$, the expenditure E_{t-k} made in period $t-k$ depreciates in value to $E_{t-k}(1 - d^1)(1 - d^2)...(1 - d^k) = a_k E_{t-k}$ in period t; and

- the expenditure E_{t-n-1} made in period $t - n - 1$ depreciates in value to $E_{t-2}(1 - d^1)(1 - d^2)...(1 - d^n) = 0$ in period t.

So, using this depreciation schedule, the capitalized value of the history of expenditures $\mathbf{E}_t$ as of period t is

$$
K_t(\mathbf{E}_t) = \sum_{i=0}^{n} a_i E_{t-i}.
$$

Moreover, using the notation in (12), we have

$$
K_1^0(\mathbf{E}_1) = \sum_{i=0}^{n-1} a_{i+1} E_{-i}
$$

$$
K_2^0(\mathbf{E}_2) = \sum_{i=0}^{n-2} a_{i+2} E_{-i}
$$

and, in general,

$$K_t^0(\mathbf{E}_t) = \sum_{i=0}^{n-t} a_{i+t} E_{-i},$$

so:

$$\frac{K_1^0(\mathbf{E}_1)}{K_1(\mathbf{E}_1)} \times \pi_1 = \pi_1^{asif},$$

$$\frac{K_2^0(\mathbf{E}_2)}{K_2(\mathbf{E}_2)} \times \pi_2 = \pi_2^{asif},$$

and in general,

$$\frac{K_t^0(\mathbf{E}_t)}{K_t(\mathbf{E}_t)} \times \pi_t = \pi_t^{asif}. \tag{17}$$

This equation (17) proves the theorem.

There are two important special cases of the preceding, which we summarize in the following corollary.

Corollary (a) *When $a_{k-1} = a_k$, the correct depreciation schedule records no depreciation in the value of intangibles purchased k periods in the past, i.e., $d^k = 0$.*

(b) *When $a_{k-1} < a_k$, the correct depreciation schedule records negative depreciation in the value of intangibles purchased k periods in the past, i.e., $d^k < 0$.*

Proof of Corollary (a) When $a_{k-1} = a_k$, then the contribution of the intangibles made $k-1$ periods ago to the residual profit stream over the current period does not decline, and so according to the previous theorem, $d^k = 0$. (b) When $a_{k-1} < a_k$, the contribution of the intangibles made $k-1$ periods ago to the residual profit stream over the current period increases, and so, according to the previous theorem, $d^k < 0$.

The importance of the corollary is the following: it shows that using a straight-line or accelerated depreciation schedule to reflect the decline in the value of intangibles is incorrect when there is either no reduction or even an increase in the contribution of expenditures on intangibles to the sub's residual profits in two successive periods.

The effects of incorrectly using straightline depreciation schedules can be considerable. As a simple example, suppose $\pi_t = E_t + E_{t-1} + E_{t-2}$ and expenditures on intangibles over time are constant and equal to E. Then, the correct "as if" profits for periods 1 and 2 are $\pi_1^{asif} = 2E$ and $\pi_2^{asif} = E$. If straight-line depreciation (over three years) were used, the residual profits assigned to the pre-buy in intangibles would be $\pi_1^{asif} = \frac{2/3E+1/3E}{E+2/3E+1/3E} \times \pi_1 = \frac{2/3E+1/3E}{E+2/3E+1/3E} \times 3E = \frac{1}{2} \times 3E = 1.5E$ and $\pi_2^{asif} = \frac{1/3E}{E+2/3E+1/3E} \times \pi_2 = \frac{1/3E}{E+2/3E+1/3E} \times 3E = \frac{1/3}{2} \times 3E = .5E$. The distortion caused by straight-line depreciation results in an understatement of π_1^{asif} by 25% $\left(\frac{.5}{2}\right)$ and an understatement of π_2^{asif} by 50% $\left(\frac{.5}{1}\right)$.

If expenditures on intangibles were to increase over time, the distortion caused by straight-line depreciation gets worse. For example, suppose $E_t = 2E_{t-1}$ and $\pi_t = E_t + E_{t-1} + E_{t-2}$ and $E_{-1} \equiv E$. Then, the correct "as if" profits for periods 1 and 2 would be $\pi_1^{asif} = E_0 + E_{-1} = 3E$ and $\pi_2^{asif} = E_0 = 2E$ whereas the residual profits assigned by straight-line depreciation would be: $\pi_1^{asif} = \frac{2/3E_0+1/3E_{-1}}{E_1+2/3E_0+1/3E_{-1}} \times \pi_1 = \frac{2/3E_0+1/3E_{-1}}{E_1+2/3E_0+1/3E_{-1}} \times (E_1 + E_0 + E_{-1}) = \frac{2/3\times2E+1/3E}{4E+2/3\times2E+1/3E} \times 7E = \frac{5/3}{17/3} \times 7E = \frac{35}{17}E$ and $\pi_2^{asif} = \frac{1/3E_0}{E_2+2/3E_1+1/3E_0} \times \pi_2 = \frac{1/3E_0}{E_2+2/3E_1+1/3E_0} \times (E_2 + E_1 + E_0) = \frac{1/3\times2E}{8E+2/3\times4E+1/3\times2E} \times 14E = \frac{28}{34}E$. The distortion caused by straight-line depreciation in this case results in an understatement of π_1^{asif} by 31.4% $\left(\frac{3-\frac{35}{17}}{3}\right)$ and an understatement of π_2^{asif} by 58.8% $\left(\frac{2-\frac{28}{34}}{2}\right)$. While these are merely numerical examples, they are suggestive of the distortions caused by an inappropriate use of straight-line depreciation. Of course, accelerated depreciation would further understate the buy-in payment.

The Corollary involves production processes that are characteristic of the returns to many investments in intangibles. It may take time, in some cases, a substantial amount of time, for past expenditures on intangibles to contribute to a firm's current profit stream. Economists who study intangibles acknowledge this. For example, Brynjolfsson [1993] states "A second explanation for the [productivity] paradox [that returns to IT investments are hard to identify in data] is that the benefits from IT can take several years to show up on the

bottom line." (page 8)

As a final observation, we note that, by using ratios of the form (1), we can calculate the "correct" depreciation schedules even when the process determining how residual profits are generated is not completely known. Specifically, it is not necessary to know anything about the distribution of the random variable $\tilde{\alpha}_t$ in order to calculate the "correct" depreciation rates specified in the previous theorem.[13] If information about the distribution of the stochastic components of the process generating residual profits is known, then it may be possible to determine the "correct" depreciation rates for a wider class of processes generating residual profits, as we show formally in one of the extensions to the model (in section 7 below).

6 Extension 1: when both the parent and sub contribute intangibles at all dates

There are straightforward extensions of the preceding analysis to cover the case where both the parent and sub contribute intangibles both before and after the buy-in date.

Suppose the process generating the sub's residual profits is now given by:

$$\tilde{\pi}_t = \tilde{\alpha}_t \times \left(\sum_{i=0}^{n} a_i^p E_{t-i}^p + \sum_{i=0}^{n} a_i^s E_{t-i}^s \right), \text{ where } a_0^p = a_0^s = 1, \qquad (18)$$

where E_t^p is the expenditures on intangibles by the parent in period t, and E_t^s is the expenditures on intangibles by the sub in period t. Here, it is clear that intangibles contributed by both the parent and sub affect the sub's residual profits.

Continue to assume that $t = 0$ is the buy-in date. The buy-in payment is given by:

$$E[\Sigma_{t=1}^{\infty} \frac{\tilde{\pi}_t^{asif}}{(1+r)^t}], \text{ where } \pi_t^{asif} = \tilde{\alpha}_t \times \sum_{i=0}^{n-t} a_{t+i}^p E_{-i}^p.$$

---

[13] Of course, one cannot be completely ignorant of the process in (9) to calculate the "correct" depreciation schedules: it is necessary to be able to know the coefficients a_i in (9).

When $\tilde{\pi}_t$ is described by (18), π_t^{asif} can be written in terms of the observable profit sequence:

$$\pi_t^{asif} = \frac{\sum_{i=0}^{n-t} a_{t+i}^p E_{-i}^p}{\sum_{i=0}^{n} a_i^p E_{t-i}^p + \sum_{i=0}^{n} a_i^s E_{t-i}^s} \times \pi_t. \tag{19}$$

We now define separate depreciation schedules $\{d_t^{kp}|t,k\}$ and $\{d_t^{ks}|t,k\})$ for the parent and sub. The meaning of these depreciation schedules is the same as above: d_t^{kp} (resp., d_t^{ks}) denotes the depreciation rate to be applied to expenditures made in period t by the parent (resp., sub) over the time interval $t+k-1$ to $t+k$. Based on these two depreciation schedules, and the history of expenditures on intangibles for both the parent and sub, we define the total capitalized value of intangibles as of date t by:

$$\begin{aligned}
K_t(\mathbf{E}_t^p, \mathbf{E}_t^s) &= E_t^p + E_{t-1}^p(1 - d_{t-1}^{1p}) + E_{t-2}^p(1 - d_{t-2}^{1p})(1 - d_{t-2}^{2p}) + \quad (20)\\
&\quad E_{t-3}^p(1 - d_{t-3}^{1p})(1 - d_{t-3}^{2p})(1 - d_{t-3}^{3p}) + ...\\
&\quad + E_t^s + E_{t-1}^s(1 - d_{t-1}^{1s}) + E_{t-2}^s(1 - d_{t-2}^{1s})(1 - d_{t-2}^{2s}) +\\
&\quad E_{t-3}^s(1 - d_{t-3}^{1s})(1 - d_{t-3}^{2s})(1 - d_{t-3}^{3s}) + ...
\end{aligned}$$

For $t > t'$, the counterpart to (12), i.e., the truncated value of the total capitalized value of expenditures on intangibles made up through period t', calculated in period t, taking as given the history of both the parent's and sub's expenditures on intangibles, is given by:

$$K_t^{t'}(\mathbf{E}_t^p, \mathbf{E}_t^s) \equiv K_t((\mathbf{0}_{t-t'}, \mathbf{E}_{t'}^p), (\mathbf{0}_{t-t'}, \mathbf{E}_{t'}^s)).$$

If we use the depreciation schedules defined by, for each of $q = p$ and $q = s$,

$$\begin{aligned}
d_t^{1q} &\equiv d^{1q} = 1 - a_1^q; \quad (21)\\
d_t^{2q} &\equiv d^{2q} = 1 - \frac{a_2^q}{a_1^q};\\
d_t^{3q} &\equiv d^{3q} = 1 - \frac{a_3^q}{a_2^q}; \text{ and in general for any } k \leq n\\
d_t^{kq} &\equiv d^{kq} = 1 - \frac{a_k^q}{a_{k-1}^q}; \text{ and}\\
d_t^{n+1q} &\equiv d^{n+1q} = 1,
\end{aligned}$$

then it is clear upon examining (19), (20), and (21), that:

$$\pi_t^{asif} = \frac{K_t^0(\mathbf{E}_t^p, \mathbf{E}_t^s)}{K_t(\mathbf{E}_t)} \times \pi_t.$$

This proves that:

Theorem 2 *When residual profit streams can be represented by the linear time series (18), the "correct" depreciation schedule is given by (21).*

7 Extension 2: other processes describing the returns to investing in intangibles

In this section we return to the "base" case where the parent contributes intangibles up to period 0 and the sub contributes intangibles after period 0, but we generalize the process (9) specifying the sub's residual profits to the following:

$$\tilde{\pi}_t = \tilde{\alpha}_t \times \sum_{i=0}^{n} a_i E_{t-i} + \tilde{\varepsilon}_t, \text{ where } a_0 = 1, \ E_t = E_t^p \text{ for } t \leq 0 \text{ and } E_t = E_t^s \text{ for } t > 0. \tag{22}$$

Suppose $\tilde{\alpha}_t$ and $\tilde{\varepsilon}_t$ are each iid over time, with known normal distributions $N(\bar{\alpha}, \sigma_\alpha^2)$ and $N(0, \sigma_\varepsilon^2)$ respectively, with $\tilde{\alpha}_t$ and $\tilde{\varepsilon}_t$ independent of each other. Then, the expectations $E[\tilde{\alpha}_t|\pi_t]$ and $E[\tilde{\varepsilon}_t|\pi_t]$ are easily formed and computed, for example, with $k_t \equiv \sum_{i=0}^{n} a_i E_{t-i}$, $E[\tilde{\alpha}_t|\pi_t] = \bar{\alpha} + \frac{k_t \sigma_\alpha^2}{k_t^2 \sigma_\alpha^2 + \sigma_\varepsilon^2}(\pi_t - k_t \bar{\alpha})$. Then, the counterpart to (17) for the time series (22) is given by:

$$\pi_t^{asif} = E[\tilde{\alpha}_t|\pi_t] \times \frac{K_t^0(\mathbf{E}_t)}{K_t(\mathbf{E}_t)} + E[\tilde{\varepsilon}_t|\pi_t]. \tag{23}$$

While, as far as we know, expressions of the form (23) are uncommon in buy-calculations, if a taxpayer could demonstrate that (22) describes the process generating a firm's residual profits, and the taxpayer has enough information to be able to calculate each of $\bar{\alpha}$, σ_α^2, and σ_ε^2, then computations of the form (23) can be used to calculate the buy-in payment.

8 Buy-in payment methods: lump sum vs. periodic royalty payments

Section §1.482-7 gives the taxpayer the option to have the sub make the buy-in payment (and the tax payments associated with the buy-in payment) as a one time lump-sum payment, as a sequence of (uncontingent) installment payments whose present value equals the lump-sum payment, or as a sequence of royalty payments, with each periodic payment determined by multiplying a royalty rate by the sub's actual post buy-in sales, with the percentage determined so that if the forecasts (as of the buy-in date) of the sub's sales are accurate, then the present value of the sequence of royalty payments will (also) coincide with the lump-sum payment. In this section, we observe that when the taxpayer elects to have the sub make a sequence of royalty payments, the taxpayer has an incentive to overstate its projections of the sub's post buy-in sales. In contrast, when the taxpayer elects to have the sub make a lump-sum payment, we observe that the taxpayer has an incentive to understate the sub's post buy-in sales.

We consider the case where the sub's forecasted and actual sales grow at constant rates, the sub's forecasted residual profits are linear in its forecasted sales, the sub's actual residual profits are linear in its actual sales, and the fraction of residual profits of each sales dollar that is attributable to pre-buy in intangibles of the parent declines at a constant rate over time. Let S_t^f and S_t^a denote respectively the sub's forecast and actual period t sales, and π_t^u denote the sub's residual profits *per dollar* of sales attributable to the pre-buy in intangibles.[14] The sub's residual profits per dollar of sales decrease at rate ε per period, actual sales grow at rate a, and forecasted sales grow at rate f. Given this notation, it is clear that $\pi_t^u S_t^f = (1 - \varepsilon)^t \pi_0^u (1 + f)^t S_0$ is the sub's forecasted residual profits in period t, and $\pi_t^u S_t^a = (1 - \varepsilon)^t \pi_0^u (1 + a)^t S_0$ is the

---

[14] We suppose that π_t^u - and hence ε - are not in dispute. To the extent that is not true, obviously the taxpayer would seek to assert that π_t^u and ε are as small as possible so as to minimize the size of the buy-in payment.

sub's actual period t residual profits, attributable to the parent's pre-buy in intangibles (here, S_0 constitutes actual sales at the buy-in date). If there were a lump-sum buy-in payment, it would be given by:

$$\sum_{t=1}^{\infty} \frac{\pi_t^u S_t^f}{(1+r)^t} = \sum_{t=1}^{\infty} \frac{(1-\varepsilon)^t \pi_0^u (1+f)^t S_0}{(1+r)^t} = \frac{(1-\varepsilon)(1+f)}{r+\varepsilon-f+\varepsilon f} \pi_0^u S_0.$$

Alternatively, suppose the taxpayer opts for making a sequence of periodic royalty payments. Then, the taxpayer would first construct the royalty rate:

$$
\begin{aligned}
R &\equiv \frac{\sum_{t=1}^{\infty} \frac{\pi_t^u S_t^f}{(1+r)^t}}{\sum_{t=1}^{\infty} \frac{S_t^f}{(1+r)^t}} = \frac{\frac{(1-\varepsilon)(1+f)}{r+\varepsilon-f+\varepsilon f} \pi_0^u S_0}{\sum_{t=1}^{\infty} \frac{(1+f)^t S_0}{(1+r)^t}} \\
&= \frac{(1-\varepsilon)(r-f)}{r+\varepsilon-f+\varepsilon f} \pi_0^u,
\end{aligned}
$$

and second, the taxpayer would take the royalty rate R and multiply it by the sub's actual sales in each post buy-in period to determine the buy-in payment related to that period's sales. If forecasted sales and actual sales are the same, then this procedure results in a sequence of buy-in payments that have exactly the same present value as the lump sum payment. This is clear in view of the way R is defined:

$$\sum_{t=1}^{\infty} \frac{R S_t^f}{(1+r)^t} = R \sum_{t=1}^{\infty} \frac{S_t^f}{(1+r)^t} = \sum_{t=1}^{\infty} \frac{\pi_t^u S_t^f}{(1+r)^t}.$$

But, consider what happens when actual sales differ from forecasted sales, i.e., when $a \neq f$. Then, the present value of the periodic payments sums to:

$$
\begin{aligned}
R \sum_{t=1}^{\infty} \frac{S_t^a}{(1+r)^t} &= R \sum_{t=1}^{\infty} \frac{(1+a)^t S_0}{(1+r)^t} = \\
R \frac{1+a}{r-a} S_0 &= \frac{(1-\varepsilon)(r-f)}{r+\varepsilon-f+\varepsilon f} \frac{1+a}{r-a} \pi_0^u S_0.
\end{aligned}
\tag{24}
$$

Differentiation shows that this expression is always decreasing in f. Thus, the taxpayer always can reduce the present value of the buy-in payment by overstating the growth in the sub's post buy-in sales. Formally, this shows:

Theorem 3 *When a taxpayer opts to have its subsidiary make the buy-in payment as a sequence of royalty payments, the taxpayer has an incentive to overstate the growth rate in its sub's sales.*

30

While this formal analysis was confined to the case where growth rates in sales was constant, and the decline in the contribution of pre-buy in intangibles to the sub's residual profits was also at a constant rate, we conjecture that the conclusion - the taxpayer has an incentive to overstate the sub's forecasted sales - holds for a variety of other processes describing the evolution of sales, and the contribution of the pre-buy in intangibles to the sub's residual profits, over time.

9   Adaptation of the "market cap" method to assessing the buy-in payment

The Introduction described some of the difficulties that arise in trying to apply the "market cap" method to determine the buy-in payment. Notwithstanding those difficulties, the present section shows that, when combined with the RPS method, the market cap method also can be used to calculate the buy-in payment.

To see this, note that in the course of determining the buy-in payment under the RPS method, the taxpayer inevitably would calculate both the expected present value of the sub's residual profits after the buy-in date attributable to the pre-buy in intangibles and also the total expected present value of the sub's residual profits after the buy-in date. We call this ratio the "buy-in ratio" in the following. Then, obviously: :

$$
\begin{aligned}
\text{buy-in payment} \quad &= \quad \frac{\text{buy-in payment}}{\text{imputed market cap of sub}} \times \text{imputed market cap of sub} \\
&= \quad \frac{\text{buy-in payment}}{\text{imputed market cap of sub}} \times \frac{\text{imputed market cap of sub}}{\text{market cap of parent}} \times \text{market cap of parent} \\
&= \quad \text{buy-in ratio} \times \frac{\text{imputed market cap of sub}}{\text{market cap of parent}} \times \text{market cap of parent}. \quad (25)
\end{aligned}
$$

What the preceding shows is that if the taxpayer can calculate a reliable estimate of the (imputed) market value of the sub as a percentage of the market cap of the parent, then the taxpayer can combine that percentage with the buy-in

ratio developed from the RPS method to calculate an alternative method of estimating the buy-in payment. To be specific, suppose the period t expected sales of the parent, S_t^p, grow at the constant rate γ_p, so $S_p^t = (\gamma_p)^t \times S_0^p$ and the period t expected sales of the sub S_s^t grow at the constant rate γ_s, so $S_s^t = (\gamma_s)^t \times S_s^0$, and the average margins of the parent's and sub's sales are, respectively, m_s and m_p, then with $\beta = \frac{1}{1+r}$, then:

$$\frac{\text{imputed market cap of sub}}{\text{market cap of parent}} = \frac{\frac{\gamma_s m_s S_s^0}{1 - \beta \gamma_s}}{\frac{\gamma_s m_s S_s^0}{1 - \beta \gamma_s} + \frac{\gamma_p m_p S_p^0}{1 - \beta \gamma_p}}. \tag{26}$$

In particular, if the parent and sub have the same growth rates and the same profit margins, this ratio is simply the ratio of the subsidiary's sales to company-wide sales:

$$\frac{\text{imputed market cap of sub}}{\text{market cap of parent}} = \frac{S_s^0}{S_s^0 + S_p^0},$$

in which case we have:

Theorem 4 If the growth rates of the sub's sales is the same as that of the parent's, and the margins on sub's sales are the same as the parent's, then the buy-in payment can be determined as the product:

$$\text{buy-in payment} = \text{buy-in ratio} \times \frac{S_s^0}{S_s^0 + S_p^0} \times \text{market cap of parent}.$$

This theorem provides a "back of the envelope" calculation of the buy-in payment. If (26) is substituted into (25), this method of calculating the buy-in payment can be applied when the growth rates and profit margins of the parent and sub differ.

While this method is not independent of the RPS method, it helps in assessing the reliability of the RPS calculations.

10   Potential future research involving cost-sharing

This paper has emphasized issues related to the calculation of the buy-in payment in cost-sharing agreements. This is not the only interesting economic

problem involving cost-sharing agreements. -7 requires that participants to a cost-sharing agreement share costs in proportion to the pattern of anticipated benefits from the intangibles produced under the agreement. While the -7 regulations give examples of how the benefits are to be calculated, difficult problems emerge in practice that are not resolved by the regulations. For example, suppose that the intangibles generated by the agreement increase both the parent's and sub's income over several years following the agreement, but - as would typically be the case - both the parent's and sub's income is random. It may be very difficult to determine what portion of the sub's or parent's random profits following the investment in intangibles are attributable to the newly developed intangibles under the agreement, or were the result of previously developed intangibles. -7 does not provide much guidance as to how to resolve these "parsing" issues.

Another unresolved issue concerns some of the resource allocation effects of cost-sharing agreements. If the taxpayer is confident that its returns to having the parent develop intangibles useful for the sub are "high," then it would appear to be desirable to postpone investing in those intangibles until after the cost-sharing agreement becomes effective. The reason for this is clear, and was described in the Introduction: after the cost-sharing agreement is in place, the sub only has to pay the parent for the cost of developing the intangibles, whereas before the agreement becomes effective, the sub has to pay the parent the market value of the intangibles. It follows that, if the expectations of "high" returns on the investment in intangibles are borne out, the parent - which receives the payments from the sub as taxable income, and is, presumably, in a higher tax jurisdiction than is the sub (who makes the payments and who also receives a reduction in taxable income for those payments) - will be subject to less taxable income by postponing the investment in intangibles until after the buy-in date. To the extent that all taxpayers view the returns to investing in intangibles as "high," this observation creates the following enigma: why don't all taxpayers

who have foreign subs operating in lower tax jurisdictions than the parent and for which the parent invests in intangibles on behalf of the sub adopt cost-sharing agreements as soon as possible? A partial answer may related to an issue not addressed in the present paper, namely that the parent - as well as the sub - also derives benefits from the intangibles that it generates. If the size of the parent's benefits are large enough, a cost-sharing agreement may not provide much of a tax advantage to the taxpayer, since the cost-sharing payment by the sub is required to be commensurate with the sub's relative benefits from the parent's investment in intangibles.

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