

Learning and managerial horizons: Beyond career concerns

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Abstract

This paper presents a unified moral hazard framework with renegotiation and anticipated managerial turnover for examining incentive dynamics given correlated performance measures and long-term actions which is applicable to CEO turnover studies and related work on the “horizon problem”. The main contribution is an explicit solution for the dynamic agency with short-term contracts and a proof of its equivalence to the renegotiation-proof contract. The results present an alternative to performance-related turnover explanations of incentive dynamics and extend the work done with career concerns models by considering general performance measures and long-term actions. The main effects identified in the optimal contracts are: a learning effect, and a horizon effect consisting of risk and incentive externalities. Both an inverted U shape of managerial performance over a manager’s tenure and V shaped performance around managerial turnover are consistent with negatively correlated accounting-based performance measures and long-term actions.

KEYWORDS: dynamic agency, renegotiation, managerial performance and turnover, horizon problem.

1 Introduction

In this paper, I use an N -period LEN (Linear contracts, Exponential utility, Normal distributions) moral hazard model to examine the link between the time-series properties of performance measures used in incentive contracts, long-term actions, and managerial performance over a manager’s tenure. In particular, firm performance around managerial turnover follows from optimal incentives given the time-series properties of contractible performance measures, or the presence of long-term actions, given that turnover is

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expected. The model provides a unified moral hazard framework both for empirical studies of the relation between turnover and performance, and for studies of the horizon problem. The LEN model is based on Dutta and Reichelstein (1999), and extends Christensen, Feltham, and Şabac (2003, 2005) to N periods (see also Christensen, Feltham, Hofmann, and Şabac 2004); the analysis of firm performance surrounding managerial turnover requires dynamic agency models with more than two periods.

The model developed in this paper allows for incentives and performance to be simultaneously endogenous given rationally anticipated turnover, and thus applies to the majority of observed managerial turnover, see Huson, Malatesta, and Parrino (2004) and Brickley (2003). Furthermore, the model extends the career concerns approach of Holmström (1999) and Gibbons and Murphy (1992) to arbitrary performance measurement systems and long-term actions. Moving away from a pre-specified correlation structure of performance measures (output in this case) in the career concerns setup allows for considering negatively auto-correlated accounting-based performance measures and highlights the different performance and incentives dynamics that obtain using different performance measurement systems; considering long-term actions allows for an examination of the horizon problem regarding long-term investments.

To illustrate the forces at work, correlated performance measures and long-term actions are considered separately. Three specific examples of correlated performance measures are considered and analyzed in detail: an auto-regressive structure, a career concerns model with learning of a productivity parameter, and an accounting model that allows for reversible accruals.¹ For the three particular cases considered, a manager's tenure with a firm naturally breaks into three parts: a first part during which posterior variance reduction results in increased performance, a middle period in which a steady state is achieved and performance tends to be constant, and an end period in which incentive risk over the remaining horizon either increases or decreases resulting in decreasing or increasing performance. The analysis shows that the dynamic of managerial incentives is different over a manager's tenure depending on the informational characteristics of the performance measurement system, in particular their time-series properties. The accounting model with positive correlation is similar to the career concerns model: managerial effort increases through the manager's tenure. On the other hand, the accounting model with negative correlation gives an inverted U shape for managerial effort during tenure.

The two examples of long-term actions considered show that in most cases, even with endogenous incentives, long-term managerial effort declines towards retirement, in particular long-term effort with uniformly

¹In the case of two periods, the career concerns model is the same as the accounting model and has been extensively used in the literature on performance measurement and incentives, e.g. Meyer (1995), Meyer and Vickers (1997), Indjejikian and Nanda (1999), Autrey, Dikolli, and Newman (2003). The LEN model has been widely used in addressing specific performance measurement and accounting issues, see Datar, Kulp, and Lambert (2001), Dutta and Reichelstein (1999, 2002, 2003), Feltham and Xie (1994), Feltham and Wu (2000), and the survey by Lambert (2001).

decreasing marginal benefits to the principal decreases towards retirement in the same way as short-term effort declines in the accounting model with negative auto-correlation. These examples continue to support predictions of V-shaped performance around turnover even for long-term effort. However, for a particular case of investments that are expensed immediately (such as R&D or advertising), managerial effort will *increase* towards retirement, and managerial effort/performance is *negatively* related to incentives (see Cheng 2004 and Bizjak, Brickley, and Coles 1993)

Managerial or firm performance surrounding turnover is given by the performance of a manager in the last periods of his tenure with the firm, and the performance of a new manager in the first periods of his tenure with the firm, while both have rational expectations with respect to their tenure with the firm. Turnover preceded by declining performance and followed by increased performance can be obtained both with negatively auto-correlated (accounting) performance measures, and with long-term actions. Hermalin and Weisbach (1998) model forced turnover and predict that turnover is more likely for a poorly performing CEO and that accounting-based performance measures are better predictors of CEO turnover than stock price performance. Huson et al. (2004) find evidence of V shaped performance around turnover for both forced turnovers and for normal (expected) turnovers; this evidence is consistent both with forced turnovers being driven by poor performance, and with normal turnovers in the accounting model with negatively correlated performance measures.

The links between managerial performance, managerial tenure, and turnover have been analyzed within various paradigms in a number of theoretical and empirical papers in strategic management, economics, and accounting. Hambrick and Fukutomi (1991) develop a qualitative model of managerial tenure as a whole and identify five stages of a CEO's tenure. The early, middle, and late stage performance predicted by their model is consistent with the Eitzen and Yetman (1972) finding of an inverted U shape for an organization's (basketball team) performance during its manager's (coach) tenure. Consistent with part of the inverted U shape prediction, Miller (1991) presents evidence that firm performance decreases with CEO tenure and attributes this to a decline in the match between CEO/firm strategy and environment.

Many authors start from the "common sense [...] that when an organization is performing poorly, replacement of the manager might be expected", Salancick and Pfeffer (1980). Some argue that CEO power determines the link between pay and performance, and tenure in office. For example, Hill and Phan (1991) present evidence that the pay/performance link for a CEO weakens with increased CEO tenure and attribute this to an increase in the CEO's power. Within a similar framework, Shen and Cannella (2002) hypothesize an inverted U shape relationship between departing CEO tenure and post-succession firm ROA, attributing it to organizational inertia and disruption surrounding the turnover event.

Others examine firm performance around CEO turnover and leave tenure outside the analysis. Hermalin and Weisbach (1998) develop an analytical model of CEO performance, board of directors selection, and CEO turnover. They concentrate primarily on board selection through bargaining between board and the CEO, and obtain endogenous turnover associated with poor performance—given that the CEO’s power is based on perceived ability, which in turn is based on observed past performance. Several papers have documented declining performance associated with (leading to) CEO turnover, see Murphy and Zimmerman (1993), Huson, Parrino, and Starks (2001), and the references therein. Engel, Hayes, and Wang (2003) document that “accounting information receives greater weight in turnover decisions when accounting-based measures are more precise and more sensitive,” consistent with the view that performance drives turnover. More recently, Huson et al. (2004) present additional evidence that firm performance has a V shape in the years around CEO turnover, even for CEOs that retire or voluntarily leave for another firm. Others address the issue of managerial horizon and its impact on performance: Dechow and Sloan (1991) provide evidence consistent with a mismatch between a CEO’s and the firm’s horizon, while Brickley, Linck, and Coles (1999) provide evidence that post-retirement incentives are a mitigating factor for the horizon problem.

None of the aforementioned literature explicitly considers moral hazard and incentives (Hermalin and Weisbach 1998 have no moral hazard in the model, while Murphy and Zimmerman 1993, Dechow and Sloan 1991, and Brickley, Linck, and Coles 1999 take incentives as exogenous). Moreover, modelling managerial turnover as a result of poor performance restricts the analysis to forced turnovers. However, a majority of CEO turnovers are normal retirements (approximately three quarters of the large sample in Huson et al. 2004, see also Brickley 2003), and it is reasonable to assume that in these cases contracts are optimally set in anticipation of retirement. The question then arises: how should performance and turnover be related in these circumstances?

Addressing the above question requires that turnover is rationally anticipated. Within a pure moral hazard agency theory framework, Gibbons and Murphy (1992) extend the career concerns model of Holmström (1999) to include explicit contracts and managerial risk aversion (see also Kim 1996 for a related approach). In the career concerns models, managerial performance and incentives are endogenously determined as a function of time, given anticipated retirement. In particular, the model in Gibbons and Murphy (1992) predicts both increasing managerial performance and stronger incentives through a manager’s tenure. The key aspect of the career concerns models is the labor market’s learning of managerial ability from the history of observed output; in the limit a steady state is attained, in which expected performance and incentives change very little from period to period. While the career concerns model of Gibbons and Murphy (1992) incorporates moral hazard and anticipated turnover, its implications are not consistent with Hu-

son et al. (2004) and others finding declining performance prior to turnover for normal (expected) turnovers. Furthermore, the career concerns model does not address performance measurement issues, does not allow for examining the link between the performance measurement system and managerial performance, and does not incorporate long-term actions.

In this paper, I examine the impact of renegotiation on the dynamic of incentives and managerial effort in a multi-period pure moral hazard model.² Renegotiation has three key consequences for dynamic moral hazard: learning, risk externalities, and incentive externalities. Renegotiated contracts are optimal given information observed prior to renegotiation, so that over time potentially more precise posterior beliefs about unobservable information are used in contracts (for example learning about uncertain managerial ability). Risk and incentive externalities arise if the periods are either statistically or technologically interdependent (where the former refers to serially correlated performance measures, and the latter to long-term actions). Because the principal and the agent rationally anticipate future renegotiation encounters, the principal's and the agent's choices are influenced by expected future contractual terms. Past actions are sunk, and ignored at each renegotiation date, as are past contractual terms (which only bear on the agent's continuation reservation utility).³ Choices made in one period ignore the potential impact on choices in earlier periods, hence the term externality.

If the contractible information is correlated over time, the principal's choice of current contractual terms has to include the impact on the agent's total risk premium, which includes a covariance term between current and future performance measures. In this case, the principal has to consider in addition to the tradeoff between risk and incentives in the current period, a risk externality imposed by future incentives on the total risk premium. Similarly, if the agent's action has a long-term impact on contractible information, it is influenced by the rationally anticipated contractual terms in future periods. In this case, the principal needs to take into account an incentive externality imposed by future incentives on the agent's current optimal action choice.

Inter-temporal correlation of the performance measures or long-term actions ensure that the solution to the agency problem is not a simple repetition of the single-period contract, in particular the sufficient

²Limited commitment and/or inter-temporal dependencies are frequently assumed away in repeated agency models, as they tend to make the analysis intractable by introducing information asymmetries at contracting time. For example, Radner (1985) assumes independent periods, Rogerson (1985) and Spear and Srivastava (1987) assume commitment. Rey and Salanie (1990) and Fudenberg et al. (1990) show that for commitment to long-term contracts not to have value, it is necessary that no information asymmetries exist at contracting time.

³As a result, renegotiation imposes constraints that make the renegotiation-proof contract suboptimal relative to a full commitment contract (one that the parties commit to at the start of the first period and is not subject to renegotiation at any later date). In other words, the renegotiation-proof contract is time consistent but not optimal in the sense used by Kydland and Prescott (1977). If the periods are both statistically and technologically independent, there is no difference between the two contracts, which take the form of repeatedly inducing the optimal action from the one period problem.

conditions of Fudenberg, Holmström, and Milgrom (1990) for short-term contracts to replicate long-term contracts are not satisfied. Limited commitment arising from institutional restrictions on contract form or duration, or from the inability of contracting parties not to commit ex ante to renegotiate ex-post to an efficient contract has been recognized as a salient feature of dynamic agency (see for example Hart and Tirole (1988), Dewatripont (1989), Fudenberg and Tirole (1990), Hermalin and Katz (1991), Ma (1994), and Dewatripont and Maskin (1995)). Renegotiation is modelled equivalently by long-term renegotiation-proof contracts as in Fudenberg and Tirole (1990) and short-term contracts (in particular, fair contracts) that provide both implicit and explicit incentives as in Christensen et al. (2003). While the effective managerial incentive, and therefore firm performance, is not affected by the equivalent contracting mechanisms, the nature of explicit/implicit incentives is sensitive to it: assuming short-term (in particular, fair) contracts gives different explicit/implicit incentives from the career concerns model of Gibbons and Murphy (1992). Demski and Frimor (2001) also examine the impact of repeated renegotiation in more than two periods and find that the negative effects of renegotiation are reduced by adding more periods. However, they are mainly concerned with “earnings management” and assume independent periods, while I exogenously assume the agent has no control over the reporting of performance measures and I focus on their inter-temporal correlation.⁴

The main contribution of the paper is to offer an alternative to performance-driven turnover in explaining managerial performance around turnover when turnover is normal retirement. In a standard multi-period agency model with anticipated turnover, managerial performance is determined by the time-series properties of performance measures and by the presence of long-term actions. In particular, negatively auto-correlated performance measures and long-term managerial actions (that do not lead to high current charges to accounting performance) should lead to an inverted U shape of managerial performance over the manager’s tenure, V-shaped performance around managerial turnover, and normal (age-related) retirement as optimal (see Şabac 2005), all of which are consistent with existing empirical evidence.

The remainder of the paper is organized as follows. Section 2 presents the model, and Section 3 presents the renegotiation-proof contract and its implementation through short-term contracts, together with the explicit solution to the short-term contracts problem. Section 4 discusses dynamic incentives for various information structures. Section 5 discusses dynamic incentives for long-term actions. Section 6 concludes

⁴In a series of multi-period papers on the design of accounting-based performance measures and residual income that use LEN models, Dutta and Reichelstein (1999, 2002) and Dutta and Zhang (2002) assume full commitment to an N -period contract. Moreover, the residual income performance measures in Dutta and Reichelstein (1999) and Dutta and Zhang (2002) reduce the agency problem to one with independent periods in which the performance measures are statistically independent and the agent’s effort affects only current period performance. By contrast, while the short-term contracts in my model are equivalent to using statistically independent performance measures, the agent’s effort affects these independent measures over several periods. Christensen et al. (2004) provide a detailed discussion of statistical and technological independence of performance measures.

the paper. Appendix A contains the proofs.

2 The principal-agent model

A risk-neutral principal owns a production technology that requires effort a_t from an agent in each of N periods $t = 1, \dots, N$. The agent's utility is time additive with multiplicatively separable effort cost $u_t(q) = -\sum_{k=t}^{\infty} \gamma^{k-t} \exp(-\hat{r}(q_k - \frac{1}{2}a_k^2))$, for a consumption stream $q = (q_t, q_{t+1}, \dots)$, where q_t represents the agent's consumption at date t , the start of period $t + 1$, $\frac{1}{2}a_t^2$ is the agent's personal effort cost in period t , and \hat{r} is the agent's risk aversion. The discount rate $\gamma = (1 + R)^{-1}$ is the same for the principal and the agent and the agent can freely borrow or lend at rate R . The output from agent's effort $a_t \in \mathbb{R}$ is, for $t = 1, \dots, N$, $z_t = b_t a_t + \lambda_t$, where λ_t is a mean zero noise term which does not depend on a_t . Neither the outcomes z_t nor the agent's actions a_t are observable, hence neither is contractible. A contractible performance measure x_t is observed at the end of each period. The agent's effort in period t affects only the mean of the current and future performance measures, so that

$$x_t = \sum_{k=1}^t m_{tk} a_k + \varepsilon_t = M_t \cdot \underline{a}_t + \varepsilon_t, \quad (1)$$

where ε_t are mean zero noise terms that are joint normally distributed. The agent's actions are *short-term* if $m_{tk} = 0$ for $t \neq k$ and *long-term* otherwise.⁵ For each $1 \leq t \leq N$, let $\underline{a}_t = (a_1, \dots, a_t)$, and $\underline{x}_t = (x_1, \dots, x_t)$ denote the histories of actions and performance for the first t periods. I use the notation $E_t[\cdot]$ for the conditional expectation given history \underline{x}_t and $\text{cov}_t(\cdot, \cdot)$ for the conditional covariance given history \underline{x}_t . The conditional variance of x_t given history \underline{x}_{t-1} is denoted $\sigma_t^2 = \text{var}(x_t | \underline{x}_{t-1})$.

Let w_t denote the agent's compensation at date t (in date t currency at the end of period t). After date N , the agent retires, provides no more productive effort, and receives no further compensation. Let W_t represent the NPV (net present value) of future compensation discounted to date t for each employment date $t = 1, \dots, N$: $W_t = \sum_{k=t}^N \gamma^{k-t} w_k$. Similarly, let K_t represent the present value of future effort cost at date t : $K_t = \frac{1}{2} \sum_{k=t}^N \gamma^{k-t} a_k^2$. Since I assume both the principal and the agent can borrow and lend at the same rate, the timing of the agent's compensation does not affect either the principal's or the agent's utility, provided the present value of total compensation at a fixed date is constant.

The agent's compensation is determined by either long-term or short-term linear contracts. A long-term

⁵Whether the agent's actions are long-term depends on both how the output is realized over time and how the performance measures capture the output's realization. The principal's net benefit, b_t , from action a_t represents the present value of expected benefits attributable to a_t . Although no restriction is imposed on b_t and m_{kt} , accounting-based performance measures should capture the net output in the long run, $b_t = \sum_k \gamma^{k-t} m_{kt}$.

linear contract at date $t - 1$ is a take-it-or-leave-it offer by the principal $c^t = (\alpha_{t-1}, \beta_t, \dots, \beta_N)$ where α_{t-1} specifies a fixed payment of α_{t-1} in date $t - 1$ currency, and β_k specifies a variable payment $\beta_k x_k$ in date k currency, $t \leq k \leq N$. A short-term contract c^t only specifies a payment $w_t = \hat{\alpha}_t + \hat{\beta}_t x_t$ in date t currency. Since the principal and the agent can freely borrow and lend, there is flexibility in the timing of compensation, and the above specification of a linear compensation scheme is without loss of generality. In other words, any sequence of payments linear in the history of reported performance measures $w_t(\underline{x}_t)$ can be rearranged so that a fixed amount is paid upfront, and the portion that is variable with respect to x_t is paid after x_t is reported. I assume there is no other contractible information. The time line for period t is presented in Figure 1.

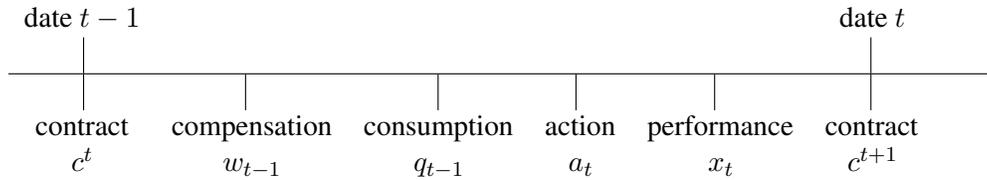


Figure 1: Time line of events in period t from date $t - 1$ to date t .

Given linear contracts, the agent's compensation is normally distributed and the agent's expected utility at date $t - 1$ is characterized by the following certainty equivalent, where $r = (1 - \gamma)\hat{r}$ is the agent's effective risk aversion, see Dutta and Reichelstein (1999) and Christensen et al. (2004):

$$\text{ACE}_{t-1} = \gamma E_{t-1}[W_t] - \gamma K_t - \gamma RP_t, \quad (2)$$

where the agent's risk premium is given by

$$RP_t = \frac{1}{2} r \sum_{k=t}^N \gamma^{k-t} \text{var}_{k-1}(E_k[W_k]). \quad (3)$$

The agent's reservation certainty equivalent is normalized to zero at the start of the first period. Utility functions, discount rates, reservation wages, the nature of the production technology, and the information structure are common knowledge.

3 Renegotiation

In this section, I present the optimal linear renegotiation-proof contract and its implementation by short-term linear contracts. Short-term contracts arise if the institutional environment restricts the writing of long-term contracts or annual compensation is set one period at a time. Additionally, short-term contracts in long-term moral hazard problems allow for the analysis of implicit incentives, that is incentives not explicitly included in compensation, but arising from the dependence of future contractual terms (fixed wages in this case) on past history.

Renegotiation-proof long-term contracts

Renegotiation of a long-term contract means that, if the agent accepts the contract offer c^0 , both the principal and the agent commit to the employment relationship for N periods, but cannot commit not to renegotiate as performance information becomes available. The renegotiation takes the form of a take-it-or-leave-it offer by the principal.⁶ An initial contract $c^0 = (\alpha_0, \beta_1, \dots, \beta_N)$, unless renegotiated, becomes at date t a contract $c^t = (\alpha_t, \beta_{t+1}, \dots, \beta_N)$, with $\alpha_t = \beta_t x_t$ the fixed payment at date t . The principal's renegotiation offer at any date t is a fixed payment at that date and a sequence of variable payments at subsequent dates, such that the agent's compensation discounted to any date t, \dots, N is linear in the agent's future performance. Thus, the principal's renegotiation offer is restricted to the same linear form as the existing contract in effect at that date. An equilibrium consists of an initial contract and a series of renegotiation offers, the agent's actions, and the principal's conjectures about the agent's actions such that: (i) the agent accepts both the initial contract and all the renegotiation offers; (ii) the principal's conjectures about the agent's actions are correct; (iii) the initial contract and the renegotiation offers are all optimal conditional on the observed performance measure history, the agent's conjectured actions, and the anticipated future renegotiation offers.

A *linear renegotiation-proof contract* is a contract such that, once agreed upon at the start of the first period, there does not exist a contract at any later renegotiation stage which is weakly preferred by both parties and at least one party strictly prefers. It is well-known, when no restriction is imposed on the contract form, that there is no loss of generality in restricting the analysis to renegotiation-proof contracts, see Fudenberg and Tirole (1990). Christensen et al. (2003, 2005) prove this result to a two-period LEN model with a single consumption date and multiplicative exponential agent utility, while Şabac (2005) shows

⁶The renegotiation concept I use here is the same as that of Fudenberg and Tirole (1990) in that both parties must agree to the renegotiated contract, but the timing is different. In my model, renegotiation takes place after the performance measure x_t is observed, while Fudenberg and Tirole have the renegotiation take place between the time the agent takes the action and the time the performance measure is observed in a single period model. Having the renegotiation take place after x_t is observed and before a_{t+1} is taken avoids the insurance/adverse selection problem of Fudenberg and Tirole.

that renegotiation-proof contracts are also without loss of generality in a general LEN model with multiple consumption dates and time-additive agent utility.⁷ The following proposition characterizes the optimal linear renegotiation-proof contract (and generalizes Şabac 2005, Proposition 2).

Proposition 1 *The actions induced by the optimal linear renegotiation-proof contract and the optimal incentives are given by the following recursive relations:*

$$a_N = m_{NN}\beta_N = \frac{m_{NN}^2 b_N}{m_{NN}^2 + r\sigma_N^2}, \quad (4)$$

$$a_t = m_{tt}\beta_t + \sum_{k=t+1}^N \gamma^{k-t} m_{kt}\beta_k, \quad (5)$$

$$\beta_t = \frac{m_{tt}b_t}{m_{tt}^2 + r\sigma_t^2} - \frac{\sum_{k=t+1}^N \gamma^{k-t} (m_{tt}m_{kt} + r\sigma_t^2 H_{kt}^t) \beta_k}{m_{tt}^2 + r\sigma_t^2}, \quad (6)$$

where H_{kt} characterizes the conditional expectations operator, $E_t[\varepsilon_k] = H_{kt} \cdot \underline{\varepsilon}_t$, with components $H_{kt} = (H_{kt}^1, \dots, H_{kt}^t)$, and $\sigma_t^2 = \text{var}_{t-1}(x_t)$.

The last period action (and incentive) are the familiar ones from the single period LEN agency problem, where the performance measure variance is the posterior variance σ_N^2 in period N . In earlier periods, the agent's action is determined both by the current period's incentive and by future periods' incentives as long as the agent's action has a long-term impact on future performance measures. The current period's optimal incentive has two components. The first component in (6) is the myopic optimal incentive for that period, $m_{tt}b_t/(m_{tt}^2 + r\sigma_t^2)$, which ignores the incentives provided by future periods' performance measures and is based on the tradeoff between the benefit of productive effort and the cost of effort plus the risk premium due to the variance of x_t alone. The second component in (6) has two parts. The first part adjusts the current incentive to the fact that future periods' incentives impact the current action. The second part is a compensation risk insurance adjustment the principal makes because there is covariance risk due to the correlation between current variable compensation $\beta_t x_t$ and future variable compensation $\beta_{t+1} x_{t+1}, \dots, \beta_N x_N$. In the renegotiation-proof contract, the principal chooses β_t so it is optimal conditional on date $t - 1$ information and future optimal incentive rates $\beta_{t+1}, \dots, \beta_N$.

The insurance adjustment in the incentive rates takes into account future optimal incentive rates and the covariance between current period performance and future periods' performance.⁸ Thus, if x_t is uncorre-

⁷The fact that a linear contract can be renegotiation-proof is due to the additive structure of the LEN framework: for every initial contract that is linear, the optimal renegotiation offers are also linear in the N performance measures from an ex-ante (start of the first period) perspective, and thus can be offered in an initial linear contract. In general, linear renegotiation offers do not allow for linear renegotiation-proof contracts, see Christensen and Feltham (2005).

⁸The impact of x_t on the conditional expectation $E_t[x_k]$ is proportional, with a positive proportionality constant, to $\text{cov}(x_t, x_k)$.

lated with future performance measures, the optimal incentive rate is the myopic one, adjusted for future incentives. In this case, the performance measure plays a pure effort incentive role since there is no covariance compensation risk. Otherwise, a positive correlation between period t performance and period k performance results in a downward adjustment to the period t incentive rate, assuming future incentive rates to be positive, because the principal trades off effort incentives for period t against increased covariance risk cost. Conversely, a negative correlation between period t performance and period k performance results in an upward adjustment to the period t incentive, because the negative covariance further reduces period t risk cost.

Short-term contracts

Short-term contracts are set every period based on available information. If neither the principal nor the agent can commit for several periods, there are no equilibria in which the agent stays for more than one period, see Christensen et al. (2003). This problem is avoided if the agent commits to stay for multiple periods, but then restrictions must be imposed on the principal at each contracting date, so that the agent does not commit to slavery. Alternatively, if the principal commits to offer the agent each period a continuation certainty equivalent that induces the agent to stay for all N periods, the agent does not need to commit to stay (see Christensen et al. 2003).

Gibbons and Murphy (1992) and Meyer and Vickers (1997) develop dynamic models with short-term contracts based on the career concerns model of Holmström (1999). In these models, the agent either participates in the labor market or commits to stay with a firm for multiple periods, while the principal offers each period a contract based on observable output subject to a constraint that reflects the agent's bargaining power relative the sharing of the surplus:

$$ACE_{t-1}(W_t) \geq \gamma B (E_{t-1}[X_t] - K_t - RP_t) , \quad (7)$$

where $X_t = \sum_{k=t}^N \gamma^{k-t} x_k$, and $z_t = x_t$, that is the firm's output is the (only) contractible performance measure. For consistency in preferences over output between the principal and the agent, $b_t = \sum_{k=t}^N m_{kt}$, that is the net marginal benefit of a_t can be realized over multiple periods, but not beyond the contracting horizon. In particular, this condition is satisfied in the case of short-term actions, with $b_t = m_{tt}$ and $m_{kt} = 0$ for $k \neq t$. The parameter B represents the agent's bargaining power: if $B = 1$, the principal's expected surplus is zero in equilibrium, as in Gibbons and Murphy (1992); if $B = 0$, the agent's certainty equivalent is zero at each contracting date in equilibrium. In a two-period model, Meyer and Vickers (1997) demonstrate

(Proposition 3, p. 564) that the agent's effort in equilibrium and the total surplus are independent of B , while for $B = 1$ Gibbons and Murphy show that they are the same as those determined by the renegotiation-proof contract. The case $B = 0$ corresponds to the "fair contracts" of Christensen et al. (2003), who also show the equivalence to the renegotiation-proof contract (see also Indjejikian and Nanda 1999); all these papers only consider the case of short-term actions, the analysis below generalizes their results to long-term actions. The following proposition characterizes the optimal short-term contracts that satisfy (7).

Proposition 2 *If the short-term contracts w_t satisfy the interim participation constraints (7), then*

$$\begin{aligned} w_t &= (1 - B) \left[\frac{1}{2} \hat{a}_t^2 + \frac{1}{2} r \text{var}_{t-1}(\hat{\beta}_t x_t + \gamma B E_t[X_{t+1}]) \right] + B \hat{E}_{t-1}[x_t] + \hat{\beta}_t (x_t - \hat{E}_{t-1}[x_t]) \\ RP_t &= \frac{1}{2} r \text{var}_{t-1}(\hat{\beta}_t x_t + \gamma B E_t[X_{t+1}]) + \gamma RP_{t+1}. \end{aligned} \quad (8)$$

The above characterization of the short-term contracts subject to (7) shows that w_t is a linear function of x_1, \dots, x_t and the agent's present value of compensation is linear in x_1, \dots, x_N :

$$W_t = \sum_{k=t}^N \gamma^{k-t} \left(B \hat{E}_{k-1}[x_k] + \hat{\beta}_k (x_k - \hat{E}_{k-1}[x_k]) \right) + (1 - B)(\hat{K}_t + RP_t). \quad (9)$$

Since the agent's effort does not affect the performance measures' variance, the agent's choice of a_t is only determined by the cost of effort and the impact of a_t on $E_{t-1}[W_t]$:

$$\begin{aligned} \frac{\partial}{\partial a_t} E_{t-1}[W_t] &= \sum_{k=t}^N \gamma^{k-t} \frac{\partial}{\partial a_t} E_{t-1} \left[B \hat{E}_{k-1}[x_k] + \hat{\beta}_k (x_k - \hat{E}_{k-1}[x_k]) \right] \\ &= \sum_{k=t}^N \gamma^{k-t} \left((B - \hat{\beta}_k) \frac{\partial}{\partial a_t} E_{t-1} \hat{E}_{k-1}[x_k] + \hat{\beta}_k \frac{\partial}{\partial a_t} E_{t-1}[x_k] \right) \\ &= \hat{\beta}_t m_{tt} + \sum_{k=t}^N \gamma^{k-t} \left((B - \hat{\beta}_k) \sum_{p=t}^{k-1} H_{kk-1}^p m_{pt} + \hat{\beta}_k m_{kt} \right). \end{aligned} \quad (10)$$

Substituting in the agent's incentive compatibility constraint $\partial/\partial a_t \text{ACE}_{t-1}(W_t) = 0$ yields the agent's optimal action choice:

$$a_t = m_{tt} \hat{\beta}_t + \sum_{k=t+1}^N \gamma^{k-t} \left(B \sum_{p=t}^{k-1} H_{kk-1}^p m_{pt} + \Gamma_{kt} \hat{\beta}_k \right), \quad (11)$$

where $\Gamma_{kt} = m_{kt} - \sum_{p=t}^{k-1} H_{kk-1}^p m_{pt}$. The agent's present value of wages (9) for periods t, \dots, N is a linear function of the performance measures x_t, \dots, x_N plus a constant that depends only on history \underline{x}_{t-1}

and the principal's conjectures regarding the agent's actions:

$$\begin{aligned}
W_t &= \alpha_t(\underline{x}_{t-1}, \hat{a}_N) + \sum_{k=t}^N \gamma^{k-t} \left(\hat{\beta}_k x_k + (B - \hat{\beta}_k) H_{kk-1} \cdot \underline{x}_{t-1} \right) \\
&= \alpha_t(\underline{x}_{t-1}, \hat{a}_N) + \sum_{k=t}^N \gamma^{k-t} \left(\hat{\beta}_k + \sum_{p=k+1}^N \gamma^{p-k} (B - \hat{\beta}_p) H_{pp-1}^k \right) x_k \\
&= \alpha_t(\underline{x}_{t-1}, \hat{a}_N) + \sum_{k=t}^N \gamma^{k-t} \beta_k^{\text{eff}} x_k,
\end{aligned} \tag{12}$$

where β_k^{eff} is the total slope of x_k in the agent's compensation and is called the *effective* (or total) incentive rate on x_k . It is instructive to compare (5) with (11). While with a renegotiation-proof long-term contract, the explicit incentive rates on period t, \dots, N performance alone determine period t effort, with the short-term contracts considered above, the explicit incentive rates on period t, \dots, N performance only determine part of the agent's effort choice, the other part being determined by how period t effort impacts fixed wages in future periods' compensation contracts. The agent's fixed wage in period k (given in (8)) consists of a term independent of performance measure history and a term dependent on performance measure history, namely $(B - \hat{\beta}_k) \hat{E}_{k-1}[x_k]$. The dependence of future fixed wages on past performance is due to the sharing of output by the agent $B \hat{E}_{k-1}[x_k]$ and by the principal's adjustment of agent compensation $-\hat{\beta}_k \hat{E}_{k-1}[x_k]$, so the agent's expected compensation is a fraction of expected output and the corresponding fraction of the effort cost and risk premium.

To summarize, at date $t-1$, the agent's action choice is determined by the *explicit incentives* $\hat{\beta}_t, \dots, \hat{\beta}_N$ and by the *implicit incentives* contained in future periods' fixed wages. Nevertheless, the agent faces total compensation that is linear in the performance measures, and the *effective incentives* for a_t are given by the coefficients $\beta_t^{\text{eff}}, \dots, \beta_N^{\text{eff}}$ of x_t, \dots, x_N in W_t . By contrast, in the renegotiation-proof contract, the agent's effort in period t is determined by the incentive rates β_t, \dots, β_N since a_t impacts only x_t, \dots, x_N . Thus, with a renegotiation-proof contract, the *explicit incentive rates* β_t, \dots, β_N are also the *effective incentive rates*, in that no other part of the contract provides incentives for a_t .

The agent's total compensation W_t can be rewritten in terms of effective incentives as $W_t = \alpha_t^{\text{eff}} + \sum_{k=t}^N \gamma^{k-t} \beta_k^{\text{eff}} x_k$ where α_t^{eff} only depends on \underline{x}_{t-1} . By backwards induction, it follows that the principal's choice of the explicit incentive β_t at date $t-1$ is equivalent to choosing the effective incentive rate β_t^{eff} on x_t , given anticipated future optimal incentive rates $\beta_{t+1}^{\text{eff}}, \dots, \beta_N^{\text{eff}}$. In equilibrium, the interim participation constraints (7) are binding, hence the principal's maximization of expected surplus does not depend on

$B < 1$:⁹

$$E_{t-1}[X_t - W_t] = (1 - B) (E_{t-1}[X_t] - K_t - RP_t) . \quad (13)$$

It follows that the principal's problem with respect to the effective incentive rates is the same as with long-term contracts subject to renegotiation. This proves the following proposition.

Proposition 3 *For any sequence of optimal short-term linear contracts that satisfy the interim participation constraints (7), the optimal effective incentive rates and the agent's optimal action choices are the same as the optimal incentive rates in the renegotiation-proof contract, and are independent of the agent's bargaining power.*

Proposition 3 generalizes both Gibbons and Murphy (1992) and Meyer and Vickers (1997) by showing that the short-term contract mechanisms based on the interim participation constraints (7) implement the same actions in equilibrium as the renegotiation-proof contract regardless of the agent's bargaining power. The net surplus (expected output less effort cost and risk premium) is the same, the only impact of the bargaining power parameter B is on how this surplus is shared between the agent and the principal and how the effective incentive is split between explicit and implicit incentives.

Special case $B = 0$, fair contracts

I now turn to the case where the principal has all the bargaining power at the interim contracting dates and the agent's reservation certainty equivalent is normalized to zero, which corresponds to setting $B = 0$ in (7). This case is considered by Meyer and Vickers (1997) and Indjejikian and Nanda (1999) and referred to by Christensen et al. (2003) as "fair contracts", based on Baron and Besanko (1987)(see also Fehr, Gächter, and Kirchsteiger 1997 and Rabin 1993). In the subsequent discussion of implicit incentives, I restrict the analysis to fair contracts because there is no sharing of the output by the agent, and I can consider the more general case in which output is not contractible and x_t represent contractible performance measures distinct from unobservable output. This has the advantage of allowing performance measure correlations that are independent of and do not impose restrictions on how output is correlated through time. Moreover, the principal's net benefit from period t agent effort, b_t , is again unconstrained as in the long-term contract setting, and may incorporate benefits realized after the end of period N . The interim participation constraints are now $ACE_{t-1}(W_t) \geq 0$ for all t . Proposition 2 carries through without modifications by simply setting

⁹This is also true for $B = 1$ as firms compete in offering an acceptable contract to the agent, thus maximizing $E_{t-1}[X_t] - K_t - RP_t$.

$B = 0$ in (8), that is

$$\begin{aligned} w_t &= \frac{1}{2} \hat{a}_t^2 + \frac{1}{2} r \hat{\beta}_t^2 \sigma_t^2 + \hat{\beta}_t (x_t - \hat{\mathbf{E}}_{t-1}[x_t]) \\ RP_t &= \frac{1}{2} r \hat{\beta}_t^2 \sigma_t^2 + \gamma RP_{t+1} = \frac{1}{2} r \sum_{k=t}^N \gamma^{k-t} \hat{\beta}_k^2 \sigma_k^2. \end{aligned} \quad (14)$$

In this case, the agent is effectively compensated each period on the current performance adjusted for expectations based on past performance:

$$\hat{x}_t = x_t - M_t \cdot \hat{a}_t - H_{tt-1} \cdot (\underline{x}_{t-1} - (M_1 \cdot \hat{a}_1, \dots, M_{t-1} \cdot \hat{a}_{t-1})) \sim x_t - H_{tt-1} \cdot \underline{x}_{t-1}, \quad (15)$$

where the equivalence is (up to an additive constant independent of history) with respect to incentives. These adjusted performance measures are statistically independent since \hat{x}_t represents the unexpected component of x_t , is independent of x_1, \dots, x_{t-1} , and $\hat{x}_1, \dots, \hat{x}_{t-1}$ are linear functions of x_1, \dots, x_{t-1} . In the short-term action case, the statistical independence of the adjusted performance measures is achieved at the cost of introducing long-term actions. For example, a_t impacts \hat{x}_t directly through its impact on x_t and $\hat{x}_{t+1}, \dots, \hat{x}_N$ indirectly through its impact on $\hat{\mathbf{E}}_t[x_{t+1}], \dots, \hat{\mathbf{E}}_{N-1}[x_N]$. Thus, even with short-term actions, the agency problem cannot be reduced to a series of fully independent one-period problems (see Christensen et al. 2004 and Christensen and Feltham 2005).

With fair contracts, the principal's problem in choosing $\hat{\beta}_t$ is the same as in determining the long-term renegotiation-proof contract in the adjusted performance measures \hat{x}_t . The following proposition characterizes the optimal (explicit) incentive rates in the fair contracts and how they relate to the induced actions.

Proposition 4 *If a_t are the actions induced by the optimal linear renegotiation-proof contract, and $\hat{\beta}_t$ represent the optimal incentive rates from the optimal fair contracts, then*

$$a_N = m_{NN} \hat{\beta}_N = \frac{m_{NN}^2 b_N}{m_{NN}^2 + r \sigma_N^2} \quad (16)$$

$$a_t = b_t - \frac{r \hat{\beta}_t \sigma_t^2}{m_{tt}} \quad (17)$$

$$\hat{\beta}_t = \frac{m_{tt} b_t}{m_{tt}^2 + r \sigma_t^2} - \frac{m_{tt} \sum_{k=t+1}^N \gamma^{k-t} \Gamma_{kt} \hat{\beta}_k}{m_{tt}^2 + r \sigma_t^2}, \quad (18)$$

where $\Gamma_{kt} = m_{kt} - \sum_{p=t}^{k-1} H_{kp}^p m_{pt}$.

In general, note that $\Gamma_{kt} = -\partial/\partial a_t \mathbf{E}_{t-1}[\hat{x}_k]$, so that Γ_{kt} captures the marginal impact of period t effort on the period k adjusted performance measure. The transformation from the correlated x_t to the statistically

independent \hat{x}_t introduces an effect similar to that of long term actions, so the marginal impact of a_t on the adjusted performance measure is a combination of its original marginal long-term effect m_{kt} on x_t and the marginal impact on $E_{k-1}[x_k]$, given by $\sum_{p=t}^{k-1} H_{kk-1}^p m_{pt}$. In particular, if the original performance measures x_t are statistically independent, contracting on the adjusted performance measures is the same as contracting on the original performance measures, and (18) and (5) coincide.

From (17) it follows that the risk premium attributable to the period t wage w_t can be written as $\frac{1}{2}r\hat{\beta}_t\sigma_t^2 = \frac{1}{2}m_{tt}(b_t - a_t)$ and the explicit incentive can be represented in terms of the optimal agent effort as $\hat{\beta}_t = m_{tt}(b_t - a_t)/(r\sigma_t^2)$. Substituting in the risk premium representation in (14) allows the representation of the principal's surplus in terms of only the optimal agent effort (see also Şabac 2005, Proposition 3)

$$U^P = \sum_{t=1}^N \gamma^t \left(b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} \frac{m_{tt}^2}{r\sigma_t^2} (b_t - a_t)^2 \right) = \sum_{t=1}^N \gamma^t \left(\frac{1}{2} b_t^2 - \frac{1}{2} \left(1 + \frac{m_{tt}^2}{r\sigma_t^2} \right) (b_t - a_t)^2 \right). \quad (19)$$

Substituting $\hat{\beta}_k$ as a function of a_k in the recursive relation for the explicit incentives (18) gives a recursive relation for the optimal agent effort a_t :

$$a_t = \frac{m_{tt}^2 b_t + \sum_{k=t+1}^N \gamma^{k-t} \sigma_t^2 \sigma_k^{-2} m_{kk} \Gamma_{kt} b_k}{m_{tt}^2 + r\sigma_t^2} - \frac{\sum_{k=t+1}^N \gamma^{k-t} \sigma_t^2 \sigma_k^{-2} m_{kk} \Gamma_{kt} a_k}{m_{tt}^2 + r\sigma_t^2}. \quad (20)$$

In the analysis of effort and incentive dynamics that follows, I will consider separately two particular cases, short-term actions and correlated performance measures, and long term actions and independent performance measures; the two particular cases emphasize the impact on incentives and managerial effort of performance measure correlation and long-term actions, respectively.

4 Incentive dynamics: the impact of correlation

In this section, I examine the dynamics of incentives and induced managerial effort when the managerial actions are short-term and the performance measures are correlated, that is $m_{tk} = 0$ for $t \neq k$, so that $\Gamma_{kt} = -H_{kk-1}^t m_{tt}$ for $t \leq k-1$. The induced managerial effort is the same for both fair contracts and the renegotiation-proof contract, while the explicit incentives differ. With a renegotiation-proof contract there are no implicit incentives, while with fair contracts implicit incentives for each period are included in future periods' fixed wages. The analysis extends the results on implicit incentives of Gibbons and Murphy (1992) to other information structures with the notable difference that I allocate all bargaining power to the principal. In order to provide explicit solutions and numerical examples, I simplify the analysis

by assuming identical periods, $b_t = b$ and $m_{tt} = m$ for all t , and I consider three specific information structures: an auto-regressive model, a career concerns model, and an accounting model.

4.1 The auto-regressive model

The first information system is a simple auto-regressive noise process characterized by

$$\varepsilon_t = \omega\varepsilon_{t-1} + \delta_t, \quad (21)$$

where $\omega \in [0, 1]$ and δ_t are independent, identically distributed mean zero terms with $\text{var}(\delta_t) = \sigma_\delta^2$, and $\varepsilon_1 = \delta_1$.¹⁰ The key characteristic of this information system is that posterior variances are constant since $E_{t-1}[\varepsilon_t] = \omega\varepsilon_{t-1}$ and $\sigma_t^2 = \text{var}_{t-1}(\varepsilon_t) = \sigma_\delta^2$. In particular, for $\omega = 1$, the performance measure noise is a random walk.

The following proposition characterizes the optimal incentive sequence for the auto-regressive model.

Proposition 5 *The optimal explicit incentives and the sequence of actions induced by the optimal sequence of fair contracts are given by the following relations*

$$\hat{\beta}_t = \frac{mb}{(1-\gamma\omega)m^2 + r\sigma_\delta^2} \left[1 - \left(\frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2} \right)^{N-t+1} \right] \quad (22)$$

$$a_t = \frac{(1-\gamma\omega)m^2 b}{(1-\gamma\omega)m^2 + r\sigma_\delta^2} + \frac{r\sigma_\delta^2 b}{(1-\gamma\omega)m^2 + r\sigma_\delta^2} \left(\frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2} \right)^{N-t+1}. \quad (23)$$

The explicit incentives are decreasing in t , while the induced actions are increasing in t .

The fact that the optimal effort induced increases as the agent nears retirement follows from the reduction of covariance risk for which the agent must be compensated as the number of periods remaining until retirement decreases. Conditional on date $t - 1$ information, the noise in the future performance measures is determined by the sequence $\delta_t, \omega\delta_t + \delta_{t+1}, \dots, \sum_{k=t}^N \omega^{N-k} \delta_k$. Since posterior variances are constant in the auto-regressive model, the only reduction in covariance risk comes from decreasing the number of remaining periods of employment. Thus, the incentive dynamics are determined by the remaining horizon and not by posterior variance reduction through learning. A numerical example is provided in Figure 2.

The fair contracts effectively substitute contracting on the correlated performance measures x_t by contracting on the statistically independent performance measures $x_t - \hat{E}_{t-1}[x_t]$. In particular, for the auto-

¹⁰I am grateful to Peter Christensen for suggesting this particular case. See also Holmström (1999) for a related model in the context of career concerns.

regressive model, these transformed performance measures are

$$\begin{aligned}\hat{x}_t &= x_t - \hat{\mathbf{E}}_{t-1}[x_t] = m(a_t - \omega a_{t-1}) - m(\hat{a}_t - \omega \hat{a}_{t-1}) + \delta_t \\ \hat{x}_{t+1} &= x_{t+1} - \hat{\mathbf{E}}_t[x_{t+1}] = m(a_{t+1} - \omega a_t) - m(\hat{a}_{t+1} - \omega \hat{a}_t) + \delta_{t+1}.\end{aligned}\tag{24}$$

Thus, a_t impacts \hat{x}_t directly and \hat{x}_{t+1} indirectly through the principal's expectation of period $t + 1$ performance as a "performance standard". As a result, it is necessary to set β_t higher than β_{t+1} to offset the negative effect of β_{t+1} on period t incentives. This example also illustrates how β_{t+1} provides an implicit incentive for a_t , while the explicit incentive is β_t .

4.2 The career concerns model

The second correlation structure corresponds to the career concerns model of Gibbons and Murphy (1992). The information system for the career concerns model is characterized by

$$\varepsilon_t = \delta_t + \theta,\tag{25}$$

where the terms δ_t are independent identically distributed with mean zero and variance σ_δ^2 , while θ is independent of the δ_t , has mean zero and variance σ_θ^2 . Thus, δ_t represents period-specific noise that is independent of all other noise components, while θ represents managerial ability or the match between manager and job, and is the same in all periods. The manager's ability or job match is not known to either the principal or the manager prior to contracting. The presence of θ implies that any two performance measures are positively correlated since $\text{cov}(x_k, x_l) = \sigma_\theta^2$ for $k \neq l$.

The following proposition characterizes the optimal incentive sequence for the career concerns model.

Proposition 6 *The last period action and explicit incentive are determined by*

$$a_N = m\hat{\beta}_N = \frac{m^2 b}{m^2 + r\sigma_N^2}.\tag{26}$$

The optimal explicit incentives β_t and the sequence of actions a_t induced by the optimal sequence of fair contracts are given by the following recursive relations

$$\hat{\beta}_t = \frac{mb(1-\gamma)}{m^2 + r\sigma_t^2} + \gamma \frac{m^2 \frac{\sigma_t^2}{\sigma_\delta^2} + r\sigma_{t+1}^2}{m^2 + r\sigma_t^2} \hat{\beta}_{t+1}\tag{27}$$

$$a_t = \frac{m^2 b \left(1 - \gamma \frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{\sigma_t^2}{\sigma_\delta^2} \right)}{m^2 + r\sigma_t^2} + \gamma \frac{\frac{\sigma_t^2}{\sigma_{t+1}^2} \frac{\sigma_t^2}{\sigma_\delta^2} m^2 + r\sigma_t^2}{m^2 + r\sigma_t^2} a_{t+1}. \quad (28)$$

The explicit incentives can be either increasing or decreasing in t , while the induced actions are increasing in t .

While optimal agent effort and effective incentives increase over time as described by Gibbons and Murphy (1992), explicit incentives increase or decrease over time depending on the model parameters.¹¹

It is useful to consider the impact of the agent's remaining tenure with the firm separately from the impact of decreasing posterior variances of the performance measures. To that end, assume that $\sigma_t = \sigma_\delta$, that is consider periods that are far enough from the first so that the posterior variance is close to its limit value. In this case, the principal's and the agent's problem are equivalent to the case of uncorrelated periods with $\varepsilon_t = \delta_t$. Thus, towards the end of the agent's tenure incentives and effort are close to the incentive and effort level from the single period problem with $\text{var}(\varepsilon) = \sigma_\delta^2$. The negative effect of the agent's human capital insurance drives the agent's optimal effort level significantly below last periods' effort at the start of his tenure. A numerical example is provided in Figure 2.

After sufficiently many periods, learning the agent's ability leads to approximately uncorrelated periods and approximately constant solutions for both effort and incentives. It follows that the main effect driving effort and incentives in the career concerns model is learning about the persistent uncertain parameter θ that describes managerial ability or job fit, which reduces the conditional variance and covariance of performance measures in later periods relative to the first periods.

4.3 The accounting model

The third correlation structure limits inter-period correlation to adjacent periods and I refer to it as the accounting model. The information system corresponding to the accounting model is characterized by

$$\varepsilon_t = \delta_t + \rho\theta_{t-1} + \theta_t, \quad (29)$$

where $\rho = \pm 1$, δ_t are period-specific, and θ_t is a common component in adjacent periods. The δ_t, θ_t terms are mutually independent, have mean zero, $\text{var}(\delta_t) = \sigma_\delta^2$, and $\text{var}(\theta_t) = \sigma_\theta^2$.

¹¹For example, if $\gamma = 0$, β_t are increasing, while for $\gamma = 1$ and $m^2(1/\sigma_\theta^2 + \sigma_\delta^2) > r$, β_t are decreasing. Gibbons and Murphy demonstrate in their model that explicit incentives are increasing as well. The different results on explicit incentives arise from the different short-term contract mechanisms. While Gibbons and Murphy assume the agent has all bargaining power, $B = 1$, with fair contracts, I assume the principal has all the bargaining power, $B = 0$.

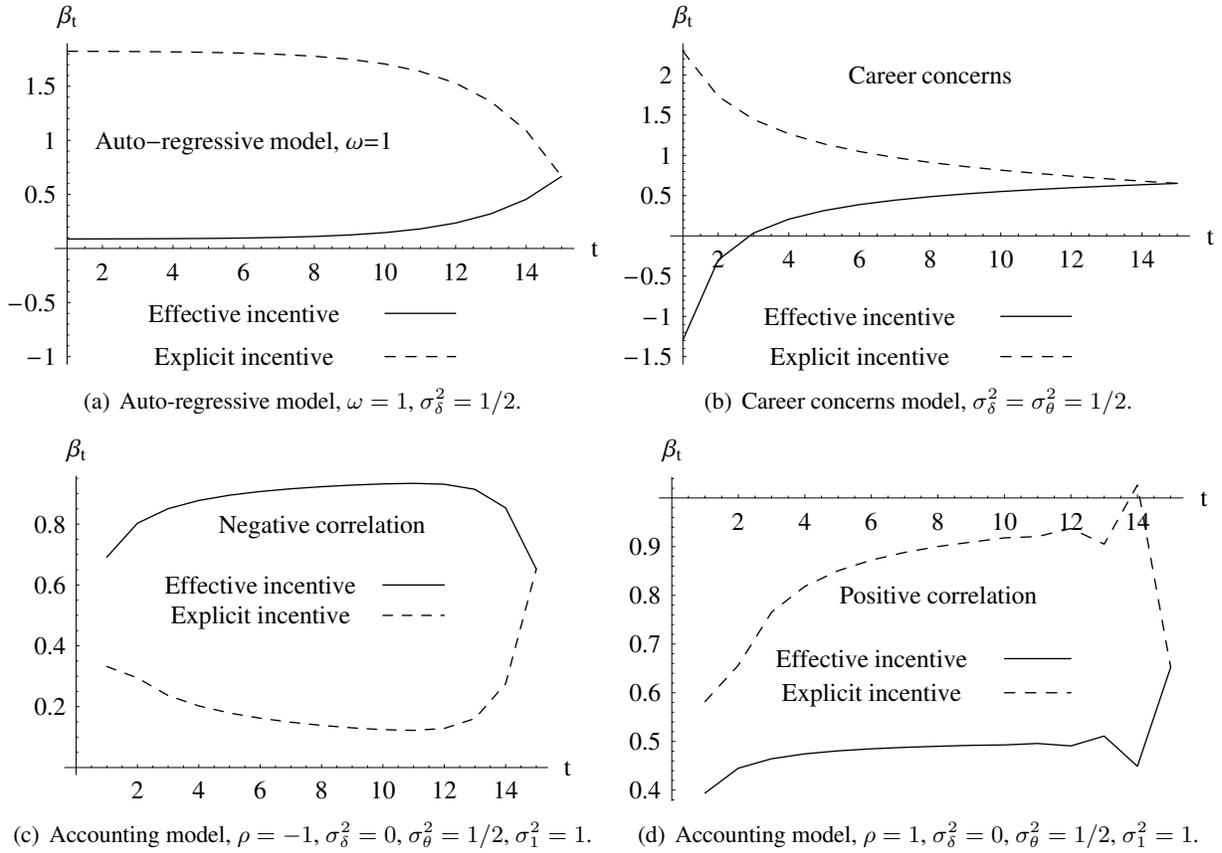


Figure 2: Short-term actions, explicit and effective incentives, $a_t = m\beta_t$: $b = m = r = 1$, $R = 5\%$, and $N = 15$.

With negative correlation, $\rho = -1$, and the common (between adjacent periods) component θ_t can be thought of as period t accrual estimation errors that have to be reversed in the next period, $t + 1$. Thus, negatively correlated noise in accounting-based performance measures reflects the nature of the accrual process. In this model, however, the accrual estimation errors, as with all the other components of the noise in the performance measure, are outside the manager's control.¹²

With positive correlation, $\rho = 1$, and the common (between adjacent periods) component θ_t can be thought of as a shock that persists from one period to the next period. The noise component θ_t first appears in period t , persists for one period and then disappears. In period $t + 1$, a new noise term θ_{t+1} appears, and so on. Note that the career concerns model and the accounting model coincide in the two periods case for positive correlation.¹³

¹²Christensen, Demski, and Frimor (2002) and Demski and Frimor (1999, 2001) examine the role played by income smoothing, or earnings management in dynamic incentives with renegotiation.

¹³In the two periods case, Christensen et al. (2005) show that the principal's surplus decreases in the correlation of the performance measures, and that accrual estimation errors may counteract the negative effects of having a high variance of managerial ability or job matching. Their findings suggest the correlation of performance measures may be endogenously determined. By contrast, in this paper, I assume the correlation structure to be exogenously given.

The following proposition characterizes the optimal incentive sequence for the accounting model.

Proposition 7 *The last period action and explicit incentive are determined by*

$$a_N = m\hat{\beta}_N = \frac{m^2b}{m^2 + r\sigma_N^2}. \quad (30)$$

The optimal explicit incentives and the sequence of actions induced by the optimal sequence of fair contracts are given by the following recursive relations

$$\hat{\beta}_t = \frac{mb \left(1 + \gamma\rho \frac{\sigma_\theta^2}{\sigma_t^2}\right)}{m^2 + r\sigma_t^2} - \gamma\rho \frac{\sigma_\theta^2 r \frac{\sigma_{t+1}^2}{\sigma_t^2}}{m^2 + r\sigma_t^2} \hat{\beta}_{t+1} \quad (31)$$

$$a_t = \frac{m^2b}{m^2 + r\sigma_t^2} - \gamma\rho \frac{\sigma_\theta^2 r}{m^2 + r\sigma_t^2} a_{t+1}. \quad (32)$$

The explicit incentives and the induced actions are generally non-monotonic in t .

Of the three particular cases examined, the accounting model generates the most complex behavior. In the auto-regressive model, posterior variances are constant, so effort and incentives are determined only by the risk imposed on the agent for the remaining employment horizon. By contrast, in the career concerns model, posterior variances decrease over time and, in the limit, later periods become uncorrelated. As a result, effort and incentives tend to become constant toward the end of a long enough tenure. Thus, while the auto-regressive model does not have this learning effect and incentives are determined by the remaining horizon, in the career concerns model there is only a learning effect and no horizon effect. In order to separate the learning effect from the horizon effect in the accounting model, it is necessary to separately examine the agent's tenure over three parts: an early part, a middle part, and an end part.

To examine the horizon effect, consider the last periods of the agent's tenure, which are far enough from the early periods so the posterior variance is approximately equal to its limit value σ_∞^2 (see Appendix A in Şabac 2005). In that case, equations (31) and (32) imply

$$a_t - a_{t-1} = -\gamma\rho \frac{\sigma_\theta^2 r}{m^2 + r\sigma_\infty^2} (a_{t+1} - a_t) \quad (33)$$

$$\hat{\beta}_t - \hat{\beta}_{t-1} = -\gamma\rho \frac{\sigma_\theta^2 r}{m^2 + r\sigma_\infty^2} (\hat{\beta}_{t+1} - \hat{\beta}_t). \quad (34)$$

Thus, the incentive insurance effect drives the agent's incentives in the last periods as follows. For positive correlation, $\rho = 1$, both sequences are alternating. This is due to the fact that, to reduce aggregate risk imposed on the agent, the principal must choose a lower or a higher incentive depending on how high or

low the next period's incentive will be. For negative correlation, $\rho = -1$, the principal can choose higher incentives in previous periods due to the increasing beneficial effect of mutual insurance offered by future performance measures.

Moving backwards in time towards the middle periods of the agent's tenure and holding the conditional variance constant, the agent's effort and incentives converge quickly towards limit values. For positive correlation, these limit values are significantly below last period's values. For negative correlation, the opposite holds, and the limit values are significantly above last period's values. In both cases, explicit incentives are almost the mirror image of effective incentives because they are adjusted to take into account future implicit incentives.

Finally, during the first periods of the agent's tenure, the dominant effect is the learning effect due to the reduction of posterior variances for the performance measures. The analysis depends on how ε_1 is specified. Suppose, in general, that $\varepsilon_1 = \psi + \delta_1 + \theta_1$, where ψ is independent of δ_t, θ_t , and $\text{var}(\psi) = \sigma_0^2$. In particular, $\sigma_1^2 = \sigma_0^2 + \sigma_\delta^2 + \sigma_\theta^2$ and the posterior variances behave as follows: if $\sigma_1^2 < \sigma_\infty^2$, then σ_t^2 is increasing towards σ_∞^2 ; if $\sigma_1^2 = \sigma_\infty^2$, then $\sigma_t^2 = \sigma_\infty^2$ for all t ; and if $\sigma_1^2 > \sigma_\infty^2$, then σ_t^2 is decreasing towards σ_∞^2 . For example, the last case occurs when prior variances are assumed identical across periods, that is $\sigma_0^2 = \sigma_\theta^2$.

If enough prior history is common knowledge at the time the agent is hired, then the common beliefs on the posterior variance σ_1^2 are such that $\sigma_1^2 \approx \sigma_\infty^2$ and no learning takes place. Then, the behavior of actions and incentives over time is the same as that discussed above for the case $\sigma_t^2 = \sigma_\infty^2$. However, even if enough prior history is common knowledge, the term ψ can represent aspects of a manager's performance that are generally controllable by the manager, but not right after the manager is hired: the manager needs a learning period to control these factors. In this case $\sigma_1^2 = \sigma_0^2 + \sigma_\infty^2 > \sigma_\infty^2$ and learning does take place in the model. While the analytical expressions for the actions and incentives during the first periods of the manager's tenure are not tractable, numerical analysis indicates that the learning effect (present when $\sigma_1^2 > \sigma_\infty^2$) works the same way as in the career concerns model, and effort is increasing over time due to reductions in the posterior variances of the performance measures.¹⁴ Increasing posterior variances lack an intuitive interpretation, and I consider only cases with decreasing posterior variances—that is, learning—in the accounting model. A numerical example is provided in Figure 2. Note that the numerical example illustrates a case with learning in the first periods.

To conclude, the manager's performance and incentives are driven by the reduction in posterior variance during the first periods, are constant during the middle periods, and are driven by a last period insurance

¹⁴In the case when posterior variances *increase* during the first periods, numerical analysis (not reported) indicates that optimally induced actions are decreasing.

effect during the last periods. Interestingly, for the negative correlation case, performance has an inverted U shape, and first-period performance is higher than last-period performance. For the positive correlation case, except for the last periods, performance generally increases towards the last period and last-period performance is higher than first-period performance as in the career concerns model.

5 Incentive dynamics: the impact of long-term actions

In this section, I examine the dynamics of incentives and induced managerial effort when the managerial actions are long-term and the performance measures are uncorrelated, that is $H_{kk-1} = 0$, so $\Gamma_{kt} = m_{kt}$ for $t \leq k-1$. As noted earlier, with serially uncorrelated performance measures, there are no implicit incentives with short-term contracts, and the explicit incentives from the fair contracts are the same as the incentives from the renegotiation-proof contract. As in the previous section, I maintain the simplifying assumption that $b_t = b$ and I assume that the performance measures' noise has constant variance $\sigma_t = \sigma$.

5.1 Two-period benefits

This case is characterized by actions with a two-period impact: $m_{tt} = m$, $m_{t+1t} = \gamma^{-1}(b - m)$, and $m_{kt} = 0$ for $k \geq t + 2$. The following proposition characterizes the optimal incentives and actions for the two-period benefit model.

Proposition 8 *The optimal incentives and the sequence of actions induced by the renegotiation-proof contract are given by the following relations*

$$\beta_t = \frac{m(b - a_t)}{r\sigma^2} \tag{35}$$

$$a_t = \frac{mb^2 - m(b - m)a_{t+1}}{m^2 + r\sigma^2}. \tag{36}$$

Solving the recursive relation (36) gives

$$a_t = \frac{b^2m}{bm + r\sigma^2} + \frac{br\sigma^2}{bm + r\sigma^2} \left[\frac{-m(b - m)}{m^2 + r\sigma^2} \right]^{N-t+1}. \tag{37}$$

The magnitude of the term $\psi = -m(b - m)/(m^2 + r\sigma^2)$ determines the behavior of the optimal actions as follows. If $m \geq 0$, then $|\psi| < 1$ and the optimal actions converge (moving back in time from the last period) to a limit value equal to the first term in (37). The convergence is non-monotonic because $\psi < 0$ and the limit value of optimal effort is higher than the last period's effort if $0 < m < b$. Thus, in this

case managerial effort is generally decreasing towards the last period. A numerical example is presented in Figure 3, (a). If $m = b$, managerial effort is constant, since in this case the periods are also technologically independent (the actions are only short-term).

If $m < 0$ and $b|m| < r\sigma^2$, then as before $|\psi| < 1$. The convergence is monotonic this time because $\psi > 0$ and the limit value of optimal effort is lower than the last period's effort. Thus, in this case managerial effort is generally increasing towards the last period. Finally, if $m < 0$ and $b|m| \geq r\sigma^2$, then $\psi \geq 1$, and managerial effort increases towards the last period (but does no longer converge back in time). A numerical example is presented in Figure 3, (b).¹⁵

The intuition for this behavior is that future periods' incentives act as positive/negative externalities for the current period's incentive rate and optimal effort. The principal sets β_t in period t and the induced managerial effort is $a_t = m\beta_t + (b - m)\beta_{t+1}$. It follows that the marginal benefit to the principal for incentive rate β_t is $\partial(ba_t)/\partial\beta_t = bm$, while the marginal cost is

$$\frac{\partial}{\partial\beta_t} \left(\frac{1}{2}r\beta_t^2\sigma^2 + \frac{1}{2}a_t^2 \right) = r\beta_t\sigma^2 + ma_t = (m^2 + r\sigma^2)\beta_t + m(b - m)\beta_{t+1}. \quad (38)$$

Thus, if $0 \leq m \leq b$, and $\beta_{t+1} \geq 0$, a higher effort a_t is induced relative to that given by the current incentive alone, and so the current incentive is reduced below the myopic level. At the same time, a higher/lower next period's incentive results in a lower/higher current period incentive, because the next period's incentive increases the marginal cost to the principal of setting the current period's incentive. If $m < 0$ and $\beta_{t+1} < 0$, this effect is reversed for optimal effort induced, since $a_t < m\beta_t$; at the same time, the next period's incentive increases the marginal cost to the principal of setting the current period's incentive, resulting in a lower current period incentive.

Alternatively, the marginal cost of risk for increasing managerial effort in the current period is

$$\frac{\partial}{\partial a_t} \left(\frac{1}{2}r\beta_t^2\sigma^2 \right) = \frac{r\sigma^2}{m^2} [a_t - (b - m)\beta_{t+1}], \quad (39)$$

so it is reduced or increased by a fixed amount proportional to the next period's incentive, leading to a lower or higher current induced effort.

¹⁵In both panels (a) and (b), the principal's net benefit is the same, $b = 2$. In the first panel, $m = -10$ in the first period corresponds to an expensed investment whose benefit is realized entirely in the second period. In the second panel, both the benefits and the costs are recognized uniformly over the two periods.

5.2 Diminishing marginal impact of effort

This case is characterized by actions with a diminishing marginal impact: $m_{kt} = \omega^{k-t}m$ for $t \leq k$, with $\omega < 1$, and $m > 0$. The following proposition characterizes the optimal incentives and actions for the diminishing marginal impact model.

Proposition 9 *The optimal incentives and the sequence of actions induced by the renegotiation-proof contract are given by the following relations*

$$\beta_t = \frac{m(b - a_t)}{r\sigma^2} \quad (40)$$

$$a_t = \frac{bm^2 + \gamma\omega r\sigma^2 a_{t+1}}{m^2 + r\sigma^2}. \quad (41)$$

Solving the recursive relation (41) gives

$$a_t = \frac{bm^2}{m^2 + r\sigma^2(1 - \gamma\omega)} - \frac{1}{m^2 + r\sigma^2(1 - \gamma\omega)} \left[\frac{\gamma\omega r\sigma^2}{m^2 + r\sigma^2} \right]^{N-t+1}. \quad (42)$$

The optimal actions converge (moving back in time from the last period) to a limit value equal to the first term in (42). The convergence is monotonic and the limit value of optimal effort is higher than the last period's effort. Thus, in this case managerial effort is decreasing towards the last period. A numerical example is provided in Figure 3 c, d.

Note the similarity between (32) and (41). The accounting model with negative correlation generates the same behavior as the long-term effort with diminishing marginal impact. In fact the two models are mathematically equivalent, assuming constant posterior variances in the accounting model and $\omega = \sigma_\theta^2 / \sigma_\infty^2$. Thus, ignoring learning effects, one cannot distinguish between the impact of negatively correlated accruals and that of long-term effort (with diminishing marginal impact on performance measures), as they both result in decreased managerial effort towards retirement.

6 Conclusions

In this paper, I analyze the impact of the time-series properties of performance measures, together with long-term actions and the contracting horizon on incentives and managerial performance in a dynamic agency with renegotiation. The multi-period LEN model combines renegotiation and correlated periods in a setting in which short-term contracts implement a long-term renegotiation-proof contract. The use of short-term contracts relies on anticipated future contracts and introduces implicit incentives into the analysis (see also

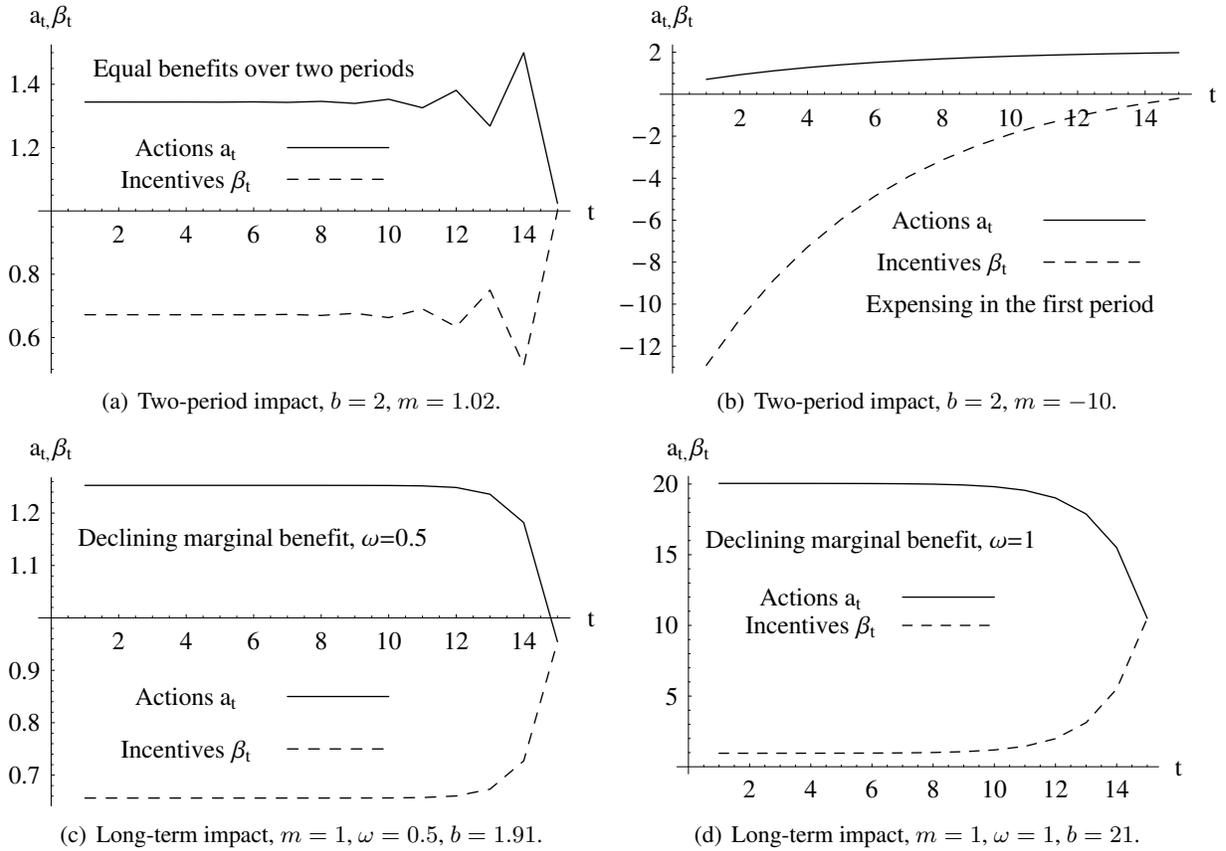


Figure 3: Long-term actions, actions and incentives: $r = \sigma^2 = 1$, $R = 5\%$, and $N = 15$.

Gibbons and Murphy (1992), Indjejikian and Nanda (1999), Christensen et al. (2003), and the references therein). Considering more than two (or three) periods allows for more interesting covariance structures and allows the separation between the beginning and the end of a manager's tenure; by contrast the related papers of Christensen et al. (2003, 2005) only consider two-period models and short-term actions.

The main result (Proposition 4) provides a general recursive solution to the short-term contract problem that determines both the dynamic of managerial actions and explicit contractual incentives by backwards induction as a function of posterior variances of the performance measures, future anticipated incentives, and the impact of current actions on future performance expectations. Thus the two main effects determining the dynamic of incentives and managerial actions are: a forward-looking horizon effect due to the anticipation of future contractual terms and a backward-looking learning effect due to observing the history of performance measures (which leads to changes in the posterior variances of the performance measures). In addition, the horizon effect has two components: an insurance, or risk externality effect, which is due to correlation in the performance measures, and an incentive externality, due to long-term managerial actions.

In addition, I show (Proposition 3) that the short-term contracts implement the renegotiation-proof con-

tract (characterized in Proposition 1). This result generalizes Meyer and Vickers (1997) to multiple periods, arbitrary correlation structures, and long-term actions.

If both the horizon effect and the learning effect converge to steady states, a manager's tenure is naturally divided into three parts: an early part in which changes in the posterior variances of performance measures dominate; a middle part in which a steady state is attained; and a late part in which the effect of anticipated incentives in the periods remaining to retirement dominates. The impact of performance measure correlation is exemplified by three special cases: an auto-regressive model in which posterior variances are constant, so only the horizon effect is present; a career concerns model similar to the one of Gibbons and Murphy (1992) in which uncertainty about a persistent productivity parameter is gradually resolved; and an accounting model in which early uncertainty is accompanied by the reversal of accrual estimation errors, so that both the learning effect and the horizon effect are present.

In each of the three special cases pertaining to correlation of performance measures, a steady state is attained in the middle periods, the learning effect is negligible in the late part of the agent's tenure (given enough prior history), and the principal optimally sets incentives based on the anticipated incentive risk borne by the agent over the remaining periods. Thus, for positively correlated performance measures, the horizon effect appears as an increase in expected performance because of the decrease of covariance risk with approaching retirement; with negatively correlated performance measure, the horizon effect appears as a decrease in expected performance because of increased covariance risk towards retirement.

While the expected performance is monotonically increasing in the auto-regressive model and the career concerns model, in the accounting model there is an increase in expected performance in the early part of the agent's tenure if there is a learning effect (declining posterior variances). In particular, an inverted U shape performance is predicted for the accounting model with negative correlation if there is a learning effect.

The impact of long-term actions is exemplified by two special cases: one in which the managerial effort impacts only the current and the next period, and one in which the managerial action has a declining marginal benefit over all future periods. With a declining marginal benefit of effort, expected performance declines towards retirement because the positive externalities associated with future incentives are reduced. In fact, the predicted behavior is undistinguishable from the horizon effect in the accounting model with negative correlation. With two-period managerial effort impact, the horizon effect is ambiguous: performance may either decrease or increase. However, for a wide range of parameters, the prediction of the model continues to support declining managerial effort towards retirement.

More generally, assuming turnover to be rationally anticipated, the model links the time-series properties of performance measures and the presence of long-term effort to observed performance around managerial

turnover. For the special cases considered in the paper, the inverted U shape of the manager's performance obtained in the accounting model with negative correlation and in the long-term action models results in V shaped performance around managerial turnover, consistent with evidence on firm performance around CEO turnover. By contrast, the models with positive correlation—including the career concerns model—generally predict increasing managerial performance throughout the manager's tenure, with the successor's performance starting lower than the predecessor's performance at the end.

While the above empirical predictions on expected performance surrounding turnover are in some cases similar to predictions made in the existing literature, their basis is very different: in my model performance is endogenously determined given rationally anticipated turnover and is not an exogenous determinant of managerial turnover as in Hermalin and Weisbach (1998) (see also Murphy and Zimmerman (1993), Engel et al. (2003), and Huson et al. (2004)). However, a direct comparison between the implications of Hermalin and Weisbach (1998) and this paper is difficult: while Hermalin and Weisbach base their results on the bargaining between the CEO and the board and have no moral hazard in the model, my model is driven by pure moral hazard and the board plays no role in it (other than being “the principal”).

Finally, using short-term contracts, implicit incentives are created, such that there is no simple link between expected performance in a given period and the explicit incentive in that period. By contrast, a renegotiation-proof contract has no implicit incentives and the expected performance depends on the current (and future) explicit incentives in the model. Since the dynamic of pay/performance sensitivities is the dynamic of explicit incentives, it strongly depends on the contracting setting; in the examples analyzed, the dynamic of explicit incentives is almost the mirror image of the dynamic of expected performance.

The above predictions have to be considered with some caution, as the model on which they are based is highly stylized: the board of directors does not play an active role and multiple managerial tasks and performance measures are not considered. Other than the above reservations, the main limitation in the analysis stems from the complexity of covariance structures for the performance measures. The tractability of the explicit solutions to the agency problem is a direct function of the complexity of conditional expectations and variances of the performance measures. For example, it would be interesting to combine a persistent productivity parameter as in the career concerns model with accrual estimation errors as in the accounting model. Intuitively, the inverted U shape of performance in the accounting model should be preserved, since only the learning effect is strengthened, but the exact solution appears intractable.

Appendix A. Proofs

Proof of Proposition 1. Since the agent's actions do not impact the performance measures' variance, and thus do not affect the risk premium, the agent's optimal action choice satisfies

$$a_t \in \operatorname{argmax} \left(\mathbf{E}_{t-1}[W_t(c)] - \frac{1}{2}a_t^2 \right), \quad (43)$$

which proves (5). When setting β_t , the principal's problem is to induce the optimal period t effort:

$$\max_{\beta_t} \left(b_t a_t - \frac{1}{2}a_t^2 - \frac{1}{2}r \operatorname{var}_{t-1}(\hat{\mathbf{E}}_t[W_t(c)]) \right). \quad (44)$$

Substituting (5) for a_t , differentiating with respect to β_t and solving the resulting first-order condition gives (6). In differentiating the risk premium term, note that

$$\frac{\partial}{\partial \beta_t} \operatorname{var}_{t-1}(\hat{\mathbf{E}}_t[W_t(c)]) = \frac{\partial}{\partial \beta_t} \operatorname{var}_{t-1} \left(\frac{\partial}{\partial x_t} \hat{\mathbf{E}}_t[W_t(c)] x_t \right). \quad (45)$$

□

Proof of Proposition 2. The proof is by backwards induction. Since the principal can adjust the fixed wage so the interim participation constraint is binding in equilibrium,

$$\hat{\mathbf{E}}_{N-1}[W_N] - K_N - RP_N = B(\hat{\mathbf{E}}_{N-1}[X_N] - K_N - RP_N). \quad (46)$$

Substituting $\hat{\mathbf{E}}_{N-1}[W_N] = \hat{\mathbf{E}}_{N-1}[w_N] = \alpha_N + \beta_N \hat{\mathbf{E}}_{N-1}[x_N]$ in the above yields

$$\alpha_N + \beta_N \hat{\mathbf{E}}_{N-1}[x_N] = B\hat{\mathbf{E}}_{N-1}[x_N] + (1 - B)(K_N + RP_N). \quad (47)$$

Solving for α_N and substituting back in w_N gives

$$w_N = (1 - B)(K_N + RP_N) + B\hat{\mathbf{E}}_{N-1}[x_N] + \beta_N(x_N - \hat{\mathbf{E}}_{N-1}[x_N]). \quad (48)$$

This proves the proposition for $t = N$ because

$$RP_N = \frac{1}{2} r \operatorname{var}_{N-1}(\hat{\mathbf{E}}_N[W_N]) = \frac{1}{2} r \operatorname{var}_{N-1}(\beta_N x_N). \quad (49)$$

Assume that (8) holds for t, \dots, N , I will now prove it for $t - 1$. As in the previous case, the participation

constraint being binding in equilibrium,

$$\hat{E}_{t-2}[W_{t-1}] - K_{t-1} - RP_{t-1} = B(\hat{E}_{t-2}[X_{t-1}] - K_{t-1} - RP_{t-1}). \quad (50)$$

Substituting $\hat{E}_{t-2}[W_{t-1}] = \hat{E}_{t-2}[w_{t-1} + \gamma W_t] = \alpha_{t-1} + \beta_{t-1}\hat{E}_{t-2}[x_{t-1}] + \gamma\hat{E}_{t-2}[W_t]$ in the above yields

$$\alpha_{t-1} + \beta_{t-1}\hat{E}_{t-2}[x_{t-1}] + \gamma\hat{E}_{t-2}[W_t] = B\hat{E}_{t-2}[X_{t-1}] + (1 - B)(K_{t-1} + RP_{t-1}). \quad (51)$$

From the induction hypothesis it follows that $\hat{E}_{t-2}[W_t] = (1 - B)(K_t + RP_t) + B\hat{E}_{t-2}[X_t]$. Since $X_{t-1} = x_{t-1} + \gamma X_t$, $K_{t-1} = \frac{1}{2}a_{t-1}^2 + \gamma K_t$, and $RP_{t-1} = \frac{1}{2}r\text{var}_{t-2}(E_{t-1}[W_{t-1}]) + \gamma RP_t$, the above equation simplifies to

$$\alpha_{t-1} + \beta_{t-1}\hat{E}_{t-2}[x_{t-1}] = B\hat{E}_{t-2}[x_{t-1}] + (1 - B)\left(\frac{1}{2}a_{t-1}^2 + \frac{1}{2}r\text{var}_{t-2}(E_{t-1}[W_{t-1}])\right). \quad (52)$$

Solving for α_{t-1} and substituting back in w_{t-1} gives

$$w_{t-1} = (1 - B)\left(\frac{1}{2}a_{t-1}^2 + \frac{1}{2}r\text{var}_{t-2}(E_{t-1}[W_{t-1}])\right) + B\hat{E}_{t-2}[x_{t-1}] + \beta_{t-1}(x_{t-1} - \hat{E}_{t-2}[x_{t-1}]). \quad (53)$$

This proves the proposition because

$$RP_{t-1} = \frac{1}{2}r\text{var}_{t-2}(\hat{E}_{t-1}[W_{t-1}]) + \gamma RP_t = \frac{1}{2}r\text{var}_{t-2}(\beta_{t-1}x_{t-1} + \gamma BE_{t-1}[X_{t-1}]) + \gamma RP_t. \quad (54)$$

□

Proof of Proposition 4. Using the characterization of the risk premium in (14), the agent's certainty equivalent at date $t - 1$ can be written as

$$\begin{aligned} ACE_{t-1} &= \gamma E_{t-1}[W_t] - \gamma K_t - \frac{1}{2}r \sum_{k=t}^N \gamma^{k-t+1} \hat{\beta}_k^2 \sigma_k^2 \\ &= \gamma E_{t-1} \left[\sum_{k=t}^N \gamma^{k-t} \hat{\beta}_k \hat{x}_k \right] - \gamma K_t - \frac{1}{2}r \sum_{k=t}^N \gamma^{k-t+1} \hat{\beta}_k^2 \sigma_k^2. \end{aligned} \quad (55)$$

The action induced by the contract satisfies, see (11) with $B = 0$,

$$a_t = \hat{\beta}_t m_{tt} + \sum_{k=t+1}^N \gamma^{k-t} \hat{\beta}_k \Gamma_{kt}, \quad (56)$$

where $\Gamma_{kt} = m_{kt} - \sum_{p=t}^{k-1} H_{kk-1}^p m_{pt}$. Since the participation constraint is binding in equilibrium, I can replace the agent's expected compensation by the expected effort cost plus the expected risk premium in the principal's problem at date $t - 1$:

$$\max_{\hat{\beta}_t} \hat{E}_{t-1} \left[\sum_{k=t}^N \gamma^{k-t} z_k - W_{t-1} \right] = \max_{\hat{\beta}_t} \hat{E}_{t-1} \left[\sum_{k=t}^N \gamma^{k-t} z_k - K_t - \frac{1}{2} r \sum_{k=t}^N \gamma^{k-t} \hat{\beta}_k^2 \sigma_k^2 \right]. \quad (57)$$

The first-order condition with respect to $\hat{\beta}_t$ gives

$$b_t \frac{\partial a_t}{\partial \hat{\beta}_t} - a_t \frac{\partial a_t}{\partial \hat{\beta}_t} - r \hat{\beta}_t \sigma_t^2 = 0. \quad (58)$$

Using the fact that $\partial a_t / \partial \hat{\beta}_t = m_{tt}$ and substituting in the principal's first-order condition yields

$$b_t m_{tt} - a_t m_{tt} - r \hat{\beta}_t \sigma_t^2 = 0, \quad (59)$$

and solving for a_t , I obtain

$$a_t = b_t - \frac{r \hat{\beta}_t \sigma_t^2}{m_{tt}}. \quad (60)$$

Substituting a_t in equation (56) and solving for the optimal incentive gives

$$\hat{\beta}_t = \frac{m_{tt} b_t}{m_{tt}^2 + r \sigma_t^2} - \frac{m_{tt} \sum_{k=t+1}^N \gamma^{k-t} \hat{\beta}_k \Gamma_{kt}}{m_{tt}^2 + r \sigma_t^2}. \quad (61)$$

□

Proof of Proposition 5. For the auto-regressive model, $E_{k-1}[\varepsilon_k] = \omega \varepsilon_{k-1}$ and, as a consequence, $H_{kk-1} = (0, \dots, 0, \omega)$. From Proposition 4, assuming identical periods and using $H_{t+1}^t = \omega$, I have that

$$\begin{aligned} \hat{\beta}_N &= \frac{mb}{m^2 + r \sigma_\delta^2} \\ \hat{\beta}_t &= \frac{mb}{m^2 + r \sigma_\delta^2} + \gamma \omega \frac{m^2}{m^2 + r \sigma_\delta^2} \hat{\beta}_{t+1}. \end{aligned} \quad (62)$$

Solving explicitly for $\hat{\beta}_t$ gives

$$\hat{\beta}_t = \hat{\beta}_N \left[\frac{1 - \left(\frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2} \right)^{N-t+1}}{1 - \frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2}} \right] = \frac{mb}{(1 - \gamma\omega)m^2 + r\sigma_\delta^2} \left[1 - \left(\frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2} \right)^{N-t+1} \right]. \quad (63)$$

Substituting $a_t = b - (r\sigma_\delta^2/m)\hat{\beta}_t$ in the above expression for $\hat{\beta}_t$ yields

$$\begin{aligned} a_t &= b - \frac{r\sigma_\delta^2 b}{(1 - \gamma\omega)m^2 + r\sigma_\delta^2} \left[1 - \left(\frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2} \right)^{N-t+1} \right] \\ &= \frac{(1 - \gamma\omega)m^2}{(1 - \gamma\omega)m^2 + r\sigma_\delta^2} + \frac{r\sigma_\delta^2 b}{(1 - \gamma\omega)m^2 + r\sigma_\delta^2} \left(\frac{\gamma\omega m^2}{m^2 + r\sigma_\delta^2} \right)^{N-t+1}. \end{aligned} \quad (64)$$

□

Proof of Proposition 6 and Proposition 7. From Proposition 4, assuming identical periods, I have that

$$\begin{aligned} \hat{\beta}_t &= \frac{mb + m^2 \sum_{k=t+1}^N \gamma^{k-t} H_{kk-1}^t \hat{\beta}_k}{m^2 + r\sigma_t^2} \\ &= \frac{mb + \gamma m^2 H_{tt-1}^t \hat{\beta}_{t+1} + m^2 \sum_{k=t+2}^N \gamma^{k-t} H_{kk-1}^t \hat{\beta}_k}{m^2 + r\sigma_t^2}, \end{aligned} \quad (65)$$

and that, for $\hat{\beta}_{t+1}$, I have

$$\hat{\beta}_{t+1} = \frac{mb + m^2 \sum_{k=t+2}^N \gamma^{k-t-1} H_{kk-1}^{t+1} \hat{\beta}_k}{m^2 + r\sigma_{t+1}^2}. \quad (66)$$

Solving for the second term in the numerator in (66) gives

$$\sum_{k=t+2}^N \gamma^{k-t-1} H_{kk-1}^{t+1} \hat{\beta}_k = -\frac{b}{m} + \left(1 + r \frac{\sigma_{t+1}^2}{m^2} \right) \hat{\beta}_{t+1}. \quad (67)$$

For the career concerns model,

$$H_{kk-1}^t = \frac{\sigma_\theta^2}{(k-1)\sigma_\theta^2 + \sigma_\delta^2} \quad \text{and} \quad \sigma_t^2 = \sigma_\delta^2 \left(1 + \frac{\sigma_\theta^2}{(k-1)\sigma_\theta^2 + \sigma_\delta^2} \right). \quad (68)$$

It follows that $H_{kk-1}^t = H_{kk-1}^{t+1}$ for $t+1 \leq k-1 \leq N-1$ and $H_{t+1t}^t = \sigma_t^2/\sigma_\delta^2 - 1$. Substituting then in (65) using (67) and (68) gives (27). Finally, I substitute $a_t = b - r\hat{\beta}_t\sigma_t^2/m$ and $\hat{\beta}_{t+1} = m(b - a_{t+1})/(r\sigma_{t+1}^2)$ in (27) to obtain (28).

For the accounting model, I have

$$H_{kk-1}^t = -\frac{(-\rho\sigma_\theta^2)^{k-t}}{\sigma_t^2 \dots \sigma_{k-1}^2}. \quad (69)$$

It follows that $H_{kk-1}^t = (-\rho\sigma_\theta^2/\sigma_t^2)H_{kk-1}^{t+1}$ and $H_{t+1t}^t = \rho\sigma_\theta^2/\sigma_t^2$. Substituting then in (65) using (67) and (69) gives (31). Finally, I substitute $a_t = b - r\hat{\beta}_t\sigma_t^2/m$ and $\hat{\beta}_{t+1} = m(b - a_{t+1})/(r\sigma_{t+1}^2)$ in (31) to obtain (32). \square

Proof of monotonicity of a_t in the career concerns model. The proof is based on the simple observation that, since $a_t = b - r\hat{\beta}_t\sigma_t^2/m$,

$$a_{t+1} - a_t = \frac{r}{m}(\hat{\beta}_t\sigma_t^2 - \hat{\beta}_{t+1}\sigma_{t+1}^2), \quad (70)$$

and the sequence $(a_t)_t$ is increasing if, and only if, the sequence $(\hat{\beta}_t\sigma_t^2)_t$ is decreasing.

To simplify notation, let $s_t = \hat{\beta}_t\sigma_t^2$. I have to show that $(s_t)_t$ is decreasing. First I rewrite the recursive relation (27) as

$$s_t = p_t + q_t s_{t+1}, \quad (71)$$

where p_t and q_t denote the sequences

$$p_t = [mb(1 - \gamma)]/(\frac{m^2}{\sigma_t^2} + r) \quad \text{and} \quad q_t = \gamma(m^2 \frac{\sigma_t^2}{\sigma_{t+1}^2 \sigma_\delta^2} + r)/(\frac{m^2}{\sigma_t^2} + r). \quad (72)$$

Since $s_t - s_{t+1} = p_t - p_{t+1} + (q_t - q_{t+1})s_{t+1} + q_{t+1}(s_{t+1} - s_{t+2})$ and all terms p_t, q_t, s_t are positive, it is sufficient—using backwards induction—to show that $s_{N-1} < s_N$ and that the sequences $(p_t)_t$ and $(q_t)_t$ are decreasing.

First, since σ_t^2 is decreasing, it follows that p_t is decreasing. Second, since $m^2/\sigma_t^2 + r$ is increasing and $\sigma_t^2/\sigma_{t+1}^2$ is decreasing, it follows that q_t is decreasing as well. To prove that $\sigma_t^2/\sigma_{t+1}^2$ is decreasing, I use the explicit expressions for σ_t^2 to show that

$$\frac{\sigma_t^2}{\sigma_{t+1}^2} = \frac{t\sigma_\theta^2 + \sigma_\delta^2}{(t-1)\sigma_\theta^2 + \sigma_\delta^2} \frac{t\sigma_\theta^2 + \sigma_\delta^2}{(t+1)\sigma_\theta^2 + \sigma_\delta^2}. \quad (73)$$

The above can be shown to be decreasing in t by differentiating with respect to t .

Finally, $s_{N-1} > s_N$ is shown through a direct calculation:

$$\begin{aligned}
s_{N-1} - s_N &= \frac{mb(1-\gamma)}{\frac{m^2}{\sigma_{N-1}^2} + r} + \gamma \frac{m^2 \frac{\sigma_{N-1}^2}{\sigma_N^2 \sigma_\delta^2} + r}{\frac{m^2}{\sigma_{N-1}^2} + r} \frac{bm}{\frac{m^2}{\sigma_N^2} + r} - \frac{bm}{\frac{m^2}{\sigma_N^2} + r} \\
&= bm \left(\frac{1}{\frac{m^2}{\sigma_{N-1}^2} + r} - \frac{1}{\frac{m^2}{\sigma_N^2} + r} \right) + \gamma \frac{bm}{\frac{m^2}{\sigma_{N-1}^2} + r} \frac{\frac{m^2}{\sigma_N^2} \left(\frac{\sigma_{N-1}^2}{\sigma_\delta^2} - 1 \right)}{\frac{m^2}{\sigma_N^2} + r}.
\end{aligned} \tag{74}$$

The first term is positive by the same argument that shows p_t to be decreasing. The second term is positive because, using the explicit expressions for σ_t^2 ,

$$\frac{\sigma_{N-1}^2}{\sigma_\delta^2} - 1 = 1 + \frac{\sigma_\theta^2}{(N-2)\sigma_\theta^2 + \sigma_\delta^2} - 1 > 0. \tag{75}$$

□

Proof of Proposition 8. Substituting $b_t = b_{t+1} = b$, $m_{tt} = m_{t+1t+1} = m$ and $\Gamma_{t+1t} = \gamma^{-1}m(b-m)$ into (20) (and using $\Gamma_{kt} = 0$ for $k \geq t+2$) proves (36). □

Proof of Proposition 9. In the accounting model with negative correlation and constant posterior variances, I have (see (69) in the proof of Proposition 7)

$$\Gamma_{kt} = -H_{kk-1}^t m = (\sigma_\theta^2 \sigma_\infty^{-2})^{k-t} m. \tag{76}$$

The model in Proposition 9 has $\Gamma_{kt} = \omega^{k-t} m$, which is the same as following the proof of Proposition 7 with $\omega = \sigma_\theta^2 \sigma_\infty^{-2}$. □

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