

# **Inter-temporal Dynamics of Corporate Voluntary Disclosures**

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## **ABSTRACT**

In this paper, we demonstrate that voluntary disclosures provided by firms in the past enhance their implicit commitment to provide similar disclosures in the future. We show that such over-time stickiness of disclosure generates various endogenous disclosure costs that reduce the propensity of firms to provide voluntary disclosures. Our analysis predicts that the extent to which disclosure is sticky across periods and the magnitude of the implied reduction in the disclosure intensity are likely to be larger for firms that operate in a relatively stable environment and for firms that have exhibited impressive operating performance in the past. These inter-temporal effects are also positively related to managerial properties, such as time horizon and degree of risk aversion.

## 1. Introduction

Most extant results on corporate voluntary disclosures are drawn from single-period frameworks.<sup>1</sup> Introducing a multi-period disclosure setting, we demonstrate that a firm's voluntary disclosure is not a time-independent activity. Rather, it is tightly integrated with the firm's past and future strategic disclosure behavior. In particular, our analysis provides an explanation for the empirically documented phenomenon of disclosure stickiness and explores its properties and implications.

We combine and extend the basic single-period setups of Verrecchia [1983] and Dye [1985] into a repeated multi-period setting. The extended model describes a firm that is traded in a rational and risk-neutral capital market for a finite number of successive periods. Each period, with some probability, the firm's manager privately observes a signal that is relevant to the estimation of that period's uncertain forthcoming cash flows, but becomes irrelevant over time as more information becomes available.

Similarly to Dye [1985], investors are uncertain about the endowment of the manager with the information, and the manager cannot credibly claim to be uninformed. Upon receiving a private signal, however, its content can be voluntarily and credibly disclosed. Similarly to Verrecchia [1983], disclosure is costly. The disclosure decision in each period is made in light of the rationally anticipated impact on the firm's market price, which, in turn, is determined by investors' rational expectations about the firm's strategic disclosure behavior. Inter-temporal dynamics occur because the firm's uncertain possession of information is assumed to be positively correlated across

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<sup>1</sup> Firms' incentives to voluntarily disclose private verifiable information have been a widespread research interest in accounting, finance and economics. For recent surveys of the voluntary disclosure literature, see Dye [2001] and Verrecchia [2001].

periods. This assumption reflects the perception that the business and the information environment of any firm is characterized by a certain degree of stability, and thus the availability of relevant information, the cost of acquiring such information and the ability to credibly disclose it to outsiders are likely to be predictable.

Using the multi-period framework, we show that a long history of not providing disclosures reduces the current disclosure intensity. Intuitively, the absence of past disclosures builds for the firm a reputation of being uninformed that moderates the adverse price reaction to the absence of disclosures in the next periods and facilitates the future withholding of information. Once this valuable reputation has been built, its collapse by the current disclosure is costly for the manager and further motivates her to continue suppressing private information from the market. The reputation is built faster when the history of not providing disclosure goes along with an impressive past operating performance. This is because high operating outcome in the past implies that any available information was apparently too favorable to be withheld, convincing the market that it is more likely information was unavailable. On the other hand, a history of intense disclosures, which reveals the firm's past possession of private information, augments the market awareness of a similar future information endowment and thereby enhances the firm's implicit commitment to provide subsequent disclosures. Our analysis therefore implies that disclosure is sticky across periods, providing a possible explanation for empirical evidence like that provided by Lang and Lundholm [1993] and Botosan and Harris [2000].

We further show that the stickiness of disclosure generates several indirect disclosure costs that reduce the manager's current propensity to provide voluntary disclosures. Recognizing that disclosure is sticky, investors rationally anticipate its impact in enhancing future costly disclosures. The incremental disclosure costs that the firm is

expected to incur in the future, due to the current disclosure, is therefore fully reflected by its current market price. This adverse price reaction to disclosure introduces an endogenous indirect disclosure cost, which reduces the incentives of the manager to provide disclosure, even if she is myopic and interested only in maximizing the current market price. For a forward looking manager, who cares not only about the current market price but also about future prices, disclosure is even more costly because it enhances the adverse price reaction to the absence of disclosure in the next periods and reduces the future leeway to avoid disclosures. When the manager is risk averse, disclosure is associated with another endogenous indirect cost, as it intensifies the dependence of the firm's future market price on the uncertain forthcoming information and thereby increases future price volatility. By demonstrating how the stickiness of disclosure endogenously generates all these indirect disclosure costs, we shed light on the role of inter-temporal forces in reducing managerial incentives to provide voluntary disclosures.

Our analysis predicts that the extent to which corporate voluntary disclosures are subject to inter-temporal dynamics depends on firm-specific characteristics and on managerial properties. In particular, the extent to which disclosure is sticky and the magnitude of the implied reduction in the intensity of disclosure are likely to be larger for firms that have exhibited impressive operating performance in the past and for firms that operate in a relatively stable business and information environment where past information endowment serves as a good indicator of a similar future information endowment. The magnitude of these inter-temporal effects also appears to be positively related to the manager's time horizon and degree of risk aversion.

The paper proceeds as follows. Section 2 provides a description of the model. In section 3, we establish the existence and uniqueness of equilibrium in the model and

analyze its properties. In section 4, we illustrate our results using a numerical example. The final section summarizes and offers concluding remarks. Highlights of the proofs appear in the appendix.

## 2. Model

Our model is a multi-period replication of the single-period disclosure setting of Dye [1985] combined with that of Verrecchia [1983]. The linkage between subsequent periods in the model stems from the history-dependent nature of the information environment. In this section, we detail the parameters and assumptions of the model, which are all common knowledge unless indicated otherwise.

Consider a firm that operates for  $N$  successive periods in a rational and risk-neutral capital market. The parameter  $N$  represents the time horizon for which the firm is expected to operate in a relatively stable business and information environment.<sup>2</sup> The firm's uncertain future cash flows over the  $N$  periods are represented by the random variables  $CF_1, CF_2, \dots, CF_N$ , which are assumed to be uncorrelated and normally distributed with means  $\mu_1, \mu_2, \dots, \mu_N$  and variances  $\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2$ , respectively. We denote the market discount rate by  $r$  ( $0 < r \leq 1$ ) and assume that the cash flows  $CF_t$  are realized and distributed as a dividend to the shareholders at the end of period  $t$ . Each period, with some probability, the manager privately observes a signal  $S_t$ . We represent the manager's information status in period  $t$  by the binary random variable  $I_t$ , which is distributed over the support  $\Omega = \{inf, ui\}$ , where *inf* stands for informed (that is, the manager possesses the private signal), and *ui* stands for uninformed. The

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<sup>2</sup> The number of periods,  $N$ , could be very large to preserve the going concern concept.

signal  $S_t$ , if received, is relevant in assessing the forthcoming cash flows,  $CF_t$ . Specifically, we assume that  $S_t = CF_t + \varepsilon_t$ , where  $\varepsilon_t$  is an independent normally distributed noise term with zero mean and variance  $\sigma_{\varepsilon_t}^2$ . The manager's private information pertains to its timing, because at the end of each period, cash flows become public and the signal becomes irrelevant.<sup>3</sup> As in Dye [1985], investors are uncertain whether the manager is informed or uninformed and the manager cannot credibly reveal her information status to the market. Upon receiving a signal, however, its content can be credibly disclosed.<sup>4</sup> Accordingly, each period, an informed manager can either credibly disclose her private signal to investors or keep quiet. The set of the manager's two disclosure alternatives in any given period  $t$  is represented by  $A = \{dis, nd\}$ , where *dis* describes the alternative of disclosing the signal  $S_t$  and *nd* describes the alternative of not disclosing it. Similarly to Verrecchia [1983], we assume that disclosure of the signal  $S_t$  is associated with positive costs  $c_t$ .

Figure 1 provides a timeline depicting the sequence of events within a given period  $t$ . At the beginning of the period prior beliefs of all players are determined. Then, with some probability, the manager privately observes the signal  $S_t$ . Conditioned on possessing the private signal and based on its realization, the manager decides whether or not to credibly disclose it at a cost  $c_t$ . Now, based on all available information, the firm's price is set in the market. Finally, at the end of the period, the periodical cash

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<sup>3</sup> Examples of such timing differences include cases of managers' predictions of future input prices, results of a market analysis regarding the introduction of a new product, future earnings forecasts, loan-loss reserves or bad-debt estimates. An extension to the model would involve the possibility of longer timing differences by assuming delayed cash realizations that occur after a few periods.

<sup>4</sup> Credible disclosure is often assumed in the literature. It is usually justified by procedures (like audit or due diligence) that verify the manager's reported information or by the potential litigation and human capital erosion costs associated with misleading disclosure.

flows,  $CF_t$ , are realized and distributed as a dividend to the shareholders and therefore are commonly known.

[Figure 1]

Subsequent periods in our model are linked through the history dependence of the information endowment. We assume that the availability of relevant information, the cost of acquiring such information and the ability to credibly disclose it to outsiders remain relatively stable across periods, and thus the information endowment is positively correlated over time.<sup>5</sup> Accordingly, following Cosimano, Jorgensen and Ramanan [2002], we model the manager's status variables,  $I_t$ , as a Markov chain, starting with  $\pi_1$  as the probability of information endowment in the first period, and assuming that the probability of information endowment in each subsequent period  $t$  is higher under the assumption that information was available in the previous period  $t-1$  than under the assumption that it was unavailable. That is, for any  $t \geq 2$ , the probability  $prob(I_t = inf | I_{t-1} = inf)$  is higher than the probability  $prob(I_t = inf | I_{t-1} = ui)$ .

Specifically, we represent the probability  $prob(I_t = inf | I_{t-1} = inf)$  by  $\pi_t$  and the probability  $prob(I_t = inf | I_{t-1} = ui)$  by  $\pi_t - \delta_t$ , where  $0 < \pi_t < 1$  and  $0 < \delta_t < \pi_t$ . The parameters  $\delta_2, \delta_3, \dots, \delta_N$  measure the magnitude of the history dependence in the information endowment, and therefore capture the level of stability in the firm's operating and information environment. Defining  $\delta_1 = \delta_{N+1} = 0$ , in any period  $t$ , the connection to past periods stems from  $\delta_t$ , while the connection to future periods stems

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<sup>5</sup> In a different setting, Dye and Sridhar [1995] assume a positive correlation for managers being privately informed across firms in the same industry.



from  $\delta_{t+1}$ . We use the notation  $\delta[i : j]$  to represent the vector  $(\delta_i, \delta_{i+1}, \dots, \delta_j)$  for any  $1 \leq i < j \leq N + 1$ , and consider one vector as lower than another if all its components are weakly lower than the corresponding components in the other vector and at least one of them is strictly lower (an incomplete ranking).

Due to the history dependence of the information endowment, investors' prior beliefs at the beginning of each period are partially determined based on information that has appeared during the preceding periods. Given that disclosure unequivocally reveals the manager's (Markovian) information status, all public events that occurred prior to the firm's last past disclosure are irrelevant. The relevant history thus includes the timing of the firm's past last disclosure and all cash flow realizations since that time. Investors could use the past cash realizations to imperfectly infer the past information status by estimating the likelihood of information being withheld. Note, however, that while past disclosure unambiguously reveals the manager was informed, the cash history provides only a noisy indication about the past information status. The manager in each period might have superior information about the firm's history, beyond the available public information, but any such superior information is irrelevant in choosing the current strategy. For any given period  $t$ , we represent the firm's disclosure history by an integer,  $h_t \in H_t = \{0, 1, 2, \dots, t - 1\}$ , which equals the serial number of the last period with disclosure or zero in the absence of past disclosure. Starting with  $h_1 = 0$ , the disclosure history evolves with time, so that  $h_{t+1} = t$  if disclosure occurs in period  $t$  and  $h_{t+1} = h_t$  otherwise for any  $t = 1, 2, \dots, N - 1$ . Using the notation  $cf[i : j]$  to represent the vector  $(cf_i, cf_{i+1}, \dots, cf_j)$  of all cash realizations between any two periods  $i$  and  $j$ , the relevant cash history in each period  $t$  with disclosure history  $h_t < t - 1$  is represented by the vector  $cf[h_t + 1 : t - 1]$ . When comparing different cash histories, we define one cash

history as lower than another cash history if all its components are weakly lower than the corresponding components in the other history and at least one of them is strictly lower.<sup>6</sup> To avoid tedious notation, we represent the history in each period  $t$  by the single datum  $h_t$  – the timing of the most recent past disclosure, but in the analysis and the formal derivation of the results, we explicitly incorporate the cash flows’ realizations since the last disclosure,  $cf[h_t + 1 : t - 1]$ , into the relevant history.

To capture the different time horizons of managers and shareholders, we limit the manager’s time horizon to include the current period and one additional period into the future.<sup>7</sup> Specifically, we consider the case where the manager is myopic and interested only in maximizing the firm’s current period market price and the case of a forward looking manager who also cares about the price in the subsequent period. We assume that the manager in period  $t$  has a mean-variance utility function

$U_t(x_t) = E[x_t] - \gamma_t VAR[x_t]$ , where  $x_t$  is a linear combination that assigns a weight 1 to the current price and a weight  $\omega_t$  to the price in the subsequent period  $t + 1$ , where  $\gamma_t \geq 0$  and  $0 \leq \omega_t \leq 1$  and  $\omega_N = 0$ .<sup>8</sup> The parameter  $\gamma_t$  represents the manager’s degree of risk aversion, where  $\gamma_t = 0$  captures risk-neutrality. The parameter  $\omega_t$  represents the strength of the manager’s preferences for the future or her time horizon. The special case  $\omega_t = 0$  describes a myopic manager, who cares only about the current market price

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<sup>6</sup> We note that the cash history ranking is not complete. That is, the results mostly pertain to local changes in one cash realization where all other past cash flows are kept fixed.

<sup>7</sup> Our model could be extended to cases where the manager values more than one future period without qualitatively affecting the results.

<sup>8</sup> When  $x_t$  is a normally distributed variable, the mean-variance utility function  $E[x_t] - \gamma_t VAR[x_t]$  is equivalent to the negative exponential function  $-e^{-2\gamma_t x_t}$ . Note, however, that this is not the case in our model, because the possibility that disclosure will not occur in the next period leads to a non-normal distribution of the future uncertain price.

of the firm. We use it to emphasize that inter-temporal dynamics are not a consequence of the manager's tenure. The manager's disclosure strategy in each period  $t$  is described by the function  $D_t : H_t \times \Omega \times \mathfrak{R} \rightarrow A$ , and investors' expectations about this strategy are described by the function  $\hat{D}_t : H_t \times \Omega \times \mathfrak{R} \rightarrow A$ . Specifically,  $D_t(h_t, i_t, s_t)$  is the manager's binary disclosure decision in period  $t$ , given the history  $h_t$ , the current information status  $i_t$ , and the realization  $s_t$  of the private signal  $S_t$ .

Investors are assumed risk neutral. Therefore, in each period, the firm's market price is determined as the expected value of the firm, conditional on all available information. At any point in time, investors determine the expected value of the firm by aggregating the firm's expected future cash flows from its operations and subtracting the disclosure-associated costs that the firm is expected to incur in the future. The investors' information set in each period  $t$  includes the realization of the signal  $S_t$ , if disclosed, or a history-based Bayesian update on the distribution of the information status  $I_t$  and the signal  $S_t$  otherwise. The market pricing rule in each period  $t$  is described by the function  $P_t : H_t \times A \times \mathfrak{R} \rightarrow \mathfrak{R}$  and the manager's expectations about it are represented by the function  $\hat{P}_t : H_t \times A \times \mathfrak{R} \rightarrow \mathfrak{R}$ . Explicitly,  $P_t(h_t, d_t, s_t)$  is the market price of the firm in period  $t$ , where  $h_t$  is the firm's history,  $d_t$  is the manager's current disclosure decision and  $s_t$  is the realization of her private signal  $S_t$  (if available and disclosed).

We look for Bayesian equilibrium. In equilibrium, the firm's managers choose the disclosure strategies  $D_t$  based on the rationally anticipated market pricing functions  $P_t$ , which, in turn, are determined by the investors' rational expectations about the managers' disclosure strategies. Equilibrium is formally defined as a set of functions

$\{ D_t, \hat{D}_t : H_t \times \Omega \times \mathfrak{R} \rightarrow A, P_t, \hat{P}_t : H_t \times A \times \mathfrak{R} \rightarrow \mathfrak{R} \}_{t=1}^N$ , which satisfies the following three conditions for any  $t = 1, 2, \dots, N$ ,  $h_t \in H_t$ ,  $i_t \in \Omega$ ,  $d_t \in A$  and  $s_t \in \mathfrak{R}$ :

$$(i) D_t(h_t, inf, s_t) \in \operatorname{argmax}_{d_t \in \{dis, nd\}} \hat{P}_t(h_t, d_t, s_t) + \omega_t EFP_t(h_t, d_t, s_t) - \gamma_t \omega_t^2 VFP_t(h_t, d_t, s_t) \text{ and}$$

$$D_t(h_t, ui, s_t) = nd;$$

$$(ii) P_t(h_t, d_t, s_t) = \begin{cases} E[\sum_{j=t}^N r^{j-t} CF_j | S_t = s_t] - c_t - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, dis) & \text{if } d_t = dis \\ E[\sum_{j=t}^N r^{j-t} CF_j | \hat{D}_t(h_t, I_t, S_t) = nd] - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, nd) & \text{if } d_t = nd \end{cases}$$

$$(iii) \hat{D}_t(h_t, i_t, s_t) = D_t(h_t, i_t, s_t) \text{ and } \hat{P}_t(h_t, d_t, s_t) = P_t(h_t, d_t, s_t);$$

where in each period  $t$ ,  $EFP_t(h_t, d_t, s_t)$  and  $VFP_t(h_t, d_t, s_t)$  are the estimates of an informed manager about the mean and the variance, respectively, of the future price in the next period  $t + 1$ , and  $EFC_{t,j}(h_t, d_t)$  is the investors' estimate of the future disclosure costs that are expected to be incurred in period  $j$  ( $j > t$ ), based on the history  $h_t$  and the disclosure decision  $d_t$  of period  $t$ , but prior to the cash realization of that period.<sup>9</sup> The sensitivity of these estimates to the disclosure decision of period  $t$  is measured by the differences  $\Delta EFP_t(h_t, s_t) = EFP_t(h_t, dis, s_t) - EFP_t(h_t, nd, s_t)$ ,

$$\Delta VFP_t(h_t, s_t) = VFP_t(h_t, dis, s_t) - VFP_t(h_t, nd, s_t) \text{ and } \Delta EFC_{t,j}(h_t) = EFC_{t,j}(h_t, dis) - EFC_{t,j}(h_t, nd).$$

The first equilibrium condition pertains to the firm's disclosure strategies  $D_t$ , requiring that the disclosure decision of an informed manager in each period maximizes her utility, based on its anticipated impact on the current and future prices, while an

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<sup>9</sup> Note that the manager uses the realization  $s_t$  of her private signal  $S_t$  to imperfectly learn about the forthcoming cash flows of period  $t$  and thereby assesses the mean and the variance of the future price when  $d_t = nd$ . However, since the realization  $s_t$  of the signal  $S_t$  is available to investors only when  $d_t = dis$ , it is irrelevant in determining their expectations about the future disclosure costs.

uninformed manager obviously cannot provide disclosure. The second equilibrium condition describes the market pricing rules  $P_t$ . The pricing rules reflect the risk-neutrality of investors by setting the market price in each period to be the expected future cash flows of the firm, net of the expected disclosure costs, conditional on all the available public information. Note also that the market price of the firm in each period without disclosure reflects the investors' expectations about the firm's disclosure strategy in that particular period, as well as their expectations about the firm's disclosure strategies in the past and in the future. Investors use their expectations about the disclosure strategies in past periods to assess the probability of past information endowment and thereby update their beliefs about the current information status  $I_t$ . Investors' expectations about the expected disclosure strategies in future periods are also important in pricing the firm, because they help to estimate the costs of the current disclosure that stem from its future consequences. After establishing their beliefs about the probability that the manager is currently informed and the costs that she faces in the case of disclosing her information, investors use their expectations about the current disclosure strategy to update their beliefs about the realization of the withheld signal. Lastly, the third equilibrium condition implies that, in each period, both the investors and the manager have rational expectations regarding each other's behavior.

### **3. Equilibrium Analysis**

We now turn to deriving the equilibrium and analyzing its properties. Throughout the analysis, we emphasize the reliance of the manager's disclosure strategy on past and future considerations. As a benchmark, denoted by the upper-case subscript  $B$ , consider the case  $\delta_2 = \delta_3 = \dots = \delta_N = 0$ , an extremely volatile environment, where the information endowment is unpredictable. Here, each period  $t$  can be viewed as a

single-period setting with time-independent disclosure strategy. Consistent with Verrecchia [1983] and Dye [1985], Observation 1 establishes the existence and uniqueness of the benchmark equilibrium and characterizes its basic form.

**Observation 1.** Assume  $\delta_2 = \delta_3 = \dots = \delta_N = 0$ . Then, there exists a unique equilibrium, where the disclosure strategy of each period  $t$ ,  $D_t^B : H_t \times \Omega \times \mathfrak{R} \rightarrow A$ , is characterized by a threshold  $s_t^B \in \mathfrak{R}$ , such that for any  $h_t \in H_t$ ,  $i_t \in \Omega$ , and  $s_t \in \mathfrak{R}$ :

$$D_t^B(h_t, i_t, s_t) = \begin{cases} dis & \text{if } i_t = inf \text{ and } s_t \geq s_t^B \\ nd & \text{otherwise} \end{cases} .$$

Observation 1 yields a standard disclosure strategy, where an informed manager chooses to disclose her private signal  $S_t$  if and only if its realization is sufficiently favorable and exceeds the history-independent threshold level  $s_t^B$ . The upper-tailed structure of the disclosure strategy is a consequence of the positive correlation between the signal and the firm's future cash flows. Proposition 1 establishes the existence and uniqueness of equilibrium for the case where  $\delta_2, \delta_3, \dots, \delta_N > 0$ , demonstrating that the upper-tailed structure of the disclosure strategies continues to hold. However, the disclosure thresholds are history dependent.

**Proposition 1.** There exists a unique equilibrium, where the disclosure strategy of each period  $t$ ,  $D_t : H_t \times \Omega \times \mathfrak{R} \rightarrow A$ , is characterized by a threshold function  $s_t^* : H_t \rightarrow \mathfrak{R}$ , such that for any  $h_t \in H_t$ ,  $i_t \in \Omega$ , and  $s_t \in \mathfrak{R}$ :

$$D_t(h_t, i_t, s_t) = \begin{cases} dis & \text{if } i_t = inf \text{ and } s_t \geq s_t^*(h_t) \\ nd & \text{otherwise} \end{cases} .$$

Being characterized by the threshold  $s_t^*(h_t)$ , the manager's disclosure strategy in period  $t$ , under history  $h_t$ , can be unequivocally represented by

$\tau_t(h_t) = Prob[S_t \geq s_t^*(h_t)]$ . The measure  $\tau_t(h_t)$  is the ex-ante (prior to observing the realization of the signal  $S_t$ ) probability of disclosure occurrence in period  $t$  with disclosure history  $h_t$ , *conditioned* on the manager being informed. Hereafter, we refer to  $\tau_t(h_t)$  as the firm's disclosure intensity in period  $t$  given a disclosure history  $h_t$ . Similarly, we refer to  $\tau_t^B = Prob[S_t \geq s_t^B]$  as the benchmark disclosure intensity for period  $t$ . Proposition 2 describes the role of inter-temporal dynamics in shifting the disclosure intensities downward below the benchmark level.

**Proposition 2.** In equilibrium, for any period  $t$  and disclosure history  $h_t \in H_t$ , the disclosure intensity  $\tau_t(h_t)$  satisfies  $\tau_t(h_t) \leq \tau_t^B$ , where the equality holds if and only if  $t = N$  and  $h_t = N - 1$ .

Using the set of functions  $\{\tau_t : H_t \rightarrow [0,1]\}_{t=1}^N$  as a condensed representation of the equilibrium, we proceed to explore the inter-temporal forces that explain its deviation from the benchmark  $\{\tau_t^B\}_{t=1}^N$ . We start by analyzing how the strategic disclosure behavior of the firm is affected by its past behavior (henceforth – “the past effect”). Then, we consider how the firm's disclosure strategy is affected by its expected future implications (henceforth – “the future effect”).

Starting with the past effect, it appears that a history of intense disclosures or poor operating performance enhances the current disclosure intensity of the firm. Given the positive correlation of the information endowment over time, the probability assigned by the market to the information endowment in period  $t$  is based on both the last

disclosure date,  $h_t$ , and the cash flow realizations following that last disclosure.

Focusing first on the disclosure history, two past public events are of importance:

disclosure in period  $h_t$  (which reveals that the manager was informed in that period)

and the absence of disclosure since that time. The former event revises upward

investors' beliefs about the likelihood that the manager is currently informed, making

all prior disclosure decisions irrelevant, while the latter has an opposite, downward,

impact. The cumulative downward impact is stronger when the number of periods

passed since the last disclosure is larger, as well as when the cash flow realizations in

these periods are higher. High past cash realizations imply that information, if it was

available, was apparently too favorable to be withheld, convincing the market that

information was most likely unavailable. Taken all together, the longer the history of

avoiding disclosure and the better the corresponding past operating performance, the

better is the manager's reputation of being currently uninformed and the lower her

propensity for making the current disclosure. The history dependence of the firm's

disclosure strategy is formally characterized in Proposition 3.

**Proposition 3** (past effect). In equilibrium, for any period  $t$  and disclosure history

$h_t \in H_t$ , the disclosure intensity  $\tau_t(h_t)$  is increasing in the disclosure history  $h_t$ ,

decreasing in the cash history  $cf[h_t + 1 : t - 1]$ , and decreasing in  $\delta[h_t + 2 : t]$ .

Our analysis demonstrates that, by augmenting the market's awareness of the existence

of information, voluntary disclosures provided by firms in the past endogenously

enhance their implicit commitment to provide similar disclosures in the future. This

provides a possible explanation for the commonly observed stickiness of disclosure,

rationalizing empirical evidence like that provided by Lang and Lundholm [1993] and



Botosan and Harris [2000].<sup>10</sup> Observe that the stickiness of disclosure in our model is not a mere reflection of the underlying positive correlation of the firm's information endowment across periods. By assumption, past information endowment increases the probability of future information endowment. Note, however, that our measure of disclosure intensity,  $\tau_t(h_t)$ , is defined as the probability that an *informed* manager will provide disclosure in period  $t$  given a disclosure history  $h_t$ . That is, our model predicts that the probability of disclosure occurrence in period  $t$ , *conditioned* on possessing the information, is positively related to disclosure history  $h_t$ . The unconditional probability of disclosure, which is used to measure the stickiness of disclosure in the empirical literature, is even more tightly related to disclosure history.

The extent to which disclosure is sticky can be measured by the sensitivity of the disclosure intensity  $\tau_t(h_t)$  to the disclosure history  $h_t$ . Accordingly, we measure the magnitude of the disclosure stickiness in each period  $t$  by the differences  $\tau_t(j) - \tau_t(i)$  for any two histories  $i, j \in H_t$ , such that  $i < j$ . Using this measurement, Corollary 1 describes how the extent to which disclosure is sticky relates to firm-specific characteristics.

**Corollary 1.** In equilibrium, for any period  $t$  and any two disclosure histories  $i, j \in H_t$ , such that  $i < j$ , the difference  $\tau_t(j) - \tau_t(i)$  is positive and increasing in  $cf[i+1:j]$  and  $\delta[i+2:j+1]$ .

Corollary 1 suggests that the magnitude of disclosure stickiness is likely to be stronger for firms that exhibited better operating performance in the past, which leads the market

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<sup>10</sup> Other explanations include the legal liability (Trueman [1997]) and the inter-temporal relations between mandatory and voluntary disclosures (Cosimano, Jorgensen and Ramanan [2002]).

to associate the absence of disclosure to the unavailability of information rather than to its unfavorable content. Corollary 1 also implies that the disclosure stickiness is expected to be of larger magnitude for firms that operate in a relatively stable business and information environment, where the past information endowment serves as a good indicator for a similar subsequent information endowment. Taking this result to its extreme, Corollary 2 indicates that when  $\delta_t$  converges to zero the past effect described in Proposition 3 and Corollary 1 ceases to exist.

**Corollary 2.** In equilibrium, for any period  $t$ , when  $\delta_t$  converges to zero,  $\tau_t(h_t)$  becomes independent of  $h_t$ ,  $cf[h_t + 1 : t - 1]$  and  $\delta[h_t + 2 : t - 1]$ , but it is still strictly below the benchmark level  $\tau_t^B$  for any  $t < N$ .

According to Corollary 2, when  $\delta_t$  converges to zero, past information endowment is completely uninformative about the information endowment in period  $t$ , and thus the past effect disappears and the disclosure intensity in each period becomes history independent. Nevertheless, Corollary 2 also implies that even in the absence of the past effect, the disclosure intensity in each period  $t$ , except for the last period, is still below the benchmark level  $\tau_t^B$  due to the future effect. Moving now to discussing the future effect, Corollary 3 highlights three important future consequences of disclosure, which the manager takes into account in choosing the current disclosure strategy.

**Corollary 3.** In equilibrium, for any period  $t$  with disclosure history  $h_t \in H_t$  and information  $s_t \in \mathfrak{R}$ , and for any subsequent period  $j$ , such that  $j > t$ , the differences  $\Delta EFC_{t,j}(h_t)$ ,  $-\Delta EFP_t(h_t, s_t)$  and  $\Delta VFP_t(h_t, s_t)$  are positive, decreasing in  $h_t$  and increasing in  $\delta_{t+1}$ , converging to zero when  $\delta_{t+1}$  converges to zero.

Corollary 3 points toward three kinds of indirect costs that are associated with the current disclosure and arise due to its anticipated future stickiness. As disclosure in the current period  $t$  enhances the likelihood of disclosure occurring in future periods, it implies that higher disclosure costs are expected to be incurred by the firm in the future.

The discounted incremental expected disclosure costs,  $\sum_{j=t+1}^N r^{j-t} \Delta EFC_{t,j}(h_t) > 0$ , are rationally anticipated by investors and therefore are already conveyed by period  $t$ 's market price. The resulting adverse price reaction to disclosure generates the first indirect cost that is associated with the current disclosure. This indirect cost is increasing in the discount rate,  $r$ , and in the firm's horizon,  $N$ . Observe that this cost exists even if the manager in period  $t$  is myopic ( $\omega_t = 0$ ) and interested only in maximizing the current price. Two additional indirect costs emerge when the manager is forward looking ( $\omega_t > 0$ ) and cares about the future random price of the firm. By enhancing the negative price reaction to the absence of future disclosure and restricting the flexibility of the manager to avoid unfavorable disclosure in the future, the current disclosure reduces the expected value of the future random price. This generates an indirect disclosure cost of  $-\omega_t \Delta EFP_t(h_t, s_t) > 0$ , which is increasing in the strength of the manager's future preferences,  $\omega_t$ . If the manager is not only forward looking, but also risk averse ( $\gamma_t > 0$ ), the current disclosure is even more costly, because it also intensifies the dependence of the future price on the forthcoming unknown information, and thereby increases its volatility.<sup>11</sup> The increase in the volatility of the future price introduces a third kind of indirect cost,  $\gamma_t \omega_t^2 \Delta VFP_t(h_t, s_t) > 0$ , which is increasing in

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<sup>11</sup> In a different model, Nagar [1999] demonstrates that risk-averse managers face disclosure costs due to the risk of not knowing how the market will interpret their disclosure.

both the manager's time horizon,  $\omega_t$ , and her degree of risk aversion,  $\gamma_t$ . In summary, the anticipated over-time stickiness of disclosure generates the three indirect disclosure costs  $-\sum_{j=t+1}^N r^{j-t} \Delta EFC_{t,j}(h_t)$ ,  $-\omega_t \Delta EFP_t(h_t, s_t)$  and  $\gamma_t \omega_t^2 \Delta VFP_t(h_t, s_t)$ , and thereby reduces the propensity of firms to provide voluntary disclosures. As the magnitude of the indirect disclosure costs is positively related to the parameters  $N$ ,  $r$ ,  $\omega_t$  and  $\gamma_t$ , so too is the implied reduction in the disclosure intensity. Furthermore, since disclosure results in a collapse of the entire accumulated past reputation of being uninformed, all three adverse future consequences from losing this reputation are positively related to its magnitude. Consequently, all three indirect costs are decreasing in the disclosure history. That is, a longer history without disclosures is associated with a lower subsequent propensity to provide disclosure, not only because of the better accumulated past reputation of being uninformed, but also due to the higher future costs associated with losing this reputation. In this sense, the future considerations underlying the disclosure strategy enhance its history dependence. Also, since the three indirect costs arise from the anticipated stickiness of disclosure, all become more significant in more stable information environments where  $\delta[t+1:N]$  is higher, so that disclosure in period  $t$  is more likely to remain sticky in the future. Proposition 4 summarizes the consequences of the future considerations underlying the disclosure strategy.

**Proposition 4** (future effect). In equilibrium, for any period  $t$  and disclosure history  $h_t \in H_t$ , the disclosure intensity  $\tau_t(h_t)$  is decreasing in  $N$ ,  $r$ ,  $\omega_t$ ,  $\gamma_t$  and  $\delta[t+1:N]$ .

The future effect described in Proposition 4 stems from the assumption that  $\delta_{t+1} > 0$ , which implies that the disclosure in period  $t$  is expected to remain sticky in the next

periods. In the extreme case where  $\delta_{t+1}$  converges to zero, the future information endowment becomes unpredictable, so current disclosure is not expected to remain sticky, and all three indirect costs vanish. Corollary 4 indicates that when  $\delta_{t+1}$  converges to zero, the future effect described in Proposition 4 disappears, but nevertheless the disclosure intensity is still below the benchmark level  $\tau_t^B$ , as long as  $h_t < t - 1$ , due to the past effect.

**Corollary 4.** In equilibrium, for any period  $t$ , when  $\delta_{t+1}$  converges to zero,  $\tau_t(h_t)$  becomes independent of  $N$ ,  $r$ ,  $\omega_t$ ,  $\gamma_t$  and  $\delta[t + 2 : N]$ , but it is still strictly below the benchmark level  $\tau_t^B$  for  $h_t < t - 1$ .

#### 4. Numerical Example

In this section, we illustrate the results of our multi-period analysis using a numerical example. Consider a five-period setting ( $N = 5$ ), where the distribution of the information status variables corresponds to the following Markovian probabilities:

$\pi_1 = 0.8$ ,  $\pi_t = 0.8$  and  $\pi_t - \delta_t = 0.2$  for any period  $t \geq 2$ . Also, let  $r = 1$ , and assume the stationary parameters  $\mu_t = 100$ ,  $\sigma_t = 20$ ,  $\sigma_{\varepsilon_t} = 10$  and  $c_t = 8$  for any period  $t$ .

Suppose also that the manager in each period is myopic and is interested only in maximizing the current period market price. That is,  $\omega_t = 0$  for any period  $t$  (and thus  $\gamma_t$  is irrelevant).<sup>12</sup> Our numerical example, therefore, illustrates inter-temporal dynamics of disclosure that do not arise from the manager's tenure, where the expected enhancing impact of disclosure on the future disclosure costs is the only indirect cost

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<sup>12</sup> This assumption significantly simplifies the calculations required to derive the numerical results, while still allowing us to demonstrate our insights.

associated with current disclosure. The resulting equilibrium disclosure intensities for any period and disclosure history are presented in Panel A of Table 1, under the assumption that all past cash realizations coincide with the mean  $\mu_t = 100$ .

[ Table 1 ]

Observe that for any given period, the lower the disclosure history the lower is the resulting disclosure intensity. For example, consider the third period, where the disclosure intensity decreases from 51% (for disclosure history 2) through 41% (disclosure history 1) to 38% (disclosure history 0). That is, disclosure is sticky – an informed manager is more likely to disclose her information in the third period if disclosure occurred in the second period than if the most recent disclosure occurred in the first period or never occurred. The impact of the cash history on the current disclosure intensity is illustrated in Panel B of Table 1. The disclosure intensities presented in Panel B relate to the third period, assuming the last past disclosure occurred in the first period, and under three alternative levels of cash realizations in the second period: low (one standard deviation below the mean), intermediate (at the mean) and high (one standard deviation above the mean). Here, as the level of past cash flows increases, the disclosure intensity decreases from 44% to 41% and to 33%. Next, observe that for a given disclosure history, the disclosure intensity might be non-monotonically changing as time progresses. Although the past effect is intensified over time implying a better reputation of being uninformed, the future effect becomes weaker because of the lower benefit from this reputation. For example, see the movement across periods under disclosure history 1, where the disclosure intensity decreases from 51% (period 2) to 41% (period 3) and to 39% (period 4) and then increases to 41% (period 5). To isolate the future effect, compare the results of periods 4 and 5 under the assumption that disclosure has occurred in the previous period

(disclosure histories 3 and 4, respectively). The only difference between the two cases is the existence of a future period in the first case, which generates indirect disclosure costs and therefore implies a lower disclosure intensity (52% versus 55%).

Lastly, we benchmark the results presented in Table 1 against the case of  $\delta_t = 0$  for any  $t \geq 2$ . The benchmark case yields an identical history-independent disclosure intensity of  $\tau_t^B = 55\%$  for any period  $t$ . Observe that all but one of the disclosure intensities presented in Table 1 are strictly below the benchmark level of 55%. The only disclosure intensity that coincides with the benchmark level is that of the last period under the assumption that disclosure has occurred in the previous period, where neither the past effect nor the future effect exist. Note that even when only one of these two effects exist (that is, in the last column and on the main diagonal of the table), all disclosure intensities are strictly below the benchmark level of 55%.

## 5. Summary and Conclusions

Credible voluntary disclosure is usually studied in the existing literature within single-period frameworks. In this paper, we consider inter-temporal dynamics of corporate voluntary disclosure strategies, demonstrating that a firm's disclosure strategy is not time isolated, but is rather integrated with the firm's past and future strategic disclosure behavior.

We show that, by augmenting the market's awareness of the existence of information, past voluntary disclosures endogenously enhance the firm's implicit commitment to provide similar disclosures in the future, implying that disclosure is sticky. We further show that the stickiness of disclosure over time endogenously generates various indirect disclosure costs. In particular, disclosure is associated with an adverse price reaction

due to the expected increase in the disclosure costs that the firm is expected to incur in the future, and therefore is costly even for a myopic manager who is interested only in maximizing the current market price of the firm. For a forward looking manager, who also cares about future prices, disclosure is associated with the additional cost of reducing her leeway to suppress unfavorable disclosures in the future. If the manager is not only forward looking, but also risk averse, then disclosure becomes even more costly, because it enhances the volatility of the firm's future prices by increasing their dependence on the forthcoming unknown information. In summary, due to different kinds of indirect disclosure costs, disclosure stickiness reduces the firm's propensity to provide voluntary disclosures.

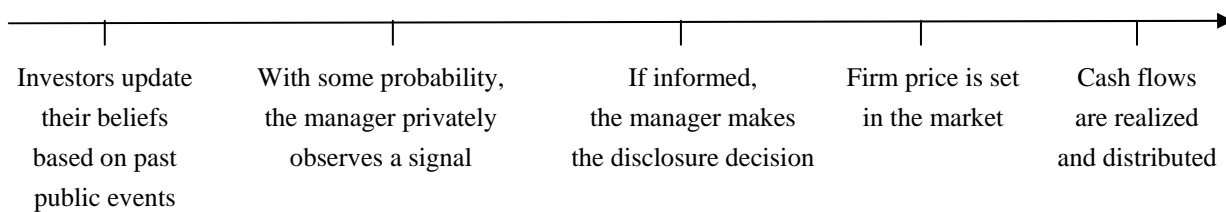
Our analysis predicts that the stickiness of disclosure across periods and the implied reduction in the disclosure intensity depend upon firm-specific characteristics, and are likely to be of larger magnitude for firms that operate in a more stable environment and for firms that have exhibited better operating performance in the past. The magnitude of these inter-temporal effects is also expected to be positively related to managerial properties, such as time horizon and degree of risk-aversion.

Obviously, the inter-temporal considerations underlying corporate discretionary disclosure strategies are much richer than what is captured in our current model. While we highlight the history dependence of the information endowment as a possible source for inter-temporal dynamics of corporate voluntary disclosures, we believe that there is a lot of potential for future research in exploring many other sources. Examples include the history dependence of disclosure costs, the history dependence of information quality, and the endowment with private relevant information about subsequent periods. Another related research path is the exploration of interactions and mutual relationships between firms' operating and disclosure strategies (see Einhorn and Ziv [2005]).



## FIGURES

**FIGURE 1**  
**Timeline per period**



## TABLES

**TABLE 1**  
**Multi-period Example**

We present the equilibrium results for a five-period numerical example, using the following Markovian probabilities:  $\pi_1 = 0.8$ ,  $\pi_t = 0.8$  and  $\pi_t - \delta_t = 0.2$  for any  $t \geq 2$ , using the stationary parameters:  $\mu_t = 100$ ,  $\sigma_t = 20$ ,  $\sigma_{\varepsilon_t} = 10$ ,  $c_t = 8$  and  $\omega_t = 0$  for any period  $t$ , and assuming  $r = 1$ . Panel A presents the equilibrium disclosure intensities for each of the periods and given any disclosure history, assuming that all past cash relations coincide with the mean  $\mu_t = 100$ . Panel B presents the equilibrium disclosure intensity for period 3 assuming the last disclosure occurred in period 1 and under three alternative levels of cash realizations in period 2.

### Panel A

	Period 1	Period 2	Period 3	Period 4	Period 5
History 0	0.50	0.41	0.38	0.38	0.40
History 1		0.51	0.41	0.39	0.41
History 2			0.51	0.42	0.42
History 3				0.52	0.45
History 4					0.55

### Panel B

Cash Flow in Period 2	Disclosure Intensity in Period 3 under Disclosure History 1
$\mu_2 - \sigma_2 = 80$	0.44
$\mu_2 = 100$	0.41
$\mu_2 + \sigma_2 = 120$	0.33

## APPENDIX

### Highlights of the Proofs

**Lemma 1.** Let  $F(\pi, x) = x + \frac{\pi z(x)}{1 - \pi(1 - \Phi(x))}$ , where  $z$  and  $\Phi$  are, respectively, the

probability density function and the cumulative distribution function for a standard normal variable. Then,  $F(\pi, x)$  is continuous and increasing in  $x \in \mathfrak{R}$  and in  $\pi \in (0,1)$ , where  $\lim_{x \rightarrow -\infty} F(\pi, x) = -\infty$  and  $\lim_{x \rightarrow \infty} F(\pi, x) = \infty$  for any  $\pi \in (0,1)$ .

#### Proof of Lemma 1

First, the function  $F$  is continuous, because it is a composition of continuous

functions. Next, we prove the monotonicity of  $F$  by showing that  $\frac{dF}{dx} > 0$  and

$\frac{dF}{d\pi} > 0$ . Using  $z'(x) = -xz(x)$  and  $\Phi'(x) = z(x)$ , we obtain

$$\frac{dF}{dx} = 1 + \frac{-\pi x z(x)(1 - \pi + \pi\Phi(x)) - \pi^2 z(x)^2}{(1 - \pi + \pi\Phi(x))^2} = 1 - \frac{\pi z(x)}{1 - \pi + \pi\Phi(x)} \left( x + \frac{\pi z(x)}{1 - \pi + \pi\Phi(x)} \right).$$

Now, note that  $\frac{dF}{dx} = 1 - \frac{\pi z(x)}{1 - \pi + \pi\Phi(x)} \left( x + \frac{\pi z(x)}{1 - \pi + \pi\Phi(x)} \right) > 1 - \frac{z(x)}{\Phi(x)} \left( \frac{z(x)}{\Phi(x)} + x \right)$ .

Sampford (1953) shows  $\forall x \in \mathfrak{R} : 0 < \frac{z(x)}{1 - \Phi(x)} \left( \frac{z(x)}{1 - \Phi(x)} - x \right) < 1$ . As  $\forall x \in \mathfrak{R} :$

$z(x) = z(-x)$  and  $\Phi(x) = 1 - \Phi(-x)$ , Sampford's inequality is equivalent to

$0 < \frac{z(-x)}{\Phi(-x)} \left( \frac{z(-x)}{\Phi(-x)} + (-x) \right) < 1$ . Thus,  $\frac{dF}{dx} > 1 - \frac{z(x)}{\Phi(x)} \left( \frac{z(x)}{\Phi(x)} + x \right) > 0$ . Also,

$\frac{dF}{d\pi} = \frac{z(x)}{(1 - \pi + \pi\Phi(x))^2} > 0$ . Finally, recall  $\lim_{x \rightarrow -\infty} z(x) = \lim_{x \rightarrow \infty} z(x) = \lim_{x \rightarrow -\infty} \Phi(x) = 0$  and

$\lim_{x \rightarrow \infty} \Phi(x) = 1$ , thus  $\lim_{x \rightarrow -\infty} F(\pi, x) = -\infty$  and  $\lim_{x \rightarrow \infty} F(\pi, x) = \infty$ .

**Lemma 2.** In each period  $t$  with disclosure history  $h_t \in H_t$ , the disclosure strategy is upper-tailed with a threshold  $s_t^*(h_t)$  that satisfies the following equation:

$$F(\varphi_t(h_t), \frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}}) = \frac{\sigma_{S_t}}{\sigma_t^2} (c_t + \lambda_t(h_t, s_t^*(h_t))), \quad (\text{A1})$$

where  $\sigma_{S_t}^2 = \sigma_t^2 + \sigma_{\varepsilon_t}^2$ ,  $\varphi_t(h_t)$  is the probability of the manager being informed in period  $t$ , as inferred by investors at the beginning of that period given a disclosure history  $h_t$ , and  $\lambda_t(h_t, s_t) = \sum_{j=t+1}^N r^{j-t} \Delta EFC_{t,j}(h_t) - \omega_t \Delta EFP_t(h_t, s_t) + \gamma_t \omega_t^2 \Delta VFP_t(h_t, s_t)$  is the indirect disclosure costs.

### Proof of Lemma 2

Consider a period  $t$  with a disclosure history  $h_t$ . Since the signal  $S_t$  is positively correlated with the firm's cash flows, the signal is disclosed if and only if it exceeds some threshold  $s_t^*(h_t)$ . Hence, the pricing rule can be rewritten as follows:

$$\begin{aligned} P_t(h_t, dis, s_t) &= E[\sum_{j=t}^N r^{j-t} CF_j | S_t = s_t] - c_t - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, dis) \\ &= \sum_{j=t}^N r^{j-t} \mu_j + \frac{\sigma_t^2}{\sigma_{S_t}} \cdot \frac{s_t - \mu_t}{\sigma_{S_t}} - c_t - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, dis) \end{aligned}$$

and

$$\begin{aligned} P_t(h_t, nd, s_t) &= E[\sum_{j=t}^N r^{j-t} CF_j | D_t(h_t, I_t, S_t) = nd] - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, nd) \\ &= \frac{1 - \varphi_t(h_t)}{1 - \varphi_t(h_t) \text{prob}(S_t \geq s_t^*(h_t))} E[\sum_{j=t}^N r^{j-t} CF_j] + \\ &\quad \frac{\varphi_t(h_t) \text{prob}(S_t \leq s_t^*(h_t))}{1 - \varphi_t(h_t) \text{prob}(S_t \geq s_t^*(h_t))} E[\sum_{j=t}^N r^{j-t} CF_j | S_t \leq s_t^*(h_t)] - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, nd) \end{aligned}$$

$$\begin{aligned}
&= \frac{1 - \varphi_t(h_t)}{1 - \varphi_t(h_t)(1 - \Phi(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}}))} \sum_{j=t}^N r^{j-t} \mu_j + \\
&\quad \frac{\varphi_t(h_t) \Phi(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}})}{1 - \varphi_t(h_t)(1 - \Phi(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}}))} \left( \sum_{j=t}^N r^{j-t} \mu_j - \frac{\sigma_t^2}{\sigma_{S_t}} \cdot \frac{z(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}})}{\Phi(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}})} \right) - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, nd) \\
&= \sum_{j=t}^N r^{j-t} \mu_j - \frac{\sigma_t^2}{\sigma_{S_t}} \cdot \frac{\varphi_t(h_t) z(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}})}{1 - \varphi_t(h_t)(1 - \Phi(\frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}}))} - \sum_{j=t+1}^N r^{j-t} EFC_{t,j}(h_t, nd)
\end{aligned}$$

When the realization of the signal  $S_t$  equals the disclosure threshold  $s_t^*(h_t)$ , the manager is indifferent between disclosing the signal or withholding it. Thus,

$$\begin{aligned}
&P_t(h_t, dis, s_t^*(h_t)) + \omega_t EFP_t(h_t, dis, s_t^*(h_t)) - \gamma_t \omega_t^2 VFP_t(h_t, dis, s_t^*(h_t)) = \\
&P_t(h_t, nd, s_t^*(h_t)) + \omega_t EFP_t(h_t, nd, s_t^*(h_t)) - \gamma_t \omega_t^2 VFP_t(h_t, nd, s_t^*(h_t))
\end{aligned}$$

This implies that the threshold  $s_t^*(h_t)$  satisfies  $F(\varphi_t(h_t), \frac{s_t^*(h_t) - \mu_t}{\sigma_{S_t}}) = \frac{\sigma_{S_t}}{\sigma_t^2} (c_t + \lambda_t(h_t, s_t^*(h_t)))$ .

**Lemma 3.** The probability  $\varphi_t(h_t)$  is increasing in  $h_t$ , decreasing in  $cf[h_t + 1 : t - 1]$ , decreasing in  $\delta[h_t + 2 : t]$ , and increasing in  $s_{h_t+1}^*(h_t), s_{h_t+2}^*(h_t), \dots, s_{t-1}^*(h_t)$ . When  $\delta_t$  converges to zero or  $t = 1$ ,  $\varphi_t(h_t)$  converges to  $\pi_t$ .

### Proof of Lemma 3

The probability  $\varphi_t(h_t)$  can be recursively defined as follows:

$$\varphi_t(h_t) = \begin{cases} \pi_1 & \text{if } t = 1 \\ \pi_t & \text{if } t > 1 \text{ and } h_t = t - 1 \\ \pi_t - \frac{1 - \varphi_{t-1}(h_t)}{1 - \varphi_{t-1}(h_t) \text{prob}(S_{t-1} \geq s_{t-1}^*(h_t) | CF_{t-1} = cf_{t-1})} \delta_t & \text{if } t > 1 \text{ and } h_t < t - 1 \end{cases}$$

Using this recursive definition, the proof now proceeds by an induction on  $t$ . The Lemma trivially holds for the first period, and from here we continue by an induction, showing that it holds for any period  $t \geq 2$ , by assuming it for the preceding period  $t - 1$ . It follows from the definition of  $\varphi_t(h_t)$  that for any history  $h_t \leq t - 2$ ,

$\varphi_t(h_t) < \pi_t = \varphi_t(t - 1)$  because  $\delta_t > 0$ . Observe now that for any history  $h_t \leq t - 2$ ,  $\varphi_t(h_t)$  is decreasing in  $cf_{t-1}$ , decreasing in  $\delta_t$  and increasing in  $s_{t-1}^*(h_t)$ . Observe also that  $\varphi_t(h_t)$  is increasing in  $\varphi_{t-1}(h_t)$ , which by the induction assumption is increasing in  $h_t$ , decreasing in  $cf[h_t + 1 : t - 2]$ , decreasing in  $\delta[h_t + 2 : t - 1]$ , and increasing in  $s_{h_t+1}^*(h_t), s_{h_t+2}^*(h_t), \dots, s_{t-2}^*(h_t)$ . Therefore,  $\varphi_t(h_t)$  is increasing in  $h_t$ , decreasing in  $cf[h_t + 1 : t - 1]$ , decreasing in  $\delta[h_t + 2 : t]$ , and increasing in  $s_{h_t+1}^*(h_t), s_{h_t+2}^*(h_t), \dots, s_{t-1}^*(h_t)$ . When  $\delta_t$  converges to zero or  $t = 1$ , it follows from the recursive definition of  $\varphi_t(h_t)$  that  $\varphi_t(h_t)$  converges to  $\pi_t$ .

**Lemma 4.** For any  $t < N$ , if  $s_k^*(h_k)$  is decreasing in  $h_k$  for any  $k > t$ , then  $\lambda_t(h_t, s_t)$  is positive, decreasing in  $h_t$ , increasing in  $s_{t+1}^*(h_t), s_{t+2}^*(h_t), \dots, s_N^*(h_t)$ , and increasing in  $N, r, \omega_t, \gamma_t$  for any  $s_t \in \mathfrak{R}$ . When  $\delta_t$  converges to zero,  $\lambda_t(h_t, s_t)$  becomes history independent but still positive. When  $\delta_{t+1}$  converges to zero or  $t = N$ ,  $\lambda_t(h_t, s_t)$  equals zero.

#### Proof of Lemma 4

$EFC_{t,j}(h_t, dis)$  and  $EFC_{t,j}(h_t, nd)$  can be recursively defined as follows for any

$1 \leq t < j \leq N$  and  $h_t \in H_t$ :

$$EFC_{t,j}(h_t, dis) = \begin{cases} \tau_{t+1}(t)c_{t+1} & \text{if } t = j - 1 \\ \tau_{t+1}(t)EFC_{t+1,j}(t, dis) + (1 - \tau_{t+1}(t))EFC_{t+1,j}(t, nd) & \text{if } t < j - 1 \end{cases}$$

$$EFC_{t,j}(h_t, nd) = \begin{cases} \tau_{t+1}(h_t)c_{t+1} & \text{if } t = j-1 \\ E_{CF_t}[\tau_{t+1}(h_t)EFC_{t+1,j}(h_t, dis) + (1 - \tau_{t+1}(h_t))EFC_{t+1,j}(h_t, nd)] & \text{if } t < j-1 \end{cases}$$

Using this recursive definition, we prove by a backward induction on  $t$ , starting with

$t = j-1$ , that  $EFC_{t,j}(h_t, nd) < EFC_{t,j}(h_t, dis)$  and the difference  $\Delta EFC_{t,j}(h_t)$  is

decreasing in  $h_t$  and increasing in  $s_{t+1}^*(h_t)$ ,  $s_{t+2}^*(h_t)$ , ...,  $s_j^*(h_t)$ . This trivially holds

for  $t = j-1$ . For  $t < j-1$ , the proof follows from applying the induction assumption to

$t+1$  and using the recursive definitions of  $EFC_{t,j}(h_t, dis)$  and  $EFC_{t,j}(h_t, nd)$ .

Now let the random variable  $FP_t(h_t, d_t)$  be the future uncertain price of period  $t+1$

as anticipated in period  $t$ , given a history  $h_t \in H_t$  and disclosure decision  $d_t \in A$ .

$$\text{That is, } FP_t(h_t, d_t) = \begin{cases} P_{t+1}(t, D_{t+1}(t, I_{t+1}, S_{t+1}), S_{t+1}) & \text{if } d_t = dis \\ P_{t+1}(h_t, D_{t+1}(h_t, I_{t+1}, S_{t+1}), S_{t+1}) & \text{if } d_t = nd. \end{cases}$$

Based on the definition of the random variable  $FP_t(h_t, d_t)$ , we get

$$EFP_t(h_t, d_t, s_t) = E[FP_t(h_t, d_t) | I_t = inf, S_t = s_t] \text{ and}$$

$$VFP_t(h_t, d_t, s_t) = VAR[FP_t(h_t, d_t) | I_t = inf, S_t = s_t]$$

For any given disclosure history  $h_{t+1} \in H_{t+1}$ , the mean of the price in period  $t+1$  is

$$\begin{aligned} & (\mu_{t+1} + \frac{\sigma_{t+1}^2}{\sigma_{S_{t+1}}} \cdot \frac{s_{t+1}^*(h_{t+1}) - \mu_{t+1}}{\sigma_{S_{t+1}}} - c_{t+1}) \Phi(\frac{s_{t+1}^*(h_{t+1}) - \mu_{t+1}}{\sigma_{S_{t+1}}}) \\ & + (\mu_{t+1} + \frac{\sigma_{t+1}^2}{\sigma_{S_{t+1}}} \cdot \frac{E[S_{t+1} | S_{t+1} > s_{t+1}^*(h_{t+1})] - \mu_{t+1}}{\sigma_{S_{t+1}}} - c_{t+1}) (1 - \Phi(\frac{s_{t+1}^*(h_{t+1}) - \mu_{t+1}}{\sigma_{S_{t+1}}}) \end{aligned} ,$$

which equals  $\mu_{t+1} - c_{t+1} + \frac{\sigma_{t+1}^2}{\sigma_{S_{t+1}}} v(\frac{s_{t+1}^*(h_{t+1}) - \mu_{t+1}}{\sigma_{S_{t+1}}})$ , where  $v(x) = x\Phi(x) + z(x)$ .

Observe that  $\frac{dv}{dx} = \Phi(x) + xz(x) + z'(x) = \Phi(x) > 0$ . Since  $v(x)$  is increasing in  $x$  and

$s_{t+1}^*(h_{t+1})$  is decreasing in  $h_{t+1}$ , we get that

$$-\Delta EFP_t(h_t, s_t) = \mu_{t+1} - c_{t+1} + \frac{\sigma_{t+1}^2}{\sigma_{S_{t+1}}} \left( v\left(\frac{s_{t+1}^*(h_t) - \mu_{t+1}}{\sigma_{S_{t+1}}}\right) - v\left(\frac{s_{t+1}^*(t) - \mu_{t+1}}{\sigma_{S_{t+1}}}\right) \right) \text{ is positive,}$$

decreasing in  $h_t$  and increasing in  $s_{t+1}^*(h_t)$ .

For any given history  $h_{t+1} \in H_{t+1}$ , the variance of the price in period  $t+1$  is

$$\frac{\sigma_{t+1}^4}{\sigma_{S_{t+1}}^4} w(s_{t+1}^*(h_{t+1})), \text{ where } w(x) = (1 - \Phi(x))^2 \text{VAR}(S_{t+1} | S_{t+1} \geq x), \text{ because the uncertainty}$$

about the price stems from the uncertainty about the realization of the signal  $S_{t+1}$ . Since

$w(x)$  is decreasing in  $x$  and  $s_{t+1}^*(h_{t+1})$  is decreasing in  $h_{t+1}$ , we get

$$\Delta VFP_t(h_t, s_t) = \frac{\sigma_{t+1}^4}{\sigma_{S_{t+1}}^4} (w(s_{t+1}^*(t)) - w(s_{t+1}^*(h_t))) \text{ is positive, decreasing in } h_t \text{ and}$$

increasing in  $s_{t+1}^*(h_t)$ .

Given the properties of  $\Delta EFC_{t,j}(h_t)$ ,  $-\Delta EFP_t(h_t, s_t)$  and  $\Delta VFP_t(h_t, s_t)$ , it follows that

$$\lambda_t(h_t, s_t) = \sum_{j=t+1}^N r^{j-t} \Delta EFC_{t,j}(h_t) - \frac{\omega_t}{1-\omega_t} \Delta EFP_t(h_t, s_t) + \frac{\gamma_t \omega_t^2}{1-\omega_t} \Delta VFP_t(h_t, s_t) \text{ is positive,}$$

decreasing in  $h_t$ , increasing in  $s_{t+1}^*(h_t)$ ,  $s_{t+2}^*(h_t)$ ,  $\dots$ ,  $s_N^*(h_t)$ , and increasing in  $N$ ,  $r$ ,

$\omega_t$ ,  $\gamma_t$  for any  $s_t \in \mathfrak{R}$ .

When  $\delta_t$  converges to zero, periods  $t, t+1, \dots, N$  create a sub-model that is

independent of the history  $h_t$ , and thus the differences  $\Delta EFC_{t,j}(h_t)$ ,  $-\Delta EFP_t(h_t, s_t)$

and  $\Delta VFP_t(h_t, s_t)$  are all independent of  $h_t$ , and so is  $\lambda_t(h_t, s_t)$ . When  $\delta_{t+1}$  converges

to zero, periods  $t+1, t+2, \dots, N$  create a sub-model that is independent of the

history  $h_t$  and of the disclosure decision of period  $t$ , and thus the differences

$\Delta EFC_{t,j}(h_t)$ ,  $-\Delta EFP_t(h_t, s_t)$  and  $\Delta VFP_t(h_t, s_t)$  all equal zero, and so is  $\lambda_t(h_t, s_t)$ .



### Proofs of Propositions 1 and 3

The proof is by an induction on the number of periods. For a single-period model, the only possible history is 0,  $\varphi_1(0) = \pi_1$  and  $\lambda_1(0, s_1) = 0$  for any  $s_1 \in \mathfrak{R}$ , and thus by Lemma 1 there is a unique solution  $s_1^*(0)$  to Equation (A1) that constitutes the unique equilibrium according to Lemma 2, trivially satisfies Proposition 3, and is decreasing in  $\pi_1$ . Now, we proceed to prove that a unique equilibrium exists for a model with  $N$  periods and that this equilibrium satisfies Proposition 3 and all thresholds are decreasing in  $\pi_1$ , based on the assumption that such equilibrium exists in a model with  $N - 1$  periods. We apply the induction assumption to the last  $N - 1$  periods, where the initial probability is  $\varphi_2(1) = \pi_2$  if disclosure occurred in the first period, and  $\varphi_2(0)$  (where  $\pi_2 - \delta_2 < \varphi_2(0) < \pi_2$ ) otherwise. When applying Equation (A1) to period  $t = 1$  and the one and only possible history  $h_1 = 0$  in that period, we get

$$F(\pi_1, \frac{s_1^*(0) - \mu_1}{\sigma_{s_1}}) = \frac{\sigma_{s_1}}{\sigma_1^2} (c_1 + \lambda_1(0, s_1^*(0))).$$

By Lemma 1, the left side of this equation is

increasing in  $s_1^*(0)$  from  $-\infty$  to  $+\infty$ . Lemma 3 implies that  $\varphi_2(0)$  is increasing in  $s_1^*(0)$ , so by the induction assumption  $s_2^*(0), s_3^*(0), \dots, s_N^*(0)$  are decreasing in  $s_1^*(0)$ . Thus, according to Lemma 4, the right side of the equation is decreasing in  $s_1^*(0)$ . Hence, the equation yields a unique solution  $s_1^*(0)$  that together with the equilibrium thresholds of the following  $N - 1$  periods constitutes the unique equilibrium in the model. This completes the proof of Proposition 1. To complete the proof of Proposition 3, we need to prove that  $\tau_t(0) < \tau_t(1)$  and  $\tau_t(0)$  is decreasing in  $cf_1$  and in  $\delta_2$  for any  $t \geq 2$ . According to Lemma 3,  $\varphi_t(0) < \varphi_t(1)$  and  $\varphi_t(0)$  is decreasing in  $cf_1$  and in  $\delta_2$ . By the induction assumption and Lemma 4,

$\lambda_t(0, s_t) > \lambda_t(1, s_t)$  and  $\lambda_t(0, s_t)$  is increasing in  $cf_1$  and in  $\delta_2$  for any  $t \geq 2$  and  $s_t \in \mathfrak{R}$ .

Using now Lemmata 1 and 2, for any  $t \geq 2$ ,  $s_t^*(0) > s_t^*(1)$  and  $s_t^*(0)$  is increasing in  $cf_1$  and in  $\delta_2$ . Accordingly,  $\tau_t(0) < \tau_t(1)$  and  $\tau_t(0)$  is decreasing in  $cf_1$  and in  $\delta_2$ .

### **Proofs of Observation 1 and Proposition 2**

By Lemma 2, the threshold  $s_t^*(h_t)$  is the solution to Equation (A1). In the benchmark case, by Lemmata 3 and 4,  $\varphi_t(h_t) = \pi_t$  and  $\lambda_t(h_t, s_t) = 0$  for any  $h_t \in H_t$  and  $s_t \in \mathfrak{R}$ .

Hence, in the benchmark case, the solution to Equation (A1) is history independent, and so is the implied  $\tau_t^B$ . When  $\delta_2, \delta_3, \dots, \delta_N > 0$ , unless  $t = N$  and  $h_t = N - 1$ ,

$\varphi_t(h_t) < \pi_t$  or  $\lambda_t(h_t, s_t) > 0$ . Using Lemma 1 now, the solution  $s_t^*(h_t)$  to Equation (A1) is higher than that obtained in the benchmark case. Accordingly,  $\tau_t(h_t) \leq \tau_t^B$ , where the equality holds only for  $t = N$  and  $h_t = N - 1$ .

### **Proof of Corollary 1**

By Proposition 3,  $\tau_t(j)$  is independent of  $cf[i+1:j]$  and  $\delta[i+2:j+1]$ , while  $\tau_t(i)$  is decreasing in  $cf[i+1:j]$  and  $\delta[i+2:j+1]$ , and thus the difference  $\tau_t(j) - \tau_t(i)$  is increasing in  $cf[i+1:j]$  and  $\delta[i+2:j+1]$ .

### **Proof of Corollary 2**

By Lemma 2, the threshold  $s_t^*(h_t)$  is the solution to Equation (A1). By Lemmata 3 and 4, when  $\delta_t$  converges to zero, both  $\varphi_t(h_t)$  and  $\lambda_t(h_t, s_t)$  in Equation (A1) become history independent, and so are the solution  $s_t^*(h_t)$  and the implied  $\tau_t(h_t)$ . However,

since  $\delta_{t+1} > 0$ , for any  $t < N$ ,  $\lambda_t(h_t, s_t)$  is positive, and thus  $s_t^*(h_t)$  is still above the benchmark threshold, so  $\tau_t(h_t) < \tau_t^B$ .

### **Proof of Corollary 3**

The proof is based on Proposition 3 and the proof of Lemma 4.

### **Proof of Proposition 4**

By Lemma 2, the threshold  $s_t^*(h_t)$  is the solution to Equation (A1). It follows from Proposition 3 and Lemma 4 that  $\lambda_t(h_t, s_t)$  in the right side of Equation (A1) is increasing in  $N$ ,  $r$ ,  $\omega_t$ ,  $\gamma_t$  and  $\delta[t+1:N]$  for any  $s_t \in \mathfrak{R}$ , and thus by Lemma 1 so is the solution  $s_t^*(h_t)$  of this equation. The implied  $\tau_t(h_t)$  is therefore decreasing in  $N$ ,  $r$ ,  $\omega_t$ ,  $\gamma_t$  and  $\delta[t+1:N]$ .

### **Proof of Corollary 4**

By Lemma 2, the threshold  $s_t^*(h_t)$  is the solution to Equation (A1). By Lemmata 3 and 4, when  $\delta_{t+1}$  converges to zero,  $\lambda_t(h_t, s_t)$  in the right side of the equation is zero, and thus the solution  $s_t^*(h_t)$  and the implied  $\tau_t(h_t)$  become independent of  $N$ ,  $r$ ,  $\omega_t$ ,  $\gamma_t$  and  $\delta[t+2:N]$ . Nevertheless, since  $\delta_t > 0$ , for any  $h_t < t-1$ ,  $\varphi_t(h_t) < \pi_t$ , and thus  $\tau_t(h_t)$  is still below the benchmark level  $s_t^B$ .

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