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Andrea Lodi*, Enrico Malaguti*,
Nicolás E. Stier-Moses[‡], Tommaso Bonino[†]

**Dipartimento di Ingegneria dell'Energia Elettrica e dell'Informazione "Guglielmo Marconi",
Università di Bologna, Bologna, Italy
andrea.lodi@unibo.it, enrico.malaguti@unibo.it*

*‡Graduate School of Business, Columbia University, New York, NY, USA
stier@gsb.columbia.edu*

*†SRM - Reti e Mobilità Srl, Bologna, Italy
tommaso.bonino@srbologna.it*

Abstract

Until the end of the 20th century it was common that cities organized, financed and managed their own public transportation systems. More recently—in Italy mainly during the last decade—many countries liberalized the service provision of public transportation. Indeed, while offering local public transportation is a political and financial duty of governments, the service provision is outsourced to a private Operator, while the government retains a control position, given to a regulatory Agency. This is a common scheme not only in transportation.

In close collaboration with the public transportation agency of the city of Bologna, Italy, we designed a methodology to optimize some aspects of the contractual relationship between that Agency and the bus Operator. We focus on the fines specified by the contract when the Operator fails to comply with the service level it has agreed to provide, and on a procedure to measure that service level. Our approach has the benefit that it aligns the incentives of both parties in the benefit of bus riders. We model the Agency-Operator relationship as a multi-stage game and find its equilibrium to establish the best operating regime. The game theoretic approach provides expressions for the fines that the Operator should be charged if it does not satisfy the contracted services, and for the optimal Agency's budget to devote to control activities. Second, to check the compliance of the Operator to the schedule specified by the contract in a resource efficient way, we compute how to position the Agency's employees to verify if buses are running according to the specification in the contract. This is achieved by counting bus services and checking other quality indicators. We formulate this NP-hard problem as a Mixed Integer Linear Program, and propose an algorithm to solve it that is effective in providing itineraries for the controllers working for the Agency.

1 Introduction

In all modern economies, it is common for governments and local authorities to regularly outsource services to the market. Depending on the economical organization of the government and the nature of the object or service to be purchased, the transaction can be performed as a one-time activity or give rise to a long-term relationship with the provider. This paper is about the latter

case, where a public entity outsources a service to be provided to the citizens through a contract with a provider.

Many, if not all, public-service contracts can be naturally fit to the following scheme. First, the contract is designed by the central/local government, often with the help of a regulatory *Agency*, and a public auction takes place. Second, companies interested in the contract participate in the auction, the “best” offer is selected, and the contract is signed. Third, the service is provided by the company that won the auction. Fourth, the central/local government controls quality of service and the compliance to the contract specifications. This is done very often with the help of the same regulatory Agency. Depending on the type, duration and sector of the contract, the control activity can be performed *while* the contract is in effect, or *after* it is completed.

Commonly, these contracts involve penalties of different types. They depend on the kind of mismatches between specifications and provision, on the reasons for which the mismatches occurred, and, of course, on how severe the mismatches have been. Although the fines charged by the government can be extremely high, it is crucial to stress that the aim of the central/local government is the overall benefit of the citizens, and consequently, there is no incentive to make money out of the fines. In other words, putting the service provider out of business by imposing fines is not an option because that would result in a severe disservice to society. This implies that the design of the contract, for example in terms of its obligations and fines, must consider trade-offs carefully: a fine that is clearly too high will not discourage the service provider to miss the contract obligations because the provider knows that enforcing it has a political cost that is not affordable for the government.

For the fines specified in the contract to work, it is essential that the Agency performs independent verifications of the service level. Indeed, knowing that the Agency measures the quality of service accurately provides incentives to the provider to increase the effort to comply with the contract. Furthermore, contracts of this kind are often periodic—i.e., a similar contract for the same public service has to be signed every few years—which provides some opportunities to tune details, generate better contracts and improve the relationship between both parties. This further supports that the controlling activities performed by the Agency to be as informed as possible regarding the working condition of the system are valuable in the long term. At the same time, control activities have a cost. The issue at hand is getting the most out of the (typically small) budget that the government allocates to these control activities; i.e., distilling enough information to provide the maximum incentive for the provider to align with the contract. This is essentially a resource allocation problem where capacity constraints are typically small.

One of the best examples of public-private contracting and of the problems related to the lack of an effective control during the contract execution is the so-called “Big Dig” project in Boston (see, e.g. Wikipedia (2013)). That was a megaproject that rerouted Interstate 93 that used to go through downtown Boston into a 3.5-mile tunnel. Concurrently, another tunnel was constructed to extend Interstate 90 to Logan International Airport. The planning started in 1982 while the actual construction was done between 1991 and 2006. The project was managed by the Massachusetts Turnpike Authority, and the design and construction was supervised by a private joint-venture and executed by dozens of smaller subprojects because of scale. The Big Dig cost was of the order of 20 billion dollars, the most expensive highway project in the United States. At that price-tag, it was plagued by problems of all possible kinds (design, scheduling, quality). Even one person died because one concrete slab fell on a car, caused by the incorrect application of epoxy glue used to hold the roof in place.

Other meaningful examples of general areas in which public-service contracts arise are

- roadworks maintenance, when part of the road network must be renovated and the amount of material and its quality is a big discriminator for the compliance with the contract,
- garbage collection, where the quality of service must be assessed in terms of number and frequency of the visits to the garbage bins, and
- public transportation systems, where contracts specify frequencies of different bus lines that must be operated.

The present paper focuses of the last example and presents a case study in the city of Bologna, Italy. Specifically, until the end of the 20th century in many countries of western Europe, including Italy, public transportation has been organized, financed and managed by central or local governments. More recently, mainly during the last decade, many countries liberalized their public transportation services. Nowadays in Italy, while offering local public transportation is a political and financial duty of regional governments, the operation of the service is outsourced to a private entity, henceforth the *Operator*. The new approach represented a big discontinuity with respect to the past when local public transportation was managed and operated directly by the government. As counterparts to operators, local governments created regulatory agencies whose tasks include the design of the local transportation service, the definition of the service contract that describes the duties of the Operator, and the organization of a public auction to choose a private Operator that will run the service.

Although the implementation details change from country to country and often from region to region in the same country, this system has been adopted in several cities and regions in the world. Just to provide some examples, we mention Syndicat Mixte des Transports en Commun de l'Agglomération Grenobloise (2012) in Grenoble, France; Hamburger Verkehrsverbund (2012) in Hamburg, Germany; Consorcio Regional de Transportes (2012) in Madrid, Spain; Veolia Transport (2012) in Nassau County, New York, United States; and Société de Transport de Montréal (2012) in Montreal, Canada.

Paper's Contribution. The article is divided in two parts that are deeply connected. We look at the relationship between the Agency and the Operator, and design a procedure to be used by the Agency to control the performance of the Operator regarding satisfying its contractual obligations. We use game theoretical techniques to model the contract between the Agency and the Operator, and characterize the equilibrium of the game to shed light in its optimal structure. The control procedure we propose is, in turn, an input needed for the computation of the equilibria of the game because the Operator decides to what extent it is going to report missed services based on the effectiveness of the control procedure. Clearly, an optimized control procedure allows the Agency to do its job with the highest possible precision within its budget, which results in an increase of welfare for society. It also leads to an equilibrium in which the Operator has a high incentive to satisfy its contractual obligations (concerning service and reporting activity).

We solve the game and show that it can be used to align the incentives of both parties. Then we solve the problem of how to optimally oversee the behavior of the Operator. Closing the loop, we use the output of the optimal control procedure to compute the fines to be charged to the Operator at equilibrium. The game is solved under specific assumptions on the investment structure of the Operator and on the effectiveness of checking if services took place.

The contracting model relies on the principal-agent problem, a traditional framework to analyze this class of phenomena. The novel feature of our model is that it adds the possibility of *cheating* on top of the classic *moral hazard* found in traditional principal-agent models. Indeed, besides choosing a level of effort, the agent also chooses a level of honesty t that captures the probability it misreports the quality-of-service. The agent trades-off paying a small fine f when reporting truthfully a service that was skipped and a lottery. In the lottery, the agent either gets away without paying anything if the principal does not realize there was a problem, or pays a big fine F if the principal becomes aware of the breach in the contract. Notice that the trade-off is conditional to the level of effort the agent invests; with enough investment there is not trade-off since the probability that a service does not run is minimal. The possibility of bankruptcy and cheating prevents the Agency to implement a first-best contract, which then goes with the best contract possible for the regulatory setting in which it is immersed.

The planning of control activities is obtained through the use of combinatorial optimization techniques applied to a Mixed Integer Linear Program (MIP) that captures the main aspects of the control procedure. Although the optimization problem we propose is NP-hard, we can solve it for problem instances taken from Bologna. The computational study we perform shows that we can efficiently find solutions that are useful to our purposes.

The use of the methodology we developed for the Agency in Bologna led to the enforcement of new controlling procedures at the operational level, to an exhaustive re-design of the next public transportation bus contract to be auctioned in 2016, and to an intensive redesign of the evaluation framework in the context of the European project *EPTA*, “European model for Public Transport Authority as a key factor leading to transport sustainability”.

Public-service contracts in other contexts than transportation essentially follow the same scheme, and, although the optimization problem associated with the control component of the scheme will of course be different, we believe that the methodology applied in the present paper is sufficiently general to be used in public-service at large.

Paper’s Organization. This introductory section is completed below by a discussion of the details of our case study (Section 1.1), and by a short literature overview on the general public-service contracts and the specific transportation context (Section 1.2). In Section 2 we analyze the contract between the Agency and the Operator from a game theoretical perspective, and compute the equilibrium of the corresponding game. To monitor the execution of the contract, in Section 3 we design an optimized control procedure to oversee the provision of service. Section 4 describes the algorithm we implemented to find the optimal control procedure while in Section 5, we apply the methodology to the case study of the city of Bologna, Italy. Section 6 closes the loop and applies the optimal control procedure to the optimal contract computed earlier. Finally, Section 7 presents our conclusions.

1.1 Public Transportation Service in Bologna, Italy

As explained above, this article conducts a case study about the service provision of local bus transportation in Bologna, Italy. The public bus Operator of the city of Bologna has to run 2.2 million services every year, which cover a total of 18 million kilometers and transport 92 million passengers. The regional government entrusts the supervision of the local transportation to a government-owned Agency called *SRM* (Reti e Mobilità, 2012). This Agency has direct responsibility for:

- the design, organization and promotion of local public transportation;
- the definition of a service contract, including the minimum requirements of the Operator such as performed services, reliability, punctuality, and information collection and provision concerning actual performance;
- the setup of an auction for selecting the Operator, which includes the pre-selection of participants based on technical and legal qualifications;
- supervision of the satisfaction of the Operator's contractual obligations.

The contract is renewed every 6 years with a possible 3-year extension. In 2010, the Agency organized the first auction in Bologna. The winner was the operator that had been providing the service previously, which had become a private company during this process. The total revenue of the Operator is approximately €150 millions per year, which is used to provide service according to a schedule provided by the Agency. This requires over 2,100 drivers. Since the Operator is still owned by the regional and local governments, the relationship between Operator and the Agency is resolutely friendly at the moment. However, some large private transport operators that have a strong presence in the European market (like, e.g., Veolia, Transdev, Arriva, etc.) are likely to participate in future auctions competing for a contract of more than one billion Euros. As a consequence, small local operators are starting to merge to grow to a regional size to compete more efficiently in the liberalized markets of today. The winner of the auction has the obligation of running the buses in exchange of receiving the bus fare and a subsidy from the regional government.

The Operator is under contractual obligation to monitor its own quality of service, which has to be reported to the Agency monthly. If the reported performance does not meet the contractual obligations, the Operator receives a fine from the Agency. As with any complex operation, sometimes it happens that some services are canceled or interrupted because, e.g., the driver does not show up, the bus breaks down, there is a road accident, etc. In some cases the failure may be out of the Operator's control and in others the Operator has a direct influence on the root of the problem. For example, although it cannot make sure that no bus will break down during service, it can drastically reduce the probability of this event by spending more resources in maintenance; similarly, it can implement a policy to replace the unavailable drivers. To align the incentives, the contract indicates that the Agency must charge fines if the Operator does not comply with the agreed service level. For every service that is skipped or canceled, the Operator has to pay a small fine to discourage the Operator from canceling services, understanding that sometimes unintended operational problems arise. Instead, when the actual performance is much lower than the minimum requirement, the Operator is charged expensive fines to provide a much stronger incentive to do as it should.

A major issue in this scheme is the asymmetry of information between the Operator and the Agency. The Operator has complete knowledge of its operations, its real service level, which services are performed and which are not, etc. Instead, the Agency does not have direct access to this information; they receive it from the Operator. The problem is that the latter might purposely alter the reports to prevent fines by declaring a level of service that is higher than the actual one. For this reason, the Agency must confirm the information reported by the Operator by doing an independent assessment. If the Agency discovers that the Operator's reports misrepresented reality, the latter receives a hefty fine. However, this fine cannot be too large; otherwise, the threat would lose credibility because neither side wants to cancel the contract under normal circumstances. It is

important to highlight that the Agency does not have a special interest in the revenue arising from fines. Although fines are part of the Agency's payoff, relatively much more importance is placed in the impact on welfare that occurs when services are not performed according to the planned schedule. The main objective of the Agency is to have the level of service specified in the contract and to receive reliable information about service provision from the Operator. In absence of a control system and the associated fines, the Operator would not be compelled to provide reliable information, not necessarily in bad faith, but simply because it is not cost-effective to perform the necessary effort. Anticipating this, the Agency optimizes the control procedure to streamline its daily operations, by minimizing the associated cost and maximizing the impact of the taxpayer money spent in overseeing the contract signed with the Operator.

Currently the Agency performs some supervision on the service provision, but these controls are not systematic, and are mainly based on complains received from customers. In these cases, the Agency sends an employe to visit the corresponding stops, along an itinerary that is arranged in an informal way. The Agency is interested in standardizing the checking procedure with the goal of assessing the performance of the Operator and validating the current contract. This activity will help the Agency to refine the contract structure if needed, and to improve the structure of fines, which is going to be used to modify the contract at the time the concession of the bus service ends and the selection process restarts.

It is important to observe that the information asymmetry between the Agency and the Operator cannot be fixed by simply employing an *Intelligent Transportation System* (ITS) that provides vehicle tracking. In fact, buses are *already* equipped with GPS devices. Actually, the most common service quality defect in Bologna is that buses do not stop long enough to allow passengers to get in (because the bus is running late); that event cannot be identified by a tracking system. In addition, drivers could be tempted to switch the GPS off to forbid the control, then claiming that the device is malfunctioning, which will be, at a later stage, hard to assess and will lead to unpleasant controversies. Finally, the control activities performed by employees of the Agency at stops and at buses is much more accurate because it involves many aspects of quality such as cleanness, courtesy, etc. As we point out in the next section, this type of control is indeed generally preferred in many places throughout the world.

To summarize, our work describes the development of a complete control framework and analyzes its impact on the design of the contractual relationship. The Agency will direct employees, referred to as *controllers*, to strategic intersections in Bologna to detect missing services so the Operator refrains from not complying with the contract on purpose. The Agency faces the decision of how much it should spend on deterrence, and where and what kind of controls it must put in place to use their budget wisely and maximize the likelihood of detecting problems. The Operator may fail to report skipped services only when and where it is unlikely to be detected.

Although the Agency can perform these controls in different ways, it is most convenient to observe services in different locations. The problem to be solved consists on deciding the locations and times that need to be checked, together with the itinerary (a sequence of stops) the controllers must follow. The problem in question is hard from an optimization standpoint, and to evaluate the proposed control procedure, we compare it to a greedy heuristic that mimics how controllers build their itineraries today.

1.2 Literature Overview

The paper essentially contains three viewpoints and perspectives: the game that encodes the contract, the case study about the public transportation system, and the mathematical programming approach used to optimize the control problem. We split the literature review accordingly.

1.2.1 Contract Regulation (in Public Service)

This work is closely related to the literature on the principal-agent model with moral hazard. This literature, exemplified by the classical work of Grossman and Hart (1983) and Rogerson (1985), analyzes how a principal can delegate the execution of a job to an agent. The principal and the agent agree on a contract that provides the necessary incentives so the latter performs the job in an effective way, under the assumption that the principal cannot perfectly monitor the effort that the agent puts in the job. This model has been the workhorse to understand many interesting economic phenomena where incentives play a crucial role.

In the contracts for public transportation described above, the regulator gives the right to the Operator to run the bus services. Because of information asymmetries, the regulator is less informed than the operator regarding the cost of providing service and the quality of the public transportation network. This implies that generally a first-best solution is not achieved. Indeed, Gagnepain and Ivaldi (2002) focus on the principal-agent framework and study the effect on welfare caused by different types of contracts between the regulator and the public transit operator. Their experiments suggest that cost-plus contracts damage welfare, whereas fixed-price mechanisms are more likely to curb the inflation of operating costs. Socorro and de Rus (2010) also considers a principal-agent model and study how to best align the incentives of both parties by comparing different financing schemes that have been used in Spain. Hansson (2010) discusses the relations between different government bodies at the local, regional and country levels and their relations to regulating public transport procurement.

In terms of the details of the contract discussed in Section 2, we chose a simple model of a sequential game where both parties first choose the long-term decisions that will govern the relationship and then play a one shot game that determines the interaction. In the literature of contract theory, some articles take on the more complex task of modeling the interaction as a repeated game (see, e.g., Green and Porter, 1984; Chassang, 2010; Yared, 2010). These papers present how private information and imperfect monitoring can lead to punishment situations that are inappropriate. Enriching our model along those lines is a possible line for future research.

Although the center of our focus, an important discussion related to our work is how these contracts are awarded and how to subsidize the difference between ticket prices and operating costs. Jansson and Wallin (1991) reports the effect of deregulation of public transport in Sweden and how different local transport agencies reacted to the new policy. Hensher and Stanley (2008) review the recent debate over the performance of auctions (i.e., competitive tendering) versus performance-based negotiated contracts when the service to be provided is complex. A key example of this tension is provided by the case of a service contract for public transportation which has quite often very large transaction costs. The high costs arise from the difficulty of specifying and then correctly monitoring and influencing the key performance measures. Hensher and Stanley argue that the focus on auctions may have led governments to place too much emphasis on cost reduction instead of the social obligation to maximize welfare. They conclude that negotiated performance-based contracts, aimed at the development of trusting partnership based on incentive and penalties structures, are

preferable to competitive tendering. Along these lines, Hensher and Stanley (2003) discuss about performance-based contracting as a more effective alternative to competitive tendering for obtaining the largest social benefit from public subsidies. Hensher et al. (2003) identify a set of important dimensions that must be considered for evaluating the service perceived by bus service passengers. Rizzi (2013) discuss different mechanisms to help defray the capital and operating expenses of public transportation and analyze which are better in aligning incentives between parties.

1.2.2 Best Practice and Strategic View in Public Transportation Systems

As previously mentioned, the public transportation service nowadays is mostly outsourced to private companies and regulated by independent agencies that are intermediaries between the Operator and the local government. The *European Union* (EU) has recommended to structure public transportation in this manner (see, e.g., European Parliament (2007)) but, as mentioned in the introduction, this structure has been widely adopted also in the rest of the world. In addition, the EU has enforced political objectives and rules in the attempt of making the Public Transportation System more sustainable (see, e.g., POLIS (2013)), and FP7-funded EU projects like EPTA aim at defining the regulatory foundation of the *7 key functions* that characterize the governance of public transportation; namely, Regulation, Planning, Tendering and Awarding procedures, Integration, Promotion, Management, and Control and Monitoring. It is easy to see that the last two of those functions precisely define the application context of the present study, and it is worth noting that such a context is much broader than running the buses in a medium-size city like Bologna. In fact, the methodology can be applied to virtually any public transportation service that is outsourced, e.g., metro or local trains for commuters (see, e.g., Caprara et al. (2010)), home-healthcare services (see, e.g., Hiermann et al. (2013)), transportation of handicapped people (see, e.g., Toth and Vigo (1997)), etc.

While, at least at the EU level, the central government explicitly puts local authorities and, as a consequence, *ad hoc* agencies in charge of the control, the way the control is implemented depends obviously on local policies. A human-based control like that implemented in Bologna instead of (or at least in conjunction with) one achieved through ITS has been found to have more acceptance in Italy, as pointed out in the previous section. However, this applies elsewhere too. The Argentinean National Agency of Transport Regulation (CNRT) in a series of documents explicitly states (AGN (2005, Section 1.1.1) and CNRT (2007, pp. 25-47)) that the local authority controls each line three times a year and takes a sample of 2,000 bus drivers per year to check their behavior (which includes courtesy, cleanness of the bus, etc.). The authority measures frequency through controllers standing at least for 3 hours in one place while they count the number of services.

A similar situation can be encountered, e.g., in London, where the bus service is provided by operators that compete for contracts to provide specified services for up to seven years, and are rewarded for exceeding defined targets so as to improve the service to the passengers. The level of service, including the ability of the operator to schedule, control and adjust services, is evaluated by a system that integrates human observations and tracking technology (see TFL (2009)).

Finally, it is clear that a human-based control is more appropriate and likely to work in developing countries where buses are sometimes operated by informal and heterogeneous organizations (for example it is typical that some buses are owned by individuals, often the driver, and not by companies). In those cases, the investment associated with installing a GPS device on each bus and delegating the measurements of quality of service might be unfeasible or too expensive.

1.2.3 Mathematical Programming

With regard to the methodology to control the quality-of-service discussed in this article, we propose to solve a routing problem that collects prizes associated with the visit of specific nodes or subsets of nodes of the bus network. Prizes correspond to the value of the information that can be collected in each location. We construct routes that can be traversed during the work-day of a controller. We find one route per controller, maximizing the total prize. To model and solve this routing problem we rely on results from the routing literature. For a general introduction to Vehicle Routing Problems (VRP), we refer the reader to, e.g., Toth and Vigo (2001).

We now comment on some problems that are specifically related to the control problem addressed by this article. First, the *Orienteering Problem* is a problem defined on a graph where arcs have a cost, nodes have a prize, and one wants to construct the simple cycle that maximizes the prize that can be collected such that the cycle not exceeds a given length (see, e.g., Fischetti et al. (1998)). This resembles the problem we study although in our case we need to construct several cycles, one for each controller. Our problem also resembles the *Prize-Collecting Steiner Tree Problem*, which consists of finding a connected subgraph of a graph that has prizes associated with the nodes (see, e.g., Ljubic et al. (2005)). In that problem, the objective value is the sum of the prizes of the nodes minus the costs of the edges included in the subgraph. Our problem also has elements of the *Distance-Constrained Vehicle Routing Problem*: a VRP with constraints on the total distance traveled by each vehicle (see, e.g., Li et al. (1992)). In our problem, though, some nodes may remain unvisited. Traditionally VRP problems do not have prizes; they call for minimizing the cost associated with the routes. Finally, our problem has also elements of the *Generalized Traveling Salesman Problem*, where given a set of cities partitioned into (non intersecting) clusters, one has to visit at least one city in each cluster (see, e.g., Fischetti et al. (1997)). This shares with our problem that nodes that represent stops of a bus line are grouped into a cluster (but that can intersect), and a prize is earned if the cluster is visited because that particular bus stop is checked. The most common solution method in all papers mentioned above is to formulate the problems as MIPs and design a *Branch-and-Cut* algorithm, which is the framework we adopt as well.

Finally, in most of the literature the control phase is not modeled and solved as an optimization problem. Usually, the quality of service measurement is estimated using surveys sent to costumers. A notable exception is the work of Borndörfer et al. (2013) who write about a highway with tolls. In their case, the control decision is how often one should check that drivers had payed the toll in a highway to minimize evasion. They use a Mixed Integer Linear Problem to compute the optimal control strategy.

2 Contract Between the Agency and Operator

To model the contract between the Agency and the Operator in the city of Bologna we rely on a principal-agent model. The contribution of this analysis lies in formalizing the interrelations of this contract with the service control operation that must be implemented by the Operator. In addition, this analysis allows the Agency to validate the fines that they have initially considered for the contract. We suggest a model for this contract and study it in detail under realistic assumptions that fit the needs of the Agency. We defer some details until later because Section 5 discusses the case-study and it is better to discuss the implementation in the city of Bologna after the specifics have been introduced. Although we customized our analysis for Bologna, most elements of this

contract are of broader appeal and apply to many other places and situations, as described in more detail in the introduction. The novel feature of our model is the inclusion of the possibility of underreporting by the Operator which is not traditional in the principal-agent literature.

The main purpose of the contract is to create a framework under which the Operator runs the bus services in exchange of a payment. The Operator has to provide service according to a fixed schedule for the bus lines. The contract follows a *net cost* paradigm: the Agency defines a service and gives the Operator a subsidy to cover part of its costs. The subsidy is currently paid according to the services planned, and not to the services really performed or to the number of passengers. The revenue generated by users paying bus fare goes directly from passengers to the Operator. Because there may be situations that prevent the Operator from running a service, sometimes out of their control, it is specified by the contract that it has to report back to the Agency what services were provided successfully and other performance indicators. If the Operator does not reach a specified level of service, it gets a fine. Finally, the role of the Agency is to make sure that the reports provided by the Operator are accurate. Because fines are levied on the Operator when not performing the required services, it has the incentive to underreport that information. For that reason, the Agency spends part of its budget controlling that the reports are truthful. If the Agency finds that the reported quality of service and the measures differ, a much larger fine is imposed on the Operator.

This section proposes a model that allows the Agency to tune the details of the contract to minimize the problems of moral hazard and cheating. Using a game theoretical model, we find the amounts of fines that are best to minimize those problems and provide incentives to the Operator to act honestly and efficiently. To model the control procedure to be implemented by the Agency, we postulate that the budget destined to those activities and the effectiveness of the control are described by polynomials, which are convenient and capture well the intricacies of the resulting control procedure, as discussed in detail in the computational study of Section 5. Hence, it is important to highlight that although the control is performed *after* designing the contract and signing it with the Operator, the optimal itineraries used for checking the quality of service are taken as inputs in the contract design phase addressed by this section.

We conducted interviews with the Agency and agreed on the key aspects of the contract. The primary objective of the Agency is to maximize commuters welfare by enforcing the provision of the planned services. The secondary objective is creating a regulatory framework so the Operator is honest and provides reliable information about its performance. We model the contract as a sequential, single-shot game between the Agency and the Operator. First, the Agency establishes the details of the contract followed by the Operator's decision of how much to invest in service provision. Then, at the same time the Operator chooses to what extent to report truthfully while the Agency chooses the effort to be incurred in controlling activities. The stages of the game mimic the time-scale of the different decisions. We provide more details on each stage in the bullet points below. Although we assume that all the aspects of the game are static and effects are uniform across the network, more general assumptions could also be incorporated at the expense of complicating the game. Because the contract does not specify provisions for those differences we agreed with the Agency to model these decisions using a game as simple as possible.

To quantify the impact in welfare when a service is planned but not provided, we take the additional total waiting time of the passengers that would have taken that bus.

1. *Agency chooses fines.* For each planned service that is not performed and reported truthfully as so by the Operator, it receives a small fine f . For each service that is not performed and

not reported, if the Agency finds out it gives the Operator a big fine of F . On caveat is that if the Agency sets F too large, it would be prohibitive for the Operator to pay it and it would bankrupt, in which case the Agency would have to organize a new auction to select a new operator. Such a consequence is deemed extreme, and unrealistic, hence undesirable. Terminating a contract early may have negative reputation effects for the city. Furthermore, there is a (small) possibility that a service that was reported as performed by the Operator and that was checked and detected missing by the Agency, actually took place but was not observed for various reasons (e.g., it was late, it took a different route, the controller missed it, etc.). The possibility of imperfect detection also imposes a limit to the value of F . Hence, we assume that F cannot be larger than an upper bound fixed a-priori (e.g., the cost of tendering the contract again).

2. *Operator chooses investment in service provision.* In this stage the Operator decides how much to invest to obtain and maintain buses, hire and train drivers, and all the other aspects of service provision. We call this budget decision b_{op} , which represents the invested money per service. We define the function $q(b_{\text{op}})$ that maps investment to the achieved quality of service, represented by the percentage of the services that are provided as planned. This relies on the observation that the more is invested in bus provision the more likely it is to provide a given service successfully. The percentage $q(b_{\text{op}})$ is to be interpreted as the probability that the Operator provides a given service. We assume that $q(0) = 0$, $q'(b_{\text{op}}) > 0$, $q''(b_{\text{op}}) < 0$ and $q(b_{\text{op}}) \rightarrow 1$ when $b_{\text{op}} \rightarrow \infty$. A finite investment cannot guarantee that $q(b_{\text{op}}) = 1$ because there are always factors outside the Operator's control.
3. *Operator chooses honesty level; Agency chooses effort to control.* In this stage both players compete in a single shot noncooperative game. The Operator decides what fraction of non-performed services t to report honestly. Hence, a fraction $1 - t$ of missed services are not reported as missing. Notice that whether to report a missed service or not is not stochastic but a decision of the Operator. The Operator may manage to save some money by withholding the information, but if discovered it must pay a much heftier fine. Concurrently, the Agency decides the budget per service b_{a} allocated to controlling activities. Since the control is done by placing employees in the street, it is labor intensive so b_{a} mainly consists of wages paid to those employees. The control procedure defines a function $p(b_{\text{a}})$ that maps the budget devoted to control to the probability that a service is checked. The function satisfies that $p(0) = 0$, $p'(b_{\text{a}}) > 0$ and $p''(b_{\text{a}}) \leq 0$. It is theoretically possible (albeit impractical) to check all services with a finite budget, hence there exists \bar{b}_{a} such that $p(\bar{b}_{\text{a}}) = 1$.

For convenience, we summarize the variables and parameters of the game in Table 1. Now that we presented the decisions, let us specify the objectives of the players. The utility functions of the Operator and the Agent are respectively defined by:

$$\pi_{\text{op}} := -(1 - q(b_{\text{op}}))(tf + (1 - t)p(b_{\text{a}})F) - b_{\text{op}} + R, \quad (2.1)$$

$$\pi_{\text{a}} := (1 - q(b_{\text{op}}))(tf + (1 - t)p(b_{\text{a}})(F - r(F)C)) + \eta - \eta^-(1 - q(b_{\text{op}})) - b_{\text{a}} - R. \quad (2.2)$$

The first term of the Operator's payoff in (2.1) represents the expected fines, the second term subtracts the investment incurred to attain the desired service level, and the last term is the total revenue. The expected fines paid by the Operator are composed of two parts which are multiplied by

Table 1: Glossary of symbols used by the game

Decisions:	
f, F	Fines charged when Operator does not perform a service and reports it (f), and is discovered not reporting it (F).
b_{op}	Investment of Operator.
t	Probability that Operator reports a missed service.
b_a	Budget for control of Agency.
Inputs:	
$q(b_{op})$	Function that describes the fraction of services performed by the Operator.
$p(b_a)$	Function that describes the fraction of services checked by the Agency.
R	Total subsidies paid to the Operator corresponding to planned services.
η	Total welfare perceived by society from transportation service.
η^-	Welfare loss incurred by users for a missed service.
C	Cost incurred by agency if operator bankrupts.
$r(F)$	Probability that Operator bankrupts if it needs to pay a fine F .

$(1 - q(b_{op}))$, which represent the probability of not performing a service. The first is for the fraction t of services that the Operator truthfully reports as missed, while the second is for services that the Operator does not declare but the Agency discovers (with probability $p(b_a)$ which represents the effort of the Agency). Notice that we assume that all probabilities are independent so it is enough to multiply the probabilities of different events. The revenue R is assumed constant because the Agency pays subsidies regardless of whether the service took place or not, and users pay fares as well because when a service is missed they wait for the next.

Regarding the Agency's payoff in (2.2), the first term is the amount of fines received by the Agency, adjusted by the probability of bankruptcy as discussed below, the second term is the total user welfare, the third is the reduction of user welfare caused by missed services, the fourth is the cost of performing controls, and the last is the revenue. (Note that in fact π_a is the social welfare of the Agency together with the individuals since R includes the ticket prices paid to the Operator.) The Agency receives the fines paid by the Operator. There is an extra term $r(F)C$ in (2.2) that represents the possibility that the Operator bankrupts when it is imposed a fine F that is too high. In that case, the Agency must restart the process of tendering to find a new private company that will operate the services. This has a huge cost, which we represent by C . The probability this happens is encoded by the function $r(F)$ which is increasing. To disregard unrealistic situations we impose that (i) $F < C$, (ii) $r(C) = 1$, and (iii) $r(F)C \leq F$. These conditions impose fines to be smaller than the cost of finding a new Operator. The constant η is the total user welfare in the system when all services are run according to the contract. This is an additive constant that represents the maximum total surplus (economic value of traveling minus time spent traveling) that society would get from transportation. Since it is fixed, it does not play a role in this model. The important coefficient is η^- , which represents the welfare loss that is caused by every service that the Operator fails to provided. It captures the additional total minutes lost by users that have to wait for the next service, converted to money. This coefficient can also be regarded as a tradeoff between the contribution to payoff arising from the fines versus welfare maximization. When the coefficient is large the Agency values welfare more than the income generated by fines, and vice-versa.

As usual for this type of games, we solve it starting from the end and go to the earlier stages progressively. We work to characterize the subgame perfect equilibria of the game, describing the

resulting utility functions along the way. The third stage is a one shot game where the Operator and the Agency choose t and b_a , respectively. We consider the decisions of the first two stages fixed and take the derivative of both utility functions with respect to the decisions at this stage. It is necessary that both vanish at an interior Nash equilibrium. Hence, at the third stage of the game,

$$\begin{aligned}\partial\pi_{\text{op}}^3/\partial t &= -(1 - q(b_{\text{op}}))(f - Fp(b_a)) = 0, \\ \partial\pi_a^3/\partial b_a &= (1 - q(b_{\text{op}}))(1 - t)p'(b_a)(F - r(F)C) - 1 = 0.\end{aligned}$$

Solving them, we get that the unique Nash equilibrium $(b_a^{\text{NE}}, t^{\text{NE}})$ of the third-stage game is

$$(b_a^{\text{NE}}, t^{\text{NE}}) = \left(p^{-1}\left(\frac{f}{F}\right), 1 - \frac{1}{(1 - q(b_{\text{op}}))p'(p^{-1}(f/F))(F - r(F)C)} \right). \quad (2.3)$$

This game does not admit equilibria at extreme solutions. At equilibrium, the utilities of both players are, respectively,

$$\begin{aligned}\pi_{\text{op}}^3 &= -(1 - q(b_{\text{op}}))f - b_{\text{op}} + R, \\ \pi_a^3 &= (1 - q(b_{\text{op}}))(f - \eta^-) - \frac{f}{Fp'(p^{-1}(\frac{f}{F}))(\frac{F}{r(F)C} - 1)} + \eta - p^{-1}(f/F).\end{aligned}$$

Furthermore, analyzing the best response function of the Operator to the Agency strategy, we see that it is optimal for the Operator to be fully honest ($t = 1$) when $b_a > p^{-1}(f/F)$ and to hide all missed services ($t = 0$) when $b_a < p^{-1}(f/F)$.

In the second stage, the Operator chooses the effort b_{op}^* that maximizes its total payoff π_{op} . Taking the derivative of the payoff anticipating the outcome of the third stage, b_{op}^* satisfies that $q'(b_{\text{op}}^*) = 1/f$, which has a unique solution because $q'(\cdot)$ is a decreasing function. The solution b_{op}^* equals $(q')^{-1}(1/f)$ if $q'(0) \geq 1/f$ and 0 otherwise. Going to the first stage, the Agency can anticipate the outcome of the last two stages of the game and compute the payoff for any choice of fines f and F using

$$\pi_a = (1 - q((q')^{-1}(\frac{1}{f}))) (f - \eta^-) - \frac{f}{Fp'(p^{-1}(\frac{f}{F}))(\frac{F}{r(F)C} - 1)} + \eta - p^{-1}(\frac{f}{F}).$$

With the assumptions on the primitives that we required, the function π_a need not be unimodal so there might be cases in which more than one value of f globally maximizes the utility for the Agency. The possible multiplicity of solutions, though, is not as much of a concern as it would be in a one-shot game because equilibrium selection is not a problem. The Agency can freely use other factors to choose among the different maximizers. The Operator does not need to guess what fines the Agency would impose at equilibrium; that is publicly available in the contract. We summarize the discussion above in the following proposition.

Proposition 1. *The subgame perfect equilibrium of the game between the Operator and the Agency*

as described earlier is given by

$$(f^*, F^*) \in \arg \max_{0 \leq f \leq F \leq C} (1 - q((q')^{-1}(\frac{1}{f}))) (f - \eta^-) - \frac{f}{F p'(p^{-1}(\frac{f}{F})) (\frac{F}{r(F)C} - 1)} - p^{-1}(\frac{f}{F}),$$

$$b_{op}^* = (q')^{-1}(1/f^*),$$

$$b_a^{NE} = p^{-1}(f^*/F^*),$$

$$t^{NE} = 1 - \frac{1}{(1 - q((q')^{-1}(\frac{1}{F^*}))) p'(p^{-1}(\frac{f^*}{F^*})) (F^* - r(F^*)C)}.$$

To specialize the model, we consider simple functions that validate all assumptions we have made earlier, namely, $p(b_a) = k b_a^\alpha$ with $k > 0$ and $0 < \alpha < 1$, $q(b_{op}) = 1 - e^{-\beta b_{op}}$ with $\beta > 0$, and $r(F) = (F/C)^2$. The level of control that we can ensure with a given budget of b_a per service determines the function that captures the probability $p(b_a)$ that a service is checked. By improving the efficiency of controls, the equilibrium of the game changes because given the same budget allocated to the control activities, we have a larger probability of checking a given service. Section 3 provides a procedure to compute $p(\cdot)$ by optimizing how the budget b_a is allocated to performing controls in an efficient way. As we will see then, the monomial approximation of $p(\cdot)$ made earlier is reasonable given the computational studies we shall conduct.

Evaluating the formulas described in Proposition 1, the Agent chooses f and F to maximize

$$\pi_a = \frac{1}{\beta} - \frac{\eta^-}{\beta f} - \left(\frac{f}{kF}\right)^{1/\alpha} \left[1 + \frac{1}{\alpha(\frac{C}{F} - 1)}\right] + \eta.$$

From the previous objective function, we derive first order optimality conditions. Indeed, we need that

$$0 = \frac{\partial \pi_a}{\partial f} = \frac{1}{f} \left\{ \frac{\eta^-}{\beta f} - \frac{1}{\alpha} \left(\frac{f}{kF}\right)^{1/\alpha} \left[1 + \frac{1}{\alpha(\frac{C}{F} - 1)}\right] \right\} \quad (2.4)$$

$$0 = \frac{\partial \pi_a}{\partial F} = \frac{1}{\alpha} \left(\frac{f}{kF}\right)^{1/\alpha} \left\{ \frac{1}{F} + \frac{1}{\alpha(C - F)} - \frac{C}{(C - F)^2} \right\} \quad (2.5)$$

Condition (2.5) requires that F is a root of $(1 - \alpha^{-1})F^2 + (\alpha^{-1} - 3)CF + C^2$. After some algebra,

$$F^* = C \frac{3 - \alpha^{-1} - \sqrt{\alpha^{-2} + 5 - 2\alpha^{-1}}}{2 - 2\alpha^{-1}},$$

which satisfies $C/2 < F^* < C$. Now, solving (2.4) and plugging the results in the formulas of Proposition 1, we complete the solution of the game as

$$f^* = \left\{ \frac{\beta}{\eta^- \alpha (kF^*)^{1/\alpha}} \left[1 + \frac{1}{\alpha(\frac{C}{F^*} - 1)}\right] \right\}^{-\frac{\alpha}{\alpha+1}},$$

$$b_{op}^* = \ln(\beta f^*)/\beta,$$

$$b_a^{NE} = (f^*/(kF^*))^{1/\alpha},$$

$$t^{NE} = 1 - \frac{\beta (f^*/(kF^*))^{1/\alpha}}{\alpha(1 - F^*/C)}.$$

The resulting level of service is $q(b_{\text{op}}^*) = 1 - 1/(\beta f^*)$ and the resulting probability of checking a service is $p(b_{\text{a}}^{\text{NE}}) = f^*/F^*$. From those formulas we verify that the budget spent by the Agency on overseeing the contract, as well as the probability that a given service is checked, increases when the impact of service provision to society increases. Similarly, the level of service at equilibrium and the honesty of the Operator increase with the impact of service provision. To support that, the optimal fine f must get larger with the impact of not having service.

In Section 5, we show how the constants required by this model can capture the details of the situation given by the case study in Bologna. Afterwards, we close the loop and provide an implementation of the contract for the city of Bologna in Section 6.

3 Control Design

In this section, we propose a model to compute a sequence of stops that controllers must visit. That itinerary indicates where and when they should check that services took place as expected. We rely on an integer program that has elements of various combinatorial optimization problems such as covering, routing and prize-collecting problems, as described in Section 1.2.3.

Let us start with the basic modeling elements. We refer to a *line* as the route in the transportation network that buses take. Instead, a *service* is a bus run on a specified line at a specified time of the day, as it appears in the bus schedule. For example, in Bologna, Italy, line 13 that goes from Ponte Savena to Normandia has services every 15 minutes during the day (see Figure 5 in Section 5).

The control procedure is performed by a limited number of *controllers* (typically 3 or 4) who are in the payroll of the Agency. These employees are assigned to the control of buses only during part of their working day and, at that time, they check the compliance of the contract in the street and on the buses. As was already mentioned, among other things the controllers record the punctuality and frequency of services which will be used to estimate the quality-of-service provided by the Operator. The main idea of the control procedure is to locate controllers at bus stops so they can check if a particular service took place, or if they stay longer, they can contrast the empirical frequency of service to the scheduled one. For this approach to work, controllers must have the capability of distinguishing among services and among lines. In other words, controllers must know which bus (identified by, e.g., the licence plate or bus id) they are seeing, and (possibly later) to which service it is associated. When frequency is low, it is easier to tell apart different services and contrast them to the schedule. However, when frequency is high, controllers can just determine frequencies and ignore if it is one particular service or another. In the latter case, it is enough to just count the number of buses seen per line during the specified time slot.

A control procedure can be either static or dynamic, depending on what is specifically observed at a stop. A static control procedure does not incorporate time explicitly; i.e., it does not consider the time when a given stop of the network must be visited. Static controls specify that a given stop must be visited for a given duration of time *at any time during that day*. This is reasonable for lines with high and relatively constant frequency. In contrast, a dynamic control procedure considers time explicitly by choosing at what exact time the controller must be present at a given stop to check on a particular service. This choice is better suited for lines with low frequency, for which being in a stop at the wrong time would be a waste of time. For example, there are some longer-distance services that are only offered during the morning or afternoon rush hour.

For the reasons mentioned above, designing a control procedure for high or low frequency lines

leads to significantly different strategies. We focus on high-frequency lines per request of the Agency because those lines carry many more passengers overall. This motivated us to use a static control procedure that records frequencies and potential problems at bus stops. We do not specifically plan for the time needed by controllers to check if a service visited a stop properly; they just do it whenever is possible. In those cases, they check if the bus stopped at the request of passengers, if the bus approached the curb, if all the passengers had time to get in, etc. For example, one common problem is that bus drivers skip a stop when they are running late. Although the procedure is designed considering high-frequency lines exclusively, controllers are nevertheless asked to record services of low-frequency lines when they see one.

For a given budget for control put forward by the Agency (mainly used to pay controllers), the design goal is to maximize the information collected. We select the locations to be visited by controllers daily, depending on the time window allocated to control. Another design goal consists in constructing different routes every day. The Agency values diversifying the control locations because that provides a more complete picture of the bus operations. To capture this, we define the value of information collected at a specified stop relative to when the stop was last visited. Precisely, a stop may be desirable because many buses go near it but nevertheless should be penalized today if it was visited yesterday. For this reason, we reduce the ‘prize’ associated with recently checked stops and lines to balance the tradeoff between visiting central stops that allow us to collect information very efficiently and visiting less-central stops that are less desirable but where it is more likely that services are skipped. As a result, we tend to cover the entire network over successive days as had been requested to us. Furthermore, this makes it harder for the Operator to guess where controllers are going to be located in any given day. That was another concern of the Agency because if the Operator could learn the locations of the controllers, it could adapt its strategy to do everything as it should when it knows it is being observed. Finally, our method has the flexibility of imposing that some particular lines or stops must be checked, which was another requirement of the Agency when we were discussing the approach. This is used to address specific customer complaints. In the rest of this section we present the general method, in Section 4 we discuss on how to solve the related optimization model, while implementation details for the case study of Bologna are given in Section 5.

3.1 The Model

From the actual transportation network, we extract an underlying oriented graph $G(S \cup \{0\}, A)$. The set of nodes includes the stops S and the Agency’s office, denoted as node 0, where the data collecting activities of the controllers start and end. We associate one or more weights to each stop $s \in S$. They represent the possible durations that a controller may spend at the node. Each stopping time comes with a *prize*, which is earned if a controller visits the stop for exactly that time. The prize captures the value of information that can be collected during that time and basically encodes the number of lines visiting the stop. We associate weights to arcs as well, which represent the travel time needed to traverse them. Note that A only includes existing network connections so G is not a complete graph, as opposed to many routing problems commonly studied in the literature. Given a stop $s \in S$, $\delta^+(s)$ (resp., $\delta^-(s)$) denotes the set of arcs leaving (resp., entering) s . Similarly, given a set of stops $\Sigma \subseteq S$, $\delta^+(\Sigma)$ (resp., $\delta^-(\Sigma)$) denotes the set of arcs connecting a node $s \in \Sigma$ (resp., $s \notin \Sigma$) with a stop $\sigma \notin \Sigma$ (resp., $\sigma \in \Sigma$).

In addition to the general prizes mentioned earlier, which capture the lines that can be observed from a stop, stops have intrinsic values. This is used to model preferences for visiting particular

stops. For example, the Agency is interested in checking the endpoints of a line because it happens relatively more often (e.g., when late) that drivers cut services short, and turn around before getting to the terminal of a line.

The problem has a structure similar to a classical *Prize-Collecting* routing problem. However, in addition to prizes associated to stops, we also consider a prize to be earned each time a different line is checked. This means that visiting two stops of the same line does not provide an additional reward associated with that line. Because of that, our computations are also related to the *Set Covering* problem because a stop ‘covers’ each line visiting the stop.

Our goal is to design as many routes on graph G as available controllers in such a way that the total prize collected at the visited nodes is maximized and the time along routes is bounded by the time dedicated to control. All routes must start and end at the controllers’ office, node $0 \in G$.

Let us now introduce the notation for the sets and parameters involved in the problem described above. We denote the units of measurement by $[\cdot]$ (we represent the units of information collected by the controls as [info]). Later, in Section 5, we provide the concrete sources and calculations we used for each of these elements in the application of the problem to the city of Bologna.

- Let K be the set of controllers, each controller $k \in K$ has an associated maximum working time W_k [min].
- Let L be the set of lines, each line $\ell \in L$ has an associated prize π_ℓ [info], which is earned (at most once) if the line is checked.
- Given the set of stops S , let $S_\ell \subset S$ be the subset of stops visited by line ℓ .
- Let τ_ℓ [min] be the time that a controller must spend at a stop of line ℓ to check the line. Such a τ_ℓ is proportional to the inverse of the frequency of the line.
- Each arc $a \in A$ has an associated travel time w_a [min].
- Let $T_s = \{1, \dots, q_s\}$ be the set of possible durations of time to spend at stop s . For each possibility, τ_s^t [min] denotes the time spent in the stop and π_s^t [info] denotes associated prize that is collected if the controller stays at stop s for at least τ_s^t units of time. The prizes include preferences for particular stops.

For a line ℓ that stops in stop s , there is enough time to check that line for a controller that selects that stop and stays τ_s^t minutes only if $\tau_s^t \geq \tau_\ell$. By distinguishing between line prizes π_ℓ and stop prizes π_s , we obtain a flexible model that allows us to place prizes on lines to be obtained at most once per line, and on stops of the line to be obtained every time a stop of the line is visited.

To implement the problem as a MIP, we associate with each line ℓ a binary variable x_ℓ taking value 1 if line ℓ is checked, and 0 otherwise. We also define binary variables $y_s^{t,k}$ that take value 1 if stop s is checked for time τ_s^t by controller k , and 0 otherwise. We say that a node is visited if one of the associated y variables is set to 1. Finally, integer variables z_a^k denote the number of times controller k travels along arc a . For convenience we summarize the characteristics of the model in Table 2.

Table 2: Glossary of symbols used by the control model

K, k, W_k	Set of controllers, one particular controller, maximum time devoted to control for a controller.
L, ℓ, π_ℓ	Set of bus lines, one particular line, and prize associated with line.
S_ℓ, τ_ℓ	Set of stops of line, minimum time to spend at stop to check line.
w_a	Travel time of arc a .
T_s, τ_s^t, π_s^t	Set of possible times to spend at stop s , and one possible duration of time and prize for the duration of index t and for the stop s .
x_ℓ	Line ℓ is checked.
$y_s^{t,k}$	Stop s is checked for time τ_s^t by controller k .
z_a^k	Number of times controller k travels along arc a .

The Integer Programming (IP) formulation of the problem can be stated as follows:

$$\max \sum_{\ell \in L} \pi_\ell x_\ell + \sum_{k \in K} \sum_{s \in S} \sum_{t \in T_s} \pi_s^t y_s^{t,k} \quad (3.1)$$

$$\text{subject to } \sum_{k \in K} \sum_{s \in S_\ell} \sum_{t \in T_s: \tau_s^t \geq \tau_\ell} y_s^{t,k} \geq x_\ell \quad \ell \in L \quad (3.2)$$

$$\sum_{a \in A} w_a z_a^k + \sum_{s \in S} \sum_{t \in T_s} \tau_s^t y_s^{t,k} \leq W_k \quad k \in K \quad (3.3)$$

$$\sum_{a \in \delta^+(0)} z_a^k \geq 1 \quad k \in K \quad (3.4)$$

$$\sum_{k \in K} \sum_{t \in T_s} y_s^{t,k} \leq 1 \quad s \in S \quad (3.5)$$

$$\sum_{a \in \delta^+(s)} z_a^k = \sum_{a \in \delta^-(s)} z_a^k \geq \sum_{t \in T_s} y_s^{t,k} \quad s \in S, k \in K \quad (3.6)$$

$$\sum_{a \in \delta^+(\Sigma)} z_a^k \geq \sum_{t \in T_s} y_s^{t,k} \quad k \in K, \Sigma \subset S, s \in \Sigma \quad (3.7)$$

$$y_s^{t,k} \in \{0, 1\} \quad s \in S, t \in T, k \in K \quad (3.8)$$

$$x_\ell \in \{0, 1\} \quad \ell \in L \quad (3.9)$$

$$z_a^k \in \mathcal{Z}_+ \quad a \in A, k \in K. \quad (3.10)$$

The objective function (3.1) captures the prize arising from checking lines (first term) and visiting stops (second term). Constraints (3.2) impose that, to earn the line prize, the line must be checked by at least one controller during an interval of sufficient length. Constraints (3.3) impose an upper bound on the working time of each controller. Note that different controllers can have different working times. Constraints (3.4) impose that each controller visits the office during its working day (because time is not explicit, this visit can be thought of as the start of the route). Constraints (3.5) impose that at most one controller visits a stop and at most one duration of time in T_s is selected. Constraints (3.6) guarantee flow conservation and impose that if a node s is visited, at some point a controller must have arrived to it and left from it. Note that a route can go by a node without

stopping to visit it. To guarantee the feasibility of itineraries, we add subtour elimination constraints in (3.7) (note that $0 \notin \Sigma \subset S$ by definition of S). Finally, constraints (3.8) and (3.9) impose that variables are binary, while constraints (3.10) require arc variables z to be (non-negative) integer. The latter constraints allow arcs to be used more than once, which is necessary because the graph is not complete.

3.2 Building Solutions that Exhibit Less Clustering

When building a route, we would not like to visit a set of stops that are very close to each other and serve the same lines because the overall aim is to collect information as complete as possible in terms of checked lines and geographic location. However, model (3.1)–(3.10) naturally produces solutions that visit stops close to each other and are visited by the same line, because this is a very efficient way of spending the available time. To make solutions of this type unfeasible, we impose incompatibility constraints among stops and prevent the selection of stops that are ‘too close’ according to a given distance function. Specifically, we say that two stops are incompatible, if their distance is lower than 1 km, or they are connected by an arc (in A) with travel time lower than 10 minutes, and are served by the same lines. To avoid a pair of incompatible stops, say s_1 and s_2 , being *checked* during the same day, we can trivially impose the constraints

$$\sum_{k \in K} \sum_{t \in T_{s_1}} y_{s_1}^{t,k} + \sum_{k \in K} \sum_{t \in T_{s_2}} y_{s_2}^{t,k} \leq 1. \quad (3.11)$$

However, it is well-known that incompatibility constraints expressed by constraints of the form of (3.11) lead to weak formulations because their continuous relaxations are loose.

A much more effective way to exclude tours that are highly clustered is by defining an incompatibility graph $I = (S, E)$ whose edges connect pairs of stops that are incompatible (i.e., close and served by the same lines). Then, we compute a cover of maximal cliques of I and include a clique constraint of the form

$$\sum_{k \in K} \sum_{t \in T_s} y_s^{t,k} \leq 1 \quad s \in C, \quad (3.12)$$

for each clique $C \subseteq S$ in the cover. The clique cover of I is computed through a greedy heuristic procedure that iteratively constructs a maximal clique (see, e.g., Malaguti et al., 2011).

3.3 Additional Constraints

In some situations, mainly where there are complaints, the Agency might require to control a specified set of lines, or a specified number of stops (usually, terminals where bus lines end). These requirements can be translated into constraints. Let us consider the following additional sets and parameters.

- Let $\bar{L} \subseteq L$ be the set of lines that must be compulsorily checked.
- Let $\bar{S} \subseteq S$ be the set of stops that must be compulsorily visited. For each stop we can additionally impose a minimum stopping time $\bar{\tau}_s$.

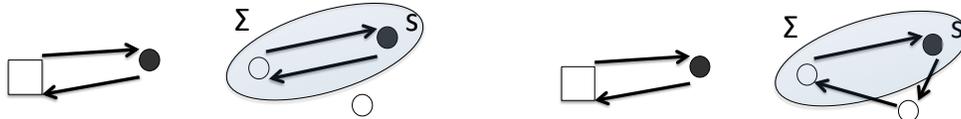


Figure 1: A solution violating a subtour elimination constraint (left) and a close solution not violating the constraint (right)

The possible additional constraints consist of

$$x_\ell = 1 \quad \ell \in \bar{L} \quad (3.13)$$

$$\sum_{k \in K} \sum_{t \in T_s: \tau_s^t \geq \bar{\tau}_s} y_s^{t,k} \geq 1 \quad s \in \bar{S}. \quad (3.14)$$

4 The Algorithm

We refer to the IP model described in Section 3 as the *control design problem*, or CDP in short. In this section we describe the components of an algorithm that solves it. We use a Branch-and-Cut (B&C) algorithm embedded in the IBM ILOG CPLEX MIP solver. The solver takes care of LP relaxations and of the branching tree. Because we cannot include all subtour elimination constraints (3.7) in the model, when a new (fractional or integer) solution is discovered, the control is passed back to our code via *callbacks* to generate valid inequalities. In particular, we generate subtour elimination constraints to iteratively obtain valid formulations.

Since there are exponentially-many subtour elimination constraints (3.7), to solve model CDP we must introduce them as needed. We implement the following separation procedure. Given a fractional solution $(\tilde{x}, \tilde{y}, \tilde{z})$ to CDP, we solve max-flow problems from the office location 0 to s , for every controller $k \in K$ and every stop $s \in S$ using the algorithm by Goldberg and Tarjan (1988). In each problem we set arc capacities to be \tilde{z}_a^k for all arcs, and get a min-cost flow f_s^k and a maximum cut $\Sigma_s^k \ni s$. If we have that

$$\sum_{t \in T_s} \tilde{y}_s^{t,k} > f_s^k,$$

we add the corresponding violated constraint (3.7)

$$\sum_{a \in \delta^+(\Sigma)} z_a^k \geq \sum_{t \in T_s} y_s^{t,k}.$$

After the addition of some violated constraints (details in Section 5), we re-optimize the problem and iterate until a solution that does not violate any of the subtour elimination constraints is obtained. The separation described above can also be used for cutting off integer solutions with subtours.

Instances with up to about 100 nodes can be solved through the B&C solver of CPLEX. For large-size instances like those of the city of Bologna, instead, a textbook implementation of the Branch-and-Cut method highlighted in the previous paragraph is ineffective. In our preliminary experiments the continuous relaxation of CDP provided good bounds, but the solver had difficulties

to produce feasible solutions without ad-hoc procedures and heuristics. This is because when we find a subtour and add the corresponding subtour elimination constraint (3.7), the solver easily finds a similar solution by re-routing on some node having all the associated y variables at 0. This situation is depicted in Figure 1, where solid circles denote stops associated with positive y variables, empty circles denote stops associated with y variables equal to 0, and the square is node 0 (office). On the left, we depict a subtour involving node s and set Σ . On the right, we show a similar solution that does not violate the subtour elimination constraint associated with s and Σ .

For this reason, the implementation of a heuristic that returns good solutions has proven to be a key factor to solve this problem successfully. This heuristic, presented below in Section 4.1, is based on selecting a subset of nodes and reducing the original sparse graph to a smaller but complete graph, where we can solve a slightly different version of model CDP, so as to produce good feasible solutions.

4.1 Primal Heuristics and Suboptimal Solutions

We use a primal heuristic algorithm based on an integration of LNS (see, Shaw, 2008; Pisinger and Ropke, 2010) and rounding to produce feasible solutions *during* the execution of the B&C algorithm. The idea is to use information from the current infeasible solution to define a smaller problem on a complete network. We solve the small problem with the CPLEX MIP solver for a limited amount of time, possibly obtaining integer solutions of good quality, which are then mapped back to the original network. This algorithm, which we refer to as *MIP-heuristic*, is executed with a specified frequency at the nodes of the B&C tree.

To run the MIP-heuristic, we select a subset $\tilde{S} \subset S$ of nodes, and define a complete graph \tilde{G} by computing the shortest path in G between any pair of nodes in \tilde{S} . (Note that we pre-compute all shortest paths off-line.) Having defined \tilde{G} , we solve model CDP on that graph using B&C but with the subtle difference that constraints (3.6) are set to equality. We can do this because \tilde{G} is a complete graph, and thus it is not necessary to allow a path to traverse a node without visiting it since there is always the option to go straight (triangle inequalities are satisfied since distances are given by shortest paths in the original graph). Another simplification is that variables z_a^k are binary, since again we can always go from any node to any other node directly. These simplifications improve efficiency because they prevent the solver from producing similar infeasible solutions after adding subtour elimination constraints, in contrast to what could happen when working with sparse graphs. Finally, as soon as we become aware of a feasible solution, we can use its value, say z_G^* , as a cut-off for model CDP when solved on \tilde{G} .

In general, CPLEX managed to produce solutions to the reduced problems effectively. These solutions are then mapped back to the original network G . This process generates feasible solutions to G which can be used to prune the B&C tree for the original problem. Similarly, the best feasible solution to the problem on G found at any stage can be used to prune the B&C tree used to compute the solution to the reduced problem, thus accelerating the computation.

To select the nodes \tilde{S} to include in the small graph, we use the information associated with the fractional and integer solutions explored by the B&C tree since the last time the heuristic was called. One possibility is to include nodes corresponding to an integer solution containing cycles. A second possibility is to include nodes corresponding to a fractional acyclic solution. A third possibility, which combines the previous two, is choosing the solution with the smallest number of fractional variables generated since the last call. According to our computational experience, we did not find that one strategy uniformly dominated other, thus we keep the simplest one which

algorithm *MIP-heuristic*

input: CDP model data, fractional solution $(\tilde{x}, \tilde{y}, \tilde{z})$, best feasible solution value z_G^*

output: integer feasible solution

initialization:

select a subset of nodes $\tilde{S} = \{s \in S : \tilde{y}_s^{t,k} \geq \eta \text{ for some } t, k\}$

define \tilde{G} by computing shortest paths among all nodes in \tilde{S}

set constraints (3.6) to equality in CDP

set a cut-off value of z_G^*

algorithm:

solve model CDP on \tilde{G} with a specified time limit

return the best feasible solution found

end

Figure 2: Summary of the MIP-heuristic algorithm

is a fractional rounding method. We select the nodes s such that in the last generated fractional solution at least one of the associated variables $y_s^{t,k}$ has value larger than a threshold η . Algorithm MIP-heuristic is given in Figure 2.

We are aware of other heuristic algorithms for solving routing problems based on the solution of a restricted instance of the problem. For example, De Franceschi et al. (2006) proposed a refinement method for VRP-related problems, where a feasible solution is improved by removing some nodes and sequences of nodes, which are then re-inserted in the solution in an optimal way, by solving an optimization problem modeled as a MIP. More in general, restricting the solution space using local information on the current solution is at the core of two methods for general MIPs that are very effective: namely, Local Branching (Fischetti and Lodi (2003)) and Relaxation Induced Neighborhood Search (Danna et al. (2005)). However, in those methods the idea is to improve a given *feasible* solution, while our method is actually designed to produce a good solution starting from a currently fractional, thus infeasible, one. In this spirit, our method is also inspired by the Feasibility Pump method (Fischetti et al. (2005)), where the idea is to start from a fractional solution, and then solve a related problem to find a feasible one close to it (with respect to some metric). More generally, our algorithm can be seen as a rather sophisticated primal heuristic that repairs a solution of the continuous relaxation of the model at hand.

In addition to solutions generated by the MIP-heuristic, we obtain other feasible solutions to CDP with the general-purpose primal heuristics implemented within CPLEX. However, these solutions, which are computed for the original graph G , may include subtours and detours so they are not really feasible. A subtour may occur if the solution includes a tour that does not visit any nodes; i.e., if it only involves arc variables z 's without a corresponding y variable set to 1. This is because there is no violated subtour elimination constraint when all the corresponding y variables are at 0. In a detour, two subsequent nodes that are visited are connected by a path that is not shortest. A solution containing a detour is depicted in Figure 3 (left), where the full circles represent visited stops. Although subtours and detours cannot be present in optimal solutions to problem CDP (and cannot appear in feasible solutions when constraints (3.6) are set to equality), they consistently appear in intermediate nodes of the B&C tree.

To remove subtours and detours from feasible solutions, we use an iterative algorithm that

enumerates all possible paths that are contained in the current integral solution and selects the shortest one that traverses all nodes visited in the solution and comes back to the office. On the right of Figure 3, we show the resulting solution after the removal of the detour, while the iterative algorithm is described in Figure 4.



Figure 3: Solution with a detour (left) and after its removal (right)

5 Case Study: Definition of the Control Methodology and the Contract for Bologna, Italy

In this section, we tune the details of the control model and the contract for the city of Bologna, Italy. We defined these details in close cooperation with the Agency that oversees public transportation there. We start with the control procedure and then use the output of the corresponding computational experiments to define the contract.

5.1 Setting up the data

Figure 5 shows a diagram with the network of buses in Bologna. Following the wish of the Agency, we focused on high frequency lines and thus considered the 29 urban lines with an average time between two consecutive services of at most 30 minutes. These lines serve 1,104 stops with 6,000 services per day, which accounts to the 81.9% of the passengers served by the Operator in Bologna.

We derive the graph $G = (S \cup \{0\}, A)$ from the real network. This graph contains two kinds of arcs: (i) bus connections and (ii) pedestrian connections. For bus connections, we connect stops along a bus line by defining an arc $a = (i, j)$ for each pair of two subsequent stops i and j . The travel time w_a of an arc a served by more than one bus line is set to the maximum travel time. This is a conservative choice because a faster bus may not be the first to come when the controller needs it. Figure 6 shows all bus connections among stops as black lines. We use pedestrian connections to connect all pairs of stops that are not already connected and that are at walking distance. We set the maximum distance to be 10 minutes. We show the full network corresponding to Bologna in Figure 6, where grey lines show pedestrian connections.

The values of the model parameters were set following the recommendations of the Agency. (We refer the reader to Agresti (2007) for details.) For each of the 1,104 stops, we consider that controllers can spend either 15, 20 or 30 minutes in the stop counting services.

- The score of a line p_ℓ to be collected potentially multiple times when observing a stop is defined as the sum of three scores $w_1^\ell + w_2^\ell + w_3^\ell$. The score w_1^ℓ is the main component,

algorithm *subtour-detour removal*

input: CDP model data, integer feasible solution $(\bar{x}, \bar{y}, \bar{z})$

output: k cycles without subtours or detours, each one crossing the same nodes visited by the input solution

algorithm:

for each controller k ,
 $T(i, j) = 0$ for all $(i, j) \in A$
 $path = \{0\}$, $last = 0$
 explore($path$, $last$, k)
 return best k -th path

endfor

end

procedure *explore*

input: current path $path$, last explored node $last$, controller k

procedure:

if ($last = 0$ and $|path| > 1$)
 if (path crossed all nodes visited by k)
 update best k -th path
 return
 for each $j \in \delta^+(last)$, $T(last, j) < \bar{z}_{last, j}^k$
 $path = path \cup \{j\}$, $last = j$, $T(last, j) = T(last, j) + 1$
 if ($path$ longer than best path)
 return
 explore ($path$, $last$, k)
 return
endfor

Figure 4: Summary of the algorithm used to remove subtours and detours

and depends on the number of passengers per year in the line. This captures the impact on commuters when services are missed or delayed. Table 3 reports scores w_1^ℓ associated with each line, which was constructed from the categorization used internally by the Agency. For instance, the Agency follows this categorization when constructing statistics and evaluating bus performance. Scores w_1^ℓ provide levers to prioritize the control tasks. The score w_2^ℓ is a corrective term proportional to the traveled kilometers per year for that line. We use it because lines covering more distance usually have a larger probability of incurring in a delay or a cancelation. The score w_3^ℓ is a corrective term proportional to the inverse of the line's number of services per year. We use it in the opposite direction compared to the previous measures. Indeed, checking a particular service is less important when the frequency of a route is large, actually, a delay or missing service for a high frequency line would have a minor impact on the service quality perceived by customers. Although one could also say that having more services makes a line more important to be checked, we have already partially

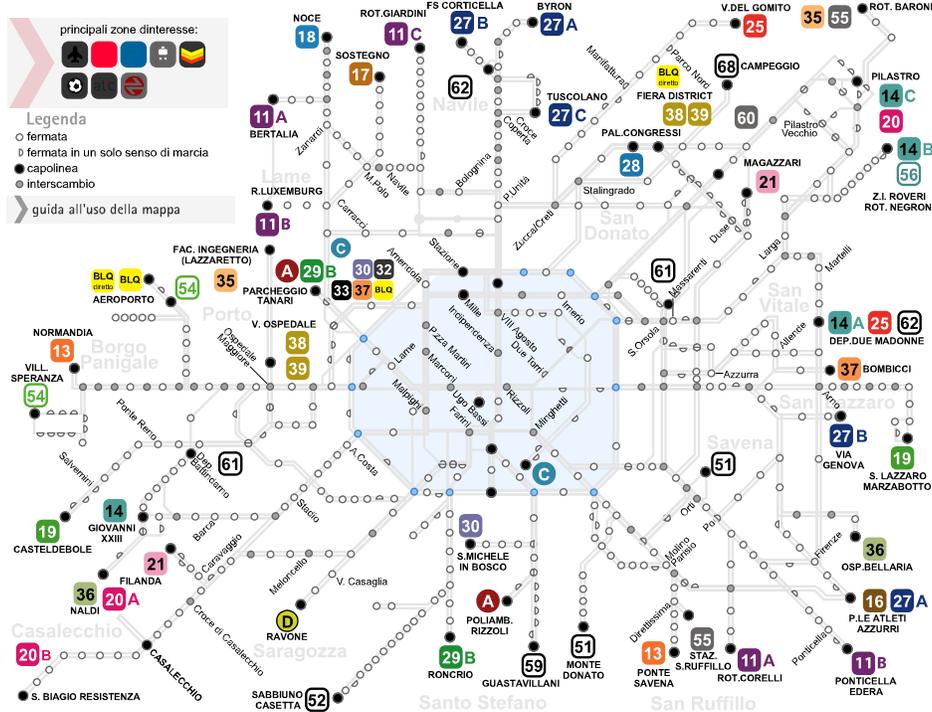


Figure 5: Bus network in Bologna

captured this effect in the other two scores.

- The prize π_s^t associated with a stop s where the controller stays for τ_s^t minutes is defined as the sum of the prizes p_ℓ associated with the lines that can be observed during that time. Concretely, $\pi_s^t := \sum_{\ell: \tau_\ell > \tau_s^t} p_\ell$.
- Finally, the prize π_ℓ , which is earned at most once daily, is set as a function of the passengers the line serves. Table 3 indicates the values, which again, describe the internal ranking used by the Agency.

To evaluate the solution returned by the model, we use a greedy algorithm that generates solu-

Table 3: Scores and prizes associated to bus lines

passengers per year	w_1	π_ℓ
< 20,000	1	15
20,001 - 100,000	2	30
100,001 - 500,001	3	45
500,001 - 1,000,000	4	60
1,000,001 - 5,000,000	5	75
> 5,000,000	6	90

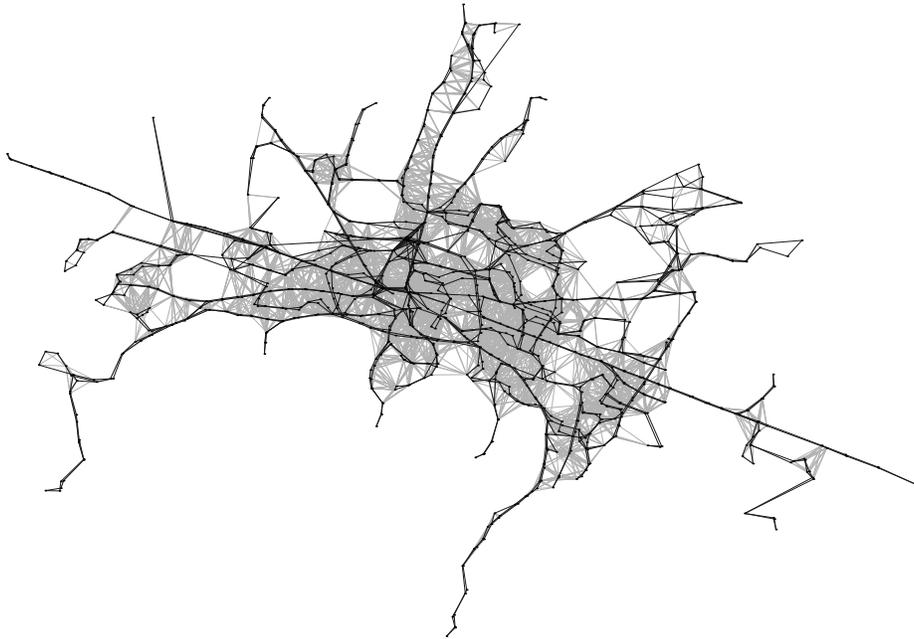


Figure 6: Graph of bus connections (black) and overall graph (grey)

tions that resemble the current behavior of the Agency’s controllers. For each controller available, the algorithm initially selects a random node in the network to start the route. Then, it selects the next node to visit among those that can be directly reached from the current one and are neither incompatible with some previously visited node nor already visited by the route under construction or by a previously constructed route. The node and the stopping time that maximize the ratio between the time needed to visit the node (including the travel time from the current node) and the earned prize is selected as next node to visit and the corresponding stopping time is set. If all neighbor nodes are already visited (by the current or previously constructed routes), the algorithm moves to the closest node that was not traversed by the current route (choosing an already traversed node would make the algorithm cycle). If no such a node exists, the algorithm closes the route, directing the controller back to the office using a shortest path. The same happens if visiting the next node and traveling back to the office requires the controller to exceed the allocated time to perform controls. Figure 7 provides a sketch of the greedy algorithm we implemented.

We run the greedy route-generation procedure 30 times, and select the best solution as a reference point to evaluate our algorithm. This is done to capture that controllers know the city well and use judgment in selecting their routes, without necessarily taking a holistic approach to construct the whole route at the same time.

5.2 Computational experiments

All algorithms were coded in C and tested on a single core of a Linux-based PC equipped with an i5-650 processor at 3.20GHz and 8GB of memory. The algorithm is implemented under the IBM ILOG CPLEX MIP solver platform, version 12.2. We implemented user-defined cuts and other

algorithm *Greedy*
input: CDP model data
output: feasible routing
initialization: mark all nodes as non-visited
algorithm:
 for each controller k
 $W = W_k$, mark all nodes as non-traversed
 randomly select a non-visited node s
 visit s for the stopping time t maximizing ratio π_s^t/τ_t , update $W = W - \tau_t$
 do
 for each node $j, (s, j) \in A$, j non-visited and compatible with a visited node,
 and each possible stopping time $t \in T_j$ such that $W - w_{sj} - \tau_t - w_{j0} \geq 0$,
 compute the ratio $r_j^t = \pi_j^t/(w_{sj} + \tau_t)$
 endfor
 choose node j and stopping time $t \in T_j$ maximizing ratio r_j^t , mark j as visited,
 update $W = W - w_{sj} - \tau_t$, move on j by setting $s = j$
 if no move was done, choose to the closest non-traversed node j
 such that $W - w_{sj} - w_{j0} \geq 0$, mark j as traversed,
 update $W = W - w_{sj}$, move on j by setting $s = j$
 while a move is done
 close the cycle by going back to node 0
 endfor
end

Figure 7: Summary of the *Greedy* algorithm

customizations using callbacks to enhance the standard solution procedures.

We tuned our solution algorithm and the parameters of the solver engine through extensive computational experiments. This required a careful calibration of the frequency of invoking the cut-generation routine. We had to balance the trade-off between obtaining strong bounds, which would suggest generating many cuts, and not overwhelming the solver with too many cuts, which would slow down the solution of LP relaxations. According to our experiments, we obtained good results from the following methodology.

- We call the cut-generating procedure only for separating *integer* solutions, and we generate at most 5 cuts per controller.
- We run the primal heuristic of Section 4.1 on every 100 nodes of the B&C with a time limit of 200 seconds. In that case, because graphs are small and complete, we call the cut generating procedure every 100 nodes of B&C, in addition to separating all integer solutions.
- We give the overall algorithm a time limit of 2 hours.

In Table 4, we report computational results for a variety of cases. We considered six different scenarios with 2, 3 and 4 controllers working for 3 and 6 hours. For every scenario, we report the optimality gap of the the greedy algorithm described in Section 5.1 with respect to the best lower

Table 4: Computational study of our algorithm to solve the CDP Problem

controllers	working hours	greedy gap %	optimality gap %	nodes tree	cuts tree	gap with no MIP-heur. %	gap MIP-heur. with random selection %
2	3	19.54	3.66	20,600	6,222	94.54	22.47
3	3	22.98	3.67	6,100	2,702	97.36	25.82
4	3	22.54	4.42	3,500	1,370	98.24	27.21
2	6	19.46	2.22	6,700	5,398	98.69	26.27
3	6	21.69	2.83	3,870	2,756	98.93	32.61
4	6	20.96	2.39	2,900	2,093	99.11	35.77

bound, the optimality gap, the number of nodes explored in the B&C tree, and the number of cuts added to the tree by the separation procedure. These two values regarding the tree are for the main problem; they do not include the B&C trees generated when solving the heuristic.

The optimality gaps for all scenarios are very low. Although in the time allocated we could not achieve optimality, for the purposes of the use of the controls by the Agency gaps lower than 5% are considered more than sufficient for the Agency. In any case, in Section 6 we compare solutions according to the objective introduced by the game presented in the first part of the paper. Notice that the greedy procedure used as a reference point achieves gaps that are many times larger, validating the use of our model. The size of the tree explored goes down with the complexity of the instance because linear relaxations become harder so there is less time to explore nodes within the time allocated.

Figure 8 considers the case of 2 controllers and shows how the quality of the best feasible solution and the best upper bound evolve for increasing running times up to 2 hours. Figures corresponding to other parameter choices are similar. Our study provides evidence that running times of one hour seem to be enough to determine good solutions that are acceptable by the Agency for its planning activities. A second hour of computation brings only marginal improvements. These results indicate that proving optimality may take a long time since the upper bounds do not seem to improve much after the initial two hours of computation.

To show the importance of the MIP-heuristic, Table 4 also reports the optimality gaps with the heuristic disabled. It can be seen that gaps are always larger than 94%. This happens because the solver cannot produce feasible solutions of good quality. Finally, to validate the choice of nodes made by the MIP-heuristic, we compared our solutions to using a random selection of 100 nodes to be compared to the strategy described in Section 4.1. The last column of Table 4 contains the corresponding optimality gap. It clearly shows that a strategy of choosing 100 nodes randomly does not work, providing evidence that the fractional solution returned by the LP solver contains crucial information that can be exploited by the heuristic when constructing the smaller graph \tilde{G} .

5.3 Dynamics: Generation of Schedules over Successive Days

As mentioned before, the controllers daily schedule should vary so as to collect different information at different times. To address this, one possibility would have been to run our model including additional controllers, representing the routes to be followed in the successive days. Although the model becomes much larger, it could be solved by giving the algorithm more CPU time. In that

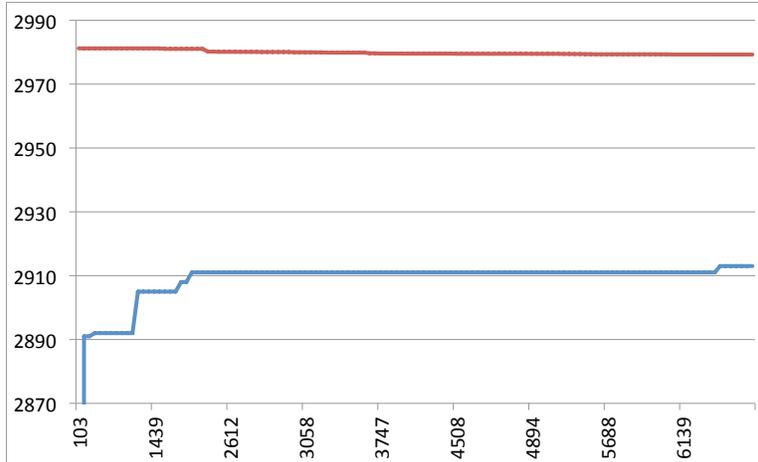


Figure 8: Profile of computational experiment. The line in the bottom represents the evolution over time (in seconds) of the value of the incumbent solution, and line on top represents the evolution of the value of the best upper bound.

case, different routes for different days are subject to the constraints of the model. The Agency opted for adopting soft constraints by penalizing the objective. Hence, we discount the value of the information observable at the recently-checked stops. Prizes associated with lines and stops are modified as follows: $\pi_s^{int} = \pi_s(1 - (1 - int)^{-1})$, where int is the interval of time (in days) since the last visit to the stop. A similar idea is implemented to discount the value of information associated with lines checked in the previous days, by modifying the prizes of lines π_ℓ in the same fashion.

Figure 9 shows the output of the algorithm for 2 consecutive working days in the case of 2 controllers and 6 hours per day. The grey hexagons in the figures represent the ancient walls around the perimeter of the city (currently, one of the main traffic avenues) and the main traffic direction outside the walls. The two cycles represent the routes followed by each controller and the black dots show where controllers stop to check bus services. The size of these dots is proportional to the stopping time. Because controllers follow different schedules daily, in a medium-term period they cover most of the network.

As explained earlier, model CDP allows the Agency to specify a set of stops or lines that will be checked for sure. This is very important in practice because the Agency wishes to integrate the day-by-day optimized control with the specific control of some stops, like some terminals or specific lines, based on customer complaints or other needs of the Agency. Concerning the constraints added to reduce clustering in the solutions, they are active in the solutions depicted in Figure 9. As soon as these constraints are removed, the model would tend to select clusters of stops close to each other.

Finally, although we designed the model focusing on high-frequency lines, in practice controllers also check low-frequency lines that can be seen from their control locations.

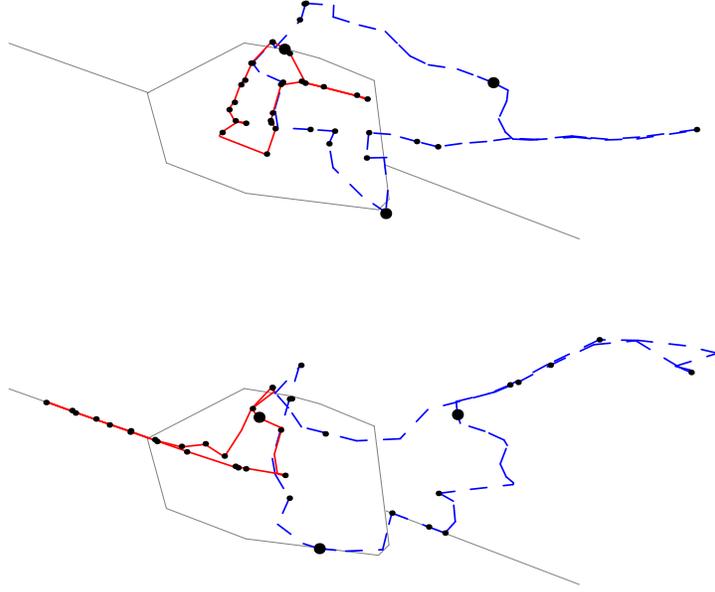


Figure 9: Pictures showing each of 2 consecutive working days for 2 controllers (solid and dashed lines, respectively). The thin lines represent the city ancient walls.

6 Closing the loop: Implementation of the Optimal Contract

This section closes the loop and goes back to the contract design problem presented in the first part of this article. We discuss how to implement the contract, in light of the optimized control procedure presented in the second part. This completes the case study by providing an illustration of how the game of Section 2 can be used to find fines that are appropriate for the contract. In the current implementation of the contract, the required level of service (intended as number of services to be performed over the total scheduled) is fixed at 99.4%. For every 0.01% under this threshold the Operator receives a fine of €150, but the fine for a non-declared canceled service ranges between €5,000 and €200,000 (depending on the reason of the failure to declare the missed service). However, as anticipated in the introduction, these values result from political negotiations and a long-term relationship, rather than a quantitative approach. Because service level in recent times has been very good, fines are rarely imposed. However, it is not certain that this level of service will be maintained in the future, specially if a different Operator were to be selected in the next auction.

We now estimate the optimal fines arising from the game, using the optimal control procedure. We start by estimating the function $p(b_a)$ that links both parts of the methodology because the probability a service is checked depends on the control procedure. The argument is the investment made by the Agency, which we express in Euros per service; that is, b_a is the money destined to pay controllers, prorated to each planned service. We take this measurement unit because it is independent of the duration of the contract. To estimate $p(b_a)$, we compute the percentage of checked services for various budget levels b_a used for controls. This can be estimated using the objective function (3.1), after setting all rewards associated with lines π_ℓ to zero. Indeed, the

Table 5: Evaluation of the solution to CDP with respect to the percentage of services checked.

controllers	working hours	services checked	% checked	% checked by greedy	relative difference %
2	3	108	1.80	1.22	47.54
3	3	146	2.43	1.27	91.33
4	3	181	3.02	1.82	65.93
2	6	185	3.08	2.27	35.68
3	6	242	4.03	3.10	30.00
4	6	294	4.90	3.62	35.35

remaining part of the objective can be used to count the services when π_s^t measures how many of them can be observed by staying at stop s for τ_s^t minutes.

Relying on the computational study, Table 5 provides a comparison between the number of services checked under the greedy solution of Section 5.1, which is a good representation of what happens currently in practice, and the solution provided by solving model CDP. Interestingly, this comparison was the initial study that the Agency in Bologna requested, so this was the starting point for the rest of the methodology developed in this study. The table reports, for each scenario (number of controllers working for number of hours), the number of services checked by the solution to CDP, the corresponding percentage over the total number of services daily, the percentage of services checked by the greedy solution, and relative difference measured as the percentage improvement of CDP over greedy. Consistent with the conclusions drawn earlier, the solution of CDP is consistently and significantly better than the greedy heuristic.

Computing the total number of hours needed for each scenario (this is the main component of the cost of performing controls), we can readily estimate the function $p(\cdot)$. In Figure 10, we add more computations to those in the table and show these curves for 2, 3 and 4 controllers. From the figure, we observe that the main driver of the probability of checking a service is the total number of working hours and not so much how these hours are allocated among controllers. This is because the central office is in Bologna so fixed costs are low and the first and last trip of a controller are not significantly more time-consuming than the other trips. Putting it all together, we need to find the coefficients of the function $p(b_a) = kb_a^\alpha$ used in Section 2 empirically with our computational study. Converting the variable ‘working hours’ to ‘Euros per service’ using the wages paid to controllers and number of services per day, regression analysis tells us that the best fit is provided by the function $p(b_a) = 0.0733b_a^{0.7236}$ with an R^2 of 0.9985.

To find the function that captures the probability that the Operator performs services, we must find the constant β . We merely estimated the cost of labor (driver) per service taking into account the hourly rate and we added the cost of equipment (bus) prorated to the number of services that it typically lasts before it is replaced. With this basic cost of service provision that does not include any extra drivers or extra equipment, we estimate that the probability to provide a particular service is of the order of 90% since the driver may not show up or the bus may not work. Solving for β , we get that $q(b_{op}) = 1 - e^{-0.09b_{op}}$. Finally, we set $r(F) = (F/C)^2$ as explained earlier.

Having estimated the necessary functions, we need to fit the other constants used as inputs to the model. The values R and η are additive constants so they are irrelevant in the outcome of the model, hence we do not need to specify them. The coefficient η^- that quantifies the impact in welfare for each missed service is estimated using the average people waiting in a stop multiplied by

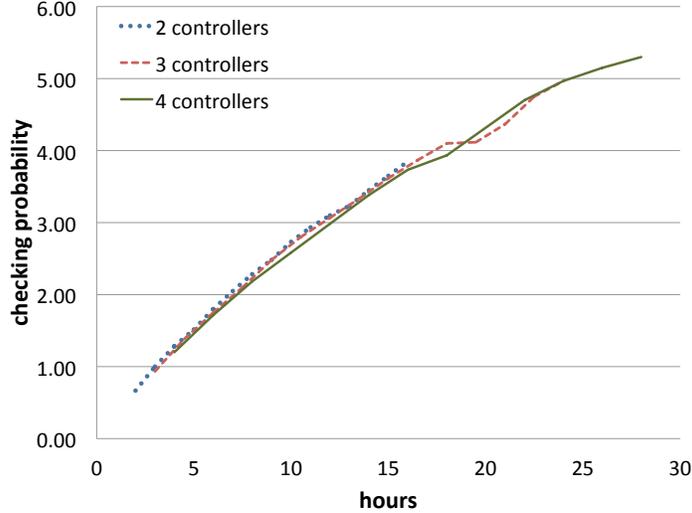


Figure 10: Probability of checking a service as a function of total working hours. The curves correspond to 2, 3 and 4 controllers.

the average number of stops in a line and the average inter-arrival time between services, which is then converted to money using the average value-of-time of the population. It is reasonable to set them to, respectively, 1.3 people per stop, 15 stops per service, 6 services per hour, and 7€/hour. Putting all together, $\eta^- = 22.75\text{€/service}$. Finally, we set C to 200,000€.

Plugging in all values in the formulas derived in Section 2, we get that fines are $F^* = 109,464\text{€}$ and $f^* = 1104\text{€}$. At equilibrium, the Operator improves the quality of service $q(b_{op})$ from 90% to 99% and hence only misses approximately 1% of the planned services. It achieves that by investing around twice the minimum operating cost specified above. The Agency invests around 0.06€/service to check that services run properly. At that investment level, the optimized controls can check approximately $p(b_a) = 1.01\%$ of the services. As shown in Figure 10, to be able to control that percentage of services, controllers need to work for approximately 2.7 hours daily. Out of the few services that are not performed, the Operator truthfully reports $t = 98.29\%$ of them; the remaining 1.71% are not reported because the probability that the Agency finds out is small.

This methodology we have developed will constitute be the basis of the contract design for the upcoming auction to be held in 2016.

7 Conclusions

We have conducted a case study in the management of public-private partnerships in bus transportation. Here, the two parties consist of: (a) the Agency who designs the contract and oversees the service, and (b) the Operator who provides the bus service. There is a strong informative asymmetry between them, whereby the Agency does not necessarily know if the Operator provides all services as scheduled by the contract. We have addressed this asymmetry by providing an incentive to the Operator to report the quality of service to the Agency truthfully. In addition, we have proposed that the Agency controls the validity of the information reported by the Operator using the solution of a prize collecting routing model. The solution provides itineraries of locations where

controllers will go to count bus services and compare them with those reported by the Operator. In the near future, the current contract will expire and the city of Bologna plans to sign a new one with an Operator. This analysis can help guide the design of some details of that contract.

Regarding the modeling methodology, we have made some assumptions about the information that is revealed by various events. When the Operator reports that a service took place, the Agency may or may not be sure about it, depending whether a controller witnessed that particular service. When the Operator reports that a service did not take place, the Agency does not put this in doubt and charges it a fine. When a controller witnesses that the count of bus services is smaller than expected, the Operator receives a fine. The assumption of perfect information for positive detection of missed bus services implies that imposing higher fines F encourages the operator to be more honest because the threat of being fined makes being dishonest too risky. Nevertheless this is compensated by the fact that large fines are unlikely to be paid because the Operator may bankrupt. Hence, in reality higher fines make the threat less credible because both parties know that the Operator is less likely to afford it.

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References

- AGN (2005). *Informe de Auditora — Actuación AGN Número 127/05*. Auditoría General de la Nación Argentina, Gerencia de Entes Reguladores y Privatizaciones. http://www.agn.gov.ar/informes/informesPDF2006/2006_078.pdf.
- Agresti, S. (2007). *Il Rapporto Contrattuale Inerente il Controllo sui Servizi di Trasporto Pubblico Locale (TLP)*. Tesi di Laurea Specialistica in Ingegneria Informatica. Università Degli Studi di Bologna.
- Borndörfer, R., J. Buwaya, G. Sagnol, and E. Swarat (2013). Optimizing toll enforcement in transportation networks: a game-theoretic approach. In *INOC2013 (International Network Optimization Conference)*, Volume 41 of *Electronic Notes in Discrete Mathematics*, pp. 253–260. Elsevier.
- Caprara, A., L. Kroon, and P. Toth (2010). Optimization problems in passenger railway systems. In J. J. Cochran, L. A. Cox, P. Keskinocak, J. Kharoufeh, and J. C. Smith (Eds.), *Wiley Encyclopedia of Operations Research and Management Science*. John Wiley & Sons, Inc.

- Chassang, S. (2010). Building routines: Learning, cooperation, and the dynamics of incomplete relational contracts. *American Economic Review* 100(1), 448–465.
- CNRT (2007). *III Carta Compromiso con el Ciudadano*. Argentinean National Agency of Transport Regulation.
- Consortio Regional de Transportes (2012). <http://www.ctm-madrid.es>.
- Danna, E., E. Rothberg, and C. Le Pape (2005). Exploring relaxation induced neighborhoods to improve mip solutions. *Mathematical Programming* 102, 71–90.
- De Franceschi, R., M. Fischetti, and P. Toth (2006). A new ilp-based refinement heuristic for vehicle routing problems. *Mathematical Programming* 105, 471–499.
- European Parliament (2007). *Regulation (EC) No 1370/2007 of the European Parliament and of the Council of 23 October 2007 on public passenger transport services by rail and by road and repealing Council Regulations (EEC) Nos 1191/69 and 1107/70*. European Parliament.
- Fischetti, M., F. Glover, and A. Lodi (2005). The feasibility pump. *Mathematical Programming* 104, 91–104.
- Fischetti, M., J. J. S. González, and P. Toth (1998). Solving the orienteering problem through branch-and-cut. *INFORMS Journal on Computing* 10, 133–148.
- Fischetti, M., J. S. González, and P. Toth (1997). A branch-and-cut algorithm for the symmetric generalized traveling salesman problem. *Operations Research* 45, pp. 378–394.
- Fischetti, M. and A. Lodi (2003). Local branching. *Mathematical Programming* 98, 23–47.
- Gagnepain, P. and M. Ivaldi (2002). Incentive regulatory policies: The case of public transit systems in france. *RAND Journal of Economics* 33(4), 605–629.
- Goldberg, A. V. and R. E. Tarjan (1988). A new approach to the maximum-flow problem. *Journal of the ACM* 35, 921–940.
- Green, E. J. and R. H. Porter (1984). Noncooperative collusion under imperfect price information. *Econometrica* 52(1), 87–100.
- Grossman, S. J. and O. D. Hart (1983). An analysis of the principal-agent problem. *Econometrica* 51(1), 7–45.
- Hamburger Verkehrsverbund (2012). <http://www.hvv.de>.
- Hansson, L. (2010). Solving procurement problems in public transport: Examining multi-principal roles in relation to effective control mechanisms. *Research in Transportation Economics* 29(1), 124–132.
- Hensher, D. and J. Stanley (2003). Performance-based quality contracts in bus service provision. *Transportation Research Part A* 37, 519–538.
- Hensher, D. and J. Stanley (2008). Transacting under a performance-based contract: the role of negotiation and competitive tendering. *Transportation Research Part A* 42, 1143–1151.

- Hensher, D., P. Stopher, and P. Bullock (2003). Service quality - developing a service quality index in the provision of commercial bus contracts. *Transportation Research Part A* 37, 499–517.
- Hiermann, G., M. Prandtstetter, A. Rendl, J. Puchinger, and G. Raidl (2013). Metaheuristics for solving a multimodal home-healthcare scheduling problem. *Central European Journal of Operations Research Forthcoming*.
- Jansson, K. and B. Wallin (1991). Deregulation of public transport in Sweden. *Journal of Transport Economics and Policy* 25(1), 97–107.
- Li, C.-L., D. Simchi-Levi, and M. Desrochers (1992). On the distance constrained vehicle routing problem. *Operations Research* 40, pp. 790–799.
- Ljubic, I., R. Weiskircher, U. Pferschy, G. W. Klau, P. Mutzel, and M. Fischetti (2005). An algorithmic framework for the exact solution of the prize-collecting steiner tree problem. *Mathematical Programming* 105, 427–449.
- Malaguti, E., M. Monaci, and P. Toth (2011). An exact approach for the vertex coloring problem. *Discrete Optimization* 8, 174–190.
- Pisinger, D. and S. Ropke (2010). Large neighborhood search. In M. Gendreau and J.-Y. Potvin (Eds.), *Handbook of Metaheuristics*, Volume 146 of *International Series in Operations Research & Management Science*, pp. 399–419. Springer US.
- POLIS (2013). *European Cities and Regions Networking for Innovative Transportation Solutions*. <http://www.polisnetwork.eu>.
- Reti e Mobilità (2012). Agenzia per la mobilità e il trasporto pubblico locale. <http://www.srbologna.it>.
- Rizzi, L. (2013). Simple model of road infrastructure financing: the impact of different road user charges. *Journal of Transport Economic and Policy*.
- Rogerson, W. P. (1985). The first-order approach to principal-agent problems. *Econometrica* 53(6), 1357–1367.
- Shaw, P. (2008). Using constraint programming and local search methods to solve vehicle routing problems. In *CP-98 (Fourth International Conference on Principles and Practice of Constraint Programming)*, Volume 1520 of *LNCS*, pp. 417–431. Springer.
- Société de Transport de Montréal (2012). <http://www.stm.info>.
- Socorro, M. P. and G. de Rus (2010). The effectiveness of the Spanish urban transport contracts in terms of incentives. *Applied Economics Letters* 17(9), 913–916.
- Syndicat Mixte des Transports en Commun de l’Agglomération Grenobloise (2012). <http://www.smtc-grenoble.org>.
- TFL (2009). *London’s Bus Contracting and Tendering Process*. Transport for London. <http://www.tfl.gov.uk/tfl/businessandpartners/buses/tenderresults/lbsl-tendering-and-contracting-feb-09.pdf>.

- Toth, P. and D. Vigo (1997). Heuristic algorithms for the handicapped persons transportation problem. *Transportation Science* 31, 60–71.
- Toth, P. and D. Vigo (2001). An overview of vehicle routing problems. In P. Toth and D. Vigo (Eds.), *The vehicle routing problem*, pp. 1–26. Philadelphia, PA, USA: SIAM.
- Veolia Transport (2012). <http://www.veolia-transport.com>.
- Wikipedia (2013). Central artery/tunnel project. http://en.wikipedia.org/wiki/Big_Dig .
Last accessed: June 15, 2013.
- Yared, P. (2010). A dynamic theory of war and peace. *Journal of Economic Theory* 145, 1921–1950.