The Design of Combinatorial Auctions for Procurement: An Empirical Study of the Chilean School Meals Auction*

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April, 2010

Abstract

In this paper we conduct an empirical investigation of a large-scale combinatorial auction (CA); the Chilean auction for school meals in which the government procures half a billion dollars worth of meal services every year. Our empirical study is motivated by two fundamental aspects in the design of CAs: (1) which packages should bidders be allowed to bid on; and (2) diversifying the supplier base to promote competition. We use bidding data to uncover important aspects of the firms’ cost structure and their strategic behavior both of which are not directly observed by the auctioneer. Based on these estimates we analyze and suggest important improvements to the auction design. Our results indicate that package bidding should be flexible enough so that firms can express their cost synergies due to economies of scale and density. However, we also found evidence that firms can take advantage of this flexibility by discounting package bids for strategic reasons and not driven by cost synergies. Because this behavior can lead to inefficiencies, certain specific combinations could be prohibited in the bidding process. Our results also suggest that market share restrictions and running sequential auctions promote competition in the long-run, without significantly increasing the short-run cost for the government due to unrealized cost synergies. Our results highlight that the simultaneous consideration of the firms’ operational cost structure and their strategic behavior is key to the successful design of a CA. More broadly, our paper is the first to provide an econometric study of a large-scale CA, providing novel and substantive insights regarding bidding behavior in this type of auctions.

Keywords: combinatorial auctions, procurement, auction design, supply chain management, empirical, public sector applications.

*Acknowledgments: We thank Cristián Martínez, Vice-Minister of Education of Chile, for his collaboration and the valuable insights he provided to us throughout this work. In addition, executives at JUNAEB and Jaime Catalán provided valuable data, information, and feedback. We are thankful for the helpful comments received from Gérard Cachon, Estelle Cantillon, Fangruo Chen, Francesco Decarolis, Awi Federguren, Nicolás Figueroa, Ken Hendricks, Ananth Iyer, Elena Katok, Anthony Kwasnica and seminar participants at Columbia, Stanford, Informs, MSOM, Penn State and Wharton Workshop in Empirical Research in OM. We also thank the Social Enterprise Program at Columbia Business School for their financial support.
1 Introduction

Auctions are increasingly becoming a common mechanism for supply chain procurement. Corporations and government use auctions to procure billions of dollars in inputs and services (Rothkopf and Whinston (2007) and Elmaghraby (2000)). Auction mechanisms are also becoming more sophisticated. In particular, combinatorial auctions (CAs), multi-unit auctions in which suppliers can bid for packages of items, have generated much interest in procurement applications.\(^1\) The use of CAs has also been a widely debated topic in other contexts as well, such as the allocation of spectrum by the Federal Communications Commission (FCC). The main advantage of CAs is that they allow suppliers (who we also refer to as firms or bidders) to express cost synergies among units in the bidding process, which often results in a lower procurement cost for the buyer (who we also refer to as the auctioneer).

Practical experience and academic research have shown that the design of an auction may have an important impact on its outcome: good designs can result in large savings, while poor designs may lead to large losses for the auctioneer (Milgrom, 2004). Typically, the performance of an auction is critically determined by the firms’ cost structure and their strategic behavior, both of which are usually not directly observed by the auctioneer. The main objective of this work is to conduct an empirical study of bidding data to uncover important aspects of the firms’ cost structure and their strategic behavior in a large-scale procurement CA. Based on this information we evaluate potential changes to the current auction design. While we study a particular CA – the Chilean auction for school meals in which the government procures half a billion dollars worth of meal services every year – our method and analysis could be used more broadly in other CAs.

In this work we study two fundamental aspects of the design of CAs. While these issues are of general interest in CAs, for concreteness, the following discussion focuses in the context of procuring a service on multiple geographic locations.\(^2\) In this case, suppliers’ cost synergies across units may be generally categorized as: (1) **economies of scale** that depend on the total number of units served and, for example, are generated by volume discounts in purchasing inputs; and by (2) **economies of density** that arise when serving nearby units, for example, due to the use of common logistics infrastructure. In addition, our analysis is focused on single-round sealed-bid first-price CAs, because that is the format used in our application and several CAs in practice have the same format.

We consider two commonly used criteria to evaluate the auction design: (1) **optimality**, that is, mini-

\(^1\)See Elmaghraby and Keskinocak (2003), Hohner et al. (2003), Metty et al. (2005), Caplice and Sheffi (2006), Bichler et al. (2006), Sandholm et al. (2006), and Sandholm (2007) for several recent applications.

\(^2\)For example, the auction we study here as well as auctions for transportation and logistic services, like some of the ones mentioned in the previous footnote, share these characteristics.
mizing the expected total payments to the bidders; and (2) **efficiency**, that is, maximizing social welfare by assigning the units to the set of bidders that achieves the most cost efficient allocation. We consider both objectives when evaluating improvements to the auction design.3

The first design issue we study is how to determine which packages should bidders be allowed to bid on. On one hand, the auctioneer should provide enough flexibility so that bidders can express their synergies in the bids. This mitigates the so called **exposure problem**: if package bidding is not allowed, a firm that exhibits synergies among two units may not bid below the costs for the individual items because of the risk of not getting the package. Indeed, under suitable conditions, allowing package bidding is a necessary condition for the efficiency (Rosenthal and Wang (1996), Rothkopf et al. (1998), Bykowsky et al. (2000)) and optimality (Levin, 1997) of the auction.

However, allowing too much flexibility on the package bids can also hurt the efficiency and optimality of the auction, while complicating the bidding process unnecessarily. Bidders in a CA may have incentives to submit package bids and offer discounts even in the absence of synergies, a phenomenon we refer to as **strategic bundling**. A firm that has an advantage in one unit, say A, may have incentives to submit a package bid with another unit, say B, to leverage its advantage in A to get B. Consequently, unit B may not be allocated to the lowest cost firm if the stand-alone bids are too high to outbid the package bid, resulting in an inefficient outcome. In addition, due to the so called **threshold problem**, package bidding can also result in a more expensive allocation. Two “local bidders” that desire units A and B respectively will be successful in their efforts if the sum of their individual bids is less than the **threshold** created by a package bid from a “global bidder”. As a consequence, a local bidder lowering its bid benefits the other local bidder. Because local bidders do not internalize these benefits, their bids may not be as competitive as they could have been if package bidding was not allowed, resulting in larger expected payments from the auctioneer (see Pekec and Rothkopf (2003) for a more detailed discussion; Chernomaz and Levin (2009) and Maréchal and Morand (2009) provide concrete examples of the threshold problem in equilibrium bidding strategies).

The previous discussion suggests that to minimize the negative impact of strategic bundling, the auctioneer should only allow package bidding among units for which synergies are sufficiently large. Because the precise cost structure of firms is often unknown to the auctioneer, we seek to estimate the magnitude of economies of scale and density using bidding data. From a design perspective, for example, if economies of density are predominant, it may be convenient to allow package bids only among nearby units. (Cantillon and Pesendorfer (2006b) analyze a similar problem in an auction for public bus routes and provide further

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3In many government procurement auctions, like the one we study here, the declared objective is efficiency. In contrast, optimality is the primary objective in auctions run by private firms.
The second design issue we study is how to promote diversification and competition among bidders in a context in which cost synergies are important. This usually involves the following trade-off for the auctioneer: if cost synergies are significant, it may be efficient and optimal in the short-run to allocate all units to one or few firms. On the other hand, this could depress competition in the bidders’ market for future auctions, as inactive firms may exit, hurting efficiency and increasing expected payments in the long-run. We investigate two mechanisms that together help to intensify competition: (1) imposing market share restrictions for bidders; and (2) awarding the units in multiple sequential auctions. The latter could intensify competition if incumbent firms that won units in previous auctions bid more aggressively due to cost advantages given by their installed base. If cost synergies are important, these mechanisms can hurt the efficiency and optimality of the auction, though, because they may prevent bidders to submit package bids containing a large number of units and fully expressing those synergies. Therefore, to study the effectiveness of these measures we compare the intensity of cost synergies with the impact that competition has on bid prices. Our analysis is focused on the effects of local competition measured as the number of local incumbents; local incumbency is measured as the number of firms that are currently operating close to a unit that is being auctioned.4

Analyzing the two design issues discussed above calls for an empirical analysis to measure the intensity of economies of scale and density as well as the effect of local incumbency and competition. Although CAs have received considerable attention from different academic communities including management science/operations research, economics, and computer science,5 to the best of our knowledge there are no other empirical studies that use field data on CAs, except for the notable work of Cantillon and Pesendorfer (2006b). However, the application studied by Cantillon and Pesendorfer (2006b) exhibits few package bids and their estimations do not suggest the presence of synergies. Therefore, as far as we know, our paper is the first to provide an econometric study of a real-world CA that exhibits an important amount of package bidding and synergies, providing novel and substantive insights into bidding behavior in CAs. In that sense, we believe our work constitutes an important contribution to the literature.

The auction we study is the Chilean auction for school meals (see Epstein et al. (2002), Epstein et al. (2004), and Catalán et al. (2009) for a detailed description). The Chilean government provides breakfast and lunch for 2.5 million children daily in primary and secondary public schools during the school year. In a developing country where about 14 percent of children under the age of 18 live below the poverty line,

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4 Although more global competitive effects could be important, our application does not have variation in this dimension because the number of bidders is relatively stable over time.
5 For surveys on work in CAs, see Pekec and Rothkopf (2003), de Vries and Vohra (2003), Hoffman (2006), and Blumrosen and Nisan (2007), and the recently edited volume by Cramton et al. (2006)
many students depend on these free meals as a key source of nutrition. Since 1999 the contracts are awarded through a single-round, sealed-bid, first-price CA. Meal services are standardized and firms compete in prices. Chile is divided into territorial units and firms can submit bids on any package of units defining the combinatorial character of this auction. Around 20 firms participate in each auction; each firm submit many bids (hundreds or even thousands) ranging from just one to several units. The CA has been used every year since its inception awarding more than $3 billion of contracts (US$577 million were awarded in 2008).

Our data set contains bids for packages of different sizes that contain units in different locations. Our empirical strategy is based on using variation in bid prices for different combinations to quantify the discounts associated to economies of scale and density. There are three important challenges in the estimation. First, firms are heterogeneous and may have local cost advantages, some of which are not observable in our data. To mitigate a potential omitted variable bias, we exploit the combinatorial character of our data to control for unobserved local cost advantages. A second challenge arises from the endogeneity of local competition, which can be related to the costs of providing service. We exploit the panel structure of our dataset to address this issue. The third challenge is that the discounts for combinations cannot be entirely attributed to cost synergies; potentially, they could also be explained by markup adjustments due to strategic bundling. To evaluate this alternative explanation, we conducted a detailed analysis to empirically test for the presence of incentives that lead to strategic bundling and how firms respond to them.

Our analysis of the Chilean auction indicates that cost synergies due to economies of scale and density are important, together they can be as high as 8% of the average bid price. The results suggest that economies of scale are stronger than economies of density. Interestingly, our estimates indicate that economies of scale and density get practically exhausted after combining seven or more units. While we do find evidence of strategic bundling, they explain only a small fraction of the discounts in our data. We also find that incumbent firms bid more aggressively and, as a consequence, all firms (including non-incumbents) bid more aggressively as local competition intensifies. The effect of local competition is comparable in magnitude to the discounts from cost synergies.

We use these insights to evaluate and recommend improvements to the auction design along the two dimensions previously discussed. These recommendations are being considered by the Chilean government to redesign future auctions. In particular, our results indicate that package bidding should be allowed so that firms can express their economies of scale and density. However, bidding on specific combinations of units for which strategic bundling seems more severe could be prohibited. Our results also suggest that market share restrictions and running sequential auctions foster supplier base diversification and promote competition in the long-run, without significantly increasing the short-run cost for the government due to unrealized
cost synergies. In fact, at the time of writing of this paper, the government was evaluating relaxing the limits on bidders’ market shares in order to enable them to further exploit cost synergies. Our results suggest that this change could potentially reduce the intensity of local competition, which in our analysis shows to be an important factor in lowering bid prices. Moreover, from a cost efficiency perspective, the benefit of allocating more units to a single firm in a given auction is small because cost synergies get exhausted. Therefore, allowing firms to operate in a larger scale could potentially increase the procurement cost for the government in the long-run. More broadly, our results highlight that the simultaneous consideration of the firms’ operational cost structure and their strategic behavior is key to the successful design of a CA.

The rest of the paper is organized as follows. Section 2 reviews related literature. Section 3 describes the Chilean auction for school meals and our data set. The regression analysis of bidding data is presented in Section 4 and results are presented in Section 5. In Section 6 we study strategic bundling. We evaluate potential redesigns to the current auction in Section 7 and conclude in Section 8.

2 Related Literature

Our paper is related to streams of literature in operations management, empirical industrial organization, and experimental economics as we now describe.

First, our paper is related to the recent literature in operations management that studies procurement and on-line auctions. Most of these papers develop models that introduce novel and important operational aspects to traditional auction settings, providing a theoretical or computational analysis. Among these papers the most related to our work are Elmaghraby (2005) that studies the effect of economies of scale and bidders’ heterogeneity in production capacity on auctions’ performance, and Chen et al. (2005) that studies auctions in a supply chain network explicitly considering transportation costs. None of these papers, however, provide an empirical analysis as we do here.

Our work is related to the extensive empirical auctions’ literature in economics (see Hendricks and Porter (2007) Athey and Haile (2006), and Paarsch and Hong (2006) for several surveys). There are several papers that study multi-unit auctions to learn about cost synergies. Ausubel et al. (1997) and Moreton and Spiller (1998) estimate synergies for wireless providers in spectrum auctions by the FCC. Gandal (1997) estimates synergies in an auction for cable television licenses. However, in all these papers the auction format is sequential and no package bidding is allowed; this is drastically different to our CA in which

package bidding plays a fundamental role. The paper most related to ours is Cantillon and Pesendorfer (2006b) that estimates a model for the CA for London buses routes. While our approach is reduced form, their approach is structural. Because the auctions for London buses routes are small (a maximum of three units are auctioned in each one), a structural approach similar to approaches previously used in single-unit auctions is computationally feasible. However, this approach is infeasible in large-scale CAs like the one we analyze. Also related to our work is Marshall et al. (2006) that estimates the synergies on bidding for the Georgia school milk market using a structural model of simultaneous first-price auctions for multiple homogeneous units.

Our work is also related to other articles that use field data to test theoretical predictions of equilibrium behavior in multi-unit auctions, such as Hortacsu (2002) and Hortacsu and Puller (2008). List and Lucking-Reiley (2000) study multi-unit auctions in field experiments. These articles, however, do not study combinatorial package bidding.

There is also previous work studying bidding behavior in multi-unit auctions using controlled laboratory experiments. Engelbrecht-Wiggans and Katok (2006) study the performance of reverse auctions versus non-competitive contracts in a procurement context, but the auction mechanism used allocates at most one unit to each bidder. In contrast, Katok and Roth (2004) compare different formats of multi-round auctions some of which allow for package bids. Consistent with theoretical predictions, their results suggest that auctions formats which permit package bidding tend to mitigate the exposure problem at the expense of reinforcing the threshold problem. Kwasnica et al. (2005) conduct experiments to compare two ascending auction formats used by the FCC to allocate spectrum, which also involve a trade-off between the exposure and threshold problems. Kagel and Levin (2005) conduct experiments to compare bidding behavior in sealed-bid versus ascending-bid uniform price multi-unit auctions. Note that the school meals auction we analyze is a single-round, sealed-bid, first-price CA, which is different from the auction formats studied in these papers. In that sense, our work is more directly related to Chernomaz and Levin (2009) that studies a similar auction format for two homogeneous units with and without package bidding. They show examples for which if cost synergies are small, allowing for package bidding can lead to a less efficient outcome.

Finally, our paper is related to previous work that estimates economies of scale and density in different contexts. Holmes (2009) and Jia (2008) estimate economies of density for Walmart. Caves et al. (1984) measure and distinguish between economies of scale and density for the airline industry. Finally, Holmes and Lee (2009) estimate economies of density in agriculture distinguishing them from local advantages that favor a specific crop. We propose a different identification strategy to distinguish between cost synergies and local advantages in the context of a CA that exploits the combinatorial character of our data.
3 The Chilean Auction for School Meals

In this section we describe the Chilean auction for school meals, we specify the design issues raised in Section 1 in this context, and we describe our data set.

3.1 Description of the Chilean Auction for School Meals

We present a brief description of the auction process (see Epstein et al. (2002), Epstein et al. (2004), and Catalán et al. (2009) for a more detailed description). Junta Nacional de Auxilio Escolar y Becas (JUNAEB) is a government agency in Chile that provides breakfast and lunch for 2.5 million children daily in primary and secondary public schools during the school year. In a developing country where about 14 percent of children under the age of 18 live below the poverty line, many students depend on these free meals as a key source of nutrition.

Since 1999 JUNAEB assigns its school meals service contracts through a single-round, sealed-bid, first-price CA, that was fully implemented for the first time that year and has been used ever since. For the purposes of the auction, Chile is divided into approximately 100 school districts or territorial units (TUs). Firms can submit bids on various groupings of TUs defining the combinatorial character of this auction. This mechanism is motivated by the belief that firms are subject to cost synergies that arise from operational advantages when serving multiple TUs. JUNAEB holds auctions in one-third of the TUs every year, awarding three-year contracts. Figure 1 presents a map of Chile with the TUs auctioned each year.

The auction process begins when JUNAEB contacts and registers potential vendors. The agency then evaluates the companies from a managerial, technical and financial point of view, and eliminates those that do not meet minimum reliability standards. Qualifying vendors are classified according to two characteristics: their financial capacity (based on data from the firms’ balance sheets), and their managerial competence. Potential vendors submit their bids through an online system. Meal plans are standardized and firms compete in prices. Upon winning a contract, the firm is responsible for managing the entire supply chain associated to all meal services in the corresponding TUs, starting from sourcing food inputs going all the way to cooking and serving the meals in the schools.

A bid can cover any combination from one to 8 TUs and specifies the price for which the firm would serve all meals included in the TUs in the combination. Vendors can submit many bids and each package bid is either fully accepted or rejected (i.e. the mechanism does not allocate a fraction of a bid); most firms submit hundreds or even thousands of bids.

The allocation is chosen by selecting the combination of bids that supply all of the TUs at a minimum
cost. The problem is formulated as an integer linear program that incorporates other considerations and side constraints. An important set of constraints put limits on the maximum number of meals that can be assigned to any given firm, both nationally and in specified geographical regions (to encourage competition in the suppliers’ market). The restriction on the maximum number of meals can vary across firms depending on the results of the multiple evaluations that JUNAEB conducts every year.

The new auction process has been used every year since its inception awarding more than $3 billion of contracts (US$577 million were awarded in 2008), yielding significant social benefits. It has been estimated that this auction process improved the price-quality ratio of the meals with yearly savings of around 22% of the budget. One of the main objectives of our work is to evaluate and propose improvements to the current auction design that can result in further efficiency gains and cost savings for the Chilean government.

3.2 Design Issues in the Chilean Auction

In this section we discuss the two design issues presented in Section 1 in the context of the school meals procurement auction: (1) which package bids should be allowed; and (2) how to diversify the supplier base to promote competition.

Package Bidding

Package bidding should be sufficiently flexible to let bidders express cost synergies among units. Indeed, in the current design all possible combinations are allowed in the bidding process. However, too much flexibility can hurt efficiency and increase expected total payments if bidders engage on strategic bundling. Therefore, we suggest that the auction mechanism only allows package bidding among units for which cost synergies are sufficiently large. Given the cost structure associated to serving a typical TU we identify two types of cost synergies:

1. **Economies of Scale.** Approximately 50% of the total cost of serving meals is related to food inputs that include perishable and non-perishable items. Most of the food is purchased centrally and firms can get important volume discounts from their providers. These discounts result in economies of scale when serving multiple TUs. Note that these synergies are only a function of the total volume of meals served and are independent of the proximity between the units served.

2. **Economies of Density.** Logistical costs associated to transportation and administration amount to around 9% of total cost. Some of these costs are fixed and can be shared by TUs that are close to each other (e.g., by sharing a local warehouse and a distribution network), resulting in economies of density. Note that these synergies are a function of the proximity of the units served.
The distinction between these two types of cost synergies is important from an auction design perspective. If economies of density are predominant, then it could be preferable to restrict combinations to units that are closely located. If economies of scale are predominant, then there are simpler auction mechanisms that would allow bidders to express them in their bids (e.g. providing prices for each unit and a discount curve). We therefore seek to identify separately these two sources of cost synergies using bidding data.\(^7\)

Promoting Competition

The school meals auction exhibits two specific characteristics that help to diversify the supplier base and promote long-term competition among bidders:

1. The current mechanism imposes strict market share restrictions in the allocation of units to bidders. There are three such restrictions: (1) a maximum number of TUs that each firm can be allocated in any given auction; this maximum is based on the financial evaluation conducted by JUNAEB every year and therefore can be different across firms, ranging from 2 to 8 TUs; (2) at any point in time, the total standing contracts of any firm cannot exceed 16% of the total number of meals included in all TUs in the country; and (3) a local market share constraint that limits the number of TUs a firm can be awarded with in pre-established geographical regions (this limit varies across regions).

2. The TUs are split into multiple sequential auctions which are conducted in consecutive years. Figure 1 shows how the different TUs across the country are grouped on multiple auctions. In general, TUs in adjacent geographic regions are awarded in different years, so that each auction awards units scattered all along the country rather than concentrated in a specific area.

The latter generates local incumbents – for most of the TUs in a given auction (except the first one in 1999), there are firms with on-going contracts for nearby TUs awarded in previous auctions. Because these local incumbents already have an installed base in the proximity of some TUs, they may have cost advantages and therefore bid more aggressively. If other firms anticipate this, standard auction theory predicts that a non-incumbent firm should also bid more aggressively (Krishna, 2002). Hence, an increase in the intensity of local competition, measured as the number of local incumbents, is likely to reduce bid prices from all firms. The market share restrictions would reinforce this local competition effect as it tends to increase the number of local incumbents.

These benefits notwithstanding, there are also potential disadvantages of the current design. In particular, the market share restrictions together with running multiple auctions may result in many firms providing

\(^7\)Approximately 25% of the operating costs is related to labor dedicated to the preparation of the meals at the schools. The number of servers per school and their salaries are heavily regulated and not subject to cost synergies, so we ignore this cost in our analysis.
service in narrowly defined geographical areas, precluding them from fully realizing their cost synergies. It is therefore useful to compare the magnitude of the local competition effects relative to the cost synergies in order to evaluate the overall effectiveness of the current auction mechanism.

3.3 Description of the Data

Our data set contains all bids presented by all firms in all auctions between 1999 and 2005. We also collected information about the firms and the TUs on each auction. We describe this data and some descriptive statistics in what follows.

For each auction, we know the identity of all participant firms, which are around 20 each year. We have data on all the bids presented by each firm. Each bid specifies a set of TUs and the price per meal for which the firm would serve all units in the combination. Firms submit from tens to several thousand bids. Typically, the total number of bids submitted by all firms is around 50 thousand. In addition, we know the set of winning bids in each auction and, therefore, at every point in time we know the identity of the firm serving each TU.

We have detailed data about TUs including the location and population of all schools in them. The number of meals per year of an average TU in the bidding data is 2.5 million. We note that TUs are heterogeneous in the density of school population, a factor that strongly affects the procurement cost. For example, there are units in urban areas that are small in size and with high school population, but also units in isolated parts of the country that cover large extensions of territory with a low population density. The average price per meal over all bids is $0.75. Recall that each TU is auctioned every three years. Therefore, each TU is auctioned at least twice in our data set, which is an important aspect for the estimation method described in Section 4.

We now describe some interesting patterns in the bidding data regarding the characteristics of packages in the bids. Figure 2 shows a histogram of the number of TUs in a bid. We observe a significant degree of variation in bid sizes and an important presence of combinatorial bidding. Firms present small bids with one TU to large ones with eight TUs; the median and mode is four TUs. Figure 3 shows a scatter plot of bid prices (per-meal, in US$) vs. the size of the package bid (measured in million meals per year). The figure suggests that bidders use volume discounts in their bids.

Figure 4 shows the average maximum distance among the TUs contained in a bid, for bids of different

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8 The average number of firms entering and exiting the market each year is close to three.
9 The current allocation mechanism uses some criteria to eliminate unrealistically low bids. In our analysis, we do not eliminate bids.
sizes. Firms make bids that include both TUs that are close to each other and TUs that are far apart. However, by comparing the pattern in the actual data with one in which TUs in each bid are selected randomly, we observe that on average firms tend to prioritize combinations of TUs that are close to each other.

The previous analysis suggests that combinations of TUs in bids exhibit some distinguishable patterns. First, combinatorial bidding and discounts are widely observed suggesting the presence of economies of scale. Second, firms tend to include TUs that are close to each other in the same bid suggesting the presence of economies of density. In the next section we test these hypothesis and quantify these effects with an econometric analysis.

4 Econometric Analysis of Bidding Data

In this and the following sections we provide an econometric study to measure the intensity of economies of scale and density as well as the effect of local incumbency and competition; these measurements will shed light on the two design issues discussed in Section 1. In particular, we use an econometric model to study the factors that explain variation in bid prices. Since most of the variation in the prices is explained by the number of meals included in the package, we normalize the total bid price dividing it by the number of meals. Throughout, we use $i$ to index territorial units (hereon units), $a$ to index sets of units (also referred to as combinations or packages), $t$ to index auctions, and $f$ to index firms. The dependent variable in our empirical analysis is the price-per-meal, defined as total bid price (in 1999 Chilean pesos) divided by the number of meals of a combination $a$ submitted by a firm $f$ in a specific auction $t$, and is denoted by $b_{aft}$ (hereon refereed to as the bid price).

There are two main factors that explain the variation in bid prices. First, there is heterogeneity in the individual prices of the units, which arises from differences in costs and markups associated with each unit. This heterogeneity is across units (e.g. some units are more expensive to operate than others), across firms (e.g. some firms could be more efficient in particular regions, or charge different markups) and across auctions (e.g. the price of inputs may vary from year to year). Second, different combinations are subject to different discounts and the data shows that larger combinations tend to have, on average, a lower bid price. We expect that a significant portion of these discounts arises from cost synergies due to economies of scale and density.

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10The distance between two TUs is the bird-fly distance between the weighted geographic centers of the respective TUs. The weighted geographic center of a TU is calculated as the weighted average latitude and longitude of the schools it contains, weighted by the school populations.

11We used the consumer price index of food to normalize prices to real 1999 Chilean pesos.
To account for these sources of variation, our specification decomposes the bid price into two parts: (1) the stand-alone prices of the units contained in the package, which captures heterogeneity across units; and (2) discounts for combinations, which captures interactions among the units contained in the package and is therefore package-specific. Let $d_{ift}$ be the stand-alone price of a unit $i$ for firm $f$ in auction $t$, representing the average bid price charged for that unit. Let $v_i$ be the size of unit $i$ (measured in million number of meals per year), and accordingly define, with a slight abuse of notation, the size of a combination $a$ as $v_a = \sum_{i \in a} v_i$. We model the bid price as:

$$b_{aft} = \sum_{i \in a} d_{ift} \cdot \frac{v_i}{v_a} - g_{ft}(a).$$  \hspace{1cm} (1)

Equation (1) decomposes the bid price as a weighted average of the stand-alone prices $d_{ift}$ minus a discount function $g_{ft}(a)$. The discount function depends on the package $a$ and could potentially be different across firms and auctions.

Before describing in further detail this specification, it is useful to link the two auction design issues we study with equation (1). Recall that the first design issue is concerned about which combination bids should be allowed in the auction. This is related to the discount function $g_{ft}(a)$; combinations should be restricted to packages $a$ for which the discount due to costs synergies is substantial. The second design issue is related to promoting local competition in the auction. An active presence of local incumbents for a unit $i$ should enhance competition and lead to lower stand-alone prices $d_{ift}$ for that unit from all firms in the long-run. The rest of this section describes details on the specifications of the discount function and stand-alone prices which capture these relevant effects.

### 4.1 Estimation of the Discount Function

Discounts for packages can be a consequence of cost reductions arising from synergies, but also perhaps of other factors that lead to adjustments in the markups charged by firms, such as strategic bundling. In this section, we construct discount functions that, in the absence of markup adjustments, would reflect economies of scale and density. In Section 6 we try to quantify what fraction of the discounts are explained by markup adjustments due to strategic bundling, as oppose to by cost synergies.

As mentioned earlier, there are two main forms of cost synergies in the procurement process: economies of scale, which only depend on the total size of the combination ($v_a$), and economies of density, which depend on the geographic location of the units in $a$. Accordingly, we define two separate discount functions: (i) $g_{scale}(v_a, \beta_{scale})$, called the scale discount function, intends to capture economies of scale and thereby
is a sole function of \(v_a\); and (ii) \(g^{\text{dens}}(a, \beta^{\text{dens}})\), called the density discount function, intends to capture economies of density and therefore could vary for each set \(a\). Recall that the discount functions represent discounts per meal. These functions are parametrized by the vectors \(\beta^{\text{scale}}\) and \(\beta^{\text{dens}}\) which measure the intensity of the respective discounts. These parameters could vary across firms and auctions, but for simplicity we assume they are common to all firms and auctions (Section 5.3 shows results from a more flexible specification to capture firms’ heterogeneity in this dimension). Because we want to measure discounts arising from the interaction among units, both discounts functions \(g^{\text{scale}}\) and \(g^{\text{dens}}\) are set to zero for bids with a single unit (i.e., when \(a\) is a singleton).

In estimating \(g^{\text{scale}}\) and \(g^{\text{dens}}\), it is important to allow a sufficiently flexible specification to capture a potential saturation effect of these discounts. Note that \(g^{\text{scale}}(v_a, \beta^{\text{dens}})\) is a function of a single variable and is therefore relatively easy to approximate with a flexible, yet parsimonious, specification. In contrast, representing \(g^{\text{dens}}(a, \beta^{\text{dens}})\) compactly is more challenging because the number of possible sets \(a\) is enormous. To overcome this difficulty, we make this function depend only on measures that summarize the geographic proximity of the units in \(a\). We describe this procedure next.

For a given set \(a\), we identified clusters of units located within a circular perimeter of 150 km. radius.\(^{12}\) Let \(Cl(a)\) be the set of clusters formed by the units in \(a\), which form a partition of \(a\). The size of each cluster \(c \in Cl(a)\) can be calculated as the sum of the sizes of its units, \(v_c = \sum_{i \in c} v_i\) (with some abuse of notation). We then define the discount function at the level of a cluster as function of its size, denoted \(g^{\text{clust}}(v_c, \beta^{\text{dens}})\). Again, this discount function is set to zero when the cluster contains a single unit, so that it indeed captures discounts from combining units. Economies of density would imply that the discount function \(g^{\text{clust}}\) is non-decreasing in the cluster size \(v_c\): as more demand is concentrated within the limits of the cluster, the cheaper it is to supply each unit. Given a specification for \(g^{\text{clust}}(v_c, \beta^{\text{dens}})\), the overall density discounts are given by:

\[
g^{\text{dens}}(a, \beta^{\text{dens}}) = \sum_{c \in Cl(a)} g^{\text{clust}}(v_c, \beta^{\text{dens}}) \cdot \frac{v_c}{v_a},
\]

that is, a weighted average of the discounts from each cluster. The following example helps to illustrate the calculations of equation (2) and the logic behind it.

Consider four units, labeled 1 through 4. To simplify the discussion, we assume all units have the same size equal to one and have similar stand-alone prices. Due to their location, \{1,2,3\} form a cluster but unit 4 does not form a cluster with any of the other units. Consider two hypothetical bids: bid A for units \{1,2,4\},

\(^{12}\) In section 5.3 we discuss results with alternative cluster definitions. We describe the clustering algorithm in Appendix A.
and bid B for \{1,2,3\}. Both bids have the same size, so scale discounts will be similar. But the clusters formed within each bid are different. Bid A has two clusters, \{1,2\} and \{4\}, one of which is a singleton and therefore does not get any discounts. Bid B has a single cluster of size 3. Let $g_v = g^{\text{cluster}}(v, \beta^{\text{dens}})$ be the discount per meal for a cluster of size $v$. The density discount for bid A is equal to $\frac{2}{3}g_2$. For bid B, the discount is equal to $g_3$, which is larger than the discount on bid A because: (1) $g_v$ is increasing in the size of the cluster; and (2) all three units in bid B benefit from the density discounts, compared to only two units in bid A. Also note that a bid for package \{1,2\} – which is a subset of bid A – could have a larger discount per meal than bid A if density discounts are sufficiently large (relative to scale).

To incorporate scale and density discounts into the bid price equation (1), we specify the discount function as:

$$g_{ft}(a) = g^{\text{scale}}(v_a, \beta^{\text{scale}}) + g^{\text{dens}}(a, \beta^{\text{dens}}) - \varepsilon_{aft},$$

(3)

where the error term $\varepsilon_{aft}$ captures idiosyncratic discounts (or charges) for combination $a$. The specific parametric forms of $g^{\text{scale}}$ and $g^{\text{dens}}$ used in the estimation are described in Section 5. Replacing (2) and (3) into (1) gives the regression equation:

$$b_{aft} = \sum_{i \in a} d_{i ft} \frac{v_i}{v_a} - \sum_{c \in C\{a\}} g^{\text{cluster}}(v_c, \beta^{\text{dens}}) \cdot \frac{v_c}{v_a} - g^{\text{scale}}(v_a, \beta^{\text{scale}}) + \varepsilon_{aft}.$$  

(4)

We seek to estimate the parameters $(\beta^{\text{dens}}, \beta^{\text{scale}})$ that characterize scale and density discounts. In order to estimate cost synergies with regression (4), there are two important challenges that need to be addressed. First, discounts for combinations cannot be entirely attributed to cost synergies if firms adjust their markups when submitting package bids. Second, firms may have local cost advantages that are not observable in the data which could confound the effect of economies of density. We now discuss how we address each of these identification issues.

The first issue, as we discussed before, is whether we can interpret the estimated discount functions $g^{\text{scale}}$ and $g^{\text{dens}}$ as cost synergies. Recall that firms may have incentives to adjust their markups and provide discounts for packages for pure strategic reasons even in the absence of cost synergies. In Section 6 we study whether markup adjustments due to strategic bundling explain part of the observed discounts. There, we conduct an additional investigation which: (1) analyzes the incentives that lead to strategic bundling and study whether these are present in our data; and (2) empirically test how firms respond to these incentives. Our results will suggest that although bidders do engage on strategic bundling, the effects on the total discounts are small. Hence, we will interpret scale and density discounts as mostly been explained by cost
synergies.

Even after correcting our discount measures for the presence of strategic bundling, there is a second identification issue related to economies of density. As noted by Holmes and Lee (2009), distinguishing economies of density from local advantages can be challenging. To explain, Figure 4 suggests that bids are more likely to include units which are located in close proximity, which could be interpreted as evidence of economies of density. However, an alternative explanation is that firms have local cost advantages in specific regions and are more likely to submit bids for units where they have an advantage. Because advantage are local, packages with co-located units are more likely to be submitted by firms with lower costs and therefore have a lower price. Since we do not observe the local cost advantages of firms, using variation across bids submitted by different firms will tend to overestimate economies of density, as we would attribute the lower price of co-located units entirely to this type of synergy.

To eliminate this source of bias in measuring economies of density, regression (4) controls for the stand-alone prices, $d_{i,f,t}$, which are firm-unit-auction fixed effects that capture firm-specific cost advantages in a unit. Hence, scale and density discounts are estimated using variation across different combinations submitted by the same firm over the same set of units in the same auction. Consequently, our estimation strategy requires a large number of combination bids submitted by the same firm in order to obtain consistent estimates of the parameters in (4). Note that the estimation provides estimates of the stand-alone prices $d_{i,f,t}$ along with $\beta_{dens}$ and $\beta_{scale}$.

4.2 Analysis of Stand-Alone Prices

The stand-alone price $d_{i,f,t}$ captures the average price charged for supplying a unit before accounting for the interactions that may arise with other units. It is affected by the firm’s cost in supplying the unit –net of any cost synergies with other units in a combination– plus a markup. The unit’s cost is affected by the characteristics of the unit (e.g. size and location of the schools) and other specific costs advantages that a firm may have. An example of firm specific advantages is when a bidder has a lower cost of supplying a unit because it has a nearby warehouse used to serve other related businesses in the area. Consequently, the stand-alone price should be lower for firms that are local incumbents; recall that we defined local incumbents as firms with ongoing contracts for nearby TUs awarded in a previous auction (consistent with our cluster definition we say that two TUs are near if the distance between them is less than 150 km.). As discussed in section 3.2, the presence of local incumbents could generate additional price reductions from all firms (including non-incumbents) through a local competition effect.

For the estimation, we define a binary variable $LocIncumb_{i,f,t}$ indicating if firm $f$ is a local incumbent.
for unit $i$ in auction $t$. Local competition is measured as the number of rival firms that are nearby incumbents, defined as $\text{LocComp}_{ift} = \sum_{f' \neq f} \text{LocIncumb}_{ift}$. We seek to estimate the following regression:

$$d_{ift} = \beta_1 \text{LocIncumb}_{ift} + \beta_2 \text{LocComp}_{ift} + \beta_x X_{ift} + u_{ift},$$

which captures the effect of local incumbents and competition on the stand-alone price $d_{ift}$. The vector $X_{ift}$ includes other controls (described shortly), and the error term $u_{ift}$ captures other unobservable factors.

Note that the dependent variable in regression (5), $d_{ift}$, is not directly observed in our data. Therefore, we follow a two-stage estimation approach. The first-stage regression (4) (which has bid price as the dependent variable) provides estimates of the stand-alone prices, $\hat{d}_{ift}$ (along with the parameters characterizing discounts). The second-step regression replaces $d_{ift}$ by its estimate $\hat{d}_{ift}$ in equation (5) to estimate the coefficients of $\text{LocIncumb}$ and $\text{LocComp}$ (as well as $\beta_x$). If $\hat{d}_{ift}$ is a consistent estimator of $d_{ift}$ and assuming the usual orthogonality conditions ($E(u_{ift}|\text{LocIncumb}, \text{LocComp}, X) = 0$), $(\beta_1, \beta_2, \beta_x)$ can be estimated consistently through Ordinary Least Squares. However, the covariates $\text{LocIncumb}$ and $\text{LocComp}$ are endogenous and its effect could be confounded by other factors not captured in the regression. To mitigate endogeneity bias, we include several controls in $X_{ift}$, which we discuss in detail in what follows.

$\text{LocComp}$ measures the number of rival firms that are incumbent to a unit and is therefore affected by the unit’s location. Units located in urban areas tend to be smaller and have more “neighboring” units, and thereby tend to have more incumbent firms. Because these territories are also more densely populated and have better transportation infrastructure, they tend to be cheaper to supply. Hence, unobservable unit costs could confound the effect of competition, generating a negative bias on the coefficient on $\text{LocComp}$. To mitigate this source of bias, we include unit fixed-effects which control for time-invariant characteristics of a unit. Note that the panel structure of our dataset –with two or more auctions observed for each unit – is essential for the identification of local competition effects. We also include the following controls to capture costs associated to a unit which can vary across auctions: (1) the size of the unit ($\text{Size}$, measured as the number of meals), which reduces the cost of serving the unit due to scale economies; (2) the fraction of “Special Meals” supplied, which increase the costs of serving the unit.

The $\text{LocIncumb}$ covariate could also lead to endogeneity bias. Similar to the bias regarding economies of density described earlier, a local incumbent firm that was previously awarded units in the “neighborhood” of unit $i$ is a potential indication that it had a local cost advantage in the vicinity. If this advantage is persistent over time, the firm would have a lower cost of serving unit $i$ in the current auction and therefore submits a lower bid. In this alternative explanation, $\text{LocIncumb}$ is a proxy for permanent local cost advantages,
which would bias the estimate of the $LocIncumb$ variable. Note that unit fixed effects do not control for this source of bias because local advantages are firm-specific. In absence of good instrumental variables for $LocIncumb$, we introduce controls into the regression which capture permanent firm local advantages. The idea is to use pre-defined regions, which can contain one or more units, and introduce firm-regions specific fixed effects which capture unobservable firm advantages on each area. With these controls, the identification of $LocIncumb$ relies in the variation across unit’s stand-alone prices from the same firm within the pre-defined areas and the variation across auctions. Defining these pre-defined areas is somewhat subjective. We used the political regions of Chile as our areas, which contain on average 7.7 units (see Figure 1 for a map describing the regions). We also did some analysis with other pre-specified areas to assess the robustness of the results. In addition, we also include the following firm characteristics that can change over time as additional controls: (1) an indicator variable on whether a firm is attempting to renew a previously awarded contract for the unit, to capture possible sunk costs in the service provision ($Renew$); (2) a firm performance measure assigned by JUNAEB each year, which captures the managerial competence of firms ($Performance$); (3) a financial grade assigned by JUNAEB based on firms’ financial classification in the range 1 to 7, 1 being the best grade ($FinGrade$); and (4) an indicator for firms that are participating in the auction for the first time ($NewFirm$). Finally, we include auction fixed-effects to capture temporal price trends.

In summary, our estimation is a two-stage regression where the first-stage estimates equation (4) and the second stage uses the estimates of the stand-alone price $\{d_{ijt}\}$ as the dependent variable to estimate equation (5) via OLS. This two-stage procedure provides estimates of the parameters characterizing scale and density discounts ($\beta_{dens}$ and $\beta_{scale}$) and the coefficients of $LocIncumb$ and $LocComp$. These together with our analysis in Section 6 are useful to study the design issues discussed in Sections 1 and 3.2. The next section describes the main estimation results.

## 5 Results

This section describes the main results of the two-stage estimation and a sensitivity analysis to assess their robustness.

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\(^{13}\)Since the objective is to control for permanent cost advantages, this fixed-effects do not vary across auctions.
5.1 First-Stage Regressions: Estimates of Discounts

We begin by providing details on the specifications for the discount functions $g_{dens}(v_c, \beta_{dens})$ and $g_{scale}(v_a, \beta_{scale})$ in the regression equation (4). Recall, the cluster size $v_c$ and package size $v_a$ are measured in million meals per year. The scale discount function $g_{scale}(.)$ is estimated by a step function with 10 equally spaced intervals of size three that cover all the range of bids. Similarly, the density discount function $g_{clust}(.)$ is estimated with a step function with 13 equally spaced intervals of size two. The step functions for $g_{clust}$ and $g_{scale}$ are implemented with binary variables indicating the size level, excluding the smallest level, so that the coefficient represents the average change in bid price at that level relative to the excluded level. For example, the results in Table 1, right column (under scale discounts), indicate that a combination $a$ of size $v_a$ in the range $[6, 9]$ is Ch$17.49 cheaper than one of size $v_a \in [0, 3]$.

Table 1 shows the estimates of the first stage regression (4), which includes covariates measuring scale and density discounts and a set of firm-unit-auction stand-alone prices $\{d_{i,Ft}\}$ (the estimates of stand-alone prices are not shown). Robust standard errors are reported in parenthesis. All the coefficients are estimated with precision, which is expected given the large sample size. The explanatory power of the regression is remarkably high (R-square equal to 0.98).

Figure 5 compares the estimates of the discount functions due to scale and density in terms of percent change on the average bid price (the average bid price is 75¢). The results suggest that the magnitude of scale discounts are economically significant (see top of Figure 5): increasing the package size to 20 million (about 8 units) generates discounts of approximately 6%. Interestingly, most of the scale discounts are seen for volumes below 18 million (about 7 units), suggesting they get exhausted after that point. Similarly to scale, they also get exhausted after 7 units. To illustrate, consider a bid for three units of size 3 million each. Scale discounts are approximately 4.5% of the average bid price. In addition, if the 3 units form a cluster, the additional discount due to density is 1.3%, for a total discount of 5.8%.

Note that typical margins in this industry are between 4% and 8%.
5.2 Second-Stage Regression: Effect of Incumbents and Local Competition

Table 2 shows the results from the second-stage regression (equation (5), replacing $d_{it}$ with its estimate from the first stage). As is common with panel data, we decompose the error term $u_{it}$ into a random-effect $\delta_{if}$ plus an idiosyncratic error, and account for this heteroskedasticity in the estimation (typically known as “random-effect” estimator, see Wooldridge (2002)). Specification (1) includes firm, auction and unit dummy variables as controls, plus all the control variables described in section 4.2 ($\text{FinGrade}$ is included with a linear effect plus a dummy variable indicating firms with the highest financial grade, $\text{FinGrade}=1$).

The coefficient on $\text{LocIncumb}$ is negative and significant, revealing that firms which operate in nearby units (awarded in previous auctions) tend to bid, on average, 2.3% lower. This is similar in order of magnitude to the discounts generated by density. The effect of local competition ($\text{LocComp}$) is also negative and significant. Increasing the number of local incumbents by 4 (about 1 standard deviation) reduces stand-alone prices by 3%. The controls $\text{Renew}$, $\text{Size}$ and $\text{SpecialMeals}$ all have the expected signs.

Column (2) in Table 2 includes firm-region fixed effects as additional controls (together with auction and unit fixed effects and all the other controls). The coefficient of $\text{LocComp}$ and $\text{LocIncumb}$ are similar to those obtained in column (2), although the statistical significance of $\text{LocIncumb}$ is smaller. Given the small difference across the estimates, it appears that the potential bias on $\text{LocIncumb}$ due to local firm advantages is small.

5.3 Sensitivity Analysis

We conducted some additional empirical analysis to validate the robustness of these results, which we briefly discuss in this section.

Our definition of clusters –using a 150 km radius – is a reasonable area to capture economies of density in this industry based on the information available to JUNAEB. To analyze the robustness of the estimates of density discounts, we considered alternative cluster sizes, with radiuses of 100, 200, 300 and 400 km.

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15 The 1999 auction was the first year in which the CA was fully implemented. Hence the measures of local incumbency ($\text{LocIncumb}$) and local competition ($\text{LocComp}$) are not well defined. For this reason, we chose to exclude this auction from the second-stage estimation.

16 We also estimated additional regressions to test a possible non-linear effect of $\text{LocComp}$. Perhaps surprisingly, all of these models suggest a linear effect of $\text{LocComp}$.

17 While the other controls are statistically significant, we do not have theory to predict their sign a-priori.

18 A Hausman test cannot reject that the coefficients of $\text{LocIncumb}$ in models (1) and (2) are similar ($t$-stat=.87). We also ran a regression that treats $\delta_{if}$ as fixed effects. The standard errors in this regression are much larger and the coefficients on $\text{LocIncumb}$ and $\text{LocComp}$ were no longer statistically significant. However, their magnitude and sign were similar to those obtained in Table 2, and a Hausman test cannot reject that the estimates are equivalent. Finally, we also estimated a regression with firm-auction fixed effects (instead of firm fixed effects) to account for unobservable firm characteristics that change over time beyond the ones we control for as described in Section 4.2; the results were also similar (although the statistical significance for $\text{LocIncumb}$ was smaller).
Figure 6 shows these estimates using the different cluster definitions. Density discounts appear to decrease rapidly as the radius of the cluster increases. The estimated discounts with a cluster radius of 150 km. can be as much as 40% larger than the discounts estimated with a cluster radius of 400 km. The discounts estimated with cluster radii of 100 km and 200 km are similar to those obtained in our main results. Overall, this analysis suggests that our cluster definition of 150 km radius is reasonable at capturing density discounts.

We also estimated the first step-regression with the logarithm of the bid price as the dependent variable. All the estimated effects were similar in order of magnitude and statistical significance.

As an alternative specification for equation (4), we estimated a more parsimonious regression which replaces the stand-alone prices \(d_{It} \) for separate indicator variables for firm, units and auction (with no interactions between them). The estimates of density discounts in this model differ substantially from those reported in our main results. This suggests that controlling for the stand-alone prices \( \{d_{It}\} \), which controls for firm’s local advantages, are important to obtain consistent estimates of the discount functions.

We studied heterogeneity of the discount curves for firms of different sizes to see if the observed pattern in the discounts varied. We used a firm classification based on FinGrade to group firms into three types: large, medium, and small (recall that FinGrade is based on firms’ financial classification). We ran regression (4) separately for the three types of firms. We found that the magnitude and shape of the scale discounts were similar across the three types. Small and medium firms tend to exhaust their scale discounts at around 15 million meals, while large firms exhaust their discounts at around 20 million. In all the regressions, the scale discounts where in the order of 4% to 7%. The estimates of economies of density where in the order of 1% to 2%, and medium firms tend to offer higher density discounts. In summary, although we do find some heterogeneity across firms of different size, our main results prevail: (1) discounts are economically significant; (2) scale discounts dominate; and (3) discounts get exhausted at around 7 units.

One possible interpretation for why the scale discounts get exhausted after 7 units is that cost synergies get exhausted after that point. However, there could also be other alternative explanations related to markup adjustments. In particular, it may be possible that firms adjust markups due to the side constraints in the allocation mechanism which limit the maximum number of meals that can be awarded to a single firm. This is specially relevant for large firms for which the average maximum number allowed is around 20 million meals, which is close to the number at which discounts get exhausted. To test this alternative explanation, we allowed for interactions between the maximum number of meals allowed (which may change across auctions for a given firm) and the scale discount function, so that large firms with different maximums may exhaust their discounts at different quantities. The estimates suggest that the restriction on the maximum meals allowed does not change significantly the quantity at which scale discounts get exhausted. We cannot
reject the null hypothesis that a one standard deviation change in the maximum restriction does not change the quantity at which the scale discounts are maximized (p-value>0.11). The next section presents a detailed study of a different potential source of markup adjustments in package bids.

6 Strategic Bundling

The regression analysis suggests that discounts for combination of units are significant. It also shows that scale discounts are larger than density discounts. While this is suggestive that cost synergies are substantial, it is not conclusive because discounts may also arise due to markup adjustments. The focus of this section is to analyze whether such adjustments due to strategic bundling are indeed present and to assess their magnitude.

Section 6.1 revises the theory describing bidders’ incentives to submit discounted package bids even in the absence of cost synergies due to strategic bundling. Based on this theory, we establish testable empirical predictions regarding strategic bundling in a CA. This requires quantifying the randomness due to asymmetric information that bidders face; Section 6.2 provides a way to do this. Section 6.3 describes an empirical test of the theoretical predictions regarding strategic bundling.

6.1 The Bidder’s Problem and Testable Predictions for Strategic Bundling

We start by providing a stylized example borrowed from Cantillon and Pesendorfer (2006a) that illustrates why bidders may have incentives to submit discounted package bids even in the absence of cost synergies. Consider a CA with two units, A and B, and two bidders, 1 and 2. Bidder 1’s costs are $5 for each unit and exhibits no cost synergies, that is, its cost of serving both units is $10. As is usual in auction theory, we assume that due to asymmetric information bidder 1 “competes” against the distribution of its opponent’s bids. Suppose bidder 1 believes its competitor with probability $\frac{1}{2}$ will submit a bid of $7 and $15 for units A and B, respectively, and with probability $\frac{1}{2}$ the bids will be $15 and $7. Its competitor does not submit a package bid. Note that a package bid from bidder 1 of $22 - \epsilon$ (for small $\epsilon > 0$) always wins at a profit of approximately $22 - 10 = $12. When package bidding is not allowed, the best strategy is to bid $15 - \epsilon$.

\[19\] More specifically, for the purpose of this test the discount curve is specified as a quadratic polynomial including main effects for volume and volume square plus their interactions with maximum volume restriction. This analysis is limited to large firms, which are the only firm types which have variation in the maximum volume restriction. For these firms, the restriction is usually determined by the market share constraint over the total standing contracts (see point 1 on page 9 for a description of the different types of market share restrictions). A given firm may have different standing contracts in different auctions. Therefore, the maximum number of meals the firm can be awarded with in a given auction changes. For other types of firms, the maximum number allowed is determined by the financial evaluation, which usually does not change across years.
for each unit and get an expected profit of approximately $15 - 5 = $10. However, when package bids are allowed, it can be shown that it is never in bidder 1’s interest to win with individual bids. Hence, bidder 1’s best response is to bid a price strictly larger than $15 for each unit and a package bid of slightly less than $22. This example shows that discounts for combinations may arise in any best response even in the absence of cost synergies; we refer to this behavior as strategic bundling.

The example is useful to illustrate the incentives that lead to strategic bundling. Note that from bidder 1’s perspective there is randomness in the bids for individual items it is competing against. Therefore, when submitting individual bids, bidder 1 needs to trade-off the possibility of winning with the ability of charging a higher markup. However, because the individual bids are perfectly negatively correlated, a package bid of $22 - \epsilon$ wins with certainty. Hence, when bidding for the package there is no such trade-off: bidder 1 wins the package with probability one even when charging the highest possible markup, that is, when bidding slightly less than $22. Therefore, bidder 1 has more ability to extract surplus from a package bid than from individual bids.

Previous research shows that the idea behind the example generalizes (see Adams and Yellen (1976), Schmalensee (1984), McAfee et al. (1989), Fang and Norman (2006), and Chu et al. (2009)). Bidders may have incentives to submit discounted package bids in any best response if the competitors’ bids for the package exhibit less variance than the bids for the individual items. With less variance, it is easier to extract surplus as we argued above. This will be the case if the competitors’ bids for individual items are not too positively correlated. More generally, the incentives to submit discounted package bids increase as the effect of the reduction in variance of a package becomes stronger. We use these insights to formulate testable predictions regarding discounts due to strategic bundling in a bidder’s best response when competing against the distribution of its opponents’ bids.

More formally, consider a multi-unit auction and let $b^f_i$ be the bid that firm $f$ submits for unit $i$. From the perspective of firm $f$, bids submitted by firms $f' \neq f$ are random due to asymmetric information; with $N$ units, the bid vector of stand-alone prices $(b^f_1, ..., b^f_N)$ can be viewed as a multivariate random vector. Suppose that these random vectors are independent and identically distributed across $f'$. In addition, suppose that the competitors of firm $f$ do not submit package bids. In this case, a bid from firm $f'$ for a package is just the sum of the stand-alone bids; for example $b^f_{i \cup j} = b^f_i + b^f_j$. To win a unit, firm $f$ needs to underbid all of its competitors; in that sense, firm $f$ competes against the minimum of the bids submitted by its opponents. Accordingly, let $b^{\min}_{i \cup f} = \min_{f' \neq f} b^{f'}_i$ be the minimum of the opponent bids for a unit $i$ and

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20 Although these articles focus on pricing by a multiproduct monopolist, Cantillon and Pesendorfer (2006b) show that when consumer valuations are additive (and under other fairly general conditions) this problem is equivalent to the bidder’s problem in a CA.
let $b_{i,j}^{-f} = b_i^{-f} + b_j^{-f}$. The results in the existing literature suggest that for $N = 2$ and in the absence of cost synergies, the package discount due to strategic bundling for firm $f$—defined as $b_f^i + b_f^j - b_{i,j}^{-f}$—increases when: (1) the correlation between the minimum of the stand-alone bids $b_{i}^{-f}$ and $b_{j}^{-f}$ decreases; and (2) the coefficient of variation of the stand-alone bid $b_{i}^{-f}$ increases. Under these circumstances, the reduction in variability of $b_{i,j}^{-f}$ (relative to the variability of $b_{i}^{-f}$ and $b_{j}^{-f}$) increases.\(^{21}\)

Note that these theoretical predictions are based on the distribution of the minimum of the opponents’ bids. Unfortunately, we do not have enough data to quantify statistical properties of the distribution of the minimum stand-alone prices directly. Nevertheless, numerical experiments show that, with normally distributed bids, the correlation structure and coefficient of variation of the underlying opponents’ bids ($b_f^i$ and $b_f^j$ in our previous notation) carries over to the distribution of the minimum of these bids. We therefore establish our hypotheses based on the underlying distribution of bids (rather than their minimum), which are feasible to test with our data as we show below.

These theoretical predictions consider packages of two units only. However, there is a natural extension to the case in which a firm combines a unit $i$ with a package $a$ (with more than one unit) that we use to develop our hypotheses.

Based on these existing theoretical results, we formulate two testable hypotheses:

- **Hypothesis H1**: The magnitude of the discounts due to strategic bundling when combining a unit $i$ with a package $a$ (possibly a singleton) increases, as the correlation among the competitors’ bid prices between the individual unit ($b_f^i$) and the package ($b_a^f$) decreases.

- **Hypothesis H2**: The magnitude of the discounts due to strategic bundling when combining a unit $i$ with a package $a$ (possibly a singleton) increases, as the coefficient of variation of the competitors’ bid price for the individual unit ($b_f^i$) increases.

In what follows we describe an empirical strategy to test H1 and H2 in our data. Finding empirical support of these hypotheses is suggestive that firms’ bidding behavior is consistent with strategic bundling.

\(^{21}\)It is worth noting that most of the literature on multiproduct monopolist studies the following problem when translated to our setting: under what conditions is it preferable to exclusively submit a package bid as oppose to individual bids? In contrast, our focus is to analyze the magnitude of discounts due to strategic bundling given a mechanism that allows for both package bids and individual bids. Nevertheless, we analyzed the extensive numerical experiments in Chu et al. (2009) and confirmed that the insights obtained in the former problem carry over to the latter setting. We are grateful to the authors for providing us with this data (this analysis can be obtained upon request). Our analysis is also related to Crawford (2008) who tests the theoretical predictions for multiproduct bundling in the cable television industry, finding evidence of strategic bundling.
6.2 Quantifying Bid Randomness

In this section, we describe how to come up with reasonable estimates of statistical measures of the joint distribution of competitors’ bid prices for the different units in the auction, such as correlations and coefficient of variations, that are useful to test H1 and H2. We also discuss how we deal with several challenges that our data set imposes when testing the hypotheses.

First, the hypotheses require quantifying statistical measures about the joint distribution of competitors’ bid prices for individual units in the auction \((b_{i1}^{f'}, ..., b_{iN}^{f'})\) in the notation above. In our context, each firm may submit many bids that contain the same unit, raising the question on how to define a bid price for an individual unit. We use the stand-alone price \(d_{if}^{ft}\) – which represents the average bid price charged for that unit – as an estimate for the bid price for unit \(i\) by firm \(f'\) in auction \(t\) (see Section 4).

Second, we derived the hypotheses under the assumption that competitors do not submit discounted package bids, therefore, their bids are additive \((b_a^{f'} = \sum_{i \in a} b_{i}^{f'}\) in the notation above). However, in our data set, firms submit discounted package bids. Although one would expect firms to apply package discounts as a best response to other firms applying such discounts, our objective in this section is to isolate the effect of a specific form of strategic interaction: incentives arising from strategic bundling. The theoretical predictions described in the previous section suggest that discounts due to strategic bundling arise even in the absence of discounted package bids from competitors. For that reason, for the purpose of testing the predictions of the theory on strategic bundling, we ignore the discount function \(g_{ft}(a)\) and only study how the stand-alone prices \(d_{if}^{ft}\) (which are additive) affect firms’ discounts.\(^{22}\)

Thus motivated, throughout the subsequent analysis we use the estimates of the stand-alone prices \(\hat{d}_{if}^{ft}\) (obtained from the first step regression (4)) to capture randomness due to asymmetric information in bid prices for individual units.\(^{23}\) From the perspective of a given firm \(f\), the stand-alone prices of another firm \(f'\) in auction \(t\), \(\hat{d}_{f't} = (\hat{d}_{1f't}, ..., \hat{d}_{N(f't)}^{ft})\), are viewed as an independent and identically distributed random vector sampled from a common multivariate distribution which we seek to characterize (\(N(t)\) is the number of units in auction \(t\)). Specifically, using the estimates \(\{\hat{d}_{if}^{ft}\}\) we calculate the sample average \((E(i,t))\),

\(^{22}\)In addition, an Analysis of Variance (ANOVA) for the estimated values \(\hat{d}_{if}^{ft}\) suggest that there is statistically significant heterogeneity in stand-alone prices among firms. In contrast, a similar analysis provides only weak evidence of heterogeneity on the discount curves \(g_{ft}(a)\) among firms of similar size. Hence, it seems reasonable to assume that the source of asymmetric information are the stand-alone prices only and to ignore randomness of the discount functions when quantifying properties of the joint distribution of bids.

\(^{23}\)Variation in stand-alone prices is not a perfect proxy for uncertainty due to asymmetric information if firms can anticipate some of the variation in \(d_{if}^{ft}\). In particular, the set of local incumbents and other firm characteristics are known to all firms and they may help anticipating some variation in bid prices across firms. As a robustness check, we also did all of the subsequent analysis using the residuals of the regression of \(d_{if}^{ft}\) on the following observable firm characteristics: LocIncumb, Renew, Performance, NewFirm, FinGrade and FinGrade=1. All the main results that follow in this section were similar.
sample standard deviation \((SD(i,t))\) and coefficient of variation \((CV(i,t))\) for unit \(i\) in auction \(t\) as:

\[
E(i,t) = \frac{1}{F_{it}} \sum_f \hat{d}_{ift}
\]

\[
SD(i,t) = \left( \frac{1}{F_{it}} \sum_f (\hat{d}_{ift} - E(i,t))^2 \right)^{1/2}
\]

\[
CV(i,t) = \frac{SD(i,t)}{E(i,t)},
\]

where \(F_{it}\) is the number of firms for which we could estimate stand-alone prices for unit \(i\) in auction \(t\).\(^{24}\)

Similarly, we calculate the sample correlation between units. To get more precise estimates, we pooled all the auctions to estimate one correlation coefficient for each pair as:

\[
Corr(i,j) = \frac{1}{SD(i)SD(j)\sum_t F_{ijt}} \sum_{f,t} (\hat{d}_{ift} - E(i))(\hat{d}_{jft} - E(j))
\]

where \(E(i)\) and \(SD(i)\) are the sample average and sample standard deviation for unit \(i\), respectively, pooling data across all auctions.\(^{25}\) \(F_{ijt}\) is the number of firms with estimated stand-alone prices for both units \(i\) and \(j\) in auction \(t\). To simplify computation, we proxy for the correlation between an individual bid \(i\) and a package bid \(a\) by a volume-weighted average correlation: \(Corr(i,a) = \sum_{j \in a} Corr(i,j) v_j/v_a\).

### 6.3 Empirical Tests of Discounts Due to Strategic Bundling

The correlation among units is a key incentive that drives discounts due to strategic bundling: the higher the correlation, the lower the benefit of combining units in terms of reducing dispersion on the bid distribution.

In the data, more than 90% of the sample correlations are positive, and the median is about 0.6. The relatively high correlations suggest that the incentives to do strategic bundling are not too strong.

To test H1 and H2 empirically, we seek to estimate the effect of variability (measured by \(CV\)) and correlation on the discounts observed in the bids (measured by \(Corr\)). To isolate the discount effect of adding a unit \(i\) to a package of units \(a\), we looked at “nested” bids submitted by a firm, for which we observe a stand-alone bid for \(i\), a bid for package \(a\) (which could also be a single unit) and a bid for the package

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\(^{24}\)Some firms did not submit bids for all units, which precluded the estimation of those stand-alone prices. A few stand-alone prices were not identified (about 1%); for example, when two units are never observed in separate package bids. We also note that we consider all firms to compute statistics of the joint distribution of bids; these provide a good approximation to statistics of the joint distribution of opponents’ bids.

\(^{25}\)We also estimated different correlation coefficients for each auction, but some correlation pairs had few observations (as low as two) and were very noisy.
\[ a' = a \cup \{i\} \]

The discount, that we normalize per meal, is calculated as:

\[
D_{ft}(i, a) = \frac{b_{at}v_a + b_{ft}v_i - b_{at}'(v_a + v_i)}{v_a + v_i}.
\] (6)

Hence, discounts could only be calculated for a subset of combinations for which we observe nesting. Because we want to isolate the effect of scale discounts generated by strategic bundling, we also limited our sample to combinations which contained units which are not co-located—units which do not form clusters and are not located in the same geographic region—to remove the effect of density discounts (we revise this point later).

We estimated the following linear regression with \( D_{ft}(i, a) \) as the dependent variable, where \( i \) is an individual unit and \( a \) is a package of units:

\[
D_{ft}(i, a) = \beta_1 CV(i, t) + \beta_2 Corr(i, a) + \beta_3 Size_i + \phi_f + \tau_t + \chi_a + e_{ftia},
\] (7)

where \( a' = a \cup \{i\} \), \( e_{ftia} \) is an error term and \( \phi_f, \tau_t, \) and \( \chi_a \) are fixed effects for firm \( f \), auction \( t \), and combination \( a \), respectively. \( \chi_a \) controls for effects from scale discounts and other factors associated with package \( a \) that may affect the discount. Similarly, \( Size_i \), which measures the number of meals of unit \( i \), is included to control for scale discounts. Given the controls included in equation (7), the estimation uses variation with respect to the different units that were combined with a given package \( a \).

Table 3 shows the estimation results. Column (1) reports the estimates with a sample that includes packages of 2 units only (i.e. where the set \( a \) is a singleton). Hence, the \( Corr \) measure in this regression is the sample correlation between the stand-alone prices of the two units in the package. The discounts tend to become larger as the coefficient of variation (\( CV \)) of the units increases and as the correlation decreases, providing support for hypotheses H1 and H2. Although the effects are statistically significant, they are small in magnitude. Increasing \( CV \) by two standard deviations increases discounts by less than 3.5% of the average discount (in this sample of nested bids the average discount is 1.5¢). Decreasing \( Corr \) by two standard deviations increases discounts by less than 4.5%.

The results are similar when the sample includes packages of up to 4 units, reported in Column (2) of Table 3. A two-standard deviation change in \( CV \) and \( Corr \) increases discounts only by 2.7% and 1%, respectively. We also tested other specifications which included more controls into regression (7) and the effects were similar.\(^{26}\)

\(^{26}\)We estimated regressions which include a firm-auction fixed effect (instead of the separate fixed effects \( \phi_f \) and \( \tau_t \)). All results were similar, although in some cases the coefficients were not statistically significant. An additional prediction that follows from the multiproduct monopolist literature mentioned before is that firms with lower costs should have more incentives to engage in

26
Recall that to remove the effect of density discounts, the empirical analysis is limited to packages including units which are not co-located. However, the correlation between two units tends to increase as the distance between them becomes smaller. Therefore, in light of our results, discounts due to strategic bundling between co-located units should be smaller than for the sample of packages we analyzed.

Overall, the results suggest that although bidders do engage on strategic bundling, these markup adjustments explain only a small fraction of scale and density discounts. Therefore, we interpret the discounts shown in Figure 5 as mostly being explained by cost synergies. Nevertheless, it is remarkable to find some evidence of strategic bundling under such a complex mechanism.

7 Recommendations for Design

In this section, we use the results in Sections 5 and 6 to evaluate the current mechanism and suggest potential changes for the Chilean schools meals auction along the dimensions discussed in Sections 1 and 3.2. These recommendations are being considered by the Chilean government to redesign future auctions.

Package Bidding

Our results show that scale and density discounts are very important (together they can be as high as 8% of the average bid price). Moreover, our results suggest that a large fraction of the discounts can be explained by cost synergies as oppose to strategic bundling. Therefore, allowing package bids seems appropriate. Moreover, firms seem to take advantage of the flexibility of the current mechanism that allows them to express cost synergies due to scale and density, both of which are present. In fact, most of the bids (about 90%) are below 18 million (a range for which the scale discounts are important and increasing in magnitude), and tend to have units which are located close to each other.

Although on average the discounts due to strategic bundling seem to be relatively small, there are specific combinations for which they could be relevant, as large as 10% of the observed discounts. The efficiency of the allocation mechanism may be improved by prohibiting packages for which the incentives for strategic bundling are significant – that is, packages of TUs with high heterogeneity in prices among firms (i.e. with a high coefficient of variation in stand-alone prices) and which tend to have a low correlation in their stand-

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27 Increasing the distance between two units by 400 km. lowers the correlation by 0.2 on average.
28 See Figure 4. Also, about 70% of the clusters identified in the bids include 2 or more units.
alone prices. For example, it may be worth evaluating prohibiting packages which include TUs in urban areas close to the center of the country together with isolated TUs in the southern extreme of the country, which exhibit these characteristics.

### Promoting Competition

Our analysis suggests that market share restrictions together with splitting the TUs into multiple sequential auctions are a useful design element to promote local competition through the presence of local incumbent firms. In fact, this effect is comparable in magnitude to the discounts from cost synergies. This suggests that it may be worth revising the years at which the TUs are auctioned in order to further increase the benefits from local incumbency in the outcome of the auction.

At the time of writing of this paper, the government was evaluating whether to relax the restrictions on bidders’ market shares in order to enable them to further exploit cost synergies. Our results suggest that this change to the design could potentially reduce supplier diversification and the intensity of local competition, which in our analysis shows to be an important factor in lowering bid prices. Moreover, from a cost efficiency perspective, the benefit of allocating more than 7 TUs to a single firm in an auction (the current limit is 8) is small, because cost synergies get exhausted at that point. Parenthetically, the government was also considering increasing the maximum number of TUs allowed in a package which is currently set at 8. By a similar argument, this would complicate the mechanism and may increase the extent to which bidders can engage on strategic bundling, without any clear efficiency gains and reduction of the expected payments to bidders. In summary, our analysis suggests that allowing firms to operate in a larger scale could potentially increase the procurement cost for the government in the long-run.

### 8 Conclusions

We have empirically analyzed the procurement combinatorial Chilean auction for school meals. Our empirical analysis provides substantive insight into bidding behavior in CAs, suggesting that firms are sophisticated in their bidding and that they take advantage of the flexibility of the bidding mechanism by expressing cost synergies and by adjusting their markups. These insights can perhaps be extrapolated to the bidding behavior in other CAs.

In addition, our results inform important auction design issues, highlighting how the simultaneous consideration of the firms’ operational cost structure and their strategic behavior is key to the successful design of a CA.

Our work leaves several interesting directions for future research. First, a significant challenge in our
approach was to distinguish whether discounts were determined by cost synergies or markup adjustments. An important factor identified in the literature that affects markups for package bids is strategic bundling; we empirically tested how much of the measured discounts can be explained by this mechanism. In future research, it could be interesting to explore whether there are other important mechanisms that affect markups for package bids and test whether they are present in the data. An alternative approach is to use a structural estimation method that poses a model of bidders’ behavior, imposing restrictions on how markups are determined and thereby identifying costs. The main drawback of this approach is that identification relies on strong assumptions on bidders’ behavior and that it is computationally demanding. However, we believe it is a useful complement to the work we have presented in this paper and it is being matter of current research.

Our analysis was focused on single-round sealed-bid first-price CAs, because the Chilean government is not planning on changing this format and several CAs in practice have the same format. However, it could be interesting to study the impact of other auction formats (e.g. a multi-round ascending auction) in future research.

Finally, while our focus has been in the Chilean auction for school meals, we believe our method of analysis can be used broadly for many other CAs, enabling the study of a wide range of issues in auctions. We hope that our study together with future similar studies in other CAs could enhance the understanding of bidders’ behavior and, as a consequence, give us insights to improve the design of this type of auctions.

References


A Clustering Algorithm

Consider a set of units \( a = (a_1, ..., a_{\text{card}(a)}) \), where \( \text{card}(s) \) is the cardinality of the set \( a \). Let \( r \) be a pre-defined cluster radius (in our main estimation we choose \( r = 150 \ km \)). For a set of units \( b \), let \( C_b \) be the weighted geographic center of \( b \). The weighted geographic center of a set of units is calculated as the weighted average latitude and longitude of all the schools it contains, weighted by the school populations. The distance between two set of units \( a \) and \( b \), \( \text{dist}(a, b) \), is the bird-fly distance between the weighted geographic centers of the respective sets.

We describe the algorithm that builds the set of clusters formed by the units in \( a \), \( Cl(a) \).

Algorithm 1 Clustering Algorithm

1: Let \( A \) be the collection of sets \( \{\{a_1\}, ..., \{a_{\text{card}(a)}\}\} \), \( d := 0 \)
2: while \( d < r \) do
3:  Let \( d := \min_{b,c \in A, b \neq c} \text{dist}(b, c) \). Let \( (b^*, c^*) := \arg\min_{b,c \in A, b \neq c} \text{dist}(b, c) \)
4:  if \( d < r \) then
5:   \( A := A \setminus (b^* \cup c^*) \), \( A := A \cup \{b^*, c^*\} \)
6:  end if
7: end while
8: \( Cl(a) := A \)
<table>
<thead>
<tr>
<th>Size</th>
<th>Estimate</th>
<th>SE</th>
<th>Size</th>
<th>Estimate</th>
<th>SE</th>
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<td>[3,6]</td>
<td>12.14</td>
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</tr>
<tr>
<td>[4,6]</td>
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<td>(0.09)</td>
<td>[6,9]</td>
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<td>[9,12]</td>
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<td>[8,10]</td>
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<td>[12,15]</td>
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<td>[10,12]</td>
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<td>(0.10)</td>
<td>[15,18]</td>
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</tr>
<tr>
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<td>(0.10)</td>
<td>[18,21]</td>
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<td>(0.35)</td>
</tr>
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<td>[14,16]</td>
<td>7.27</td>
<td>(0.11)</td>
<td>[21,24]</td>
<td>26.26</td>
<td>(0.36)</td>
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<td>[18,20]</td>
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<td>[27,]</td>
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<tr>
<td>[22,24]</td>
<td>6.78</td>
<td>(0.33)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>[24,]</td>
<td>7.03</td>
<td>(1.18)</td>
<td></td>
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Table 1 – Results from first step regression (equation (4)). Robust standard errors shown in parenthesis. Size measured in million meals per year. Number of observations is 409,831. Centered R-square is equal to 0.98.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<td><strong>LocIncumb</strong></td>
<td>-13.29**</td>
<td>-10.14</td>
</tr>
<tr>
<td></td>
<td>(3.876)</td>
<td>(4.555)</td>
</tr>
<tr>
<td><strong>LocComp</strong></td>
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<td>-3.578*</td>
</tr>
<tr>
<td></td>
<td>(1.504)</td>
<td>(1.545)</td>
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<tr>
<td><strong>Renew</strong></td>
<td>-18.02**</td>
<td>-8.839</td>
</tr>
<tr>
<td></td>
<td>(6.412)</td>
<td>(6.734)</td>
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<tr>
<td><strong>Size</strong></td>
<td>-21.09</td>
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<tr>
<td></td>
<td>(11.53)</td>
<td>(11.08)</td>
</tr>
<tr>
<td><strong>Special meals</strong></td>
<td>497.8**</td>
<td>497.7**</td>
</tr>
<tr>
<td></td>
<td>(36.21)</td>
<td>(34.77)</td>
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<tr>
<td><strong>Performance</strong></td>
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<td>-32.83**</td>
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<td></td>
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<tr>
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<td>(4.394)</td>
<td>(5.261)</td>
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<td>-7.362**</td>
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<td>(2.074)</td>
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<tr>
<td><strong>FinGrade=1</strong></td>
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<td>-58.97**</td>
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<tr>
<td></td>
<td>(9.086)</td>
<td>(11.36)</td>
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<td><strong>Observations</strong></td>
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<td>2839</td>
</tr>
<tr>
<td><strong>R-Square</strong></td>
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<td>0.741</td>
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Table 2 – Results from second-stage regression (equation (5)). Both specifications include dummy variables for firm, auction and unit, and the controls specified in Section 4.2. Specification (2) also includes dummy variables for firm-region (which absorb the firm dummies). Standard errors in parentheses. * and ** indicate statistical significance at 0.05 and 0.01 confidence levels.
<table>
<thead>
<tr>
<th></th>
<th>(1) Packages of 2 units</th>
<th>(2) Packages of 4 or less units</th>
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</thead>
<tbody>
<tr>
<td>CV</td>
<td>2.640*</td>
<td>3.352**</td>
</tr>
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<td></td>
<td>(1.262)</td>
<td>(0.443)</td>
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<tr>
<td>Correlation</td>
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<td>-0.295*</td>
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<tr>
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<td>(0.241)</td>
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<tr>
<td>Observations</td>
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<td>77619</td>
</tr>
<tr>
<td>R-square</td>
<td>0.43</td>
<td>0.64</td>
</tr>
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Table 3 – Results of the regression analysis on strategic bundling (equation (7)). Both regressions include the control variable Size and dummies for firm, auction and combination. Standard errors in parentheses. *, ** indicate statistical significance at the 0.05 and 0.01 confidence levels.
Figure 1 – Map of Chile. Points indicate the centers of the territorial units (TUs); colors indicate the year in which they are auctioned.
Figure 2 – Histogram of number of units in a bid.

Figure 3 – Bid price as a function of the size of the bid. Includes all bids in auctions 1999, 2000 and 2001.
Figure 4 – Average maximum distance among TUs contained in a bid, for bids with different number of units. Dashed line indicates the average maximum distance when the units in the package are sampled at random.
Figure 5 – Estimates of discount curves due to scale (top) and density (bottom). Dotted line indicates the 95% confidence interval of the estimate.
Figure 6 – Estimates of density discounts for different cluster sizes.