

Adverse Selection, Reputation and Sudden Collapses in Secondary Loan Markets ^{*}

V.V. Chari

UMN, MPLS Fed

chari002@umn.edu

Ali Shourideh

Wharton

shouride@wharton.upenn.edu

Ariel Zetlin-Jones

CMU

azj@andrew.cmu.edu

June 28, 2013

Abstract

Loan originators often securitize some loans in secondary loan markets and hold on to others. New issuances in such secondary markets collapse abruptly on occasion, typically when collateral values used to secure the underlying loans fall and these collapses are viewed by policymakers as inefficient. We develop a dynamic adverse selection model in which small reductions in collateral values can generate abrupt inefficient collapses in new issuances in the secondary loan market by affecting reputational incentives. We find that a variety of policies intended to remedy market inefficiencies do not help resolve the adverse selection problem.

^{*}We are grateful to Kathy Rolfe for editorial assistance and Hugo Hopenhayn, Roozbeh Hosseini, Larry Jones, Patrick Kehoe, Guido Lorenzoni, Chris Phelan as well as seminar participants at ASU, Kellogg, Yale, the 2009 SED Meeting, New York and Minneapolis Fed, the Conference on Money and Banking at the University of Wisconsin, and XII International Workshop in International Economics and Finance in Rio for helpful comments. Chari and Shourideh are grateful to the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

A central question in the economics of financial markets is to account for fluctuations in the volume of trade. In late 2007 and also in the late 1920s, the volume of new issuances in the secondary loan market fell sharply. In this paper we develop a theory of the volume of new issuances of secondary loans and show that reductions in collateral values can generate a fall in this volume. Our theory of fluctuations in trade volume follows the literature in that it is based on changes in informational frictions. These frictions arise because asymmetric information among buyers and sellers creates an adverse selection problem. We argue that adverse selection alone is unlikely to generate fluctuations in trade volumes but that adverse selection in the presence of reputational concerns can induce such fluctuations. We use our model to show that in the face of collapses in trading volume in secondary loan markets, asset purchase policies are either ineffective or require the use of tax revenues from other sources.

Sudden collapses in secondary loan markets raise concerns in part due to concerns that loans are inefficiently allocated across financial institutions and also that a reduction in trading volume in secondary markets affects the volume of primary loans and therefore adversely affects economic activity. The variety of policy proposals initiated and implemented in the wake of the collapse in secondary loan markets during the financial crisis, clearly shows that policymakers were concerned about this sudden collapse. We focus on theories of the volume of trade based on informational frictions because, as we argue, reductions in collateral values naturally generate reductions in trading volume. Such reductions in collateral values occur when housing prices show signs of declining relative to previous expectations or economic activity show signs of slowing.

Secondary loan markets reallocate part or all of loans from originating institutions to other institutions. We model this reallocation by assuming that some institutions have a comparative advantage in originating loans and other have a comparative advantage in holding and managing loans. This reallocation, in our model, is disrupted by informational frictions in the sense that loan originators differ in their ability to originate high-quality loans and are better informed about this ability than are potential purchasers. This informational friction creates an *adverse selection* problem.

Models with adverse selection seem like a promising avenue for generating fluctuations in the volume of trade in newly issued securitized loans when the values of the underlying loans fluctuate. In such models, the volume of trade falls when the magnitude of informational asymmetries increases. In our model, such informational asymmetries increase when the default value of a loan falls. We refer to the default value of a loan as the *collateral value*.

In this sense, adverse selection induces the volume of new issuances of securitized loans to fall when the collateral value of the underlying loan portfolio falls.

Realizing this promise requires that adverse selection problems persist over time so that buyers remain uncertain about loan originator ability. Ensuring that this uncertainty persists in a model is particularly essential given that a relatively small number of institutions dominate the secondary loan market in the United States and there is considerable evidence that measures of their ability persist over time (see [Ross \(2010\)](#) and [Fang \(2005\)](#)). These features of the secondary loan market suggest that originators may have incentives to acquire a reputation for originating high-quality loans.

Understanding these incentives requires developing a dynamic model with adverse selection. Such models face two challenges in delivering on their promise when the ability of loan originators to originate high quality loans is persistent over time. One challenge, the *persistence challenge*, is to ensure the adverse selection problem persists over time. The second challenge, the *correlation challenge*, is to ensure that fluctuations in collateral values induce fluctuations in volume of trade across high and low quality assets.

Both challenges arise in dynamic adverse selection models in which reputational concerns play no role. In such models, loan purchasers design contracts for originators which induce different types of originators to select different contracts. Because purchasers induce self-selection, future purchasers can infer the ability of originators from their past contract choices. Thus, in such models, the adverse selection problem does not persist. Furthermore, absent reputational concerns, contracts designed by purchasers have the feature that while the volume of trade of high quality loans is increasing in collateral values the volume of trade of low quality loans is independent of collateral values. This feature implies that fluctuations in collateral values should affect only the volume of high quality loans.

Our main contribution is to show that sufficiently strong reputational concerns can address both challenges. When reputational concerns are strong, low ability originators have strong incentives to mimic the behavior of their high ability counterparts. We show that such mimicking implies that purchasers can induce at best partial self-selection of originators. The extent of such partial self-selection depends on the beliefs of purchasers about the ability levels of originators. When purchasers are relatively optimistic about this ability level, low ability originators completely pool with high ability originators in the sense that both types of originators choose to sell their loans at the same contract. When purchasers are relatively pessimistic about originators' ability level, low ability originators partially pool with their high ability counterparts in the sense some low ability originators choose to mimic high ability originators by holding their loans while others choose to sell their loans.

These mimicking incentives imply that adverse selection problems persist because future

purchasers are unable to infer perfectly the ability of originators from their past contract choices. These incentives also imply that changes in collateral values change the behavior of low ability originators as well as their high ability counterparts. In particular, a fall in collateral values induces a decline in the volume of trade of both high and low quality loans.

We conduct our analysis primarily in an infinite horizon model. We do so because in finite horizon models, fluctuations in the aggregate volume of trade depend on the exogenously specified initial distribution of buyers' beliefs about bank quality. In the infinite horizon model, in the long run, the distribution of buyers' beliefs is endogenously determined. In a deterministic version of this model, we show that an unanticipated permanent decline in collateral values leads to a decline in the volume of trade by both high- and low-quality banks in all periods. Absent reputational concerns, such a decline in collateral values leads at best to a decline in the period of impact.

In order to study the effects of temporary and anticipated shocks, we analyze a stochastic version of our infinite horizon model with aggregate shocks to collateral values. We show that aggregate trading volume is relatively low when the collateral value is relatively low. We also show in the stochastic version of the model that relative to the deterministic model, low-quality banks have stronger incentives to hold onto their loans in the hope that they will be able to sell at favorable prices in the future when collateral values are relatively high. In this sense, anticipation effects exacerbate adverse selection problems.

We argue that allowing for reputational concerns in adverse selection models can alter policy prescriptions in important ways. We show that when reputational concerns are weak, a version of our adverse selection model with a different equilibrium concept can have multiple equilibria and that policy interventions such as offers by the government to purchase securities at an actuarially fair market price can improve allocative efficiency by eliminating inefficient equilibria. Such policies resemble the policy of deposit insurance in bank run models in the sense that the government policy can eliminate bad equilibria without necessarily committing the government to expend resources. In this sense, this version provides one possible rationale for policy interventions. We argue that the different equilibrium concept used to generate multiple equilibria is inappropriate in the context of our model.

Since our model generates sudden collapses in the aggregate volume of trade, we ask how an asset purchase policy would affect equilibrium outcomes. We show that if the government's purchase prices are set at an actuarially fair level, banks choose not to sell their loans to the government so that volume of trade stays depressed and the policy is ineffective. If the prices are set sufficiently above an actuarially fair level, then the policy does generate an increase in the volume of trade but the government necessarily has to use revenues from other sources to make up for its losses in the secondary loan market. This finding shows that

incorporating reputational concerns into policy analysis can lead to substantially different implications for the outcomes of various policies.

1.1 Related Literature

Our work here is related to an extensive literature on adverse selection in asset markets, including the work of Myers and Majluf (1984), Glosten and Milgrom (1985), Kyle (1985), and Garleanu and Pedersen (2004) as well as to the related securitization literature, specifically, the work of DeMarzo and Duffie (1999) and DeMarzo (2005). See also Eisfeldt (2004), Kurlat (2009), Guerrieri et al. (2010) and Guerrieri and Shimer (2011) for analyses of adverse selection in dynamic environments. We add to this literature by analyzing how reputational incentives affect adverse selection problems.

Our assumption that buyers have less information concerning the loan quality of a bank is in line with a descriptive literature that argues that secondary loan markets feature adverse selection (see, for example, the work of Dewatripont and Tirole (1994), Ashcraft and Schuermann (2008), and Arora et al. (2009)). Also, a growing literature provides data on the presence of adverse selection in asset markets. For example, Ivashina (2009) finds evidence of adverse selection in the market for syndicated loans. Downing et al. (2009) find that loans that banks held on their balance sheets yielded more on average relative to similar loans which they securitized and sold. Drucker and Mayer (2008) argue that underwriters of prime mortgage-backed securities are better informed than buyers and present evidence that these underwriters exploit their superior information when trading in the secondary market. Specifically, the tranches that such underwriters avoid bidding on exhibit much worse than average ex post performance than the tranches that they do bid on.

A recent paper by Elul (2011) presents evidence that is consistent with our model. Elul (2011) shows that returns on securitized loans and loans held by originators were similar before 2006 and that returns on securitized loans were lower than returns on comparable loans after 2006. This evidence is consistent with our model in the following sense. Our model implies that when collateral values underlying loans are relatively high, most high-quality banks with high costs of managing the loans choose to sell their loans; but when collateral values are relatively low, such banks choose to hold their loans. Before 2006, land values were rising, so it seems reasonable to suppose that collateral values were relatively high. After 2006, land values stopped rising and in some cases fell, so it seems reasonable to suppose that collateral values were lower than they had been.

Finally, Mian and Sufi (2009) present evidence that securitized loans were more likely to default than nonsecuritized loans. This evidence is consistent with our model in the sense

that for all realizations of the aggregate shock, the default rate of securitized loans is at least as high as that of held loans, and for some realizations the default rate of securitized loans is higher than that of held loans.

Our work is also related to an extensive literature on reputation. [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1982\)](#) argue that equilibrium outcomes are better in models with reputational incentives than in models without them. In the banking literature, [Diamond \(1989\)](#) develops this argument. More recently, [Mailath and Samuelson \(2001\)](#) analyze the role of reputational incentives in infinite horizon economies and provide conditions under which they can improve outcomes. In contrast, [Ely and Välimäki \(2003\)](#) and [Ely et al. \(2008\)](#) describe models in which reputational incentives can worsen outcomes. Our work here combines the results in this literature by showing that reputational models can have multiple equilibria. In some of these equilibria, reputational incentives can generate better outcomes; in others, they can generate worse. Furthermore, using techniques from the global games literature, we develop a refinement that produces a unique, fragile equilibrium. Perhaps the work most closely related to ours is that of [Ordoñez \(2008\)](#). An important difference between our work and his is that our model has equilibria that are worse than the static equilibrium, so that reputational incentives can lead to outcomes that are ex post less efficient than those in a model without these incentives.

Our analysis of policy is closely related to recent work by [Philippon and Skreta \(2011\)](#) who analyze a variety of policies in a model with adverse selection. The main difference with our work is that we focus on the incentives induced by reputation, whereas they analyze a static model.

2 Static Model of Adverse Selection in Secondary Loan Markets

In this section, we introduce and analyze a static model of secondary loan markets that feature adverse selection. We show that such a model has a unique separating equilibrium and we show that fluctuations in collateral values induce fluctuations in aggregate trading volume.

2.1 Model

Here we introduce a static model of a secondary loan market which features comparative advantage and adverse selection. This model should be interpreted as describing the last period of a finite horizon model. Consider an economy with a large number of loan originators

referred to as banks and a large number of buyers. Both banks and buyers are risk neutral. Each bank is endowed with a loan portfolio. We normalize the size of the loan portfolio to be 1. The loans in a portfolio can also be thought of more generally as an investment opportunity such as a project, a mortgage, or an asset-backed security.

Loans are risky in the sense that they yield a return of \bar{v} if there is no default on the loan and a return of \underline{v} in the event of default on the loan. We refer to \underline{v} as the *collateral value* of the loan. Let $v = \bar{v} - \underline{v}$ denote the *spread* in returns. Note for future reference that a fall in the collateral value, \underline{v} , induces an increase in the spread, v . It turns out that in fluctuations in collateral values induce fluctuations in outcomes solely through fluctuations in the spread, v . The probability of no default is denoted by π . For simplicity, we assume that the probability of no default is the same for all loans in a given bank's portfolio. Banks are heterogeneous in the sense that the probability of no default differs across different banks. We assume that there are two types of banks so that the probability of no default can take on one of two values $\pi \in \{\underline{\pi}, \bar{\pi}\}$ with $\underline{\pi} < \bar{\pi}$. We refer to a bank that has a loan portfolio of type $\bar{\pi}$ as a *high-quality bank* and one with a loan portfolio of type $\underline{\pi}$ as a *low-quality bank*.

The bank chooses how much of its loan portfolio to sell in the secondary market. Let x denote the fraction of the loan portfolio that the bank sells. Let t denote the payment the bank receives from buyers for selling x loans so that $p = t/x$ is the price per loan. In Appendix A, we show that the payoff for a bank of type π which sells a fraction x of its loan portfolio at payment t is given by

$$t + (1 - x)(\pi v - c) \tag{1}$$

subject to a normalization and the profit of a buyer who purchases a fraction of loans x for a payment t from a bank of type π is given by

$$x\pi v - t.$$

We assume that the buyers have a comparative advantage in managing loans. We model this comparative advantage by assuming that the cost for the bank of managing a loan is $c > 0$ and we normalize the management cost for buyers to be 0. This comparative advantage provides the main economic role for the secondary market. We intend the management costs of the loan to represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank's portfolio, among other potential factors.

We introduce adverse selection by assuming that the bank knows the type of loans in its portfolio and that potential buyers do not. Buyers believe that the bank is high-quality with probability μ and low-quality probability $1 - \mu$. We let μ denote the *reputation* of the bank.

In the dynamic models that follow, this reputation will evolve endogenously. Banks differ in their reputation levels. Since the sales of an individual bank do not affect other banks, we focus on the secondary loan market for a given bank with reputation μ . We will also model buyers as engaging in Bertrand-style price competition, so it suffices to restrict the number of potential buyers to two.

In the secondary loan market, buyers simultaneously offer contracts to the bank. A contract specifies a fraction of loans the buyer will purchase and the payment for such a purchase. Since we have two types of banks, a *contract*, z , consists of a four-tuple (x_h, t_h, x_l, t_l) . Here a pair (x_i, t_i) is an offer that is intended for a bank of type $i = l, h$. After buyers make contract offers, the bank chooses whether to accept one of the contracts or to reject both contracts. If the bank chooses to accept one of the buyer's contracts, the bank chooses which of the pairs offered by that buyer to accept. Since the bank can choose which offer to accept, without loss of generality we can restrict attention to contracts that are incentive compatible and satisfy participation constraints in the sense that they satisfy

$$t_h + (1 - x_h)(\bar{\pi}v - c) \geq t_l + (1 - x_l)(\bar{\pi}v - c) \quad (2)$$

$$t_l + (1 - x_l)(\underline{\pi}v - c) \geq t_h + (1 - x_h)(\underline{\pi}v - c) \quad (3)$$

$$t_h + (1 - x_h)(\bar{\pi}v - c) \geq \bar{\pi}v - c \quad (4)$$

$$t_l + (1 - x_l)(\underline{\pi}v - c) \geq \underline{\pi}v - c \quad (5)$$

The incentive constraint 2 ensures that facing a contract z , the high-quality bank prefers the offer (x_h, t_h) intended for the high-quality bank and the incentive constraint 3 ensures that the low-quality bank prefers the offer (x_l, t_l) intended for the low-quality bank. The participation constraints 4 and 5 ensure that the bank is not better off by rejecting the contract. Let Z denote the set of contracts which satisfy incentive compatibility and the participation constraints.

A well known problem with adverse selection models is that equilibria in pure strategies sometimes do not exist (see for example [Rothschild and Stiglitz \(1976\)](#)). [Dasgupta and Maskin \(1986\)](#) show that in many such environments, mixed strategy equilibria do exist. We follow [Dasgupta and Maskin \(1986\)](#) and [Rosenthal and Weiss \(1984\)](#) in allowing for mixed strategies on the part of both buyers and banks.

In the static model, we restrict attention to equilibria which have pure strategies by banks and possibly mixed strategies by buyers. A strategy for buyer $j = 1, 2$ is a distribution function $F_j(z)$ over the set Z . A strategy for the bank consists of an action $\delta_j(z_1, z_2; \pi)$ for $j = 1, 2$ where $\delta_j = 1$ denotes a decision to accept contract j and $\delta = 0$ denotes a decision not to accept contract j . Facing F_{-j} and δ , the profits earned by a buyer offering contract

z are given by

$$\int \mu \delta_j(z, z_{-j}; \bar{\pi}) [x_h \bar{\pi} v - t_h] + (1 - \mu) \delta_j(z, z_{-j}; \underline{\pi}) [x_l \underline{\pi} v - t_l] dF_{-j}(z_{-j}) \quad (6)$$

An *equilibrium* consists of strategies for buyers and a strategy for the bank such that for all z_j in the support of F_j there is no alternative contract \hat{z}_j which earns strictly higher profits and the bank's strategy specifies that it chooses the contract that offers the highest payoff. An equilibrium is *symmetric* if both $F_1 = F_2 = F$.

We say that an equilibrium is *separating* if the offers accepted by low- and high-quality banks are different. In such a situation, after trades have occurred, the type of the bank is known. Separating equilibria are of interest in dynamic versions of our model because future buyers could exploit knowledge of choices by the bank in previous periods and thus affect the behavior of the bank.

We say that a contract features no cross-subsidization across types of banks if each offer (x_i, t_i) makes zero profits. A contract features cross-subsidization if one of the offers makes positive profits and the other makes negative profits.

We turn now to characterizing a symmetric equilibrium. The equilibrium has three key properties. At each contract in the support of F , the low-quality bank sells its entire loan portfolio ($x_l = 1$), buyers make zero profit, and the incentive constraint for the low type, [3](#), holds with equality.¹ These features imply that each equilibrium contract can be indexed by the payment to the low-quality bank, t_l . Given t_l and $x_l = 1$, the offer (x_h, t_h) can be determined from the zero profit condition and the incentive constraint for the low type.

When the bank's reputation is below a threshold, $\tilde{\mu}$, it is straightforward to use arguments similar to those in [Rothschild and Stiglitz \(1976\)](#) to establish that a pure strategy equilibrium exists with no cross-subsidization. The equilibrium outcome, referred to as the least-cost separating outcome, has the following form:

$$z = (x_h \bar{\pi} v, x_h, \underline{\pi} v, 1) \quad (7)$$

where x_h is given by the equality version of the incentive constraint for the low-quality bank:

$$x_h \bar{\pi} v + (1 - x_h) (\underline{\pi} v - c) = \underline{\pi} v.$$

The threshold $\tilde{\mu}$ is given by

$$\tilde{\mu} = \frac{(\bar{\pi} - \underline{\pi}) v}{(\bar{\pi} - \underline{\pi}) v + c}. \quad (8)$$

Above $\tilde{\mu}$, when the bank's reputation is relatively high, it is possible to use arguments like those in [Rothschild and Stiglitz \(1976\)](#) to show that the least-cost separating outcome

¹In a slightly different environment, [Dasgupta and Maskin \(1986\)](#) showed that any equilibrium must have these properties.

is not an equilibrium. We follow [Rosenthal and Weiss \(1984\)](#) in showing that a mixed strategy equilibrium with cross-subsidization exists for high levels of bank reputation. This equilibrium has a continuous distribution over buyers' contracts F . Since any equilibrium contract can be indexed by the payment to the low-quality bank, t_l , it follows that we may describe the distribution F over a single dimensional variable, t_l . With some abuse of notation, we use F to denote the distribution over t_l . This distribution has support given by $[\underline{\pi}v, \hat{p}(\mu)]$ where

$$\hat{p}(\mu) = \mu\bar{\pi}v + (1 - \mu)\underline{\pi}v \quad (9)$$

denotes the price per loan in a pooling outcome in which both types of banks sell their entire loan-portfolio. In the appendix we show that the equilibrium distribution F has the form

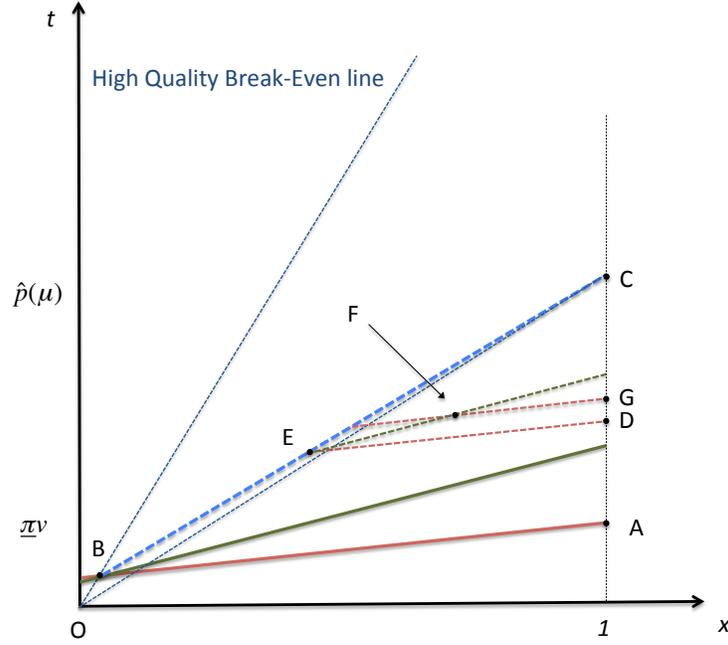
$$F(t_l) = \left(\frac{t_l - \underline{\pi}v}{\mu(\bar{\pi} - \underline{\pi})v} \right)^{\frac{\mu c}{(1-\mu)(\bar{\pi}-\underline{\pi})v} - 1}. \quad (10)$$

Proposition 1 *The static model has a separating equilibrium. If $\mu \leq \tilde{\mu}$ equilibrium contract satisfies 7 and has no cross-subsidization. If $\mu \geq \tilde{\mu}$, then the equilibrium has mixed strategies by buyers, the distribution over contracts is given by 10 and has cross-subsidization.*

The proof of the proposition when the bank's reputation is low, or $\mu \leq \tilde{\mu}$, is a straightforward application of methods in [Rothschild and Stiglitz \(1976\)](#). The proof of the proposition when the bank's reputation is high, or $\mu \geq \tilde{\mu}$ is somewhat more subtle and parallels the methods in [Rosenthal and Weiss \(1984\)](#). Figure 1 provides a plot of the nature of the equilibrium and helps provide some intuition for the proof when $\mu \geq \tilde{\mu}$. In this figure, we have plotted the payments t against the fraction of loans purchased x . The equilibrium contracts, z , feature offers for low-quality banks which range from the points A to C in which the low-quality bank sells all of its loan portfolio at different payments. Associated with any point on the line AC , say the point F , the companion offer for the high-quality bank is the point D on the line BC where the line DF represents the incentive constraint for the low quality bank. The pair of offers (D, F) is constructed so that the contract makes zero profits. Notice that since payments at the point F are strictly greater than $\underline{\pi}v$, buyers lose profits on the low-quality banks at offer F . The point D is chosen so that the profits that the buyers make on the high-quality banks exactly offsets the losses from at point F .

The proof of the proposition is by contradiction. Figure 1 helps illustrate the nature of the contradiction. The point (\hat{x}_h, \hat{t}_h) represents an offer for high-quality banks associated with a deviation contract. Consider the best companion offer for the low-quality bank associated with this deviation. Clearly this companion offer must lie on the low-quality bank's indifference curve running through (\hat{x}_h, \hat{t}_h) . To see that this companion offer must have $\hat{x}_l = 1$, note that a unit increase in \hat{x}_l allows holding costs to be reduced c units.

Figure 1: Equilibrium in the Static Model when bank reputation is high



This surplus can be shared between the buyer and the low-quality bank without violating incentives. It follows that associated with the deviation (\hat{x}_h, \hat{t}_h) , the best companion offer is represented by the point E . Next consider the point D which has the property that the high quality bank is indifferent between the offer D and the deviation offer (\hat{x}_h, \hat{t}_h) . Suppose now that buyer 2 follows the equilibrium strategy and buyer 1 deviates to (\hat{x}_h, \hat{t}_h) and the associated companion offer E . Buyer 1 attracts the high quality bank whenever the (mixed-strategy) offer by buyer 2 yields the bank lower utility than D and attracts the low-quality bank whenever the (mixed-strategy) offer by buyer 2 yields the bank lower utility than E . Notice that the probability that buyer 2 makes an offer worse than D to the high quality bank is equal to the probability that buyer 2 makes an offer worse than point F to the low-quality bank. We have just established that the probability that low-quality banks are attracted to the deviation is higher than under the contract (D, F) . Losses from a low-quality bank are higher under the deviation than under (D, F) while profits from a high-quality bank are higher. Thus, for a deviation not to be profitable, it must be that the losses from attracting low-quality banks with higher probability offset the increased profits from high-quality banks. In Appendix A, we show that when the distribution function satisfies 10, the losses exceed the increased profits.

Dynamic versions of this adverse selection model turn out to have multiple equilibria. In terms of uniqueness of equilibria in the static model, it is straightforward to show that the

equilibrium is unique for reputation levels below $\tilde{\mu}$. In the Technical Appendix, we prove that this equilibrium is unique if buyers are restricted to contracts which are monotone in the sense that if a contract z_1 is preferred by a low-quality bank to a contract z_2 then z_1 is also preferred by a high-quality bank to z_2 . Following [Rosenthal and Weiss \(1984\)](#), we conjecture that the equilibrium in the static model is unique for reputation levels above $\tilde{\mu}$ with unrestricted contracts as well.

The uniqueness of equilibrium in our model depends critically on strategic behavior by buyers. To see the role of this strategic behavior, suppose instead that we had simply imposed as a condition of equilibrium that buyers make zero profits. This version of the model has multiple equilibria. One equilibrium has all buyers offering a low price per loan of $\bar{\pi}v$ and only low-quality banks selling. For sufficiently high reputation levels, this version has another equilibrium in which all buyers offer the pooling price $\hat{p}(\mu)$ and both high- and low-quality banks sell all of their loans. We discuss the role of this type of multiplicity when we turn to policy implications.

The equilibrium of the dynamic version of our model depends in an important way on the nature of the equilibrium in the static model when the reputation level of the bank is 0 or 1. From [10](#), it is straightforward to show that as μ converges to 1, the distribution function for F converges to the distribution function which puts all mass on $\bar{\pi}v$. In this limit, buyers offer a payment of $\bar{\pi}v$ and buy the bank's entire loan portfolio.

In terms of the role of mixed strategies in our equilibrium, a recent literature, see for example [Guerrieri et al. \(2010\)](#), has developed an alternative resolution to the problem of non-existence of pure strategy equilibria in adverse selection models. [Guerrieri et al. \(2010\)](#) et al develop a search and matching model with capacity constraints which has a pure strategy equilibrium. Their equilibrium coincides with the least-cost separating outcome for all reputation levels. In this outcome, equilibrium contracts do not depend on the reputation level except when the bank is known for sure to be of high-quality. In this sense, the outcomes are discontinuous in the reputational levels of banks. We prefer our mixed strategy equilibrium because the distribution over equilibrium contracts is continuous for all reputation levels. Furthermore, an attractive feature of the mixed strategy equilibrium is that as the probability that the buyers believe the bank is high-quality goes to 1, the equilibrium outcomes converge to an efficient outcome which maximizes the welfare of the high-quality bank subject to the incentive constraints and the break-even constraint for buyers. An open question is whether our results on the persistence of adverse selection problems in dynamic environments hold in environments similar to that in [Guerrieri et al. \(2010\)](#).

Next, we turn to the promise of adverse selection models in generating fluctuations in the volume of trade associated with changes in collateral values. For low levels of reputation,

the expected volume of trade, T , in our model is given by

$$T = \mu \frac{c}{(\bar{\pi} - \underline{\pi})v + c} + (1 - \mu). \quad (11)$$

To see this result, note that the high-quality bank sells a fraction of its loans, $x_h = c/[(\bar{\pi} - \underline{\pi})v + c]$ and the low-quality bank sells all of its loans. For high levels of reputation, straightforward computation using the form of F yields that the expected volume of trade in our model is given by

$$T = \mu \left[1 - \frac{1 - \mu}{\mu c} \left[\frac{1}{(\bar{\pi} - \underline{\pi})v} + \frac{c}{[(\bar{\pi} - \underline{\pi})v]^2} \right]^{-1} \right] + (1 - \mu). \quad (12)$$

It is easy to show that at the threshold $\tilde{\mu}$, the expected volume of trade is the same in [11](#) and [12](#). From [8](#), we see that the threshold $\tilde{\mu}$ increases as either v or $(\bar{\pi} - \underline{\pi})$ increases. This result together with the form of [11](#) and [12](#) establishes that the expected volume of trade for an individual bank declines as either v or $(\bar{\pi} - \underline{\pi})$ increases. Recall that the value v is defined as the difference between the value of a loan when there is no default, \bar{v} , and the collateral value \underline{v} . Thus, we have shown that if the collateral value, \underline{v} decreases or the dispersion in bank quality, $(\bar{\pi} - \underline{\pi})$, increases the expected volume of trade for a bank of a given reputation, μ falls. Since the expected volume of trade for banks of all reputation levels falls, it follows that the aggregate volume of trade falls as well. We summarize this discussion in the following proposition.

Proposition 2 (*The Promise of Adverse Selection Models in Generating Trade Volume Fluctuations*) *Decreases in collateral values and increases in the dispersion of bank quality reduces expected trade volumes for banks of all reputational levels and decreases aggregate trade volume.*

While the static adverse selection model is promising in generating a fall in the expected volume of trade when collateral values fall, note that all of the decline is due to a decline in volume of trade of high-quality banks. Low-quality banks always sell their entire loan portfolio. In this sense, the static adverse selection model cannot generate a decline in the volume of trade by all quality-types of banks so that the static adverse selection model faces a correlation challenge. Next we turn to dynamic models in part to ask whether reputational effects can address this challenge.

3 A Dynamic Model of Adverse Selection in Secondary Loan Markets

In this section, we develop a dynamic model in which reputational concerns induce adverse selection to persist. We will show that the equilibrium outcomes in the dynamic model feature incomplete information revelation in the sense that the quality of the bank cannot be perfectly inferred from its past actions. In particular, we show that the equilibrium features partial pooling for low values of bank reputation in the sense that future buyers are able to obtain some information about the bank's quality from its past actions. We also show that the equilibrium features complete pooling for high values of bank reputation in the sense that future buyers are able to obtain no information about the bank's quality from its past actions. The incompleteness of information revelation implies that fluctuations in collateral values can induce fluctuations in the aggregate volume of trade as well as in the volume of trade by banks of different qualities in all periods. In this sense we show that reputational concerns can help address the persistence and correlation challenges.

Consider a dynamic version of our adverse selection model. In each period $t = 1, 2, \dots, T$, banks originate a loan portfolio of size normalized to be 1 and choose whether to hold or sell their portfolio. Buyers offer contracts intended for high- and low-quality banks exactly as in our static model. Banks discount future payoffs at rate β .

We make a variety of simplifications which are intended to allow us to focus on the role of reputation and to suppress other links between buyers and banks over time. We assume that loans originated in any period can only be sold in that period and that each bank interacts with a new set of buyers in each period. In order for reputation to play a role, we assume that buyers observe the contracts chosen by an individual bank in all previous periods. Buyers use these observations to update their beliefs about the quality level of an individual bank. These simplifications imply that the dynamic version of our model is simply a T -period repetition of our static model so that the only variable that links behavior over time is the endogenously determined reputation level of the bank.

To allow us to focus on the role of reputation and abstract from other sources of learning over time, we assume that the quality types of banks is completely persistent in the sense that it is the same in all periods and that buyers do not observe the returns on loans in previous periods. If the quality levels of banks are independent over time, past outcomes are irrelevant for inferring banks' quality levels so that clearly some persistence of quality types over time is essential for reputation over bank quality to play a role. We conjecture that our results are robust to allowing for incomplete persistence and to allowing for other sources of learning.

A Markov equilibrium for this economy consists of (possibly mixed) strategies by both banks and buyers, updating rules for buyers which satisfy Bayes rule and is defined in the usual fashion.

3.1 Dynamic Model without Reputational Concerns

We begin by analyzing a dynamic adverse selection model without reputational concerns. Formally, we suppress reputational concerns by setting banks' discount factor β equal to 0. Note that buyers still update their beliefs about the quality level of an individual bank from past contract choices.

Since the discount factor β is zero and since each buyer interacts with an individual bank only in one period, all decisions in any period are unaffected by future payoffs. It follows that the equilibrium in the first period coincides with the equilibrium in the static model. Since this equilibrium features complete separation of banks by quality type, it also features complete learning by buyers. In all subsequent periods, buyers believe that the bank is either of high- or low-quality depending on its first period contract choice. The equilibrium in all subsequent periods has the bank selling its entire loan portfolio at a price of $\bar{\pi}v$ if buyers believe the bank is of high-quality and at a price of $\underline{\pi}v$ if buyers believe the bank is of low-quality. The volume of trade in all periods except the first one is independent of the collateral value and the dispersion in bank quality. We summarize this discussion in a proposition.

Proposition 3 *Suppose that the discount factor of banks, β , equals 0. The equilibrium of the dynamic model features full separation and complete learning in the first period. The volume of trade in all periods except the first is independent of the collateral value and the dispersion in bank quality.*

This proposition illustrates starkly that absent reputational concerns adverse selection cannot persist and raises challenges for theories based on adverse selection in accounting for fluctuations in the volume of trade.

3.2 Dynamic Model with Reputational Concerns

The main focus of our analysis is an infinite horizon model. It is convenient to begin our analysis with a two period model. This two period model is useful for illustrating the nature of the equilibria and the sense in which fluctuations in collateral values can induce fluctuations in the volume of trade. Fluctuations in the aggregate volume of trade, however, depend on the initial distribution of buyers' beliefs about bank quality. In the infinite horizon

model, this distribution of buyers' beliefs is endogenously determined by past actions of banks. This endogenous determination allows us to study fluctuations in the volume of trade which do not depend on particular assumptions about the initial distribution.

3.2.1 A Two Period Model with Reputational Concerns

Consider a two period version of our dynamic model. We will show that if banks' discount factor is sufficiently high, no equilibrium features complete separation in the first period so that no equilibrium can have complete learning. The proof of this result is by contradiction. Suppose that an equilibrium exists with complete separation. It suffices to consider the bank's payoffs in the last period when the buyers beliefs are 1 and 0. The equilibrium in the last period coincides with the static equilibrium and the associated payoffs are given by

$$\begin{aligned} V(1; \bar{\pi}) &= \bar{\pi}v \\ V(0; \bar{\pi}) &= (1 - \tilde{\mu})\bar{\pi}v + \tilde{\mu}(\bar{\pi}v - c) \\ V(1; \underline{\pi}) &= \bar{\pi}v \\ V(0; \underline{\pi}) &= \underline{\pi}v. \end{aligned}$$

Incentive compatibility of buyers' contracts in the first period implies

$$t_h + (1 - x_h)(\bar{\pi}v - c) + \beta V(1; \bar{\pi}) \geq t_l + (1 - x_l)(\bar{\pi}v - c) + \beta V(0; \bar{\pi}) \quad (13)$$

$$t_l + (1 - x_l)(\underline{\pi}v - c) + \beta V(0; \underline{\pi}) \geq t_h + (1 - x_h)(\underline{\pi}v - c) + \beta V(1; \bar{\pi}) \quad (14)$$

where we have suppressed time indices for convenience. Adding the constraints 13 and 14 and substituting for the value functions gives

$$x_l - x_h \geq \beta \tilde{\mu}.$$

Clearly, if $\beta \tilde{\mu} > 1$, then $x_l > 1$ so that no incentive compatible contract can achieve full separation in the first period. We summarize this discussion in the following proposition.

Proposition 4 (*Patience and Persistence of Adverse Selection*) *Suppose the discount factor of banks, β , is larger than $1/\tilde{\mu}$. Then no equilibrium in the two period economy has complete separation of high- and low-quality banks in the first period.*

This proposition shows that if banks are sufficiently patient, then reputational concerns imply that adverse selection problems cannot be quickly resolved. It also implies that any equilibrium must necessarily involve at best partial revelation of information over time. This slow revelation of information implies that adverse selection problems can persist and that declines in collateral values and increases in dispersion of bank quality can lead to fluctuations in the volume of trade in periods other than the very first one.

Proposition 4 (patience and persistence) implies that no equilibrium can have high- and low-quality banks accepting distinct offers and banks using pure strategies. The reason is that any such equilibrium will feature complete learning by buyers in the second period. This implication leads us to consider equilibria in our two period model in which either both types of banks accept the same offer or banks choose mixed strategies. For simplicity, we consider pure strategies by buyers and we construct equilibria in which buyers make zero profits. We focus on equilibria in which the high-quality bank uses pure strategies and the low-quality bank uses possibly mixed strategies.

We will show that there are equilibria in which for sufficiently high reputation levels, the equilibria features *complete pooling* in the sense that both high- and low-quality banks accept the same offer. For low reputation levels, the equilibria feature *partial pooling* in the sense that the low-quality bank mixes between offers while the high-quality bank does not. While there are many such equilibria, for expositional purposes, we begin by focusing on one which maximizes the quantity of loans sold. We will call this equilibrium the *maximal trade equilibrium*.

Consider equilibrium outcomes which feature complete pooling and maximize the probability of trade for high reputation levels so that both types of banks sell their entire loan portfolio at the same price. Since buyers make zero profits, the price per loan is given by the pooling price, $\hat{p}(\mu)$ from 9. This equilibrium is supported by future buyers beliefs associated with deviation offers (x', t') given by

$$\mu'(x', t') = \begin{cases} 0 & \text{if } t' + (1 - x')(\bar{\pi}v - c) \leq \hat{p}(\mu) \\ 1 & \text{o.w.} \end{cases} .$$

Note that these beliefs say that future buyers believe that the bank is low-quality if and only if the bank accepts an offer which is statically less favorable for the high-quality bank than the pooling contract.²

In the static model, such a pooling contract cannot be an equilibrium because buyers could make offer cream-skimming contracts that attract only high-quality banks. In particular, one such cream-skimming contract has a deviating buyer making the same offer to both types of banks in which the buyer offers to buy less than the whole loan portfolio and setting payments at a level that is attractive only to high-quality banks. In the dynamic model, reputational gains imply that the low-quality bank gains future profits by accepting such a cream-skimming offer. Thus, such attempted cream-skimming contracts attract banks of both quality types and make negative profits. In the Appendix, we show that this type of argument can be extended to cream-skimming contracts which have the deviating buyer

²In the Appendix, we discuss refinements of our equilibrium concept including the Intuitive Criteria.

making different offers to the two types of banks.

Consider next equilibrium outcomes when the reputation levels of banks are low and feature partial pooling. In this equilibrium, buyers offer a contract $z = (\bar{x}_h, \bar{t}_h, 1, \bar{t}_l)$ where \bar{x}_h satisfies

$$\bar{x}_h = \frac{(\tilde{\mu} - \mu)(1 - \mu^*)}{\tilde{\mu}(1 - \mu) - \mu(1 - \mu^*)}. \quad (15)$$

Here, μ^* is the threshold in bank reputation above which a high quality bank prefers to sell its entire loan portfolio at the pooling price $\hat{p}(\mu)$ rather than hold its entire loan portfolio, or $\hat{p}(\mu^*) = \bar{\pi}v - c$. Thus, μ^* satisfies

$$\mu^* = 1 - \frac{c}{(\bar{\pi} - \underline{\pi})v}. \quad (16)$$

It is straightforward to show that the threshold, μ^* is strictly smaller than our previous threshold from the static model, $\tilde{\mu}$.

Low-quality banks choose the offer $(1, \bar{t}_l)$ with probability α_l and (\bar{x}_h, \bar{t}_h) with probability $1 - \alpha_l$, and high-quality banks accept (\bar{x}_h, \bar{t}_h) . The mixing probability for low-quality banks, α_l , is such that after observing a trade at (\bar{x}_h, \bar{t}_h) , future buyers believe the bank is high-quality with probability $\tilde{\mu}$ so that α_l satisfies

$$\tilde{\mu} = \frac{\mu}{\mu + (1 - \mu)(1 - \alpha_l)}. \quad (17)$$

Since only low-quality banks accept the offer $(1, \bar{t}_l)$, after observing a sale at this offer, future buyers believe the bank is high-quality with probability 0. Since the low-quality bank receives a reputation of zero after accepting the offer $(1, \bar{t}_l)$, the probability α_l can be interpreted as the probability the low-quality bank reveals its type.

Given \bar{x}_h , the payments \bar{t}_h and \bar{t}_l are chosen so that the low-quality bank is indifferent between the two offers, or,

$$\bar{t}_l + \beta V(0; \underline{\pi}) = \bar{t}_h + (1 - \bar{x}_h)(\underline{\pi}v - c) + \beta V(\tilde{\mu}; \underline{\pi}), \quad (18)$$

and buyers make zero profits.

$$\mu(\bar{x}_h\bar{\pi}v - \bar{t}_h) + (1 - \mu)(1 - \alpha_l)(\bar{x}_h\underline{\pi}v - \bar{t}_h) + (1 - \mu)\alpha_l(\underline{\pi}v - \bar{t}_l) = 0 \quad (19)$$

The equilibrium is supported by future buyers beliefs associated with deviations offer (x', t') given by

$$\mu'(x', t') = \begin{cases} 0 & \text{if } t' + (1 - x')(\bar{\pi}v - c) \leq \bar{t}_h + (1 - \bar{x}_h)(\bar{\pi}v - c) \\ 1 & \text{o.w.} \end{cases}.$$

As in the case of high first period reputation levels, these beliefs say that future buyers believe the bank is low-quality if and only if the bank accepts an offer which is statically less favorable for the high-quality bank than the equilibrium contract.

Straightforward algebra can be used to show that $\bar{t}_l > \underline{\pi}v$ so that this equilibrium features cross-subsidization in the sense that high-quality banks receive less than actuarially fair prices per loan and low-quality banks receive higher than actuarially fair prices per loan both when they accept (\bar{x}_h, \bar{t}_h) and when they accept $(1, \bar{t}_l)$.

The logic of the proof is similar to the case of high first period reputational levels, reputational concerns ensure cream-skimming contracts earn negative profits. In this equilibrium, buyers lose more profit per low-quality bank when such banks trade at (\bar{x}_h, \bar{t}_h) than at $(1, \bar{t}_l)$. Thus, buyers have an incentive to induce a more profitable tie-breaking rule for low-quality banks than that implied by α_l . In the Appendix, with an additional technical assumption that $1/2 \leq c/[(\bar{\pi} - \underline{\pi})v]$, we prove that deviation contracts which induce a better tie breaking rule earn non-positive profits. The reason is that contracts which leave the offer for the high-quality bank unchanged but offer a slightly higher payment to the low-quality bank attract a disproportionate number of low-quality banks relative to the equilibrium and \bar{t}_l is greater than $\underline{\pi}v$. We summarize this characterization of equilibria in the following proposition.

Proposition 5 *Suppose $1/2 \leq c/[(\bar{\pi} - \underline{\pi})v]$ and the discount factor of banks, β , is larger than $1/\tilde{\mu}$. The equilibrium in the first period of the dynamic model has cross-subsidization and at best partial separation. When $\mu \leq \mu^*$, there is partial learning as the low-quality bank reveals its type with probability α_l satisfying 17. When $\mu \geq \mu^*$, the equilibrium has no learning with both quality types selling their entire loan portfolio at the pooling price.*

Other equilibria with a lower fraction of loans sold by the high-quality bank do exist in our dynamic model. For low reputation levels, there are similar partial-pooling equilibria with a lower fraction of loans sold at the offer intended for the high-quality bank. In the Technical Appendix, we prove that for each $\mu \leq \mu^*$, there is a lower bound, $\underline{x}(\mu) \leq \bar{x}_h$ where for any $x_h \in [\underline{x}(\mu), \bar{x}_h(\mu)]$, a partial-pooling equilibrium exists where t_h and t_l are chosen so that 18 and 19 hold with equality. Additionally, when reputation levels are low, there is a different partial pooling equilibrium in which the high-quality bank holds its entire loan portfolio and the low-quality bank accepts an offer $(1, \underline{\pi}v)$ with some probability α_l . In this equilibrium, after observing a decision by a bank to hold its entire portfolio, buyers believe the bank is high-quality with probability μ_h where the value of μ_h is chosen to satisfy

$$\underline{\pi}v + \beta V(0; \underline{\pi}) = \underline{\pi}v - c + \beta V(\mu_h; \underline{\pi}).$$

Such a μ_h exists since $\beta \geq 1/\tilde{\mu}$ and the probability that the low-quality bank holds its loan portfolio, $1 - \alpha_l$, is chosen so that buyers beliefs are consistent with Bayes rule.

When reputational levels are sufficiently high, there are additional pooling equilibria in which banks of both quality types sell a fraction less than 1 of their loan portfolio at the

same pooling price. Let $(x, x\hat{p}(\mu))$ denote such an equilibrium. This equilibrium is supported by future buyers beliefs associated with deviation offers (x', t') given by

$$\mu'(x', t') = \begin{cases} 0 & \text{if } t' + (1 - x')(\bar{\pi}v - c) \leq \max \{x'\hat{p}(\mu) + (1 - x')(\bar{\pi}v - c), x\hat{p}(\mu) + (1 - x)(\bar{\pi}v - c)\} \\ 1 & \text{o.w.} \end{cases}$$

These beliefs say that future buyers believe the bank is low-quality if and only if the bank accepts an offer which is either statically less favorable for the high-quality bank than the equilibrium or below the market-odds line. In the Technical Appendix, we prove that for μ sufficiently close to 1, a *minimal trade* equilibrium exists in which high- and low-quality banks sell none of their loan portfolio so that $x = 0$.

Next we analyze the effects of aggregate shocks to collateral values or to the dispersion of bank quality on trading volume in our dynamic model with reputational concerns. We show that such changes are associated with changes in trading volume in periods beyond the initial period so that the dynamic model overcomes the persistence challenge, and that expected volume of loans sold by low-quality banks also declines when collateral values fall so that the dynamic model also overcomes the correlation challenge.

It is immediate from Proposition 5 that adverse selection persists when $\mu \geq \mu^*$ and persists with ex-ante probability $\mu + (1 - \mu)(1 - \alpha_l)$ when the reputational level of the bank is below μ^* . To illustrate the effect of changes in collateral values (or dispersion in bank quality), we focus on the equilibrium described in Proposition 5 and evaluate the change in the expected volume of trade in periods 1 and 2 when there is a decline in collateral values. First consider an unanticipated temporary decline in collateral values in period 2. Since the decline is unanticipated, equilibrium outcomes in period 1 are unchanged. For a given distribution of reputations in period 1, the reputations of banks with reputation levels above μ^* do not change. For banks with reputation levels below μ^* , if the bank is high-quality its period 2 reputation is $\tilde{\mu}$ and if the bank is low-quality with probability α_l its period 2 reputation is 0 while with probability $(1 - \alpha_l)$ its period 2 reputation is $\tilde{\mu}$. As a result, there remains a distribution over μ in the second period with some banks at or above μ^* and below 1. As a result, when the collateral value declines in period 2, there is a decline in aggregate volume as discussed in Proposition 2. This stands in contrast to the same exercise in the dynamic model without reputational concerns in which case such a decline led to no fluctuations in aggregate volume in the period 2. This comparative static is intended to highlight the persistence of adverse selection in the presence of reputational concerns and the importance of reputational concerns in generating fluctuations in trade volumes associated with changes in collateral values in periods beyond the initial period.

Next consider a temporary decline in collateral values which leaves the collateral value in period 2 unchanged. We use this comparative static to show how reputational concerns aid in overcoming the correlation challenge. In period 1, banks of both quality types with reputation levels above μ^* sell their entire loan portfolio. When the reputation level is below μ^* , the expected volume of trade in the partial pooling equilibrium is given by

$$T = [\mu + (1 - \mu)(1 - \alpha_l)] \bar{x}_h + (1 - \mu)\alpha_l.$$

Using the definitions of $\tilde{\mu}$ and α_l , straightforward algebra can be used to express this volume as

$$T = 1 - \mu\tilde{\mu} \frac{1 - \bar{x}_h}{\tilde{\mu}^2}. \quad (20)$$

When the collateral value in period 1 declines, the fraction of loans sold at the offer (\bar{x}_h, \bar{t}_h) , \bar{x}_h declines, which can be seen in 15 since μ^* increases. Note both high- and low-quality banks decrease their expected volume of trade since the probability that low-quality banks accept offer (\bar{x}_h, \bar{t}_h) is unchanged and high-quality banks continue to only accept offer (\bar{x}_h, \bar{t}_h) . Thus, a decline in the collateral value causes the expected volume of trade, T in 20, to decline. From 16, we see that the threshold μ^* increases as either v or $(\bar{\pi} - \underline{\pi})$ increases. This result together with the form of 20 establishes that the expected volume of trade for an individual bank declines as either v or $(\bar{\pi} - \underline{\pi})$ increases. We have established that a temporary decline in collateral values causes banks of both quality types for all reputation levels to (weakly) decrease the fraction of loans they sell. In this sense, the dynamic model of adverse selection with reputational concerns can induce a decline in volume of trade among both high- and low-quality loans.

The effects of a permanent decline in collateral values are similar in each period to the temporary declines we have analyzed. One key difference is a permanent decline in collateral values induces an opposing force which tends to increase the expected volume of trade. When a decline in period 2 collateral values is known to occur in period 1, the reputation which banks receive after accepting offer (\bar{x}_h, \bar{t}_h) , which is $\tilde{\mu}$ increases. Since the mixed strategy of low-quality banks is chosen so that receiving a reputation of $\tilde{\mu}$ after accepting (\bar{x}_h, \bar{t}_h) is consistent with Bayes rule, as in 17, the probability that low-quality banks accept the offer $(1, \bar{t}_l)$, which is α_l , must also increase. In this sense, there is a countervailing force which tends to raise the expected volume of trade for low-quality banks with reputation levels below μ^* . Nevertheless, straightforward algebra can be used to show that the decline in \bar{x}_h dominates this effect so that the expected volume of trade T given in 20, declines when there is a permanent decline in collateral values, and, moreover, the expected volume of trade for a low-quality bank with a given reputation $\mu \leq \mu^*$, which is given by $(1 - \alpha_l)\bar{x}_h + \alpha_l$, also

declines. We summarize these comparative statics in the following proposition.

Proposition 6 (*Fluctuations in Trade Volume with Reputational Concerns*) *If banks are sufficiently patient, decreases in collateral values and increases in the dispersion of bank quality reduce expected trade volumes for banks of all reputational levels and decrease aggregate trade volume in both periods.*

This proposition illustrates that if reputational concerns are sufficiently strong, adverse selection can persist and fluctuations in collateral values induce fluctuations in the volume of trade in both periods. In this sense, reputational concerns can help address the persistence and correlation challenges confronting theories of trade volume based on adverse selection.

The dynamic model with reputational concerns has another source of fluctuations in the volume of trade resulting from the existence of multiple equilibria. In some of these equilibria, reputation helps to sustain a larger volume of trade for individual banks than the static model. In other equilibria, reputational concerns induce a lower volume of trade for individual banks. To see that reputational concerns can induce a larger volume of trade than in a static model, recall that in the static model, the expected volume of trade is given by 12 when $\mu \geq \tilde{\mu}$ and by 11 when $\mu \leq \tilde{\mu}$. Clearly this volume is less than 1 for all $\mu < 1$ and, using the definition of $\tilde{\mu}$, 11 can be expressed as

$$T = 1 - \mu\tilde{\mu}.$$

In the partial pooling equilibrium (as defined in Proposition 5), for reputations $\mu \in [\mu^*, 1]$, the volume of trade is $T = 1$. For such reputations, reputational concerns clearly lead to an increased volume of trade. For reputation levels below μ^* , the volume of trade is given by 20. Recall that as μ converges to μ^* , the fraction of loans sold by the high-quality bank in the partial pooling equilibrium converges to 1 and is increasing in μ . As a result, there exists a range of μ below μ^* such that the volume of trade in the first period of the dynamic model is larger than in the static model. It can be shown that when the threshold μ^* is sufficiently small, the expected volume of trade in the first period of the dynamic model is larger than in the static model for all $\mu \in [0, \mu^*]$. In the dynamic model, the equilibrium then features a higher expected volume of trade associated with a higher degree of cross-subsidization. This degree of cross-subsidization is sustained only because of reputational concerns in the sense that buyers cannot use cream-skimming contracts to undercut the equilibrium contract. We have then shown that reputational concerns may relax adverse selection problems in the sense that when banks are sufficiently patient, there is an equilibrium that has an expected volume of trade which is larger both for individual banks and in the aggregate for any given distribution of reputations than in the static model.

There are also equilibria in which banks with particular reputation levels have a lower expected volume of trade than in the static model. For example, for low reputation levels, we have argued that there is an equilibrium in which the high-quality bank holds its loans and the low-quality bank chooses to hold its entire loan portfolio with positive probability. In this equilibrium for $\mu \leq \mu^*$, the expected volume of trade is given by

$$T = (1 - \mu)\alpha_l,$$

which is strictly below that found in the static model. In this sense, we find that reputational concerns can amplify adverse selection problems and lead to outcomes with lower expected volume of trade than would occur in a static model.

The fact that there are multiple equilibria associated with different expected trade volumes implies another source of fluctuations in trade volume. Sunspot-like coordination problems in which banks and buyers switch from a high volume equilibrium to a low volume equilibrium, even in the absence of fluctuations in collateral values can lead to declines in expected trade volume. In [Chari et al. \(2010\)](#), in a similar environment, we show that fluctuations in collateral values are likely to trigger such switches in equilibrium outcomes. Specifically, we adapt techniques from the global games literature to our environment and show that there is a unique equilibrium which has a critical threshold in the collateral value above which the equilibrium features high aggregate trading volume and below which the equilibrium features low aggregate trading volume. In this sense, reputational concerns introduce another source of fluctuations in aggregate trading volume.

3.2.2 The Infinite Horizon Model with Reputational Concerns

We turn now to the main focus of our analysis: the infinite horizon model with reputational concerns. We build on our two period model and show that as in that model, the equilibrium outcomes in the infinite horizon feature partial pooling for low values of bank reputation and complete pooling for high values of reputation. Hence, the implication that the equilibrium features at best partial separation for low values of reputation and no separation for high values of reputation is a robust prediction of the model. In a stochastic version of our infinite horizon model, we show that fluctuations in collateral values or fluctuations in bank quality dispersion induce fluctuations in expected volume in all periods. In this sense, we show that reputational concerns can address the persistence and correlation challenges.

Proposition 4, which shows that if banks are sufficiently patient, a separating equilibrium does not exist, can be extended in a straightforward manner. We have the following proposition.

Proposition 7 (*Patience and Persistence of Adverse Selection*) Suppose the discount factor of banks is such that, $\beta/(1-\beta)$, is larger than $1/\tilde{\mu}$. Then no equilibrium in the infinite horizon economy has complete separation of high- and low-quality banks in any period.

Next we turn to characterizing a stationary markov equilibrium which is a markov equilibrium in which equilibrium outcomes in any period depend only on the beliefs of buyers about the bank's quality. In particular, equilibrium outcomes do not depend on calendar time. In the Technical Appendix, we show that our model has prove that there exists a stationary Markov equilibrium in the infinite horizon.

In this equilibrium, when the bank's reputation is low, or $\mu_t < \mu^*$, the equilibrium outcome is partial pooling where the high-quality bank accepts the offer $(x_h(\mu_t), t_h(\mu_t)) = (0, 0)$ so that it holds its entire loan portfolio while the low-quality bank mixes between $(x_h(\mu_t), t_h(\mu_t))$ and $(1, \underline{\pi}v)$. When a bank with reputation $\mu_t < \mu^*$ holds its entire loan portfolio, future buyers assign the bank a reputation $\mu_{t+1} = \mu^*$. The mixed strategy of a low-quality bank with reputation μ_t , $\alpha_l(\mu_t)$ is chosen so that future buyers beliefs are consistent with Bayes rule, or

$$1 - \frac{c}{(\bar{\pi} - \underline{\pi})v} = \mu^* = \frac{\mu_t}{\mu_t + (1 - \mu_t)(1 - \alpha_l(\mu_t))}. \quad (21)$$

When a bank with low reputation sells its loan portfolio at price $\underline{\pi}v$, future buyers assign the bank a reputation $\mu_{t+1} = 0$.

When the bank's reputation is high, or $\mu_t \geq \mu^*$, the equilibrium outcome is complete pooling where both high- and low-quality banks accept an offer of $(x_h(\mu_t), t_h(\mu_t)) = (\bar{x}(\mu_t), \bar{x}(\mu_t)\hat{p}(\mu_t))$. The fraction of the bank's loan portfolio which they sell, $\bar{x}(\mu_t)$, is given by $(1 - \mu^*)/\beta$ for $\mu_t = \mu^*$ and is given by

$$\bar{x}(\mu_t) = \min \left\{ \frac{\beta [(\bar{\pi} - \underline{\pi})v + c]}{\beta [(\bar{\pi} - \underline{\pi})v + c] + (1 - \mu_t)(\bar{\pi} - \underline{\pi})v \left[(1 - \beta) \frac{c + (\bar{\pi} - \underline{\pi})v}{c} - 1 \right]}, 1 \right\} \quad (22)$$

for all $\mu_t > \mu^*$. The value $\bar{x}(\mu^*) = (1 - \mu^*)/\beta$ is chosen for analytical convenience in the sense that the value functions implied by this equilibrium is constant for all reputation levels μ_t below μ^* and is equal to $\underline{\pi}v/(1 - \beta)$ for low-quality banks and $(\bar{\pi}v - c)/(1 - \beta)$ for high quality banks. When a bank accepts the equilibrium offer, the bank's reputation is unchanged over time so that $\mu_{t+1} = \mu_t$.

In terms of beliefs following out of equilibrium actions, the beliefs of future buyers for all $(\hat{x}, \hat{t}) \neq (x_h(\mu_t), t_h(\mu_t))$ are given by

$$\mu_{t+1}(\hat{x}, \hat{t}; \mu_t) = \begin{cases} 0 & \text{if } \hat{t} + (1 - \hat{x})(\bar{\pi}v - c) \leq \max \{ \hat{x}\hat{p}(\mu_t) + (1 - \hat{x})(\bar{\pi}v - c), t_h(\mu_t) + (1 - x_h(\mu_t))(\bar{\pi}v - c) \} \\ 1 & \text{o.w.} \end{cases}$$

The logic of the proof parallels the proof used in the two period version of the model in the sense that similar arguments are used to prove that there are no profitable deviations by buyers.

Proposition 8 *Suppose the discount factor of banks is such that $\beta/(1 - \beta)$ is larger than $1/\tilde{\mu}$. There exists a stationary markov equilibrium with at best partial separation. When $\mu_t \leq \mu^*$, there is partial learning as the low-quality bank reveals its type with probability $\alpha_l(\mu_t)$ satisfying 21. When $\mu_t \geq \mu^*$, the equilibrium has no learning with both quality types selling a fraction of their loan portfolio equal to $\bar{x}(\mu_t)$.*

This proposition shows that if reputational concerns are sufficiently strong, adverse selection persists.

Next we use this proposition to ask how the aggregate volume of trade changes following aggregate shocks to collateral values or to dispersion in bank quality. To answer this question, we begin by characterizing properties of the invariant distribution over buyers' beliefs about bank quality. Let H denote this invariant distribution. Proposition 8 implies that this invariant distribution has a mass point (possibly of size 0) at beliefs $\mu = 0$, no mass for beliefs strictly between 0 and μ^* , and a mass point (possibly of size 0) at μ^* . Note that the invariant distribution depends on the nature of the initial distribution and is not uniquely determined (below we consider models in which the quality of banks changes exogenously and show that such models do have unique invariant distributions).

Consider the aggregate volume of trade in such an invariant distribution. This aggregate volume is given by

$$T = H(0) + (H(\mu^*) - H(0)) \bar{x}(\mu^*) + \int_{\mu^*}^1 \bar{x}(\mu) dH(\mu). \quad (23)$$

Consider now an unanticipated permanent shock in say period 1 to collateral values which raises v from v_0 to v_1 . Let T_t denote aggregate trading volume in period t . We will show that a fall in collateral values reduces aggregate trading volume in all periods $t \geq 1$. Proposition 8 implies that banks with reputation of 0 continue to sell their entire loan portfolio in all periods. Let $\mu^*(v)$ be given by the first equation in 21. Note that $\mu^*(v_0) < \mu^*(v_1)$. Let $\bar{x}(\mu, v)$ be given by 22. Straightforward algebra can be used to show that $\bar{x}(\mu, v)$ is decreasing in v . Equilibrium outcomes in period 1 for banks with reputation levels between $\mu^*(v_0)$ and $\mu^*(v_1)$ feature partial pooling in which the low-quality bank sells its loans with probability α_l given by 21. Expected trading volume for banks with such reputation levels is given by

$$(H(\mu^*(v_0)) - H(0)) (1 - \mu^*(v_0)) \alpha_l(\mu, v_1) + \int_{\mu^*(v_0)}^{\mu^*(v_1)} (1 - \mu) \alpha_l(\mu, v_1) dH(\mu). \quad (24)$$

Equilibrium outcomes in period 1 for banks with reputation levels above $\mu^*(v_1)$ feature complete pooling in which bank of both quality types sell a fraction $\bar{x}(\mu, v_1)$ of their loans. Expected trading volume for these banks is given by

$$\int_{\mu^*(v_1)}^1 \bar{x}(\mu, v_1) dH(\mu). \quad (25)$$

Using 24 and 25 to compute aggregate trading volume and subtracting this volume from aggregate volume in period 0 given by 23, we have the decline in trading volume is given by

$$\begin{aligned} T_0 - T_1 &= \int_{\mu^*(v_0)}^{\mu^*(v_1)} [\bar{x}(\mu, v_0) - (1 - \mu)\alpha_l(\mu, v_1)] dH(\mu) + \int_{\mu^*(v_1)}^1 [\bar{x}(\mu, v_0) - \bar{x}(\mu, v_1)] dH(\mu) \\ &\quad + (H(\mu^*(v_0)) - H(0)) [\bar{x}(\mu^*(v_0), v_0) - (1 - \mu^*(v_0))\alpha_l(\mu^*(v_0), v_1)]. \end{aligned} \quad (26)$$

Since $\bar{x}(\mu, v_0)$ is decreasing, the second term in 26 is positive. In the Technical Appendix, we show that the first term and third terms are also positive.

Consider next volume of trade in periods $t = 2, 3, \dots$. In all such periods, the distribution has no mass strictly between 0 and $\mu^*(v_1)$ and has a mass point at $\mu^*(v_1)$. The volume of trade at this mass point is given by

$$\bar{x}(\mu^*(v_1), v_1) \int_{\mu^*(v_0)}^{\mu^*(v_1)} [\mu + (1 - \mu)(1 - \alpha_l(\mu, v_1))] dH(\mu).$$

The change in trade from period 0 to period t is then given by

$$\begin{aligned} T_0 - T_t &= \int_{\mu^*(v_1)}^1 [\bar{x}(\mu, v_0) - \bar{x}(\mu, v_1)] dH(\mu) + \int_{\mu^*(v_0)}^{\mu^*(v_1)} [\bar{x}(\mu, v_0) - (1 - \mu)\alpha_l(\mu, v_1) - [\mu + (1 - \mu)(1 - \alpha_l(\mu, v_1))] \\ &\quad + (H(\mu^*(v_0)) - H(0)) [\bar{x}(\mu^*(v_0), v_0) - (1 - \mu^*(v_0))\alpha_l(\mu^*(v_0), v_1) - [\mu^*(v_0) + (1 - \mu^*(v_0))(1 - \alpha_l(\mu^*(v_0), v_1))]]] dH(\mu). \end{aligned}$$

In the Technical Appendix, using straightforward algebra, we show that $T_0 > T_t$. We then have the following proposition.

Proposition 9 *If reputational concerns are sufficiently strong in the sense that $\beta/(1 - \beta) \geq 1/\tilde{\mu}$, following an unanticipated permanent shock to collateral values, the aggregate volume of trade declines in all periods.*

This proposition shows that persistent shocks to collateral values induce persistent changes in the volume of trade. This result is in contrast to that in a model with no reputational concerns where persistent shocks to collateral values induce at best transient fluctuations in the volume of trade. The central reason for this difference in results is that reputational concerns induce adverse selection to persist.

3.2.3 The Infinite Horizon Model with Reputational Concerns and Aggregate Fluctuations

In Proposition 9, we analyzed the effects of unanticipated permanent shocks to collateral values on aggregate volume of trade. In this section we turn to an analysis of our model with stochastic fluctuations in collateral values in order to study the effects of temporary shocks and the effects of anticipated shocks. To do so, we introduce stochastic fluctuations in collateral values that are perfectly correlated across banks. We assume that the spread v_t is independently distributed over time with identical distribution $G(v_t)$ with support $[v_l, \infty)$ where the lower bound v_l is given by $c/(\bar{\pi} - \underline{\pi})$. At this lower bound, there is no adverse selection problem in the sense that for all reputation levels, the high-quality bank is willing to sell its entire loan portfolio at the pooling price $\hat{p}(\mu)$. Let Ev denote the expected value of the spread v .

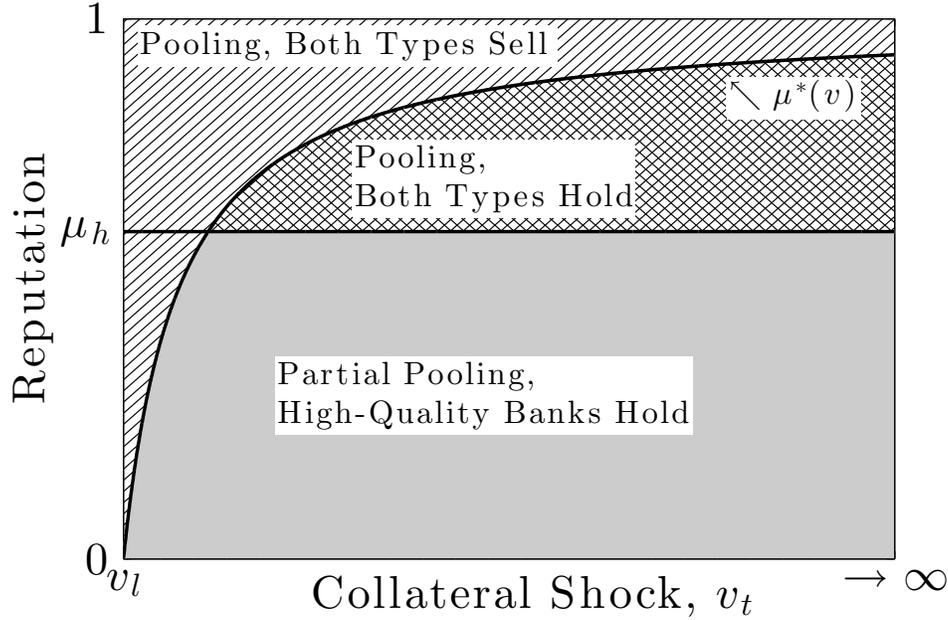
We make two changes to the dynamic model to simplify the analysis. The first change is to allow the quality of the banks to change according to a markov process. In particular, at the end of each period, with probability λ , each bank draws a new quality level. These draws are independent across banks and buyers can observe whether a bank has drawn a new quality level. If the bank draws a new quality level, the bank becomes a high-quality bank with probability μ_0 and a low-quality bank with probability $1 - \mu_0$. This change of the model implies that the model has a unique invariant distribution of buyers' beliefs. Second, we simplify the strategies of buyers and sellers by restricting contracts to satisfy $x = 0$ or $x = 1$. One interpretation of this version of the model is that the loan portfolio of banks is indivisible and thus banks can either sell their entire loan portfolio or they must hold their loans. We conjecture that our results do not depend on this simplification.

In the Technical Appendix, we show that if banks are sufficiently patient, or,

$$\beta \geq \frac{c}{c + (\bar{\pi} - \underline{\pi}) Ev}$$

no equilibrium features complete separation of banks in any period. We also show that a stationary markov equilibrium exists. For any given bank, the equilibrium outcomes depend on its reputation level and on the current realization of the spread v_t . Figure 2 illustrates the nature of the equilibrium. To understand this equilibrium, consider fixing some value of the spread, v_t and varying the reputation levels of banks. When a bank's reputation level is below a threshold, the outcomes feature partial pooling and above this threshold the outcomes feature complete pooling. In this sense, equilibrium outcomes resemble the equilibrium described in Proposition 8. In the partial pooling region, high-quality banks hold their loans and low-quality banks mix over holding and selling. In this region, low-

Figure 2: Equilibrium Regions in the Infinite Horizon Model with Stochastic Spreads



quality banks are induced to hold their loans because by doing so they acquire a higher reputation level. In future periods, such banks use their reputation levels to sell at favorable prices. Next consider fixing a bank's reputation level and increasing in the realized spread. Figure 2 shows that there is some threshold value of the spread at which at least one quality type of bank switches from selling its entire loan portfolio to holding its loans.

The outcomes in the stochastic model are different from those in the deterministic model since the stochastic model also features a region in which both high- and low-quality banks hold their loans. This complete pooling region consists of relatively high spreads and reputation levels between $\mu^*(v)$ and μ_h . Low-quality banks are induced to hold their loans because by doing so they maintain their reputations and are able to sell their loans for favorable prices in the future when spreads are relatively low. This result illustrates the sense in which anticipations of future shocks affect the nature of the equilibrium.

In the deterministic model, in the analogous region, the equilibrium features partial pooling. The deterministic model cannot have an equilibrium in which in the analogous region, both high- and low-quality banks hold their loans. In such a purported equilibrium, maintaining reputation would have no value for low-quality banks. The reason is that in any stationary equilibrium, low-quality banks would have to hold their loans forever and would strictly prefer to sell their for a payment for $\underline{\pi}v$. In the stochastic model, in contrast, maintaining reputations is valuable.

In terms of how buyers' beliefs evolve, note in the partial pooling region, conditional on a sale, future buyers believe the bank is low-quality and conditional on the bank holding its loans, future buyers believe the bank is high quality with probability μ_h . The mixed strategies of the low-quality banks are chosen so that the beliefs of buyers are consistent with Bayes' rule in the sense that

$$\mu_h = \frac{\mu_t}{\mu_t + (1 - \mu_t)(1 - \alpha_l(\mu_t))}.$$

The value, μ_h is chosen to ensure that in the partial pooling region, low-quality banks are indifferent between holding their loans and selling their loans, or

$$\underline{\pi}v_t + \beta(1 - \lambda)V(0; \underline{\pi}) + \beta\lambda V(\mu_0; \underline{\pi}) = \underline{\pi}v_t - c + \beta(1 - \lambda)V(\mu_h; \underline{\pi}) + \beta\lambda V(\mu_0; \underline{\pi})$$

so that

$$\beta(1 - \lambda) [V(\mu_h; \underline{\pi}) - V(0; \underline{\pi})] = c$$

where V is the continuation value of payoffs. In the complete pooling regions, the bank's reputation remains unchanged.

Consider the support of the invariant distribution of buyers beliefs about bank quality. If $\mu_0 \leq \mu_h$, this support consists of three reputation levels, $0, \mu_0, \mu_h$. If $\mu_0 > \mu_h$, this support consists of μ_0 . We use this support to show how aggregate shocks to collateral values affect the volume of trade. Suppose first that $\mu_0 \leq \mu_h$. Figure 2 shows that as the spread v_t rises above a critical value of v , high-quality banks at reputation levels μ_0 switch from selling their loans to holding their loans and low-quality banks switch from selling their loans with probability 1 to holding them with positive probability. Thus the aggregate volume of trade declines at this critical value of v . Figure 2 also shows that as the spread v rises above another critical value, both high- and low-quality banks at reputation level μ_h switch from selling their loans to holding their loans. Thus the aggregate volume of trade declines at this critical value of v as well.

Suppose next that $\mu_0 > \mu_h$. Figure 2 shows that above a critical value of v , both types of banks switch from selling their loans to holding their loans. Thus the aggregate volume of trade declines at this critical value of v .

These arguments establish the following proposition.

Proposition 10 *If banks are sufficiently patient and if shocks to collateral values are independent over time, then the aggregate volume of trade is declining in the spread v and these declines are discontinuous.*

4 Implications for Policy

In the wake of the 2007 collapse of secondary loan markets, policymakers proposed a variety of programs intended to remedy perceived inefficiencies in the market for securitized assets. Under some of these policies, the government would purchase asset-backed securities at prices above existing market value.

In this section, we show that an adverse selection model without reputational concerns can produce sudden collapses in aggregate trading volume. In the face of such sudden collapses, policies like the ones proposed lead to desirable outcomes. The sudden collapses in such models arise if we use an equilibrium concept different from ours. We argue that our equilibrium concept is more appropriate for environments with adverse selection. As we have shown, our model with reputational concerns generates sudden collapses in aggregate trading volume. We show that policies like the ones proposed may not lead to desirable outcomes. In this sense, incorporating reputational concerns into policy analysis can lead to substantially different implications for the outcomes of various policies.

Consider a version of our model with the same payoffs for banks and buyers and suppose that the discount factor is sufficiently small so that we can ignore reputational considerations. We adopt a notion of equilibrium which is similar to that of competitive equilibrium in the sense that both banks and buyers take the price per unit of loan, $p(\mu)$, as parametrically given. Given this price, banks of high- and low-quality types choose x_h and x_l quantity of loans to sell and buyers choose to buy y quantity of loans. Because of private information, we need to augment the usual notion of competitive equilibrium by having the payoffs of buyers depend on the choices of banks so that this equilibrium has externalities. A *competitive equilibrium with externalities* consists of a price per unit of loan $p(\mu)$, and quantities of loans sales and purchases, (x_h, x_l, y) such that banks of each quality type maximize their payoffs and buyers choose y to maximize their profits given by

$$y \left[\mu 1_{[x_h > 0]} (\bar{\pi}v - p(\mu)) + (1 - \mu) 1_{[x_l > 0]} (\underline{\pi}v - p(\mu)) \right]$$

and markets clear.

Next we show that for $\mu \geq \mu^*$, this model has two competitive equilibria. In one equilibrium, the *high trade* equilibrium, the price per loan is $p(\mu) = \mu \bar{\pi}v + (1 - \mu) \underline{\pi}v = \hat{p}(\mu)$ and banks of both quality types sell their entire loan portfolios. In the other equilibrium, the *low trade* equilibrium, the price per loan is $\underline{\pi}v$ and only low-quality banks sell their loan portfolios. This model can clearly generate sudden collapses in trade volume associated with a switch from the high to the low trade equilibrium. Also note that the low trade equilibrium is inefficient in the sense that the high trade equilibrium pareto dominates the low trade

equilibrium.

We use this multiplicity to provide one rationalization for asset purchase policies under which the government commits to buy all loans at a price higher than would prevail absent its intervention. Specifically, suppose the government commits to buy all loans at a price $\hat{p}(\mu)$. With this policy, the high trade outcome is the unique equilibrium. Indeed, under this policy, the government does not actually have to buy any loans. In this sense, this policy resembles deposit insurance in the [Diamond and Dybvig \(1983\)](#) model. Such an asset purchase policy is clearly desirable because it is pareto improving.

We find this way of modeling sudden collapses in trade unattractive because we find the equilibrium concept to be unappealing. In this model, buyers have strong incentives to offer contracts intended to separate high-quality banks from low-quality banks. That is buyers have strong incentives to offer nonlinear contracts. This nonlinearity is a pervasive feature of models with adverse selection. For example, in this model, a buyer who makes an offer to purchase a small quantity of loans for a price per loan close to $\bar{\pi}v$ will attract only high-quality banks. Even if we restricted ourselves to linear contracts, a buyer who offers to buy the entire loan portfolio at a price per loan slightly less than $\hat{p}(\mu)$ will attract both types of banks.³ These considerations suggest that an equilibrium concept like competitive equilibrium with externalities is unsuitable for environments like ours.

We have shown that our model with reputational concerns can also generate sudden collapses in the aggregate volume of trade. We use our model to analyze the effects of asset purchase policies. For simplicity, we focus on our two period model. In that model, for sufficiently high reputation levels, the first period has a maximal trade equilibrium in which both high- and low-quality banks sell their entire loan portfolios and a minimal trade equilibrium both quality types of banks hold their entire loan portfolios. Consider an asset purchase policy under which the government commits to purchase all loans at a price $\hat{p}(\mu)$. Under this policy, our model has an equilibrium in which neither bank sells its loans to the government. To see this result, note that since the minimal trade outcome is an equilibrium, it must be that if a buyer deviates and offers the contract $(1, \hat{p}(\mu))$, the high-quality bank would not accept the deviation contract. To see that the low-quality bank would not accept the contract either, note that its payoffs if it accepts the contract are given by

$$\hat{p}(\mu) + \beta V(0; \underline{\pi}) = \hat{p}(\mu) + \beta \underline{\pi}v$$

³It has been suggested to us that models with linear contracts and capacity constraints could possibly generate multiple equilibria. We think that this conjecture is unlikely to be true based on the results of [Guerrieri et al. \(2010\)](#) who develop an adverse selection model with capacity constraints and matching frictions and obtain a unique equilibrium.

and its payoffs if it rejects the contract are given by

$$\underline{\pi}v - c + \beta V(\mu; \underline{\pi}) = .$$

Since something is true, the first express is less than the second.

$$\begin{aligned} \mu(\bar{\pi} - \underline{\pi})v + \underline{\pi}v + \beta \underline{\pi}v &\leq \underline{\pi}v - c + \beta V(\mu; \underline{\pi}) \\ \frac{c + \beta \underline{\pi}v}{(\bar{\pi} - \underline{\pi})v} &\leq \beta \frac{V(\mu; \underline{\pi})}{(\bar{\pi} - \underline{\pi})v} - \mu \end{aligned}$$

In our model, of course, if the government offered to purchase assets at a sufficiently attractive price above $\hat{p}(\mu)$, it could eliminate the minimal trade equilibrium. Such a policy is very different from deposit insurance like policies in the sense that such policies must necessarily be financed by resources obtained from general taxpayers. A more subtle policy that ensures maximal trade volume is a policy under which the government offers to buy the entire loan portfolio in both periods at a price per loan of $\hat{p}(\mu)$ and forbids all trades in the either period at a price per loan greater than $\hat{p}(\mu)$. Under this policy, it is relatively straightforward to show that both quality banks will sell their loans to the government in both periods and the government will break even. Forbidding private trades at a price per loan above $\hat{p}(\mu)$ is essential. Absent this aspect of policy, buyers will offer contracts that attract only high-quality banks. Low-quality banks will sell to the government and high-quality banks will sell part of their loan portfolio to buyers. Other subtle policies will also ensure maximal trade volume.

We have shown that in the face of sudden collapses in trading volume, an asset purchase policy in which the government offers to buy all loans at a pooling price in the current period will not induce high-quality banks to sell their loans. We have also shown that an asset purchase policy which offers to buy all loans at a sufficiently attractive price will induce high-quality banks to sell their loans but will necessarily require the use of other resources by the government. More subtle policies which involve greater interference in private markets will induce high-quality banks to sell their loans without the use of other resources by the government.

5 Conclusion

Appendix

A Proofs

A.1 Static Model

In this section, we show that we can re-write bank and buyer payoffs as stated in the static model and prove the existence of an equilibrium in the static version of our model. In the Technical Appendix, we prove a uniqueness result for high reputation levels.

A.1.1 Normalization of Bank and Buyer Payoffs

Suppose buyers offer contracts of the form (x, \hat{t}) . If a bank accepts such a contract, the banks' payoffs are given by

$$\hat{t} + (1 - x)(\pi\bar{v} + (1 - \pi)\underline{v} - c)$$

which can be expressed as

$$\hat{t} - x\underline{v} + (1 - x)(\pi(\bar{v} - \underline{v}) - c) + \underline{v}.$$

Buyers profits from (x, \hat{t}) if a bank of type π accepts are given by

$$x(\pi\bar{v} + (1 - \pi)\underline{v}) - \hat{t} = x\pi(\bar{v} - \underline{v}) - (\hat{t} - x\underline{v}).$$

If we use $t = \hat{t} - x\underline{v}$ and $v = \bar{v} - \underline{v}$, then these payoffs can be expressed as

$$t + (1 - x)(\pi v - c) + \underline{v}$$

and

$$x\pi v - t.$$

It is then immediate that we may normalize \underline{v} and analyze the effect of changes in the collateral value, \underline{v} , by analyzing changes in the loan spread, v .

A.1.2 Proof of Existence Proposition 1.

Suppose $\mu \leq \tilde{\mu}$. We will show that an equilibrium exists in which the contract z^* satisfies $t_l = \underline{\pi}v$, $x_l = 1$, $t_h = x_h^*\bar{\pi}v$ and x_h satisfies the equality version of the incentive constraint 3. Clearly z^* earns zero profits. Suppose buyer 2 offers z^* . Let \hat{z} denote buyer 1's contract. We will show if \hat{z} is not equal to z^* , then buyer 1 earns non-positive profits. Suppose by way of contradiction that \hat{z} earns strictly positive profits, that is,

$$\mu\delta_1(\hat{z}, z^*; \bar{\pi})(\hat{x}_h\bar{\pi}v - \hat{t}_h) + (1 - \mu)\delta_1(\hat{z}, z^*; \underline{\pi})(\hat{x}_l\underline{\pi}v - \hat{t}_l) > 0.$$

We will show that under the contradiction hypothesis, $\hat{x}_h\bar{\pi}v - \hat{t}_h$ must be positive and \hat{z} must attract the high-quality bank or $\delta_1(\hat{z}, z^*; \bar{\pi}) = 1$. To see this result, note that if $\hat{x}_l\underline{\pi}v > \hat{t}_l$,

the low quality bank will choose buyer 2's contract so that $\hat{x}_h \bar{\pi} v - \hat{t}_h$ must be positive and $\delta_1(\hat{z}, z^*; \bar{\pi}) = 1$. If $\hat{x}_l \underline{\pi} v < \hat{t}_l$, clearly $\hat{x}_h \bar{\pi} v - \hat{t}_h$ must be positive and $\delta_1(\hat{z}, z^*; \bar{\pi}) = 1$. Since $\delta_1(\hat{z}, z^*; \bar{\pi}) = 1$, \hat{z} must satisfy

$$\hat{t}_h + (1 - \hat{x}_h)(\bar{\pi} v - c) \geq x_h \bar{\pi} v + (1 - x_h)(\bar{\pi} v - c). \quad (28)$$

Fixing \hat{x}_h , choosing \hat{t}_h so that 28 holds with equality increases buyer 1's profits, so we assume that 28 holds with equality at \hat{z} . We can then re-write 28 as

$$\hat{t}_h - \hat{x}_h \bar{\pi} v = (x_h - \hat{x}_h) c.$$

Since $\hat{t}_h - \hat{x}_h \bar{\pi} v$ is negative, it follows that \hat{x}_h must be greater than x_h . Next, for any such (\hat{x}_h, \hat{t}_h) , since it is possible to reduce \hat{t}_l at any \hat{x}_l , the most profitable companion offer must satisfy the low-quality bank's incentive constraint with equality, that is,

$$\hat{t}_l + (1 - \hat{x}_l)(\underline{\pi} v - c) = \hat{t}_h + (1 - \hat{x}_h)(\underline{\pi} v - c). \quad (29)$$

We show that the most profitable companion offer has $\hat{x}_l = 1$. To see this result, note that for any $\hat{x}_l < 1$, increasing \hat{x}_l by $\varepsilon > 0$ and increasing \hat{t}_l by $\varepsilon(\underline{\pi} v - c)$ maintains the low-quality bank's incentive constraint, but increases profits by εc .

Consider now any \hat{x}_h associated with the deviation \hat{z} . We have shown that the best deviation associated with \hat{x}_h has \hat{t}_h satisfying 28, $\hat{x}_l = 1$, and \hat{t}_l satisfying 29 with $\hat{x}_l = 1$. The profits associated with this deviation are given by

$$\mu (\hat{x}_h \bar{\pi} v - \hat{t}_h) + (1 - \mu) (\underline{\pi} v - \hat{t}_l)$$

Using 28 with equality and 29 straightforward algebra shows that these profits can be written as

$$(\hat{x}_h - x_h) \frac{(\bar{\pi} - \underline{\pi}) v}{\tilde{\mu}} [\mu - \tilde{\mu}]$$

Since $\mu \leq \tilde{\mu}$ and $\hat{x}_h > x_h$, it follows that profits from such a deviation are negative.

Suppose now that $\mu \geq \tilde{\mu}$. We will show a mixed strategy equilibrium exists where the distribution over contracts $F(z)$ is given by 10. Clearly for every z in the support of F , profits are zero. We show that there is no \hat{z} which earns strictly positive profits. The argument is by contradiction. Suppose profits associated with \hat{z} are strictly positive so that

$$\int \mu \delta_1(\hat{z}, z; \bar{\pi}) [\hat{x}_h \bar{\pi} v - \hat{t}_h] + (1 - \mu) \delta_1(\hat{z}, z; \underline{\pi}) [\hat{x}_l \underline{\pi} v - \hat{t}_l] dF(z) > 0.$$

Using the same argument as in the proof of the first part of the proposition, it follows that $\hat{x}_h \bar{\pi} v \geq \hat{t}_h$. Next we show that without loss of generality, we can restrict ourselves to deviations \hat{z} such that for some z in the support of F , the high-quality bank is indifferent between z and \hat{z} , or

$$\hat{t}_h + (1 - \hat{x}_h) (\bar{\pi}v - c) = t_h + (1 - x_h) (\bar{\pi}v - c). \quad (30)$$

To see this result, notice that if for all z , the left hand side of 30 is strictly greater than the right hand side, then \hat{t}_h can be reduced and profits can be raised. If the left hand side of 30 is strictly below the right hand side for all z , then $\delta_1(\hat{z}, z; \bar{\pi}) = 0$ for all z and profits cannot be strictly positive.

Repeating the argument as in the proof of the first part of the proposition, it follows that we can restrict attention to deviations that satisfy the incentive constraint of the low-quality bank holding with equality, or

$$\hat{t}_l + (1 - \hat{x}_l) (\underline{\pi}v - c) = \hat{t}_h + (1 - \hat{x}_h) (\underline{\pi}v - c), \quad (31)$$

and $\hat{x}_l = 1$.

We now evaluate profits for a deviation of the form $\hat{x}_h = x_h + \varepsilon$ and \hat{t}_h and \hat{t}_l given by 30 and 31. For any contract z in the support of F , the probability of attracting either a high- or low-quality bank to contract z is given by $F(t_l)$. Since the high-quality bank is indifferent between z and \hat{z} , the probability of attracting the high quality bank to \hat{z} is given by $F(t_l)$. It is straightforward to show that the deviation contract has \hat{t}_l satisfying

$$\hat{t}_l = t_l + (\bar{\pi} - \underline{\pi})v\varepsilon.$$

Thus, the probability of attracting the low-quality bank to the deviation is given by $F(\hat{t}_l) = F(t_l + (\bar{\pi} - \underline{\pi})v\varepsilon)$. Holding fixed (x_h, t_h) , we then define profits from this deviation as a function of ε as

$$\begin{aligned} \Pi(\varepsilon) &= (1 - \mu) F(t_l + (\bar{\pi} - \underline{\pi})v\varepsilon) (\underline{\pi}v - t_l - (\bar{\pi} - \underline{\pi})v\varepsilon) \\ &\quad + \mu F(t_l) ((x_h + \varepsilon)\bar{\pi}v - t_h - (\bar{\pi}v - c)\varepsilon) \end{aligned}$$

Next we prove that profits are globally concave and attain a maximum at $\varepsilon = 0$. First, note that

$$\begin{aligned} \Pi'(\varepsilon) &= (1 - \mu)(\bar{\pi} - \underline{\pi})v f(t_l + (\bar{\pi} - \underline{\pi})v\varepsilon) (\underline{\pi}v - t_l - (\bar{\pi} - \underline{\pi})v\varepsilon) \\ &\quad - (1 - \mu)F(t_l + (\bar{\pi} - \underline{\pi})v\varepsilon) (\bar{\pi} - \underline{\pi})v + \mu F(t_l)c \end{aligned}$$

Then, setting $\varepsilon = 0$ yields

$$\Pi'(0) = (1 - \mu)(\bar{\pi} - \underline{\pi})v f(t_l) (\underline{\pi}v - t_l) - (1 - \mu)F(t_l) (\bar{\pi} - \underline{\pi})v + \mu F(t_l)c$$

Under the definition of F from 10, it follows that

$$f(t_l) = \left(\frac{\mu c - (1 - \mu)(\bar{\pi} - \underline{\pi})v}{(1 - \mu)(\bar{\pi} - \underline{\pi})v} \right) \frac{1}{t_l - \underline{\pi}v} F(t_l) \quad (32)$$

so that $\Pi'(0) = 0$. Re-writing 32 as

$$f(t_l) = \left(\frac{\mu - \tilde{\mu}}{\tilde{\mu}(1 - \mu)(t_l - \underline{\pi})} \right) F(t_l)$$

and noting that $\mu \geq \tilde{\mu}$ ensures that $f(t_l) \geq 0$ for all t_l .

We now prove that $\Pi''(\epsilon) < 0$ for all ϵ so that $\epsilon = 0$ is a global maximum. We have

$$\begin{aligned} \Pi''(\epsilon) = & (1 - \mu) ((\bar{\pi} - \underline{\pi})v)^2 f'(t_l + (\bar{\pi} - \underline{\pi})v\epsilon) (\underline{\pi}v - t_l - (\bar{\pi} - \underline{\pi})v\epsilon) \\ & - 2(1 - \mu) ((\bar{\pi} - \underline{\pi})v)^2 f(t_l + (\bar{\pi} - \underline{\pi})v\epsilon) \end{aligned}$$

Now, note that

$$f'(t_l) = \frac{f(t_l)}{t_l - \underline{\pi}v} \left[\frac{\mu c}{(1 - \mu)(\bar{\pi} - \underline{\pi})v} - 2 \right].$$

Hence,

$$\begin{aligned} \Pi''(\epsilon) = & (1 - \mu) ((\bar{\pi} - \underline{\pi})v)^2 f(t_l + (\bar{\pi} - \underline{\pi})v\epsilon) \\ & \times \left[-2 + (\underline{\pi}v - t_l - (\bar{\pi} - \underline{\pi})v\epsilon) \left(\frac{\mu c}{(1 - \mu)(\bar{\pi} - \underline{\pi})v} - 2 \right) \left(\frac{1}{t_l + (\bar{\pi} - \underline{\pi})v\epsilon - \underline{\pi}v} \right) \right] \\ = & - (1 - \mu) ((\bar{\pi} - \underline{\pi})v)^2 f(t_l + (\bar{\pi} - \underline{\pi})v\epsilon) \frac{\mu c}{(1 - \mu)(\bar{\pi} - \underline{\pi})v} \\ < & 0. \end{aligned}$$

A.2 Dynamic Model

In this section, we provide a proof of Proposition 5. In particular, we prove that if $\mu \geq \mu^*$, there exists a complete pooling equilibrium and when $\mu \leq \mu^*$, there exists a partial pooling equilibrium.

A.2.1 Existence of Complete Pooling when $\mu \geq \mu^*$.

We first prove that $\mu \geq \mu^*$ is a necessary condition for existence of the pooling equilibrium. Suppose for contradiction that $\mu < \mu^*$ and there is a pooling equilibrium with $x_h = 1$ and $t_h = \hat{p}(\mu)$. Recall that the value function associated with equilibrium outcomes in the static model yields constant utility to both types when $\mu \leq \tilde{\mu}$. As a result, $V(0; \pi) = V(\mu; \pi)$ when $\mu \leq \tilde{\mu}$. Consider then a deviation contract with $\hat{x}_h = \hat{x}_l$ and $\hat{t}_h = \hat{t}_l$ where the contract offer is statically preferred by the high-quality bank which and satisfies satisfy $\hat{t}_h \leq \hat{x}_h \hat{p}(\mu)$. Since $\mu < \mu^*$, in which case the high-quality bank statically prefers to hold rather than sell its entire portfolio at $\hat{p}(\mu)$, such a contract exists. Since there is no reputational loss to the high-quality bank at such a deviation, the deviation attracts the high-quality bank for sure. Thus, independent of whether this contract attracts the low-quality bank, since the offer lies

below the market-odds line the deviation earns strictly positive profits. We have then shown that both quality types selling their entire loan portfolio at price $\hat{p}(\mu)$ is not an equilibrium when $\mu < \mu^*$.

We next prove that for $\mu \geq \mu^*$, complete pooling with $x = 1$ is an equilibrium. The proof is by contradiction. Let $z^* = (1, \hat{p}(\mu), 1, \hat{p}(\mu))$. Clearly z^* earns zero profits if both quality types of banks accept z^* . As in the proof in the static model, suppose buyer 2 offers z^* and buyer 1 offers a different contract \hat{z} contract. We will show that if $\hat{z} \neq z^*$, then buyer 1 earns non-positive profits. Suppose by way of contradiction that \hat{z} earns strictly positive profits so that

$$\mu \delta_1(\hat{z}, z^*; \bar{\pi})(\hat{x}_h \bar{\pi} v - \hat{t}_h) + (1 - \mu) \delta_1(\hat{z}, z^*; \underline{\pi})(\hat{x}_l \underline{\pi} v - \hat{t}_l) > 0.$$

As in the proofs for the static model, $\delta_1(\hat{z}, z^*; \bar{\pi}) > 0$ and $\hat{x}_h \bar{\pi} v - \hat{t}_h$ must be positive. The reason is that if $\delta_1(\hat{z}, z^*; \underline{\pi})$ is positive, then $\hat{x}_l \underline{\pi} v - \hat{t}_l$ is negative (this follows from the fact that any contract with $\hat{t}_l \leq \hat{x}_l \underline{\pi} v$ yields less utility to the low-quality bank both statically and dynamically than the equilibrium under the given belief function). As a result, the deviation contract offered by buyer 1 must attract high-quality banks and must make positive profit on each high-quality bank it attracts.

In any contract, \hat{z} with $\delta_1(\hat{z}, z^*; \bar{\pi}) > 0$, the offer intended for the high-quality bank must yield statically more utility to the high-quality bank than the equilibrium offer. The reason for this is that offers which are statically less preferred by the high-quality bank are also dynamically less preferred because of the belief function and so such offers do not attract the high-quality bank. Thus, such deviation contracts must satisfy

$$\hat{t}_h + (1 - \hat{x}_h)(\bar{\pi} v - c) \geq \hat{p}(\mu). \quad (33)$$

This expression along with $\mu \geq \mu^*$ (which implies $\hat{p}(\mu) \geq \bar{\pi} v - c$) can be re-written to yield a lower bound on \hat{t}_h given by $\hat{t}_h \geq \hat{x}_h \hat{p}(\mu)$. In other words, any deviation offer which is attractive to the high-quality bank must also lie above the market-odds line.

We now prove that any contract which the high-quality bank prefers statically over the equilibrium is also preferred dynamically over the equilibrium by the low-quality bank. To see this, note that $\hat{x}_h \bar{\pi} v \geq \hat{t}_h$ and [33](#) imply

$$\hat{x}_h \geq \frac{1}{c}(\hat{p}(\mu) - \bar{\pi} v + c). \quad (34)$$

Then, the utility the low-quality bank would obtain by accepting the offer for the high-quality bank is given by

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi} v - c) + \beta V(1; \underline{\pi}).$$

Using 33 we obtain

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \geq \hat{p}(\mu) + \beta V(\mu; \underline{\pi}) + \beta V(1; \underline{\pi}) - \beta V(\mu; \underline{\pi}) - (1 - x_h)(\bar{\pi} - \underline{\pi})v$$

and using the lower bound on \hat{x}_h from 34 and the definition of μ^* , we can further decrease the bound to

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \geq \hat{p}(\mu) + \beta V(\mu; \underline{\pi}) + \beta V(1; \underline{\pi}) - \beta V(\mu; \underline{\pi}) - \frac{1 - \mu}{1 - \mu^*} (\bar{\pi} - \underline{\pi})v.$$

Since $\beta \geq 1 \geq \tilde{\mu}$, using the form of the value function implied by the static model, it is straightforward to show that

$$\beta V(1; \underline{\pi}) - \beta V(\mu; \underline{\pi}) \geq \frac{1 - \mu}{1 - \mu^*} (\bar{\pi} - \underline{\pi})v.$$

Hence,

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \geq \hat{p}(\mu) + \beta V(\mu; \underline{\pi}). \quad (35)$$

We have shown that any offer which is statically preferred by the high-quality bank satisfies $\hat{t}_i \geq \hat{x}_i \hat{p}(\mu)$ and $\delta_1(\hat{z}; z^*; \pi) > 0$ for $\pi = \bar{\pi}, \underline{\pi}$. Since \hat{z} makes strictly positive profits, it follows that

$$\hat{p}(\mu) > \hat{t}_l + (1 - \hat{x}_l)(\bar{\pi}v - c).$$

Incentive compatibility of the deviation contract for the low-quality bank implies that the contract must then satisfy

$$\hat{t}_l + (1 - \hat{x}_l)(\underline{\pi}v - c) + \beta V(0; \underline{\pi}) \geq \hat{t}_h + (1 - \hat{x}_h)(\underline{\pi}v - c) + \beta V(1; \underline{\pi}).$$

Since the deviation to earn strictly positive profits, the offer for the low-quality bank must also satisfy $\hat{x}_l \hat{p}(\mu) \geq \hat{t}_l$. We will show these conditions give rise to a contradiction. Such a deviation satisfies

$$\hat{p}(\mu) \geq \hat{t}_l + (1 - \hat{x}_l)(\bar{\pi}v - c) > \hat{t}_l + (1 - \hat{x}_l)(\underline{\pi}v - c)$$

and since the value function is monotone,

$$\beta V(\mu; \underline{\pi}) \geq \beta V(0; \underline{\pi}).$$

Then adding these inequalities we obtain

$$\hat{p}(\mu) + \beta V(\mu; \underline{\pi}) > \hat{t}_l + (1 - \hat{x}_l)(\underline{\pi}v - c) + \beta V(0; \underline{\pi}).$$

But from 35, the deviation also satisfies

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \geq \hat{p}(\mu) + \beta V(\mu; \underline{\pi})$$

Hence

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \geq \hat{p}(\mu) + \beta V(\mu; \underline{\pi}) > \hat{t}_l + (1 - \hat{x}_l)(\underline{\pi}v - c) + \beta V(0; \underline{\pi})$$

which contradicts incentive compatibility of the contract. Thus, such a profitable deviation does not exist which completes the proof.

A.2.2 Existence of a Partial Pooling Equilibrium when $\mu \leq \mu^*$

Suppose $\mu \leq \mu^*$ and $\beta \geq \frac{1}{\bar{\mu}}$. We prove that there exists an equilibrium in which buyers offer a contract $z^* = (\bar{x}_h, \bar{t}_h, 1, \bar{t}_l)$ where

$$\bar{x}_h = \frac{\tilde{\mu} - \mu}{\tilde{\mu} \frac{1-\mu}{1-\mu^*} - \mu}$$

and \bar{t}_l and \bar{t}_h are chosen so that

$$\bar{t}_l + \beta V(0; \underline{\pi}) = \bar{t}_h + (1 - \bar{x}_h)(\underline{\pi}v - c) + \beta V(\tilde{\mu}; \underline{\pi})$$

and

$$\mu(\bar{x}_h \bar{\pi}v - \bar{t}_h) + (1 - \mu)(1 - \alpha_l)(\bar{x}_h \underline{\pi}v - \bar{t}_h) + (1 - \mu)\alpha_l(\underline{\pi}v - \bar{t}_l) = 0.$$

where α_l is the probability that the low-quality bank chooses the offer $(1, \bar{t}_l)$ which is chosen so that 17 holds.

We again prove existence by way of contradiction. Suppose buyer 2 offers z^* and buyer 1 offers a contract \hat{z} which earns strictly positive profits. Since we have allowed banks to use mixed strategies, write profits of the deviation contract as

$$\begin{aligned} \Pi(\hat{z} = \mu\delta_1(\hat{z}, z^*; \bar{\pi}) & [\alpha_1((\hat{x}_h, \hat{t}_h); \bar{\pi})(\hat{x}_h \bar{\pi}v - \hat{t}_h) + \alpha_1((\hat{x}_l, \hat{t}_l); \bar{\pi})(\hat{x}_l \bar{\pi}v - \hat{t}_l)] \\ & + (1 - \mu)\delta_1(\hat{z}, z^*; \underline{\pi}) [\alpha_1((\hat{x}_h, \hat{t}_h); \underline{\pi})(\hat{x}_h \underline{\pi}v - \hat{t}_h) + \alpha_1((\hat{x}_l, \hat{t}_l); \underline{\pi})(\hat{x}_l \underline{\pi}v - \hat{t}_l)] \end{aligned}$$

where we use $\alpha_1((x_i, t_i); \pi)$ to denote the probability that a bank of quality type π chooses offer $i = h, l$ of contract z offered by buyer 1 conditional on $\delta_1(z_1, z_2; \pi) > 0$.

Using the same argument as in the proof of existence in the static environment, $\hat{x}_h \bar{\pi}v \geq \hat{t}_h$ and $\delta_1(\hat{z}, z^*; \bar{\pi}) > 0$. Furthermore, fixing $\delta_1(\hat{z}, z; \pi)$, choosing (\hat{x}_l, \hat{t}_l) so that the low-quality bank's incentive constraint holds with equality and induces $\alpha_1((\hat{x}_l, \hat{t}_l); \underline{\pi}) = 1$ increases profits. Thus we will restrict attention to deviation contracts which induce pure strategy by banks if accepted.

Since $\delta_1(\hat{z}, z; \bar{\pi}) > 0$, the deviation offer must attract the high-quality type so that

$$\hat{t}_h + (1 - \hat{x}_h)(\bar{\pi}v - c) \geq \bar{t}_h + (1 - \bar{x}_h)(\bar{\pi}v - c). \quad (36)$$

We will proceed in two cases. In the first, \hat{z} differs from z^* in the offer intended for the high-quality type (and possibly for the low-quality type). Such offers are intended to earn profits by cream-skimming. In the second case, \hat{z} will make the same offer for high-quality types z^* but will make a different offer to low-quality banks so that $(\hat{x}_l, \hat{t}_l) \neq (1, \bar{t}_l)$. Such

deviations are intended to induce a more profitable tie-breaking rule by low-quality banks.

First consider any deviation contract \hat{z} with an offer for the high-quality type which differs from the equilibrium. Re-writing 36, we have

$$\hat{t}_h \geq \bar{t}_h - \bar{x}_h (\bar{\pi}v - c) + \hat{x}_h (\bar{\pi}v - c).$$

Then, under the belief function, the utility the low-quality bank would obtain at the deviation offer is given by

$$\hat{t}_h + (1 - \hat{x}_h) (\underline{\pi}v - c) + \beta V(1; \underline{\pi}).$$

Using the lower bound on \hat{t}_h , we obtain

$$\begin{aligned} & \hat{t}_h + (1 - \hat{x}_h) (\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \\ & \geq \bar{t}_h + (1 - \bar{x}_h) (\underline{\pi}v - c) + \beta V(0; \underline{\pi}) + \beta V(1; \underline{\pi}) - \beta V(0; \underline{\pi}) - \bar{x}_h (\bar{\pi} - \underline{\pi})v + \hat{x}_h (\bar{\pi} - \underline{\pi})v \\ & \geq \bar{t}_l + \beta V(0; \underline{\pi}) + (\beta - \bar{x}_h) (\bar{\pi} - \underline{\pi})v \end{aligned}$$

where the last inequality follows by minimizing \hat{x}_h and substituting for \bar{t}_l and the value functions at the extreme points $\mu = 0, 1$. Since $\beta \geq 1 \geq x_h^*$, it follows that

$$\hat{t}_h + (1 - \hat{x}_h) (\underline{\pi}v - c) + \beta V(1; \underline{\pi}) \geq \bar{t}_l + \beta V(0; \underline{\pi}). \quad (37)$$

We now show that the deviation contract must have the property that the high-quality bank statically prefers the equilibrium offer intended for itself over the deviation offer intended for the low-quality bank or,

$$\bar{t}_h + (1 - \bar{x}_h) (\bar{\pi}v - c) \geq \hat{t}_l + (1 - \hat{x}_l) (\bar{\pi}v - c).$$

Suppose instead the companion offer for the low-quality bank is preferred by the high-quality bank to the equilibrium. Such an incentive compatible deviation must attract both high- and low-quality banks, or $\delta_1(\hat{z}_h, z^*; \bar{\pi}) = 1$ and $\delta_1(\hat{z}_l, z^*; \underline{\pi}) = 1$. Furthermore, since \hat{z} is incentive compatible straightforward algebra implies that $\hat{x}_l \geq \hat{x}_h$ where we have used the fact that the off-path equilibrium belief function assigns constant reputation following such trades with $\mu' = 1$.

Next, since both contracts are preferred statically by the high-quality bank, they must both satisfy $\hat{x}_i \hat{p}(\mu) \leq \hat{t}_i$. We prove this by demonstrating that both of the following inequalities hold:

$$\bar{t}_h + (1 - \bar{x}_h) (\bar{\pi}v - c) \geq \hat{p}(\mu) \quad (38)$$

$$\bar{t}_h + (1 - \bar{x}_h) (\bar{\pi}v - c) \geq \bar{\pi}v - c. \quad (39)$$

Inequality 38 follows from the fact that when $\mu \leq \mu^*$, the static indifference curve of the high-quality bank is steeper than the market-odds line. The inequality 39 holds exactly by

construction of \bar{x}_h and is indeed the upper bound of x_h that can be sustained in such a partial pooling equilibrium. Straightforward algebra can be used to prove both of these inequalities hold and is therefore omitted.

We have shown that if both offers of the deviation \hat{z} are preferred statically by the high type to the equilibrium, then $\delta_1(\hat{z}_h, z^*; \bar{\pi}) = 1$ and $\delta_1(\hat{z}_l, z^*; \underline{\pi}) = 1$, that $\hat{x}_l \geq \hat{x}_h$ and that $\hat{x}_i \hat{p}(\mu) \leq \hat{t}_i$. We now show that these conditions imply that \hat{z} earns non-positive profits. Profits of this deviation are then given by

$$\begin{aligned} \mu (\hat{x}_h \bar{\pi} v - \hat{t}_h) + (1 - \mu) (\hat{x}_l \underline{\pi} v - \hat{t}_l) &\leq \mu (\hat{x}_h \bar{\pi} v - \hat{p}(\mu) \hat{x}_h) + (1 - \mu) (\hat{x}_l \underline{\pi} v - \hat{p}(\mu) \hat{x}_l) \\ &\leq \mu (\hat{x}_l \bar{\pi} v - \hat{p}(\mu) \hat{x}_l) + (1 - \mu) (\hat{x}_l \underline{\pi} v - \hat{p}(\mu) \hat{x}_l) \\ &= \hat{x}_l (\mu \bar{\pi} v + (1 - \mu) \underline{\pi} v - \hat{p}(\mu)) \\ &= \hat{x}_l (\hat{p}(\mu) - \hat{p}(\mu)) = 0. \end{aligned}$$

As a result, since \hat{z} earns strictly positive profits, it must be that

$$\hat{t}_h + (1 - \hat{x}_h) (\bar{\pi} v - c) \geq \bar{t}_h + (1 - \bar{x}_h) (\bar{\pi} v - c) > \hat{t}_l + (1 - \hat{x}_l) (\bar{\pi} v - c).$$

Under the equilibrium beliefs, the reputation assigned to a bank accepting an offers (\hat{x}_l, \hat{t}_l) which the high-quality type does not prefer over the equilibrium is zero. Since reputation in this region of offers is constant, repeating arguments used in proof of existence in the static model, we can restrict attention to deviation offers that satisfy $\hat{x}_l = 1$ and the incentive constraint

$$\hat{t}_l + \beta V(0; \underline{\pi}) \geq \hat{t}_h + (1 - \hat{x}_h) (\underline{\pi} v - c) + \beta V(1; \underline{\pi}).$$

It immediately follows from 37 that

$$\hat{t}_l + \beta V(0; \underline{\pi}) \geq \bar{t}_l + \beta V(0; \underline{\pi})$$

which yields $\hat{t}_l \geq \bar{t}_l$ as a consequence.

We now show that profits from such a deviation are negative. The profits from any such deviation are given by

$$\Pi = \mu (\hat{x}_h \bar{\pi} v - \hat{t}_h) + (1 - \mu) (\underline{\pi} v - \hat{t}_l)$$

Using incentive compatibility and re-arranging terms, we obtain

$$\Pi \leq \hat{x}_h (\mu \bar{\pi} v + (1 - \mu) (\underline{\pi} v - c)) - \hat{t}_h - (1 - \mu) (\beta [V(1; \underline{\pi}) - V(0; \underline{\pi})] - c)$$

Substituting the lower bound for \hat{t}_h from 36 and re-arranging terms, we obtain

$$\begin{aligned} \Pi &\leq \hat{x}_h (-(1 - \mu) ((\bar{\pi} - \underline{\pi}) v + c) + c) - \bar{t}_h + \bar{x}_h (\bar{\pi} v - c) \\ &\quad - (1 - \mu) (\beta [V(1; \underline{\pi}) - V(0; \underline{\pi})] - c) \end{aligned} \tag{40}$$

Since $\mu \leq \mu^* < \tilde{\mu}$, it follows that

$$(1 - \mu) ((\bar{\pi} - \underline{\pi})v + c) + c < 0$$

so that the right hand side of 40 is maximized at $\hat{x}_h = 0$. It is possible to show that the remaining terms on the right hand side of 40 are negative, or

$$-t_h^* + x_h^* (\bar{\pi}v - c) - (1 - \mu) (\beta (\bar{\pi} - \underline{\pi})v - c) \leq 0.$$

We have shown that any deviation contract \hat{z} where the offer for the high quality bank differs from the equilibrium offer earns non-positive profits.

We now prove that any deviation where $(\hat{x}_h, \hat{t}_h) = (\bar{x}_h, \bar{t}_h)$ also yields negative profits. As in the earlier case, the best such deviation has $\hat{x}_l = 1$. So we may consider any such deviation as perturbation from the equilibrium offer for the low-quality bank $(1, \bar{t}_l + \varepsilon)$ where $\varepsilon \geq 0$. Profits from such a deviation are given by

$$\frac{1}{2}\mu (\bar{x}_h \bar{\pi}v - \bar{t}_h) + (1 - \mu)(\underline{\pi}v - \bar{t}_l - \varepsilon).$$

The reason we obtain the above expression for profits is that buyer 1 attracts half of the high-quality banks (when a bank is indifferent between two buyers we assume the bank randomizes equally) but buyer 1 is now offering strictly higher utility to the low-quality bank and so attracts all of the high quality banks. Equivalently, we have $\delta_1(\hat{z}, z^*; \bar{\pi}) = 1/2$, and $\delta_1(\hat{z}, z^*; \underline{\pi}) = 1$. In the Technical Appendix, we show prove that even when $\varepsilon = 0$ such a deviation earns negative profits when $\mu^* \leq 1/2$.

We have thus proven that any deviation contract \hat{z} must earn negative profits, which completes the proof.

References

- ARORA, S., B. BOAZ, M. BRUNNERMEIER, AND R. GE (2009): “Computational Complexity and Information Asymmetry in Financial Products,” Working Paper, Princeton University, Department of Computer Science.
- ASHCRAFT, A. AND T. SCHUERMANN (2008): “Understanding the securitization of sub-prime mortgage credit,” 2:3, 191–309.
- CHARI, V. V., A. SHOURIDEH, AND A. ZETLIN-JONES (2010): “Adverse selection, reputation and sudden collapses in secondary loan markets,” Tech. rep., National Bureau of Economic Research.

- DASGUPTA, P. AND E. MASKIN (1986): “The existence of equilibrium in discontinuous economic games, I: Theory,” *The Review of Economic Studies*, 53, 1–26.
- DEMARZO, P. (2005): “The pooling and tranching of securities: A model of informed intermediation,” *Review of Financial Studies*, 18, 1.
- DEMARZO, P. AND D. DUFFIE (1999): “A liquidity-based model of security design,” *Econometrica*, 67, 65–99.
- DEWATRIPONT, M. AND J. TIROLE (1994): *The Prudential Regulation of Banks*, MIT Press.
- DIAMOND, D. (1989): “Reputation acquisition in debt markets,” *The Journal of Political Economy*, 97, 828–862.
- DIAMOND, D. W. AND P. H. DYBVIK (1983): “Bank Runs, Deposit Insurance, and Liquidity,” *The Journal of Political Economy*, 401–419.
- DOWNING, C., D. JAFFEE, AND N. WALLACE (2009): “Is the Market for Mortgage-Backed Securities a Market for Lemons?” *Review of Financial Studies*, 22, 2257.
- DRUCKER, S. AND C. MAYER (2008): “Inside Information and Market Making in Secondary Mortgage Markets,” Tech. rep., working paper, Renaissance Technologies LL3 and Duke University.
- EISFELDT, A. (2004): “Endogenous Liquidity in Asset Markets,” *The Journal of Finance*, 59, 1–30.
- ELUL, R. (2011): “Securitization and Mortgage Default,” Philadelphia Fed Working Paper.
- ELY, J., D. FUDENBERG, AND D. LEVINE (2008): “When is reputation bad?” *Games and Economic Behavior*, 63, 498–526.
- ELY, J. AND J. VÄLIMÄKI (2003): “Bad Reputation*,” *Quarterly Journal of Economics*, 118, 785–814.
- FANG, L. H. (2005): “Investment bank reputation and the price and quality of underwriting services,” *The Journal of Finance*, 60, 2729–2761.
- GARLEANU, N. AND L. PEDERSEN (2004): “Adverse selection and the required return,” *Review of Financial Studies*, 17, 643–665.

- GLOSTEN, L. AND P. MILGROM (1985): “Bid, ask and transaction prices in a market-maker market with heterogeneously informed traders,” *Journal of Financial Economics*, 14, 71–100.
- GUERRIERI, V. AND R. SHIMER (2011): “Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality,” Tech. rep., mimeo.
- GUERRIERI, V., R. SHIMER, AND R. WRIGHT (2010): “Adverse Selection in Competitive Search Equilibrium,” *Econometrica*, 78, 1823–1862.
- IVASHINA, V. (2009): “Asymmetric Information Effects on Loan Spreads,” *Journal of Financial Economics*, 92, 300–319.
- KREPS, D. AND R. WILSON (1982): “Reputation and Imperfect Information,” *Journal of Economic Theory*, 27, 253–279.
- KURLAT, P. (2009): “Lemons, Market Shutdowns and Learning,” *MIT Working Paper*.
- KYLE, A. (1985): “Continuous auctions and insider trading,” *Econometrica*, 53, 1315–1335.
- MAILATH, G. AND L. SAMUELSON (2001): “Who wants a good reputation?” *The Review of Economic Studies*, 68, 415–441.
- MIAN, A. AND A. SUFI (2009): “The Consequences of Mortgage Credit Expansion: Evidence From the US Mortgage Default Crisis,” *The Quarterly Journal of Economics*, 124, 1449.
- MILGROM, P. AND J. ROBERTS (1982): “Predation, Reputation, and Entry Deterrence,” *Journal of Economics Theory*, 27, 280–312.
- MYERS, S. AND N. MAJLUF (1984): “Corporate financing and investment decisions when firms have information that investors do not have,” .
- ORDOÑEZ, G. (2008): “Fragility of Reputation and Clustering in Risk Taking,” Working Paper, University of California Los Angeles.
- PHILIPPON, T. AND V. SKRETA (2011): “Efficient Interventions in Markets with Adverse Selection,” *American Economic Review*, forthcoming.
- ROSENTHAL, R. W. AND A. WEISS (1984): “Mixed-strategy equilibrium in a market with asymmetric information,” *The Review of Economic Studies*, 51, 333–342.

ROSS, D. G. (2010): “The dominant bank effect: How high lender reputation affects the information content and terms of bank loans,” *Review of Financial Studies*, 23, 2730–2756.

ROTHSCHILD, M. AND J. STIGLITZ (1976): “Equilibrium in competitive insurance markets: An essay on the economics of imperfect information,” *The Quarterly Journal of Economics*, 629–649.