# On Fragmented Markets

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**Abstract**

**PRELIMINARY**
Centralized markets reduce the costs of search for buyers and sellers. More importantly, their 'thickness' increases the chance of order execution at competitive prices. In spite of the incentives to consolidate, some markets, securities markets being the most notable, have fragmented into multiple trading venues. We argue in this paper that fragmentation is an unavoidable feature of *any* centralized exchange except in certain special circumstances.

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# 1 Introduction

A centralized exchange reduces the costs of clearing, settlement and search compared with a market consisting of multiple trading venues. Even were these costs to decline because of technological innovation, a centralized exchange should still dominate a fragmented market because traders would choose the venue with the greatest 'thickness' or liquidity. A thick or liquid exchange is one that offers the highest probability of order execution and the most competitive prices. Each additional trader on an exchange reduces the execution risk for other potential traders, attracting more traders. This positive feedback should encourage trade to be concentrated in a single exchange.

In spite of the incentives to consolidate, some markets, securities markets being the most notable, have spawned multiple trading venues.[1] Traditional exchanges now face a host of competitors such as ECNS (electronic communication networks), ATS (alternative trading systems) and the trading desks of broker/dealer firms. The Toronto Stock Exchange (now known as the TMX group), for example, which once enjoyed a virtual monopoly on trading, faces a fragmented market of competing venues such as Alpha, Pure and MATCH Now. Importantly these alternative venues use a variety of pricing rules, may not broadcast the bids they receive and in some cases allow traders to restrict who they will transact with.

It has been argued that fragmentation enhances efficiency because competition between exchanges forces them to narrow their bid-ask spreads (e.g. Pagano (1989), (Biais, Martimort, and Rochet (2000)). Indeed, fragmentation in securities markets can be traced to regulation in the 80's and 90's designed to limit the abuse of market power by operators of centralized exchanges.[2] Alternative trading venues may also arise to cater to the needs of a heterogeneous clientele. Investors, for example, differ in their preferences for order size, anonymity and likelihood of execution (Harris (1993), Ambrus and Argenziano (2009) and Petrella (2009)). Congestion or 'crowding out' effects can also cause market fragmentation as agents trade-off thickness in one venue for less competition in another (see Ellison and Fudenberg (2003), Ellison, Fudenberg and Mobius (2004)).[3]

[1]For fragmentation in labor markets see Roth and Xing (1994).

[2]Regulation National Market System in the US and Market in Financial Instruments Directive (MiFID) in Europe.

[3]We do not include misco-ordination in this list. Miscoordination allows multiple venues to

This paper argues that a tendency towards fragmentation is an unavoidable feature of *any* centralized exchange. Within the model in which we make this point, the usual arguments for why a centralized exchange will or should fragment do not apply. These arguments are:

1. The exchange is operated by a profit seeking mediator who has an incentive to restrict trade to increase rents. This is not the case in our set up.

2. There is a desire for anonymity or preferences for certain order sizes on the part of some traders. The agents in our model have no preference for anonymity and are all interested in the same order size. Indeed, what is traded in our model is a homogenous good.

3. One or more agents benefit by changing the time at which trades take place. In our model there is a single time period, so timing of trades is irrelevant.

4. Prices do not properly impound available information. Ours is a private values model with no common values component.

The reasons for fragmentation in this list can all be eliminated *in principle* by a suitable (but possibly impractical) mechanism. In the first case, the operator could be mandated to implement the constrained efficient mechanism. In the remaining cases, a mechanism that allowed agents to use a richer message space to communicate preferences could be employed. In our case, the tendency to fragmentation *cannot* be retarded by deploying a suitable mechanism unless the mechanism is a money pump for its participants. This is the mainspring of our claim that fragmentation is an unavoidable feature of a centralized exchange.

Our setting is the Myerson and Satterthwaite (1983) model of bilateral trade with buyers and sellers. Each seller has one unit of a homogenous good and each buyer is interested in purchasing at most one unit of the same good. The private information of each buyer is their marginal value for the good and the private information of each seller is the opportunity cost of their endowment. We model trade within the centralized exchange as being conducted via a direct *deterministic* mechanism. The virtue of this formulation is that our conclusions are not tied to a particular choice of trading rule. We require the mechanism to be ex-post individually rational and

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coexist but does not explain why a single venue would fragment into multiple venues.

ex-post (weakly) budget balanced, features enjoyed by many observed trading rules. We also require the mechanism to be robust or insensitive to the beliefs of agents. Like Hagerty and Rogerson (1987) we model this by requiring the mechanism to be dominant strategy incentive compatible. These conditions do not exclude the possibility that the mechanism can depend on the designer's beliefs. For example, the designer could select a single price at which all trade must take place *a -priori* that depends on the designer's beleifs about the distribution of types of the agents. In what follows we will also examine mechanisms that are robust to the beliefs of the designer as well.

Our argument relies on a notion of the core of a game of incomplete information introduced in Peivandi (2013). Roughly speaking, a centralized exchange, modeled as a mechanism, is *blocked* by a coalition of agents and an alternative deterministic mechanism if the alternative mechanism gives to each member of the blocking coalition at least as much surplus as they would obtain if they remain in the central exchange. Furthermore, no agent outside the blocking coalition wishes to participate in the alternative mechanism. The blocking mechanism is also required to be dominant strategy incentive compatible and ex-post (weakly) budget balanced mechanism. Thus, it is not enough for the blocking agents to enjoy gains from trade they must be able to implement a self-financing mechanism to secure those gains.

Under this notion of blocking we prove two results. First, the only class of deterministic mechanisms that cannot be blocked are posted price mechanisms. In this class, a price $p$ is selected. If a pair of buyer and seller wish to transact they do so at price $p$. Any fixed price $p$ will *not* do. The price $p$ must be tailored to the underlying distribution of types. Furthermore, not any distribution will do. The distribution must satisfy some regularity conditions like the monotone hazard rate condition. For example, under the monotone hazard rate condition a price $p$ for which the hazard rate of the buyer is non-positive and the hazard rate of the seller is not less than one that cannot be blocked. The upshot is that the only mechanisms immune to blocking are sensitive to the underlying distribution of types.

One may prefer a mechanism to be robust not just against the beliefs of the agents but against the beliefs of the designer as well. If this additional condition is imposed, then, no deterministic mechanism (within the class considered) is immune to blocking. We show that every mechanism can be blocked by a simple deterministic mechanism called a positive spread posted price mechanism. In a positive spread posted price mechanism, two prices $p_1 \leq p_2$ are posted. If buyer and seller agree to trade, the seller is paid $p_1$ and the buyer must pay $p_2$. The spread of a posted price mechanism is $p_2 - p_1$ and this what the designer pockets. A posted price mechanism is one where $p_1 = p_2 = p$. Thus, every mechanism can be blocked by a mechanism that gives the operator of the blocking mechanism positive expected profit. This is the justification for our assertion that fragmentation is an unavoidable feature of a centralized exchange.

The conclusion that no deterministic mechanism (within the class considered) is immune to blocking implies no collection of smaller exchanges can be immune to blocking. The outcome of a collection of smaller exchanges can be replicated by a suitable mechanism. This mechanism would also be blocked. This extreme conclusion highlights the fact that incentive compatibility and budget balancedness conspire to make markets fissiparous. In reality, there will be forces pushing in favor of centralization and the final outcome will be a balance of these opposing tendencies.

In the next section of this paper we introduce notation and give a precise definition of the notion of blocking employed in this paper. Subsequently we contrast the notion of blocking employed here with prior notions of the core of games with incomplete information. The following section states and proves the main result.

# 2 Notation and Definitions

Let $N = \{1, 2, 3, ..., n\}$ be the set of agents. The value of agent $i$ for a unit of the good is $v_i \in V_i$ where $V_i \subset \mathbb{R}^+$ is a finite set. Now, $v_i$ of agent $i$ is the private information of agent $i$ and we assume that the $v_i$'s are independently distributed. Each agent $i$ has an endowment $\omega_i \in \{0, 1\}$ of the good, which is common knowledge. Note that if $\omega_i = 1$, then agent $i$ is a seller and if $\omega_i = 0$ then $i$ is a buyer.

Preferences of each agent are quasilinear, that is, agent $i$'s payoff from receiving $q$ units of the good for a payment of $t$ is $qv_i - t$.

A direct mechanism is defined in terms of two functions. The first is an allocation rule that stipulates as a function of the profile of reports of the private information of the agents an allocation of the good. If $Q$ is the allocation rule we denote the component of $Q$ that corresponds to agent $i$'s allocation by $q_i$. Thus, $q_i : \prod_{i \in N} V_i \to \mathbb{R}^+$. As agent $i$ has an endowment of $\omega_i$ we will require that any allocation rule be feasible in the sense that for all $i \in N$ and all profiles $\mathbf{v} \in \prod_{i \in N} V_i$ that

1. $1 \geq q_i(\mathbf{v}) + \omega_\mathbf{i} \geq \mathbf{0}$, and,

2. $\sum_{i \in N} q_{\mathbf{v}} = 0$.

The second is a rule that stipulates as a function of the profile of reports of the private information of the agents the *per-unit* price of quantity each agent must make. If $P$ is the payment rule, the component of $P$ that corresponds to agent $i$'s per-unit payment is denoted $p_i$. Thus, $p_i : \prod_{i \in N} V_i \rightarrow \mathbb{R}^+$.

We will require the mechanism to be dominant strategy incentive compatible. To define this formally, it is convenient to denote a generic element of $\prod_{i \in N} V_i$ by $v$ and its $i^{th}$ component by $v_i$. Let $v = ((v_i, v_{-i})$ and $\hat{v} = ((\hat{v}_i, v_{-i})$. Observe that $\hat{v}$ differs from $v$ in that agent $i$ changes the report of his marginal value only. Thus, agents can misreport their marginal value opportunity cost but not their role as buyer or seller. Note that we only need to impose incentive compatibility on deviations to profiles that result in feasible outcomes. The mechanism $(Q, P)$ is dominant strategy incentive compatible if:

$$q_i(v)(v_i - p_i(v)) \geq q_i(\hat{v})(v_i - p_i(\hat{v})).$$

for all $v$ and $\hat{v}$. The mechanism $(Q, P)$ is ex-post individually rational if for all message profiles $v \in \prod_{i \in N} V_i$ and all $i \in N$

$$q_i(v)(v_i - p_i(v)) \geq 0.$$

The mechanism $(Q, P)$ is (weakly) ex-post budget balanced if for all $v \in \prod_{i \in N} V_i$

$$\sum_{i \in N} p_i(v) q_i(v) \geq 0.$$

Given a mechanism $(Q, P)$, the utility that agent $i \in N$ under profile $v$ enjoys is

$$u_i(v, Q, P) = q_i(v)(v_i - p_i(v)).$$

The expected utility that agent $i \in N$ enjoys when her type is $v_i$ is

$$E_{v_{-i}}[u_i(\{v_i, v_{-i}\}, Q, P)].$$

Fix a subset $A \subseteq N$ of the agents and for each $i \in A$ a *positive measure* subset $V_i' \subseteq V_i$. Call $V_i'$ the **critical** set of types for agent $i$. For each $i \in A$ let $T_i$ be the event that each agent $j \in A \setminus \{i\}$ has a type in $V_j'$. We shall say the set $A$ *blocks* $(Q, P)$ with respect to $\Pi_{i \in A} V_i'$ if there exists a feasible, incentive compatible, ex-post individually rational mechanism $(\hat{Q}, \hat{P})$:

$$\hat{p}_i : \prod_{i \in A} V_i \to \mathbb{R}^+ \quad \text{and} \quad \hat{q}_i : \prod_{i \in A} V_i \to \mathbb{R}^+ \ \forall i \in A$$

that satisfies five conditions. To understand these conditions it is helpful to imagine that before participating in the mechanism $(Q, P)$, each agent in $A$ (and only $A$) is invited to participate in $(\hat{Q}, \hat{P})$. If every agent in $A$ accepts the invitation, then, the agents in $A$ enjoy the outcome delivered by $(\hat{Q}, \hat{P})$. If at least one of the agents in $A$ declines the invitation, then all agents are required to participate in $(Q, P)$.

1. If $v_i \in V_i'$, then,

$$E_{-i}[u_i(\{v_i, v_{-i}\}, Q, P)|T_i] \leq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \ \forall i \in A \tag{1}$$

2. If $v_i \notin V_i'$ then,

$$E_{-i}[u_i(\{v_i, v_{-i}\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i]. \tag{2}$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$

$$\sum_{i \in A} \hat{q}_i(\bar{v}) = 0. \tag{3}$$

4. 

$$E[\sum_{i \in A} \hat{q}_i(\bar{v}) \hat{p}_i(\bar{v}) | \bar{v} \in \prod_{i \in A} V_i'] > 0. \tag{4}$$

Note that the block is not formed when some agents in $A$ choose not to participate in the blocking mechanism. Therefore, Condition (1) and (2) state that if each $i \in A$ has a type in $V_i'$, then all agents in $A$ simultaneously choosing to participate in $(\hat{Q}, \hat{P})$ is an equilibrium. Condition (3) ensures that the sum of the net trades is zero. Condition (4) requires that on some profile, the blocking mechanism generates a positive surplus. There is a technical and a substantive reason for this condition.

First, it eliminates the possibility that all mechanisms are blocked by themselves. Second, there must be an incentive for someone to offer the blocking mechanism.

## 3 Prior Notions of Blocking

The set of mechanisms that cannot be blocked in the sense defined can be interpreted as the core of the underlying game of incomplete information. This is not the first notion of a core for games of incomplete information. The various notions differ in the information that agents considering blocking condition on. We will focus our comparison on notions of the core that like this one make use of the incentive compatibility of some mechanism. To describe them it is useful to imagine two mediators. Mediator 1 proposes a Bayesian incentive compatible mechanism to all the agents in $N$. It is important to note that the mechanism proposed by Mediator 1 is Bayesian incentive compatible under the assumption that reports made in this mechanism do not influence the outcomes in other mechanisms. For the given profile of types, let $x$ be the resulting allocation and call this the status quo.

Mediator 2 now enters and offers an incentive compatible mechanism (possibly randomized) that selects a subset $S$ of agents and an allocation for the agents in $S$. Mediator 2 succeeds in blocking the status quo if the interim expected utility of each agent in $S$ under her proposed mechanism is at least as large as the utility she would enjoy in the status quo with strict inequality for at least one agent. An important thing to note about Mediator 2's mechanism is that it can choose the status quo, $x$. This means that incentive compatibility in Mediator 2's mechanism *assumes* truthful reporting to Mediator 1.

The various prior notions of the core differ in the restrictions they place on Mediator 2. Dutta and Vohra (2005), for example, restrict Mediator 2 to randomizing between an allocation for a single subset $S$ and the status quo. Like the notion used here there is a subset of agents and a subset of their types. The mechanism proposed by Mediator 1 is blocked if each agent within the subset has a type within the prescribed set of types such that such that the relevant agents are the only ones that prefer the alternative to the status quo. Dutta and Vohra (2005) call this a credible objection. Thus, a coalition has a credible objection if it can identify an informational event such that the types of agents involved in the event are the only ones that prefer the alternative proposed to the status quo, given that the other types behave as prescribed in the objection. Myerson (2007), using the virtual utility construct, proposes a core notion that, in addition to the credibility requirements, considers random coalition formation and random allocations for each coalition. Finally, Serrano and Vohra (2007) set up the Dutta-Vohra and Myerson objections as communication mechanisms played by the agents in each coalition, and derive their objections inequalities from the equilibria of such communication games.

Recall that prior notions of blocking do not allow an agent's report to Mediator 1 to take into account the effect this will have on the choice of the status quo when it comes time for Mediator 2 to choose a mechanism. This makes the status quo exogenous in that the decision to participate in the blocking mechanism is made *after* the outcome of the incumbent mechanism is announced. In the notion of blocking used here, agents must decide *simultaneously* which mechanism to participate in. For the applications we have in mind we think this timing is more natural.

# 4 Main Result

As stated earlier, our main result is that only mechanisms immune to blocking (under conditions on the distribution of types) is a posted price mechanism. To provide some intuition restrict attention to positive (or zero) spread posted price mechanisms. Consider a positive spread posted price mechanism with a spread of $\delta > 0$. It is not hard to see that such a mechanism can always be blocked by positive spread posted price mechanism with a slightly smaller spread. Thus, the only mechanisms (within the class considered) that might be immune to blocking are posted price mechanisms. The only question that remains is what the posted price should be. To see how to determine that price, consider the case of one buyer and one seller. Agent 1 is a seller with an opportunity cost of $c \in [0, 1]$ and $\omega_1 = 1$. Agent 2 is a buyer with a value of $v \in [0, 1]$ and $\omega_2 = 0$. Assume $v$ and $c$ are private information that are distributed independently with atomless density functions $f(v)$ and $g(c)$ respectively. Denote the corresponding distribution functions by $F$ and $G$. Endowments, however, are common knowledge.

Let $p \in [0, 1]$ be the posted price of the zero spread mechanism. Consider a positive spread posted price mechanism $(p', p'')$ with $p' < p''$ as a possible blocking mechanism. As there are only two agents (one buyer and one seller), the blocking coalition will consist of just these two agents. It remains to identify a critical set of types. We can do this by 'reverse' engineering. There are three cases:

1. **Case:** $p' < p'' < p$
   A buyer with type $v \geq p''$ strictly prefers the blocking mechanism conditional on a seller being present. Thus, the critical set of types of the buyer will be $[1, p'']$. Now, we find the critical set of types for the seller that would make them interested in joining the blocking mechanism. A seller with type $c < p'$ will join the blocking mechanism only if:

$$(p - c)Pr(v \geq p | v \geq p'') \leq (p' - c) \;\Rightarrow\; \frac{1 - F(p)}{1 - F(p'')} \leq \frac{p' - c}{p - c}.$$

   The right hand side is maximized at $c = 0$, therefore the posted price $p$ *cannot* be blocked by the positive spread posted price mechanism $(p', p'')$ if the following holds:

$$\frac{1 - F(p)}{1 - F(p'')} > \frac{p'}{p}.$$

   This is equivalent to $p(1 - F(p)) > p'(1 - F(p''))$. Therefore, if for all $p' < p$,

$$p(1 - F(p)) > p'(1 - F(p')) \tag{5}$$

   the posted price mechanism cannot be blocked with prices lower than $p$.

2. **Case 2:** $p < p' < p''$
   In this case the seller with opportunity cost $c < p'$ joins the blocking mechanism conditional on a buyer being present. A buyer with type $v > p''$ joins the mechanism if

$$(v - p)Pr(c \leq p | c \leq p') \leq v - p''.$$

   As in Case 1 this does not happen if:

$$\forall\, p' > p \; (1 - G(p)) > (1 - p')G(p'). \tag{6}$$

3. **Case 3:** $p' < p < p''$
   In this case no agent would join the blocking mechanism.

This analysis shows that under suitable assumptions on the distribution function of types, there exists a posted price mechanism that cannot be blocked by any positive spread posted price mechanism. For example, equations (5) and (6) imply that if functions $x(1-F(x))$ and $(1-x)G(x)$ are convex and $\arg\max_{x\in[0,1]} x(1-F(x)) > \arg\max_{x\in[0,1]}(1-x)G(x)$, any posted price mechanism with a price

$$p \in [\arg\max_{x\in[0,1]}(1-x)G(x), \arg\max_{x\in[0,1]} x(1-F(x))]$$

is immune to blocking.

Hagerty and Rogerson (1987) showed that any deterministic, dominant strategy incentive compatible, (weakly) ex-post budget balanced and ex-post individually rational mechanisms can be implemented as a posted price. In this sense, the simple argument above goes a long way to establishing our main result for the case of one buyer and one seller. In the next subsection, we *characterize* all (not necessarily deterministic) dominant strategy incentive compatible, (weakly) ex-post budget balanced and ex-post individually rational mechanisms that are immune to blocking by a positive spread posted price mechanism. We then use this characterization to determine which posted price mechanisms are immune to blocking.

## 4.1 A Characterization

Recall that the hazard rate of the buyer is defined as $v - \frac{1-F(v)}{f(v)}$ while the hazard rate of the seller is defined as $c + \frac{G(c)}{g(c)}$.

**Theorem 1.** *Assume that the hazard rate of both buyer and seller are increasing. Suppose there exists $p \in [0,1]$ such that $p - \frac{1-F(p)}{f(p)}$ and $1 - p - \frac{G(p)}{g(p)}$ are both negative. Then, a posted price mechanism with price $p$ is immune to blocking by a positive spread posted price mechanism.*

*Proof.* Let $\mathcal{M}$ be any dominant strategy incentive compatible, ex-post (weakly) budget balanced mechanism for the case of bilateral trade. Denote by $u_b(v,c)$ and $u_s(v,c)$ the buyer's and seller's payoff, respectively under $\mathcal{M}$. We first identify conditions under which $\mathcal{M}$ is immune to blocking by a positive spread posted price mechanism.

**Lemma 1.** *$\mathcal{M}$ is immune to blocking by a positive spread posted price mechanism if and only if for all $0 \le y < x \le 1$ the following holds:*

$$E[u_b(x,c)|c \le y] + E[u_s(v,y)|v \ge x] \ge x - y. \tag{7}$$

**Proof.** If for some $0 \leq y < x \leq 1$ inequality (1) is violated, we construct a posted price blocking mechanism. Let $V_b = [x, 1]$ and $V_s = [0, y]$ be the critical set of types for buyer and the seller respectively. As inequality (1) is violated, there exists $0 \leq p_1 < p_2 \leq 1$ such that the following holds:

$$E[u_b(x, c)|c \leq y] = x - p_2, \tag{8}$$

$$E[u_s(v, y)|v \geq x] = p_1 - y. \tag{9}$$

For a candidate blocking mechanism we choose the positive spread posted price mechanism with prices $(p_1, p_2)$. This mechanism is clearly dominant strategy incentive compatible and budget balanced. We now verify that all types in the critical set weakly prefer the blocking mechanism to the mechanism $\mathcal{M}$.

Let $a(v, c)$ be the probability of trade in $\mathcal{M}$ when the the profile of types is $(v, c)$. Recall, from Myerson and Satterthwaite (1983) that $u_b(\alpha, \beta) = \int_0^\alpha a(t, \beta)dt$ and $u_s(\alpha, \beta) = \int_\beta^1 a(\alpha, t)dt$. Therefore, for all $1 \geq v' \geq x$ and $y \geq c' \geq 0$ the following holds:

$$\begin{aligned}
&E[u_b(v', c)|c \leq y] = E[u_b(x, c)|c \leq y] + \int_x^{v'} E[a(v, c)|c \leq y]dv \\
&\leq E[u_b(x, c)|c \leq y] + (v' - x) = v' - p_2, && (10) \\
&E[u_s(v, c')|v \geq x] = E[u_s(v, y)|v \geq x] + \int_{c'}^{y} E[a(v, c)|v \geq x]dc \\
&\leq E[u_s(v, y)|v \geq x] + (y - c') = p_1 - c'. && (11)
\end{aligned}$$

Equations (10) and (11) ensure that all types in the critical set weakly prefer the blocking mechanism to $\mathcal{M}$. It is straightforward to check that when an agent's type is outside the critical set, this agent does not prefer the blocking mechanism to $\mathcal{M}$.

To prove the reverse we show that if there is a positive spread posted price blocking mechanism, inequality (1) is violated for some $0 \leq y < x \leq 1$ . Let $0 \leq p_1 < p_2 \leq 1$ be the prices in the blocking mechanism and $V_b$ and $V_s$ be the associated critical set of types. As the sets $V_b$ and $V_s$ have positive measure, there exists $x \geq p_2$ and $y \leq p_1$ such that $x \in V_b$ and $y \in V_s$. For all such $x, y$ the following must hold:

$$E[u_b(x,c)|c \in V_s] \leq E[(x-p_2)I_{\{c \leq p_1\}}|c \in V_s]. \tag{12}$$

The left hand side of (12) is the expected payoff to the buyer when she participates in $\mathcal{M}$ knowing that the seller has a type in the critical set $V_s$. The right hand side is the expected payoff to the buyer when she chooses to participate in the blocking mechanism conditional on the seller's type being in the critical set and the seller participating in the blocking mechanism. A similar observation yields:

$$E[u_s(v,y)|v \in V_b] \leq E[(p_1-y)I_{\{v \geq p_2\}}|v \in V_b]. \tag{13}$$

Thus, rewriting inequality (12) yields:

$$\begin{aligned}
&\frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(c \in V_s)} \leq \frac{(x-p_2)Pr(V_s \cap [0,p_1])}{Pr(c \in V_s)} \\
&\iff \frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \\
&\iff \frac{\int_{c \in V_s \cap [0,p_1]} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \\
&\iff E[u_b(x,c)|c \in V_s \cap [0,p_1]] \leq x - p_2
\end{aligned} \tag{14}$$

Similarly, the following inequality holds:

$$E[u_s(v,y)|v \in V_b \cap [p_2,1]] \leq p_1 - y. \tag{15}$$

Inequalities (14) and (15) allow us to assume $V_b \subseteq [p_2,1]$ and $V_s \subseteq [0,p_1]$. Let $x^* = \inf V_b$ and $y^* = \sup V_s$. As the distribution of types is atomless, we may assume $x^* \in V_b$ and $y^* \in V_s$. The following inequalities hold:

$$E[u_b(x^*,c)|c \in V_s] \leq x^* - p_2, \tag{16}$$

$$E[u_s(v,y^*)|v \in V_b] \leq p_1 - y^*. \tag{17}$$

Note that the payoff to a seller with type $c \in V_s \cap [p_1,1]$ is zero in the blocking mechanism. Therefore, if a seller has type in $c \in V_s \cap [p_1,1]$, it must receive a payoff of zero in $\mathcal{M}$, i.e., almost surely $\forall v \in V_b \; u_s(v,c) = 0$. Similarly with a buyer whose type is in $V_b \cap [p_2, 1]$. If $a(x^*, y)$ is constant for all $y \leq y^*$, then $u_b(x^*, c) = u_b(x^*, c')$ for any two $c, c' \in V_s$. It follows from (16) that $u_b(x^*, c) = x^* - p_2$ for all $c \leq y^*$. Hence

$$E[u_b(x^*, c)|c \leq y^*] \leq x^* - p_2, \tag{18}$$

Similarly, if $a(x, y^*)$ is constant for all $x \geq x^*$ we deduce that

$$E[u_s(v, y^*)|v \geq x^*] \leq p_1 - y^*. \tag{19}$$

Thus, if $a(x, y)$ is constant in the relevant ranges, the proof is complete. Suppose, for a contradiction, this is not true. Consider the case $x > x^*$ (a similar argument applies when $y < y^*$). For all $x > x^*$ the following holds:

$$\begin{aligned} E[u_b(x, c)|c \in V_c] &= E[u_b(x^*, c)|c \in V_c] + \int_{x^*}^{x} E[a(s, c)|c \in V_c] ds \\ &\leq (x^* - p_2) + (x - x^*) = x - p_2 \end{aligned} \tag{20}$$

If inequality (20) holds with equality for any $\bar{x} > x^*$, it must be the case that for all $x > x^*$ and almost all $c \in V_c$ , $a(x, c) = 1$. To see why, not that equality for $x = \bar{x}$ implies that $a(x, c) = 1$ for all $x^* < x \leq \bar{x}$. However, $a(\cdot, c)$ is monotone in its first component by dominant strategy incentive compatibility. Therefore, $a(x, c) = 1$ for all $x > x^*$. This means that $a(x, c)$ is constant and (18) applies.

Suppose then that inequality (20) is strict for all $x > x^*$. Therefore, $E[u_b(x, c)|c \in V_c] < x - p_2$ for all $x > x^*$. Hence, $x \in V_b$ for all $x > x^*$. A similar argument shows that $y \in V_s$ for all $y < y^*$. This proves the lemma. ■

Consider a posted price mechanism that selects a price according to density $h(p)$. Lemma 1 implies that this mechanism is blocked by a positive spread posted price mechanism if for all $1 \geq x > y \geq 0$ the following holds:

$$\begin{aligned}
&\int_y^x (x-p)h(p)dp + \frac{\int_0^y \int_0^p (x-p)g(c)h(p)dcdp}{G(y)} \\
&+ \int_y^x (p-y)h(p)dp + \frac{\int_x^1 \int_p^1 (p-y)h(p)f(v)dvdp}{1-F(x)} \\
&\geq x-y
\end{aligned} \tag{21}$$

The right hand side of inequality (21) can be rewritten as follows:

$$(x-y)(H(x)-H(y)) + \int_x^1 (p-y)\frac{1-F(p)}{1-F(x)}h(p)dp + \int_0^y (x-p)\frac{G(p)}{G(y)}h(p)dp$$

Using integration by parts inequality (21) can be written as follows:

$$\begin{aligned}
&\int_0^y \int_x^1 H(v)(v - \frac{1-F(v)}{f(v)} - y)f(v)g(c)dvdc + \int_0^y \int_x^1 H(c)(c + \frac{G(c)}{g(c)} - x)f(v)g(c)dvdc \\
&\geq (x-y)G(y)(1-F(x))
\end{aligned} \tag{22}$$

Inequality (22) provides a necessary and sufficient condition for immunity of a trade mechanisms to blocking by a positive spread posted price mechanism.

To complete the proof consider a randomized posted price mechanism that randomizes only over prices for which both hazard rates are negative. Note that if $H(v) = 1$ for all $v \geq x$ and $H(v) = 0$ for all $v \leq y$, then inequality (22) holds with equality. Such a randomized posted price mechanism sets $H(v) < 1$ in the first part of the integral only if $v - \frac{1-F(v)}{f(v)} \leq 0$ and it sets $H(v) > 0$ in the second integral only if $\frac{G(c)}{g(c)} - 1 \geq 0$. Note that

$$v - \frac{1-F(v)}{f(v)} \leq 0 \Rightarrow v - \frac{1-F(v)}{f(v)} - y \leq 0 \text{ and } \frac{G(c)}{g(c)} - 1 \geq 0 \Rightarrow \frac{G(c)}{g(c)} - x \geq 0.$$

Therefore, inequality (22) holds for this mechanism. This proves the theorem. ■

The next result explains why restricting attention to positive spread posted price is without loss.

**Theorem 2.** *Every only deterministic, dominant strategy incentive compatible, (weakly) ex-post budget balanced and ex-post individually rational mechanism that generates positive expected profits can be implemented as a positive spread posted price mechanism.*

The proof follows Hagerty and Rogerson (1987) and so is omitted.

## 4.2 The General Case

We now allow for the possibility of more than one buyer and seller.

**Theorem 3.** *Fix a deterministic dominant strategy incentive compatible, ex-post individually rational and ex-post (weakly) budget balanced mechanism that is robust to the beliefs of the designer. For this mechanism there is an atomless distribution over types under which the mechanism can be blocked by a group of agents.*

*Proof.* Suppose the mechanism cannot be blocked under any atomless distribution over types. We show that such a mechanism must be ex-post efficient. The theorem follows from the fact that such a mechanism does not exist. Let $I \subset N$ be the set of sellers and $J \subset N$ be the set of buyers. Consider a profile of valuations $x = (x_I, x_J) \in \prod_{i \in N} V_i$. Let $I' \subseteq I$ and $J' \subseteq J$ be the subset of the sellers and buyer that should trade in an efficient allocation. Note that $|I'| = |J'|$. Let $T_i$ be the event that the types of the sellers in $I' \setminus i$ are below the $x_{I' \setminus i}$ and the type of buyers in $J' \setminus i$ are below $x_{J' \setminus i}$s for all agents in $I' \cup J'$. Formally,

$$T_i = \{v \in \prod_{i \in N} V_i | \forall k \in I' \setminus \{i\} \ v_k \leq x_k \text{ and } \forall k \in J' \setminus \{i\} \ v_k \geq x_k\}.$$

If the following inequality is violated one can construct a blocking mechanism as in the the proof of Lemma 1:

$$\sum_{i \in I' \cup J'} E[u_i(x_i, v_{-i}) | T_i] \geq \sum_{k \in J'} x_k - \sum_{k \in I'} x_k. \tag{23}$$

Inequality (23) must hold for all possible atomless distributions. Consider a sequence of the atomeless distributions that converge to the distribution that puts probability one on the event that the type profile is $x$. Therefore, the following must hold:

$$\sum_{i \in I' \cup J'} E[u_i(x)|T_i] \geq \sum_{k \in J'} x_k - \sum_{k \in I'} x_k. \tag{24}$$

Inequality (24) implies that the mechanism must be efficient. ■

# 5 Conclusion

There have been a number observed instances of centralized markets fragmenting into smaller ones. A variety of explanations have been offered for this phenomenon. This paper offers a new one, that we argue is more fundamental. If trade can be modeled as a direct mechanism, the earlier explanations can all be voided by the adoption of a rich a mechanism with a rich enough message space. Ours cannot. Essentially, two fundamental features of centralized exchange; incentive compatibility and budget balance conspire to make the centralized exchange unstable.

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