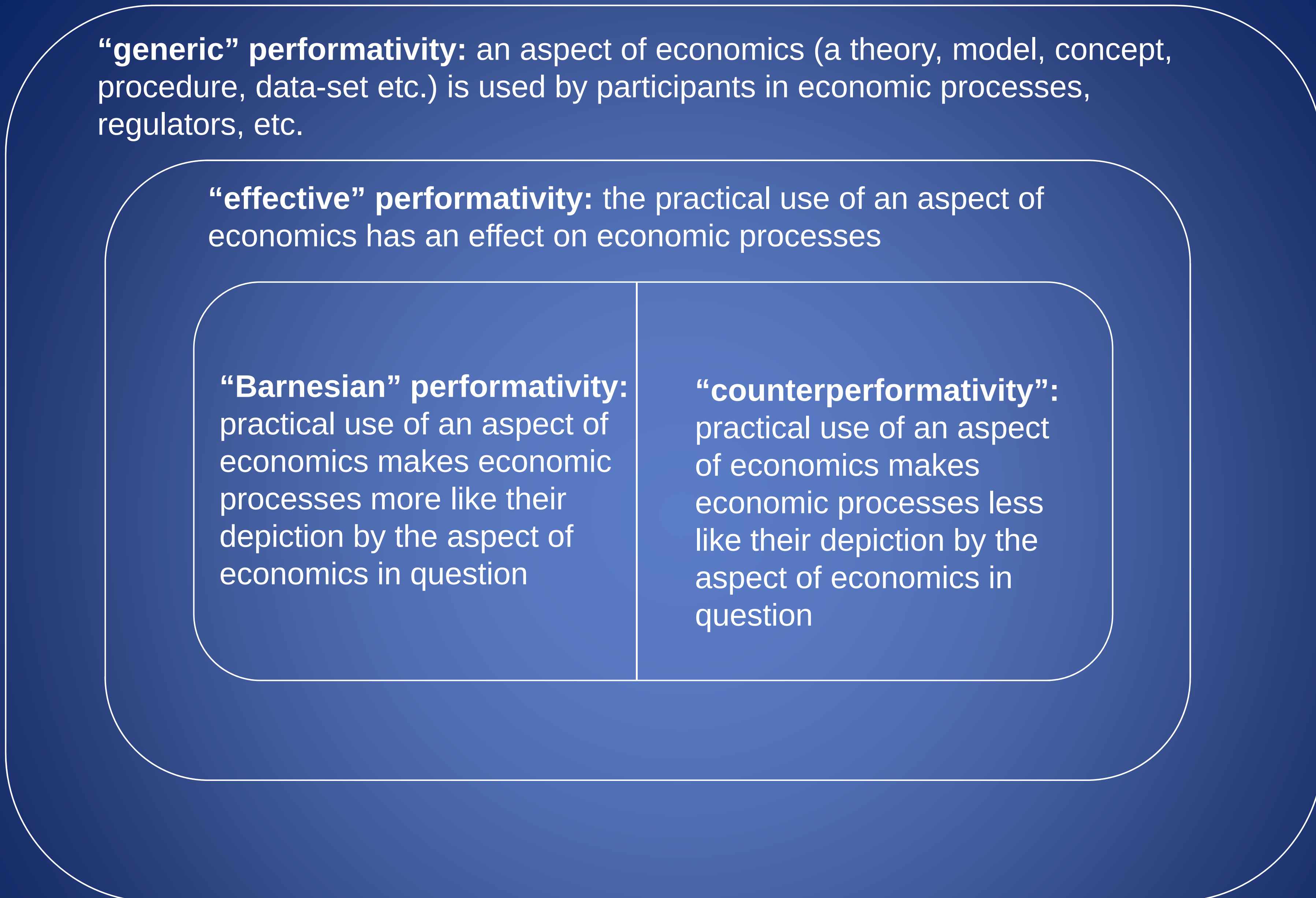
Markets, Models, and Crises

Donald MacKenzie

1. Performativity
2. Black-Scholes-Merton option pricing theory
3. Performativity, counter-performativity and the credit crisis

Part 1: Performativity

Performative utterance (J.L. Austin):
'I apologize'
'I name this ship the Queen Elizabeth'



A possible classification of the performativity of economics. The depicted sizes of the subsets are arbitrary: I have not attempted to estimate the prevalence of the different forms of performativity.

Part 2: Black-Scholes-Merton Option Pricing Theory

Option A financial derivative that gives a right but not an obligation, e.g. right:

- to buy 100 shares of ABC corporation at an ‘exercise price’ of $50 a share (‘call option’)

- to sell 100 shares of XYZ corporation at an exercise price of $30 a share (‘put option’)

HOW DO MARKET PROCESSES DETERMINE PRICE OF OPTIONS?

Black, Scholes, Merton: 1970-73. Given certain assumptions (price of underlying asset follows ‘lognormal’ random walk, no transaction costs, etc.), an option can be hedged perfectly by ‘replicating portfolio’: continuously-adjusted position in underlying asset and borrowing/lending cash.

A position consisting of an option hedged with ‘replicating portfolio’ is thus riskless. Must earn exactly the riskless rate of interest. If not, opportunity for **arbitrage**: for making profit with no risk and no net outlay.

Lognormal random walk: changes in natural logarithms of price of asset normally distributed.

Black-Scholes equation (1969-70):

$$\frac{\partial w}{\partial t} = rw - rx\frac{\partial w}{\partial x} - \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 w}{\partial x^2}$$

w is option price, *x* stock price, σ volatility of stock, *r* riskless rate of interest, *t* time. Stock pays no dividends & option ‘European’ (can be exercised only at expiry). Characteristics of option give boundary condition (at expiry, *w* is a known function of *x*).

Volatility: the extent of the fluctuations of the price of an asset (annualized standard deviation of continuously-compounded returns on the asset).

Riskless rate: the rate of interest paid by a borrower who creditors are certain will not default.

Solution (1970) for European call option, which gives the right to buy stock at 'exercise price' *c* at time *t**:

$$w = xN\left[\frac{\ln(x/c) + (r + {}^{1}/_{2}\,\sigma^{2})(t^{*} - t)}{\sigma\sqrt{t^{*} - t}}\right]$$

$$- c[\exp\{r(t - t^{*})\}]N\left[\frac{\ln(x/c) + (r - {}^{1}/_{2}\,\sigma^{2})(t^{*} - t)}{\sigma\sqrt{t^{*} - t}}\right]$$

where *N* is the (cumulative) distribution function of a normal distribution, and ln indicates natural logarithm.

Option trading on Chicago Board Options Exchange (established 1973) and, later, Chicago Mercantile Exchange:

Bodies and theorems

$$w = xN\left[\frac{\ln(x/c) + (r + {}^{1}/_{2}\,\sigma^{2})(t^{*} - t)}{\sigma\sqrt{t^{*} - t}}\right]$$

$$-c[\exp\{r(t - t^{*})\}]\,N\left[\frac{\ln(x/c) + (r - {}^{1}/_{2}\,\sigma^{2})(t^{*} - t)}{\sigma\sqrt{t^{*} - t}}\right]$$

AN EXAMPLE OF — BLACK'S SHEETS

EXPIRY DATES — RISKLESS RATE OF INTEREST — VOLATILITY — DIVIDENDS

EXPIRATION	ANN INT	ANN DEV	DIV AMT	EX DATE
JLY 16 76	5.650%	21.00%	$.4700	8/ 4/76
OCT 15 76	6.230%	21.00%	$.4700	11/ 3/76
JAN 21 77	6.630%	21.00%		

STRIKE PRICES — FRIDAY OF CURRENT WEEK

	06/04/76			06/11/76			06/18/76			06/25/76			07/02/76		
	JLY 76	OCT 76	JAN 77	JLY 76	OCT 76	JAN 77	JLY 76	OCT 76	JAN 77	JLY 76	OCT 76	JAN 77	JLY 76	OCT 76	JAN 77
45															
40	5.31 99	5.85 88	6.50 82	5.25 97	5.78 87	6.43 82	5.19 98	5.70 87	6.36 82	5.14 99	5.63 87	6.29 82	5.09 100	5.56 88	6.22 82
45	1.43 80	2.52 64	3.39 56	1.29 79	2.43 64	3.32 58	1.14 84	2.34 66	3.24 58	.977 84	2.25 68	3.17 57	.787 83	2.16 69	3.09 57
46.7	.727 39	1.77 49	2.63 50	.609 33	1.69 44	2.56 49	.484 30	1.61 44	2.49 49	.352 26	1.53 43	2.42 48	.213 21	1.44 42	2.35 48
50	.127 09	.796 28	1.51 34	.083 07	.736 24	1.45 33	.046 04	.676 23	1.39 32	.019 02	.617 22	1.33 32	.004 01	.559 21	1.27 31
53.3	.013 01	.313 12	.809 21	.006 01	.278 11	.766 20	.002 00	.244 10	.724 20	.000 00	.212 09	.682 19	.000 00	.181 08	.641 18
55	.003 00	.187 08	.580 16	.001 00	.162 07	.544 15	.000 00	.139 06	.509 14	.000 00	.117 06	.475 14	.000 00	.097 05	.442 14
60	.000 00	.014 02	.197 07	.000 00	.027 01	.180 06	.000 00	.021 01	.163 06	.000 00	.016 01	.147 03	.000 00	.012 01	.132 05
66.7	.000 00	.003 00	.040 02	.000 00	.002 00	.035 01	.000 00	.001 00	.030 01	.000 00	.001 00	.026 01	.000 00	.000 00	.022 01
70	.000 00	.001 00	.017 01	.000 00	.000 00	.015 01	.000 00	.000 00	.012 01	.000 00	.000 00	.010 00	.000 00	.000 00	.008 00
46															
40	6.28 98	6.73 90	7.33 85	6.23 99	6.66 90	7.27 85	6.18 99	6.59 90	7.20 85	6.13 100	6.52 91	7.13 86	6.09 100	6.45 91	7.06 86
45	2.04 67	3.12 63	4.00 63	1.90 68	3.03 63	3.92 63	1.76 68	2.94 63	3.85 63	1.60 70	2.84 63	3.77 63	1.42 73	2.75 63	3.69 63
46.7	1.14 47	2.26 52	3.15 55	1.00 46	2.17 51	3.08 55	.854 44	2.08 51	3.01 54	.693 42	1.99 50	2.93 54	.508 39	1.90 50	2.86 54
50	.243 18	1.08 31	1.87 39	.175 14	1.01 30	1.80 38	.111 09	.937 29	1.74 38	.057 06	.867 28	1.68 37	.018 02	.797 27	1.61 36
53.3	.031 03	.451 16	1.04 25	.016 02	.406 15	.990 24	.006 01	.363 14	.941 24	.001 00	.321 13	.892 23	.000 00	.281 12	.843 22
55	.009 01	.278 11	.758 20	.004 00	.246 10	.716 19	.001 00	.214 09	.675 18	.000 00	.185 08	.634 18	.000 00	.157 07	.594 17
60	.000 00	.056 03	.271 08	.000 00	.046 02	.250 08	.000 00	.037 02	.229 07	.000 00	.029 02	.208 07	.000 00	.022 01	.189 07
66.7	.000 00	.005 00	.059 02	.000 00	.003 00	.052 02	.000 00	.002 00	.046 02	.000 00	.002 00	.040 02	.000 00	.001 00	.034 01
70	.000 00	.001 00	.026 01	.000 00	.001 00	.022 01	.000 00	.001 00	.019 01	.000 00	.000 00	.016 01	.000 00	.000 00	.014 01
47															
40	7.27 99	7.64 92	8.20 88	7.22 100	7.58 93	8.14 88	7.17 100	7.51 93	8.07 88	7.13 100	7.45 94	8.00 89	7.09 100	7.38 94	7.94 89
45	2.76 77	3.78 69	4.66 66	2.63 78	3.69 69	4.58 66	2.50 80	3.60 69	4.50 66	2.37 82	3.51 70	4.42 66	2.22 87	3.41 70	4.35 66
46.7	1.67 58	2.81 58	3.73 58	1.52 59	2.72 58	3.65 58	1.37 59	2.63 58	3.58 58	1.20 59	2.53 58	3.50 59	1.00 60	2.44 58	3.42 59
50	.430 27	1.42 37	2.28 44	.334 24	1.34 36	2.21 43	.239 17	1.26 36	2.14 43	.147 13	1.18 35	2.07 42	.065 06	1.10 34	2.00 41
53.3	.068 05	.630 20	1.31 29	.040 03	.576 19	1.26 29	.019 02	.523 18	1.20 28	.006 01	.470 17	1.14 27	.001 00	.419 16	1.09 27
55	.022 02	.401 14	.972 23	.011 01	.360 13	.924 23	.004 00	.319 12	.876 22	.001 00	.290 11	.828 21	.000 00	.243 10	.781 21
60	.000 00	.098 04	.366 11	.000 00	.074 03	.339 10	.000 00	.061 03	.313 10	.000 00	.049 02	.288 09	.000 00	.038 02	.264 08
66.7	.000 00	.008 00	.085 03	.000 00	.006 00	.076 03	.000 00	.005 00	.067 03	.000 00	.003 00	.059 02	.000 00	.002 00	.051 02
70	.000 00	.002 00	.039 02	.000 00	.002 00	.034 01	.000 00	.001 00	.029 01	.000 00	.001 00	.025 01	.000 00	.000 00	.021 01
48															
40	8.26 100	8.58 95	9.10 90	8.22 100	8.52 95	9.03 91	8.17 100	8.45 95	8.97 91	8.13 100	8.39 96	8.90 91	8.09 100	8.33 96	8.84 91
45	3.57 85	4.50 78	5.36 73	3.46 87	4.41 78	5.29 73	3.35 89	4.32 78	5.21 73	3.24 91	4.23 76	5.13 73	3.14 95	4.14 76	5.05 74
46.7	2.31 70	3.43 65	4.36 63	2.17 71	3.34 65	4.28 65	2.03 72	3.24 65	4.20 65	1.87 74	3.15 65	4.12 65	1.69 78	3.05 65	4.04 64
50	.707 33	1.82 44	2.75 49	.585 30	1.74 43	2.67 49	.458 27	1.65 42	2.60 48	.325 24	1.56 42	2.52 48	.188 18	1.47 41	2.45 47
53.3	.135 09	.857 29	1.63 34	.088 07	.793 28	1.57 33	.049 04	.729 27	1.50 33	.020 02	.666 26	1.44 32	.004 01	.603 25	1.38 31
55	.049 04	.562 18	1.23 28	.027 02	.511 17	1.17 27	.012 01	.461 16	1.12 26	.003 00	.411 15	1.06 26	.000 00	.363 14	1.01 28
60	.001 00	.135 06	.485 13	.000 00	.115 05	.453 13	.000 00	.096 04	.421 12	.000 00	.080 04	.390 11	.000 00	.064 03	.360 11
66.7	.000 00	.015 01	.120 04	.000 00	.011 01	.108 04	.000 00	.008 00	.096 03	.000 00	.006 00	.085 03	.000 00	.004 00	.075 03
70	.000 00	.004 00	.057 02	.000 00	.003 00	.050 02	.000 00	.002 00	.043 02	.000 00	.001 00	.038 01	.000 00	.001 00	.032 01

STOCK PRICES

IF YOU HAVE ANY QUESTIONS, PLEASE CALL FISCHER BLACK AT 617-253-6691

BLACK-SCHOLES OPTION PRICES

DELTAS = $\partial w / \partial x$

= number of shares that need purchased to hedge each call option sold

Traders embrace Black-Scholes-Merton because of:

- authority of financial economics (?)
- public availability (qv PC vs Apple)
- cognitive simplicity (only one free parameter: volatility)

Black’s sheets and other implementations of the Black-Scholes-Merton model used:

1) (sometimes) to set option prices

2) to identify overpriced options to sell or (sometimes) underpriced options to buy

3) as a guide to hedging (‘delta’)

4) in ‘spreading’: simultaneous purchase of ‘underpriced’ option and sale of ‘overpriced’ option on same stock.

From 1973, option prices fell towards Black-Scholes levels, but this not a simple performative effect of use of model: ‘Chicago-style’ competitive market at least equally important.

See Moore and Juh, *J. Finance* 51 (2006) and Mixon, *J. Fin. Economics* 94 (2009)

However, option theory provided options trading with legitimacy:

‘Black-Scholes was really what enabled the exchange to thrive. ... [I]t gave a lot of legitimacy to the whole notion of hedging and efficient pricing, whereas we were faced, in the late 60s-early 70s with the issue of gambling. That issue fell away, and I think Black-Scholes made it fall away. It wasn’t speculation or gambling, it was efficient pricing. I think the SEC [Securities and Exchange Commission] very quickly thought of options as a useful mechanism in the securities markets and it’s probably - that’s my judgement - the effects of Black-Scholes. I never heard the word “gambling” again in relation to options’ [interview: Burton R. Rissman, former counsel, Chicago Board Options Exchange].

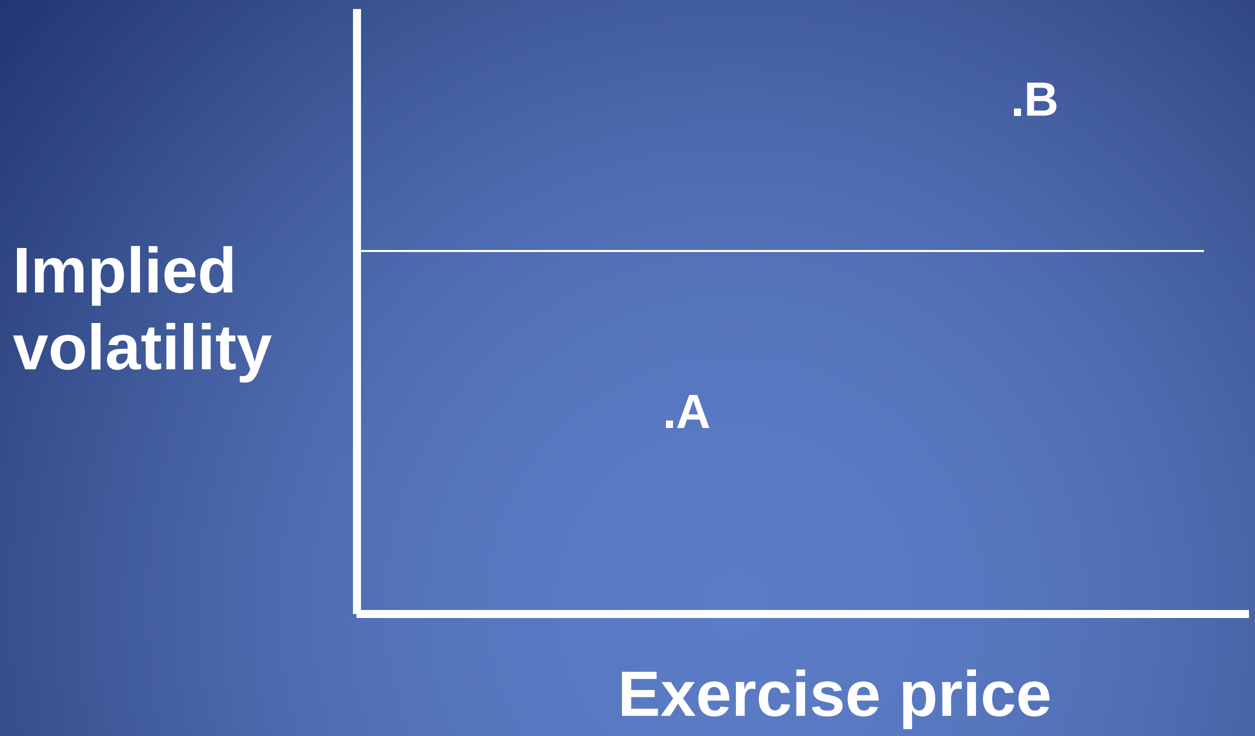
Black-Scholes-Merton and similar models also facilitating trading and risk management by making it easier to **talk** about options.

‘Implied volatility’: the volatility of the underlying asset consistent with price of option on the asset. Reduces plethora of contracts to metric that is (a) simple and (b) meaningful. E.g. implied volatility of S&P 500 options:

15% or less: ‘normal’ conditions.

30%: serious unease

40% or more: crisis.



If the Black-Scholes-Merton model is correct, the implied volatility of all options on the same stock with the same time to expiration should be the same, so the graph of implied volatility against strike price should be a flat line. Rubinstein used this as a test of the empirical validity of the model. “Spreaders” used it as a way of profiting from price discrepancies. They used the model to identity relatively cheap options to buy (such as point A on the graph) and, simultaneously, relatively expensive options to sell (point B). Such trading could be expected to have the effect of flattening the graph.

Fit even better when Rubinstein’s test applied to index options data from 1980s.

‘When judged by its ability to explain the empirical data, option pricing theory is the most successful theory not only in finance, but in all of economics’ (Steve Ross, 1987)

A nice smooth tale of performativity ...?

Portfolio insurance: use of Black-Scholes-like option pricing theory to synthesize a put (option to sell), and thus create a ‘floor’ to the value of a portfolio.

Grows rapidly in 1980s: by 1987, $60-90 billion under portfolio insurance.

Put synthesis demands stock or index-futures sales as stock price falls.

Index future: derivative whose value tracks the price of the stocks comprising an index such as S&P 500.

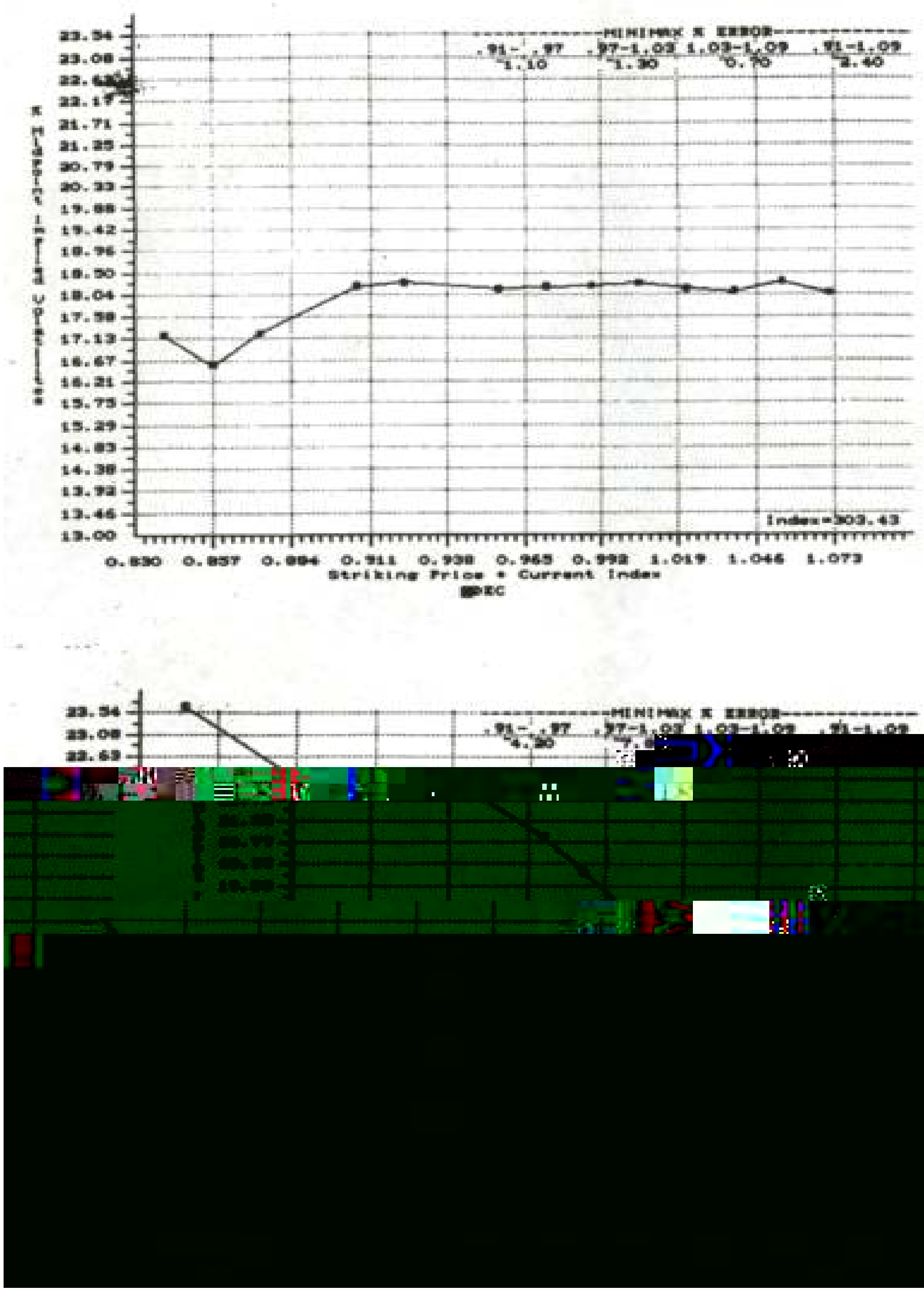
October 19-20, 1987: worst crisis of US financial markets since 1929. (Monday 19^{th}: Dow falls 22.6%). Trading disruptions (NYSE and main stock-derivatives exchanges nearly forced to close) and fears of ramifying chain of bankruptcies and bank failures.

If portfolio insurance exacerbated 1987 crash–serious counterperformative effect. Crash grotesquely unlikely on lognormal random walk underpinning Black-Scholes: fall in price of S&P 500 2-month futures a -27σ event, prob. 10^{-160} (Jackworth and Rubinstein) ‘the crowd detected a pattern of a guy who had to sell [as] the market went lower. So what do you do? You push lower ... and you see him getting even more nervous. ... It’s chemistry between participants. And here’s what happened. You understand, these guys are looking at each other for ten years ... They go to each other’s houses and they’re each other’s best friends and everything Now one of them is a broker. He has an order to sell. They can read on his face if he’s nervous or not. They can read it. They’re animals. They detect things. So this is how it happened in the stock market crash. They kept selling. They see the guys sell more ...’ (Taleb interview).

Empirical history of option pricing:

1. *up to mid 1970s*: model fits reality only approximately
2. *mid 1970s to summer 1987*: reality adjusts to fit model; the Barnesian performativity of option theory
3. *Oct. 1987 to present*: reality deviates systematically from model; volatility skew. (Some evidence that skew 'too big': not just the result of empirical deviation of prices from lognormality.) The model did not *fully* create a world in its image, and it had 'counterperformative' effects.

Skew: a pattern of option prices in which implied volatility is not independent of exercise price (as it should be on the Black-Scholes model).

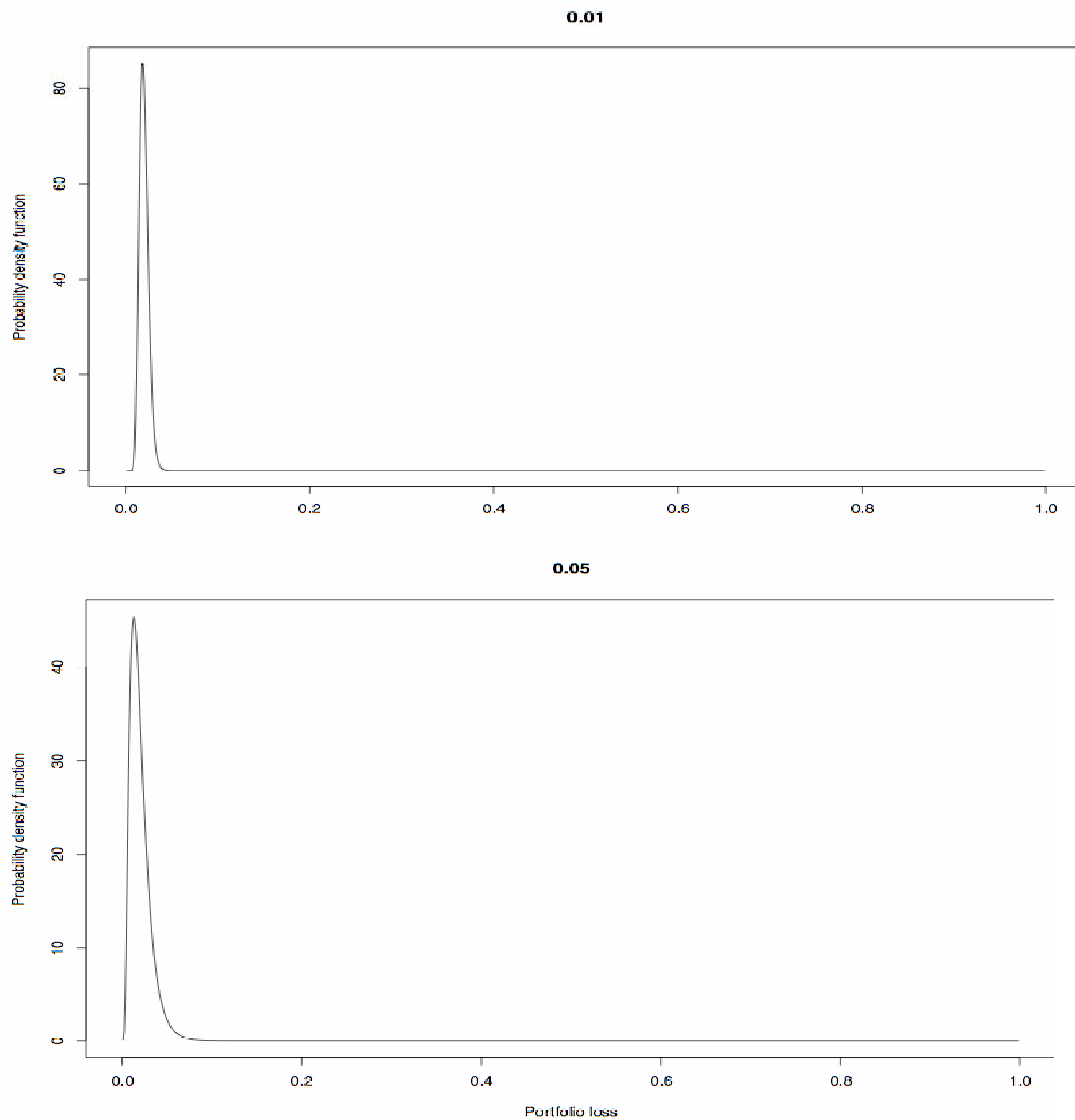


Volatility skew of CBOE S&P index options, 9.00 a.m., July 1, 1987 (upper graph), and 10.00 a.m., January 2, 1990. From Rubinstein (1994, pp. 776-77) courtesy of Mark Rubinstein and Blackwell Publishing Ltd.

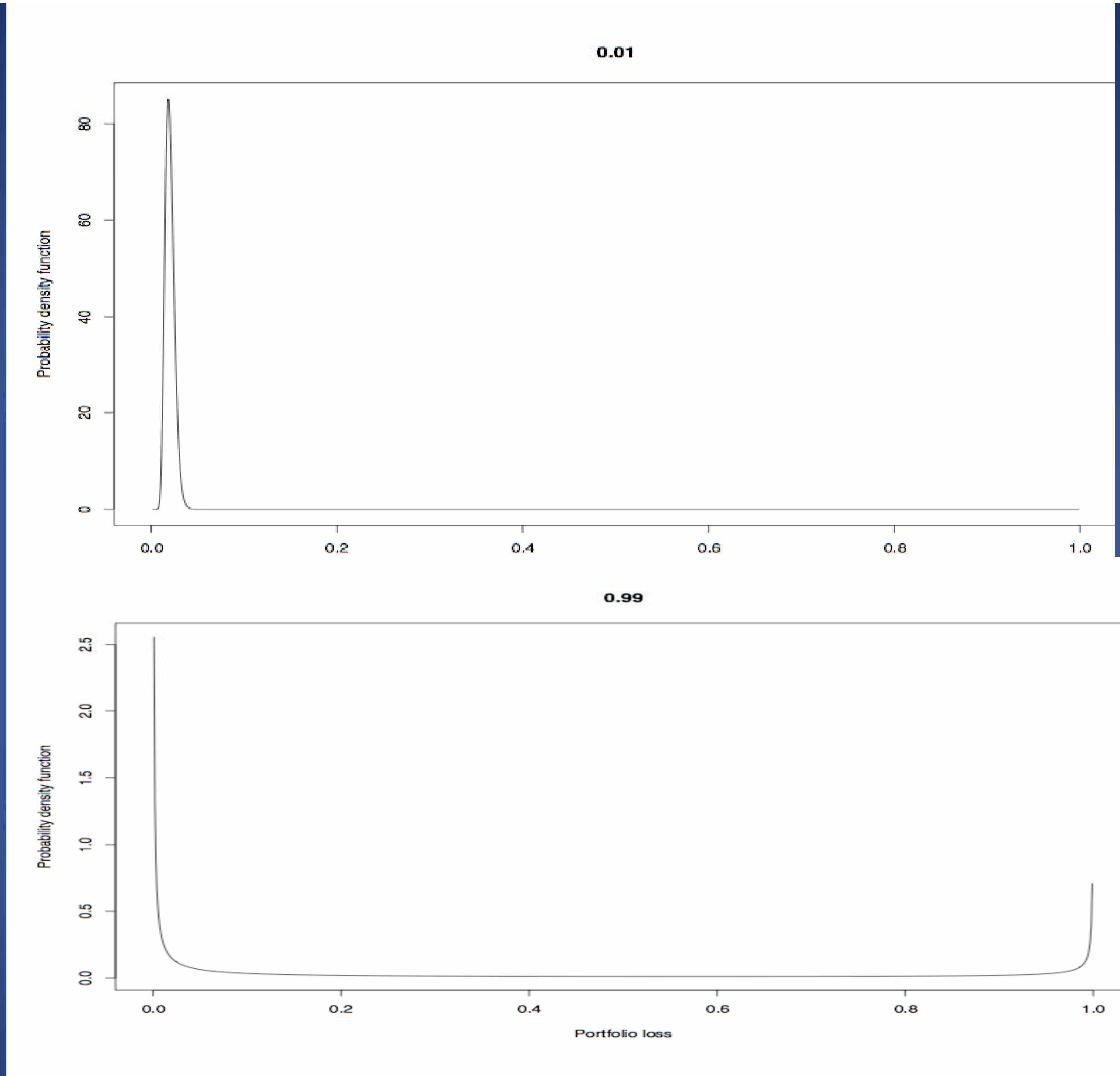
Part 3: Performativity, counterperformativity and the credit crisis (work-in-progress!)

The most obvious analogue of Black-Scholes-Merton is the Gaussian copula model used to value CDOs (collateralized debt obligations)

Slides that follow use the simplest Gaussian copula model (Vasicek's single-period large homogeneous pool model) to show probability distribution of losses on large, highly granular portfolio of assets, each with default probability 0.02, recovery rate of zero, and identical pair-wise asset-value correlations, as the level of that correlation varies.



0.01
Probability density function
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
1.0
0.05
Probability density function
0
10
20
30
40
0.0
0.2
0.4
0.6
0.8
1.0
Portfolio loss


0.01
Probability density function
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
1.0
0.99
Probability density function
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
Portfolio loss

Effective performativity of Gaussian copula. Eg: used by S&P (from 2001), Fitch (from 2003), and (to some extent) Moody's to rate CDOs;
ratings (esp. of ABS CDOs) crucial to crisis.

No clear evidence of **Barnesian performativity**: there's **always** been a 'correlation skew' analogous to volatility skew. Hypotheses:
1. 'arbitrage' **organizationally** harder in credit derivatives than in options (might have to hold position for 5-10 years)
2. early crisis of strategies loosely equivalent to arbitraging correlation skew (2005 'correlation crisis')

Counterperformativity of use of Gaussian copula by rating agencies to rate ABS CDOs. Assuming modest correlations and low default probabilities of underlying assets sets in train processes that dramatically undermine those assumptions:

1. ABSs change mortgage market
2. ABS CDOs change ABS market

	CDO Evaluator three-year default prob. assumptions, as of June 2006 (percent)	Realized incidence of default, as of July 2009 (percent)
AAA	0.008	0.10
AA+	0.014	1.68
AA	0.042	8.16
AA-	0.053	12.03
A+	0.061	20.96
A	0.088	29.21
A-	0.118	36.65
BBB+	0.340	48.73
BBB	0.488	56.10
BBB-	0.881	66.67

***CDO Evaluator* three-year default probability assumptions versus realized default rate of US subprime mortgage-backed securities issued from 2005 to 2007.** Sources: Adelson (2006a); Erturk and Gillis (2009).

Counterperformativity of logistic regression or hazard rate models used to rate the underlying mortage-backed securities: as use of these models become more central to mortgage lending, they became (much) less accurate.

Estimates from yearly regression of interest rates on FICO (borrowers' credit scores) & LTV (loan-to-value ratio)

	§$_{FICO}$	§$_{LTV}$	R^2(in %)	Observations
1997	-0.004	0.030	3	240 67
1998	-0.007	0.035	7	600 94
1999	-0.007	0.020	8	104 847
2000	-0.010	0.035	14	116 778
2001	-0.012	0.038	20	136 483
2002	-0.011	0.071	18	162 501
2003	-0.012	0.079	32	318 866
2004	-0.010	0.097	40	610 753
2005	-0.009	0.110	48	793 725
2006	-0.011	0.115	50	614 820

Source: adapted from table II of Uday Rajan, Amit Seru and Vikrant Vig, "The Failure of Models that Predict Failure: Distance, Incentives and Defaults" (October 2008), ssrn.com/abstract=1296982. Underlying dataset: 16.5 million US residential mortgages in private-label securitisations. Statistical significance of all ßs 0.0005 or better.

Note the analogy between naïve versions of the ‘performativity of economics’ and linear views of technological innovation:

The ‘science’ (economics) comes first

It is applied (its implications are deduced)

The resultant innovations diffuse unchanged.

Current controversy (e.g. critique of performativity by Mirowski and Nik-Khah in *Performing Economics*, ed. MacKenzie, Muniesa, Siu) tends to read it this way.

We have known for some time that:

Science is a resource, not a determinant

Application is not deduction

'Users' innovate (Fleck: 'innofusion')

Same holds for the performativity of economics.

Even the numbers on Black's sheets aren't merely solutions of the Black-Scholes equation: dividends; American puts; estimating volatility.

Esp. after 1987, Black-Scholes becomes 'practitioner Black-Scholes', with volatility skew (supply & demand; industry structure; judgement).

'American option': can be exercised at any point up to its expiry.

Is Barnesian performativity just self-fulfilling prophecy?

Key role of arbitrage, both as process ‘enforcing’ the model, and as bound on models that can be self-fulfilling prophecies.

Qv credit derivatives and single-factor Gaussian copula. Closer to self-fulfilling prophecy case. Arbitrage options by constructing replicating portfolio. Doing same for 5- or 10-year CDO (Collateralized Debt Obligation) institutionally problematic: marking-to-market, annual bonuses etc.

‘Totem’ service: calibrate your CDO model against others’.