Abstract

We develop new measures of time-varying risk aversion and economic uncertainty that can be calculated from observable financial information at high frequencies. Our approach has four important elements. First, we formulate a dynamic no-arbitrage asset pricing model that consistently prices all assets under assumptions regarding the joint dynamics among asset-specific cash flow dynamics, macroeconomic fundamentals and risk aversion. Second, both the fundamentals and cash flow dynamics feature time-varying heteroskedasticity and non-Gaussianity to accommodate dynamics observed in the data, which we document. This allows us to distinguish time variation in economic uncertainty (the amount of risk) from time variation in risk aversion (the price of risk). Third, despite featuring non-Gaussian dynamics, the model retains closed-form solutions for asset prices. Fourth, our approach exploits information on realized volatility and option prices for the two main risky asset classes, equities and corporate bonds, to help identify and differentiate economic uncertainty from risk aversion. We find that equity variance risk premiums are very informative about risk aversion, whereas credit spreads and corporate bond volatility are highly correlated with economic uncertainty. Model-implied risk premiums beat standard instrument sets predicting excess returns on equity and corporate bonds. A financial proxy to our economic uncertainty predicts output growth negatively and significantly, even in the presence of the VIX.

*PRELIMINARY*

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1 Introduction

It has become increasingly commonplace to assume that changes in risk appetites are an important determinant of asset price dynamics. For instance, the behavioral finance literature (see, e.g., Lemmon and Portnaiguina (2006) and Baker and Wurgler (2006) for a discussion) has developed “sentiment indices,” and there are now a wide variety of “risk aversion” or “sentiment” indicators available, created by financial institutions (see Coudert and Gex (2008) for a survey). The “structural” dynamic asset pricing literature has meanwhile proposed time-varying risk aversion as a potential explanation for salient asset price features (see Campbell and Cochrane (1999) and a large number of related articles), whereas reduced-form asset pricing models, aiming to simultaneously explaining stock return dynamics and option prices, have also concluded that time-varying prices of risk are important drivers of stock return and option price dynamics (see Bakshi and Wu, 2010; Bollerslev, Gibson, and Zhou, 2011; Broadie, Chernov, and Johannes, 2007). Risk aversion has also featured prominently in recent monetary economics papers that suggest a potential link between loose monetary policy and the risk appetite of market participants, spurring a literature on what structural economic factors would drive risk aversion changes (see, e.g., Rajan, 2006; Adrian and Shin, 2009; Bekaert, Hoerova, and Lo Duca, 2013). In international finance, Miranda-Agrippino and Rey (2015) and Rey (2015) suggest that global risk aversion is a key transmission mechanism for US monetary policy to be exported to countries worldwide and is a major source of asset return comovements across countries (see also Xu, 2017). Finally, several papers on sovereign bonds (e.g. Bernoth and Erdogan, 2012) have stressed the importance of global risk aversion in explaining their dynamics and contagion across countries.

Our goal is to develop a measure of time-varying risk aversion that is relatively easy to estimate and compute, so that it can be compared to other indices and tracked over time. However, the measure should also correct for deficiencies plaguing many of the current measures. First, it must control for macro-economic uncertainty; we want to separately identify both the aversion to risk (the price of risk) and the amount of risk. To do so, we build on dynamic asset pricing theory. Essentially, our risk aversion measure constitutes a second factor in the pricing kernel that is not driven by macroeconomic fundamentals. The modeling framework therefore is related, but not identical, to the habit models of Campbell and Cochrane (1999), Menzly, Santos and Veronesi (2004) and Wachter (2006). As in Bekaert, Engstrom and Xing (2009) and Bekaert, Engstrom and Grenadier (2010), we allow for a stochastic risk aversion component that is not perfectly correlated with fundamentals. As an important byproduct, we also derive a measure of economic uncertainty, which constitutes an alternative to recent measures (e.g. Juardo, Ludvigson, and Ng, 2015). In the model, asset prices are linked to cash flow dynamics and preferences in an internally consistent fashion. In contrast, a number of articles develop time-varying risk aversion measures motivated by models that really assume “constant” prices of risk and hence are inherently inconsistent (see, for example, Bollerslev, Gibson, and Zhou, 2011), or fail to fully model the link between fundamentals and asset prices (see e.g. Bekaert and Hoerova, 2016). Third, as is well-known, asset prices and returns display
dynamics with highly non-Gaussian distributions that are time varying. In fact, a number of articles (see Bollerslev and Todorov, 2011; Liu, Pan, and Wang, 2004; Santa-Clara and Yan, 2010) suggest that compensation for rare events (“jumps”) accounts for a large fraction of equity risk premiums. To accommodate these non-linearities in a tractable fashion, we use the Bad Environment-Good Environment (BEGE, henceforth) framework developed in Bekaert and Engstrom (2017). Shocks are modeled as the sum of two variables with de-meaned gamma distributions, whose shape parameters vary through time. The model delivers conditional non-Gaussian shocks, with changes in “good” or “bad” volatility also changing the conditional distribution of the process. Finally, our data include macroeconomic fundamentals, asset prices, and options prices. The dynamic asset pricing and options literatures indirectly reveal the difficulty in interpreting many existing risk aversion indicators. Often they use information such as the VIX or return risk premiums that are obviously driven by both the amount of risk and risk aversion. Disentangling the two is not straightforward. Articles such as Drechsler and Yaron (2008), Bollerslev et al. (2009) and Bekaert and Hoerova (2016) point towards the use of the VIX in combination with the (conditional) expected variance as particularly informative about risk preferences. Therefore, this paper is also related to the literature on extracting information about risk and risk preferences from option prices (for a survey, see Gai and Vause, 2006).

The use of different asset classes in deriving a single measure of risk aversion imposes the important assumption that different markets are priced in an integrated setting. This may not (always) be the case. During the 2007-2009 global crisis, it was widely recognized that arbitrage opportunities surfaced between asset classes and sometimes within an asset class (for instance, between Treasury bonds of different maturities, see e.g. Hu, Pan, and Wang, 2013). There may well be a link between risk aversion and the existence of arbitrage opportunities. That is, in uncertain, risk averse times, there is insufficient risky capital available, which causes different asset classes to be priced incorrectly (see, for example, Gilchrist, Yankov, and Zakrajsek, 2009). While consistent pricing across risky asset classes is a maintained assumption in our benchmark model, we can easily test for consistent pricing by examining risk aversion measures implied by different asset classes. We provide an example by comparing risk aversions filtered from risky assets only and from both risky assets and Treasury bonds.

The remainder of the paper is organized as follows. Sections 2 and 3 presents the model and estimation strategy in detail. Section 4 briefly outlines the data we use. Section 5 extracts risk aversion and uncertainty from asset prices and discusses the links between the risk aversion estimates and various financial variables. We also examine the behavior of the indices around the Bear Stearns and Lehman Brothers bankruptcies. In Section 6, we link our measures of risk appetite and uncertainty to alternative indices including ones produced by practitioners. In Section 7, we discuss the case of risk aversion involving Treasury bonds. Concluding remarks are in Section 8.
2 Modeling Risk Appetite and Uncertainty

In this section, we first define our concept of risk aversion in general terms in Section 2.1. We then build a dynamic model with stochastic risk aversion and macro-economic factors affecting the cash flows processes of two main risky asset classes, corporate bonds and equity. The state variables are described in Section 2.2 and the pricing kernel in Section 2.3.

2.1 General Strategy

An ideal measure of risk aversion would be model free and not confound time variation in economic uncertainty with time variation in risk aversion. There are many attempts in the literature to approximate this ideal, but invariably various modeling and statistical assumptions are necessary to tie down risk aversion. For example, in the options literature, a number of articles (Ant-Sahalia and Lo, 2000; Engle and Rosenberg, 2002; Jackwerth, 2000; Bakshi, Kapadia and Madan, 2003; Britten-Jones and Neuberger, 2000) appear at first glance to infer risk aversion from equity options prices in a general fashion, but it is generally the case that the utility function is assumed to be of a particular form and/or to depend only on stock prices.

Another strand of the literature relies on general properties of pricing kernels. Using a strictly positive pricing kernel or stochastic discount factor, \( M_{t+1} \), no-arbitrage conditions imply that for all gross returns, \( R_t \),

\[
E_t [M_{t+1} R_{t+1}] = 1
\]

It is then straightforward to derive that any asset’s expected excess return can be written as an asset specific risk exposure (“beta”, or \( \beta_t \)) times a price of risk (or \( \lambda_t \)), which applies to all assets (see also Coudert and Gex, 2008):

\[
E_t [R_{t+1}] - R_f^t = \beta_t \lambda_t
\]

where \( R_f^t \) is the risk free rate, \( \beta_t = -\frac{\text{Cov}_t(R_{t+1}, M_{t+1})}{\text{Var}_t(M_{t+1})} \), and \( \lambda_t = \frac{\text{Var}_t(M_{t+1})}{E_t(M_{t+1})} \).

Unfortunately, this price of risk is not equal to time-varying risk aversion, and in particular may confound economic uncertainty with risk aversion. In a simple power utility framework, it is easy to show that the price of risk is linked to both the coefficient of relative risk aversion and the volatility of consumption growth, the latter being a reasonable measure of economic uncertainty.

Our approach is to start from a fairly general utility function defined over both fundamentals and non-fundamentals. Our measure of risk aversion simply is then the coefficient of relative risk aversion implied by the utility function. We specify a fairly general consumption process accommodating time variation in economic uncertainty and use the utility framework to price assets, given general processes for the cash flows of assets. Therefore, while certainly not model free, our risk aversion process is consistent with a wide set of economic models that respect no-arbitrage conditions. Moreover, we can use any risky asset for which we can model cash flows to help identify risk aversion. The identification of the risk aversion process takes
into account that economic uncertainty varies through time and controls for non-Gaussianities in cash flow processes.

Consider a period utility function in the HARA class:

$$U\left(\frac{C}{Q}\right) = \left(\frac{C}{Q}\right)^{1-\gamma}$$

where $C$ is consumption and $Q$ is a process that will be shown to drive time-variation in risk aversion. Essentially, when $Q$ is high, consumption delivers less utility and marginal utility increases. For the general HARA class of utility functions,

$$Q = \left(\frac{a}{\gamma} - \frac{b}{C}\right)^{-1} = f(C)$$

where $a$ and $\gamma$ are positive parameters, and $b$ is an exogenous benchmark parameter or process. Note that $\gamma$ (the curvature parameter) is not equal to risk aversion in this framework. In principle, all parameters ($a$, $\gamma$, $b$) could have time subscripts, but we only allow time-variation in $b$. Note that the $Q$ process depends on consumption, but we do not allow $b$ to depend on consumption. This excludes internal habit models, for example.

The coefficient of relative risk aversion for this class of models is given by

$$RRA = -\frac{CU''(C)}{U'(C)} = aQ$$

and is thus proportional to $Q$. Note that $\frac{dQ}{dC} = -b\left(\frac{a}{\gamma}C - b\right)^{-2} < 0$; in good times when consumption increases, risk aversion decreases.

For pricing assets, we need to derive the log pricing kernel which is the intertemporal marginal rate of substitution in a dynamic economy. We assume an infinitely lived agent, facing a constant discount factor of $\beta$, and the HARA period utility function given above. The pricing kernel is then given by

$$m_{t+1} = \ln(\beta) + \ln\left[\frac{U'(C_{t+1})}{U'(C_t)}\right] = \ln(\beta) - \gamma \Delta c_{t+1} + \gamma \Delta q_{t+1}$$

where we use $t$ to indicate time, lower case letters to indicate logs of uppercase variables, and $\Delta$ to indicate log changes.

To get more intuition for this framework, note that the Campbell and Cochrane (1999) (CC henceforth) utility function is a special case. CC use an external habit model, with utility being a power function over $C_t - H_t$, where $H_t$ is the habit stock. Of course, we can also write

$$C_t - H_t = \frac{C_t}{Q_t}$$

with $Q_t = \frac{C_t}{C_t - H_t}$. So the CC utility function is a special case of our framework with $a = \gamma$ and $b = H$. As $C_t$ gets closer to the habit stock, risk aversion increases. $Q_t$ is thus the inverse of
the surplus ratio in the CC article. CC also model \( q_t \) exogenously but restrict the correlation between \( q_t \) and \( \Delta c_t \) to be perfect. The “moody investor” economy in Bekaert, Engstrom, and Grenadier (2010) is also a special case. In that model, \( q_t \) is also exogenously modeled, but has its own shock; that is, there are preference shocks not correlated with fundamentals. In our general quest to identify risk aversion, we surely must allow for such shocks to hit \( q \) as well. The model in Brandt and Wang (2003) is also a special case but the risk aversion process specifically depends on inflation in addition to consumption growth. In fact, DSGE models in macro-economics routinely feature preference shocks (see e.g. Besley and Coate, 2003).

In sum, our approach specifies a stochastic process for \( q \) (risk aversion), which constitutes a second factor in the pricing kernel that is not fully driven by fundamentals (consumption growth).

2.2 Economic Environment: State Variables

2.2.1 Macroeconomic Factors

In canonical asset pricing models agents have utility over consumption, but it is well known that consumption growth and asset returns show very little correlation. Moreover, consumption data are only available at the quarterly frequency. Because the use of options data is key to our identification strategy and these data are only available since 1986, it is important to use macro-economic data that are available at the monthly frequency. We therefore chose to use industrial production, which is available at the monthly frequency, as our main macroeconomic factor. In the macro-economic literature, much attention has been devoted recently to the measurement of “real” uncertainty (see e.g. Jurado, Ludvigson and Ng, 2015) and its effects on the real economy (see e.g. Bloom, 2009). We add to this literature by using a novel econometric framework to extract two macro risk factors from industrial production: “good” uncertainty, denoted by \( p_t \), and “bad” uncertainty, denoted by \( n_t \).

Specifically, the change in log industrial production index, \( \theta_t \), has time-varying conditional moments governed by two state variables: \( p_t \) and \( n_t \). The conditional mean is modeled as a persistent process to accommodate a time-varying long-run mean of output growth:

\[
\theta_{t+1} = \bar{\theta} + \rho (\theta_t - \bar{\theta}) + m_p (p_t - \bar{p}) + m_n (n_t - \bar{n}) + u_\theta^{\theta}_{t+1},
\]

where the growth shock is decomposed into two independent centered gamma shocks,

\[
u_\theta^{\theta}_{t+1} = \sigma_\theta p \omega_{p,t+1} - \sigma_\theta n \omega_{n,t+1}.
\]

The shocks follow centered gamma distributions with time-varying shape parameters,

\[
\omega_{p,t+1} \sim \tilde{\Gamma} (p_t, 1) \quad \quad (10)
\]
\[
\omega_{n,t+1} \sim \tilde{\Gamma} (n_t, 1), \quad \quad (11)
\]

where \( \tilde{\Gamma} (x, 1) \) denotes a centered gamma distribution with shape parameter \( x \) and a unit scale
parameter. The shape factors, $p_t$ and $n_t$, follow autoregressive processes,

$$p_{t+1} = \bar{p} + \rho_p(p_t - \bar{p}) + \sigma_{pp}\omega_{p,t+1}$$
$$n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1},$$

where $\rho_x$ denotes the autoregressive term of process $x_{t+1}$, $\sigma_{xx}$ the sensitivity to shock $\omega_{x,t+1}$, and $\bar{x}$ the long-run mean. We denote the macroeconomic state variables as, $Y_{t}^{mac} = [\theta_t \ p_t \ n_t]'$, and the set of unknown parameters are $\bar{\theta}, \rho_\theta, m_p, m_n, \bar{n}, \sigma_{\theta p}, \sigma_{\theta n}, \rho_p, \sigma_{pp}, \rho_n, \text{ and } \sigma_{nn}$.

In this model, the conditional mean has an autoregressive component, but macro risks can also affect expected growth. This can both accommodate cyclical effects (lower conditional means in bad times), or the uncertainty effect described in Bloom (2009). The shocks reflect the BEGE framework of Bekaert and Engstrom (2017), implying that the conditional higher moments of output growth are linear functions of the bad and good uncertainties. For example, the conditional variance and the conditional unscaled skewness are as follows,

- **Conditional Variance:** $E_t \left[ \left( \nu_{t+1}^\theta \right)^2 \right] = \sigma_{\theta p}^2 p_t + \sigma_{\theta n}^2 n_t$,
- **Conditional Unscaled Skewness:** $E_t \left[ \left( \nu_{t+1}^\theta \right)^3 \right] = 2\sigma_{\theta p}^3 p_t - 2\sigma_{\theta n}^3 n_t$.

This reveals the sense in which $p_t$ represents “good” and $n_t$ “bad” volatility: $p_t$ ($n_t$) increases (decreases) the skewness of industrial production growth.

The industrial production process is a key determinant of the consumption growth process, but we model consumption growth jointly with the cash flow processes for equities imposing the economic restriction that those processes are cointegrated.

### 2.2.2 Cash Flows and Cash Flow Uncertainty

To model the cash flows for equities and corporate bonds, we focus attention on two variables that exhibit strong cyclical movements, namely earnings (see e.g. Longstaff and Piazzesi, 2004) and corporate defaults (see e.g. Gilchrist and Zakrajšek, 2012).

**Corporate Bond Loss Rate** To model corporate bonds, we use data on default rates. Suppose a portfolio of one-period nominal bonds has a promised payoff of $exp(c)$ at $(t + 1)$, but will in fact only pay an unknown fraction $F_{t+1} \leq 1$ of that amount. Let $l_t = \ln(1/F_t) \geq 0$ be the log loss function. Then the actual nominal payment will be $exp(c - l_{t+1})$. We use default data on corporate bonds to measure this loss rate and provide more detail on the pricing of defaultable bonds in the pricing section (Section 2.3).

The log loss rate, $l_t$, is defined as the logarithm of the current aggregate default rate multiplied by the loss-given-default rate. The dynamic system of the corporate bond loss rate is modeled as follows:

$$l_{t+1} = l_0 + \rho_l l_t + \rho_p p_t + \rho_n n_t + \sigma_l \omega_{l,t+1} + \sigma_n \omega_{n,t+1} + u_{l,t+1}^l$$
$$u_{l,t+1} = \sigma_{ul}\omega_{l,t+1}$$
where

\[ v_{t+1} = v_0 + \rho_{vv} v_t + \sigma_{vl} \omega_{l,t+1}. \]  

The conditional mean depends on an autoregressive term and the good and bad uncertainty state variables \( p_t \) and \( n_t \). The loss rate total disturbance is governed by three independent heteroskedastic centered gamma shocks: the good and bad environment macro shocks \( \{\omega_{p,t+1}, \omega_{n,t+1}\} \) and the (orthogonal) loss rate shock \( \omega_{l,t+1} \). The loss rate shock follows a centered gamma distribution where the shape parameter \( v_t \) varies through time.

This dynamic system allows macro-economic uncertainty to affect both the conditional mean and conditional variance of the loss rate process. However, it also allows the loss rate to have an autonomous autoregressive component in its conditional mean (making \( l_t \) a state variable) and accommodates heteroskedasticity not spanned by macro-economic uncertainty. Therefore, \( v_t \) can be viewed as “financial” cash flow uncertainty. Note that the shock to \( v_t \) is the same as the shock for the loss process itself. If \( \sigma_{ll} \) and \( \sigma_{vl} \) are positive, as we would expect, the loss rate and its volatility are positively correlated; that is, in bad times with a high incidence of defaults, there is also more uncertainty about the loss rate, and because the gamma distribution is positively skewed, the (unscaled) skewness of the process increases. We would also expect the sensitivities to the good (bad) environment shocks, \( \sigma_{lp} \) (\( \sigma_{ln} \)) to be negative (positive): defaults should decrease (increase) in relatively good (bad) times.

The conditional variance of the loss rate is \( \sigma_{ll}^2 p_t + \sigma_{ln}^2 n_t + \sigma_{vl}^2 v_t \), and its conditional unscaled skewness is \( 2 \left( \sigma_{lp}^3 p_t + \sigma_{ln}^3 n_t + \sigma_{vl}^3 v_t \right) \). The set of unknown parameters are \( l_0, \rho_{ll}, \rho_{lp}, \rho_{ln}, \sigma_{lp}, \sigma_{ln}, \sigma_{ll}, v_0, \rho_{vv}, \) and \( \sigma_{vl} \).

**Log Earnings Growth** Log earnings growth, \( g_t \), is defined as the change in log real earnings of the aggregate stock market. It is modeled as follows:

\[
\begin{align*}
g_{t+1} & = g_0 + \rho_{gg} g_t + \rho_{gg}' Y_{mac}^t + \sigma_{gg} \omega_{p,t+1} + \sigma_{gn} \omega_{n,t+1} + \sigma_{gl} \omega_{l,t+1} + u_{g,t+1}^g \quad (18) \\
v_{t+1}^g & = \sigma_{gg} \omega_{g,t+1} \quad (19) \\
\omega_{g,t+1} & \sim N(0,1). \quad (20)
\end{align*}
\]

The conditional mean is governed by an autoregressive component and the three macro factors; the time variation in the conditional variance comes from the good and bad uncertainty factors, and the loss rate uncertainty factor. The earnings shock is assumed to be Gaussian and homoskedastic, which cannot be rejected by the data in our sample.\(^1\) A key implicit assumption is that the conditional variance of earnings growth is spanned by macro-economic

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\(^1\)More specifically, we conduct the Kolmogorov-Smirnov test for Gaussianity and the Engle test for heteroskedasticity using the residuals of log earnings growth \( u^g \) (this section), log consumption-earnings ratio \( u^\kappa \) (later), and log dividend-earnings ratio \( u^\eta \) (later). We fail to reject the null that the residuals series, after controlling for heteroskedastic fundamental shocks, are Gaussian and homoskedastic.
uncertainty and the financial uncertainty present in default rates. The set of unknown parameters is \( \{g_0, \rho_{gg}, \rho'_{gy}, \sigma_{gp}, \sigma_{gn}, \sigma_{gl}, \sigma_{gg}\} \).

**Log Consumption-Earnings Ratio** We model consumption as stochastically cointegrated with earnings so that the consumption-earnings ratio becomes a relevant state variable. Define \( \kappa_t \equiv \ln \left( \frac{C_t}{E_t} \right) \) which is assumed to follow:

\[
\begin{align*}
\kappa_{t+1} &= \kappa_0 + \rho_{\kappa \kappa} \kappa_t + \rho'_{\kappa y} Y_{t+1}^{mae} + \sigma_{\kappa p} \omega_{p,t+1} + \sigma_{\kappa n} \omega_{n,t+1} + \sigma_{\kappa l} \omega_{l,t+1} + u_{\kappa,t+1}^\kappa \quad (21) \\
u_{\kappa,t+1} &= \sigma_{\kappa \kappa} \omega_{\kappa,t+1} \quad (22) \\
\omega_{\kappa,t+1} &\sim N(0, 1). \quad (23)
\end{align*}
\]

Similarly to earnings growth, there is an autonomous conditional mean component but the heteroskedasticity of \( \kappa_t \) is spanned by other state variables. The set of unknown parameters is \( \{\kappa_0, \rho_{\kappa \kappa}, \rho'_{\kappa y}, \sigma_{\kappa p}, \sigma_{\kappa n}, \sigma_{\kappa l}, \sigma_{\kappa \kappa}\} \).

**Log Dividend Payout Ratio** The log dividend payout ratio, \( \eta_t \), is expressed as the log ratio of dividends to earnings. Recent evidence in Kostakis, Magdalinos, and Stamatogiannis (2015) shows that the monthly dividend payout ratio is stationary. We model \( \eta_t \) analogously to \( \kappa_t \) and \( g_t \):

\[
\begin{align*}
\eta_{t+1} &= \eta_0 + \rho_{\eta \eta} \eta_t + \rho'_{\eta y} Y_{t+1}^{mae} + \sigma_{\eta p} \omega_{p,t+1} + \sigma_{\eta n} \omega_{n,t+1} + \sigma_{\eta l} \omega_{l,t+1} + u_{\eta,t+1}^\eta \quad (24) \\
u_{\eta,t+1} &= \sigma_{\eta \eta} \omega_{\eta,t+1} \quad (25) \\
\omega_{\eta,t+1} &\sim N(0, 1). \quad (26)
\end{align*}
\]

The set of unknown parameters is \( \{\eta_0, \rho_{\eta \eta}, \rho'_{\eta y}, \sigma_{\eta p}, \sigma_{\eta n}, \sigma_{\eta l}, \sigma_{\eta \eta}\} \).

### 2.2.3 Pricing Kernel State Variables

In the model we introduced above, the real pricing kernel depends on consumption growth and changes in risk aversion. To price nominal cash flows (or to price default free nominal bonds), we also need an inflation process. We discuss the modeling of these variables here.

**Consumption Growth** By definition, log real consumption growth, \( \Delta c_{t+1} = \ln \left( \frac{C_{t+1}}{C_t} \right) = g_{t+1} + \Delta \kappa_{t+1} \). Therefore, consumption growth is spanned by the previously defined state variables and shocks.

**Risk Aversion** The state variable capturing risk aversion, \( q_t \equiv \ln \left( \frac{C_t}{C_t^{\text{mae}}} \right) \) is, by definition, nonnegative. We impose the following structure,

\[
\begin{align*}
q_{t+1} &= q_0 + \rho_{qq} q_t + \rho_{qy} Y_{t+1}^{mae} + \sigma_{q p} \omega_{p,t+1} + \sigma_{q n} \omega_{n,t+1} + \sigma_{q l} \omega_{l,t+1} + u_{q,t+1}^q \quad (27) \\
u_{q,t+1} &= \sigma_{qq} \omega_{q,t+1} \quad (28)
\end{align*}
\]
\[ \omega_{q,t+1} \sim \tilde{\Gamma}(q_t, 1). \]

The risk aversion disturbance is comprised of three parts, exposure to the good uncertainty shock, exposure to the bad uncertainty shock, and an orthogonal preference shock. Thus, given the distributional assumptions on these shocks, the model-implied conditional variance is \( \sigma^2_{q} p_t + \sigma^2_{n_t} n_t + \sigma^2_{qq} q_t \), and the conditional unscaled skewness \( 2 \left( \sigma^3_{q} p_t + \sigma^3_{n_t} n_t + \sigma^3_{qq} q_t \right) \). We model the pure preference shock also with a demeaned gamma distributed shock, so that its variance and (unscaled) skewness are proportional to its own level. Controlling for current business conditions, when risk aversion is high, so is its conditional variability and unscaled skewness. The higher moments of risk aversion are perfectly spanned by macroeconomic uncertainty on the one hand and pure sentiment (\( q_t \)) on the other hand. Note that our identifying assumption is that \( q_t \) itself does not affect the macro variables and \( u_{q,t+1} \) represents a pure preference shock. The conditional mean is modeled as before: an autonomous autoregressive component and dependence on \( p_t \) and \( n_t \). The set of unknown parameters describing the risk aversion process is \( \{q_0, \rho_{qq}, \rho_{qp}, \rho_{qn}, \sigma_{qp}, \sigma_{qn}, \sigma_{qq}\} \).

**Inflation** To price nominal cash flows and nominal bonds, we must specify an inflation process. The conditional mean of inflation depends on an autoregressive term and the three macro factors \( Y_{mac}^t \). The conditional variance and higher moments of inflation are proportional to the good and bad uncertainty factors \( \{p_t, n_t\} \). The inflation innovation \( u_{\pi}^{t+1} \) is assumed to be Gaussian and homoskedastic. There is no feedback from inflation to the macro variables:

\[
\begin{align*}
\pi_{t+1} &= \pi_0 + \rho_{\pi \pi} \pi_t + \rho'_{\pi y} Y_{mac}^t + \sigma_{\pi p} \omega_{p,t+1} + \sigma_{\pi n} \omega_{n,t+1} + u_{\pi}^{t+1} \quad (30) \\
u_{\pi}^{t+1} &= \sigma_{\pi \pi} \omega_{\pi,t+1} \\
\omega_{\pi,t+1} &\sim N(0, 1). \quad (32)
\end{align*}
\]

The set of unknown parameters is \( \{\pi_0, \rho_{\pi \pi}, \rho'_{\pi y}, \sigma_{\pi p}, \sigma_{\pi n}, \sigma_{\pi \pi}\} \).

**2.2.4 Matrix Representation**

The dynamics of all state variables introduced above can be written compactly in matrix notation. We define the macro factors \( Y_{mac}^t = \begin{bmatrix} \theta_t \\ p_t \\ n_t \end{bmatrix} \) and other state variables \( Y_{other}^t = \begin{bmatrix} \pi_t \\ l_t \\ g_t \\ \kappa_t \\ \eta_t \\ v_t \\ q_t \end{bmatrix} \). Among the ten state variables, the industrial production growth \( \theta_t \), the inflation rate \( \pi_t \), the loss rate \( l_t \), earnings growth \( g_t \), the log consumption-earnings ratio \( \kappa_t \) and the log divided payout ratio \( \eta_t \) are observable, while the other four state variables, \( \{p_t, n_t, v_t, q_t\} \) are latent. There are eight independent centered gamma and Gaussian shocks in this economy. The system can be formally described as follows (technical details are relegated to the Appendix):

\[
Y_{t+1} = \mu + AY_t + \Sigma \omega_{t+1}, \quad (33)
\]

where constant matrices, \( \mu \) (10 \times 1), \( A \) (10 \times 10) and \( \Sigma \) (10 \times 8), are implicitly defined, \( Y_t = \begin{bmatrix} Y_{mac}^t \\ Y_{other}^t \end{bmatrix} \) (10 \times 1) is a vector comprised of the state variable levels, and
\[ \omega_{t+1} = \begin{bmatrix} \omega_{p,t+1} & \omega_{n,t+1} & \omega_{n,t+1} & \omega_{g,t+1} & \omega_{n,t+1} & \omega_{q,t+1} & \omega_{q,t+1} \end{bmatrix}' (8 \times 1) \] is a vector comprised of all the independent shocks in the economy.

Note that, among the eight shocks, four shocks follow the gamma shock dynamics laws—the good uncertainty shock \( \omega_{p,t+1} \), the bad uncertainty shock \( \omega_{n,t+1} \), the loss rate shock \( \omega_{l,t+1} \), and the risk aversion shock \( \omega_{q,t+1} \). The remaining four shocks are standard homoskedastic Gaussian shocks (i.e., \( N(0,1) \)). Importantly, given our preference structure, the state variables driving the time variation in the higher order moments of these shocks are the only ones driving the time variation in asset risk premiums and their higher order moments. Economically, we therefore rely on time variation in risk aversion—as in the classic Campbell-Cochrane model and its variants (see e.g. Bekaert, Engstrom and Grenadier, 2010; Wachter, 2006)—and time variation in economic uncertainty—as in the Bansal-Yaron (2004) model—to explain risk premiums. The model implications for conditional asset return variances turn out to be critical in identifying the dynamics of risk aversion (see also Le and Singleton, 2013).

Our specific structure admits conditional non-Gaussianity yet generates affine pricing solutions\(^2\). The model is tractable because the moment generating functions of gamma and Gaussian distributed variables can be derived in closed form, delivering exponentiated affine functions of the state variables. In particular,

\[
E_t \left[ \exp(\nu'Y_{t+1}) \right] = \exp \left[ \nu'S_0 + \frac{1}{2} \nu'S_1 \Sigma_{\text{other}} S_1' \nu + f_S(\nu)Y_t \right],
\]

(34)

where \( S_0 \ (10 \times 1) \) is a vector of drifts; \( S_1 \ (10 \times 4) \) is a selection matrix of 0s and 1s which picks out the Jensen’s inequality terms of the four Gaussian shocks; \( \Sigma_{\text{other}} \ (4 \times 4) \) represents the covariance of the Gaussian shocks. The matrix \( f_S(\nu) \) is a non-linear function of \( \nu \), involving the feedback matrix, and the scale parameters of the gamma-distributed variables. See Appendix A.1 for more details.

2.3 Asset Pricing

In this section, we present the model solutions. First, we formally define the real and pricing kernel as a function of the previously defined state variables. Assuming complete markets, this kernel prices any cash flow pattern spanned by our state variable dynamics. Second, asset prices of two risky assets—defaultable corporate bonds and equities—are derived. The solution of the model shows that asset prices are (quasi) affine functions of the state variables, which is crucial in developing the estimation procedure in this paper. In particular, we derive approximate expressions for endogenous returns to use in estimating the model parameters, and a risk appetite index.

We also show how to price nominal bonds, but they do not feature in our main estimation procedure because they often function as flights-to-safety assets and it is conceivable that much more intricate modeling is necessary not to break the implicit assumption of a unique pric-

\(^2\)Previous research by Bekaert, Engstrom and Xing (2009) and Bekaert and Engstrom (2017) also combines time variation in economic uncertainty with changes in risk aversion.
ing kernel (and one risk aversion process) pricing all risky assets. We test market integration between risky assets and Treasury bonds formally in Section 7.

2.3.1 The pricing kernel

Taking the ratio of marginal utilities at time \( t + 1 \) and \( t \), we obtain the intertemporal marginal rate of substitution which constitutes the real pricing kernel denoted by \( M_{t+1} \). As Equation (6) indicates, it has the same form as the pricing kernel in the Campbell and Cochrane model, however, the kernel state variables and kernel shocks are quite different. Unlike the CC model, changes in the log surplus consumption ratio (the inverse of risk aversion) are not perfectly correlated with the consumption growth shock, and consumption growth is heteroskedastic. The real pricing kernel in our model follows an affine process as well:

\[
m_{t+1} = m_0 + m_2 Y_t + m_1 \Sigma \omega_{t+1},
\]

where \( m_0, m_1 (10 \times 1), m_2 (10 \times 1) \) are constant scalar or matrices that are implicitly defined using Equations (18)–(23) and (27)-(29). To price nominal assets, we define the nominal pricing kernel, \( \tilde{m}_{t+1} \), which is a simple transformation of the log real pricing kernel, \( m_{t+1} \),

\[
\tilde{m}_{t+1} = m_{t+1} - \pi_{t+1},
\]

\[
= \tilde{m}_0 + \tilde{m}_2 Y_t + \tilde{m}_1 \Sigma \omega_{t+1},
\]

where \( \tilde{m}_0, \tilde{m}_1 (10 \times 1) \) and \( \tilde{m}_2 (10 \times 1) \) are implicitly defined. The nominal risk free rate, \( \tilde{r}_f \), is defined as \(-\ln \{ E_t [\exp (\tilde{m}_{t+1})] \} \) which can be expressed as an affine function of the state vector.

2.3.2 Asset prices

In this section, we further discuss the pricing of the two risky assets—corporate bonds and equities. The Appendix contains detailed proofs and derivations.

**Defaultable Nominal Bonds** Above, we assume that a one period nominal bond faces a fractional (logarithmic) loss of \( l_t \). Given the structure assumed for \( l_t \) and Equation (34), the log price-coupon ratio of the one-period defaultable bond portfolio is

\[
pc^t_1 = \ln \{ E_t [\exp (\tilde{m}_{t+1} - l_{t+1})] \}
\]

\[
= b^0_1 + b^1_1' Y_t,
\]

where \( b^0_1 \) and \( b^1_1' \) are implicitly defined. Consider next a portfolio of multi-period zero-coupon defaultable bonds with a promised terminal payment of \( C \) at period \((t + N)\). As for the one-period bond, the actual coupon payment will be less than or equal to the promised payment with the actual coupon, and the ex-post nominal payoff is given by \( \exp (c - l_{t+n}) \). We ignore the possibility of early default or prepayment. Then, the price-coupon ratio of a one-period
defaultable bond at period \((t + N - 1)\), \(PC_{t+N-1}^1\), is exp \(\left(b_0^1 + b_1^1 Y_{t+N-1}\right)\). Given the Euler equation and the law of iterated expectations, it then follows by induction that all farther dated zero-coupon nominally defaultable corporate bond prices are similarly affine in the state variables:

\[
PC_t^N = \ln \left\{ E_t[\tilde{M}_{t+1}PC_{t+1}^{N-1}] \right\}, \\
= b_0^N + b_1^NY_t.
\]

The assumed zero-coupon structure of the payments before maturity implies that the unexpected returns to this portfolio are exactly linearly spanned by the shocks to \(Y_t\).

**Equities** Equity is a claim to the dividend stream; let \(P_t\) denote the ex-dividend price of the claim, then, the price-dividend ratio, \(PD_t\), is given by:

\[
PD_t = E_t\left[ \frac{\tilde{M}_{t+1}(P_{t+1} + D_{t+1})}{D_t} \right] = \sum_{n=1}^{\infty} E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right],
\]

When \(n = 1\), \(F_t^1 = E_t[\exp(m_{t+1} + \Delta d_{t+1})]\) can be expressed as an exact exponential affine function of the state vector. Recursively, the \(n\)-th summation term yields the following identity:

\[
F_t^n = E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right] = E_t \left[ \exp(m_{t+1} + \Delta d_{t+1}) F_{t+1}^{n-1} \right].
\]

Therefore, by induction, any summation term with \(n > 1\) can also be expressed as an exponential affine function of the state vector. Therefore, the price-dividend ratio is the sum of an infinite number of exponential affine functions of the state vector.

### 2.3.3 Asset Returns

Given that the log price-coupon ratio of a defaultable nominal corporate bond can be expressed as an exact affine function of the state variables, it immediately implies that the log nominal return (before maturity), \(\tilde{\gamma}_{t+1}^{ch} = pc_{t+1} - pc_t\), can be represented in closed-form. For equities, the log nominal equity return is derived as follows, \(\tilde{\gamma}_{t+1}^{eq} = \ln \left( \frac{PD_{t+1} + D_{t+1}}{PD_t + D_t} \Pi_{t+1} \right)\). It is therefore a non-linear but known function of the state variables. We approximate this function by a linear function (See the Appendix for details). Note that this procedure is very different from the very popular Campbell-Shiller (1988) model to approximate returns with a linear expression. Because they approximate the return expression and then price future
cash flows with approximate expected returns, their procedure accumulates pricing errors. We approximate a known quasi-affine pricing function in deriving a return expression.

To account for the approximation error, we allow for two asset-specific homoskedastic shocks that are orthogonal to the state variable innovations. As a result, the log nominal asset returns have the following dynamic factor expression,

\[ \tilde{r}_{t+1} = \tilde{\xi}^i_t + \tilde{\xi}'_t Y_t + \tilde{r}'_t \Sigma_t + \tilde{\epsilon}'_{t+1}, \tag{45} \]

where \( \tilde{r}_{t+1} \) is the log nominal asset return from to \( t+1 \), \( \forall i = \{eq, cb\} \); \( \tilde{\xi}'_t \) (10 × 1) is the loading vector on the state vector; \( \tilde{r}'_t \) (10 × 1) is the loading vector on the state variable shocks, and \( \tilde{\epsilon}'_{t+1} \) is a homoskedastic noise term with unconditional volatility \( \sigma_i \).

Rather than exploiting the pricing restrictions on prices, we exploit the restrictions the economy imposes on asset returns, physical variances and risk-neutral variances. Given Equation (45) and the pricing kernel, the model implies that one period expected log excess returns are given by:

\[ RP^i_t \equiv E_t(\tilde{r}'_{t+1}) - \tilde{r}'_t = \left\{ \begin{array}{l} \sigma_p(\tilde{r}'^i) + \ln \left[ \frac{1 - \sigma_p(\tilde{m}_1 + \tilde{r}'^i)}{1 - \sigma_p(\tilde{m}_1)} \right] \} p_t \\
+ \left\{ \sigma_n(\tilde{r}'^i) + \ln \left[ \frac{1 - \sigma_n(\tilde{m}_1 + \tilde{r}'^i)}{1 - \sigma_n(\tilde{m}_1)} \right] \} n_t \\
+ \left\{ \sigma_v(\tilde{r}'^i) + \ln \left[ \frac{1 - \sigma_v(\tilde{m}_1 + \tilde{r}'^i)}{1 - \sigma_v(\tilde{m}_1)} \right] \} v_t \\
+ \left\{ \sigma_q(\tilde{r}'^i) + \ln \left[ \frac{1 - \sigma_q(\tilde{m}_1 + \tilde{r}'^i)}{1 - \sigma_q(\tilde{m}_1)} \right] \} q_t \\
- m'_1 S_1 \Sigma_1 \text{other} s'_1 \tilde{r}^i - \frac{1}{2} [\tilde{r}'^i \Sigma_1 \text{other} s'_1 \tilde{r}^i + \sigma_i^2] \right\}. \tag{46} \]

As shown earlier, \( \tilde{m}_1 \) and \( \tilde{r}^i \) are vectors containing the sensitivities of the log nominal pricing kernel and the log nominal asset returns to the state variable shocks, respectively. The symbols \( \sigma_p(x), \sigma_n(x), \sigma_v(x) \) and \( \sigma_q(x) \) represent linear functions of state variables’ sensitivities to the good uncertainty shock (\( \omega_{p,t+1} \)), the bad uncertainty shock (\( \omega_{n,t+1} \)), the loss rate shock (\( \omega_{l,t+1} \)) and the risk aversion shock (\( \omega_{q,t+1} \)). For instance, because \( \tilde{m}_1 = \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ -\gamma \\ -\gamma \\ 0 \\ 0 \end{array} \right] \) and \( \Sigma_{*8} = \left[ \begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \), \( \sigma_q(\tilde{m}_1) = \tilde{m}_1 \Sigma_{*8} = \gamma \sigma_{qq} > 0 \), where \( \gamma > 0 \) follows from concave utility and \( \sigma_{qq} > 0 \) implies positive skewness of risk aversion in Equation (27). It immediately implies that an asset with a negative sensitivity to the risk aversion shock exhibits a higher risk premium when risk aversion is high. That is, for such an asset, \( \sigma_q(\tilde{r}'^i) < 0 \); then, it can be easily shown that \( \sigma_q(\tilde{r}'^i) + \ln \left[ \frac{1 - \sigma_q(\tilde{m}_1 + \tilde{r}'^i)}{1 - \sigma_q(\tilde{m}_1)} \right] \approx \sigma_q(\tilde{r}'^i) - \frac{\sigma_q(\tilde{r}'^i)}{1 - \sigma_q(\tilde{m}_1)} > 0 \). Expected excess returns thus vary through time and are affine in \( p_t, n_t, v_t \) (macroeconomic and cash flow uncertainties) and \( q_t \) (market-specific risk aversion).

\(^3\)Matrix \( \Sigma_{*j} \) is the j-th column of the shock coefficient matrix in the state variable process, or \( \Sigma \) in Equation 33.
The physical conditional return variance is obtained given the return loadings of Equation (45):

\[
VAR_i \equiv VAR_t(\tilde{r}_{i,t+1}) = \left(\sigma_p(\tilde{r}_i)\right)^2 p_t + \left(\sigma_n(\tilde{r}_i)\right)^2 n_t + \left(\sigma_v(\tilde{r}_i)\right)^2 v_t + \left(\sigma_q(\tilde{r}_i)\right)^2 q_t + \tilde{r}_i S_1 \Sigma_{\text{other}} S_1' \tilde{r}_i + \sigma_i^2,
\]

(47)

where \(S_1\) is defined in Section 2.2.4. See Appendix A.1 for more details. The expected variance under the physical measure is time-varying and affine in \(p_t, n_t, v_t\) and \(q_t\).

The one-period risk-neutral conditional return variance is:

\[
VAR_{i,Q} \equiv VAR_t^Q(\tilde{r}_{i,t+1}) = \left(\frac{\sigma_p(\tilde{r}_i)}{1 - \sigma_p(m_1)}\right)^2 p_t + \left(\frac{\sigma_n(\tilde{r}_i)}{1 - \sigma_n(m_1)}\right)^2 n_t + \left(\frac{\sigma_v(\tilde{r}_i)}{1 - \sigma_v(m_1)}\right)^2 v_t + \left(\frac{\sigma_q(\tilde{r}_i)}{1 - \sigma_q(m_1)}\right)^2 q_t + \tilde{r}_i S_1 \Sigma_{\text{other}} S_1' \tilde{r}_i + \sigma_i^2.
\]

(48)

Note that the functions in Equation (48) are affine transformations from the ones in Equation (47), using the “\(\sigma(m)\)” functions. Under normal circumstances, we would expect that the relative importance of “bad” uncertainty, the loss rate’s uncertainty and risk aversion increases under the risk neutral measure relative to the importance of “good” uncertainty. In Equation (48), this intuition can potentially be formally established as \(\sigma_n(m), \sigma_l(m), \sigma_q(m)\) are positive and \(\sigma_p(m)\) is negative. For example, as derived above, \(\sigma_q(m) = \gamma \sigma_{qq}\) is strictly positive.

3 The Identification and Estimation of Risk Aversion and Uncertainty

In what follows, we describe our general estimation philosophy which is focused on retrieving a risk aversion process that can be traced at high frequencies, and then outline the methodology in detail. The first step is the identification of macro-economic and cash flow uncertainties; the second step is the actual estimation of the remainder of the model parameters and the identification of risk aversion.

3.1 General Estimation Philosophy

While there are 10 state variables in the model, there are only four latent state variables that drive risk premiums and conditional physical and risk neutral variances in the model as described in Equations (46)–(48). Three of these state variables, good uncertainty, \(p_t\), bad uncertainty, \(n_t\) and cash flow uncertainty, \(v_t\), describe economic uncertainty. We want to ensure that these variables are identified from macro-economic and cash flow information alone and are not contaminated by asset prices. We therefore pre-estimate these variables. This constitutes the first step in the estimation methodology.
Given the dynamics of these variables, there are a variety of ways that we can retrieve risk aversion from the model and data on corporate bonds and equities. However, an important goal of the paper is to make risk aversion observable, even at high frequencies. Under the null of the model, asset prices, risk premiums and variances are an exact function of the state variables, including risk aversion. It thus follows that (market-wide) risk aversion should be spanned by a judiciously chosen set of asset prices and risk variables. Given our desire to generate a high frequency risk aversion index, we select these instruments to be observable at high frequencies and to reflect risk and return information for our two asset classes. In particular, we assume

\[ q_t = \chi' z_t, \]

where \( z_t \) is a vector of 6 observed asset prices and ones. The instruments include (1) term spread (the difference between the 10-year and 3-month Treasury bond yield), (2) the credit spread (the difference between Moody’s BAA yield and 10-year Treasury bond yield), (3) a “detrended” dividend yield, (4) the realized equity return variance, (5) the risk-neutral equity return variance, and (6) the realized corporate bond return variance.

The term spread may reflect information about the macro-economy (see e.g. Harvey, 1988) and was also included in the risk appetite index of Bekaert and Hoerova (2016). The credit spread and dividend yield have direct price information from the corporate bond and equity market respectively and thus reflect partially information about risk premiums. Ideally, we would include information on both risk-neutral and physical variances for both equities and corporate bonds, but we do not have data on the risk neutral corporate bond return variance. We use the realized variance for both markets, rather than say an estimate of the physical conditional variance, because realized variances are effectively observed, whereas conditional variances must be estimated. Given a loading vector \( \chi \), the risk aversion process can be computed daily from observable data.

So far, the methodology is reminiscent of the FAVAR literature (see Bernanke, Boivin, and Eliasz, 2005) and Stock and Watson (2002), where unobserved macro-factors are identified using large date sets of observable macro-data using a spanning assumption. However, in contrast to the above literature and all “principle component” type analysis, we exploit the restrictions the economy imposes on risk premiums, and physical and risk neutral variances to estimate the loadings of the time-varying risk aversion process. That is, our risk aversion estimate is forced to have the (dynamic) properties of risk aversion implied by the above model: it is an element of the pricing kernel, which must, in turn, correctly price asset returns and be consistent with observed measures of return volatility under both the physical and risk-neutral measures. To do so, we adopt a GMM procedure detailed in Section 3.3. Imposing the model restrictions and no arbitrage through a positive pricing kernel also differentiates the estimation from the approach taken in Bekaert and Hoerova (2016).
3.2 Identifying Economic Uncertainty

Given that there is no feedback from risk aversion to the three uncertainty state variables, we can pre-estimate the uncertainty factors without using financial asset prices.

First, we use the monthly log real growth rate of industrial production to measure $\theta_t$. In the system for $\theta_t$, described in Equations (8)–(13), there are three state variables, which we collect in $Y_{t}^{mac}$,

$$Y_{t}^{mac} = [\theta_t \; \tilde{p}_t \; \tilde{n}_t]'$$

We denote the filtered shocks,

$$\omega_{t}^{mac} = [\omega_{p,t} \; \omega_{n,t}]'$$

The system is estimated using Bates (2006)’s approximate MLE procedure (see the Appendix for details).

Second, we must determine the latent cash flow uncertainty factor $v_t$, which represents the conditional variance of the log corporate bond default rate. Recall that we assume loss-given-default is a constant, and thus the log corporate bond default rate is the log loss rate plus a constant. The dynamics of the variables are described in Equations (14)–(17). Note that conditional on the model parameters, the residuals of the $v_t$ process are observed and thus the conditional variance can be estimated recursively as in a GARCH process. Thus, the estimation here is exact maximum likelihood, using the correct de-centered gamma density function for the $\omega_{t+1}$ shock. Denote the estimated loss rate shape parameter as $\hat{v}_t$, and the loss rate shock as $\hat{\omega}_{t+1}$.

3.3 Identifying Risk Aversion

To identify the risk aversion process and the parameters in the spanning condition (Equation (49) above), we exploit the restrictions the model imposes on return risk premiums (equities and corporate bonds), physical variances (equities and corporate bonds) and risk neutral variances (for equities only). The estimation is a GMM system in which we use the same instruments as the ones used to span risk aversion ($z_t$). Apart from the $\chi$ parameters, we must also identify the parameters in the kernel ($\beta$, the discount factor, and $\gamma$, the curvature parameter), and the scale parameter of the preference shock, $\sigma_{qq}$. Note that the level of risk aversion is also driven by the $q_t$ process, so that $\gamma$ and $\beta$ are not well identified. We impose $\gamma = 2$ and $\beta = 0.999$. The GMM system thus has 8 unknown parameters,

$$\Theta = [\chi_0; \chi_{tsprd}; \chi_{csprd}; \chi_{DY5yr}; \chi_{rvareq}; \chi_{varreq}; \chi_{rvarcb}; \sigma_{qq}]$$

where the notation is obvious, and $DY5yr$ refers to the detrended dividend yield, described later. Before the moment conditions can be evaluated, we must identify the state variables and their shocks, the pricing kernel, and the return shocks. The estimation is therefore intricate and we now describe the various steps in some detail. For each candidate $\hat{\Theta} = [\hat{\chi}', \hat{\sigma}_{qq}]$ vector:

\(^4\)In the remainder of the paper, a hat superscript is used to indicate estimated variables or matrices.
1. Identify the implied risk aversion series given the loading choices, \( \hat{q}_t = \hat{\chi}' z_t \). We impose a lower boundary of \( 10^{-6} \) on \( q_t \) during the estimation. This is consistent with the theoretical assumption, as \( q_t \) is motivated from a habit formation model \( (q_t = \ln (Q_t) = \ln \left( \frac{C_t}{\xi_t - H_t} \right) > 0) \). It is also consistent with the distributional assumption for \( q_t \) which is the positive shape parameter of the \( \omega_q \) shock.\(^6\)

2. Identify the state variable processes \( (Y_t) \) and shocks \( (\Sigma \omega_{t+1}) \).
   The parameters of the following state variable processes, \( \{\theta_t, p_t, n_t, l_t, v_t\} \), are pre-determined according to Section 3.2. For the remaining cash flow state variables \( \{\pi_t, g_t, \kappa_t, \eta_t\} \), we estimate the parameters in each iteration using simple projections. To identify the risk aversion-specific shock in the risk aversion process, we first project \( \hat{q}_{t+1} \) on \( \hat{\pi}_t, \hat{\rho}_t, \hat{\omega}_{t+1} \) and \( \hat{\omega}_{n,t+1} \) to obtain the residual term \( \hat{u}^q_{t+1} \), and then divide it by \( \hat{\sigma}_{qq} \) to obtain the preference shock \( \hat{\omega}_{q,t+1} \) (see Equations (27)–(29)). We later exploit the implied residual variance and unscaled skewness calculated using the distributional properties of gamma shocks as two moment conditions. Now, given the choice of \( \hat{\chi} \), a full set of state variables levels, \( \hat{Y}_t = \left[ \hat{Y}^\text{mac} t, \pi_t, l_t, g_t, \kappa_t, \eta_t, \hat{\nu}_t, \hat{\rho}_t \right]' \), and the eight independent shocks, \( \hat{\omega}_{t+1} \) including \( \hat{\omega}^q_{t+1} \), can be identified.\(^7\)

3. Identify the nominal pricing kernel.
   Consumption growth in this model is (endogenously) implied by two state variables, real log earnings growth and (changes in) the log consumption-earnings ratio. Given consumption growth \( (i.e., g_t + \Delta \kappa_t) \), the risk aversion process \( \hat{q}_t, \gamma \) and \( \beta \), the monthly nominal kernel is obtained:
   \[
   \hat{m}_{t+1} = \ln(\beta) - \gamma \Delta \kappa_{t+1} + \gamma (\hat{q}_{t+1} - \hat{q}_t) - \pi_{t+1}.
   \]
   Constant matrices related to the log nominal kernel—\( \hat{m}_0, \hat{m}_1, \hat{m}_2 \) (as in the affine representation of the kernel; see Equation (37))—are implicitly identified.

4. Estimate the return loadings.
   In this step, we obtain the loadings of nominal asset returns on the state variable shocks, controlling for time-varying conditional means. Note that there are 8 state variables \( \{\theta_t, p_t, n_t, \pi_t, g_t, \kappa_t, \eta_t, q_t\} \) affecting the pricing kernel. The remaining state variables, \( \{l_t, \eta_t\} \), correspond to cash flow state variables in the corporate bond and equity markets. We estimate the loadings by simple projections, assuming the asset-specific approximation shock is homoskedastic:
   \[
   \tilde{r}^i_{t+1} = \xi_i^0 + \xi_i^t \hat{Y}_t + \tilde{r}^i \hat{\omega}_{t+1} + \tilde{\epsilon}^i_{t+1}, \tag{50}
   \]
   where \( \tilde{r}^i_{t+1} \) is the log nominal return for asset \( i \), \( \hat{\Sigma} \) and \( \hat{\omega}_{t+1} \) are identified previously.

---

\( ^6 \)\(^7 \)However, for the best model, the minimum \( q \) is 0.32 and the boundary is non-binding.
and $\varepsilon_{i,t+1}^i$ has mean 0 and variance $\sigma_{i,t}^2$. To obtain asset moments, $\tilde{\varepsilon}^i$ is the crucial shock loading vector, but we also need $\hat{\sigma}_i$.

5. Obtain the model-implied endogenous moments.

We derive three moments for the asset returns: 1) the expected excess return implied by the model (using the pricing kernel), $R^{eq}_t$; 2) the physical (conditional expected) return variance, $VAR^i$, which only depends on the return definition in Equation (50) and 3) the risk neutral conditional variance, $VAR^{eq,Q}_t$, which also uses the pricing kernel. The expressions for these variables are derived in Equations (46)–(48) where $p_t$, $n_t$, $v_t$, $q_t$, $\tilde{\varepsilon}_t^i$, $\Sigma_{other}$ and $\sigma_i$ have been estimated in previous steps.

6. Obtain the moment conditions $\varepsilon(\Theta; \Psi_t)$.

Given data on asset returns and options, we use the derived moments to define 7 error terms that can be used to create GMM orthogonality conditions. There are three types of errors we use in the system. First, neither risk premiums nor physical conditional variances are observed in the data, but we use the restriction that the observed returns/realized variances minus their expectations under the null of the model ought to have a conditional mean of zero:

$$
\varepsilon_1(\Theta; \Psi_t) = \begin{bmatrix}
(r_{t+1}^{eq} - \tilde{r}_t^{eq}) - \tilde{R}_t^{eq} \\
RVAR_t^{eq} - \tilde{VAR}_t^{eq} \\
(r_{t+1}^{cb} - \tilde{r}_t^{cb}) - \tilde{R}_t^{cb} \\
RVAR_t^{cb} - \tilde{VAR}_t^{cb}
\end{bmatrix},
$$

(51)

where $\tilde{r}_{t+1}^i$ is the realized nominal return from $t$ to $t + 1$, $r_f$ is the risk free rate, and $RVAR_t^{eq}$ is the realized nominal variance from $t$ to $t + 1$ defined as the sum of the squares of the log high-frequency returns from $t$ to $t + 1$ (see the Data section for details). Here $\Psi_t$ denotes the information set at time $t$. The risk neutral variance can be measured from options data (see Bakshi, Kapadia, and Madan, 2003), and so we use the error:

$$
\varepsilon_2(\Theta; \Psi_t) = \begin{bmatrix}
QVAR_t^{eq} - \tilde{VAR}_t^{eq,Q}
\end{bmatrix},
$$

(52)

where $QVAR_t^{eq}$ is the ex-ante risk-neutral variances of $r_{t+1}^{eq}$ calculated from the data. We assume that $\varepsilon_2(\Theta; \Psi_t)$ reflects model and measurement error, orthogonal to $\Psi_t$. Finally, we also construct two moment conditions to identify $\sigma_{qq}$, exploiting the model dynamics for $u_{t+1}^q$ (i.e., the shock to the risk aversion process as in Equation (27)):

$$
\varepsilon_3(\Theta; \Psi_t) = \begin{bmatrix}
(\tilde{u}_{t+1}^q)^2 - (\tilde{\sigma}_{qq})^2 \tilde{q}_t \\
(\tilde{u}_{t+1}^q)^3 - 2(\tilde{\sigma}_{qq})^3 \tilde{q}_t
\end{bmatrix},
$$

(53)

Let $\varepsilon_{1,2}(\Theta; \Psi_t) = \begin{bmatrix} \varepsilon_1(\Theta; \Psi_t)' & \varepsilon_2(\Theta; \Psi_t) \end{bmatrix}$ Under our assumptions these errors are mean zero given the information set, $\Psi_t$. We can therefore use them to create the usual GMM moment conditions. Given our previously defined set of instruments, $z_t$ ($7 \times 1$, including
We define the moment conditions as:

\[
E \left[ g_t(\Theta; \Psi_t, z_t) \right] \equiv E \left[ \begin{bmatrix} \varepsilon_{1,2}(\Theta; \Psi_t) \otimes z_t & \varepsilon_3(\Theta; \Psi_t) \end{bmatrix} \right]_{5 \times 1 \otimes 7 \times 1} = 0 \quad \text{(54)}
\]

Note that to keep the set of moment conditions manageable, we only use two moment conditions for the identification of \( \sigma_{qq} \). Denote \( g_t(\Theta; \Psi_t, z_t) \) (37 \( \times \) 1) as the vector of errors at time \( t \), and \( g_T(\Theta; \Psi, z) \) (37 \( \times \) 1) the sample mean of \( g_t(\Theta; \Psi_t, z_t) \) from \( t = 1 \) to \( t = T \). Then, the GMM objective function is,

\[
J(\Theta; \Psi, z) \equiv T g_T'(\Theta; \Psi, z) W g_T(\Theta; \Psi, z),
\]

where \( W \) is the weighting matrix. We use the standard GMM procedure, first using an identity weighting matrix, yielding a first stage set of parameters \( \hat{\Theta}_1 \). We then compute the usual optimal weighting matrix as the inverse of the spectral density at frequency zero of the orthogonality conditions, \( \hat{S}_1 \), using 5 Newey-West (1987) lags:

\[
\hat{S}_1 = \sum_{j=5}^{j=5} \frac{5 - |j|}{5} \hat{E}[g_t(\hat{\Theta}_1; \Psi_t, z_t) g_{t-j}(\hat{\Theta}_1; \Psi_{t-1}, z_{t-1})]',
\]

Then, the inverse of \( \hat{S} \) is shrunk towards the identity matrix with a shrinkage parameter of 0.1 in obtaining the second-step weight matrix \( W_2 \):

\[
W_2 = 0.9 \hat{S}_1^{-1} + 0.1 I_{37 \times 37},
\]

where \( I_{37 \times 37} \) is a identity matrix of dimension 37 \( \times \) 37. This gives rise to a second-round \( \hat{\Theta}_2 \) estimator. To ensure that poor first round estimates do not affect the estimation, we conduct one more iteration, compute \( \hat{S}_2(\hat{\Theta}_2) \), and produce a third-round GMM estimator, \( \hat{\Theta}_3 \). Lastly, the asymptotic distribution for the third-step GMM estimation parameter is,

\[
\sqrt{T}(\hat{\Theta}_3 - \Theta_0) \to d N(0, Avar(\hat{\Theta}_3)), \text{ where } Avar(\hat{\Theta}_3) = (G_T'(\hat{\Theta}_3) \hat{S}_2^{-1} G_T(\hat{\Theta}_3))^{-1}
\]

where \( G_T \) denotes the gradient of \( g_T \).

Because the estimation involves several steps and is quite non-linear in the parameters, we increase the chance of finding the true global optimum by starting from 24,000 different starting values for \( \hat{\chi} \) drawn randomly from a large set of possible starting values for each parameter. The global optimum is defined as the parameter estimates generating the lowest minimum objective function value.
4 Data

Because we combine macro and cash flow data to estimate the dynamics of the state variables, with financial data in the GMM estimation, we use the longest data available for the various estimations of the state variable dynamics. The estimation of the macroeconomic uncertainty state variables uses the period from January 1947 to February 2015, and the estimation of the loss rate uncertainty state variable uses data from January 1982 to February 2015. For the GMM estimation, the sample spans the period from June 1986 to February 2015 (T=345 months). All estimations are conducted at the monthly frequency.

4.1 State variables

Our output variable—delivering three state variables \((\theta_t, p_t\) and \(n_t)\)—is the change in log real industrial production where the monthly real industrial production index is obtained from the Federal Reserve Bank at St. Louis. Inflation \((\pi)) is defined as the change in the log of the consumer price index (CPI) obtained from the Bureau of Labor Statistics (BLS).

The fifth state variable, the log corporate bond loss rate \((l)\), is defined as the log of the default rate on all U.S. corporate bonds multiplied with the loss-given-default rate (LGD). As commonly assumed in empirical research, LGD is a constant parameter, and is set to 1 (without loss of generality). Specifically, the monthly default rate is obtained by first dividing the total dollar amount of speculative-grade debt that is in default by the total par amount of speculative-grade debt outstanding (source: Moody’s “Corporate Default and Recovery Rates”). Then, we take the average of these monthly corporate bond default rates from the past six months.

The sixth state variable, real earnings growth \((g)\), is defined as the change in log real earnings per capita. Real earnings is the product of real earnings per share and the number of shares outstanding during the same month. The seventh state variable, the log consumption-earnings ratio \((\kappa)\), uses real consumption and real earnings. Real monthly consumption is defined as the sum of seasonally-adjusted real personal consumption expenditures on nondurable goods and services; the consumption deflator is different from the CPI. The source for consumption is the U.S. Bureau of Economic Analysis (BEA). The source for earnings is Shiller’s website. To obtain per capita units, we divide real consumption and real earnings by the population numbers provided by the BEA.

The eighth state variable, the log dividend payout ratio \((\eta)\), uses the log ratio of real dividends and real earnings. Therefore, given \(g\) and \(\kappa\), consumption growth is implicitly defined; given \(g\) and \(\eta\), dividend growth is implicitly defined. Real dividend and earnings per share are available from Shiller’s website. We use the 12-month trailing dividends and earnings, i.e., \(E_{t-12} = E_{t-12} + ... + E_{t-1}\) where \(E_t\) denotes the monthly earnings. There are no true monthly earnings data because almost all firms report earnings results only quarterly. According to Shiller’s website, the monthly dividend and earnings data provided are inferred from the S&P four-quarter totals, which are available since 1926. Calculating 12-month trailing values of earnings and dividends is common practice to control for the strong seasonality in the data.
Total market shares are obtained from CRSP.

4.2 Financial Variables

Daily equity returns are the continuously compounded value-weighted nominal market returns with dividends from CRSP. The monthly return \((r_{eq}^{m})\) is the sum of daily returns within the same month. To create excess returns, we subtract the one-month Treasury bill rate, also from CRSP. We use the square of the month-end VIX index (divided by 120000) as the one-period risk-neutral conditional variance of equity returns \((QVAR_{eq}^{m})\) which is obtained from the Chicago Board Options Exchange (CBOE) and is only available from the end of January 1990. We use the VXO index prior to 1990, also from CBOE. We construct the monthly one-period physical conditional variance of equity returns \((PVAR_{eq}^{m})\) in two steps. First, we calculate the monthly realized variance as the sum of the squared daily equity returns within the same month; then, we project the monthly realized variance onto the lagged risk-neutral variance and the lagged realized variance to obtain the monthly \(PVAR_{eq}^{m}\), as in Bekaert, Hoerova, and Lo Duca (2013).

The daily corporate bond market return is the continuously compounded log change in daily Dow Jones corporate bond total return index (source: Global Financial Data). The monthly return \((r_{cb}^{m})\) is the sum of daily returns within the same month. The conditional variance under the physical measure \((PVAR_{cb}^{m})\) is the projection of monthly realized variance onto the lagged realized variance and the lagged credit spread (defined as the difference between the month-end BAA yield and the 10-year zero-coupon Treasury yield).

We also obtain the 10-year log Treasury bond market return \((r_{tb}^{10})\) from DataStream. We calculate the monthly realized variance as the sum of the squared daily bond returns within the same month; then, we project the monthly realized variance onto the lagged risk-neutral variance and the lagged realized variance to obtain the monthly \(PVAR_{tb}^{10}\). The risk-neutral variance of Treasury bond returns is obtained as follows. Prior to 2003, the monthly risk-neutral conditional variance of Treasury bond returns \((QVAR_{tb}^{m})\) is calculated using the Black-Scholes formula with the 10-year Treasury bond option data with expiration as close as possible to 90 days. After 2003, we use the TYVIX series from CBOE, a 10-year U.S. Treasury Bond Volatility Index which is calculated analogously to CBOE’s VIX. We find that the Black-Scholes risk-neutral variance (our calculation) is 0.98 correlated with the TYVIX for the period after January 2003.

In attempting to span risk aversion, we use some observed financial variables. The term spread is the difference between the 10-year Treasury yield and the 3-month Treasury yield, where the yield data is obtained from the Federal Reserve Bank of St. Louis. The credit spread is the difference between Moody’s BAA yield and the 10-year Treasury bond yield. The detrended dividend yield is calculated as the difference between the raw dividend yield and an moving average term that takes the 5 year average of monthly dividend yields, starting one year before, or \(DY_{5yr,t} = DY_{t} - \sum_{i=1}^{60} DY_{t-i} - 12 - i\) where \(DY_{t}\) denotes the dividend yield level at time \(t\) (the ratio of 12-month trailing dividends and the equity market price).
5 Estimation Results

In this section, we describe the estimation of the state variable processes, and the actual risk aversion process.

5.1 State Variable Dynamics

5.1.1 Macro-economic factors

We estimate the model in Equations (8)–(13) over the full sample (January 1947 to February 2015) using the approximate likelihood procedure of Bates (2006) (for more details, see the Appendix). While we entertained a number of alternative model specifications, the current model was best in terms of the standard BIC criterion. The parameter estimates are reported in Table 1. Industrial production features slight positive auto-correlation and high realizations of “bad” volatility decrease its conditional mean significantly. The $p_t$ process is extremely persistent (almost a unit root) and quasi Gaussian, forcing us to fix its unconditional mean at 500 (for such values, skewness and kurtosis are effectively zero). The $n_t$ process has a much lower mean featuring an unconditional skewness coefficient of $0.50$ ($\frac{2}{\sqrt{1.14}}$) and excess kurtosis of $0.37$ ($\frac{6}{1.14}$). It is also less persistent than the $p_t$ process.

We graph the conditional mean and the $p_t$ and $n_t$ process in Figure 1 together with NBER recessions. The strong countercyclicality of the $n_t$ process and the procyclicity of the conditional mean of “technology” or output growth are apparent from the graph. We also confirmed it by running a regression of the three processes (conditional mean, $p_t$, and $n_t$) on a constant and a NBER dummy. The NBER dummy obtains a highly statistically significant positive (negative) coefficient for the $n_t$ (conditional mean) equation. The coefficient is in fact positive in the $p_t$ equation as well, but not statistically significant. In fact, the $n_t$ regression features an adjusted $R^2$ of almost 45%.

In Figure 2 we plot the conditional variance of industrial production and its conditional skewness. Clearly, macro-economic uncertainty is highly countercyclical, and thus exposure to such uncertainty may render asset prices countercyclical as well. Interestingly, the scaled skewness coefficient is procyclical. This arises from the fact that, while unscaled skewness is countercyclical, the countercyclicality of the variance in the denominator dominates.

5.1.2 Cash flow dynamics

The key variable here is the corporate bond loss rate, of which the dynamics are described by Equations (14)–(17). Estimation here is considerably simpler because the previous estimation delivered filtered estimates of $p_t$, $n_t$, $\omega_{p,t}$ and $\omega_{n,t}$. Therefore, we can essentially use a linear projection to retrieve the estimates for Equation (14) and then use regular maximum likelihood to estimate the conditional variance process specified in Equations (15)– (16). The results are recorded in Table 2.

The loss rate process is persistent with the autocorrelation coefficient close to 0.88. The $p_t$-process does not significantly affect the loss rate process, neither through the conditional
mean or through shock exposures. However, the $\omega_{n,t}$ shock has a statistically significant effect on the loss rate process; moreover $n_t$ affects the loss rate’s conditional mean with a statistically significant positive coefficient. The conditional variance is also persistent (with an autoregressive coefficient of 0.91).

In Figure 3, we plot the conditional moments of the loss rate process, including the $v_t$ process. Note that $v_t$ is only weakly countercyclical. In fact, a regression of $v_t$ on a constant and a NBER dummy, yields a NBER coefficient of 0.202 ($t$ Stat = 0.941). Not surprisingly, the conditional mean of the loss rate is countercyclical, partly through its positive dependence on the $n_t$ process. The conditional volatility also appears countercyclical, which is the combined result of a weakly countercyclical $v_t$ process and a strongly countercyclical $n_t$ process ($\sigma_{n_t}$ being positive). The loss rate process is naturally positively skewed through the positively skewed $u_t$-shocks and its positive dependence on $\omega_n$. This is confirmed by Figure 3 showing the average conditional skewness to be 0.63. However, the scaled skewness dips in recessions, because the conditional variance is so strongly countercyclical. In Figure 4 we decompose the conditional variance of the loss rate in its contributions coming from $v_t$, $p_t$ and $n_t$. The dominant source of variation is $v_t$ but its relative importance drops in recessions when the relative importance of $n_t$ increases, reaching almost 40% in the Great Recession. The $p_t$ process has a negligible effect on the loss rate variance. Clearly, the loss rate variance has substantial independent variation not spanned by macro-economic uncertainty.

With the loss rate process estimated, the dynamics of the other cash flow state variables (earnings growth, the consumption earnings ratio and the payout ratio) follow straightforwardly. We can simply use linear projections of the variables onto previously identified state variables and shocks. The results are contained in Table 3. Earnings growth is less persistent than the two ratio variables. All variables load positively on industrial production growth but the coefficients are not statistically significant. The $n_t$ state variable has a positive effect on the conditional mean of the consumption-earnings and dividend-earnings ratio, indicating that in recessions these ratios are expected to be larger than in normal times. This makes economic sense as consumption and dividends are likely smoothed over the cycle whereas earnings are particularly cycle sensitive (see also Longstaff and Piazessi, 2004). The same intuition explains why the ratio variables load positively on $\omega_n$ shocks and earnings growth loads negatively on this shock. The $\omega_p$ and $\omega_l$ shocks do not have a significant effect on these state variables.

The projections implicitly define the variable specific shocks, which are assumed (and demonstrated, see Footnote 1) to be homoskedastic. Table 3 indicates that they still feature substantial and significant variability. We do not impose any correlation structure on these shocks, and Table 4 shows that they are quite correlated. Essentially, because earnings growth is quite variable, the ratio variables are positively correlated with one another and negatively correlated with earnings growth. When we do asset pricing with the model, this correlation structure must be accounted for (see below). The correlations with the other state variable shocks and between these state variable shocks ($\omega_p$, $\omega_n$, $\omega_l$) ought to be zero in theory and the table shows that they are economically indeed close to zero.
5.2 Risk Aversion

Here we report results regarding the estimation process for risk aversion. Recall that we assume risk aversion to be spanned by 6 financial instruments. In Table 5, we report some properties of these financial instruments. First, all of them are highly persistent. This is the main reason we use a stochastically detrended dividend yield series (AR(1)=0.982) rather than the actual dividend yield series (AR(1)= 0.991), which shows a secular decline over part of the sample that induces much autocorrelation. This decline is likely due to American tax policy and therefore not likely informative about risk aversion (see e.g. Boudoukh et.al, 2007). Second, the various instruments are positively correlated but the correlations never exceed 85% so that we should not worry about multi-collinearity. Perhaps surprisingly, the term spread is also positively correlated with the 5 other instruments, even though it is generally believed that high term spreads indicate good times, whereas the yield and variance instruments would tend to be high in bad times. Third, 4 of the instruments show significant positive skewness. This is critical as we have assumed that the risk aversion dynamics are positively skewed through its gamma distributed shock (see Equation (29)), and we need the linear spanning model to be consistent with the assumed dynamics for risk aversion.

Table 6 reports the reduced form estimates in the spanning relation. The system estimates 8 parameters with 37 moment conditions. The test of the over-identifying restrictions fails to reject but we investigate the fit of the model along various dimensions in more detail later. The significant determinants of the risk aversion process are the dividend yield, realized equity return and corporate bond return variances and the equity return risk neutral variance. The positive coefficient on the risk neutral and the negative coefficient on the physical realized equity return variances is consistent with the idea that the variance risk premium may be quite informative about risk aversion in financial markets (see also Bekaert and Hoerova, 2016). The implied risk aversion process shows a 0.40 correlation with the NBER indicator and is thus highly counter-cyclical.

In Table 7, we estimate the dynamic properties of the risk aversion process according to Equation (27). All the parameters are estimated by OLS, except for the $\sigma_{qq}$ parameter, which is delivered by the GMM estimation (see Section 3.3). The process shows moderate persistence (an autocorrelation coefficient of 0.63) but the conditional mean surprisingly shows a significant positive loading on $p_t$, which accounts for 77% of the variation in the conditional mean. Risk aversion shocks do not load significantly on the macro-economic uncertainty shocks and therefore most of their variation is driven by the risk aversion specific shock. It appears that economic models that impose a very tight link between aggregate fundamentals and risk aversion, such as pure habit models (Campbell and Cochrane, 1999) are missing important variation in actual risk aversion. In addition, risk aversion is much less persistent than the risk aversion implied by these models; the autocorrelation coefficient of the surplus ratio process in the CC model is 0.99 at the monthly level; the first-order autocorrelation coefficient of $q_t$ derived in this paper is 0.63.

While the test of the over-identifying restrictions fails to reject, Table 8 examines in more
detail how well the estimated dynamic system fits critical asset price moments in the data. The model over-estimates the equity premium but is still within one standard error of the data moment. In contrast, the corporate bond risk premium is under-estimated by about 2 standard errors relative to the data moment. The model implied variance moments are all quite close to their empirical counterparts. Finally, the table also reports the model-implied variance and unscaled skewness of the risk aversion innovation, $\sigma^2_q q_t$ and $2\sigma^3_q q_t$ (respectively).

Of all the asset return moments examined here, the only observed one is the risk neutral variance (the VIX index). Because we have filtered state variables, we can therefore compare how well this process fits the actual observed risk neutral variance at each point in time. Figure 5 graphs the empirical and model implied risk neutral variance. While the model fails to match the distinct spikes of the VIX in several crisis periods, the fit is remarkably good, with the correlation between the two series being 87.26%.

6 Risk Aversion, Uncertainty and Asset Prices

In this section, we first characterize the link between risk aversion and macroeconomic uncertainty, on the one hand and asset prices, on the other hand. We compare the time variation in risk aversion and macroeconomic uncertainty and document how our measures correlate with extant measures of uncertainty and risk aversion.

6.1 Risk Aversion, Macro-Economic Uncertainty and the First and Second Moments of Asset Returns

Figure 6 graphs the risk aversion process, which in our model is:

$$ra_t^{BEX} = \gamma \exp(q_t)$$

(57)

The weak countercyclicality of the process is immediately apparent with risk aversion spiking in all three recessions, but also showing distinct peaks in other periods. The highest risk aversion of 11.58 is reached at the end of January in 2009, at the height of the Great Recession. But the risk aversion process also peaks in the October 1987 crash, the August 1998 crisis (Russia default and LTCM collapse), after the TMT bull market ended in August 2002 and in August 2011 (Euro area debt crisis).

How important is risk aversion for asset prices? In this article’s model, the priced state variables for risk premiums and variances are those entering the conditional covariance between asset returns and the pricing kernel and therefore are limited to the risk aversion $q_t$, the macroeconomic uncertainty state variables, $p_t$ and $n_t$ and the loss rate variability $v_t$. In Table 9 we

---

7Bootstrapped standard errors for the five asset price moments (equity risk premium, equity physical variance, equity risk-neutral variance, corporate bond risk premium, and corporate bond physical variance) use different block sizes to accommodate different serial auto-correlations, to ensure that the sampled blocks are approximately i.i.d. In particular, Politis and Romano (1995) (and later discussed in Politis and White, 2004) suggest looking for the smallest integer after which the correlogram appears negligible, where the significance of the autocorrelation estimates is tested using the Ljung-Box Q Test (Ljung and Box, 1978).
report the loadings of risk premiums and variances on the 4 state variables. To help interpret these coefficients, we scaled the projection coefficients by the standard deviation of the state variables so that they can be interpreted as the response to a one standard deviation move in the state variable. For the equity premium, by far the most important state variable is $q_t$ which has an effect more than 10 times larger than that of $n_t$. The effects of $p_t$ and $v_t$ are trivially small. The economic effect of a one standard deviation change in $q_t$ is large representing 54 basis points at the monthly level (almost as high as the average equity premium). For the corporate bond premium, $n_t$ and $q_t$ are again the most important state variables, with $n_t$ now generating the largest effect. A one standard deviation increase in $n_t$ increases the corporate bond risk premium by 8 basis points at a monthly basis, about 1/3 of the average monthly premium. The coefficients for variances are somewhat harder to interpret, but $n_t$ and $q_t$ remain the most important state variables with the former (latter) more important for corporate bond (equity) variances. Because the relationship between asset prices and state variables is affine, we also compute a variance decomposition, coefficient $\times \frac{\text{Cov}(x_t, \text{Mom}_t)}{\text{Var}(\text{Mom}_t)}$ where $x \in \{p, n, v, q\}$ and $\text{Mom}$ represents an asset price moment like the equity risk premium, or corporate bond physical variance. These variance proportions add up to one. In the model, 95% of the equity risk premium’s variance is driven by risk aversion; only 29% of the corporate bond risk premium is driven by risk aversion, while more than 70% is accounted for by “bad” macro-economic uncertainty. Similarly, the physical equity variance is predominantly driven by risk aversion (73%) while 99% of the corporate bond return’s physical variance is driven by bad macroeconomic uncertainty. Nevertheless, macro-economic uncertainty also accounts for 27% of the variance of the physical equity variance. It would be logical that the risk neutral variance would load more on risk aversion and less on macroeconomic uncertainty than the physical variance and this is indeed the case, with risk aversion accounting for 87% of the variance of the risk neutral variance.

Bekaert, Hoerova and Lo Duca (2013) argue that the variance risk premium houses much information about risk aversion. Is this true in our model? To answer this question, we compute the model-implied variance risk premium as the difference between the risk neutral variance and the physical variance. A projection on the 4 state variables reveals that 97.4% of the variance of the variance risk premium is accounted for by risk aversion. Conversely, regressing risk aversion on the variance premium, the coefficient is 149.83 with a t-stat of 112.41, and the $R^2$ is 97.3%. Through the lens of our model, the variance premium is clearly a good proxy for risk aversion.

Finally, because the state variables perfectly explain conditional first and second moments of asset returns in the model, they should help predict realized returns and variances in the data. We test this in Table 10. We regress realized monthly excess returns and variances in both the equity and corporate bond markets on 1) the 4 state variables, or 2) the model-implied conditional moment (either the conditional risk premium or the conditional variance). Not imposing the model restrictions on how the state variables combine to the model implied conditional moment, only slightly decreases the adjusted $R^2$, except for equity returns where the $R^2$ decreases from 5.6% to 0.1%. In this case, the coefficient on the model implied moment is about 0.67.
and not significantly different from 1, but it is borderline significantly different from zero. For corporate bond risk premiums, the coefficient is higher than 1, and not significantly different from one. Not surprisingly, the $R^2$s are higher for the realized equity and corporate bond variances hovering around 21–22%. The coefficients on the model-implied conditional moments are too high mostly because we under-estimate the conditional variances. When investigating the coefficients of individual state variables, risk aversion significantly predicts both returns and both realized variances. Bad economic uncertainty predicts the realized variances and equity returns, but not bond returns. The loss rate variance only significantly predicts bond return variances. The last line reports correlations of the implied risk premiums (the fitted values) with the NBER recession indicator, showing all of them to be significantly countercyclical.

Given the vast literature on return predictability, it is informative to contrast the predictive power of our model implied premiums with the predictive power of the usual instruments used in the literature. We do this exercise out of sample as the literature has shown huge biases due to in sample over-fitting (Goyal and Welch, 2008). Our model premium candidates are either derived from a projection of excess returns on the 4 state variables (Model 2) or the actual model-implied risk premium (Model 1). We consider three empirical models, depending on the instruments used: 1) dividend yield, 2) dividend yield, term spread and credit spread, 3) physical uncertainty and variance risk premium estimate. These instruments are equity market centric and for corporate bond returns, we replace the physical uncertainty by the physical uncertainty derived from corporate bond returns. We then generate out-of-sample predictions for the risk premiums according to the various empirical models by starting the sample after five years of data and then running rolling samples to generate predictions from the five-year point to one month before the end of the sample. For the model implied risk premiums, Model 1 uses whatever the model predicts the premium to be. For Model 2, the projections are also conducted in a rolling fashion, but note that the construction of the state variables uses information from the full sample. With those competing risk premium estimates in hand, we then run simple horse races by estimating:

$$\tilde{r}_{t+1} - r_f = a \text{ Mod}(t,i) + (1-a) \text{ Emp Mod}(t,j) + e_{t+1},$$

for $i = 1, 2, j = 1, 2, 3. \quad (58)$

The results are reported in Table II. For the model implied risk premiums (Model 1), the “$a$” coefficients are quite close to 1.0. Only when confronted with the empirical model facing the spread variables, is its coefficient significantly below 1.0 for the corporate bond risk premium. Model 2 fares less well, with the coefficients only being above 0.5 for the equity return regressions when the model is pitted against empirical models 2 and 3. We conclude that our model seems to capture the predictable variation better than the fitted values extracted from standard instruments used in the literature. While the model premium is not strictly out of sample, the model imposes numerous restrictions relative to the empirical models.
6.2 An Uncertainty Index

An advantage of the risk aversion process we estimated is that because of its dependence on financial instruments it can be computed at even a daily level. Unfortunately, our filtered macro-economic uncertainty variables were extracted from industrial production which is only available at the monthly level. Here we consider whether we can use the financial instruments to approximate macro uncertainty. First, total macro-economic uncertainty, the conditional variance of industrial production growth, is a function of both $p_t$ and $n_t$, $\sigma^2_{p_t}p_t + \sigma^2_{n_t}n_t$ where the coefficient estimates of $\sigma_p$ and $\sigma_n$ are provided in Table 11. In Table 12, we show the coefficients from a regression of uncertainty on the financial instruments used to span risk aversion. The $R^2$ is almost 48% and uncertainty loads significantly on all instruments except for the realized equity variance. Unlike the risk aversion process, uncertainty loads very strongly on credit spreads and the physical corporate bond variance. The term spread also has a significant negative effect on uncertainty (and no effect on risk aversion). This makes sense as flattening yield curves are associated with future economic downturns. The table also reports regressions from the two components in macro-economic uncertainty, bad and good uncertainty, onto the instruments. Clearly, the variation in macro-economic uncertainty is dominated by the bad component and the coefficients for the bad component projection coefficients are very similar to those of total uncertainty. From this analysis, we create an uncertainty index which represents the part of total uncertainty that is explained by the financial instruments:

$$unc^BEX_t = \chi^{unc}zt.$$  \hfill (59)

In Figure 7, we graph the financial instrument proxies to uncertainty and risk aversion for comparison. The correlation between actual uncertainty and risk aversion is 49%; when we use the proxy the correlation increases to 68%. Obviously, most of the time crisis periods feature both high uncertainty and high risk aversion. There are exceptions however. For example, the October 1987 crash happened during a time of relatively low economic uncertainty. It also appears that at the end of the 90s, macro-uncertainty seems to secularly increase, consistent with the Great Moderation ending around that time (see also Baele et al., 2015).

Bloom (2009) has argued that uncertainty precedes bad economic outcomes. We regress future real industrial production growth at various horizons on our uncertainty index—its financial proxy and the actual one—and the risk aversion process. In addition, we use the VIX as suggested in Bloom (2009). The results are in Table 13. We use Hodrick (1992) standard errors to accommodate the overlap in the data. Panel A shows univariate results. All indices predict growth with a negative sign at the one month, one quarter and one year horizons. Our financial instrument uncertainty index generates the highest $R^2$ by far. This suggests that it is indeed macro uncertainty predicting output growth, with the VIX having much lower predictive power in univariate regressions. This result is confirmed in multivariate regressions. In Panel B, we use our risk aversion index ($ra^{BEX}$), financial uncertainty index ($unc^{BEX}$), and the VIX simultaneously. The financial uncertainty index comes in statistically significant for all
horizons. Risk aversion (with a positive sign) and the VIX squared (with a negative sign) are only significant at the annual horizon. Thus, our uncertainty index dominates the VIX index in terms of its predictive power for real activity. Panel C shows that the financial uncertainty proxy also dominates the actual economic uncertainty ($\text{unc}^{true}$), which is only significant at the quarterly horizon.

### 6.3 Correlations with Extant Measures

In this section, we examine how correlated our risk aversion and uncertainty indices are with existing measures. For risk aversion, we consider three categories: risk aversion indices based on “fundamental” habit models, sentiment indices and commercially available risk aversions indices. For uncertainty, we focus on the recent uncertainty index developed by Jurado, Ludvigson and Ng (2015) and the Baker, Bloom and Davis (2016) political uncertainty measure.

Recall that in an external habit model such as Campbell and Cochrane (1999), the curvature of the utility function is a negative affine function of the log “consumption surplus ratio,” which in turns follows a heteroskedastic autoregressive process with shocks perfectly correlated with consumption growth. We follow Wachter (2006) and create a “fundamental” risk aversion process from consumption data and CC’s parameter estimates, which we denote by RA$^{CC}$. Table 14 shows that it is only weakly correlated with our risk aversion measure but the correlation is still significantly different from zero. Clearly, the asset pricing literature should start accepting that risk aversion shows higher frequency movements inconsistent with the focus on low frequency changes tightly linked to consumption growth as in the extant habit models. Work by Bekaert, Engstrom and Grenadier (2010) and Martin (2017) also suggests the existence of more variable risk aversion in financial markets.

The behavioral finance literature suggests that the sentiment of retail investors may drive asset prices and cause non-fundamental price swings. As a well-known representative of this work, we use the sentiment index from Baker and Wurgler (2006). The index is based on the first principal component of six (standardized) sentiment proxies including: the closed-end fund discount, the NYSE share turnover, the number and the average first-day returns of IPOs, the share of equity issues in total equity and debt issues, and the dividend premium (the log-difference of the average market-to-book ratios of payers and nonpayers). We denote their index by Sent$^{BW}$. High values mean positive sentiment so we expect a negative correlation with our risk aversion indicator, and indeed the correlation is significantly negative but still relatively small at -0.16.

Because the Baker-Wurgler index relies on financial data, it may not directly reflect the sentiment of retail investors. Lemmon and Portnaiguina (2006) and Qiu and Welch (2006) therefore suggest using a consumer sentiment index such as the Michigan Consumer Sentiment Index (MCSI). The correlation with this index is also negative, as expected, and larger in absolute magnitude at -0.26; note that we obtain higher correlation with a pure consumer sentiment index than Sent$^{BW}$ derived from financial variables.

Finally, many financial services companies create their own risk appetite indices. As
a well-known example, we obtained data on the Credit Suisse First Boston Risk Appetite Index (RAI). The indicator draws on the correlation between risk appetite and the relative performance of safe assets (proxied by seven to ten-year government bonds) and risky assets (equities and emerging market bonds). The underlying assumption is that an increasing risk preference shifts the demand from less risky investments to assets associated with higher risks, thus pushing their prices up relative to low-risk assets (and vice versa). The indicator is based on the cross-sectional linear regression of excess returns of 64 international stock and bond indices on their risk, approximated by historic volatility. The slope of the regression line represents the risk appetite index. The index shows a -0.48 correlation with our index and is thus highly correlated with our concept of risk aversion.

Our uncertainty measure only uses industrial production data. Jurado, Ludvigson and Ng (2015) use the weighted sum of the conditional volatilities of 132 financial and macroeconomic series, with the bulk of them being macroeconomic. They have three versions of the measure depending on the forecasting horizon, but we focus on the one month horizon, which is most consistent with our model ($MUC_{JLN}$). The correlation with our uncertainty index is highly significant and substantial at 80%.

Macroeconomic uncertainty may be correlated with political uncertainty, which has recently been proposed as a source of asset market risk premiums (Pastor and Veronesi, 2013). Baker, Bloom and Davis (2016) create a policy uncertainty measure, based on newspaper coverage frequency, which we denote by $UC_{BBD}$. The index shows a highly significant correlation of 0.34 with our uncertainty index. One advantage of $UC_{BBD}$ relative to the Jurado et al. (2015) measure is that it can also be computed at daily frequency. However, our financial proxy to uncertainty can also be computed at the daily frequency.

Monthly indices may hide important variation within the month in uncertainty and risk aversion. To demonstrate this, Figure 8 shows how the indices behaved around two critical events in the recent global financial crisis: the Bear Stearns collapse and bail out and the Lehman Brothers bankruptcy. In general, Bear Stearns’ woes generated less effect on our measures than did Lehman Brothers, as expected. To be more specific, Figure 8 plots 2-month intervals of daily risk aversion indices (top plots) and daily financial uncertainty indices (bottom plots) around the two events. By the end of February and March 2008, the $q_t$ index reached 0.57 and 0.44, respectively; the difference is small, considering the substantive time variation in the full sample. However, our daily risk aversion index climbed to 1 on March 16th, the day of Bear Stearns bailout. The uncertainty index also kept increasing until that day. Uncertainty and risk aversion drop steeply afterwards. During August and September 2008, both risk aversion and uncertainty gradually increase, with $q_t$ rising to above 1 on the day of Lehman Brothers’ bankruptcy—which is the same value reached during Bear Stearns’ collapse. However, as the magnification of the Lehman Brothers bankruptcy became clear to financial market participants, both risk aversion and uncertainty continues to rise, with $q_t$ rising to 3.88 on October 10th which corresponds to the coordinated global action by central banks to lower interest rate. 

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8On October 8th, the Federal Reserve and the central banks of the EU, Canada, UK, Sweden and Switzerland cut their rates by half a point. China’s central bank cut its rate by .27 of a point. This was done to lower LIBOR,
The Curious Case of Treasury Bond Markets

We only used risky asset classes to create the risk appetite index. Under the null of the model, the pricing kernel should price all asset returns. A very important asset class is Treasury bond returns. Given a process for inflation, it is straightforward to price Treasury bond returns with our model. It is also the case that for bonds returns we cannot compute physical return variances, but also have risk neutral variances (see Data section). Therefore, we considered formally whether the different asset classes are “integrated,” meaning priced by the same pricing kernel. Interestingly, the term structure literature has a long tradition with “preferred habitat” theories, suggesting different clienteles and different pricing for different parts of the term structure (see also Guibaud, Nosbusch and Vayanos, 2013). However, for our purposes, the problem with considering Treasuries as determining general risk aversion is that they are often viewed as the benchmark “safe” assets and are subject to occasional flights to safety (see Baele et al, 2017). This makes it ex-ante unlikely that a simple pricing model such as ours can jointly price the three assets classes.

To formally test this, we conduct two exercises. First, we simply use our pricing kernel derived from equity and corporate bond returns and test whether it can price bond returns and bond variance swaps. To be specific, market integration is tested by evaluating the fit of 3 bond return moments (risk premium, physical and risk-neutral variance) where the model-implied moments are priced by our pricing kernel. According to the test results in Table 15, we reject the market integration hypothesis because none of the Treasury bond market moments is within the 95% confidence intervals of the empirical point estimates.

Second, we created an “integrated” risk aversion process using asset moments from both risky (equity and corporate bond) and “safe” (Treasury bond) asset markets. To be more specific, we include two more instruments from the Treasury bond market to span the integrated risk aversion process: realized and risk-neutral Treasury bond return variances. In addition, we filter this risk aversion index using moments from risky and safe assets; thus, three asset moments are added to the GMM system: the risk premium, physical variance and risk-neutral variance of Treasury bonds. The resulting risk aversion index is negatively correlated with our risk aversion index (with a -0.157 correlation) and procyclical instead of countercyclical (with a -0.267 correlation with the NBER indicator). We conclude that the current model is inadequate to price Treasury bonds and their role in asset markets deserves more scrutiny.

Conclusion

We formulate a no arbitrage model where fundamentals such as industrial production, consumption earnings ratios, corporate loss rates, etc. follow dynamic processes that admit time-variation in both conditional variances and the shape of the shock distribution. The agent thus lowering the cost of bank borrowing. Overnight bank lending rates dropped in response, indicating a potential turning point in the crisis. (Source: Guardian, “Global rate cuts helps ease overnight interbank rates,” October 8, 2008)

Detailed estimation results and time series are available upon requests.
in the economy takes this time-varying uncertainty into account when pricing equity and corporate bonds, but also faces preference shocks imperfectly correlated with fundamentals. The state variables in the economy that drive risk premiums and higher order moments of asset prices involve risk aversion, good and bad economic uncertainty and the conditional variance of loss rates on corporate bonds. We use equity and corporate bond returns, physical equity and corporate bond return variances and the risk neutral equity variance to estimate the model parameters and simultaneously derive a risk aversion spanning process. Risk aversion is a function of 6 financial instruments, namely the term spread, credit spread, a detrended dividend yield, realized and risk-neutral equity return variance, and realized corporate bond return variance.

We find that risk aversion loads significantly and positively on the risk neutral equity variance and the realized corporate bond variance, and negatively on the realized equity return variance. Risk aversion is much less persistent than the risk aversion process implied by standard habit models. It is the main driver of the equity premium and the equity return risk neutral variance. It also accounts for 73% of the conditional variance of equity returns with the remainder accounted for by bad macro uncertainty. These proportions are reversed for the corporate bond risk premium and the corporate bond physical variance is almost entirely driven by bad macro uncertainty. Hence, different asset markets reflect differential information about risk appetite versus economic uncertainty. Our model-implied risk premiums beat standard predictors of equity and corporate bond returns in an out-of-sample horse race.

While our risk aversion measure is correlated with some existing risk appetite and sentiment indices, the simplest approximation may be the variance risk premium in equity markets which is 98.7% correlated with our risk appetite index.

Because the spanning instruments are financial data, we can track the risk aversion index at higher frequencies. Similarly, we obtain a financial proxy to economic uncertainty (the conditional variance of industrial production growth) which can be obtained at the daily frequency as well. This measure is 80% correlated with the well-known Jurado, Ludvigson and Ng (2015) measure, extracted from macro data. The financial proxy to economic uncertainty predicts output growth negatively and significantly and is a much stronger predictor of output growth than is the VIX. We plan to make both our risk aversion and uncertainty indices available on our websites and update them regularly, which could potentially be useful for both academic researchers and practitioners.
Appendices

A The state variables

A.1 Matrix representation of the state variables

In this section, we show the matrix representation of the system of ten state variables in this economy. The ten state variables, as introduced in Section 3, are as follows,

\[ Y_t = [\theta_t, p_t, n_t, \pi_t, l_t, g_t, \kappa_t, \eta_t, v_t, q_t]^\prime, \]

where \( \{p_t, n_t\} \) denote the upside uncertainty factor and the downside uncertainty factor, as latent variables extracted from the system of output growth (i.e., change in log real industrial production index); \( \pi_t \) represents the inflation rate; \( l_t \) represents the log of corporate loss rate; \( g_t \) represents the log change in real earnings; \( \kappa_t \) represents the log consumption-earnings ratio; \( \eta_t \) represents the log dividend payout ratio; \( v_t \) represents the cash flow uncertainty factor, as the latent variable extracted from the system of corporate loss rate; \( q_t \) represents the latent risk aversion of the economy. The state variables have the following matrix representation:

\[ Y_{t+1} = \mu + AY_t + \Sigma \omega_{t+1}, \quad (A.1) \]

where \( \omega_{t+1} = [\omega_{p,t+1}, \omega_{n,t+1}, \omega_{\pi,t+1}, \omega_{l,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}, \omega_{q,t+1}]^\prime (8 \times 1) \) is a vector comprised of eight independent shocks in the economy. Among the eight shocks, the conditional variance, skewness and higher-order moments of the following four centered gamma shocks—\( \omega_{p,t+1}, \omega_{n,t+1}, \omega_{l,t+1}, \omega_{q,t+1} \)—are assumed to be proportional to \( p_t, n_t, v_t, q_t \) respectively. The underlying distributions for the rest four shocks are assumed to be Gaussian with unit standard deviation.

The constant matrices are defined implicitly,

\[ \mu = \begin{bmatrix} (1 - \rho_p)\bar{\theta} - m_p\bar{p} - m_n\bar{n} \equiv \theta_0 \\ (1 - \rho_n)\bar{n} \equiv n_0 \\ \pi_0 \\ l_0 \\ g_0 \\ \kappa_0 \\ \eta_0 \\ v_0 \\ q_0 \end{bmatrix}, \quad (A.2) \]

\[ A = \begin{bmatrix} \rho_p & m_p & m_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_n & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{n\theta} & \rho_{np} & \rho_{nk} & \rho_{nk} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{np} & \rho_{nk} & \rho_{nk} & 0 & 0 & 0 & 0 & 0 \\ \rho_{n\pi} & \rho_{np} & \rho_{nk} & \rho_{nk} & 0 & 0 & 0 & 0 & 0 \\ \rho_{l\theta} & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ \rho_{l\pi} & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ \rho_{l\kappa} & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ \rho_{l\eta} & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ \rho_{l\nu} & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{lp} & \rho_{ln} & \rho_{ln} & 0 & 0 & 0 & 0 & 0 \\ \rho_{q\pi} & \rho_{qp} & \rho_{qn} & \rho_{qn} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{qp} & \rho_{qn} & \rho_{qn} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{qp} & \rho_{qn} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{qp} & \rho_{qn} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (A.3) \]

\[ \Sigma = \begin{bmatrix} \sigma_{\theta\theta} & -\sigma_{\theta\eta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\theta\eta} & \sigma_{\eta\eta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi\pi} & \sigma_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi l} & \sigma_{\pi l} & 0 & \sigma_{lll} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi l} & \sigma_{\pi l} & 0 & \sigma_{lll} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi\kappa} & \sigma_{\pi\kappa} & 0 & \sigma_{\kappa\kappa} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi\kappa} & \sigma_{\pi\kappa} & 0 & \sigma_{\kappa\kappa} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi\eta} & \sigma_{\pi\eta} & 0 & \sigma_{\eta\eta} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi\eta} & \sigma_{\pi\eta} & 0 & \sigma_{\eta\eta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{pp} & \sigma_{pp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (A.4) \]
Given the moment generating functions (mgf) of gamma and Gaussian distributions, we show that the model is affine, \( \forall \nu \in \mathbb{R}^I \),

\[
M_Y(\nu) := E_t \left[ \exp(\nu' Y_{t+1}) \right] = \exp(\nu' \mu + \nu' A Y_t) E_t \left[ \exp(\nu' \Sigma_{\omega t+1}) \right]
\]

\[
= \exp \left[ \nu' S_0 + \frac{1}{2} \nu' S_1 \Sigma_{\text{other}} S_1' \nu + f_\nu(Y_t) \right], \tag{A.5}
\]

where \( S_0 = \mu \) (10 \( \times \) 1),

\[
S_1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \tag{A.6}
\]

\[
\Sigma_{\text{other}} = \begin{bmatrix}
\sigma_{\pi \pi} & \sigma_{\pi g} & \sigma_{\pi \kappa} & \sigma_{\pi \eta} \\
\sigma_{g \pi} & \sigma_{g g} & \sigma_{g \kappa} & \sigma_{g \eta} \\
\sigma_{\kappa \pi} & \sigma_{\kappa g} & \sigma_{\kappa \kappa} & \sigma_{\kappa \eta} \\
\sigma_{\eta \pi} & \sigma_{\eta g} & \sigma_{\eta \kappa} & \sigma_{\eta \eta}
\end{bmatrix}
\]

\( \text{cov-var matrix of} \ \{\omega_\pi, \omega_g, \omega_\kappa, \omega_\eta\} \), \tag{A.7}

\[
f_\nu(Y) = \nu' A + \begin{bmatrix}
0 \\
-\sigma_p(\nu) - \ln(1 - \sigma_p(\nu)) \\
-\sigma_n(\nu) - \ln(1 - \sigma_n(\nu)) \\
0 \\
0 \\
0 \\
-\sigma_v(\nu) - \ln(1 - \sigma_v(\nu)) \\
-\sigma_q(\nu) - \ln(1 - \sigma_q(\nu))
\end{bmatrix}, \tag{A.8}
\]

\[
\sigma_p(\nu) = \nu' \Sigma_{\nu 1}, \tag{A.9}
\]

\[
\sigma_n(\nu) = \nu' \Sigma_{\nu 2}, \tag{A.10}
\]

\[
\sigma_v(\nu) = \nu' \Sigma_{\nu 4}, \tag{A.11}
\]

\[
\sigma_q(\nu) = \nu' \Sigma_{\nu 8}, \tag{A.12}
\]

where \( M_{\bullet j} \) denotes the \( j \)-th column of the matrix \( M \).

**A.2 Consumption growth**

Consumption growth in this economy is endogenous and can be expressed in an affine function:

\[
\Delta c_{t+1} = g_{t+1} + \Delta \kappa_{t+1} \tag{A.13}
\]

\[
= c_0 + c_2 Y_t + c_1' \Sigma_{\omega t+1}, \tag{A.14}
\]

where \( c_0 = g_0 + \kappa_0, \) \( c_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}' \), and

\[
c_2 = \begin{bmatrix}
\rho_{g \theta} + \rho_{n \theta} \\
\rho_{g y} + \rho_{n y} \\
\rho_{g \kappa} + \rho_{n \kappa} \\
0 \\
0 \\
\rho_{g \theta} \\
0 \\
0
\end{bmatrix}. \tag{A.16}
\]
B Asset Pricing

In this section, we solve the model analytically. First, given consumption growth and changes in risk aversion, the log of real pricing kernel of the economy is derived as an affine function of the state variables. Next, we show that asset prices of claims on cash flows from three different asset markets can be expressed in (quasi) affine equations. The model is solved using the non-arbitrage condition. The goal of this section is to derive the analytical solutions for the expected excess returns, the physical variance of asset returns and the risk-neutral variance of asset returns in closed forms. The implied moments are crucial for the estimation procedure.

B.1 The real pricing kernel

The log real pricing kernel for this economy is given by,

\[
E_t[\exp(\nu m_{t+1})] = \exp [\nu m_0 + \nu m_2 Y_t] \\
\cdot \exp \left\{ \frac{1}{2} \nu^2 \left[ m_1' S_1 \Sigma_{\text{other}} S_1' m_1 \right] \right\},
\]

where \( m_0, m_1, m_2, S_1, \) and \( \Sigma_{\text{other}} \) are constant matrices defined earlier, and

\[
\sigma_p(m_1) = m_1' \Sigma_{s1},
\]

\[
\sigma_n(m_1) = m_1' \Sigma_{s2},
\]

\[
\sigma_e(m_1) = m_1' \Sigma_{s4},
\]

\[
\sigma_s(m_1) = m_1' \Sigma_{s8}.
\]

Accordingly, the model-implied short rate \( r_{ft} \) is,

\[
\begin{aligned}
rf_t &= - \ln \{ E_t[\exp(m_{t+1})] \} \\
     &= -m_0 - m_2' Y_t \\
&\quad + \left[ \sigma_p(m_1) + \ln (1 - \sigma_p(m_1)) \right] p_t \\
&\quad + \left[ \sigma_n(m_1) + \ln (1 - \sigma_n(m_1)) \right] n_t \\
&\quad + \left[ \sigma_e(m_1) + \ln (1 - \sigma_e(m_1)) \right] e_t \\
&\quad + \left[ \sigma_s(m_1) + \ln (1 - \sigma_s(m_1)) \right] q_t \\
&\quad - \frac{1}{2} \left[ m_1' S_1 \Sigma_{\text{other}} S_1' m_1 \right],
\end{aligned}
\]

To price nominal assets, we define the nominal pricing kernel, \( \tilde{m}_{t+1} \), which is a simple transformation of the log real pricing kernel, \( m_{t+1} \),

\[
\tilde{m}_{t+1} = m_{t+1} - \pi_{t+1},
\]

\[
\tilde{m}_{t+1} = \tilde{m}_0 + \tilde{m}_2' Y_t + \tilde{m}_1' \Sigma_{\omega t+1},
\]

\[
\tilde{m}_0 + \tilde{m}_2' Y_t + \tilde{m}_1' \Sigma_{\omega t+1}.
\]
where \( \bar{m}_0 = m_0 - \pi_0 \), \( \bar{m}_1 = m_1 - [0 0 0 1 0 0 0 0 0 0] \)′, and

\[
\bar{m}_2 = m_2 - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\]  

(B.17)

The nominal risk free rate \( rf_t \) is defined as \( -\ln \{ E_t [\exp(\bar{m}_{t+1})] \} \).

**B.2 Valuation ratio**

It is a crucial step in this paper to show that asset prices are (quasi) affine functions of the state variables. It is especially not obvious for equity price-dividend ratio, of which we provide proofs below. First, we rewrite the real dividend growth in a general matrix expression:

\[
\Delta d_{t+1} = g_{t+1} + \Delta \eta_{t+1} = h_0 + h_2' \mathbf{Y}_t + h_1' \Sigma \omega_{t+1},
\]

(B.18)

where \( h_0 = g_0 + \eta_0 \), \( h_1 = [0 0 0 0 1 0 1 0 0] \)′, and

\[
h_2 = \begin{bmatrix} p_{00} + p_{0q} \\ p_{0p} + p_{pp} \\ p_{0n} + p_{pn} \\ 0 \\ 0 \\ 0 \\ p_{0g} \\ 0 \\ p_{0n} - 1 \end{bmatrix}.
\]  

(B.19)

The price-dividend ratio, \( PD_t = E_t \left[ M_{t+1} \left( \frac{D_{t+1} + g_{t+1}}{D_t} \right) \right] \), can be rewritten as,

\[
PD_t = \sum_{n=1}^{\infty} E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right].
\]  

(B.20)

Let \( F^n_t \) denote the \( n \)-th term in the summation:

\[
F^n_t = E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right],
\]  

(B.21)

and \( F^n_t D_t \) is the price of zero-coupon equity that matures in \( n \) periods.

To show that equity price is an approximate affine function of the state variables, we first prove that \( F^n_t (\forall n \geq 1) \) is exactly affine using induction. First, when \( n = 1 \),

\[
F_1^1 = E_t [\exp (m_{t+1} + \Delta d_{t+1})] \\
= E_t \left[ \exp \left( (m_0 + h_0) + (m_2' + h_2') \Sigma \omega_t + (m_1' + h_1') \Sigma \omega_{t+1} \right) \right] \\
= \exp \left( (m_0 + h_0) + (m_2' + h_2') \Sigma \omega_t \right) \\
\cdot \exp \left\{ \left[ -\sigma_p (m_1 + h_1) - \ln (1 - \sigma_p (m_1 + h_1)) \right] p_t + \left[ -\sigma_n (m_1 + h_1) - \ln (1 - \sigma_n (m_1 + h_1)) \right] n_t \right\} \\
\cdot \exp \left\{ \left[ -\sigma_p (m_1 + h_1) - \ln (1 - \sigma_p (m_1 + h_1)) \right] v_t + \left[ -\sigma_n (m_1 + h_1) - \ln (1 - \sigma_n (m_1 + h_1)) \right] q_t \right\} \\
\cdot \exp \left\{ \frac{1}{2} \left[ (m_1' + h_1') S_1 \Sigma_{other} S_1' (m_1 + h_1) \right] \right\}
\]
\[ e_0^1 = m_0 + h_0 + \frac{1}{2} \left[ (m_1' + h_1') S_1 \Sigma^{other} S_1'(m_1 + h_1) \right], \] and

\[ \begin{bmatrix} 0 \\ -\sigma_p(m_1 + h_1) - \ln(1 - \sigma_p(m_1 + h_1)) \\ -\sigma_n(m_1 + h_1) - \ln(1 - \sigma_n(m_1 + h_1)) \end{bmatrix} \]

Now, suppose that the \((n-1)\)-th term \(F_t^{n-1} = \exp\left( e_0^{n-1} + e_1^{n-1} Y_t \right)\), then

\[
F_t^n = E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right] = E_t \left[ E_{t+1} \left[ \exp(m_{t+1} + \Delta d_{t+1}) \exp \left( \sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1} \right) \right] \right] = E_t \left[ \exp(m_{t+1} + \Delta d_{t+1}) E_{t+1} \left[ \exp \left( \sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1} \right) \right] \right] = E_t \left[ \exp(m_{t+1} + \Delta d_{t+1}) \exp \left( e_0^{n-1} + e_1^{n-1} Y_{t+1} \right) \right] = \exp \left( e_0^n + e_1^n Y_t \right),
\]

where \(e_0^n\) and \(e_1^n\) are defined implicitly.

Hence, the price-dividend ratio is approximately affine:

\[
P D_t = \sum_{n=1}^{\infty} E_t \left[ \exp \left( \sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j} \right) \right] = \sum_{n=1}^{\infty} F_t^n = \sum_{n=1}^{\infty} \exp \left( e_0^n + e_1^n Y_t \right).
\]

\section*{B.3 Log nominal equity return}

We apply first-order Taylor approximations to the log nominal equity return, and obtain a linear system,

\[
\hat{r}_{t+1}^* = \ln \left( \frac{P_{t+1}^* + D_{t+1}^*}{P_t} \Pi_{t+1} \right)
\]
= \ln \left( \frac{PD_{t+1} + 1}{PD_t} \right) \ln \left( \frac{D_{t+1}}{D_t} \right) \ln (\Pi_{t+1})
= \Delta d_{t+1} + \pi_{t+1} + \ln \left[ 1 + \sum_{n=1}^{\infty} \exp \left( e_0^n + e_1^n \bar{Y}_{t+1} \right) \exp \left( e_0^n + e_1^n \bar{Y}_t \right) \right] 
\approx \Delta d_{t+1} + \pi_{t+1} + \text{const.} + \sum_{n=1}^{\infty} \exp \left( e_0^n + e_1^n \bar{Y} \right) e_1^n \pi_{t+1} - \sum_{n=1}^{\infty} \exp \left( e_0^n + e_1^n \bar{Y} \right) e_1^n \pi_t 
= \tilde{\rho}_{eq} e_0 + \tilde{\xi}_1 \pi_t + \tilde{\rho}_{eq} \pi_{t+1} + \tau_{eq} \sigma_{\omega t+1}, \quad (B.30)

where \( \tilde{\rho}_{eq} \) is the log nominal return of asset \( i \) from \( t \) to \( t+1 \), \( \tilde{\xi}_1 \) is constant, \( \tilde{\xi}_{eq} \) is a vector of state vector coefficients, and \( \tilde{\rho}_{eq} \) is a vector of shock coefficients. Thus, this step involves linear approximation.

More generally, to acknowledge the errors that are potentially caused by the linear approximations (the Taylor approximation in log price-dividend ratio in the return equation), we write down the return innovations for asset \( i \) with an idiosyncratic shock:

\[ \tilde{r}_{t+1} = E_t (\tilde{r}_{t+1}) = \tilde{\rho}_{eq} \Sigma \omega_{t+1} + \xi_{t+1}, \quad (B.31) \]

where \( E_t (\tilde{r}_{t+1}) \) is the expected return, \( \tilde{\rho}_i \) (10 \times 1) is the asset \( i \) return loadings on selected state variables innovations (the choice of which depends on the asset classes), and \( \xi_{t+1} \) is the Gaussian noise uncorrelated with the state variable shocks but may be cross-correlated (with other asset-specific shocks). The Gaussian shock \( \xi_{t+1} \) has an unconditional variance \( \sigma_i^2 \).

### B.4 Model-implied moments

In this section, we derive three model-implied asset conditional moments—expected excess returns, physical and risk-neutral conditional variances of nominal asset returns. The moments are crucial in creating the moment conditions during the third step of model estimation.

#### B.4.1 One-period expected excess return

We impose the no-arbitrage condition, \( 1 = E_t [\exp(\tilde{m}_{t+1} + \tilde{r}_{t+1})] \) (\( \forall i \in \{\text{equity, treasury bond, corporate bond}\} \)), and obtain the expected excess returns. Expand the law of one price (LOOP) equation:

\[
1 = E_t [\exp(\tilde{m}_{t+1} + \tilde{r}_{t+1})] \\
= \exp \left[ E_t (\tilde{m}_{t+1}) + E_t (\tilde{r}_{t+1}) \right] \\
\cdot \exp \left\{ -\sigma_p (\tilde{m}_1 + \tilde{r}_1) - \ln \left( 1 - \sigma_p (\tilde{m}_1 + \tilde{r}_1) \right) \pi_t + [-\sigma_n (\tilde{m}_1 + \tilde{r}_1) - \ln \left( 1 - \sigma_n (\tilde{m}_1 + \tilde{r}_1) \right)] \pi_t \right\} \\
\cdot \exp \left\{ -\sigma_p (\tilde{m}_1 + \tilde{r}_1) - \ln \left( 1 - \sigma_p (\tilde{m}_1 + \tilde{r}_1) \right) \pi_t + [-\sigma_n (\tilde{m}_1 + \tilde{r}_1) - \ln \left( 1 - \sigma_n (\tilde{m}_1 + \tilde{r}_1) \right)] \pi_t \right\} \\
\cdot \exp \left\{ \frac{1}{2} (\tilde{m}_1 + \tilde{r}_1)' S_1 \Sigma_{\alpha} S_1' (\tilde{m}_1 + \tilde{r}_1) + \sigma_i^2 \right\}, \quad (B.32)
\]

where \( \tilde{m}_1, \tilde{r}_1, \sigma_i, S_1, \) and \( \Sigma_{\alpha} \) are constant matrices defined earlier, and

\[
\sigma_p (\tilde{m}_1 + \tilde{r}_1) = (\tilde{m}_1 + \tilde{r}_1)' \Sigma_{\alpha 1}, \\
\sigma_n (\tilde{m}_1 + \tilde{r}_1) = (\tilde{m}_1 + \tilde{r}_1)' \Sigma_{\alpha 2}, \\
\sigma_p (\tilde{m}_1 + \tilde{r}_1) = (\tilde{m}_1 + \tilde{r}_1)' \Sigma_{\alpha 4}, \\
\sigma_n (\tilde{m}_1 + \tilde{r}_1) = (\tilde{m}_1 + \tilde{r}_1)' \Sigma_{\alpha 8}. \quad (B.33)
\]

Given the nominal risk free rate derived earlier using real pricing kernel and inflation, the nominal excess return is,

\[
E_t (\tilde{r}_{t+1}) - \tilde{F}_{\Pi} = \left\{ \sigma_p (\tilde{r}_1) + \ln \left( \frac{1 - \sigma_p (\tilde{m}_1 + \tilde{r}_1)}{1 - \sigma_p (\tilde{m}_1)} \right) \right\} \pi_t \\
+ \left\{ \sigma_n (\tilde{r}_1) + \ln \left( \frac{1 - \sigma_n (\tilde{m}_1 + \tilde{r}_1)}{1 - \sigma_n (\tilde{m}_1)} \right) \right\} \pi_t \\
+ \left\{ \sigma_p (\tilde{r}_1) + \ln \left( \frac{1 - \sigma_p (\tilde{m}_1 + \tilde{r}_1)}{1 - \sigma_p (\tilde{m}_1)} \right) \right\} \pi_t 
\]
\[ + \left\{ \sigma_q(\bar{r}^i) + \ln \left[ \frac{1 - \sigma_q(\bar{m}_1 + \bar{r}^i)}{1 - \sigma_q(\bar{m}_1)} \right] \right\} q_i \]
\[ - \bar{m}_i^t S_1 \Sigma^{other} S_1^t \bar{r}^i - \frac{1}{2} \left[ \bar{r}^i S_1 \Sigma^{other} S_1^t \bar{r}^i + \sigma_i^2 \right] \quad (B.34) \]

where
\[ \sigma_p(\bar{r}^i) = \bar{r}^i \Sigma \bar{r}^i, \quad (B.35) \]
\[ \sigma_n(\bar{r}^i) = \bar{r}^i \Sigma \bar{r}^i, \quad (B.36) \]
\[ \sigma_c(\bar{r}^i) = \bar{r}^i \Sigma \bar{r}^i, \quad (B.37) \]
\[ \sigma_q(\bar{r}^i) = \bar{r}^i \Sigma \bar{r}^i, \quad (B.38) \]
\[ \sigma_p(\bar{m}_1 + \bar{r}^i) = (\bar{m}_1^t + \bar{r}^i) \Sigma \bar{r}^i, \quad (B.39) \]
\[ \sigma_n(\bar{m}_1 + \bar{r}^i) = (\bar{m}_1^t + \bar{r}^i) \Sigma \bar{r}^i, \quad (B.40) \]
\[ \sigma_c(\bar{m}_1 + \bar{r}^i) = (\bar{m}_1^t + \bar{r}^i) \Sigma \bar{r}^i, \quad (B.41) \]
\[ \sigma_q(\bar{m}_1 + \bar{r}^i) = (\bar{m}_1^t + \bar{r}^i) \Sigma \bar{r}^i. \quad (B.42) \]

**B.4.2 One-period physical conditional return variance**

The physical variance is easily obtained given the loadings:
\[ VAR_i(\bar{r}^i_{t+1}) = \left( \sigma_p(\bar{r}^i_{t+1}) \right)^2 p_t + \left( \sigma_n(\bar{r}^i_{t+1}) \right)^2 n_t + \left( \sigma_c(\bar{r}^i_{t+1}) \right)^2 v_t + \left( \sigma_q(\bar{r}^i_{t+1}) \right)^2 q_t \]
\[ + \bar{r}^i S_1 \Sigma^{other} S_1^t \bar{r}^i + \sigma_i^2. \quad (B.43) \]

**B.4.3 One-period risk-neutral conditional return variance**

To obtain the risk-neutral variance of the asset returns, we use the moment generating function under the risk-neutral measure:
\[ \text{mgf}_t^Q(\bar{r}_{t+1}; \nu) = \frac{E_t [\exp (\bar{m}_{t+1} + \nu \bar{r}_{t+1})]}{E_t [\exp (\bar{m}_{t+1})]} \]
\[ = \exp \left\{ E_t (\bar{m}_{t+1}) + \nu E_t (\bar{r}_{t+1}) \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_p(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_p(\bar{m}_1 + \nu \bar{r}^i) \right) \right] p_t \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_n(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_n(\bar{m}_1 + \nu \bar{r}^i) \right) \right] n_t \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_c(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_c(\bar{m}_1 + \nu \bar{r}^i) \right) \right] v_t \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_q(\bar{m}_1 + \nu \bar{r}^i) - \ln \left( 1 - \sigma_q(\bar{m}_1 + \nu \bar{r}^i) \right) \right] q_t \right\} \]
\[ \cdot \frac{1}{2} \left[ (\bar{m}_1^t + \nu \bar{r}^i) S_1 \Sigma^{other} S_1^t (\bar{m}_1 + \nu \bar{r}^i) + \nu^2 \sigma_i^2 \right] \]
\[ \cdot \frac{1}{2} \left[ \left( \bar{m}_1^t + \nu \bar{r}^i \right) S_1 \Sigma^{other} S_1^t \left( \bar{m}_1 + \nu \bar{r}^i \right) + \nu^2 \sigma_i^2 \right] \]
\[ \exp \left\{ E_t (\bar{m}_1^t) \right\} \]
\[ \exp \left\{ \left[ -\sigma_p(\bar{m}_1) - \ln (1 - \sigma_p(\bar{m}_1)) \right] p_t \right\} \]
\[ \exp \left\{ \left[ -\sigma_n(\bar{m}_1) - \ln (1 - \sigma_n(\bar{m}_1)) \right] n_t \right\} \]
\[ \exp \left\{ \left[ -\sigma_c(\bar{m}_1) - \ln (1 - \sigma_c(\bar{m}_1)) \right] v_t \right\} \]
\[ \exp \left\{ \left[ -\sigma_q(\bar{m}_1) - \ln (1 - \sigma_q(\bar{m}_1)) \right] q_t \right\} \]
\[ = \exp \left\{ \nu E_t (\bar{r}_{t+1}) \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_p(\nu \bar{r}^i) - \ln \left( 1 - \sigma_p(\bar{m}_1 + \nu \bar{r}^i) \right) \right] p_t \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_n(\nu \bar{r}^i) - \ln \left( 1 - \sigma_n(\bar{m}_1 + \nu \bar{r}^i) \right) \right] n_t \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_c(\nu \bar{r}^i) - \ln \left( 1 - \sigma_c(\bar{m}_1 + \nu \bar{r}^i) \right) \right] v_t \right\} \]
\[ \cdot \exp \left\{ \left[ -\sigma_q(\nu \bar{r}^i) - \ln \left( 1 - \sigma_q(\bar{m}_1 + \nu \bar{r}^i) \right) \right] q_t \right\} \]
where $A(\nu) = \exp \left\{ \frac{1}{2} \left[ (\bar{m}_1' + \nu \bar{r}^i) \Sigma^\text{other} S'_1 (\bar{m}_1 + \nu \bar{r}^i) - \bar{m}_1' S_1 \Sigma^\text{other} S'_1 \bar{m}_1 + \nu^2 \sigma_i^2 \right] \right\}$, and

$$
\sigma_p(\bar{m}_1' + \nu \bar{r}^i) = (\bar{m}_1' + \nu \bar{r}^i) \Sigma_{s1}, \\
\sigma_n(\bar{m}_1' + \nu \bar{r}^i) = (\bar{m}_1' + \nu \bar{r}^i) \Sigma_{s2}, \\
\sigma_v(\bar{m}_1' + \nu \bar{r}^i) = (\bar{m}_1' + \nu \bar{r}^i) \Sigma_{s4}, \\
\sigma_q(\bar{m}_1' + \nu \bar{r}^i) = (\bar{m}_1' + \nu \bar{r}^i) \Sigma_{s8}.
$$

The first-order moment is the first-order derivatate at $\nu = 0$:

$$
E_t^Q(\bar{r}^i_{t+1}) = \frac{\partial m g f_t^Q(\bar{r}^i_{t+1}; \nu)}{\partial \nu} |_{\nu = 0} = E_t(\bar{r}^i_{t+1}) + \sigma_p(\bar{m}_1) \sigma_p(\bar{r}^i) \frac{p_t}{1 - \sigma_p(\bar{m}_1)} p_t + \frac{\sigma_n(\bar{m}_1) \sigma_n(\bar{r}^i)}{1 - \sigma_n(\bar{m}_1)} n_t + \frac{\sigma_v(\bar{m}_1) \sigma_v(\bar{r}^i)}{1 - \sigma_v(\bar{m}_1)} v_t + \frac{\sigma_q(\bar{m}_1) \sigma_q(\bar{r}^i)}{1 - \sigma_q(\bar{m}_1)} q_t + \bar{m}_1' S_1 \Sigma^\text{other} S'_1 \bar{r}^i.
$$

Note the similarity between $E_t(\bar{r}^i_{t+1}) - E_t^Q(\bar{r}^i_{t+1})$ from this equation and the equity premium derived before using the no-arbitrage condition. The second-order moment is derived,

$$
VAR_t^Q(\bar{r}^i_{t+1}) = E_t^Q(\bar{r}^i_{t+1})^2 - (E_t^Q(\bar{r}^i_{t+1}))^2 = \frac{\partial^2 m g f_t^Q(\bar{r}^i_{t+1}; \nu)}{\partial \nu^2} |_{\nu = 0} - \left( \frac{\partial m g f_t^Q(\bar{r}^i_{t+1}; \nu)}{\partial \nu} |_{\nu = 0} \right)^2 = \left( \frac{\sigma_p(\bar{r}^i)}{1 - \sigma_p(\bar{m}_1)} \right)^2 p_t + \left( \frac{\sigma_n(\bar{r}^i)}{1 - \sigma_n(\bar{m}_1)} \right)^2 n_t + \left( \frac{\sigma_v(\bar{r}^i)}{1 - \sigma_v(\bar{m}_1)} \right)^2 v_t + \left( \frac{\sigma_q(\bar{r}^i)}{1 - \sigma_q(\bar{m}_1)} \right)^2 q_t + \bar{r}^i \Sigma^\text{other} S'_1 \bar{r}^i + \sigma_i^2.
$$

### C Variables and parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_t$</td>
<td>change in log real industrial production index or growth</td>
</tr>
<tr>
<td>$p_t$</td>
<td>positive uncertainty factor</td>
</tr>
<tr>
<td>$n_t$</td>
<td>negative uncertainty factor</td>
</tr>
<tr>
<td>$\omega_{p,t}$</td>
<td>“good environment” shock</td>
</tr>
<tr>
<td>$\omega_{n,t}$</td>
<td>“bad environment” shock</td>
</tr>
<tr>
<td>$Y_t^\text{mac}$</td>
<td>technology factors consisting of ${\theta_t, p_t, n_t}$</td>
</tr>
<tr>
<td>$\omega_t^\text{mac}$</td>
<td>technology shocks consisting of ${\omega_{p,t}, \omega_{n,t}}$</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>change in log historical consumer price index</td>
</tr>
<tr>
<td>$u^\pi_t$</td>
<td>independent state variable shock of $\pi$</td>
</tr>
<tr>
<td>$\omega^\pi_{n,t}$</td>
<td>inflation shock</td>
</tr>
<tr>
<td>$\bar{l}_t$</td>
<td>log corporate bond loss rate</td>
</tr>
<tr>
<td>$u^l_t$</td>
<td>independent state variable shock of $l$</td>
</tr>
<tr>
<td>$\omega^l_{n,t}$</td>
<td>loss rate shock</td>
</tr>
<tr>
<td>$g_t$</td>
<td>change in log earnings</td>
</tr>
<tr>
<td>$u^g_t$</td>
<td>independent state variable shock of $g$</td>
</tr>
<tr>
<td>$\omega^g_{p,t}$</td>
<td>earnings shock</td>
</tr>
<tr>
<td>$\kappa_t$</td>
<td>log consumption-earnings ratio</td>
</tr>
<tr>
<td>$u^\kappa_t$</td>
<td>independent state variable shock of $\kappa$</td>
</tr>
<tr>
<td>$\omega^\kappa_{n,t}$</td>
<td>log consumption-earnings ratio shock</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>log dividend payout ratio</td>
</tr>
<tr>
<td>$u^\eta_t$</td>
<td>independent state variable shock of $\eta$</td>
</tr>
<tr>
<td>$\omega^\eta_{n,t}$</td>
<td>log dividend payout ratio shock</td>
</tr>
<tr>
<td>$v_t$</td>
<td>loss rate shock shape parameter</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>risk aversion</td>
</tr>
<tr>
<td>$u^\gamma_t$</td>
<td>independent state variable shock of $q$</td>
</tr>
</tbody>
</table>
\( \omega_{q,t} \) risk aversion shock  
\( Y_t \) a vector of 10 state variables  
\( \omega \) a vector of 8 independent shocks  
\( \Delta c_t \) change in log consumption  
\( m_t \) log real pricing kernel  
\( \tilde{m}_t \) log nominal pricing kernel  
\( y_t \) nominal short rate  
\( PC_t \) price-to-coupon ratio of one period defaultable bond  
\( PD_t \) price-dividend ratio  
\( r_t^i \) log asset return for assets \( i \)  
\( E_{t-1} (r_t^i) \) expected return for assets \( i \)  
\( u_t^i \) asset-specific shock of assets \( i \)  
\( \text{VAR}_t^i \) model-implied one-period physical conditional return variance of assets \( i \)  
\( \text{VAR}_t^{iQ} \) model-implied one-period risk-neutral conditional return variance of assets \( i \)  
\( z_t \) a vector of observable asset prices / instruments  
\( \text{PVAR}_t^i \) empirical one-period physical conditional return variance of assets \( i \) for \( t+1 \)  
\( \text{QVAR}_t^i \) empirical one-period risk-neutral conditional return variance of assets \( i \) for \( t+1 \)

### Table C.2: Parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\theta} )</td>
<td>unconditional mean of growth</td>
</tr>
<tr>
<td>( m_p )</td>
<td>sensitivity of output growth on lagged upside uncertainty</td>
</tr>
<tr>
<td>( m_n )</td>
<td>sensitivity of output growth on lagged downside uncertainty</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>unconditional mean of positive uncertainty factor</td>
</tr>
<tr>
<td>( \bar{n} )</td>
<td>unconditional mean of negative uncertainty factor</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>autocorrelation coefficient of positive uncertainty factor</td>
</tr>
<tr>
<td>( \rho_n )</td>
<td>autocorrelation coefficient of negative uncertainty factor</td>
</tr>
<tr>
<td>( \sigma_{p\theta} )</td>
<td>scale parameter of growth to “good environment” shock</td>
</tr>
<tr>
<td>( \sigma_{n\theta} )</td>
<td>scale parameter of growth to “bad environment” shock</td>
</tr>
<tr>
<td>( \sigma_{p\eta} )</td>
<td>scale parameter of positive uncertainty factor to “good environment” shock</td>
</tr>
<tr>
<td>( \sigma_{n\eta} )</td>
<td>scale parameter of negative uncertainty factor to “bad environment” shock</td>
</tr>
<tr>
<td>( j_0 )</td>
<td>constant in Variable ( j ) process</td>
</tr>
<tr>
<td>( \rho_{jy} )</td>
<td>autocorrelation coefficient of Variable ( j )</td>
</tr>
<tr>
<td>( \rho_{jy^p} )</td>
<td>sensitivity coefficient of Variable ( j ) to positive uncertainty factor</td>
</tr>
<tr>
<td>( \rho_{jy^n} )</td>
<td>sensitivity coefficient of Variable ( j ) to negative uncertainty factor</td>
</tr>
<tr>
<td>( \rho_{jy^\theta} )</td>
<td>sensitivity coefficient of Variable ( j ) to output growth factor</td>
</tr>
<tr>
<td>( \sigma_{jy} )</td>
<td>[( \rho_{jy^p}, \rho_{jy^n}, \rho_{jy^\theta} )]</td>
</tr>
<tr>
<td>( \sigma_{jy}^p )</td>
<td>scale parameter of Variable ( j ) to “good environment” shock</td>
</tr>
<tr>
<td>( \sigma_{jy}^n )</td>
<td>scale parameter of Variable ( j ) to “bad environment” shock</td>
</tr>
<tr>
<td>( \sigma_{jy}^{**} )</td>
<td>unconditional volatility of ( u_t^j )</td>
</tr>
<tr>
<td>( \sigma_{jy}^{***} )</td>
<td>scale parameter of the state variable gamma shock ( u_t^j )</td>
</tr>
<tr>
<td>( \sigma_{\theta\theta} )</td>
<td>scale parameter of the ( \theta_t ) to the loss shock</td>
</tr>
<tr>
<td>( \mu )</td>
<td>constant vector in the state variable system (10 x 1)</td>
</tr>
<tr>
<td>( A )</td>
<td>autocorrelation vector in the state variable system (10 x 10)</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>scale / volatility parameter matrix of the 8 shocks (10 x 8)</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>constant in the consumption growth process</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>sensitivity vector of consumption growth to state variable shocks</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>sensitivity vector of consumption growth to state variable levels</td>
</tr>
<tr>
<td>( m_0 )</td>
<td>constant in the real pricing kernel process</td>
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<td>( m_1 )</td>
<td>sensitivity vector of real pricing kernel to state variable shocks</td>
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<tr>
<td>( m_2 )</td>
<td>sensitivity vector of real pricing kernel to state variable levels</td>
</tr>
<tr>
<td>( \gamma_t )</td>
<td>return loadings on state variable shocks</td>
</tr>
<tr>
<td>( \sigma_i )</td>
<td>unconditional volatility of ( u_t^i )</td>
</tr>
<tr>
<td>( \chi )</td>
<td>risk aversion loadings on observed asset prices</td>
</tr>
</tbody>
</table>

* for all \( j \in \{ \pi, l, g, \kappa, \eta, v, q \} \):  
** for all \( j \in \{ \pi, g, \kappa, \eta \} \):  
*** for all \( j \in \{ l, q \} \):
References


Tables and Plots

Table 1: The Dynamics of the Macro Factors

This table reports parameter estimates from the model below using the monthly log growth rate of U.S. industrial production from January 1947 to February 2015. This system involves latent processes (good shape parameter governing positive skewness \( \theta_t \) and bad shape parameter governing negative skewness \( \bar{n}_t \)) and is estimated using the MLE-filtration methodology described in Bates (2006).

\[
\begin{align*}
\theta_{t+1} &= \bar{\theta} + \rho_\theta (\theta_t - \bar{\theta}) + m_p (p_t - 500) + m_n (n_t - \bar{n}) + \omega_{t+1}^	heta \\
p_{t+1} &= 500 + \rho_p (p_t - 500) + \sigma_{pp} \omega_{p,t+1} \\
n_{t+1} &= \bar{n} + \rho_n (n_t - \bar{n}) + \sigma_{nn} \omega_{n,t+1}
\end{align*}
\]

where

\[
\omega_{t+1} = \sigma_{\theta p} \omega_{p,t+1} - \sigma_{\bar{n} n} \omega_{n,t+1}
\]

\[
\omega_{p,t+1} \sim \tilde{\gamma}(p_t, 1)
\]

\[
\omega_{n,t+1} \sim \tilde{\gamma}(n_t, 1)
\]

\[
\sigma_{pp} > 0 \\
\sigma_{nn} > 0.
\]

Standard errors are in parentheses. Note that “\( \omega_{n,t} \) loading” in Column “\( \theta_t \)” is -\( \sigma_{\theta n} \); “\( \omega_{n,t} \) loading” in Column “\( p_t \)” (“\( n_t \)” is +\( \sigma_{pp} (+\sigma_{nn}) \).

<table>
<thead>
<tr>
<th></th>
<th>( \theta_t )</th>
<th>( p_t )</th>
<th>( n_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.00002</td>
<td>500</td>
<td>16.14206</td>
</tr>
<tr>
<td></td>
<td>(0.00045)</td>
<td>(fix)</td>
<td>(2.14529)</td>
</tr>
<tr>
<td>AR</td>
<td>0.13100</td>
<td>0.99968</td>
<td>0.91081</td>
</tr>
<tr>
<td></td>
<td>(0.03094)</td>
<td>(0.01918)</td>
<td>(0.01350)</td>
</tr>
<tr>
<td>( m_p )</td>
<td>0.00001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00034)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_n )</td>
<td>-0.00020</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \omega_{p,t} ) loading</td>
<td>0.00011</td>
<td>0.55277</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00001)</td>
<td>(0.07073)</td>
<td></td>
</tr>
<tr>
<td>( \omega_{n,t} ) loading</td>
<td>-0.00174</td>
<td>2.17755</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00014)</td>
<td>(0.15027)</td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>2861.30797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BIC</td>
<td>-5648.85</td>
<td>AIC</td>
<td>-5700.62</td>
</tr>
</tbody>
</table>
Table 2: The Dynamics of the Corporate Loss Rate

This table reports parameter estimates for the corporate loss rate model using monthly data from January 1982 to February 2015. The mean equation of the loss rate is as follows,

\[ l_{t+1} = l_0 + \rho_l l_t + \rho_{lp} p_t + \rho_{ln} n_t + \sigma_{lp} \omega_{p,t+1} + \sigma_{ln} \omega_{n,t+1} + u_{t+1} \]

\[ u_{t+1} = \sigma_l \omega_{l,t+1} \]

\[ \omega_{l,t+1} \sim \tilde{\gamma}(v_t, 1) \]

where the variance equation is,

\[ v_{t+1} = v_0 + \rho_v v_t + \sigma_v \omega_{v,t+1} \]

The mean equation is estimated by projection, the variance equation by MLE. Standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Mean Equation</th>
<th>l_0</th>
<th>\rho_l</th>
<th>\rho_{lp}</th>
<th>\rho_{ln}</th>
<th>\sigma_{lp}</th>
<th>\sigma_{ln}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.3463</td>
<td>0.8779</td>
<td>0.0001</td>
<td>0.0047</td>
<td>-0.0004</td>
<td>0.0177</td>
</tr>
<tr>
<td>(0.0911)</td>
<td>(0.0209)</td>
<td>(0.0002)</td>
<td>(0.0010)</td>
<td>(0.0004)</td>
<td>(0.0039)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Equation</th>
<th>\sigma_l</th>
<th>v_0</th>
<th>\rho_v</th>
<th>\sigma_v</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_0</td>
<td>0.0599</td>
<td>0.8544</td>
<td>0.9051</td>
<td>0.1820</td>
</tr>
<tr>
<td>(0.0008)</td>
<td>(0.1867)</td>
<td>(0.0205)</td>
<td>(0.0203)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Cash Flow Dynamics

Parameters of cash flow processes are shown in Equation (18) for the log earnings growth, Equation (21) for the log consumption-earnings ratio, Equation (24) for the log dividend-earnings ratio, and Equation (30) for the inflation rate. Estimation is by simple linear projection. Bold (italic) coefficients have <5% (10%) p-values. Robust errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>earnings growth</th>
<th>log CE</th>
<th>log DE</th>
<th>inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>drift</td>
<td>0.0285</td>
<td>0.1101</td>
<td>-0.1005</td>
</tr>
<tr>
<td>(0.0273)</td>
<td>(0.0453)</td>
<td>(0.0337)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>AR</td>
<td>0.6619</td>
<td>0.9400</td>
<td>0.9188</td>
</tr>
<tr>
<td>(0.0432)</td>
<td>(0.0088)</td>
<td>(0.0103)</td>
<td>(0.0500)</td>
</tr>
<tr>
<td>\theta_t</td>
<td>0.7932</td>
<td>0.8494</td>
<td>1.2237</td>
</tr>
<tr>
<td>(0.6214)</td>
<td>(0.8104)</td>
<td>(0.8420)</td>
<td>(0.0315)</td>
</tr>
<tr>
<td>\rho_t</td>
<td>-5.29E-05</td>
<td>4.42E-05</td>
<td>-2.55E-05</td>
</tr>
<tr>
<td>(5.66E-05)</td>
<td>(7.10E-05)</td>
<td>(7.48E-05)</td>
<td>(2.84E-06)</td>
</tr>
<tr>
<td>\eta_t</td>
<td>-0.0006</td>
<td>0.0049</td>
<td>0.0053</td>
</tr>
<tr>
<td>(0.0004)</td>
<td>(0.0005)</td>
<td>(0.0006)</td>
<td>(1.94E-05)</td>
</tr>
<tr>
<td>\omega_{p,t+1} loading</td>
<td>-8.63E-05</td>
<td>4.94E-05</td>
<td>5.34E-05</td>
</tr>
<tr>
<td>(1.16E-04)</td>
<td>(1.42E-04)</td>
<td>(1.46E-04)</td>
<td>(5.87E-06)</td>
</tr>
<tr>
<td>\omega_{n,t+1} loading</td>
<td>-0.0029</td>
<td>0.0061</td>
<td>0.0063</td>
</tr>
<tr>
<td>(0.0011)</td>
<td>(0.0014)</td>
<td>(0.0014)</td>
<td>(5.60E-06)</td>
</tr>
<tr>
<td>\omega_{z,t+1} loading</td>
<td>0.0010</td>
<td>-0.0014</td>
<td>-0.0014</td>
</tr>
<tr>
<td>(0.0009)</td>
<td>(0.0011)</td>
<td>(0.0011)</td>
<td></td>
</tr>
<tr>
<td>Gaussian shock volatility</td>
<td>0.0465</td>
<td>0.0568</td>
<td>0.0582</td>
</tr>
<tr>
<td>(0.0018)</td>
<td>(0.0021)</td>
<td>(0.0022)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Adjusted $R^2$ (conditional mean)</td>
<td>55.58%</td>
<td>98.19%</td>
<td>97.89%</td>
</tr>
</tbody>
</table>
Table 4: Shock Correlations

| ω_{p,t+1} | good uncertainty shock | Γ(p_t, 1); |
| ω_{n,t+1} | bad uncertainty shock | Γ(n_t, 1); |
| ω_{l,t+1} | loss rate-specific shock | Γ(v_t, 1); |
| ω_{g,t+1} | log earnings growth-specific shock | N(0, 1); |
| ω_{κ,t+1} | log C/E-specific shock | N(0, 1); |
| ω_{η,t+1} | log D/E-specific shock | N(0, 1); |
| ω_{q,t+1} | risk aversion-specific shock | Γ(q_t, 1). |

Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

| tsprd  | 1 | 0.3524 | 0.2583 | 0.1266 | 0.1240 | 0.2949 |
| csprd  | 1 | 0.5063 | 0.4793 | 0.5999 | 0.5340 |
| DY5yr  | 1 | 0.1675 | 0.1641 | 0.3101 |
| rvareq | 1 | 0.8430 | 0.5942 |
| qvareq | 1 | 0.5374 |
| rvarcb | 1 |

Table 5: Financial Instruments Spanning Risk Aversion

This table presents summary statistics of the 6 financial instruments that are used to span our risk aversion measure: “tsprd” is the difference between 10-year treasury yield and 3-month Treasury yield; “csprd” is the difference between Moody’s BAA yield and the 10-year zero-coupon Treasury yield; “DY5yr” is the detrended dividend yield where the moving average takes the 5 year average of monthly dividend yields, starting one year before; “rvareq” and “rvarcb” are realized variances of log equity returns and log corporate bond returns, calculated from daily returns; “qvareq” is the risk-neutral conditional variance of log equity returns; for the early years (before 1990), we use VVO and authors’ calculations. Bold (italic) coefficients have <5% (10%) p-values. Block bootstrapped errors are shown in parentheses. The sample period is from 1986/06 to 2015/02 (345 months).

| tsprd | 0.0179 | 0.0231 | -0.0030 | 0.0029 | 0.0040 | 0.0002 |
| csprd | 0.0116 | 0.0075 | 0.0061 | 0.0059 | 0.0010 | 0.0003 |
| rvarcb | 0.0810 | 0.2515 | 0.1882 | 1.5951 | 0.5123 | 0.6872 |
| AR(1) | 0.9669 | 0.9642 | 0.9822 | 0.4311 | 0.7461 | 0.5775 |
| SE | 0.0137 | 0.0143 | 0.0083 | 0.0489 | 0.0360 | 0.0442 |
Table 6: Reduced-Form Risk Aversion Parameters

This table presents the two-step GMM estimation results for risk aversion, \( q_t = \chi' \mathbf{z}_t \), estimated using equity market and corporate bond market asset prices. The utility function curvature \( \gamma \) is fixed at 2. The first-step weight matrix is an identity matrix; the second-step weight matrix builds on the Newey-West spectral density function with 5-month lags, and then is shrunk towards an identity matrix where the shrinkage parameter is 0.1. The GMM system also consistently estimates \( \sigma_{qq} \). Therefore, the system has 8 unknown parameters. The p-value of Hansen’s overidentification test (J test) is calculated from the asymptotic \( \chi^2 \) distribution with the degree of freedom being 29 (37-8). Bold (italic) coefficients have <5% (10%) p-values. Efficient standard errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>( q_t )</th>
<th>Efficient GMM Estimators</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.264</td>
</tr>
<tr>
<td></td>
<td>(0.510)</td>
</tr>
<tr>
<td>( \chi_{tsprd} )</td>
<td>-0.442</td>
</tr>
<tr>
<td></td>
<td>(4.507)</td>
</tr>
<tr>
<td>( \chi_{csprd} )</td>
<td>-7.599</td>
</tr>
<tr>
<td></td>
<td>(7.186)</td>
</tr>
<tr>
<td>( \chi_{dy5yr} )</td>
<td>6.550</td>
</tr>
<tr>
<td></td>
<td>(8.158)</td>
</tr>
<tr>
<td>( \chi_{rvareq} )</td>
<td><strong>-43.232</strong></td>
</tr>
<tr>
<td></td>
<td>(1.879)</td>
</tr>
<tr>
<td>( \chi_{qvareq} )</td>
<td><strong>104.599</strong></td>
</tr>
<tr>
<td></td>
<td>(8.288)</td>
</tr>
<tr>
<td>( \chi_{rvarcb} )</td>
<td><strong>239.663</strong></td>
</tr>
<tr>
<td></td>
<td>(114.473)</td>
</tr>
</tbody>
</table>

Correlation with the NBER Indicator

| \( \rho(q_t, NBER_t) \)          | **0.397**                |
|                                  | (0.045)                  |

Model Specifications

| Hansen’s J                        | 27.919                   |
| p-value                           | 0.5222                   |


Table 7: Structural Risk Aversion Parameters.

This table presents the model-implied risk aversion process parameters. “Projection”: coefficient estimates are obtained from simple projection. “GMM”: the scale parameter of the risk aversion innovation which is estimated in the GMM framework (Table 6). The second and third panels report the variance decomposition results of the conditional mean and shock structure of $\hat{q}_{t+1}$, denotes with “VARC”. In the second panel, $VARC = \beta_x \text{cov}(\hat{y}, x) \text{var}(\hat{y})$ where $\hat{y} = \hat{E}_t(\hat{q}_{t+1})$. VARC in the third panel is calculated using the residual, $\hat{q}_{t+1} - \hat{E}_t(\hat{q}_{t+1})$. Bold (italic) coefficients have <5% (10%) p-values. Robust and efficient standard errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

$$\hat{q}_{t+1} = q_0 + \rho_q \hat{q}_t + \rho_p \hat{p}_t + \rho_n \hat{n}_t + \sigma_p \hat{\omega}_p,t+1 + \sigma_n \hat{\omega}_n,t+1 + \omega_{q,t+1}^{q},$$

$$u_{q,t+1} = \sigma_q \hat{\omega}_q,t+1,$$

$$\omega_{q,t+1} = \tilde{\Gamma}(q_1,1).$$

<table>
<thead>
<tr>
<th>Structural Risk Aversion Parameters, $q_{t+1}$</th>
<th>○ Projection</th>
<th>○ GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$p_t$</td>
<td>$n_t$</td>
</tr>
<tr>
<td>Est</td>
<td>-0.1040</td>
<td>0.0006</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.0835)</td>
<td>(0.0002)</td>
</tr>
</tbody>
</table>

Conditional Mean Variance Decomposition (60.70% of Total Variance)

| VARC | 77.15% | 1.68% | 21.17% |

Shock Structure Variance Decomposition (39.30% of Total Variance)

<table>
<thead>
<tr>
<th>VARC</th>
<th>$\omega_{p,t+1}$</th>
<th>$\omega_{n,t+1}$</th>
<th>$\omega_{q,t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35%</td>
<td>0.30%</td>
<td>99.35%</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Fit of Moments.

This table evaluates the fit of conditional moments of equity and corporate bond returns. That is, Column “Model” reports the averages of the relevant model-implied conditional moments. The “Empirical Averages” represent the sample averages of the excess returns (for “Mom 1” and “Mom 4”), the sample average of empirical conditional variances (for “Mom 2”, “Mom 3”, and “Mom 5”). In “Mom 6” and “Mom 7”, “Risk Aversion Innovation” is $\omega_{q,t+1}^{q}$ in Equation (27). The variance and unscaled skewness rows compare the average model-implied conditional moments with the unconditional moments. Bolded number(s) denote a distance of less than 1.645 standard errors from the corresponding point estimate, and italicized number(s) a distance of more than 1.645 but less than 1.96 standard errors. Block bootstrapped standard errors are shown in parentheses; we allow the block size to vary for different moments, block sizes=[0 6 15 1 10] for Mom 1 to Mom 5, respectively. Asymptotic standard errors (standard deviation divided by square root of the number of observations) are reported for Mom 6 and Mom 7. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Empirical Average</th>
<th>Boot.SE/SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom 1</td>
<td>Equity Risk Premium</td>
<td>0.00749</td>
<td>0.00530</td>
</tr>
<tr>
<td>Mom 2</td>
<td>Equity Physical Variance</td>
<td>0.00310</td>
<td>0.00286</td>
</tr>
<tr>
<td>Mom 3</td>
<td>Equity Risk-neutral Variance</td>
<td>0.00369</td>
<td>0.00397</td>
</tr>
<tr>
<td>Mom 4</td>
<td>Corporate Bond Risk Premium</td>
<td>0.00289</td>
<td>0.00388</td>
</tr>
<tr>
<td>Mom 5</td>
<td>Corporate Bond Physical Variance</td>
<td>0.00027</td>
<td>0.00024</td>
</tr>
<tr>
<td>Mom 6</td>
<td>Risk Aversion Innovation Variance</td>
<td>0.00823</td>
<td>0.00906</td>
</tr>
<tr>
<td>Mom 7</td>
<td>Risk Aversion Innovation Unscaled Skewness</td>
<td>0.00246</td>
<td>0.00246</td>
</tr>
</tbody>
</table>
Table 9: Asset Prices and the State Variables.

The asset conditional moments are explained by \( \{p_t, n_t, v_t, q_t\} \). The coefficients are scaled by the standard deviation of the state variable in the same column, and then multiplied by 10000 for reporting purposes. \( \text{VARC} \) is coefficient \( \frac{\text{Cov}(x_t, \text{Mom}_t)}{\text{Var(\text{Mom}_t})} \) where \( x \in \{p, n, v, q\} \) and \( \text{Mom} \) is from Mom 1 to Mom 5. The variance decomposition is reported in a bold italic font.

<table>
<thead>
<tr>
<th>Moment</th>
<th>( p_t )</th>
<th>( n_t )</th>
<th>( v_t )</th>
<th>( q_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom 1</td>
<td>Equity Risk Premium</td>
<td>0.1286</td>
<td>4.6231</td>
<td>0.0039</td>
</tr>
<tr>
<td></td>
<td>( \text{VARC} )</td>
<td>\textbf{0.014}%</td>
<td>\textbf{4.449}%</td>
<td>\textbf{0.000}%</td>
</tr>
<tr>
<td>Mom 2</td>
<td>Equity Physical Variance</td>
<td>0.0651</td>
<td>2.5845</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>( \text{VARC} )</td>
<td>\textbf{-0.031}%</td>
<td>\textbf{26.640}%</td>
<td>\textbf{0.000}%</td>
</tr>
<tr>
<td>Mom 3</td>
<td>Equity Risk-neutral Variance</td>
<td>0.0652</td>
<td>2.5454</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>( \text{VARC} )</td>
<td>\textbf{0.011}%</td>
<td>\textbf{12.646}%</td>
<td>\textbf{0.000}%</td>
</tr>
<tr>
<td>Mom 4</td>
<td>Corporate Bond Risk Premium</td>
<td>0.0752</td>
<td>7.6016</td>
<td>-0.0200</td>
</tr>
<tr>
<td></td>
<td>( \text{VARC} )</td>
<td>\textbf{-0.143}%</td>
<td>\textbf{71.227}%</td>
<td>\textbf{-0.001}%</td>
</tr>
<tr>
<td>Mom 5</td>
<td>Corporate Bond Physical Variance</td>
<td>0.0009</td>
<td>0.3137</td>
<td>0.0007</td>
</tr>
<tr>
<td></td>
<td>( \text{VARC} )</td>
<td>\textbf{-0.090}%</td>
<td>\textbf{99.865}%</td>
<td>\textbf{-0.001}%</td>
</tr>
</tbody>
</table>

Table 10: Predicting Realized Excess Returns and Variances.

This table reports the regression coefficients of realized excess returns and realized variances of equity and corporate bond. The coefficients are scaled by the standard deviation of the state variable in the same row, and then multiplied by 100 for reporting purposes. ‘Model-Implied Moments’ are risk premiums (for realized excess returns) and physical variances (for realized variances). Bold (italic) coefficients have <5% (10%) p-values. The \( R^2 \) is adjusted. Standard errors are shown in parentheses. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>( r_{t+1}^e - r_f t )</th>
<th>( RVAR_{t+1}^a )</th>
<th>( r_{t+1}^{cb} - r_f t )</th>
<th>( RVAR_{t+1}^{cb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_t )</td>
<td>-0.1648</td>
<td>0.0190</td>
<td>-0.1098</td>
</tr>
<tr>
<td></td>
<td>(0.2584)</td>
<td>(0.0312)</td>
<td>(0.0922)</td>
</tr>
<tr>
<td>( n_t )</td>
<td>\textbf{-1.2623}</td>
<td>\textbf{0.1695}</td>
<td>0.0044</td>
</tr>
<tr>
<td></td>
<td>(0.2928)</td>
<td>(0.0354)</td>
<td>(0.1045)</td>
</tr>
<tr>
<td>( v_t )</td>
<td>-0.3478</td>
<td>0.0410</td>
<td>-0.1089</td>
</tr>
<tr>
<td></td>
<td>(0.2359)</td>
<td>(0.0285)</td>
<td>(0.0842)</td>
</tr>
<tr>
<td>( q_t )</td>
<td>\textbf{0.6562}</td>
<td>\textbf{0.1047}</td>
<td>\textbf{0.1262}</td>
</tr>
<tr>
<td></td>
<td>(0.1866)</td>
<td>(0.0226)</td>
<td>(0.0666)</td>
</tr>
<tr>
<td>Model-Implied Moments</td>
<td>\textbf{0.6671}</td>
<td>\textbf{3.8517}</td>
<td>\textbf{1.6610}</td>
</tr>
<tr>
<td></td>
<td>(0.4018)</td>
<td>(0.4017)</td>
<td>(0.8376)</td>
</tr>
<tr>
<td>Corr w/ NBER</td>
<td>\textbf{0.45}</td>
<td>\textbf{0.54}</td>
<td>\textbf{0.61}</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>5.6%</td>
<td>0.1%</td>
<td>21.8%</td>
</tr>
<tr>
<td></td>
<td>1.2%</td>
<td>0.8%</td>
<td>22.0%</td>
</tr>
</tbody>
</table>
Table 11: Out-Of-Sample Exercise.

This table analyzes in-sample (see Section 3.3) and out-of-sample risk premium estimates of equity returns and corporate bond returns. “Realized” indicates the realized excess returns. “Mod (1)” indicates the in-sample (full-sample) estimates of model-implied risk premiums, the dynamics of which are determined by \{p_t, n_t, v_t, q_t\}. “Mod (2)” indicates the out-of-sample estimates of model-implied risk premiums. Define a 60-month rolling window from \(t - 60\) to \(t - 1\), then project one-period ahead excess returns on the 4 state variables, use the coefficient estimates to obtain \(E_t(\tilde{r}_{t+1} - r_{ft})\), repeat. We also consider three out-of-sample empirical models that use three instruments sets (subsets of \(z_t\)), respectively: (1) dividend yield, (2) divided yield + term spread + credit spread, (3) physical uncertainty plus variance risk premium estimate. The table then reports the optimal combination of Mod and Emp Mod estimate. Least Square standard errors are shown in parentheses. Bold (italic) coefficients have <5% (10%) p-values. R^2 is adjusted. The (full) sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th></th>
<th>Least-Square Estimate of (a) in (\tilde{r}<em>{t+1} - r</em>{ft} = a \times \text{Mod}(t, i) + (1 - a) \times \text{Emp Mod}(t, j) + \epsilon_{t+1})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i = 1, 2; j = 1, 2, 3)</td>
</tr>
<tr>
<td>◦ Equity:</td>
<td>◦ Corporate Bond:</td>
</tr>
<tr>
<td>Mod (1)</td>
<td>Mod (2)</td>
</tr>
<tr>
<td>Emp Mod (1)</td>
<td>0.8262  0.4843</td>
</tr>
<tr>
<td></td>
<td>(0.1095) (0.0995)</td>
</tr>
<tr>
<td>Emp Mod (2)</td>
<td>0.9613  0.6618</td>
</tr>
<tr>
<td></td>
<td>(0.0934) (0.0870)</td>
</tr>
<tr>
<td>Emp Mod (3)</td>
<td>0.8305  0.6216</td>
</tr>
<tr>
<td></td>
<td>(0.0810) (0.0801)</td>
</tr>
</tbody>
</table>

Table 12: Uncertainty Index.

This table presents regression results of the filtered macroeconomic uncertainty (from industrial production growth) on the set of instruments used to determine risk aversion. “Total” is the total industrial production growth variance, which is a function of \(p_t, n_t, \sigma^2_{p_t} + \sigma^2_{n_t}\). “\(\times 10^3\)” in the header means that the coefficients and their SEs reported are multiplied by 1000 for reporting convenience. Bold (italic) coefficients have <5% (10%) p-values. Robust and efficient standard errors are shown in parentheses. The \(R^2\) is adjusted. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th></th>
<th>((\times 10^3)) Total</th>
<th>((\times 10^3)) VARC</th>
<th>((\times 10^3)) Upside</th>
<th>((\times 10^3)) Downside</th>
<th>((\times 10^3)) VARC</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.006</td>
<td>0.006</td>
<td>-0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.000)</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{tsprd})</td>
<td>-0.573</td>
<td>-2.38%</td>
<td>-0.005</td>
<td>5.21%</td>
<td>-0.568</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.002)</td>
<td>(0.113)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{csprd})</td>
<td>1.919</td>
<td>61.33%</td>
<td>-0.006</td>
<td>4.05%</td>
<td>1.925</td>
</tr>
<tr>
<td></td>
<td>(0.246)</td>
<td>(0.005)</td>
<td>(0.247)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{DY5yr})</td>
<td>1.083</td>
<td>19.83%</td>
<td>-0.063</td>
<td>84.69%</td>
<td>1.146</td>
</tr>
<tr>
<td></td>
<td>(0.234)</td>
<td>(0.005)</td>
<td>(0.235)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{rvareq})</td>
<td>-0.318</td>
<td>-4.81%</td>
<td>-0.022</td>
<td>-0.41%</td>
<td>-0.296</td>
</tr>
<tr>
<td></td>
<td>(0.399)</td>
<td>(0.008)</td>
<td>(0.401)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{qvarc})</td>
<td>1.243</td>
<td>14.28%</td>
<td>0.066</td>
<td>8.08%</td>
<td>1.177</td>
</tr>
<tr>
<td></td>
<td>(0.673)</td>
<td>(0.014)</td>
<td>(0.676)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{revarc})</td>
<td>14.829</td>
<td>11.76%</td>
<td>0.158</td>
<td>-1.61%</td>
<td>14.671</td>
</tr>
<tr>
<td></td>
<td>(5.910)</td>
<td>(0.124)</td>
<td>(5.933)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>47.72%</td>
<td>44.35%</td>
<td>47.95%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 13: On the Predictive Power of Risk Aversion Index and Uncertainty Index on Future Output Growth.

This table reports the coefficient estimates of the following predictive regression,

$$\theta_{t+k} = a + b'x_t + \text{ra}_{t+k},$$

where $\theta_{t+k}$ represents future industrial production growth during period $t + 1$ and $t + k \left( \sum_{r=1}^{k} \theta_{t+r} \right)$ and $x_t$ represents a vector of current predictors. We consider (1) our risk aversion index $\text{ra}_{BEX}$, (2) our uncertainty index $\text{unc}_{BEX}$ (financial proxy), (3) the risk-neutral conditional variance (the square of the month-end VIX (after 1990) / VXO (prior to 1990) index divided by 120000), QVAR, and (4) the true total macroeconomic uncertainty filtered from industrial production index $\text{unc}_{true}$. The coefficients are scaled by the standard deviation of the predictor in the same column for reporting purposes. Hodrick (1992) standard errors are reported in parentheses, and adjusted $R^2$s are in %. Bold (italic) coefficients have <5% (10%) p-values.

<table>
<thead>
<tr>
<th></th>
<th>ra_{BEX}</th>
<th>unc_{BEX}</th>
<th>QVAR</th>
<th>unc_{true}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Univariate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1m</td>
<td>-0.002</td>
<td>-0.003</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>7.9%</td>
<td>19.2%</td>
<td>6.5%</td>
<td>12.9%</td>
</tr>
<tr>
<td>1q</td>
<td>-0.005</td>
<td>-0.008</td>
<td>-0.005</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>13.9%</td>
<td>34.9%</td>
<td>15.3%</td>
<td>26.3%</td>
</tr>
<tr>
<td>4q</td>
<td>-0.006</td>
<td>-0.016</td>
<td>-0.008</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td></td>
<td>2.0%</td>
<td>15.2%</td>
<td>3.7%</td>
<td>6.2%</td>
</tr>
<tr>
<td><strong>B. Multivariate (1)</strong></td>
<td>R^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1m</td>
<td>0.000</td>
<td>-0.003</td>
<td>0.001</td>
<td>19.4%</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>1q</td>
<td>0.002</td>
<td>-0.008</td>
<td>-0.001</td>
<td>35.3%</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>4q</td>
<td><strong>0.013</strong></td>
<td><strong>-0.021</strong></td>
<td><strong>-0.005</strong></td>
<td>18.4%</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td><strong>C. Multivariate (2)</strong></td>
<td>R^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1m</td>
<td>0.000</td>
<td>-0.002</td>
<td>0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>1q</td>
<td>0.002</td>
<td>-0.006</td>
<td>-0.001</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>4q</td>
<td><strong>0.013</strong></td>
<td><strong>-0.022</strong></td>
<td>-0.005</td>
<td>0.001 17.4%</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>
Table 14: Alternative Risk Aversion and Uncertainty Measures.

This table reports the correlation between our risk aversion and economic uncertainty indices and existing measures. For risk aversion (Panel A), we consider three categories. A.1) We follow Wachter (2006) to create a fundamental risk aversion process from inflation-adjusted (real) quarterly consumption growth \( \sum_{j=0}^{4} \Delta c_{t-j} \); A.2) we consider the well-known sentiment index by Baker and Wurgler (2006) from the behavior finance literature, and the Michigan Consumer Sentiment Index (that directly measures the consumer sentiment); A.3) we also consider an industry index, the Credit Suisse First Boston Risk Appetite Index. For economic uncertainty (Panel B), we consider B.1) the macroeconomic uncertainty index created by Jurado, Ludvigson, and Ng (2015), and B.2) the Economic Policy Uncertainty Index created by Baker, Bloom, and Davis (2016). Correlations are calculated using overlapping samples at the monthly frequency. Standard errors are shown in parentheses. Bold correlation coefficients have <5% p-values.

<table>
<thead>
<tr>
<th>A. Correlations with Extant Risk Aversion Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1) “Fundamental” Habit Model:</td>
</tr>
<tr>
<td>Wachter (2006) / Campbell and Cochrane (1999)</td>
</tr>
<tr>
<td>A.2) Sentiment Index:</td>
</tr>
<tr>
<td>Baker and Wurgler (2006)</td>
</tr>
<tr>
<td>Michigan Consumer Sentiment Index</td>
</tr>
<tr>
<td>A.3) Industry Index</td>
</tr>
<tr>
<td>Credit Suisse First Boston Risk Appetite Index</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Correlations with Extant Uncertainty Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1) Macroeconomic Uncertainty:</td>
</tr>
<tr>
<td>Jurado, Ludvigson, and Ng (2015)</td>
</tr>
<tr>
<td>B.2) Political Uncertainty:</td>
</tr>
<tr>
<td>Baker, Bloom, and Davis (2016)</td>
</tr>
</tbody>
</table>

Table 15: Market Integration Test.

This table tests market integration by evaluating the fit of Treasury bond return moments priced using the pricing kernel extracted from risky assets. Column “Model” reports the averages of the relevant model-implied conditional moments. The “Empirical Averages” represent the sample averages of the excess returns (“Mom 1”), the sample average of empirical conditional variances (“Mom 2”, “Mom 3”). Bolded number(s) denote a distance of less than 1.645 standard errors from the corresponding point estimate, and italicized number(s) a distance of more than 1.645 but less than 1.96 standard errors. Block bootstrapped standard errors are shown in parentheses; we allow the block size to vary for different moments, block sizes=[0 14 13] for Mom 1 to Mom 3, respectively. The sample period is 1986/06 to 2015/02 (345 months).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Empirical Average</th>
<th>Boot.SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury Bond Risk Premium</td>
<td>-0.00368</td>
<td>0.00285</td>
<td>(0.00117)</td>
</tr>
<tr>
<td>Treasury Bond Physical Variance</td>
<td>0.00111</td>
<td>0.00035</td>
<td>(0.00004)</td>
</tr>
<tr>
<td>Treasury Bond Risk-neutral Variance</td>
<td>0.00113</td>
<td>0.00043</td>
<td>(0.00003)</td>
</tr>
</tbody>
</table>
Figure 1: Filtered state variables extracted from industrial production growth. The shaded regions are NBER recession months from the NBER website.
Figure 2: Model-implied conditional moments for industrial production growth. The shaded regions are NBER recession months from the NBER website.
Figure 3: Conditional moments of the loss rate. The shaded regions are NBER recession months from the NBER website.
Figure 4: Decomposition of the conditional variance of the loss rate. The shaded regions are NBER recession months from the NBER website. The green dashed line depicts the ratio of the unconditional variance of $u_{t+1}'$ to the unconditional variance of the total loss rate disturbance, or $1-R^2$ from the projection.
Figure 5: Model-implied and empirical risk-neutral conditional variances of equity market returns. The shaded regions are NBER recession months from the NBER website.

Figure 6: The Time Variation in the Risk Aversion. Risk aversion is $\gamma \exp(q_t)$. The shaded regions are the NBER recession months from the NBER website.
Figure 7: Risk aversion index (solid blue/left y-axis) and Uncertainty index (dashed red/right y-axis). According to Section 4, both the risk aversion index denoted as $ra_{BEX} = \gamma \exp(q_t)$ and the uncertainty index denoted as $unc_{BEX}$ are functions of a set of financial instruments. See Equations (49) and (59). Correlation between the two series is 67.94%. The shaded regions are NBER recession months from the NBER website.
Figure 8: $q_t$ and Uncertainty Index at Daily Frequency Around the Bear Stearns and Lehman Brothers Collapses in 2008. We calculate and plot our daily indices one month before and after the collapses.