Abstract

We develop a model of investor information choices and asset prices in which the availability of information about fundamentals is time-varying. A competitive research sector produces more information when there are more investors willing to pay for that research. This feedback, from the number of investors willing to pay for information to more information production, generates two regimes in equilibrium, one having high prices and low volatility, the other the opposite. Information dynamics move the market between regimes, creating large price drops with no change in fundamentals. When calibrated to market data, the model suggests an important role for information dynamics in financial crises.

1 Introduction

Most research linking investor information acquisition and asset prices assumes a constant information environment. But why should the level of potentially available information remain constant in a market that is perpetually in flux? Changes in technology and regulation can generate persistent shocks to what an investor can learn about company fundamentals; and changes in what can be learned should influence investors as they decide whether to acquire costly information. Pushing this idea a step further, we investigate what happens when the information environment itself changes in response to investor demand for information.
In other words, we posit that the news media, financial intermediaries, company executives, regulators, and prominent investors are not simply passive streams of information: the level of information they provide depends on investor demand. We then find that asset prices can change dramatically in response to changes in the supply and demand for information.

To capture these ideas, we develop a dynamic model of information and asset prices in which the level of available information changes in response to exogenous shocks. In our model, an information shock is exogenous as long as it does not depend on the current state of the economy. Examples of exogenous changes to the information environment include regulatory changes (e.g., the Sarbanes-Oxley Act or Regulation Fair Disclosure), changes in accounting standards, or voluntary disclosure decisions by firms or governments. Such exogenous shocks trigger an endogenous response in the number of investors who choose to become informed. As more investors become informed, more information about fundamentals becomes available. This happens because a competitive information production sector, with a zero marginal cost of transmitting information once it has been discovered, will produce more information when more investors are willing to pay for it. This mechanism magnifies the asymmetry between informed and uninformed investors, it tends to increase price volatility, and it can amplify small information shocks into large price drops.

Before describing the model further, we provide some motivating examples. In 2009, the Greek government revised its estimated budget deficit. This revision triggered a large increase in investor demand for information about Greek debt, as reflected for example in media attention and internet searches. Figure 1 shows the large and persistent increase in the number of Bloomberg articles mentioning Greece starting in 2009, and it provides at least circumstantial evidence that greater demand for information was met with greater supply. More information followed in the form of further revisions to official statistics, revelations about falsified data, stories of investment banks complicit in masking true conditions, research reports by industry analysts and non-governmental organizations, and a downgrade to junk by Standard & Poor’s. The price of Greece’s debt dropped sharply as the volatility of its sovereign credit default swap spreads rose. Consistent with this scenario, our model will describe circumstances in which feedback between the demand and supply of information leads to large price drops and increased volatility, even in the absence of fundamental shocks.

In June 2007, Bear Stearns disclosed that two of its hedge funds were on the brink of failure, fueling investor demand for information about the type of subprime mortgages in which the funds had invested. Indeed, Gorton and Ordoñez (2014) and Dang, Gorton, and Holmström (2012, 2019) have argued that the demand for information about “safe” collateral...
triggered the ensuing crisis. In our narrative, as more investors chose to incur the cost of becoming informed, more information became available — through revised credit ratings, academic and industry research, media scrutiny and regulatory reports — as research producers, including rating agencies, news media, sell-side research shops, began to produce more information as more investors began to demand it. The less informed investors, fearing an informational disadvantage, fled to safer assets. Abstracting from the specifics of this setting, we calibrate our model to stock market data and find that the dynamics of information precision alone, even without negative news about fundamentals, can produce crisis-like effects.

Motivated by these types of examples, our model combines exogenous shocks to the quality of available information, an endogenous response by investors who may choose to become informed at a cost, and feedback from investor information choices to information producers, and ultimately to the amount of information that is produced. Most of our analysis uses a reduced-form representation of the feedback mechanism, but an appendix provides a microfoundation for the mechanism through a competitive information production sector that supplies investor demand for information. Information about fundamentals falls in three categories: publicly known, privately knowable at a cost, and completely unknowable.\footnote{Publicly known information includes a product release that is covered in the New York Times. Privately knowable but costly information includes the performance of a firm’s supplier network, which can be analyzed with painstaking analysis of public information. And information that is unknowable includes the outcome of a future medical trial relative to expectations.} In the interest of clarity, we only treat the case in which the fraction of knowable information varies, while the portion of knowable information that is publicly known is fixed.
In more detail, we develop an overlapping generations (OLG) model with a single risky asset, which pays a dividend each period, and a riskless asset. In each period, a new generation of investors observes the information environment, decides whether to become informed at a cost, sets optimal demands, and trades to clear an exogenous net supply of shares. Market clearing determines the price. At the end of the period, these investors receive their dividend and sell their shares at the new price. The notion of “generations” should not be taken literally in our setting; the OLG framework simply provides a tractable dynamic setting to model changes in information, and it ensures that investors care about future prices as well as the next dividend.

 Crucially, in making their information choices at the start of the period, investors take into account the distribution of exogenous shocks to information precision and the feedback from information choices in the current period to future precision. The future precision will affect the end-of-period asset price and thus investors’ capital gains. Incorporating such time variation in information precision into a rational expectations setting is technically challenging, and this is one of the key contributions of the paper. Using this framework, we show that information shocks can lead to large drops in prices and increases in volatility, and these effects are particularly noteworthy given that our investors have rational expectations. Mamaysky (2020) argues that a portion of the volatility and price drops observed during the COVID-19 crisis are attributable to exactly this dynamic. The large fundamental shock of the coronavirus pandemic also triggered an information shock, which in turn caused asset prices to become hypersensitive to newsflow.

 The interplay between information and asset prices is often studied through single-period models of the type in Grossman and Stiglitz (1980), Hellwig (1980), Admati (1985), and a large subsequent literature. But there are several important features available in a dynamic model that are inaccessible in a single-period model, and these merit discussion. The first two important features we have already highlighted: feedback from investor information choices to information precision, and exogenous shocks to precision. Our emphasis is on feedback effects, and these cannot be captured in a single-period model. Persistent exogenous shocks are also important — they include changes in accounting standards, like the introduction of mark-to-market accounting, changes in regulatory policies on disclosures, or voluntary disclosures by firms of governments. In a single-period model, exogenous shocks are often approximated by changes in model parameters, but such changes are necessarily outside the model and, in particular, not contemplated by the agents in the model. In contrast, our agents’ beliefs take into account that the economy can transition between different
information regimes; such transitions are therefore a feature of the model itself.

A third important feature of a dynamic model is that it can capture two distinct aspects of an increase in available information: greater information reduces uncertainty about the next dividend but can increase volatility in future prices and thus in capital gains. The first of these effects is clear — the information we model is information about dividends. To appreciate the second effect, note that in the absence of dividend information, price volatility is driven entirely by supply volatility; but when some investors have dividend information, this information is partly reflected in the price, so volatility in the signal adds to volatility in the price. In a single-period model, the price merely determines the cost of a claim to an end-of-period dividend. With overlapping generations, investors earn the change in price over the period as well as a dividend, so the variance in this return affects their investment decisions at the beginning of the period. The two information effects, on dividends and on end-of-period prices, are potentially offsetting and lead to more complex tradeoffs than can be captured in a single-period setting. We will see that this dual role of information in dynamic models can lead to starkly different conclusions than those of static models.

To the best of our knowledge, our model is the first to capture a stochastic information environment, endogenous investor information choices, and feedback from these choices to available information. Spiegel (1998) and Dutta and Nezlobin (2017) develop overlapping generations models in which all investors have the same information. Watanabe (2008) extends Spiegel’s (1998) model by introducing asymmetric information. Biais, Bossaerts, and Spatt (2010) also model asymmetric information in an OLG setting. In their model, as in Watanabe’s (2008), the fraction of informed investors and the precision of their signals are fixed and exogenous. Wang (1993) develops a continuous-time model of trading among differentially informed investors with a fixed fraction of informed investors and a fixed information environment; Wang (1994) is a discrete-time version of the model that investigates trading volume. In Avdis (2016), the fraction informed is endogenous but does not affect the information environment. The model of Veldkamp (2006) includes a dynamic information market, but its investors are indifferent to end-of-period prices, leading to starkly different implications than our model. The OLG model of Farboodi and Veldkamp (2017) incorporates a changing information environment, but the change is limited to a deterministic increase in investor information processing capacity over time. Signal precision also changes deterministically over time in Brennan and Cao (1997).

\[\text{Similar tradeoffs arise in the multiperiod models of Avdis (2016) and Dutta and Nezlobin (2017), but those models do not include feedback effects.}\]
Through the feedback from information demand to information precision in our model, information shocks are amplified and can produce crisis-like dynamics or, less dramatically, business cycle fluctuations. Amplification in our setting arises purely through an information channel. This adds to other amplification mechanisms, such as financial frictions (as in Bernanke and Gertler 1989, Kiyotaki and Moore 1997, and Adrian and Shin 2010) or leverage (as in Lorenzoni 2008 and Bianchi 2011). We are not suggesting that other mechanisms are less important, but rather highlighting the role that information feedback alone can play.

Information revelation is at the center of the crisis explanation of Gorton and Ordoñez (2014). In their account, a crisis results when lenders choose to acquire information about borrowers’ collateral; with less information available, borrowers with poor collateral have access to credit, and the increased supply of credit sustains higher growth. We work in an entirely different framework, but one contrast is particularly noteworthy. In Gorton and Ordoñez (2014), the information revealed is bad news; following an aggregate shock, some unobservable amount of collateral becomes bad, thus inducing more information acquisition. In our setting, only the precision of information changes — a shock may bring more news or less news, but not specifically good or bad news. An increase in precision leads to a price drop when it magnifies the information asymmetry between informed and uninformed investors, leading the uninformed to reduce their demand for the risky asset. Of course, a crisis is more likely to be precipitated by bad news, and adding a directional shock would likely further amplify the effects we observe; but our model isolates the role that the dynamics of information precision alone can play, without a negative shock to fundamentals.

To illustrate these effects, we calibrate our model to stock market data. The equilibrium dynamics of the calibrated model fluctuate between two regimes, one with low volatility and high prices, and one with high volatility and low prices. The model can spend long intervals in each regime. A transition from one to the other can be sudden and result in a price drop of 10%, with no change in fundamentals. The two regimes emerge from investor information choices; we do not impose them in setting up the model. Furthermore our investors are fully rational: they understand that the economy can transition from one regime to the other.

This pattern has important implications: in times of market stress, a policy change that makes more information available is potentially destabilizing. We emphasize “policy change” because the information environment in our model is persistent, so the effects we study go beyond a single announcement. Releasing positive information may help calm markets, but our model indicates that this effect must be weighed against the increased volatility that can accompany increased information. There is an extensive literature studying the downsides
of increased disclosures, and these include reducing risk-sharing opportunities, distorting incentives for managers, inducing agents to underweight private information, and crowding out incentives for the production of additional information; see Goldstein and Yang (2017) for a survey. But the impact on prices and volatility we identify in our dynamic model is new in this context. Indeed, working in a single-period setting, Goldstein and Yang (2017) show that greater disclosure decreases return volatility, again highlighting the difference in perspective in a multiperiod model.

We present our model in Section 2. Section 3 solves the model and states our main theoretical results. Section 4 studies changes in the level of knowable information and shows that feedback can lead to two information regimes, using parameters calibrated to market data. Section 5 explores the mechanisms leading to large price changes across regimes and considers information asymmetry, the cost of information production, and strategic complementarity in information acquisition. Appendix A microfounds our feedback mechanism, and subsequent appendices provide proofs of our theoretical results. A Supplementary Appendix provides some additional proofs and details of our calibration and numerical calculations.  

2 Model

2.1 Dividends and Timing

A single infinitely-lived security pays a dividend in each period. The dividend paid at the end of period $t$ is given by

$$D_{t+1} = \bar{D} + \rho(D_t - \bar{D}) + M_{t+1} = (1 - \rho)\bar{D} + \rho D_t + M_{t+1}. \tag{1}$$

The innovation $M_{t+1}$ decomposes as

$$M_{t+1} = m_t + \theta_t + \epsilon_{t+1},$$

with the following interpretation: $m_t$ is known to informed investors; $\theta_t$ is public information; $\tilde{m}_t$ is the knowable portion of the innovation; and $\epsilon_{t+1}$ is unknowable at the beginning of period $t$. These are mean zero, normally distributed random variables, independent across

3Available at [https://sites.google.com/view/hmamaysky](https://sites.google.com/view/hmamaysky)
time\(^4\) with variances given by

\[
\text{var}(\tilde{m}_t) = f_t \text{var}(M) \quad \text{and} \quad \text{var}(\epsilon_{t+1}) = (1 - f_t)\text{var}(M). \tag{2}
\]

and

\[
\text{var}(m_t) = \phi \text{var}(\tilde{m}_t) \quad \text{and} \quad \text{var}(\theta_t) = (1 - \phi)\text{var}(\tilde{m}_t). \tag{3}
\]

Thus\(^5\)

\[
f_t = \text{fraction of dividend innovation that is knowable;}
\]

\[
1 - \phi = \text{fraction of knowable part of dividend innovation that is public.}
\]

The economy contains overlapping generations of agents. The new generation is in the market for two periods, \(t\) and \(t + 1\). Before making investment decisions in period \(t\), all agents observe \(f_t, \theta_t, D_t\), (and \(\phi\)), and the time-\(t\) informed agents observe \(m_t\).\(^6\) A fraction \(\lambda_t \in [0, 1]\) of agents are informed at time \(t\). Becoming informed entails paying a fixed cost \(c_I\); a fixed portion of this cost, \(c_M\), goes to pay a news production sector to discover new information. Under our model parameterization, informed agents find it optimal to pay both \(c_I\) and \(c_M\). The time-\(t\) uninformed agents, representing \(1 - \lambda_t\) of the population, in addition to observing \(\theta_t\) and \(D_t\), also observe the market clearing price \(P_t\). Since the market-clearing price contains information about \(m_t\) through the demands of the informed traders, the uninformed also make rational inferences from the price about the innovation \(m_t\). The price is not fully revealing about \(m_t\) because of the presence of unobservable supply shocks. In this respect, for a given \(f_t\) and \(\phi\), our information environment is the same as in Grossman and Stiglitz (1980). After observing all available (public or private) information, investors set their demands as functions of the price, which determines the price through market clearing. At time \(t + 1\), investors receive the dividend, sell their shares at the time \(t + 1\) price, and the process repeats. Figure\(^2\) summarizes the timing of the model; the agent demands \(q\) and asset supply \(X\) are discussed soon.

\(^4\)More precisely, they are conditionally independent given all \(f_t\).

\(^5\)Boot and Thakor (2001) distinguish three types of information disclosure. In our setting, these correspond roughly to an increase in \(f_t\phi\), an increase in \(f_t\), and a decrease in \(\phi\), respectively.

\(^6\)We have solved the model with time-varying \(\phi_t\) but, for clarity, we assume it is constant in this paper.

Electronic copy available at: https://ssrn.com/abstract=3324789
Figure 2: Sequence of events in each period.

2.2 Information Environment

The innovation of our paper is to allow the information environment, as represented by \( f_t \), to evolve over time in response to exogenous shocks and in response to information decisions made by past generations of investors. We show below that a straightforward microfoundation leads to simple dynamics of information precision: \( f_t \) follows an AR(1) process combined with a feedback effect from today’s fraction informed \( \lambda_t \) to tomorrow’s precision, or

\[
  f_{t+1} = a_f + \kappa_f (f_t - a_f) + b_f \lambda_t + \epsilon_{f,t+1}.  
\]

(4)

for constants \( a_f, \kappa_f \) and \( b_f \), as well as a noise term \( \epsilon_{f,t+1} \). We assume that the information shocks \( \epsilon_{f,t+1} \) and fundamental shocks \( M_{t+1} \) are independent. This allows us to cleanly separate the effect of changing information precision on equilibrium dynamics from the effect of changing fundamentals. To be consistent with the interpretation of \( f_t \) as a measure of signal precision in (2), we need to restrict \( f_t \) to values between 0 and 1. We therefore apply a mapping \( \Pi_D \) to the right side of this equation, where \( \Pi_D \) maps the real line to a set \( D \subseteq [0, 1] \).

We thus arrive at our model of the information environment:

\[
  f_{t+1} = \Pi_D (a_f + b_f \lambda_t + \kappa_f (f_t - a_f) + \epsilon_{f,t+1}) .  
\]

(5)

\footnote{In the simplest case, \( \Pi_D(x) = \min(1, \max(0, x)) \) projects \( x \) to \([0, 1]\). For some of our theoretical results in Section 3 and for our numerical results, we will discretize \( f_t \) to finite subsets of the unit interval, but for now we keep the discussion general.}
This specification provides the simplest model that captures persistent, stochastic time variation in the information environment and, most importantly, feedback from the fraction informed $\lambda_t$ to the available information.

To generate the $f_t$ dynamics in (4) and (5), we assume that the dividend innovation $M_{t+1}$ consists of a large number of iid pieces of information. This information can be about local economic conditions that affect a firm’s profitability or the economy’s output, technological innovation across different product lines, consumer demand, managerial talent, competitor performance, relevant industry and macro trends, and so on. Each unit of information can be in one of two states: observable or unobservable. The state of being observable or unobservable is persistent. For example, informed investors may push a company or government to disclose a certain piece of information, and once the company or government agrees, it is likely to continue to disclose this information, thus making it observable. However, at some point the disclosure policy may change, and previously disclosed information may become undisclosed, and thus unobservable. Observability does not depend only on disclosure. For example, technological innovation may make certain characteristics of an oil well observable, even if they were unobservable in the past. Similarly, a company may build a canopy over its distribution facility rendering satellite imagery no longer informative. In both of these examples, the change in observability is persistent. We assume any observable unit of information has a $\phi$ probability of being only privately observable and a $1 - \phi$ probability of being publicly observable.

A profit maximizing, competitive information production sector can discover, at a per unit cost $c_P$, previously unobservable units of information, and then reveal these to its clients. Once a unit of information is discovered, the marginal cost of revealing it to investors is zero. Furthermore, we assume that discovered units become observable. We refer to this sector as the news producers, though in addition to financial journalists, it can also contain stand-alone, sell-side, or buy-side research firms, ratings agencies, or bloggers on outlets like Seeking Alpha who are compensated for the number of views their posts receive. In our model calibration, informed investors optimally choose to spend a portion $c_M$ of their cost of becoming informed $c_I$ to purchase the information produced by the news sector. Investors who purchase information from the news producers are legally obligated not to share information they obtain from news outlets with one another; thus the only way to obtain information is to buy it from the producers. The zero marginal cost condition and the legal obligation not to disclose purchased information mirror the assumptions of Veldkamp (2006), and provide a strong incentive to produce more information when more investors
demand it.

As we show in Appendix A, these assumptions yield the \( f_t \) process in (4). The AR coefficient \( \kappa_f \), which determines the degree of persistence of the information state, equals one minus the sum of two probabilities: (1) the probability that a given unit of information transitions from unobservable to observable, and (2) the probability that a unit transitions from observable to unobservable. The coefficient \( a_f \), which would be the steady-state level of information precision in the absence of the feedback effect, \( \lambda_t \), equals the probability of transitioning from unobservable to observable, conditional on a transition taking place. Finally, we show in the appendix that \( b_f = c_M/c_P \), and therefore the feedback effect \( b_f \lambda_t \) reflects the incentive of news producers to discover more information: \( b_f \) is positive and increasing in the amount spent by investors to buy news, \( c_M \); it is decreasing in the cost of producing a new unit of information, \( c_P \); and the overall effect is increasing in the number of informed \( \lambda_t \).

2.3 Investor Optimization Problem

At the beginning of period \( t \), a unit mass of new (young) investors enter the market, each endowed with wealth \( W_t \), known at time \( t \). For an investor who buys \( q \) shares of the risky asset at price \( P_t \) at the beginning of the period and sells the shares at the end of the period at price \( P_{t+1} \), terminal wealth is given by

\[
W_{t+1} = R(W_t - qP_t) + q(D_{t+1} + P_{t+1}) = RW_t + q(D_{t+1} + P_{t+1} - RP_t),
\]

where \( R > 1 \) is the gross return on a riskless asset. It will be convenient to define the per period net profit from owning a single share of the stock as

\[
\pi_{t+1} \equiv D_{t+1} + P_{t+1} - RP_t,
\]

in which case the budget constraint becomes \( W_{t+1} = RW_t + q\pi_{t+1} \). Agents who enter at time \( t \) consume their wealth at \( t + 1 \) and leave the market. These agents set their demands for shares of the risky asset at time \( t \) by solving

\[
J_t^q \equiv \max_{q} \mathbb{E} \left[ \mathbb{E}[W_{t+1} | T_t, f_{t+1}] - \frac{\gamma}{2} \text{var}(W_{t+1} | T_t, f_{t+1}) \bigg| T_t \right], \quad t \in \{I, U\}, \tag{8}
\]
where $\mathcal{I}^U_t = \{ f_t, \lambda_t, D_t, \theta_t, P_t, W_t \}$ is the uninformed agents’ information set at time $t$, $\mathcal{I}^I_t = \mathcal{I}^U_t \cup \{ m_t \}$ is the informed agents’ information set, and $\gamma > 0$ is a risk aversion parameter. Similar objectives are used in Peress (2010), Van Nieuwerburgh and Veldkamp (2014), and Mondria (2010), and can be interpreted as expressing a preference for early resolution of uncertainty, in the sense of Kreps and Porteus (1978). Maximizing (8) is equivalent to maximizing

$$
\mathbb{E} \left[ v \left\{ \mathbb{E} \left[ -\exp(-\gamma W_{t+1}) \mid \mathcal{I}^I_t, f_{t+1} \right] \right\} \mid \mathcal{I}^I_t \right],
$$

with $v(u) = -\frac{1}{\gamma} \log(-u)$, if $W_{t+1}$ is conditionally normal, as it will be in our equilibrium. We could allow investors to condition on past values of variables in their information sets in (8), but past information will be irrelevant, given our independence assumptions. We will use the notation $\mathbb{E}[-\cdot] \equiv \mathbb{E}[-\cdot \mid \mathcal{I}^I_t]$ to denote conditioning on the time $t$ common information set.

In addition to investor demands for shares of the risky asset, we need to specify the supply. As in the OLG model of Allen, Morris, and Shin (2006), we assume that $X_t$, the stochastic part of the supply of the risky asset, is independent and identically distributed from one period to the next. As explained in Allen et al. (2006), i.i.d. supply can be interpreted as the result of trading by price-insensitive noise traders who reverse their trades at the end of each period. New investors each period thus only clear a new exogenous supply shock.

We assume each $X_t$ is normally distributed with mean zero and variance $\sigma^2_X$. Furthermore, we assume that there exists a positive net supply $\bar{X}$ of the risky asset, and that this fixed supply is constant over time.

### 2.4 Equilibrium

Given a function $\lambda : [0, 1] \mapsto [0, 1]$, yielding the fraction informed $\lambda(f_t)$, a *market equilibrium* is defined by a price process $P_t$ and demands $q^I_t$ and $q^U_t$, depending on the price and other time-$t$ information $\mathcal{I}^I_t$ and $\mathcal{I}^U_t$, that clear the market,

$$
\lambda_t q^I_t + (1 - \lambda_t) q^U_t = \bar{X} + X_t,
$$

and for which $q^\iota_t$ solve (8), $\iota \in \{I, U\}$, for all $t$.

Market clearing and investor optimality define a market equilibrium, given a function $\lambda$.

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8Our model extends easily to allow persistent supply shocks, at the expense of adding an additional state variable, which complicates our numerical examples. See Avdis (2016) for a model in which supply persistence influences investors’ decisions to become informed.
that determines the fraction of investors who are informed. Next we define what it means for this fraction to be determined endogenously. As in our discussion of Figure 2, we suppose that investors at the beginning of the period can choose to become informed at a cost $c_I$, incurred at the beginning of the period but after observing the current information state $f_t$. Investors’ decisions to become informed or remain uninformed thus define a mapping from the information state to the fraction informed, which is precisely $\lambda$. We will use the following:

**Definition 2.1 (Endogenous fraction informed)** Given the $f_t$ dynamics in (5), we call $\lambda$ the endogenous fraction informed if it satisfies the following conditions for each $f \in [0,1]$:

(i) $\lambda(f) = 0$ and $E[J_{t-1} - R_{c_I}| f_t = f] < E[J_{t-1}^U| f_t = f]$; or

(ii) $0 \leq \lambda(f) \leq 1$ and $E[J_{t-1} - R_{c_I}| f_t = f] = E[J_{t-1}^U| f_t = f]$; or

(iii) $\lambda(f) = 1$ and $E[J_{t-1} - R_{c_I}| f_t = f] > E[J_{t-1}^U| f_t = f]$.

In case (ii), the fraction $\lambda(f)$ is the point at which the marginal investor is indifferent between becoming informed and remaining uninformed. Cases (i) and (iii) cover the possibility that one choice dominates the other and is therefore selected by all investors.

### 3 Model Solution and Variance Beliefs

The main challenge in our model is to combine a time varying information environment with agents’ rational expectations. In this section, we state the main results on the solution of the model. We show that for arbitrary fixed $\lambda(\cdot)$ and what we call variance beliefs, the model admits a market equilibrium. Then, again for fixed $\lambda(\cdot)$, we give conditions for the existence of self-consistent variance beliefs and thus a rational expectations market equilibrium. Given a market equilibrium, we give conditions for an endogenous fraction informed $\lambda(\cdot)$. With this endogenous $\lambda(\cdot)$, it is possible that the initial variance beliefs are not self-consistent. Therefore, we combine the results to give conditions for an information equilibrium, in which conditions for a rational expectations market equilibrium and an endogenous fraction informed are jointly satisfied.

For some of the results in this section (Propositions 3.2–3.4), and for numerical calculations, we discretize the state space by restricting $D$ — the set of values that $f_t$ can take — to be a finite subset of $[0,1]$. This discretization allows us to represent any function of
as an \(n\)-dimensional vector. Our numerical procedure is discussed in the Supplementary Appendix.

### 3.1 Market Equilibrium

Proceeding with the first of these statements, we will show that, for any choice of \(\lambda\), the model admits a market equilibrium in which the price process takes the form

\[ P_t = a_t + b_t m_t + g \theta_t - c_t X_t + d D_t, \tag{10} \]

where \(g\) and \(d\) are constants, and \(a_t, b_t, c_t\) are functions of the information state \(f_t\) but do not otherwise depend on \(t\).

To characterize investor demands, we will initially solve a more general version of their optimization problem (\ref{eq:opt8}), in which we do not assume that investors know the coefficients of the price process (the meaning of this will be clear momentarily). We then show how this leads to a market equilibrium.

If prices are given by (10), we can write terminal wealth \(W_{t+1}\) in (6) as

\[ W_{t+1} = RW_t + q(1 + d)D_{t+1} + q(P_{t+1} - d D_{t+1} - R P_t) \]

\[ = RW_t + q[(1 + d)D_{t+1} + a_{t+1} + b_{t+1} m_{t+1} + g \theta_{t+1} - c_{t+1} X_{t+1} - R P_t]. \tag{11} \]

(12)

Note that \(m_{t+1}, \theta_{t+1}\) and \(X_{t+1}\) are independent of \(D_{t+1}\), and of any time \(t\) information. With a view to solving (\ref{eq:opt8}), we evaluate the conditional mean of terminal wealth as

\[ \mathbb{E}[W_{t+1} | I, f_{t+1}] = q(1 + d)(\mu_D + \rho D_t + \theta_t + \mathbb{E}[m_t | I_t]) + a(f_{t+1}) - R P_t \]

\[ + RW_t, \quad t \in \{I, U\}. \tag{13} \]

In the above, we write \(a_t\) from (10) as \(a(f)\) to make explicit its dependence on the state variable, and discuss it further below. For the conditional variance, we use (11)–(12) to write

\[ \text{var}(W_{t+1} | I_t, f_{t+1}) = q^2(1 + d)^2 \text{var}[D_{t+1} | I_t, f_{t+1}] + q^2 \text{var}[P_{t+1} - d D_{t+1} | I_t, f_{t+1}] \]

\[ = q^2(1 + d)^2 [\text{var}(m_t | I_t) + (1 - f_t) \sigma_M^2] + q^2 V_B(f_{t+1}). \tag{14} \]

In the second equality, we have used (1)–(2) and introduced the variance belief function \(V_B\). If prices are indeed given by (10), then the “correct” (rational expectations) belief is given
by
\[ V_B(f) = b(f)^2 \phi \sigma_f^2 M + g^2(1 - \phi)f \sigma_f^2 M + c(f)^2 \sigma_X^2 \quad \forall f, \quad (16) \]
as can be seen by comparing the last term in (14) and (15). However, we initially allow investors to have an arbitrary, strictly positive variance belief \(V_B\), which is shared by all investors.

With arbitrary \(V_B\), we do not have equality in (15); instead, we posit that investors solve their optimization problems (8) as though (15) held. In other words, investors solve (8) but with the conditional variance replaced by the right side of (15). Furthermore, we show in Appendix B.2 that given \(V_B\), \(a(f)\) in (13) is fully determined by the investor’s optimization problem.

A market equilibrium with variance belief \(V_B\) is then a price process and investor demand functions that clear the market and solve (8) with this modification.

**Proposition 3.1**  Under the \(f_t\) model (5), for any variance belief function \(V_B(\cdot)\) bounded above and bounded away from zero, and any \(\lambda(\cdot)\), there exists a market equilibrium with a price process of the form (10) in which \(a_t, b_t, \) and \(c_t\) are functions of \(f_t\) and do not otherwise depend on \(t\), and the constants \(d\) and \(g\) are given by \(d = \rho/(R - \rho)\) and \(g = 1/(R - \rho)\).

The proof of this proposition is in Appendix B.1. The market equilibrium of Proposition 3.1 is not in general a rational expectations equilibrium because the variance belief \(V_B\) may not coincide with the true conditional variance \(\text{var}[P_{t+1} - dD_{t+1}|I_t, f_{t+1}]\) in (14). But we can think of agents in the model as learning over time. Starting from an initial belief, investors set their demands and clear the market at a price of the form in (10). They (or the next generation) then observe the realized variance given by the right side of (16). They update their beliefs by setting \(V_B\) equal to this realized variance, and the process repeats. This in fact is how we solve our model numerically.

Given price coefficient functions \(b\) and \(c\), the belief updating equation (16) defines a new \(V_B\), and given \(V_B\), Proposition 3.1 defines new coefficients \(b\) and \(c\). (The coefficient \(a\) depends on \(V_B\) but does not enter in the update of \(V_B\).) Combining the two steps yields a mapping from an initial pair of functions \((b, c)\) to an updated pair \((b, c)\). We have self-consistent beliefs, i.e., a rational expectations market equilibrium, at a fixed point of this mapping.

We prove the existence of a fixed point in the appendix under mild restrictions on model parameters. For technical reasons, in this analysis we limit the values of \(f_t\) to a finite (but arbitrarily large) subset \(D\) of the unit interval. Appendix B.3 gives our most general
parameter restrictions. For simplicity, here we state a special case that is easy to verify and holds in our numerical examples.

**Proposition 3.2** Suppose that $R \in [1, 1.2]$ and the model parameters satisfy

$$(1 + d)\gamma \sigma_M \sigma_X = \frac{R \gamma \sigma_M \sigma_X}{R - \rho} \leq 0.28.$$  

Then for any fixed $\lambda(\cdot)$, there exists a fixed point of the variance belief updating mapping. This fixed point defines a self-consistent variance belief and thus a rational expectations market equilibrium with prices of the form in (10).

The point of condition (17) is that we need $(1 + d)\gamma \sigma_M \sigma_X$ to be small (see in particular footnote 9), and with the mild bounds on $R$ we can show that 0.28 is small enough. This condition translates to upper bounds on market volatilities $\sigma_M$ and $\sigma_X$, dividend persistence $\rho$, and risk aversion $\gamma$. See Appendix B.3 for more general conditions and the proof of the result.

Two special cases of Proposition 3.2 are worth mentioning. If we fix $f_t \equiv 0$ and $\lambda \equiv 0$ we have an OLG model without asymmetric information, similar to the one in Spiegel (1998). As in Spiegel’s (1998) model, the coefficients in the price function can be expressed through solutions of quadratic equations. If we fix $f_t$ and $\lambda$ at constant strictly positive values, we get a model similar to Watanabe’s (2008), which has asymmetric information but a fixed information environment and no feedback.

### 3.2 Information Equilibrium

Propositions 3.1 and 3.2 take $\lambda(\cdot)$ as exogenous. We need to show that our notion of the endogenous fraction informed in Definition 2.1 is meaningful. Given a variance belief $V_B$, for every $f$, we need to find a $\lambda$ that makes investors exactly indifferent between paying the cost

9In this case, the equation for a self-consistent variance belief reduces to solving a quadratic equation with two real roots, which is given by:

$$V_B^2 + \left[2(1 + d)^2 \sigma_M^2 - \frac{R^2}{\gamma^2 \sigma_X^2}\right] V_B + (1 + d)^4 \sigma_M^4 = 0.$$  

The two roots describe two market equilibria, one with high price variance and one with low price variance, and we need an upper bound on the left side of (17) to ensure that both roots are positive. However, the high variance equilibrium is unstable under arbitrarily small parameter perturbations; only the low variance equilibrium is robust to such changes. In our numerical experiments, we find that if we start from a low value of $V_B(\cdot)$ we converge to the low variance equilibrium.
cation $c_I$ of becoming informed or staying uninformed; if no such $\lambda$ exists, we set $\lambda$ equal to zero or one according to Definition 2.1. The details of this calculation are given in Appendix C.1.

The following proposition shows that this procedure does indeed generate an endogenous fraction informed. Because changing $\lambda$ changes the evolution of $f_t$, we need to be a bit more explicit about how we map these variables to the finite set $D$ in (5), particularly when the shocks $\epsilon_{f,t}$ have a continuous distribution; these details are discussed in the proof of the following proposition in Appendix C.2.

**Proposition 3.3** Suppose the shocks $\epsilon_{f,t}$ have a density. Then for any strictly positive variance belief there exists an endogenous $\lambda$ in the sense of Definition 2.1.

This result holds, in particular, at a self-consistent variance belief. That is, given an exogenous $\lambda(\cdot)$ and the associated self-consistent variance belief $V_B$, we can solve for a new endogenous $\lambda(\cdot)$. However, once we change $\lambda$, the variance belief may no longer be self-consistent. When we solve the model with endogenous beliefs numerically, we start with an arbitrary variance belief, we then solve for the endogenous fraction informed (as provided by Proposition 3.3), we then calculate the realized variance (16) using the endogenous $\lambda$, update the variance belief and repeat. We can formulate this process as starting with a pair of coefficient functions $(b,c)$, from which we calculate $\lambda$ and then a new $(b,c)$. Combining the two steps yields a mapping from an initial $(b,c,\lambda)$ to a new $(b,c,\lambda)$. A fixed-point of this mapping defines an information equilibrium, in the sense that it yields a rational expectations market equilibrium in which investors do not want to deviate from their information choices.

**Existence of an Information Equilibrium**

In this section, which is technical and can be skipped on a first reading, we establish that the procedure discussed in the prior paragraph has a fixed point. Proposition 3.3 ensures the existence of an endogenous $\lambda$, but it does not guarantee uniqueness, so we need a somewhat more general formulation to establish existence of an information equilibrium. For any coefficient functions $(b,c)$, let $\Lambda_o(b,c)$ denote the set of $\lambda$ satisfying Definition 2.1 which we know from Proposition 3.3 is nonempty. Let $\Lambda(b,c)$ denote the set of all convex combinations of elements of $\Lambda_o(b,c)$. If there is just one $\lambda$ in $\Lambda_o(b,c)$, then $\Lambda(b,c) = \Lambda_o(b,c) = \{\lambda\}$. In our numerical experiments, instances of multiple $\lambda$ satisfying Definition 2.1 occur rarely, and we have never encountered multiple solutions $\lambda$ when using self-consistent variance

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10 More precisely, we start with a variance belief within the region where Proposition 3.2 ensures the existence of a fixed point.
beliefs. However, because we have not proved the uniqueness of $\lambda$, we need to work with the potentially larger set $\Lambda(b, c)$ in establishing the existence of an information equilibrium $(b, c, \lambda)$.

A convex combination $\lambda \in \Lambda(b, c)$ represents a mixed equilibrium in the following heuristic sense. For each $f \in D$, we can write

$$\lambda(f) = w_f \lambda_1(f) + (1 - w_f) \lambda_2(f),$$

with $w_f \in [0, 1]$ and $\lambda_1, \lambda_2 \in \Lambda_0(b, c)$, $\lambda_2(f) > \lambda_1(f)$. Interpret this to mean that a fraction $w_f$ of investors thought equilibrium $\lambda_1(f)$ would be selected, and a fraction $1 - w_f$ thought $\lambda_2(f)$ would be selected. At the outcome $\lambda(f)$, the marginal investor is not indifferent between becoming informed or not. A fraction $w_f$ of investors, expecting an outcome of $\lambda_1(f)$, will see $\lambda(f)$ as too high, and in response a fraction $w_f(\lambda(f) - \lambda_1(f))$ of investors will switch from informed to uninformed. Similarly, a fraction $1 - w_f$, expecting $\lambda_2(f)$, will see $\lambda(f)$ as too low, resulting in a fraction $(1 - w_f)(\lambda_2(f) - \lambda(f))$ switching from uninformed to informed. But then (18) implies that these effects offset each other, leaving the fraction informed at $\lambda(f)$.

We establish existence of an information equilibrium — a joint equilibrium in $(b, c, \lambda)$ — within the broader class of information choices in $\Lambda(b, c)$. For the following, let $M(\lambda)$ be the set of market equilibrium parameters $(b, c)$ consistent with the fraction informed function $\lambda = \{\lambda(f), f \in D\}$; these are the fixed points in Proposition 3.2. The following proposition is proved in the Supplementary Appendix.

**Proposition 3.4** Suppose the conditions of Propositions 3.2 and 3.3 hold. Then there exists an information equilibrium $(b, c, \lambda)$, meaning that $(b, c) \in M(\lambda)$ and $\lambda \in \Lambda(b, c)$. In other words, $(b, c)$ defines a market equilibrium given $\lambda$, and $\lambda$ defines a (possibly mixed) endogenous fraction informed given $(b, c)$.

4 Price and Volatility Cycles

As discussed in the introduction, our model is motivated by the idea that as more investors become informed, more information may become available. This type of feedback can arise at the onset of market stress in response to heightened investor attention. In this section, we will show that this dynamic can lead to periods of low and high volatility and high and low prices driven purely by changes in the information state, with no change in fundamentals. In other
words, we can generate transitions similar to business cycles or even financial crises through changes in the level of information, without necessarily the release of negative information.

4.1 Dynamics of Information Precision

To provide insight into the model, we turn to a numerical example. We postpone details on parameter values to our discussion of model calibration in Section 4.2. The solid line in Figure 3 shows $\lambda$ as a function of $f$ in model (5). We calculate this curve by starting from a flat variance belief function and iteratively updating the variance belief and $\lambda$ as discussed in Section 3. This iterative process converges very quickly in our numerical experiments.

At low levels of information precision $f$, the figure shows a flat section where $\lambda(f) = 0$; with little information available, no investor chooses to bear the cost of becoming informed. Once $f$ increases to just above 0.4, we have a positive fraction of investors informed, and this fraction generally increases with the precision $f$\footnote{For some parameter values, at $f$ near 1 we have a small decline in $\lambda(f)$. The possibility of a decline in $\lambda(f)$ as $f$ increases reflects the dual roles of information in a multiperiod model. Becoming informed benefits an investor by reducing uncertainty about the end-of-period dividend. However, as more investors become informed, the variance of the end-of-period asset price increases, so the net effect on the variance of an investor’s end-of-period wealth is indeterminate.}

Figure 3: The solid line shows the fraction informed $\lambda(f_t)$ in information state $f_t$, and the dashed line shows the mapping from $\lambda$ to $f_{t+1}$ without exogenous shocks. Each circle shows a point where $f_t = f_{t+1}$ when the shocks in (5) are zero, labeled with its $(f, \lambda)$ value. The figure uses parameter values from Table 1.
To interpret the dashed line in Figure 3, we shut off the exogenous shocks in the evolution of $f_t$ by setting $\epsilon_{f,t+1} \equiv 0$ in (5). The dashed line then shows the mapping from $\lambda$ to the next value of $f$. That is, starting from any $f_t = f$ on the horizontal axis, reading up to the solid line then across to the dashed line and back down to the horizontal axis yields $f_{t+1}$. Points where the two lines cross are fixed-point combinations of $(f, \lambda(f))$ in a model without exogenous shocks. In other words, the three circled points in the figure are cases where $f_t = f_{t+1}$ when $\epsilon_{f,t+1} = 0$.

Consider, for example, the circled point near $f = 0.48$, $\lambda(f) = 0.071$. Starting at that $f$, the endogenous fraction informed $\lambda(f)$ is precisely the value that keeps the information state at $f$ under the evolution in (5) without endogenous shocks. The model still has feedback from $\lambda$ to $f$ (and $f$ to $\lambda$), but $f_t$ remains fixed. The same argument applies to the intersection near $f = 0.88$. In the lower left, the curves intersect throughout an interval where $\lambda(f) = 0$, and we have a fixed point at $(a_f,0)$ because the dynamics in (5) drive $f_t$ to $a_f$ when $\lambda_t = \epsilon_{f,t+1} = 0$.

But this perspective is somewhat misleading in a way that illustrates a difference between a genuinely dynamic model and a static one — a difference that will be important to the implications of the model. Without exogenous shocks, the three fixed points in the figure would seem to be equally valid solutions. But the middle fixed point, near $f = 0.48$, is unstable: starting just to the right of the intersection will drive $f_t$ to the fixed point near $f = 0.88$, whereas starting just to the left will drive $f_t$ to the interval where $\lambda(f) = 0$. The middle fixed point is in some sense illusory, though it is a valid fixed point without shocks.

If we reintroduce shocks in the evolution (5) and study the long-run distribution of $f_t$ using the endogenous $\lambda$ curve in the figure, we find that $f_t$ spends significant time near $f = 0.175$, and it spends significant time near $f = 0.88$, but the region near $f = 0.48$ holds no particular attraction for the dynamic model with exogenous shocks. Figure 4 shows the steady-state $f_t$ distribution (indicated by the blue circles in the left panel of the figure), calculated using a Markov chain representation. The distribution is bimodal, showing that the economy spends the majority of its time in the vicinity of the two stable fixed points from Figure 3 but not near the middle, unstable fixed point in Figure 3.

The red triangles in the left panel of Figure 4 show the steady-state distribution of $f_t$ when $\lambda$ is held fixed at its equilibrium mean of 0.0731. The distribution is unimodal, indicating that $f_t$ now spends most of its time near the middle of the interval (which happens to be in the vicinity of the unstable fixed point in Figure 3). This contrast points out the crucial role

\footnote{See the Supplementary Appendix for details.}
played by the endogeneity of $\lambda$ and the feedback through $b_f \lambda$ (in equation 5) in generating the two regimes, leading to price and volatility cycles.

4.2 Model Calibration

In calibrating the model to the aggregate market, we take one period in the model to represent one month. We estimate a monthly dividend process of the form (1) using daily dividend data for the S&P 500 index from 1998–2018, then aggregating this up to the quarterly level (to mitigate seasonality effects), and estimating an ARMA(1,1) process for the quarterly dividend. From this we back out the monthly parameters $\rho = .967$ and $\sigma_M = 0.0471$. See the Supplementary Appendix for details.

We adopt the normalization $\bar{D} = 1$ and $\bar{X} = 1$, so dividends and share supplies are measured in units of their monthly averages. We calibrate $\sigma_X^2$ to match monthly turnover, meaning the number of shares traded per month divided by the shares outstanding. Recall from Section 2.3 that in each period $t$, investors buy the new supply $X_t$ originating from liquidity demanders, and investors from the previous period sell back $X_{t-1}$ shares to the previous period’s liquidity demanders unwinding their trades. The total trading volume in period $t$ is therefore $|X_t| + |X_{t-1}|$. Using the normality of the supply shocks, the expected
volume per period becomes

\[ E[|X_t| + |X_{t-1}|] = 2\sigma_X \sqrt{\frac{2}{\pi}} \approx 1.596\sigma_X. \] (19)

In the Supplementary Appendix, we find that the average weekly turnover of the Dow Industrials index is 0.065\(^{13}\). To model a period of stress, we assume that turnover, or the turnover expectation of market participants, is four times higher than normal, so \( \sigma_X = 4 \times 0.065 \times 1/2 \times \sqrt{\pi/2} = 0.1629 \).\(^{14}\)

We use a monthly gross risk-free return of \( R = 1.0015 \) and set the risk-aversion parameter at \( \gamma = 0.46 \). This yields an annualized excess return of roughly 15%, which is not unreasonable for periods of stress. We choose a per month cost of being informed of \( c_I = 0.2627 \), which should be compared with a monthly aggregate average dividend of 1 (since \( \bar{X} = \bar{D} = 1 \)).

This high cost of information is comparable to the 2/20 fee structure of many hedge funds, and leads to an equilibrium number of informed of under 18% of the overall population.

For the dynamics of the \( f_t \) process in (5), we set \( a_f = 0.175 \), \( \kappa_f = 0.91 \) and \( b_f = 0.384 \). In the context of our information production microfoundation (see Section A), these choices of \( a_f \) and \( \kappa_f \) imply the probability of a unit of information transitioning from observable to unobservable is roughly five times larger than the probability of transitioning from unobservable to observable. A value of \( \kappa_f \) close to 1 makes the information state persistent by making both of the above probabilities small, and a positive \( b_f \) produces positive feedback from the fraction informed to the level of accessible information. When \( f_t \) is low, \( \lambda(f_t) = 0 \), and \( f_t \) is pulled toward a steady state level of \( a_f = 0.175 \).\(^{15}\) We fix \( \phi = 0.35 \), implying that 65% of the knowable information is publicly known. This introduces a high degree of information asymmetry between informed and uninformed investors, and leads to a large drop in price in the high-information regime; see the discussion in Section 5.2.

Beyond these qualitative considerations, these specific parameters were chosen to produce plausible model dynamics. Finally, for the shocks \( \epsilon_{f,t+1} \), we use a three-point distribution

\(^{13}\)Lo and Wang (2000, Table 3) show that from 1987-1996, weekly turnover on a value-weighted index of NYSE and AMEX common shares was 1.25%. Therefore the monthly turnover on this index was 52/12 \( \times 1.25\% = 5.42\% \).

\(^{14}\)The trading volume of SPY, which tracks the S&P 500 index and is one of the most liquid exchange-traded funds, has spiked by a factor of four during stress periods. For example, in 1/18/2011 the trailing month’s average daily trading volume was 106.2 million and in 8/26/2011 the trailing month’s daily trading volume was 407.0 million.

\(^{15}\)Fama and French (2000) show the \( R^2 \)'s of year-ahead firm-level earnings forecasts to be between 5% and 20%.
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Table 1: Calibrated parameters for model (5)

taking values $\{-0.135, 0, 0.135\}$ with probabilities $\{0.03, 0.94, 0.03\}$, so shocks are rare\(^{16}\)

Model parameters are summarized in Table 1. Our results are robust to changes in model parameters. For a wide range of values in our non-dividend parameters (since $\rho$ and $\sigma_M$ are estimated from actual data) in Table 1, there exists a $\phi$ close to its base value of 0.35 which generates the bimodal $f_t$ distribution and the large price drops that we discuss below. In fact, in many cases the resultant price drops are larger than the one that occurs under our base parameterization\(^{17}\)

### 4.3 Price Drops and Volatility Spikes

Figure 5 shows model quantities calculated using the parameters in Table 1. The first three panels show the price coefficient functions $a$, $b$, and $c$ from (10). The lower right-hand panel shows the expected net profit from owning one share of the stock. When no investor is informed, no dividend information is reflected in the price, and $b = 0$. As $f$ increases to the point where some investors become informed, $b$ and $c$ both increase, which drive up the price variance\(^{18}\) The increase in $c$ reflects a higher compensation for accommodating supply shocks and is attributable to higher price variance and a growing informational disadvantage of the uninformed relative to the informed. Furthermore, as informed enter the market due to a higher $f$, $a(f)$ falls sharply, a phenomenon we will study in detail.

The left panel of Figure 6 shows the expected stock price $P_0 \equiv a(f) + d\bar{D}$. The price response is dramatic: a small increase in $f$ leads to a price drop of 10%. This price drop results from an increase in information precision and an endogenous response of the number of informed investors. In stress times, more information production (and, as we will see

\(^{16}\)In Proposition 3.3, we assumed the shocks have a density to ensure continuity of the price coefficients in $\lambda$. A discrete distribution is simpler to work with numerically, and we achieve continuity by interpolating when we take expectations over state transitions of $f$, as discussed in the Supplementary Appendix. The interpolation has a similar effect as smoothing the shock distribution; moreover, a discrete distribution can be approximated arbitrarily closely by a density.

\(^{17}\)This analysis is available from the authors.

\(^{18}\)As $b$ measures the sensitivity of the price to dividend information, the monotonicity of $b$ parallels an empirical finding in Brancati and Macchiavelli (2019) that prices become more information-sensitive when information precision increases.
in Section 5.2 greater information asymmetry) can destabilize the market. We will see in Section 4.4 that the conditional variances of net profit $\pi$ in the right panel of the figure help explain the price drop.

Figure 7 shows an example of a simulated path of the model for 1,000 periods, or 83 years if each period is one month. The figure plots the evolution of the price $P_t$ and variance $V_B$, respectively. In this example, the market starts in the low volatility and high price regime, transitions to the high volatility and low price regime, and then transitions back. The figure shows a large drop in price associated with the spike in variance, resulting from an increase in $f_t$. This price drop is much larger than the within-regime price volatility, which is driven primarily by dividend fluctuations.

It is customary to associate large declines in market values with the arrival of bad news. Following a 10% decline (the price drop in Figure 6) in an individual stock price or the overall market, one would expect media and expert accounts of what bit of bad news — a

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19The variance $V_B$ in (16) increases monotonically in $f$, so the two variance regimes in Figure 7 correspond to two $f$ regimes.
product failure, a CEO scandal, a change in government policy — triggered the fall. But in our setting it is simply more news — in the form of increased precision $f_t$ — that drives investors, not necessarily good or bad news.

In the model of Gorton and Ordoñez (2014), the onset of a crisis is defined by the release of information about collateral quality. But the additional information in Gorton and Ordoñez (2014) is negative information: collateral quality is inferior to what was previously believed. This revelation leads to either a collapse in lending or a diversion of productive resources to information acquisition, thus reducing growth in both cases. The mechanism in our model is entirely different and does not rely on adverse information about fundamentals.

In practice, an increase in the quantity of information without an accompanying positive or negative implication for fundamentals is rare, and this makes it difficult to disentangle a change in precision from a directional effect of news. However, our model serves to isolate the information component; our calibration indicates that this component alone can have a material effect. When the additional information is bad news as well as more precise news, we can expect the effects to be even greater.

The potential for increased volatility from increased information has policy implications. A regulatory change that leads to persistently higher information precision for informed investors is potentially destabilizing in times of market stress. Interestingly, in their analysis

20 Their model, based on a very different framework, also exhibits an equilibrium with information cycles. In their setting, low information booms alternate with high information crashes.

21 In a macro context, Cesa-Bianchi and Fernandez-Corugedo (2018) find that an increase in economic uncertainty results in a decrease in the risk premium, which is consistent with our results.

22 The information disclosed about regulatory stress tests is disclosed publicly, but the design of scenarios
of disclosure of the results of regulatory stress tests for banks, Goldstein and Leitner (2018) conclude that disclosure is valuable only under adverse conditions. Our results do not conflict but rather reflect different considerations, as the objective in Goldstein and Leitner (2018) is optimal risk sharing among banks, and the information disclosed separates weak and strong banks.

The rest of this section explains the key features exhibited by our model, particularly the large price drop illustrated in Figure 6 and the two regimes illustrated in Figure 7.

### 4.4 Decomposing Price and Volatility

The price drop in Figure 6 is driven by the drop in the \( a() \) curve in Figure 5. In Appendix B.2 we show that \( a_t \) can be decomposed into two components,

\[
a_t = \frac{(1 + d)\mu_D}{R - 1} - \sum_{i=1}^{\infty} \frac{1}{R^i} E_t[\pi_{t+i}],
\]

where, \( \pi_{t+i} \) is the net profit from holding one share of the stock from \( t + i - 1 \) to \( t + i \), as in (7), and its conditional expectation is taken given \( f_t \). From this expression, we see that the \( a_t + dD_t \) component of \( P_t \) is the present value of all future expected dividend payments minus a discount reflecting the expected present value of all future net profits.\(^{23}\)

In the and the interpretation of the results are technical matters that are arguably accessible only to informed investors who have acquired the necessary expertise.

\(^{23}\)If the second term in (20) is zero, we get \( a_t + dD_t = \sum_{i=1}^{\infty} \frac{1}{R^i} E[D_{t+i} | D_t] \) from (1).
context of the Campbell (1991) and Vuolteenaho (2002) return variance decomposition, the second term in \( a_t \) represents the effect of a time-varying discount rate on the stock price. To understand how a change in information precision \( f_t \) creates a price drop, we need to understand the effect of \( f_t \) on the second term in \( a_t \).

The stock’s expected net profit over a single period is given by

\[
E_t[\pi_{t+1}] = \gamma \times \tilde{X} \times \frac{\lambda \left( \frac{1}{q_D} \right) + (1 - \lambda) \left( \frac{1}{q_U} \right)}{\text{average uncertainty}}^{-1}.
\]

(21)

The quantities \( q_U \) and \( q_U \) defined in equations (29) and (30) of the appendix, represent the expected conditional variance of the net profit \( \pi \), conditional on the information sets of the informed and uninformed investors, respectively. Equation (21) thus reflects the average return uncertainty faced by investors, weighted by the fractions of informed and uninformed in the economy, and scaled by \( \gamma \tilde{X} \).

The right panel of Figure 6 shows the expected conditional variances \( q_I \) and \( q_U \) of the net profit \( \pi \) for informed and uninformed investors. The shape of these curves reflects the tradeoff engendered by increased information precision. When \( f \) is low, an increase in \( f \) decreases the expected variance of net profits for informed and uninformed investors because more is known about next period’s dividend, the \( D_{t+1} \) term in (7). As long as \( f \) is low enough so that \( \lambda(f) = 0 \) (to the left of the vertical, dashed line in the graphs) this is the only effect, and higher information precision lowers uncertainty. However, past the no-informed point, with \( f \) large enough that \( \lambda(f) > 0 \), the uncertainty of next period’s net profit starts to increase, due to the increasing variance of \( P_{t+1} \) in the expression for \( \pi_{t+1} \) in (7). This effect outweighs the decrease in the variance of next period’s dividend, and thus increases the conditional variances of the net profit. For high enough \( f \) the increased information about next period’s dividend begins to dominate, and the expected conditional variance begins to fall again. This pattern depends crucially on the dynamic structure of our model: in a single-period setting, where investors care about the next dividend but not future prices, more precise information always reduces investment uncertainty.

Through (21), the common shape of \( q_I \) and \( q_U \) is inherited by \( E_t[\pi_{t+1}] \), as illustrated in the bottom-right panel of Figure 5. Recall from our discussion of Figures 3 and 4 that \( f_t \) spends most of its time near \( f = 0.175 \) or near \( f = 0.88 \). From the bottom-right panel of

24This is shown in equation (45) of the appendix. This expression generalizes the corresponding quantity derived from equation (A10) in Grossman and Stiglitz (1980).
Figure 5, we see that $E_t[\pi_{t+1}]$ is greater near $f = 0.88$ than it is near $f = 0.175$, indicating an increase in the expected profit from holding the stock as we move from the low-information regime to the high-information regime. This increase in expected profit is associated with a decrease in the current price of the stock, and it contributes to the price drop we see in Figure 6.

Notice, however, that the change in expected net profit across regimes is quite small, as indicated by the vertical scale in the lower-right panel of Figure 5. How does a small change in expected profit get amplified into a 10% price drop? The answer lies in the combination of the price discount reflected in (20) and the persistence of the two $f_t$ regimes.

The right panel of Figure 4 shows that transitions between $f_t \approx 0.175$ and $f_t \approx 0.88$ occur rarely. The chart shows the low-to-high transition probability $P[f_{t+i} > 0.5|f_t = 0.175]$ (solid line) and the high-to-low transition probability $P[f_{t+i} < 0.5|f_t = 0.88]$ (dashed line) as functions of the number of months $i$. Both probabilities grow very slowly, reaching only 6-8% even after 240 months, confirming that transitions between regimes are infrequent, and generating the bimodal stationary distribution in the left panel. As a consequence of this persistence, we expect the inequality $E_t[\pi_{t+1}|f_t = 0.88] > E_t[\pi_{t+1}|f_t = 0.175]$ (which we observed in Figure 5) to extend to $E_t[\pi_{t+i}|f_t = 0.88] > E_t[\pi_{t+i}|f_t = 0.175]$, for large $i$. Recall the present value of such terms is subtracted from the price $P_t$ through the $a_t$ coefficient in (20). Thus, even a relatively small single-period difference in expected profits around $f = 0.175$ and $f = 0.88$ is amplified to a large change in the price because the $f_t$ process spends long periods in each of the two regimes before moving towards the other.

Are such infrequent regime transitions plausible? Barro (2009) estimates country level crises occur with a 1.7% per year probability. Assuming independence across time, a given country has a 29% (i.e., $1 - (1 - 0.017)^{20}$) probability of experiencing at least one crisis over a 20-year period. As we saw in Figure 4, the probability of a low to high state transition in our model is approximately 7% over a 20-year period. Our calibration therefore suggests that one out of four country-level crises may be accompanied by the information-driven price drop of our model. If crises are typically associated with positive shocks to $f_t$, the actual ratio may be higher.

\footnote{The same argument predicts a sharp decline in the $a(t)$ curve around the unstable fixed point near $f = 0.48$ in Figure 3. Starting to the right of 0.48, $f_t$ will tend to move toward 0.88, whereas starting to the left of 0.48, $f_t$ will tend to move toward 0.175.}
5 Exploration of the Mechanism

This section further investigates the features of our model that drive its behavior. Section 5.1 shows why the large price drop in Section 4 depends on feedback from the fraction informed to the information state. Sections 5.2 and 5.3 connect the price drop with the degree of information asymmetry and the cost of information production. Section 5.4 contrasts our model with models of strategic complementarity in information acquisition.

5.1 The Effect of Feedback on the Price Discount

We saw in the previous section that large price drops in our model result from the combination of two properties: differences in the conditional variances $q_{D}^I$ and $q_{D}^U$ across $f$ regimes, and persistence of these $f$ regimes. We saw in Figure 4 that feedback through an endogenous $\lambda(f)$ produces persistence in $f_t$. We now examine how shutting off the feedback affects the conditional variance $q_{D}^U$ of the uninformed.

Figure 8: $a()$ curves and conditional return variance $q_{D}^U$ faced by uninformed investors in alternative models. The figure uses parameter values from Table 1.

The right panel of Figure 8 compares $q_{D}^U(f)$ in variants of our model. The solid line corresponds to our base case model and reproduces the dashed line in the right panel of Figure 6. Because our model extends over multiple periods (and as discussed in Section 4.3), an increase in information precision can increase the conditional return variance faced by the uninformed because the increase in price variance can outweigh the decrease in dividend.
uncertainty. The fact that $q^{U \rightarrow D}$ is larger in the high information regime than in the low information regime is a crucial factor in the price drop in Figure 6. The other curves in the right panel Figure 8 show the effect of shutting off the feedback effect (by setting $b_f = 0$ in (5)) or removing the endogeneity of the fraction informed (by setting $\lambda = 0$ or setting $\lambda$ at its mean value of 0.0731). Each of these changes lowers the return variance at higher values of $f_t$ and thus decreases the price discount in the high $f_t$ regime. As shown by the $a()$ curves in the left panel of Figure 8, the prices in these variants are insensitive to $f_t$.

5.2 The Role of Time-Varying Information Asymmetry

Information asymmetry plays an important role in generating price and volatility cycles in our model. Large price drops occur when the economy transitions from low- to high-information asymmetry states. Whether this can happen is dictated by $\phi$, the fraction of knowable information that is private. Figure 9 compares equilibrium $a()$ curves for different values of $\phi$; the case $\phi = 0.35$ is the one we have analyzed thus far.

When $\phi = 0$ and all knowable information is public, the economy is characterized by no information asymmetry — the knowable information is equally known to all agents. The $a()$ curve corresponding to this no-asymmetry case is the highest one (shown as a solid line), indicating the smallest price discount relative to the present value of future dividends. The $a()$ curve in this case is quite insensitive to $f_t$. The $\phi = 1$ case represents the highest informational asymmetry possible in the model, and corresponds to the lowest $a()$ curve, representing a large price discount needed to induce the informationally disadvantaged uninformed agents to participate in risk sharing, regardless of the information state $f_t$. Only for intermediate values of $\phi$ can the economy transition from low- to high-information asymmetry states. Such regime shifts are accompanied by large price changes.

The reason that prices in the case of $\phi \in \{0, 1\}$ do not change much across different values of $f$ can be seen from Figure 10, which shows the steady-state distribution of $f$ in the different $\phi$ models. When $\phi = 0$, there are no informed investors since all knowable information is public. With $\lambda = 0$ in (5), any positive $\epsilon_{f,t+1}$ shock quickly decays, pulling $f_t$ back to its low-information fixed point. This dynamic is seen in the unimodal distribution, with the peak centered at $f = 1$, when $\phi = 0$. Similarly, when $\phi = 1$, all knowable information is private, and $\lambda$ is relatively large. Via the $b_f$ term in the dynamics of $f_{t+1}$ in (5), a relatively high $\lambda$ produces a steady state distribution that is unimodal at $f = 1$. Any negative $\epsilon_{f,t+1}$ quickly dissipates as $f$ is pulled back to one. In both cases, $E_t[\pi_{t+1}|f_t = f]$ will be close to

Electronic copy available at: https://ssrn.com/abstract=3324789
Figure 9: $a()$ curves as functions of $f$ for different values of $\phi$. The figure uses parameter values from Table 1.

$E_t[\pi_{t+1}|f_t = 1]$ for any $f$, and $a_t$ in (20) is consequently insensitive to $f$.

For intermediate values of $\phi$ (0.35 in our calibration), the equilibrium $\lambda$ is in an intermediate range, and the tendencies of $f_t$ towards $a_f$ and towards the high-information fixed-point are balanced. The steady state $f_t$ distribution becomes bimodal as can be seen in Figure 10. Therefore, a sequence of shocks can occasionally push the economy from one information regime to the other. And yet both regimes are very persistent. As in Section 4.4, this persistence amplifies differences in expected net profit $E_t[\pi_{t+1}]$ at different values of $f$ to produce large price changes.

This effect results from an increase in information asymmetry, rather than just from an increase in information precision. When $f_t$ is low, there is little information but also no information asymmetry because all agents are uninformed. In this case, prices are high. But as $f_t$ increases, private information becomes more revealing and some investors start to acquire it at a cost. The uninformed then find themselves at a growing informational disadvantage and the price falls.

The important feature of the $f_t$ dynamics associated with a price drop is the presence of two stable fixed points, which is true in the bimodal distribution of the middle panel of Figure 10. It is possible to have two stable fixed points, and yet have very little steady-state probability associated with one of them; in this case, there can still be a large price drop from the low- to high-$f$ state. An example of this is in the right panel of Figure 12.
5.3 The Effects of Cheaper Information

Connecting the previous discussion of information asymmetry to our news production sector from Section 2.2, we can ask what happens in the economy when information gets easier to produce. We can proxy for this by assuming that the per unit cost of news production $c_P$ falls. Dropping the cost of news production $c_P$, while keeping the expenditure on news producers $c_M$ fixed, results in an increasing $b_f$ (equation 25 in the appendix shows that $b_f = c_M/c_P$). That is the feedback $b_f\lambda_t$ from this period’s informed to next period’s signal precision $f_{t+1}$ becomes larger. Figure 11 shows that cheaper information pushes the economy towards the low-price-high-volatility regime. In fact, with a sufficiently low cost of information production, i.e. a very high $b_f$, the entire weight of the steady-state distribution gravitates to the low-price-high-volatility state (see Figure 12).

Cheaper news increases the feedback effect from $\lambda_t$ to $f_{t+1}$. When information production is expensive, there is no feedback, and the economy is always in the high price, low volatility regime. When information production becomes cheaper and $\phi$ has an intermediate value, the economy can be in a low volatility, highly symmetric information regime, or a high volatility, highly asymmetric regime. This corresponds to the case analyzed in Section 5.2.

To analyze the effect of cheaper information on welfare, we look at the utility of the uninformed $J^U$, which is the utility of all agents in equilibrium because it is always either higher than that of the informed (when $\lambda = 0$) or equal to that of the informed. \footnote{This would not be true if $\lambda = 1$, which doesn’t happen in our calibration.} In the left panel of Figure 13, we see that higher $b_f$ (cheaper information production) increases $J^U(f)$.
Figure 11: The figure shows $a()$ curves as function of $f_t$ across different media regimes. The figure uses parameter values from Table I.

at each $f$. However, higher $b_f$ also pushes probability mass into the high $f$ region, where $J_U(f)$ is lower. When we take the expectation of $J_U(f)$ over the steady-state distribution of $f$, the net effect, shown in Figure 12, is to lower expected utility. In particular, then, a lower cost of information production, i.e. a higher $b_f$, reduces welfare. As before, this conclusion is a consequence of the degree of information asymmetry determined by $\phi$. Greater information production is welfare-reducing when a substantial fraction of that information remains private.

5.4 Lack of Strategic Complementarity

A key property of the Grossman and Stiglitz (1980) setting is that the value of becoming informed decreases as the number of informed investors increases. Subsequent work has investigated conditions in which the value of becoming informed increases as more investors become informed. Sources of this type of strategic complementarity identified in the literature include high fixed costs and low marginal costs in information production (Veldkamp 2006); certain deviations from normally distributed uncertainty (Chamley 2007); settings in which investors learn about supply as well as cash flows (Ganguli and Yang 2009 and Avdis 2016); other settings with multiple sources of information (Manzano and Vives 2011 and
Figure 12: Steady-state $f_t$ distributions across media regimes. The figure uses parameter values from Table I. The solid circles indicate fixed points of the $f \rightarrow \lambda^*$ mapping.

Figure 13: The left panel shows $J_U$ at different levels of $f_t$. The right panel shows the expected utility for uninformed investors, weighted by the steady-state $f_t$ distribution. The figure uses parameter values from Table I.
Goldstein and Yang (2015); and settings in which information acquisition affects cash flows (Dow et al. 2017). With few exceptions, these are static models, but they often result in multiple equilibria, with different asset prices in different equilibria.

In our dynamic setting, large price changes occur within the model, rather than through a change of equilibrium selection in a static model. But we will see that the contrast with earlier work goes beyond this feature. In order to explore strategic complementarity, we need to vary \( \lambda \) exogenously, shutting off its dependence on the information state \( f_t \); at an (interior) endogenous \( \lambda \), the marginal investor is, by definition, indifferent between becoming informed or remaining uninformed. With a fixed \( \lambda > 0 \) and \( b_f > 0 \), the dynamics in (5) tend to push the informativeness \( f_t \) to higher values at larger values of \( \lambda \), which might suggest that the value of becoming informed increases with the fraction informed \( \lambda \). But Figure 14 shows that this is not generally the case. Using parameter values from Table 1, the figure shows the expected utility gain \( E_t[J_{I_t} - J_{U_t}] \) from information acquisition as a function of \( f_t \), at fixed levels of \( \lambda \). The difference \( E_t[J_{I_t} - J_{U_t}] \) measures the expected benefit of becoming informed in the next period, given the current information state.

Across a wide range of values of \( f_t \), the curves are decreasing in \( \lambda \), meaning that in most states the value of becoming informed next period actually decreases as the current fraction informed increases, through the mechanisms discussed in Section 5.1. In the figure, this property holds wherever the value of becoming informed is positive — that is, wherever the curves are above the horizontal line at zero. (The same is true when we compare \( J_{I_t} - J_{U_t} \) at different levels of \( \lambda \).) With fixed \( \lambda \), our model thus exhibits strategic substitutability in information acquisition wherever information acquisition is beneficial. Our model produces large price drops not because of an inherent strategic complementarity in the structure of the model but as a consequence of feedback from an endogenous \( \lambda \).

The underlying source of the feedback effect in our \( f_t \) dynamics (5), as developed through the microfoundation in Appendix A, is the response of a competitive information production sector to increased demand when the marginal cost of transmitting already discovered information is zero. A similar cost structure of information production is highlighted by Veldkamp (2006) as a source of strategic complementarity, so it is interesting to contrast the implications of our models. The models have different objectives and differ in many respects; but, most notably, in Veldkamp (2006) higher prices are associated with a larger fraction of informed investors, whereas we saw in Section 4.3 that in our model an increase in \( \lambda_t \) can precipitate a large price drop. The key difference is the dual role of information acquisition.
in our dynamic model discussed in Section 4.4 more precise information decreases dividend uncertainty but can increase future price variance. The second effect is absent in Veldkamp (2006), where investors earn dividends but do not earn capital gains from reselling their shares, making them indifferent to price variance. With no dependence on the next period’s prices, the analysis reduces to a sequence of single-period problems.

To summarize, a static version of the feedback in the \( f_t \) dynamics (5), with \( b_f \lambda(f_t) \) replaced with a constant \( b_f \lambda \), does not produce complementarity in information acquisition. As we have argued, the price response to information generated by our model relies on the persistence of two information regimes and the dual role of information in a multiperiod model.

6 Conclusions

We have developed a model of a financial market in a dynamic information environment. The model combines exogenous shocks to the level of potentially available information, an endogenous response by investors, and feedback from investor information choices to the information environment through information production by a competitive research sector. The dynamic structure of the model leads to a dual role for information, in which greater information reduces uncertainty about the next dividend but may increase price variance.
We show that the equilibrium dynamics of our model, calibrated to market data, are characterized by two regimes, one with high prices and low volatility, and one with low prices and high volatility. A transition from the first regime to the second is reminiscent of a financial crisis but with no change in fundamentals — the price drop is driven by the dynamics of information and an increase in information asymmetry.

Furthermore, we show that in our calibration, the effect of an increased feedback from today’s informed to future information is welfare decreasing in the steady-state of the economy. This is true despite the fact that for any given level of the information state, more feedback makes the current set of investors better off.

A Information Production

We decompose the time $t+1$ dividend $M_{t+1}$ into a total of $\tilde{N}$ units of information $\eta_i$, so that $M_{t+1} = \eta_1 + \cdots + \eta_{\tilde{N}}$. These units are iid normally distributed with variance $\text{var}(M)/\tilde{N}$. To capture the idea that each piece of information is small, we assume $\tilde{N}$ is large. Let $f_t$ be the fraction of the $\tilde{N}$ units that are observable at time $t$, and $1 - f_t$ be the fraction of unobservable units. Then the components of $M_{t+1}$ can be written as:

$$\tilde{m}_t = \sum_{i\in \text{observable}} \eta_i,$$

$$\epsilon_{t+1} = \sum_{j\in \text{unobservable}} \eta_j.$$

And we’ll have $\text{var}(\tilde{m}_t) = f_t \text{var}(M)$ and $\text{var}(\epsilon_{t+1}) = (1 - f_t) \text{var}(M)$.

We assume that in every period: (a) with probability $\pi_{o\to u}$ any previously observable piece of information may become unobservable next period; (b) with probability $\pi_{u\to o}$ any previously unobservable piece of information becomes observable next period; and (c) a certain number $\epsilon_{f,t+1}$ of information units transitions at random from unobservable at time $t$ to observable at time $t+1$, or vice versa. It is possible for a given $t$ that $\epsilon_{f,t} = 0$. This shock proxies for large aggregate changes in observability, as opposed to the micro changes that are captured by $\pi_{u\to o}$ and $\pi_{o\to u}$. Note that $\epsilon_{f,t+1}$ should be specified to ensure that $f_{t+1} \in [0,1]$.

When $b_f = 0$, we can derive the $f_t$ dynamics in (1) and (5) in terms of these two transition probabilities. Given $f_t$, and in the absence of any information production, next period’s fraction of information units that are knowable will be

$$f_{t+1} \approx \frac{(1 - f_t) \times \pi_{u\to o}}{\text{unobservable} \rightarrow \text{observable}} + \frac{f_t \times (1 - \pi_{o\to u})}{\text{observable} \rightarrow \text{observable}} + \epsilon_{f,t+1}.$$
The approximation becomes exact for large $N$, where the fraction of units that change states approaches the probabilities $\pi^{u,o}$ and $\pi^{o,u}$. Matching this expression to the corresponding parts in our $f_t$ dynamics in (4) requires

$$(1 - f_t) \times \pi^{u,o} + f_t \times (1 - \pi^{o,u}) = a_f + \kappa_f \times (f_t - a_f).$$

Equating the coefficient of $f_t$ on the two sides yields

$$\pi^{u,o} = (1 - \kappa_f) \times a_f, \quad \pi^{o,u} = (1 - \kappa_f) \times (1 - a_f),$$

which we solve to get

$$\kappa_f = 1 - \pi^{u,o} - \pi^{o,u}, \quad a_f = \frac{\pi^{u,o}}{\pi^{u,o} + \pi^{o,u}}.$$

The persistence parameter $\kappa_f$ is therefore greater when the transition probabilities for individual units are smaller. The ratio defining $a_f$ is the stationary probability that an individual unit is observable, so (22) says that $f_t$ mean-reverts to the average fraction of observable units.

Given the model calibration from Table 1, the relationships above imply

$$\pi^{u,o} = 0.01575, \quad \pi^{o,u} = 0.07425.$$

It is about five times more likely that a currently observable piece of information becomes unobservable, than a currently unobservable piece of information becomes observable.

**Information Production Sector**

Each news outlet $j$ can discover $I_j$ units of information, i.e. $\eta_i$’s from above, at a fixed cost per unit of $c_P$. We assume each outlet discovers a unique set of information. Once a unit of information is discovered, the marginal cost of revealing it to investors is zero, and that unit becomes a generic observable unit with a $1 - \pi^{o,u}$ probability of remaining observable in the next period.

Recall from Section 2.2 that informed investors pay an amount $c_M$ (out of the total cost $c_I$ of becoming informed) to acquire information from the news outlets. We show in Section C.3 that in our calibration for a low enough $c_M$ all informed investors prefer spending $c_M$
on acquiring information from the news producers, over consuming \( c_M \).\(^{29}\) In deciding how much information to produce, news producers forecast next period’s demand for news. A market price of a unit of information is determined to clear the news production market.

Each news outlet believes that the number of informed investors at time \( t + 1 \) will be \( \mathbb{E}_t[\lambda_{t+1}] \), and that each of the \( t + 1 \) informed will choose to buy all of the outlet’s production \( I_j \) at a price of \( p \) per unit of information. Since each outlet is small and ex-ante identical, it assumes the price \( p \) is fixed. Each outlet’s profit is given by

\[
\mathbb{E}_t[\lambda_{t+1}] I_j p - c_P I_j.
\]

The \( \lambda_{t+1} \) term is present due to the zero marginal cost of transmitting information, once it’s been discovered. Since the market is competitive, each outlet must operate at zero profit, which implies

\[
p = \frac{c_P}{\mathbb{E}_t[\lambda_{t+1}]}.
\]

Because of the zero marginal cost of sharing information with additional investors, the per unit price of information is decreasing in the number of informed investors.\(^{30}\) This is similar to Veldkamp (2006), except in our model investors buy more information as the price falls, whereas in Veldkamp (2006) the cost of becoming informed varies but investors cannot choose the quantity of information they acquire.

Each investor chooses to spend \( c_M \) of the total cost \( c_I \) on buying information from all the producers. The choice to spend \( c_M \) will be discussed in Section \ref{sec:choice}. Therefore, the budget constraint becomes

\[
c_M = I \times p,
\]

where \( I = \sum_j I_j \). Combining this with the zero profit condition we get

\[
\frac{c_M}{I} = \frac{c_P}{\mathbb{E}_t[\lambda_{t+1}]}.
\]

\(^{29}\) We also show that uninformed investors would not choose to opt into an information set where they become informed but choose to consume \( c_M \) rather than pay it to the information production sector.

\(^{30}\) A similar result obtains if we assume monopolistic competition, i.e. each media outlet produces a differentiated piece of information, while taking as given the prices of all other news outlets. This case is analyzed in Perloff and Salop (1985) and Veldkamp (2006), though it does not add to the intuition here. Monopolistic competition leads to a higher constant price, and lower \( I_j \), than the competitive case.
Figure 15: Comparison of $E_t[\lambda_{t+1}]$ against $\lambda_t$. The figure uses parameter values from Table 1.

So the aggregate news production will be given by

$$I = \frac{c_M}{c_P} E_t[\lambda_{t+1}].$$  (23)

The information producers are assumed to be myopic and believe that tomorrow’s number of informed is equal to today’s number of informed.

$$E_t[\lambda_{t+1}] \approx \lambda_t$$  (24)

Figure 15 shows that in our equilibrium, this is a very accurate approximation because, in the steady state, the economy spends very little time in the $\lambda_t \in [0, 0.05]$ region in which the approximation is less precise (see equilibrium $\lambda$ in Figure 3 and the equilibrium steady-state distribution in Figure 4). With this we augment the $f_{t+1}$ dynamics in the prior section with an additional $c_M/c_P \lambda_t$ units of information. Setting

$$b_f = \frac{c_M}{c_P}$$  (25)

yields the reconciliation of our process with that of (5). Section C.3 discusses the magnitude of feasible $c_M$s in our calibration; and since $c_P$ is a free parameter in the model, from (25) we see that given a $c_M$ any $b_f$ is attainable for some $c_P$.

The foregoing discussion describes the interior behavior of $f_t$, given in (4). At the left boundary, we disallow realizations of $\epsilon_{f_{t+1}}$ that will push $f_{t+1}$ below zero. At the right
boundary, we similarly disallow realizations of $\epsilon_{f,t+1}$ that will push $f_{t+1}$ above one. When the $b_f \lambda_t$ term would push $f_{t+1}$ above one, we assume this effect is exactly offset by $\epsilon_{f,t+1}$.

B Market Equilibrium

B.1 Proof of Proposition 3.1 (Existence of a Market Equilibrium)

Investor Demands for the Risky Asset

We prove Proposition 3.1 by solving explicitly for the coefficients of the price in (10). To allow for arbitrary variance beliefs, we write the investor optimization problem (8) as

$$
\hat{J}_i \equiv \max_q \mathbb{E} \left\{ \mathbb{E} \left[ W_{t+1} | \mathcal{I}_t^i, f_{t+1} \right] \right\}, \quad i \in \{I, U\},
$$

(26)

where, using (15),

$$
\hat{\text{var}}(W_{t+1} | \mathcal{I}_t^i, f_{t+1}) = q^2 (1 + d)^2 \left[ \text{var}(m_t | \mathcal{I}_t) + (1 - f_t)\sigma_M^2 \right] + q^2 V_B(f_{t+1}).
$$

(27)

If the variance belief $V_B$ is self-consistent, then (27) yields the conditional variance, but (26) makes explicit investors’ objectives with arbitrary variance beliefs.

We can write the terminal wealth in (6) as $W_{t+1} = RW_t + q\pi_{t+1}$. Recalling that $W_t$ is known to time-$t$ investors, we set $\hat{\text{var}}(\pi_{t+1} | \mathcal{I}_t^i, f_{t+1}) = \hat{\text{var}}(W_{t+1} | \mathcal{I}_t^i, f_{t+1})/q^2$ to get

$$
\hat{\text{var}}(\pi_{t+1} | \mathcal{I}_t^i, f_{t+1}) = (1 + d)^2 \left[ \text{var}(m_t | \mathcal{I}_t) + (1 - f_t)\sigma_M^2 \right] + V_B(f_{t+1}).
$$

(28)

The first-order condition for optimality in (26) becomes

$$
q^i_t = \frac{1}{\gamma} \mathbb{E} \left\{ \mathbb{E} \left[ \pi_{t+1} | \mathcal{I}_t^i, f_{t+1} \right] | \mathcal{I}_t^i \right\} = \frac{1}{\gamma} q^N_t,
$$

(29)

where $q^N_t$ is the conditional expectation of the net profit, and $q^D_t$ is the expectation of its conditional variance, given a price variance of $V_B$. Through (28), the conditional variances, reflecting the cash-flow component and the time-$t$ price variance, take the form

$$
q^D_t = (1 + d)^2 (1 - f_t)\sigma_M^2 + \mathbb{E}_t V_B(f_{t+1}),
$$

$$
q^D_t = q^I_t + (1 + d)^2 \text{var}(m_t | P_t, \theta_t).
$$

(30)
Evaluating the conditional mean in the numerator of (29) as in [13], the demands for time-$t$ informed and uninformed agents become

\[
q^I = \frac{q_N^I}{\gamma q_D^I} = \frac{1}{\gamma q_D^I} \left[ (1 + d)(\mu_D + \rho D_t + \theta_t + m_t) + E_t a(f_{t+1}) - RP_t \right],
\]

\[
q^U = \frac{q_N^U}{\gamma q_D^U} = \frac{1}{\gamma q_D^U} \left[ (1 + d)(\mu_D + \rho D_t + \theta_t + E[m_t|P_t, D_t, \theta_t]) + E_t a(f_{t+1}) - RP_t \right].
\]

(31)

For the informed, we have used the fact that $E[m_t|I^I_t] = m_t$ and $\text{var}(m_t|I^I_t) = 0$. For the uninformed, we evaluate (31) using $\text{var}(m_t|I^U_t) = 0$. We then impose market clearing (9), taking

\[\text{Market Clearing and Price Coefficients}\]

We now impose market clearing (9), taking $\lambda$ as given. We substitute investor demands $q^I$ in (9), use the price function from (10), and collect terms. We do not have to expand $q_D^I$ or $q_D^U$ in the following because these depend on $f_t$ but not on $D_t$, $m_t$, $\theta_t$, or $X_t$. Equation (9) becomes

\[
\lambda q_D^I \left[ (1 + d)(\mu_D + \rho D_t + \theta_t + m_t) + E_t a(f_{t+1}) - RP_t \right] + (1 - \lambda) q_D^U \left[ (1 + d)(\mu_D + \rho D_t + \theta_t + E[m_t|P_t, \theta_t]) + E_t a(f_{t+1}) - RP_t \right] = \gamma q_D^I q_D^U X_t + \gamma q_D^I q_D^U \bar{X}.
\]

(34)

Collecting the $D_t$ terms and then the $\theta_t$ terms yields the constants

\[
d = \frac{\rho}{R - \rho} \quad \text{and} \quad g = \frac{1}{R - \rho}.
\]

(35)

---

31 Say $E[m|P] = K(bm + cX)$ for $K = \text{cov}(m, P)/\text{var}(P)$ and $\text{var}(m|P) = \text{var}(m - E[m|P])$. Since $m - E[m|P] = (1 - Kb)m - KcX$ then $\text{var}(m - E[m|P]) = (1 - Kb)^2 \text{var}(m) + K^2 c^2 \text{var}(X)$. This equals $(1 - 2Kb + K^2 b^2) \text{var}(m) + K^2 c^2 \text{var}(X) = (1 - 2Kb) \text{var}(m) + K^2 (b^2 \text{var}(m) + c^2 \text{var}(X))$. Note that $K = b \text{var}(m)/(b^2 \text{var}(m) + c^2 \text{var}(X))$ and therefore $K^2 (b^2 \text{var}(m) + c^2 \text{var}(X)) = b^2 \text{var}(m)/(b^2 \text{var}(m) + c^2 \text{var}(X)) = K^2 b \text{var}(m)$. And therefore $\text{var}(m|P) = (1 - Kb) \text{var}(m)$.
Collecting the constant terms in (34) yields

\[ a_t = \frac{1}{R} \left[ (1 + d)\mu_D - \frac{\gamma q_D^U q_D^U}{\lambda q_D^U + (1 - \lambda)q_D^I} \bar{X} + \mathbb{E}_t a(f_{t+1}) \right]. \]  

(36)

The function \( a \) appears on both sides. Assuming for a moment that a solution \( a_t = a(f_t) \) exists, we can proceed to solve for \( b \) and \( c \) because \( a \) plays no role in the inference the uninformed make from the price in (33). We return to solve (36) after solving for \( b \) and \( c \).

Collecting the \( m_t \) terms in (34) we get

\[ b_t = \frac{1 + d}{R} \times \frac{\lambda q_D^U + (1 - \lambda)q_D^I K_t b_t}{\lambda q_D^U + (1 - \lambda)q_D^I}. \]  

(37)

Collecting the \( X_t \) terms and simplifying — mainly dividing the resulting equation by (37) — we find

\[ b_t/c_t = \frac{\lambda(1 + d)}{\gamma q_D}, \quad \text{with} \quad c_t = \frac{\gamma q_D^U}{R} \text{ if } \lambda = 0. \]  

(38)

We can now combine these equations to solve for \( b \) and \( c \) through the following steps, each of which involves only known quantities on the right side:

\[ q_D^U(f) = (1 + d)^2(1 - f)\sigma_M^2 + \mathbb{E}[V_B(f_{t+1})|f_t = f], \]  

(39)

\[ r(f) = \frac{\lambda(f)(1 + d)/(\gamma q_D^I(f))}{r^2(f)f\phi\sigma_M^2 + \sigma_X^2}, \]  

(40)

\[ \mathcal{R}^2(f) = \frac{r^2(f)f\phi\sigma_M^2 + \sigma_X^2}{r^2(f)f\phi\sigma_M^2 + \sigma_X^2}, \]  

(41)

\[ q_D^U(f) = q_D^I(f) + (1 + d)^2f\phi\sigma_M^2(1 - \mathcal{R}^2(f)). \]  

(42)

\[ b(f) = \frac{1 + d}{R} \frac{\lambda(f)q_D^U(f) + (1 - \lambda(f))q_D^I(f)\mathcal{R}^2(f)}{\lambda(f)q_D^U(f) + (1 - \lambda(f))q_D^I(f)} \]  

(43)

\[ c(f) = \begin{cases} \frac{b(f)}{r(f)}, & \lambda(f) > 0; \\ \frac{\gamma q_D^U(f)}{R}, & \lambda(f) = 0. \end{cases} \]  

(44)

Equation (39) restates the first line of (30); (40) is the ratio in (38); (41) rewrites the expression for \( \mathcal{R}^2 \) in (33); (42) follows from the second line of (30); (43) and (44) come from (37) and (38).
B.2 Solving for the $a()$ curve

We now return to (36). Using the price function from (10) and the net profit from (7), we see that

$$E_t[\pi_{t+1}] = E_t[D_{t+1} + P_{t+1} - RP_t]$$

$$= \mu_D + \rho D_t + E_t[a_{t+1}] + d(\mu_D + \rho D_t) - Ra_t - RdD_t$$

$$= (1 + d)\mu_D + E_t[a_{t+1}] - Ra_t$$

$$= \gamma x \frac{q^D q^U}{\lambda q^D + (1 - \lambda)q^D},$$

where the third step follows from the definition of $d$ in (35) and the fourth step follows from $a_t$ in (36). Using (45) we can rewrite $a_t$ in (36) as

$$a_t = \frac{1}{R} \left[ (1 + d)\mu_D - E_t[\pi_{t+1}] + E_t a(f_{t+1}) \right]$$

$$= \frac{1}{R} \left[ (1 + d)\mu_D - E_t[\pi_{t+1}] + E_t \left\{ \frac{1}{R} \left[ (1 + d)\mu_D - E_{t+1}[\pi_{t+2}] + E_{t+1} a(f_{t+2}) \right] \right\} \right]$$

$$= \cdots = \frac{(1 + d)\mu_D}{R - 1} - \sum_{i=1}^{\infty} \frac{1}{R^i} E_t[\pi_{t+i}]$$

(46)

If the variance belief $V_B$ is bounded above and bounded away from zero, then $|E_t[\pi_{t+i}]|$ is bounded and the expression in (46) is well-defined and finite. The quantities in (39)–(44) and (45) are all functions solely of the information state $f$, so the conditional expectations in (36) and (46) are taken with respect to the evolution of the information state in (5), for given $\lambda$. Equation (46) shows $a_t$ is equal to the present value of all future expected dividend payments minus a discount reflecting the expected present value of all future net profits.

B.3 Statement and Proof of Proposition 3.2 (Existence of a Rational Expectations Equilibrium)

In this section, we prove the existence of a rational expectations equilibrium by showing that the variance belief updating mapping has a fixed point. This demonstrates the existence of self-consistent variance beliefs given an exogenously specified $\lambda()$ curve.\footnote{Note that this exogenous $\lambda()$ curve need not result in equivalent utilities for informed and uninformed investors. We endogenize the $\lambda()$ curve in Section C.}
We now precisely state Proposition 3.2 for model (5). With \( f_t \) restricted to a finite set \( D \), we represent any function of \( f_t \) as a vector of dimension \( n = |D| \). We suppose \( \lambda() \) is fixed (not necessarily constant) with \( 0 \leq \lambda(f) \leq 1 \) for all \( f \). Let \( F(b,c) \) be the mapping that sends the initial coefficients \((b,c)\) to updated coefficients \((b',c')\) through (16) and (39)–(44). A fixed point refers to the coefficients \( b \) and \( c \) such that \((b,c) = F(b,c)\).

Assume there exists a scalar \( \bar{c} > 0 \) satisfying the following four polynomial conditions:

\[
\begin{align*}
\gamma \sigma_X (2\bar{q} + (1 + d)^2 \sigma_M^2 \phi) - \bar{c} R \sigma_X (4 - (1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \phi) & \leq 0, \\
4\gamma R \sigma_X^2 \bar{q} \bar{c} - 4R^2 \sigma_X^2 \bar{c}^2 + (1 + d)^2 \sigma_M^2 \phi \left( 1 + \gamma R \sigma_X \bar{c} \right)^2 & \leq 0, \\
\gamma^2 \sigma_X^2 \left( \bar{q} + (1 + d)^2 \sigma_M^2 \phi \right) & \leq 1, \\
\gamma \left( (1 + d)^2 \sigma_M^2 \eta + \bar{c}^2 \sigma_X^4 \right) - Rc & \leq 0,
\end{align*}
\]

where \( \bar{q} \) in (47), (48), and (49) is a quadratic function in \( \bar{c} \),

\[
\bar{q} = (1 + d)^2 \sigma_M^2 \delta + \bar{c}^2 \sigma_X^4, \tag{51}
\]

and the constants \( \eta \) and \( \delta \) in (50) and (52) are defined by

\[
\eta := \max_{f \in D} \left\{ \frac{1}{R^2} E[f_{t+1}|f_t = f] + 1 - (1 - \phi)f \right\},
\]

\[
\delta := \max_{f \in D} \left\{ \frac{1}{R^2} E[f_{t+1}|f_t = f] + 1 - f \right\}. \tag{52}
\]

Our simplest conditions would require \( \delta \leq \eta \leq 2 \), which is natural for \( f \in [0,1], \phi \in [0,1] \) and \( R > 1 \). The precise statement of Proposition 3.2 is that the mapping \( F \) has a fixed point in \([0,\bar{b}]^n \times [0,\bar{c}]^n\) with \( n = |D| \), \( \bar{c} \) satisfying (47)–(50), and

\[
\bar{b} = \frac{1 + d}{R}.
\]

The existence of a \( \bar{c} \) satisfying (47)–(50), and thus the existence of fixed point for mapping \( F \), only depends on model parameters.

As a shortcut for checking that these conditions hold, we show in Appendix B.3.1 that if \( R \in [1,1.2] \) and condition (17) holds, then \( F \) has a fixed point in \([0,\bar{b}]^n \times [0,\bar{c}]^n\) with \( \bar{b} = (1 + d)/R \) and \( \bar{c} = R/(2\gamma \sigma_X^2) \).
Proof of Proposition 3.2

To prove the result, we use Brouwer’s fixed point theorem, which states that if $F$ is a continuous function mapping a compact convex set $S$ to itself, then $F$ has a fixed point in this set, meaning there exists $x \in S$ for which $x = F(x)$; see, for example, p.29 of Border (1989).

We show conditions for the Brouwer’s fixed point theorem are satisfied with the belief updating mapping $F$ and the compact convex set $[0, \bar{b}]^n \times [0, \bar{c}]^n$, $n = |D|$. First, it is evident that $F$ is continuous as each mapping in (39)–(44) is continuous in its input. It is also evident that $F(b, c) \geq 0$ since each step in (39)–(44) returns a nonnegative value. Next, for any input $(b, c)$, we have $b'(f) \leq (1 + d)/R = \bar{b}$ for all $f$: it is easy to see $q_D'(f) \geq 0$, $\mathcal{R}^2(f) \in [0, 1]$, and $q_D''(f) \geq 0$, so the second factor in (43) is in $[0, 1]$ and the bound on $b'(f)$ follows.

It only remains to show that if $(b, c) \in [0, \bar{b}]^n \times [0, \bar{c}]^n$ and $(b', c') = F(b, c)$, then $c'(f) \leq \bar{c}$ for all $f$. For this we first establish two useful bounds. To lighten notation, in the following we abbreviate conditional expectations of the form $\mathbb{E}[V_B(f_{t+1}, \phi) | f_t = f]$ as $\mathbb{E}_t[V_B(f)]$. Recalling (16), we get the bound

$$
\mathbb{E}_t[V_B(f)] = \mathbb{E}_t[(b)(f)^2 f] \phi \sigma_M^2 + g^2(1 - \phi)\mathbb{E}_t[f] \sigma_M^2 + \mathbb{E}_t[c(f)^2] \sigma_X^2
\leq \bar{b}^2 \mathbb{E}_t[f] \phi \sigma_M^2 + g^2(1 - \phi)\mathbb{E}_t[f] \sigma_M^2 + \bar{c}^2 \sigma_X^2
= (1 + d)^2 \frac{1}{R^2} \mathbb{E}_t[f] \sigma_M^2 + \bar{c}^2 \sigma_X^2.
$$

(53)

The inequality uses the assumption $(b, c) \in [0, \bar{b}]^n \times [0, \bar{c}]^n$, and the last equality uses the relationship $\bar{b} = (1 + d)/R = g$, which follows (35). Combining this with (39), we can bound $q_D'(f)$ by

$$
q_D'(f) = (1 + d)^2(1 - f) \sigma_M^2 + \mathbb{E}_t[V_B(f)]
\leq (1 + d)^2 \sigma_M^2 \delta + \bar{c}^2 \sigma_X^2 = \bar{q},
$$

(54)

where $\bar{q}$ is given in (51). The inequality follows from the definition of $\delta$ in (52). We now proceed to prove the desired bound $c'(f) \leq \bar{c}$ for all $f$ by the following two cases.

I. The Case Without Informed Investor: $\lambda(f) = 0$

We first prove the bound for $c'(f)$ for the case $\lambda(f) = 0$. By the second case in (44), $c'(f)$ is now given by $c'(f) = \gamma q_D'(f)/R$. With $\lambda(f) = 0$, (40), (41), and (43) lead to
\[ r(f) = \mathcal{R}^2(f) = b'(f) = 0. \] Plugging these into (39) and (42) we have
\[
q_D'(f) = q_D^I(f) + (1 + d)^2 f \phi \sigma_M^2 \\
= (1 + d)^2 (1 - f) \sigma_M^2 + \mathbb{E}_t[V_B(f)] + (1 + d)^2 f \phi \sigma_M^2.
\]

With the bound for \( \mathbb{E}_t[V_B(f)] \) in (53), we can derive
\[
q_D'(f) \leq (1 + d)^2 \sigma_M^2 \left(1 - f + \frac{1}{R^2} \mathbb{E}_t[f] + f \phi\right) + c^2 \sigma_X^2 \\
\leq (1 + d)^2 \sigma_M^2 \eta + c^2 \sigma_X^2,
\]
where the last inequality follows from the definition of \( \eta \) in (52). Then the updated coefficient \( c'(f) \) satisfies
\[
c'(f) = \frac{\gamma} {R} q_D'(f) \leq \frac{\gamma} {R} ((1 + d)^2 \sigma_M^2 \eta + c^2 \sigma_X^2) \leq \bar{c},
\]
which follows from condition (50).

**II. The Case With Informed Investors: \( \lambda(f) > 0 \)**

Next, we prove the bound \( c(f) \leq \bar{c} \) under the case \( \lambda(f) > 0 \). Applying the first case in (44), we get
\[
\frac{b(f)}{r(f)} = \frac{\gamma}{R} \left( \frac{q_D^I(f) + \lambda(f)q_D^II(f) + (1 - \lambda(f))q_D^I(f)\mathcal{R}^2(f)}{\lambda(f)q_D^I(f) + (1 - \lambda(f))q_D^I(f)} \right). \tag{55}
\]

We need to derive a bound for the quantity in the parenthesis above.

We substitute \( q_D^II(f) \) and \( \mathcal{R}^2(f) \) in (55) using (42), (40), and (41). Then the right-hand side of (55) can be expressed as
\[
\frac{\gamma}{R} \left( \frac{\lambda q_D^I(1 + d)^2 \sigma_M^2 f \phi + (q_D^I)^2 \gamma^2 \sigma_X^2 (1 + d)^2 \sigma_M^2 f \phi + (q_D^I)^3 \gamma^2 \sigma_X^2}{\lambda^2 (1 + d)^2 \sigma_M^2 f \phi + \lambda q_D^I \gamma^2 \sigma_X^2 (1 + d)^2 \sigma_M^2 f \phi + (q_D^I)^2 \gamma^2 \sigma_X^2} \right). \tag{56}
\]

where we have dropped the dependence on \( f \) to simplify notation. In the following, we establish the needed bound for \( c'(f) \) by combining expression (56), bound for \( q_D^I(f) \) in (54), and conditions (47)–(49). In particular, we need a bound of (56) that is valid for all \( \lambda \in (0, 1] \), as we do not make \( \lambda \) endogenous in this proof.

We first consider the case \( f = 0 \) or \( \phi = 0 \). If this holds, (56) directly reduces to \( \gamma q_D^I(f) / R \)
itself. By (54), \( c'(f) \) satisfies
\[
c'(f) = \frac{\gamma}{R} q_D'(f) \leq \frac{\gamma}{R} \bar{q} = \frac{\gamma}{R} \left( (1 + d)^2 \sigma_M^2 \delta + \bar{c}^2 \sigma_X^2 \right).
\]
As we have \( \delta \leq \eta \) by (52), the desired bound for \( c'(f) \) can be established as
\[
c'(f) \leq \frac{\gamma}{R} \left( (1 + d)^2 \sigma_M^2 \delta + \bar{c}^2 \sigma_X^2 \right) \leq \frac{\gamma}{R} \left( (1 + d)^2 \sigma_M^2 \eta + \bar{c}^2 \sigma_X^2 \right) \leq \bar{c},
\]
where the last inequality directly follows from condition (50).

Next, we consider the case (56) for \( f > 0 \) and \( \phi > 0 \). Here we need to consider the maximum of (56) over all \( \lambda \in (0, 1] \). We compute the derivative of (56) with respect to \( \lambda \). As the numerator and denominator of (56) are linear and quadratic functions in \( \lambda \) respectively, the numerator of its derivative is a quadratic function in \( \lambda \). Furthermore, it is easy to check this quadratic function is concave and the denominator of the derivative is always positive. Thus the maximum of (56) is attained at the larger root of its derivative, as long as that root falls in \((0, 1]\). Through algebraic manipulations, the larger root of the derivative is given by
\[
\bar{\lambda} = \frac{-\tau + q_D' \gamma \sigma_X \sqrt{\tau + (1 + d)^2 \sigma_M^2 \phi}}{(1 + d)^2 \sigma_M^2 \phi},
\]
where \( \tau = q_D' \gamma^2 \sigma_X^2 \left( q_D^2 + (1 + d)^2 \sigma_M^2 \phi \right) \). Also, \( \bar{\lambda} \in (0, 1] \) is guaranteed by condition (49) and the bounds \( q_D' \leq \bar{q} \) and \( f \leq 1 \). Thus the maximum of (56) is indeed attained at \( \lambda = \bar{\lambda} \).

Letting \( \lambda = \bar{\lambda} \) in (56), we can derive
\[
c'(f) \leq \frac{\gamma \sigma_X \left( 2q_D' + (1 + d)^2 \sigma_M^2 \phi \right) + 2\sqrt{(1 + d)^2 \sigma_M^2 \phi}}{R \sigma_X \left( 4 - (1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \phi \right)}.
\]
Denote the right side by \( \kappa(f, q_D') \). It is positive as condition (47) directly implies the denominator is greater than zero. As \( \kappa(f, q_D') \) clearly increases in both \( q_D' \) and \( f \) at their upper bounds \( \bar{q} \) and one, respectively. Thus to establish the needed bound \( c'(f) \leq \bar{c} \), it suffices to show
\[
c'(f) \leq \kappa(f, q_D') \leq \kappa(1, \bar{q}) \leq \bar{c}.
\]
Setting \( q_B = \bar{q} \) and \( f = 1 \) on the right side of (58), the needed inequality \( \kappa(1, \bar{q}) \leq \bar{c} \) becomes

\[
2\sqrt{(1 + d)^2 \sigma_M^2 \phi + \bar{q}^2 \sigma_X^2 (\bar{q} + (1 + d)^2 \sigma_M^2 \phi)} \leq \bar{c} R \sigma_X (4 - (1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \phi) - \gamma \sigma_X (2\bar{q} + (1 + d)^2 \sigma_M^2 \phi).
\]  

(60)

By condition (47), the right side of (60) is positive, so this inequality is equivalent to the one obtained by squaring both sides. Taking squares and simplifying, we can show the quadratic terms \( \bar{q}^2 \) cancel out, and inequality (60) is equivalent to

\[
4\gamma R \sigma_X^2 \bar{c} \bar{q} \leq 4R^2 \sigma_X^2 \bar{c} \bar{e} - (1 + d)^2 \sigma_M^2 \phi (1 + \gamma R \sigma_X^2 \bar{e})^2,
\]

which holds as long as \( \bar{c} \) and \( \bar{q} \) satisfy condition (48). By (59), this proves the needed bound \( c'(f) \leq \bar{c} \) for \( \lambda(f) > 0 \). Combining the two cases with \( \lambda(f) = 0 \) and \( \lambda(f) > 0 \), we have proved \( c' \leq \bar{c} \) holds when conditions (47)–(50) are satisfied. The existence of a fixed point for the variance belief updating mapping now follows by Brouwer’s theorem.

### B.3.1 The Simplified Condition in (17)

Finally, we prove that (17) is a simple sufficient condition for the existence of fixed point. We follow the general conclusions established above and show that as long as \( R \in [1, 1.2] \) and condition (17) hold, then the value \( \bar{c} = R/(2\gamma \sigma_X^2) \) satisfies conditions (47)–(50). Thus by the proposition, a fixed point of belief updating exists in \([0, (1 + d)/R]^n \times [0, R/(2\gamma \sigma_X^2)]^n\).

Plugging \( \bar{c} = R/(2\gamma \sigma_X^2) \) into (47)–(50), algebraic computation shows that they simplify to following equivalent conditions:

\[
(1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \leq \frac{3R^2}{3R^2 + 4\delta + 6}, \quad (1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \leq \frac{2R^4}{(4 + 4R^2 + R^4)\phi + 8R^2\delta},
\]

(61)

\[
(1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \leq \frac{4 - R^2}{4(\delta + \phi)}, \quad (1 + d)^2 \gamma^2 \sigma_M^2 \sigma_X^2 \leq \frac{R^2}{4\eta},
\]

(62)

respectively. A nice property of these new conditions is that they are all upper bounds for the product \((1 + d)\gamma \sigma_M \sigma_X\). Thus it suffices to propose a bound for \((1 + d)\gamma \sigma_M \sigma_X\) that is small enough such that all the four conditions are satisfied. Since all the bounds in the right-hand sides of (61) and (62) are decreasing in \( \phi, \delta, \) and \( \eta \), it suffices to consider their values at \( \phi = 1 \) and \( \delta = \eta = 2 \), as by (52) we clearly have \( \delta \leq \eta \leq 2 \). On the other hand, as the bounds do not monotonically depend on the risk-free return \( R \), we impose a relatively
mild condition \( R \in [1, 1.2] \). Plugging these values into the right-hand sides of (61) and (62), the minimum of the four upper bounds approximately equals to 0.283, thus (17) is sufficient for the four conditions. Consequently, the corresponding \( \bar{c} = R/(2\gamma\sigma_X^2) \) is a valid upper bound for the existence of fixed point.

C Information Equilibrium

C.1 Equating Expected Utilities of Informed and Uninformed Given Belief Variance \( V_B \)

In this section we discuss the procedure for solving for an endogenous \( \lambda \) given variance beliefs. Given the demands in (30) and variance beliefs, we have, for \( \iota \in \{I, U\} \),

\[
\begin{align*}
\mathbb{E}[E[W_{t+1} | \mathcal{I}_t, f_{t+1}] | \mathcal{I}_t] &= q' \times q_N + RW_t = \frac{(q_N')^2}{\gamma q_D} + RW_t, \\
\mathbb{E}[\text{var}(W_{t+1} | \mathcal{I}_t, f_{t+1}) | \mathcal{I}_t] &= (q')^2 \times q_D = \frac{(q_N')^2}{\gamma^2 q_D}.
\end{align*}
\]

We can therefore write the agent’s value function in (8), conditional on \( \mathcal{I}_t \) as

\[
J_{\iota t} = RW_t + \frac{1}{2\gamma} \frac{(q_N')^2}{q_D^2}, \quad \iota \in \{I, U\}.
\]

To find an endogenous \( \lambda \) in the sense of Definition 2.1, we need to evaluate the conditional expectation of \( J_{I t} - J_{U t} \) given the information state \( f_t \). We can pull the denominators \( q_D \) out of the conditional expectation because we know from (39) and (42) that they are purely functions of the information state. For the numerator terms, using the demands from (31), the price process from (10) and the condition on \( a_t \) in (36), it is straightforward to show that

\[
q_N' = \mathbb{E}_t[\pi_{t+1}] + (1 + d)\mathbb{E}[m_t | \mathcal{I}_t] - Rb_t m_t + Rc_t X_t
\]

where \( \mathbb{E}_t[\pi_{t+1}] \) — which is a function of \( \lambda \) — is given by (45). The \( \theta_t \) and \( D_t \) terms drop out, as do the terms involving \( a_t \).\(^{33}\) Note that \( q_N' \) equals the expected net profit \( \mathbb{E}_t[\pi_{t+1}] \), which only conditions on \( f_t \), adjusted for the information set of agent \( \iota \).

\(^{33}\)In particular, we do not need to evaluate \( a(\cdot) \) to find the endogenous \( \lambda(\cdot) \), which is useful in solving the model numerically.
Since $E[m_t|I^L_t] = m_t$ we have
\[
E[(q^L_N)^2|f_t] = (E_t[\pi_{t+1}])^2 + (1 + d - Rb_t)^2 \phi f_t \sigma^2_M + R^2 c^2_t \sigma^2_X. \tag{65}
\]
And from (32) we have $E[m_t|I^U_t] = K_t b_t m_t - K_t c_t X_t$. From this we have that
\[
E[(q^U_N)^2|f_t] = (E_t[\pi_{t+1}])^2 + [(1 + d) K_t b_t - Rb_t]^2 \phi f_t \sigma^2_M + [Rc_t - (1 + d) K_t c_t]^2 \sigma^2_X
\]
\[
= (E_t[\pi_{t+1}])^2 + [(1 + d) K_t - R]^2 (b^2_t \phi f_t \sigma^2_M + c^2_t \sigma^2_X). \tag{66}
\]
Combining these expressions with $J^L_t$ in (64), we get an expression for the difference in conditional expectations
\[
\Delta_f = E[J^L_t - Rc_t|f_t = f] - E[J^U_t|f_t = f]. \tag{67}
\]
When this difference is positive, the marginal investor has an incentive to become informed. For a given $f$ we numerically solve for the $\lambda \in [0, 1]$ which sets $\Delta_f = 0$. If this difference is always strictly positive we set $\lambda = 1$, and if it is always strictly negative we set $\lambda = 0$.

### C.2 Proof of Proposition 3.3 (Existence of $\lambda()$ given $V_B$)

Recall that we have restricted $f_t$ to a finite set $\mathcal{D}$. We need to be more explicit about the mapping to $\mathcal{D}$ in (5). Suppose $\mathcal{D} = \{s_1, \ldots, s_n\} \subset [0, 1]$. Partition the extended real line using
\[
-\infty = c_0 < c_1 < \cdots < c_n < c_{n+1} = \infty,
\]
and let $\Pi_\mathcal{D} : (c_j, c_{j+1}] \mapsto s_{j+1}, j = 0, 1, \ldots, n$. We prove the proposition for this choice of $\Pi_\mathcal{D}$.

We can write the difference in expected utilities (67) as
\[
\Delta_f = \frac{1}{2\gamma} \int E \left[ \frac{q^L_N f_t}{q^L_D} - \frac{q^U_N f_t}{q^U_D} \right] \, df_t = \frac{1}{2\gamma} \left( E[q^L_N|f_t = f] E[q^L_D|f_t = f] - E[q^U_N|f_t = f] E[q^U_D|f_t = f] \right) - Rc_t. \tag{68}
\]
The terms on the right depend on the mapping $\lambda : \mathcal{D} \mapsto [0, 1]$. However, for each $f$, $\Delta_f$ depends on $\lambda$ only through $\lambda(f)$. This follows from the expressions in (39)–(44). We may therefore write $\Delta_f$ as $\Delta_f(\ell)$, with the interpretation that $\ell$ is the value of $\lambda(f)$. The proposition will follow once we show that $\Delta_f(\cdot)$ is continuous: given continuity, either $\Delta_f(\ell^*) = 0$ at some $\ell^* \in [0, 1]$ (in which case we set $\lambda(f) = \ell^*$), or $\Delta_f(\ell) < 0$ for all $\ell \in [0, 1]$ (in which
case we set \( \lambda(f) = 0 \), or \( \Delta f(\ell) > 0 \) for all \( \ell \in [0, 1] \) (in which case we set \( \lambda(f) = 1 \)). This specification satisfies the conditions in Definition 2.1.

To establish continuity of \( \Delta \), we use the representation in (68). It is evident that, holding \( f \) fixed, each of the operations in (40)–(43) is continuous in \( \ell = \lambda(f) \). But \( \lambda \) is also implicit in (39) through the conditional expectation of the variance belief, which takes the form

\[
E[V_B(f_{t+1})|f_t = s_i] = \sum_{s_j \in D} P(f_{t+1} = s_j|f_t = s_i)V_B(s_j).
\]

With \( \lambda(s_i) = \ell \), the transition probabilities take the form

\[
P(f_{t+1} = s_j|f_t = s_i) = P(a_f + b_f \ell + \kappa_f(f_t - a_f) + \epsilon_{f,t+1} \in (c_j, c_{j+1})|f_t = s_i)
\]

\[
= P(\epsilon_{f,t+1} \in (c_j - [a_f + b_f \ell + \kappa_f(s_i - a_f)], c_{j+1} - [a_f + b_f \ell + \kappa_f(s_i - a_f)]),
\]

which is the integral of the density of \( \epsilon_{f,t+1} \) over the indicated interval and is therefore continuous in the endpoints and in \( \ell \). It follows that \( E[V_B(f_{t+1})|f_t = f] \) is continuous in \( \lambda(f) \), and therefore that the mapping (39)–(43) is continuous in \( \lambda(f) \), including, in particular, \( q_D^I(f) \) and \( q_D^{U}(f) \).

Next we turn to (44) and verify that \( c(f) \) is continuous at \( \lambda(f) = 0 \). Using (55), we can write, for \( \lambda(f) > 0 \),

\[
c(f) = \frac{\gamma}{R} q_D^I(f) \left( \frac{\lambda(1 + d)^2 \sigma_M^2 \phi + q_D^I(f) \gamma^2 \sigma_X^2 (1 + d)^2 \sigma_M^2 \phi + (q_D^I(f))^2 \gamma^2 \sigma_X^2}{\lambda^2 (1 + d)^2 \sigma_M^2 \phi + \lambda q_D^I(f) \gamma^2 \sigma_X^2 (1 + d)^2 \sigma_M^2 \phi + (q_D^I(f))^2 \gamma^2 \sigma_X^2} \right).
\]

As \( \lambda(f) \to 0 \), we have \( R^2(f) \to 0 \) and

\[
c(f) \to \frac{\gamma}{R} ((1 + d)^2 \sigma_M^2 \phi + q_D^I(f)) = \frac{\gamma}{R} q_D^{U}(f),
\]

which coincides with the value specified for \( c(f) \) in (44) at \( \lambda(f) = 0 \).

### C.3 Out of Equilibrium Utility and the Value of \( c_M \)

Given our model equilibrium, we analyze the incentive of an atomic agent to become informed but to forego the output of the information production sector. Because the agent is atomic, the agent’s deviation will not affect the equilibrium. By foregoing the information production output, the agent only spends \( c_I - c_M \) on acquiring information; we refer to such agents as semi-informed. The semi-informed learn only a portion \( m_o \) (suppressing the \( t \) subscript) of
Figure 16: Comparison of the uninformed utility $J_U$ to that of the semi-informed utility $J_{SI}$ averaged over the steady-state conditional probabilities as shown in (70). The red, vertical line shows the threshold $f_t$ at which $\lambda > 0$. The figure uses parameter values from Table 1.

$$m_t = m_o + m_u \quad \text{with } m_o \perp m_u,$$

where $m_u$ is the output of the information production sector. An agent evaluating this semi-informed information set $I_{SI}$ solves the same problem as in (8). The solution is similar to the derivation in Section 3 and is in the Supplementary Appendix. We now verify the conditions under which uninformed and informed agents prefer their information sets to being semi-informed.

In particular, we verify in our model calibration that for a low enough $c_M$, the expected utility of the semi-informed $J_{SI} + Rc_M$ and that of the uninformed $J_U + Rc_I$ satisfy

$$J_U(f) + Rc_I \geq J_{SI}(f) + Rc_M,$$

for all $f$. When $\lambda = 0$, the utility of the uninformed dominates that of the informed. When $\lambda \in (0, 1)$, the utility of the uninformed equals that of the informed. In our calibration $\lambda < 1$ for all $f$ so we do not need to consider the case when the informed are better off. Therefore if the uninformed would never prefer to be semi-informed, neither would the informed agents.
In the context of our microfoundation, we can think of the variance of $m_o$ as being $(f_t - b_f \lambda_{t-1}) \var(m_t)$ (recall that $\var(m_t) = f_t \var(M)$). The loss in precision from not paying $c_M$ for the information sector’s output is $b_f \lambda_{t-1}$, where $\lambda_{t-1} = \lambda^*(f_{t-1})$ is the time $t-1$ endogenous fraction resulting from the prior information state $f_{t-1}$.

Given the information sets $\mathcal{I}^U$ and $\mathcal{I}^I$ from Section 2.3, agents observe $f_t$. They are unaware of $f_{t-1}$. We assume agents make decisions consistent with the steady-state $f_t$ distribution from Figure 4. Writing $	ilde{J}^{SI}(f_t, f_{t-1})$ for the utility the semi-informed would attain if they observed both $f_t$ and $f_{t-1}$, we then have $J^{SI}(f_t) = \mathbb{E}[	ilde{J}^{SI}(f_t, f_{t-1})|f_t]$. To compare $J_U(f) + Rc_I$ and $J^{SI}(f) + Rc_M$, agents check the condition

$$J_U(f) + Rc_I - \sum_{f'} \tilde{J}^{SI}(f, f') \mathbb{P}(f_{t-1} = f'| f_t = f) > Rc_M,$$

where

$$\mathbb{P}(f_{t-1} = f'| f_t = f) = \frac{\mathbb{P}(f_t = f| f_{t-1} = f') \times \mathbb{P}(f_{t-1} = f')}{\mathbb{P}(f_t = f)}.$$

In this expression, $\mathbb{P}(f_{t-1} = f')$ and $\mathbb{P}(f_t = f)$ are the steady-state probabilities of $f'$ and $f$, respectively. For fixed $f$, the distribution of $\epsilon_{f,t}$ in Table 1 implies that $\mathbb{P}(f_t = f| f_{t-1} = f')$ is nonzero for three values of $f'$ associated with the three possible shocks, $\epsilon_f$, 0, and $-\epsilon_f$. The sum in (70) therefore reduces to a sum over these three possible shocks.

Figure 16 shows the left side of (70) for all values of $f$. We see that this difference is always positive, with a minimum value of 0.0072. Therefore for any $c_M < 0.0072/R$, neither the uninformed nor informed agents in our equilibrium would choose to deviate from their equilibrium information choice.

We note from Figure 16 the welfare benefit to the informed relative to the semi-informed is low (this is the part of the curve to the right of the vertical red line which is the threshold $f_t$ at which the fraction informed turns positive). In our calibration $c_I = 0.2627$ and the maximum benefit of being informed relative to semi-informed is roughly 0.04. The intuition is that the increase in information precision from the $b_f \lambda_t$ part of the $f_{t+1}$ dynamics in (4) is small. And yet, despite the small welfare contribution of this feedback term, the feedback has a pronounced effect on the market equilibrium.

References


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