Dynamic Matching Markets and An Application
to Residential Real Estate

In the residential real estate market, sellers arrive dynamically over time to put their units up for sale. These assets may differ in their attributes, including location, size, style, acreage, etc. Sellers themselves differ in their own financial constraints, carrying costs, and their delay tolerances, i.e., how long they are willing to wait until they sell their unit. Buyers arrive dynamically over time, differing in their preferences of house attributes, their budgets, and their delay tolerances. Buyer and seller decisions, i.e., how to price, which bid to accept, and whether to wait for better outcomes in the future, depend on the available inventory, its characteristics, the heterogeneity of buyers and sellers, the potential mismatch between buyers and sellers, and their beliefs for potential future arrivals of better units or less patient buyers, etc. This market evolves sequentially, and is subject to other frictions, such as for example the fact that buyers cannot be simultaneously bidding for too many units. This paper studies a microstructure model of this market, explicitly accounting its dynamics and the heterogeneity of buyers, sellers, and inventory. It strives to answer questions such as what explains the fact that similar units sell for different prices; how does the depth of the market depends on the supply and demand imbalance and the buyer and seller preferences; how much faster will a house sell if you lower the price by 5%.

The modeling and analysis of a dynamic matching market lies in the interface of the economics literature on market design, the CS/OR literature on matching, the finance/OR literature on market microstructure, and the stochastic networks literature. This paper leverages modeling approaches and tools from each of these areas, as well as the area of quantitative pricing and revenue management.

We propose a market microstructure model of such a stochastic and dynamic matching market. We analyze the market dynamics and its equilibrium under the simplifying approximation where buyers and sellers use linear bidding strategies. We characterize steady-state properties such as
market depth, price dispersion, and anticipated delays in selling or buying a unit. We characterize congestion and matching patterns for sellers and buyers, taking into account market dynamics, heterogeneity, and supply and demand imbalance manifested in the competition among buyers and sellers. Furthermore we show the effects of market primitives with comparative statics results.

In the sequel, we provide some detail about the model and results that we obtain.

**Model.** We propose a sequential meeting, Nash bargaining microstructure model of this decentralized, dynamic, heterogeneous market. The model evolves in discrete time over an infinite horizon. In each period, a new batch of random size of buyers and sellers arrive to the market, and consider whether to enter the market on the demand or supply side, respectively, as a function of the state of the market upon their arrival. Active buyers and sellers also make a decision whether to continue and stay in the market or leave (abandon). In our setting, the matching of buyers and sellers and the price formation occurs as follows: in each period, after buyers and sellers make entry/exit decisions, they contact potential matches under market frictions such as limited monitoring capability. The number of meetings of buyer and seller happening in one period is determined by an aggregate matching function, which is an often used modeling tool in economics literature, and is based on inventory of the two sides of the market. With standard assumptions of linear searching technology and random meeting mechanism, each active agent meets one potential match from the pool of active agents on the other side of the market in one period with a probability determined by the ratio of active buyers to active sellers in the current period. Then, when a meeting is formed, buyer and seller decides whether to match and at what transaction price by Nash bargaining based on their dynamic valuations. These not only incorporate their heterogeneity in the nominal valuation of the good/service, but also their dynamic opportunity cost or continuation value of market participation in the future. After transacted buyers and sellers exit the market, all remaining agents suffer a delay penalty because of their failure to transact in the current period, and then carry over to the next period. Both buyers and sellers are trying to maximize discounted surplus.

**Results.** We obtain the equilibrium steady-state characteristics of market depth, price dispersion, and the optimal strategies for buyers and sellers in the dynamic matching market with the
aforementioned microstructure model.

**Linear strategies.** We show that when the range of the valuations for buyers and sellers is small, bidding strategies are linear. Motivated by this result we study a market under an assumption that buyers and sellers always employ linear strategies. This assumption leads to a more tractable model, and is motivated by markets with "almost homogeneous" goods and agents, e.g., market of apartments in the same neighborhood. Numerical tests show the approximation is close to the true market equilibrium when valuation range is moderate.

**Equilibrium characterization.** Given the arrival features on the supply and demand sides and the market primitives, the strategies of buyers and sellers and the distribution of their types within the market's steady state is endogenous. Under linear strategies, we solve for the steady state equilibrium. For uniformly distributed valuation distributions, we obtain near closed-form characterization of the market equilibrium, in which delay increases with sellers' costs and decreases with buyers' valuations as power law, or exponentially in some cases.

Besides insights on congestion, our results also characterize the matching pattern between buyers and sellers when dynamics play a role, and clarify the effect of attractiveness (meaning having low cost as a seller or having high valuation as a buyer) in several aspects that would be otherwise ambiguous. The resulting assortativeness in dynamic matching patterns and surplus under the aligned preference structure add to the search and matching literature in economics.

**Symmetric market.** If the market is "symmetric" with respect to buyers and sellers primitive parameters, we can characterize the steady state equilibrium in closed form, as well as establish its existence and uniqueness. As a result, we can provide a series of comparative statics results on how the equilibrium would react to different kinds of changes in market primitives. For example, how would meeting technology, interest rate, or participation cost affect depth, price dispersion, and distribution over types in the inventory.

For downstream analysis, the modeling and analytical insights described above provide the essential ingredients for formulating and solving optimal search/pricing problems for buyers and sellers in such dynamic matching markets.