Capacity Sharing between Competitors*

Liang Guo
(Chinese University of Hong Kong)
Xiaole Wu
(Fudan University)

* We thank Teck-Hua Ho and Jun Zhang for helpful discussions. The financial support offered by the National Natural Science Foundation of China is gratefully acknowledged (Grant Number 71571047). Address for correspondence: CUHK Business School, Chinese University of Hong Kong, Shatin, Hong Kong, China; Fudan University, Shanghai, China. Email: liangguo@baf.cuhk.edu.hk and wuxiaole@fudan.edu.cn.

Capacity Sharing between Competitors

ABSTRACT

Market competition may lead to mismatch between supply and demand. That is, overpricing may give rise to underselling, and underpricing may yield stockout. Capacity sharing is a common practice to align excessive capacity with excessive demand. Yet the strategic interaction between competition and capacity sharing has not been adequately addressed. In this paper we investigate optimal strategies and firm profitability of capacity sharing between competing firms under both ex ante and ex post contracting, depending on whether the capacity sharing price is determined before or after price setting in the buyer market. We show that, with symmetric capacity, committing to an overly high capacity sharing price may not necessarily improve firm payoffs. Capacity sharing softens price competition under either contracting scheme, whereas the optimal capacity transfer price and equilibrium profits may be non-monotonically influenced by buyer loyalty. The equilibrium outcome under ex ante contracting is more sensitive to variations in market parameters than ex post contracting. As a result, ex ante contracting is more likely to be preferred when the endowed capacity is low or buyer loyalty is high. However, when firms' capacity is asymmetric, capacity sharing may intensify equilibrium competition and hurt firm profitability through reversing the firms' relative pricing aggressiveness.

Key words: capacity sharing; contracting timing; co-opetition; subcontracting; transshipment

1 Introduction

Mismatch between supply and demand is a prevalent problem in many markets. A major airline may overprice and undersell its abundant capacity, whereas a small airline may adopt a low-price strategy and attract consumers that cannot be served by its limited seats. One common practice to solve this type of mismatch problem is to trade flight capacity among competing airlines through so-called code-sharing clauses (Wassmer et al. 2010, Hu et al. 2013). Under such arrangements, underpriced airlines can borrow overpriced airlines' excessive capacity. Similarly, inventory trade among retailers is prevalent in numerous markets such as automobiles, apparels, computers, furniture, information technology products, shoes, sporting goods, and toys (Comez et al. 2012). Such practice of competitive capacity sharing can be seen in many other settings (e.g., automotive spare parts, car rentals, hotels, telecommunication, trucking service). For instance, in the maritime shipping market, shipping forwarders may purchase capacity from each other to better match supply with demand (Li and Zhang 2015). Another remarkable example is the emergence of online matching and trading platform (e.g., www.hotel-overbooking.com) where overbooking hotels can purchase rooms from, and relocate their guests to, (nearby) underselling hotels.

Capacity sharing is also becoming more common in the manufacturing sector. Competing firms can establish joint ventures in production facilities and trade their capacity according to mutually agreed contract terms. Fiat and PSA equally share the production capacity under the alliance Sevel-Nord, and have the option to trade each other's capacity when they sell the vehicle through their respective brands and distribution networks (Bidault and Schweinsberg 1996, Jolly 1997). DuPont and Honeywell jointly operate a world-scale factory to manufacture automotive refrigerants that they market and sell separately (Honeywell 2010). Samsung and Sony, fierce rivals in the LCD-TV market, established a joint venture S-LCD to produce advanced and cost-effective LCD panels in their supply chain (Ihlwan 2006). Other examples include the joint production of electric cars between BMW and PSA Peugeot Citroen (Eisenstein 2011). Competitors may share existing capacity without formal joint venture as well. For instance, starting in 2015, Mazda's new Mexican assembly plant is planned to produce not only its automobile models but also a subcompact vehicle for Toyota (Reuters 2012, Apostolides 2014).

Some important questions on capacity sharing remain inadequately addressed. First, it is not clear when competing firms should determine the price of sharing capacity. Should they formally contract on the terms of capacity sharing before the need for sharing arises, or should they postpone the agreement until the emergence of capacity stockout? In many markets the mismatch problem between capacity and demand is temporary and determined by firms' short-term marketing decisions (e.g., pricing). That is, depending on the outcome of competitive interaction, a firm may overprice and hence become the potential capacity lender, or underprice to become the potential borrower. As a result, ex post contracting after demand realization has the advantage of flexibility in adjusting the price of capacity. On the other hand, ex ante contracting and pre-commitment may influence subsequent market interaction and thus may be pursued for strategic consideration.

Second, what capacity sharing price should be set if firms adopt the ex ante contracting approach? If the committed capacity sharing price is too low, a firm may regret when it turns out to lose the market competition and thus has residual capacity. Conversely, overly high capacity sharing prices may reduce not only the gain of capacity buyers, but also may backfire on capacity lenders by decreasing the demand for capacity.

Third, would capacity sharing soften or intensify price competition between the firms? Selling excessive capacity can reduce the opportunity cost of overpricing, thus mitigating price cut incentive. On the other hand, purchasing residual capacity from competitors can overcome own capacity constraint, thus increasing underpricing tendency to pursue buyers. Therefore, it does not follow immediately how capacity sharing may influence competitive interaction. Moreover, this strategic consideration may influence not only firms' capacity sharing strategies (i.e., when and what capacity sharing price to set), but also the overall profitability of capacity sharing (i.e., whether to adopt or abandon capacity sharing at all).

Fourth, how do market characteristics affect the strategies and profitability of capacity sharing? For example, as buyer loyalty increases, should firms agree on a higher or lower capacity transfer price? Does it become more profitable to adopt the ex ante or the ex post contracting approach, or else the no sharing strategy? Similarly, what would be the impacts of firms' endowed capacity and of firm asymmetry?

We tackle these issues in this paper. We study capacity sharing between two firms that engage in price competition. Each firm has some fixed demand from loyal buyers, and seeks to undercut the rival to compete for the switching buyers. Overpricing can lead to excessive capacity, whereas underpricing may yield excessive demand. We consider two alternative schemes on the timing of capacity price setting, and compare them with the benchmark without capacity sharing. In the ex ante contracting scheme the firms agree upon the capacity sharing price before they compete in the buyer market. The sequence of moves is reversed in the ex post contracting scheme. We fully derive the firms' equilibrium strategies under each scheme, and compare the ex ante profits to determine which one should be preferred. Moreover, we consider both the symmetric capacity scenario under which each firm can ex post become the capacity lender or the borrower, and the asymmetric capacity scenario under which only one firm may be constrained by own capacity.

Interesting results emerge from our analysis. Under symmetric capacity the firms should commit to a sufficiently high capacity price that maximizes the probability of capacity sharing, i.e., equal to the anticipated lowest price in the mixed strategy equilibrium of market competition. Because of the efficiency gain from matching supply with demand, capacity sharing softens competition and improves firm profitability in either contracting scheme. Nevertheless, due to the commitment effect, the equilibrium capacity price and market competition under ex ante contracting are more sensitive to changes in market parameters. Equilibrium profit is hence higher under ex ante contracting than under ex post contracting when the endowed capacity is low or buyer loyalty is high.

Under asymmetric capacity the firm with excessive capacity continues to balance between the margin and the probability of capacity sharing, whereas the firm with capacity constraint prefers to commit to a sufficiently low capacity price. Capacity sharing between asymmetric firms makes the firm without capacity constraint less price aggressive, but the firm with capacity constraint more eager for price cut. This means that the relative pricing aggressiveness between the firms can be reversed in comparison to that without capacity sharing. As a result, interestingly, capacity sharing can intensify equilibrium competition, and thus lead to strictly lower equilibrium profit for the firm without capacity constraint.

There is a literature on capacity sharing among firms that do not compete with each other. For example, Van Mieghem (1999) considers capacity transfer after demand realization between a manufacturer and a subcontractor that operate in distinct markets, and finds that only state-dependent contracts can coordinate their ex ante capacity investments. Wu et al. (2013) and Wu et al. (2014) study optimal contracts when a subcontractor can share capacity with a manufacturer that sells to the end market. Yu et al. (2015) investigate conditions under which the cooperation of capacity sharing in queueing systems can be beneficial for a set of independent firms.

This research is related to studies on efficient transfer prices in interfirm inventory transshipment. Rudi et al. (2001) and Hu et al. (2007) investigate the determination of linear transshipment prices prior to demand realization that can lead to system-optimal inventory/capacity decisions. Anupindi et al. (2001) and Granot and Sošić (2003) focus on inventory transshipment prices, in a system of independent retailers, that are specified after demand realization. Huang and Sošić (2010) compare the efficiency implications of these two alternative approaches that differ in the timing of setting the transshipment price.

Another stream of studies investigate strategic subcontracting or outscourcing of production between competitors. Spiegel (1993) shows that horizontal subcontracting between competing firms can emerge in equilibrium and lead to more efficient production allocation, if and only if their costs are convex and asymmetric. Similarly, the explanation offered by Baake et al. (1999) for cross supplies between rivals is based on the saving of fixed production costs. They show that cross-firm purchases can modify the sequence of the firms' production decisions into a Stackelberg setting (Chen 2010). Caldieraro (2016) finds that strategic production outcourcing can occur between an entrant and an incumbent selling differentiated products. He also shows that the firms may prefer high transfer prices to mitigate price competition. Shulman (2014) investigates the incentives of retailers to resell their authorized products to an unauthorized direct competitor, and shows that such product diversion can arise as a prisoner's dilemma.

We contribute to the literature on "co-opetition" in inter-organizational relationships, i.e., the co-existence of cooperation and competition (Brandenburger and Nalebuff 1996). Previous studies typically adopt a hybrid approach with both cooperative and non-cooperative games (e.g., Brandenburger and Stuart 2007, Gurnani et al. 2007). Anupindi et al. (2001) and Granot and Sošić (2003) consider models with independent, non-cooperative inventory decisions and subsequent cooperative inventory transshipment after demand is realized. Hu et al. (2013) study cooperative negotiation of fixed proration rates for revenue-sharing airlines when the airlines can operate independent inventory control systems to maximize their respective expected revenues.

This paper departs substantially from these previous studies. We consider capacity sharing between competing firms, and compare the optimal strategies and firm profitability of alternative sharing schemes. Thus our focus differs from the inventory transshipment literature in Operations Management, which generally ignores firm competition and takes the alternative perspective of system efficiency. Our findings also differ from those in the horizontal subcontracting literature in Economics and in Marketing. In particular, relative to Spiegel (1993), Chen (2010), Shulman (2014), and Caldieraro (2016), we show that competitive capacity sharing can happen even between symmetric competitors. In addition, we find that capacity sharing can soften price competition under symmetric capacity but may intensify it under asymmetric capacity. Another differential feature of our setup is that the occurrence and the direction of capacity sharing is ex post determined, i.e., whether a firm will become the capacity borrower or the lender is the endogenous outcome of market competition, even when the capacity transfer price is ex ante contracted. Moreover, we adopt a non-cooperative game-theoretical approach in which firms are fully strategic in all decisions. This stands in contrast to standard models of co-opetition where cooperation is exogenously assumed.

The organization of the rest of the paper is as follows. In the next section we lay out the model assumptions. The analysis and the main insights are presented in Section 3 and 4. The last section concludes the paper. Proofs are presented in the Appendix.

2 The Model

Consider a market with two competing firms, A and B, selling an undifferentiated product (or service) to some potential buyers. The market can be a retailing market in which the sellers are retailers and the buyers are end consumers. Alternatively, our setting can be interpreted as B2B markets with competing manufacturers and downstream retailers/dealers.[1] The sellers compete in price to win the buyers' demand. Let the sellers' prices be p_A and p_B, respectively. Their costs of production/selling, both fixed and marginal, are assumed to be identical and, without loss of generality, normalized to zero.

There is a unit mass of buyers in the market. The demand for each buyer is fixed and normalized to one unit.[2] The buyers have an identical reservation value r. The buyers can be categorized into three groups. A group of buyers of size α ($\alpha < 1/2$) consider buying from firm A as long as the price p_A is below their reservation value, but never consider purchasing from firm B. Similarly, another group of buyers of size α consider buying only from firm B. These buyers are akin to the uninformed consumers as in Varian (1980) or the loyal consumers as in Narasimhan (1988). The remaining buyers of size $1 - 2\alpha$ are indifferent between the firms and desire to buy from the firm charging the lower price. When both firms offer the same price, we assume that half of these indifferent buyers seek to buy from firm A and the other half from firm B. Thus these buyers are akin to the informed consumers as in Varian (1980) or the switchers as in Narasimhan (1988). In the alternative B2B interpretation, the loyal buyers can represent retailers/dealers who have signed an exclusive dealing contract with the respective seller, whereas the switchable buyers are not tied by such exclusive arrangement and can purchase from either supplier.

The firms have an endowed capacity of k_A and k_B units, respectively. We consider symmetric capacity in the basic model. In other words, the firms have equal capacity, i.e., $k_A = k_B = k$. We focus on the interesting case in which the firms' capacity satisfies $1/2 < k < 1 - \alpha$. There are two considerations for the first condition, $k > 1/2$. This condition implies that the total industrial capacity exceeds the total demand, ensuring that capacity sharing can completely solve the potential stockout problem without leaving any buyer unserved. In addition, this condition can allow us to rule out uninteresting equilibria.[3] The second condition, $k < 1 - \alpha$, guarantees that

[1]We are grateful for the AE for suggesting this alternative interpretation. Where no confusion arises, we will use "firms" or "sellers" to denote the upstream competitors, and "buyers" to denote the downstream consumers or retailers/dealers.

[2]In a B2B setting, this assumption implies that the buyers (e.g., retailers, dealers) face inelastic demand from their respective end consumers.

[3]Consider, for example, the benchmark case with symmetric capacity. If $k < \frac{(1-\alpha)^2}{2-3\alpha}$, the firms are *de facto* local monopoly under the unique equilibrium in which both firms charge the reservation price r. If $\frac{(1-\alpha)^2}{2-3\alpha} < k < 1/2$, there exist an infinite number of degenerate equilibria, including the one in which the firms always charge r without

capacity constraint is not an immaterial issue in the firms' competition.[4] If instead $k \geq 1-\alpha$, each firm's capacity is large enough to meet the demand of both its own loyal buyers and the switchers, and as a result, capacity constraint is *de facto* irrelevant and the model is reduced to that in Varian (1980) or Narasimhan (1988).

Because capacity is limited, we need to specify, in case of stockout, how capacity is allocated among the buyers. We assume that, when a firm runs out of capacity to meet its demand, the randomized-rationing rule is employed in the allocation of capacity among the rationed buyers (Tirole 1988). That is, the buyers who prefer to buy from a firm are allocated the firm's capacity with equal probability. Moreover, loyal buyers of a firm with stockout do not consider the rival firm, either because of sufficiently low preference for the other firm or due to binding exclusive contracts. However, when the switchers cannot buy from the firm they desire to buy, they can still resort to the other firm to buy the product at the higher price.

For example, consider the scenario when firm A charges a lower price than firm B does. Both firm A's loyal buyers and the switchers, a total size of $1-\alpha$, prefer to buy from firm A. However, due to limited capacity, some of them would be rationed and cannot buy immediately from firm A. Denote the size of the rationed loyal buyers as w and that of the rationed switchers as s. The rationing probability is $1-k/(1-\alpha)$. Therefore, the rationed loyal buyers and the rationed switchers have a size of $w \equiv \alpha\,(1-k/(1-\alpha))$ and $s \equiv (1-2\alpha)\,(1-k/(1-\alpha))$, respectively. Note that $w+s = 1-\alpha-k$. If there is no capacity sharing between the two firms, the rationed loyal buyers would leave the market without purchase. The rationed switchers can go to the higher-priced firm B to buy the product. It can be readily verified that $k > \alpha + s$. This means that firm B's capacity is sufficient to meet the demand of both its own loyal buyers and the rationed switchers. Furthermore, we can show that firm B's leftover capacity, $k-(\alpha+s)$, is higher than the size of firm A's rationed loyal buyers, w. This suggests that the firms can potentially share their residual capacity to clear the residual demand of the market.

In this paper we consider voluntary capacity sharing between the competing firms. We assume that a firm with stockout can purchase the other firm's residual capacity to satisfy the demand of its rationed loyal buyers. We consider linear transfer price for capacity sharing, which is easy to implement in practice. Purchasing capacity is desirable for the stockout firm if the transfer price, denoted as λ, is lower than its profit margin. On the other hand, selling excess capacity is profitable

engaging in any price cut. Similarly, if $k < 1/2$, under ex ante contracting, in the unique equilibrium the firms would set the capacity transfer price $\lambda^* = r$ and the firms would always price at $p_A = p_B = r$.

[4] Note that Betrand competition under capacity constraints typically yields mixed-strategy equilibria (e.g., Tirole 1988, page 214-215). So we would still obtain mixed-strategy equilibria even if we consider continuous demand. On the other hand, the current setup with discrete demand can substantially simplify the analysis, while generating insights that would qualitatively hold under alternative setups with continuous demand.

Stage 1:	The firms contract and commit to the capacity transfer price λ.
Stage 2.1:	The firms simultaneously set their prices, p_A and p_B, respectively.
Stage 2.2:	The higher-priced firm's loyal buyers purchase from the higher-priced firm. The switchers and the lower-priced firm's loyal buyers seek to purchase from the lower-priced firm.
Stage 2.3:	If stockout occurs for the lower-priced firm, the rationed switchers resort to the higher-priced firm for purchase.
Stage 2.4:	The lower-priced firm decides whether and how many units to buy from the higher-priced firm, at the per-unit transfer price λ, to meet the demand of its rationed loyal buyers.

Table 1: Sequence of Moves under the Ex Ante Contracting Scheme

as long as the transfer price λ is positive. We assume that the determination of the transfer price λ is non-cooperative. In particular, with probability one half, each firm is assigned the opportunity to make a take-it-or-leave-it offer to the other firm (and the other firm can decide whether to accept or reject the offer).[5] This simple setup allows us to focus on the outcome of bargaining while abstracting from the bargaining process.[6]

We consider two capacity sharing schemes, depending on whether the capacity transfer price λ is agreed before or after the firms set their prices in the buyer market. The sequence of moves under either contracting scheme is detailed in Table 1 or Table 2, respectively. In the *ex ante contracting* scheme, the firms agree and commit to the capacity transfer price λ before they engage in market competition. After the firms set their prices p_A and p_B and fulfil their respective demand using their own capacity, the lower-priced firm can decide whether and how much to purchase from the other firm to meet the lower-priced firm's residual demand. The sharing of capacity is exercised at the ex ante committed capacity transfer price. Alternatively, the firms can set the capacity transfer price after market competition. This second scheme is referred to as the *ex post contracting* scheme. In particular, the firms first set their prices p_A and p_B and deliver their own capacity to meet their respective demand. It is only after stockout arises that they approach each other to determine the transfer price for capacity sharing.[7]

Some discussions on the model setup are warranted. First, we assume that the firms' decisions

[5] If we assume that one of the firms (e.g., the capacity lender) has a higher probability to be the offer maker, the firms would be asymmetric under ex post contracting, but still symmetric under ex ante contracting. Therefore, we intentionally assume equal chance of offer making between the firms to ensure comparability across the capacity sharing schemes.

[6] Nevertheless, except for the ex ante contracting scheme under asymmetric capacity, this setup leads to the same equilibrium outcome as arising from standard negotiation processes (e.g., Nash bargaining) with symmetric bargaining power.

[7] As an example of the ex post contracting scheme, hotels with overbooked reservations can now purchase rooms from other hotels through online trading platform such as www.hotel-overbooking.com.

Stage 1.1:	The firms simultaneously set their prices, p_A and p_B, respectively.
Stage 1.2:	The higher-priced firm's loyal buyers purchase from the higher-priced firm. The switchers and the lower-priced firm's loyal buyers seek to purchase from the lower-priced firm.
Stage 1.3:	If stockout occurs for the lower-priced firm, the rationed switchers resort to the higher-priced firm for purchase.
Stage 2.1:	The firms agree on the capacity transfer price λ.
Stage 2.2:	The lower-priced firm decides whether and how many units to buy from the higher-priced firm, at the per-unit transfer price λ, to meet the demand of its rationed loyal buyers.

Table 2: Sequence of Moves under the Ex Post Contracting Scheme

cannot be changed once they are made and committed. This is an inevitable assumption to differentiate dynamic games from static ones, because the model would be equivalent to a static game where all of the firms' decisions (e.g., pricing, capacity sharing) are made simultaneously in one period if it is assumed instead that ex post adjustment in decisions is allowed. For example, as long as the prices p_A and p_B are set, the lower-priced firm cannot subsequently raise its price and the higher-priced firm cannot reduce its price. This is the standard assumption in Betrand competition (e.g., Varian 1980, Narasimhan 1988), which can be justified in practice by the existence of menu/administrative costs in short-term price adjustments. Nevertheless, as is common in practice, the firms commit to their charged prices but not to the supply of sufficient capacity. That is, the firms do not have any binding responsibility to meet rationed demand.[8]

Second, it is important to clarify that capacity sharing is completely voluntary throughout all stages of the game. This means that, even after the capacity transfer price λ has been contracted, the capacity borrower can still freely decide whether and how many units of capacity to purchase from the capacity lender who can decide with full discretion whether to deliver the requested units.

Another assumption is that capacity transfer can happen only after the rationed switchers turn to the higher-priced firm for purchase.[9] This implies that the residual demand for the capacity borrower comes only from the rationed loyals (i.e., w). Alternatively, the inter-firm capacity transfer may take place before the rationed switchers buy from the higher-priced firm, i.e., the sequence of moves between Stage 2.3 and Stage 2.4 in Table 1, or that between Stage 1.3 and Stage 2 in Table 2, is reversed. Under this alternative timing the rationed switchers would stay with the lower-priced firm and the amount of shared capacity would be larger (i.e., $w + s$). Nevertheless, our current

[8] If otherwise, a firm with stockout would be charged an infinitely high transfer price for capacity sharing by the rival firm, which is apparently not the case in reality.

[9] We implicitly assume that the delay in capacity transfer is mild such that the disutility of postponed consumption for both the loyals and the switchers is negligible. Nevertheless, the firms should have no incentive to severely delay the transfer of capacity, because otherwise the residual demand for capacity sharing might be gone.

assumption would emerge in equilibrium when the timing between the rationed switchers' purchase and capacity transfer is endogenized. In particular, when the firms can decide whether to share their capacity before or after the rationed switchers buy from the higher-priced firm, it would be dominant for the higher-priced firm to delay the capacity transfer. As a result, given that capacity sharing is ex post voluntary, only the late-delivery timing can arise endogenously.[10]

In Section 4 we extend the basic model to consider asymmetric capacity. In particular, we examine the case when one firm does not have capacity constraint but the other firm does. Without loss of generality, we let firm A's capacity be sufficiently high (i.e., $k_A > 1-\alpha$), and firm B's capacity satisfy $k_B = k \in (1/2, 1-\alpha)$. Thus firm A can use its own capacity to completely satisfy all its potential demand from both its loyal buyers and the switchers. As a result, when firm A is the lower-priced firm, capacity sharing would be unnecessary. It is only when firm B is the lower-priced firm that the firms may mutually benefit from capacity sharing.

3 Symmetric Capacity

We start with the benchmark when the firms cannot share their capacity. We then analyze the case when the firms can ex ante set and commit to their capacity transfer price. In Section 3.3 we address the case when the capacity transfer price is mutually determined through ex post agreement. We then compare the ex ante and the ex post contracting schemes to investigate the firms' optimal timing to determine the capacity sharing price. We will use backward induction to solve the game. A brief summary of the main results is presented in Table 3.

3.1 No Capacity Sharing

The firms seek to balance two conflicting incentives in price competition. First, a firm has an incentive to undercut the rival's price to compete for the switchers. At the same time a firm also wants to maintain its profitability without cutting its price too much. Following standard reasoning (Varian 1980, Narasimhan 1988), no pure-strategy equilibrium exists and the unique equilibrium is in mixed strategy. Let $F(p)$ and $f(p)$ denote the cumulative distribution function and the probability density function of the firms' symmetric pricing strategy in the mixed-strategy equilibrium, respectively. It can be shown that the equilibrium price support is continuous with $p \in [L, r]$, where r is the buyers' reservation value, and L represents the lower bound and is to be

[10]Moreover, with symmetric capacity, even if the firms can decide ex ante (before the start of the game) when the capacity lender should furnish its excess capacity to the rival firm, it would still be optimal for both firms to commit to the late-delivery timing.

	Symmetric Capacity	Asymmetric Capacity
Benchmark	Π_o first decreases and then increases in α, and decreases in k	Π_{Ao} first decreases and then increases in α, and decreases in k; Π_{Bo} increases in α, and can first increase and then decrease in k
Ex Ante Contracting	$\Pi(\lambda)$ first increases and then decreases in λ; $\lambda^* = L_a$ first decreases and then increases in α, and decreases in k; $\Pi_a - \Pi_o > 0$ first increases and then decreases in α, and decreases in k	$\Pi_A(\lambda)$ first increases and then decreases in λ, and can be lower than Π_{Ao}; $\Pi_B(\lambda)$ decreases in λ, and is greater than Π_{Bo}
Ex Post Contracting	$\Pi_p - \Pi_o > 0$ first increases and then decreases in α, and decreases in k	Π_{Ap} can be lower than Π_{Ao}; $\Pi_{Bp} > \Pi_{Bo}$
Ex Ante vs. Ex Post	$\Pi_a > \Pi_p$ when k is sufficiently small or α is sufficiently large	$\Pi_{Aa} > \Pi_{Ap}$ when α is sufficiently large; $\Pi_{Ba} < \Pi_{Bp}$

Table 3: Summary of Main Results

determined in the equilibrium.

Note that when a firm charges a lower price than the competitor, its sales would be equal to k units because its total demand $1 - \alpha$ exceeds its capacity. Conversely, when a firm's price is higher than the rival's, it will be able to sell not only to its loyal buyers but also to the switchers who are rationed by the lower-priced firm. This means that the higher-priced firm's sales would be $\alpha + s$ units. As a result, a firm's expected profit of setting price p is

$$\pi(p) = \int_L^p p(\alpha + s) f(p')dp' + \int_p^r pkf(p')dp' = pF(p)(\alpha + s - k) + pk. \tag{1}$$

By the definition of the mixed-strategy equilibrium, the firms should make the same profit for all price levels in the equilibrium support. This means that the first order derivative of the expected profit with respect to p is zero. In other words, we have $\pi'(p) = [F(p) + pf(p)](\alpha + s - k) + k = 0$. Solving this differential equation by using the boundary condition $F(r) = 1$ leads to

$$F(p) = \frac{k - (\alpha + s)r/p}{k - \alpha - s}.$$

The equilibrium lower bound of price support is given by $L_o = r(\alpha + s)/k$. This is because a firm is able to guarantee a profit of $r(\alpha + s)$ by setting a price equal to the buyers' reservation value r, whereas its maximum sales of undercutting the competitor can only be k units. Moreover, the firms' equilibrium profit is $\Pi_o = r(\alpha + s)$, where s represents the size of the rationed switchers.

Proposition 1 *In the benchmark case without capacity sharing: (i)* Π_o *decreases in* α *if* α *is low and increases in* α *if* α *is high; (ii)* Π_o *decreases in* k.

This proposition presents the impacts of the size of loyal buyers and the firms' capacity on the equilibrium profit. Surprisingly, the firms do not necessarily benefit from an increase in the number of loyal buyers. This result stands in sharp contrast to that in conventional models of price competition without capacity constraint (e.g., Varian 1980, Narasimhan 1988). In the standard models the higher-priced firm can sell only to its loyal buyers such that a higher α always increases profitability. However, here the higher-priced firm can also sell to the switchers who prefer to, but could not, buy from the lower-priced firm. Recall that the size of the rationed switchers is given by $s = (1 - 2\alpha)(1 - k/(1 - \alpha))$. It is evident that a higher α decreases not only the base size of the switchers but also the rationing probability. These two negative effects can reinforce each other. That is, the negative impact of α on the size of the rationed switchers through reducing the rationing probability is larger as the size of the switchers increases, and vice versa. Therefore, the adverse effect of α on the rationed switchers is convex. In addition, when α is sufficiently low, increasing α has a more significant effect on s than on the size of the loyal buyers. When α becomes sufficiently high, its impact on s would loom smaller and an increasing α would lead to a higher equilibrium profit. This explains why the overall impact of α on the equilibrium profit is non-monotonic and exhibits an interesting "U" shape.

On the contrary, a higher capacity always decreases the firms' equilibrium profit. Intuitively, larger capacity decreases the probability that the buyers are rationed by the lower-priced firm. As a result, the firms' guaranteed profit of charging r is reduced, and so is the equilibrium profit.

The impacts on the lower bound of price support, $L_o = \Pi_o/k = r(\alpha + s)/k$, are similar. In particular, the lowest equilibrium price decreases with α if $\alpha \leq 1 - \sqrt{k}$ and increases with α if $\alpha \geq 1 - \sqrt{k}$, whereas the lower bound always becomes lower as k increases. This suggests that, interestingly, price competition can become more intense as the firms have more loyal buyers. Again this is driven by demand switching between the firms caused by capacity rationing. This is in contrast to the traditional result, when capacity constraint is absent, that price competition is always less intense with more loyal buyers (e.g., Varian 1980, Narasimhan 1988).

3.2 Ex Ante Contracting

The sequence of moves under the ex ante contracting scheme is presented in Table 1. The firms reach agreement on the capacity transfer price before they engage in price competition in the buyer market. We will start with the second stage of the game to solve for the firms' equilibrium pricing decisions, conditional on the capacity transfer price λ. We will then investigate what capacity transfer price would arise in equilibrium in the first stage, taking into account how the transfer price may influence the subsequent price competition.

3.2.1 Stage 2: Price Competition

Let p_h and p_l be the firms' prices charged to the buyers in the second stage, where $p_h > p_l$. Similarly, denote the profits of the higher-priced firm and the lower-priced firm in the second stage as π_h and π_l, respectively. Consider first the lower-priced firm's decision about whether and how much residual capacity to purchase from the higher-priced firm. It is straightforward that the lower-priced firm would buy w units of capacity if and only if its profit margin is higher than the agreed capacity transfer price. Therefore, if $p_l > \lambda$, capacity sharing does take place ex post and the firms' profits are $\pi_h = p_h(\alpha + s) + \lambda w$ and $\pi_l = p_l k + (p_l - \lambda)w$, respectively. The higher-priced firm can sell not only to its loyal buyers and the rationed switchers, but also has leftover capacity to lend to the lower-priced firm to fulfil the demand of the lower-priced firm's rationed loyal buyers. However, if $p_l < \lambda$, borrowing capacity from the competitor would be too costly to satisfy the residual demand of the lower-priced firm. So capacity sharing does not arise ex post and the rationed loyal buyers will remain unserved. The firms' profits would then be $\pi_h = p_h(\alpha + s)$ and $\pi_l = p_l k$, respectively.

Similar to the benchmark case, the unique equilibrium for the price competition game is in mixed strategy and the price support is continuous between r and some lower bound L. Given the symmetry of the setup, the firms have the same equilibrium profit in the second stage of the game, $\Pi(\lambda)$, conditional on the capacity transfer price. Because of the possibility of no ex post capacity sharing, there are two possible cases to consider, depending on whether the equilibrium lower bound of price support, L, is higher or lower than the committed capacity transfer price λ.

We start with the first possible case, $L \geq \lambda$, in which the firms' lowest equilibrium price is always higher than the capacity price and hence capacity sharing always arises. In anticipation of this, the firms' expected profit of setting price p is

$$\pi(p) = \int_L^p \left[p(\alpha + s) + \lambda w\right] f(p')dp' + \int_p^r \left[pk + (p - \lambda)w\right] f(p')dp'. \tag{2}$$

The two terms in the right-hand side of the above equation are the anticipated profits when the firm turns out to be the ex post higher-priced firm or the lower-priced firm, respectively. The equilibrium pricing strategy $F(p)$ can be solved by applying the conditions for the mixed-strategy equilibrium: $\pi'(p) = 0$ and $F(r) = 1$.

Let us then consider the alternative case, $L \leq \lambda$, in which the firms' lowest equilibrium price can be lower than the capacity transfer price and hence the firms may not always trade their capacity. In this case the firms' expected profit depends on whether the charged price is higher or lower than λ. To proceed, let the cumulative distribution function of the firms' symmetric pricing strategy be

$$F(p) = \begin{cases} F_1(p), p \leq \lambda \\ F_2(p), p \geq \lambda, \end{cases}$$

and the corresponding probability density function be

$$f(p) = \begin{cases} f_1(p), p \leq \lambda \\ f_2(p), p \geq \lambda. \end{cases}$$

The firms' expected profit by setting the price p with $p \geq \lambda$ is:

$$\pi(p) = \int_L^\lambda p(\alpha+s) f_1(p')dp' + \int_\lambda^p [p(\alpha+s)+\lambda w]\, f_2(p')dp' + \int_p^r [pk + (p-\lambda)w]\, f_2(p')dp', \quad (3)$$

where the first term in the right-hand side represents the case when the rival's price is so low to make capacity sharing unprofitable, and the second and the third terms are the anticipated profits when capacity sharing is profitable and the firm is the capacity lender or the borrower, respectively. Similarly, we can solve for the equilibrium pricing strategy $F_2(p)$ by applying the conditions $\pi'(p) = 0$ and $F_2(r) = 1$.

When a firm sets the price p with $p \leq \lambda$, there would not be ex post capacity sharing, and its expected profit would be:

$$\pi(p) = \int_L^p p(\alpha+s) f_1(p')dp' + \int_p^\lambda pk f_1(p')dp' + \int_\lambda^r pk f_2(p')dp'. \quad (4)$$

Again we can apply the equilibrium conditions $\pi'(p) = 0$ and $F_1(\lambda) = F_2(\lambda)$ to solve for the equilibrium pricing strategy $F_1(p)$. The equilibrium lower bound of price support can then be derived by applying the condition $F_1(L) = 0$.

Proposition 2 *In the second-stage price competition under the ex ante contracting scheme:*

(i) If $\lambda \leq \frac{r(\alpha+s)}{k-w}$, the lower bound of the equilibrium price support $L_a \geq \lambda$, and the firms' equilibrium profit $\Pi(\lambda) = r(\alpha+s) + \lambda w$ increases in λ;

(ii) If $\lambda \geq \frac{r(\alpha+s)}{k-w}$, the lower bound of the equilibrium price support $L_a \leq \lambda$, and the firms' equilibrium profit $\Pi(\lambda) = r(\alpha+s) + \frac{(\alpha+s)(r-\lambda)w}{k-\alpha-s-w}$ decreases in λ.

Unsurprisingly, capacity sharing can increase the firms' equilibrium profit by satisfying the residual demand of the rationed loyal buyers, i.e., $\Pi(\lambda) > \Pi_o$ for any $\lambda \in (0, r)$. However, the firms do not necessarily benefit from an increase in the capacity transfer price λ. As Proposition 2 shows, a higher λ first increases and then decreases the firms' equilibrium second-stage profit. This non-monotonic impact is driven by two offsetting forces that the transfer price exerts on the firms' equilibrium profit. On one hand, a higher λ means an increasing revenue of lending capacity to the competitor if the firm turns out to charge a higher price than the competitor does. This would then increase the guaranteed profit of charging the price equal to the reservation value r. On the other hand, a higher λ would decrease the likelihood of ex post capacity sharing, because the lower-priced firm may find that capacity sharing is too costly to be profitable. Therefore, as Proposition 2 (i) indicates, when λ is sufficiently low and always below the lowest equilibrium price, a higher capacity transfer price always improves the firms' profitability. Conversely, as part (ii) of the proposition demonstrates, when λ becomes sufficiently high, the firms' equilibrium profit would decrease with a higher capacity transfer price.

It can also be readily shown that a higher λ first increases and then decreases the equilibrium lower bound of price support. This means that the capacity sharing price can exert an "inverted-U" influence on the intensity of price competition. The driving force for this result is the same as that for the above-discussed non-monotonic impact on the firms' profitability. Under the mixed-strategy equilibrium, the lower bound of price support is determined by the guaranteed payoff of charging the highest possible price r, such that a higher (lower) ensured profit mitigates (enhances) the firms' incentive to cut their market prices.

3.2.2 Stage 1: Contracting Over Capacity Transfer Price

We now investigate the equilibrium capacity transfer price that the firms may agree with in the first stage. Note that, for any λ, both firms have the same equilibrium profit in the second stage of the game, $\Pi(\lambda)$. As a result, no matter which firm has the chance to make a take-it-or-leave-it offer to the other firm, it will propose λ to maximize $\Pi(\lambda)$. It then follows from Proposition 2 that the equilibrium capacity sharing price is such that the lower bound of price support in the second-stage equilibrium equals this capacity sharing price. As a result, under the equilibrium capacity price, the firms ensure that capacity sharing would happen with probability one.

Proposition 3 *Under the ex ante contracting scheme, the equilibrium capacity transfer price equals the lower bound of equilibrium price support, i.e.,* $\lambda^* = L_a = \frac{r(\alpha+s)}{k-w}$. λ^* *first decreases and then increases in* α, *and* λ^* *decreases in* k.

This proposition sheds light on how competing firms should ex ante set their capacity sharing price. It suggests that the firms should seek to increase the price of capacity transfer while maximizing the chance that the capacity price is not too high relative to the lowest price of the firms. In other words, the optimal capacity transfer price should be equal to the anticipated lower bound of equilibrium price. The lower bound of price support is influenced by the intensity of price competition in the market. As we have shown in the benchmark, in our setup with limited capacity, price competition will become more intensified and the firms' profitability will be hurt, as the size of loyal buyers becomes neither sufficiently small nor sufficiently large, or as the firms' endowed capacity increases. Therefore, the number of loyal buyers should have a "U-shaped" impact on the firms' optimal capacity transfer price, whereas more capacity should promote the firms to commit to a lower capacity transfer price.

Next we compare the equilibrium outcome under ex ante contracting with that in the benchmark case. It is evident that the equilibrium profit under the ex ante contracting scheme is higher than that without capacity sharing, i.e., $\Pi_a \equiv \Pi(\lambda^*) > \Pi_o$. It is without much surprise that capacity sharing can improve firm profitability. It can also be readily verified that capacity sharing can soften price competition, i.e., $L_a > L_o$. Intuitively, the prospect of lending excessive capacity to the competitor mitigates the firms' incentive to compete for the switchers.

We perform comparative statics analysis to examine the impacts on the benefit of capacity sharing. This can allow us to figure out conditions under which the firms should engage in capacity sharing, particularly when doing so involves some fixed costs.

Proposition 4 *The benefit of capacity sharing through ex ante contracting,* $\Pi_a - \Pi_o$*, first increases and then decreases in* α*, and decreases in* k*.*

This proposition reveals how the improvement in firm profitability over the benchmark may change with the size of loyal buyers and with the firms' endowed capacity, respectively. It shows that the impact of the number of loyal buyers on the benefit of capacity sharing is non-monotonic and displays an "inverted-U" shape, whereas that of increasing the capacity is negative. The benefit of capacity sharing, $\Pi_a - \Pi_o = \lambda^* w$, is driven by the number of the rationed loyal buyers. Recall that the size of the rationed loyals is given by $w = \alpha(1 - k/(1-\alpha))$. A higher α increases the base size of the loyal buyers, and decreases the number of switchers and hence reduces the rationing probability. As a result of the interplay of these two forces, a higher α first increases and then decreases the benefit of capacity sharing. In contrast, an increase in k unambiguously leads to a lower rationing probability and thus decreases the extent to which capacity sharing improves profitability.

As demonstrated in Figure 1, when the size of loyal buyers becomes overly small or overly large, or when the firms' capacity is sufficiently large, the benefit of capacity sharing may vanish to zero.

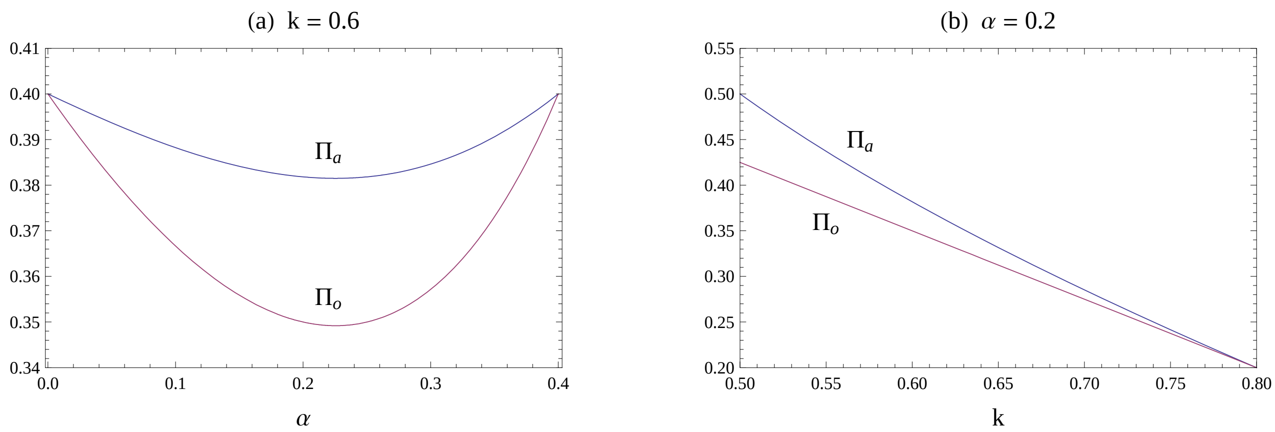

Figure 1: Impacts on the Benefit of the Ex Ante Contracting Scheme under Symmetric Capacity

This implies that, if the firms need to incur some fixed costs to engage in capacity sharing (e.g., costs of bargaining, contracting, capacity transshipment, etc.), they will do so only if the loyal buyers are neither too few nor too many, or only if their capacity is not too much.

3.3 Ex Post Contracting

Recall that the sequence of moves under the ex post contracting scheme is presented in Table 2. The firms determine whether and how to trade excessive capacity, after they engage in price competition and figure out the need for capacity sharing. We will first study the firms' ex post incentive for capacity sharing in the second stage of the game, conditional on the outcome of price competition. We will then examine how the firms may compete in the first stage, taking into account the impact on the prospect of ex post capacity sharing.

3.3.1 Stage 2: Contracting Over Capacity Transfer Price

Let p_h and p_l be the firms' first-stage prices, where $p_h > p_l$. Similar to Section 3.2, the firms will trade w units of capacity if and only if the capacity transfer price λ is not higher than the lower price between the firms, p_l. As a result, if it is the higher-priced firm to make a take-it-or-leave-it offer, $\lambda = p_l$ will be asked. Conversely, if the lower-priced firm makes the proposal, $\lambda = 0$ will be offered. In each case, the proposed offer will be accepted by the rival firm. Therefore, the firms would reach an expected transfer price that is equal to $p_l/2$. Thus the higher-priced firm's expected profit in the second stage would be $\pi_h = p_h(\alpha + s) + p_l w/2$, and that of the lower-priced firm would be $\pi_l = p_l k + p_l w/2$, respectively. The firms equally share the revenue of capacity sharing.

3.3.2 Stage 1: Price Competition

We analyze how the firms may set their prices, in anticipation of the expected outcome of capacity sharing in Stage 2. To characterize the unique equilibrium in price competition, note that a firm's expected profit of charging p is

$$\pi(p) = \int_L^p [p(\alpha + s) + p'w/2]\, f(p')dp' + \int_p^r (pk + pw/2) f(p')dp', \tag{5}$$

where the two terms in the right-hand side are the anticipated profits when the firm is the ex post capacity lender or the borrower, respectively. We can then solve for the equilibrium pricing strategy by applying the conditions for the mixed-strategy equilibrium: $\pi'(p) = 0$ and $F(r) = 1$. This yields the equilibrium price distribution $F(p) = \frac{k+w/2}{k+w/2-\alpha-s} - \frac{\alpha+s}{k+w/2-\alpha-s}\left(\frac{r}{p}\right)^{\frac{k+w/2-\alpha-s}{k-\alpha-s}}$, the lower bound of price support $L_p = r\left(\frac{\alpha+s}{k+w/2}\right)^{\frac{k-\alpha-s}{k+w/2-\alpha-s}}$, and the firms' equilibrium profit $\Pi_p = r(\alpha + s)\left(\frac{\alpha+s}{k+w/2}\right)^{\frac{w/2}{\alpha+s-w/2-k}}$.

The properties of this equilibrium outcome are similar to those under ex ante contracting. For example, the firms' equilibrium profit is higher than that without capacity sharing, i.e., $\Pi_p > \Pi_o$. We can numerically demonstrate that the equilibrium profit Π_p first decreases and then increases in α, and always decreases in k. Moreover, the impacts of α and k on the benefit of capacity sharing, $\Pi_p - \Pi_o$, are qualitatively similar to those presented in Proposition 4. Therefore, these insights on capacity sharing are robust to alternative timing of contracting over the capacity transfer price.

3.4 Ex Ante versus Ex Post Contracting

We now compare the ex ante and the ex post contracting schemes. The central question we address is when the firms should contract over their capacity transfer price.

Proposition 5 *When k converges to $1/2$, $\Pi_a - \Pi_p > 0$. When k converges to $1-\alpha$, $\Pi_a - \Pi_p > 0$ if and only if α is greater than some threshold $\alpha' \in (0, 1/2)$.*

This proposition presents conditions under which the firms should adopt one of the contracting schemes over the other. When the firms' endowed capacity is sufficiently small, they should unambiguously choose the ex ante contracting scheme and commit to their capacity transfer price before they compete in the product market. However, when the firms have sufficient supply of

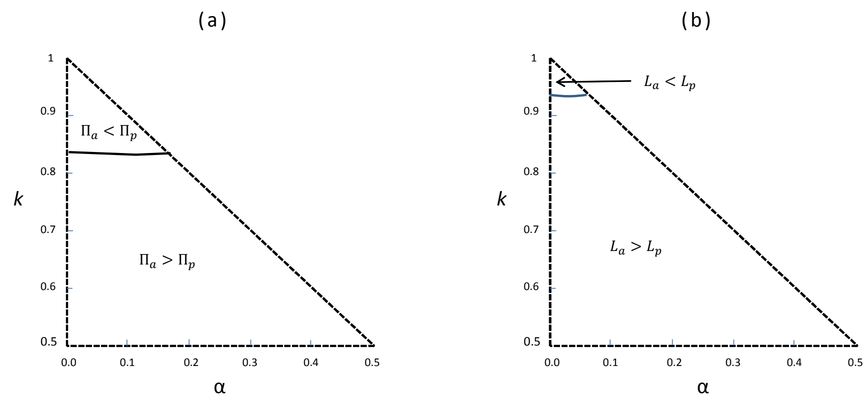

Figure 2: Equilibrium Profits and Lower Bound of Price Support under Symmetric Capacity

capacity, they should pre-determine the capacity sharing price only if the market has a sufficiently large number of loyal buyers; they should instead maintain the flexibility to ex post determine the capacity sharing price if the level of buyer loyalty is not too high. Figure 2 (a) illustrates how the market parameters k and α may influence which contracting scheme should lead to a higher equilibrium profit.

To understand these results, note that, under either contracting scheme, a firm's equilibrium profit is equal to its expected payoff when it charges the highest possible price r and ensures to be the capacity lender for the other firm. Note also that capacity sharing always takes place in equilibrium under either contracting scheme. This implies that the difference in the equilibrium profits hinges on the difference in the ex ante expected capacity transfer prices. Under ex ante contracting, the firms commit to the capacity transfer price λ that will not vary with the firms' prices. When the capacity sharing price is ex post agreed, it is expected to be equal to half of the lower of the firms' product prices. This gives rise to differential impacts on the firms' ex ante gain from capacity sharing. In particular, the firms can utilize the pre-determined capacity sharing price to directly influence the subsequent competition. If the capacity transfer price is not too high, an increasing λ can mitigate market competition and push up the lower bound of price support. However, an overly high capacity sharing price can decrease the probability of ex post capacity sharing, making market competition more intense. Thus the equilibrium capacity transfer price is equal to the equilibrium lowest price L_a. This commitment effect is absent under the ex post contracting scheme. As a result, the equilibrium capacity transfer price (and the equilibrium level of market competition) under ex ante contracting is more sensitive to the market parameters than under ex post contracting.

As demonstrated in Figure 2 (b), the lower bound of price support under ex ante contracting is higher than that under ex post contracting when k is sufficiently low or when α is sufficiently high. This is driven by the commitment effect discussed above. When k converges to $1/2$ and the product market is least competitive, a high capacity transfer price can be committed without worrying much about reducing the chance of capacity sharing. The firms' incentive to cut product prices can then be mitigated more under the ex ante contracting scheme, i.e., $L_a > L_p$. In contrast, when k converges to $1-\alpha$ and the market becomes sufficiently competitive, the firms have to commit to a sufficiently low capacity sharing price to maximize the probability of capacity sharing. The equilibrium lower bound of price support can then be lower than that under ex post contracting, unless the size of loyal buyers is large enough such that the market is not overly competitive.

Therefore, the firms are less aggressive in price competition under ex ante contracting than under ex post contracting if the market is sufficiently uncompetitive, but the reverse would be true in a sufficiently competitive market. This explains why the firms' equilibrium profit under ex ante contracting is higher than that under ex post contracting when the market is not very competitive (i.e., k is sufficiently low or α is high enough).

As shown in Figure 2, there exist scenarios under which, even though the lower bound of price support is higher under ex ante contracting, the equilibrium profit under ex post contracting is higher. This is due to the flexibility in modifying the capacity transfer price under ex post contracting. In contrast to the ex ante contracting scheme, the capacity transfer price under ex post contracting can be adjusted upward if the firms turn out to charge high product prices. Therefore, if ex ante contracting does not sufficiently soften competition (i.e., L_a is not too high relative to L_p), the ex ante expected capacity transfer price can actually be higher under ex post contracting. This is a favorable force for the equilibrium profit under the ex post contracting scheme.

4 Asymmetric Capacity

In this section we investigate the alternative scenario of asymmetric capacity, in which firm A is not constrained in supplying its potential demand (i.e., a total size of $1-\alpha$), whereas firm B has limited capacity $k \in (1/2, 1-\alpha)$. All other assumptions under symmetric capacity are maintained. As a result, the transfer of capacity can only be unilateral from firm A to firm B, and capacity sharing is relevant only when firm B is the firm with relatively lower price than firm A.

4.1 No Capacity Sharing

The equilibrium structure is similar to that for the asymmetric setup considered in Narasimhan (1988). In particular, the only equilibrium is in mixed strategy, and the equilibrium price support for both firms is between r and some lower bound L. Nevertheless, there is a mass point at r for the firm that is less price aggressive relative to the competitor.

To determine the equilibrium price support, let us consider the firms' incentives to cut price to compete for the switchers. Firm A can guarantee a profit of $r(\alpha + s)$ by setting its price equal to r, whereas its maximum demand, by charging a lower price than firm B, is $1 - \alpha$. Thus firm A will never set a price lower than $\frac{r(\alpha+s)}{1-\alpha}$. In contrast, firm B's ensured profit is equal to $r\alpha$, and the maximum sales it can seize can only be k units. This means that it is dominated for firm B to charge a price lower than $r\alpha/k$. Therefore, the equilibrium lower bound of price support for both firms is $L_o = \max\left\{\frac{r(\alpha+s)}{1-\alpha}, r\alpha/k\right\}$, because in equilibrium no firm wants to charge a price strictly lower than the competitor with probability one.

The firms' equilibrium payoffs are given by $\Pi_{Ao} = L_o(1 - \alpha)$ and $\Pi_{Bo} = L_o k$. The firm that is more aggressive in price cut can earn an equilibrium profit that is higher than what can be gained from its "guaranteed demand" by charging the highest possible price r (i.e., $\alpha + s$ for firm A and α for firm B, respectively).

Lemma 1 *In the benchmark case without capacity sharing under asymmetric capacity: (i) When* $1/2 < k < \min\left\{\frac{\alpha(1-\alpha)}{1-2\alpha}, 1-\alpha\right\}$, $L_o = r\alpha/k$, $\Pi_{Ao} = r\alpha(1-\alpha)/k > r(\alpha+s)$ *and* $\Pi_{Bo} = r\alpha$; *(ii) When* $\max\left\{1/2, \frac{\alpha(1-\alpha)}{1-2\alpha}\right\} < k < 1-\alpha$, $L_o = \frac{r(\alpha+s)}{1-\alpha}$, $\Pi_{Ao} = r(\alpha+s)$ *and* $\Pi_{Bo} = \frac{r(\alpha+s)k}{1-\alpha} > r\alpha$.

When firm B's capacity is sufficiently small (or α is high enough), its incentive for price cut would be lower than that of firm A. The equilibrium lower bound of price support is then given by $L_o = r\alpha/k$, and firm A earns an equilibrium profit higher than its guaranteed payoff. However, when firm B has enough capacity (or α is low enough), it would become more aggressive than firm A. This would lead to $L_o = \frac{r(\alpha+s)}{1-\alpha}$, and firm B's equilibrium payoff is higher than that from its ensured demand. In other words, the pricing incentive of the less aggressive firm determines the firms' common price support, whereas the more aggressive firm enjoys a pricing advantage over the rival. Intuitively, the equilibrium lower bound of price support increases in α and decreases in k.

Similar to the symmetric capacity scenario, firm A's equilibrium profit Π_{Ao} first decreases and then increases in α, and decreases in k. In contrast, firm B's equilibrium profit Π_{Bo} unambiguously increases in α, but can first increase and then decrease in k. Therefore, an increase in α has qualitatively different impacts on the firms' equilibrium profits here. This is because, as in the case of symmetric capacity, firm A can enjoy the demand from the rationed switchers when the rival is capacity constrained. This demand switching effect does not apply to firm B because firm A does not have the problem of capacity constraint. This asymmetry is also the driving force for the differential impacts of k on the firms' equilibrium profits. From firm A's perspective, a higher k implies that the competitor not only becomes more price aggressive but also has less rationed switchers. However, although an increase in firm B's capacity k makes firm A more aggressive in price competition by reducing the size of the rationed switchers, it can improve firm B's ability to satisfy its demand. The second effect is particularly relevant when firm B is more aggressive than firm A and hence is more likely to win the demand of the switchers. These two effects can interact with each other to yield the "inverted-U" impact on firm B's equilibrium profit.

4.2 Ex Ante Contracting

We start with solving for the firms' equilibrium pricing strategies in the second stage, conditional on the capacity transfer price λ. This would allow us to derive the firms' subgame-perfect equilibrium profits, $\Pi_A(\lambda)$ and $\Pi_B(\lambda)$. Similar to the scenario of symmetric capacity, there are two possible cases, depending on whether the capacity transfer price, λ, is lower or higher than the equilibrium lower bound of price support L. Consider first the case $L \geq \lambda$. The firms' expected profits of setting price p are, respectively,

$$\pi_A(p) = \int_L^p [p(\alpha + s) + \lambda w] \, dF_B(p_B) + \int_p^r p(1-\alpha) dF_B(p_B), \tag{6}$$

$$\pi_B(p) = \int_L^p p\alpha dF_A(p_A) + \int_p^r [pk + (p-\lambda)w] \, dF_A(p_A). \tag{7}$$

Firm A shares its capacity with firm B only when firm A charges a higher product price than firm B does. The equilibrium can be solved by applying the following conditions: $\pi_i'(p) = 0$, $F_i(L) = 0$, $i = A, B$, and $F_A(r) = 1 > F_B(r)$ or $F_B(r) = 1 > F_A(r)$.

Consider then the alternative case $L \leq \lambda$. A firm's expected profit depends on whether its charged price is lower or higher than the capacity transfer price. If the product price is $p \leq \lambda$,

$$\pi_A(p) = \int_L^p p(\alpha + s) dF_{B1}(p_B) + \int_p^\lambda p(1-\alpha) dF_{B1}(p_B) + \int_\lambda^r p(1-\alpha) dF_{B2}(p_B), \tag{8}$$

$$\pi_B(p) = \int_L^p p\alpha dF_{A1}(p_A) + \int_p^\lambda pk dF_{A1}(p_A) + \int_\lambda^r pk dF_{A2}(p_A). \tag{9}$$

If the product price is $p \geq \lambda$,

$$\pi_A(p) = \int_L^\lambda p(\alpha+s)dF_{B1}(p_B) + \int_\lambda^p [p(\alpha+s)+\lambda w]\, dF_{B2}(p_B) + \int_p^r p(1-\alpha)dF_{B2}(p_B), \tag{10}$$

$$\pi_B(p) = \int_L^\lambda p\alpha dF_{A1}(p_A) + \int_\lambda^p p\alpha dF_{A2}(p_A) + \int_p^r [pk+(p-\lambda)w]\, dF_{A2}(p_A). \tag{11}$$

Similarly, we can solve for the equilibrium by applying the following conditions: $\pi_i'(p) = 0$, $F_{i1}(L) = 0$, $F_{i1}(\lambda) = F_{i2}(\lambda)$, $i = A, B$, and $F_{A2}(r) = 1 > F_{B2}(r)$ or $F_{B2}(r) = 1 > F_{A2}(r)$.

Proposition 6 *In the second-stage price competition under the ex ante contracting scheme and under asymmetric capacity: (i) If $\lambda \le \frac{r(\alpha+s)}{1-\alpha-w}$, the lower bound of the equilibrium price support is $L_a = \frac{r(\alpha+s)+\lambda w}{1-\alpha}$, firm A's equilibrium profit $\Pi_A(\lambda) = L_a(1-\alpha)$ increases in λ, and firm B's equilibrium profit $\Pi_B(\lambda) = L_a k + (L_a - \lambda)w$ decreases in λ; (ii) If $\lambda \ge \frac{r(\alpha+s)}{1-\alpha-w}$, the lower bound of the equilibrium price support is $L_a = \max\left\{\frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)}, r\alpha/k, \frac{r(\alpha+s)}{1-\alpha}\right\}$, firm A's equilibrium profit $\Pi_A(\lambda) = L_a(1-\alpha)$ decreases (weakly) in λ, and firm B's equilibrium profit $\Pi_B(\lambda) = L_a k$ decreases (weakly) in λ.*

Similar to the symmetric capacity scenario, firm A's equilibrium profit first increases and then decreases in the capacity transfer price. This is also driven by a higher λ increasing the revenue, but decreasing the likelihood, of capacity lending. In comparison, firm B, the capacity borrower, is always hurt by an increasing capacity transfer price.

Recall that, under symmetric capacity, capacity sharing always softens market competition and improves the firms' profitability for any capacity transfer price. Does this intuitive result still hold under asymmetric capacity?

Proposition 7 *Under asymmetric capacity, firm A's equilibrium profit under the ex ante contracting scheme (i.e., $\Pi_A(\lambda)$) is lower than that without capacity sharing (i.e., Π_{Ao}), if and only if $1/2 < k < \min\left\{\frac{\alpha(1-\alpha)}{1-2\alpha}, 1-\alpha\right\}$ and λ is sufficiently low.*

Interestingly, this proposition suggests that capacity sharing under the ex ante contracting scheme can be harmful to firm A's profitability. There are two conditions for this surprising result to arise. The first condition is that firm B's capacity is sufficiently small (or the size of loyal buyers is large). This ensures that, as shown in Lemma 1, without capacity sharing firm B is less aggressive in price competition than firm A is and thus firm A can seize the demand from the switchers without overly cutting its price. This relative pricing advantage implies that firm A can make a higher profit than its guaranteed payoff $r(\alpha+s)$. However, capacity sharing increases firm A's expected payoff of maintaining high prices without enhancing its payoff of undercutting the competitor. In contrast, price cut becomes relatively more attractive to firm B because the increasing demand can be satisfied by borrowed capacity. Firm B would become more aggressive than firm A, and firm A would then lose its relative pricing advantage it enjoys in the case without capacity sharing. As a result, if this strategic effect is strong enough while the main effect of increasing the capacity transfer price is limited (i.e., λ is sufficiently low), capacity sharing can intensify competition and firm A would be worse off under the ex ante contracting scheme.

This surprising result stands in contrast to that under symmetric capacity. Both firms are always the same aggressive in price competition in the symmetric scenario, i.e., no firm has a higher incentive for price cut than the competitor does. Similarly, either firm can be the capacity lender or the borrower in the symmetric case, and capacity sharing does not change the symmetry in the firms' pricing aggressiveness. Therefore, capacity sharing can exert only the main effect of matching excessive supply with unserved demand, and thus can unambiguously soften competition for both firms. It is only when the firms are asymmetric in their incentives for price cut that capacity sharing may exert the strategic influence of reversing the relative aggressiveness between the firms. Nevertheless, note that firm B cannot be hurt by capacity sharing. Because capacity sharing is unilateral and can arise only if firm B charges a lower product price than firm A does, it can make the capacity lender less aggressive and the borrower more aggressive, but not vice versa. As a result, firm B can gain, but cannot lose, its relative pricing advantage over firm A.

This result suggests that firms should be cautious in setting their capacity transfer price. This is also a critical issue for scrutiny when the capacity transfer price is determined exogenously or by a third party. For example, in some regulated industry (e.g., telecommunication), the infrastructure sharing price between an incumbent with large capacity and an entrant with limited capacity is determined by the government. Our analysis above shows that the high-capacity incumbent may be hurt by capacity sharing arrangements if the capacity sharing price is too low.

We now determine the equilibrium capacity transfer price in the first stage of the game. When firm A is to make the offer, it is desirable to increase the capacity transfer price while maximizing the probability of capacity sharing. Note also that, for any $\lambda \in (0, r)$, firm B is weakly better off than in the case of no capacity sharing. This means that firm A will optimally set $\lambda = \frac{r(\alpha+s)}{1-\alpha-w}$, which will be accepted by firm B. It can be shown that firm A's optimal offer increases in α and decreases in k. Alternatively, when firm B gets the opportunity to make the proposal, it will offer the lowest possible capacity transfer price at which firm A is indifferent between acceptance and rejection, because firm B's profit $\Pi_B(\lambda)$ decreases in λ. As a result, if $1/2 < k < \min\left\{\frac{\alpha(1-\alpha)}{1-2\alpha}, 1-\alpha\right\}$, firm B has to set $\lambda = \frac{r[\alpha(1-\alpha)-(\alpha+s)k]}{wk}$. However, if $\max\left\{1/2, \frac{\alpha(1-\alpha)}{1-2\alpha}\right\} < k < 1-\alpha$, firm B will be able to set $\lambda = 0$. In other words, the optimal offer for firm B can be represented as $\lambda = \max\left\{\frac{r[\alpha(1-\alpha)-(\alpha+s)k]}{wk}, 0\right\}$.

4.3 Ex Post Contracting

The firms may engage in capacity sharing if and only if firm B charges a lower product price than firm A does. As a result, if $p_A \le p_B$, the firms' profits are $\pi_A = p_A(1-\alpha)$ and $\pi_B = p_B\alpha$. If $p_A \ge p_B$, firm A will offer a capacity transfer price that is equal to $\lambda = p_B$, whereas firm B will propose $\lambda = 0$. In anticipation of this, the firms' expected profits of setting price p in the first stage are, respectively,

$$\pi_A(p) = \int_L^p [p(\alpha+s) + p_B w/2]\, dF_B(p_B) + \int_p^r p(1-\alpha) dF_B(p_B), \tag{12}$$

$$\pi_B(p) = \int_L^p p\alpha dF_A(p_A) + \int_p^r (pk + pw/2) dF_A(p_A). \tag{13}$$

Similarly, we can apply the following conditions to solve for the equilibrium: $\pi_i'(p) = 0$, $F_i(L) = 0$, $i = A, B$, and $F_A(r) = 1 > F_B(r)$ or $F_B(r) = 1 > F_A(r)$.

Proposition 8 *Under the ex post contracting scheme and under asymmetric capacity, the equilibrium lower bound of price support is* $L_p = \max\left\{ r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}}, \frac{r\alpha}{k+w/2} \right\}$, *and the firms' equilibrium profits are* $\Pi_{Ap} = L_p(1-\alpha)$ *and* $\Pi_{Bp} = L_p(k+w/2)$, *respectively.*

The two components in L_p reflect firm A's and firm B's incentive to undercut the competitor, respectively. In comparison to the case without capacity sharing, firm A has a higher incentive to be the higher-priced firm, and thus the lowest price it is willing to charge becomes higher. In contrast, due to the gain from capacity sharing, now the payoff of charging a lower price than the rival is higher for firm B. In other words, the ex post contracting scheme makes firm A less aggressive and firm B more aggressive. Then what is the payoff implication of this differential change in the firms' pricing aggressiveness?

Proposition 9 *Under asymmetric capacity, firm A's equilibrium profit under the ex post contracting scheme,* Π_{Ap}, *can be lower than that without capacity sharing,* Π_{Ao}.

Interestingly, this proposition shows that firm A's equilibrium profit can actually be hurt by capacity sharing under the ex post contracting scheme. That is, even though lending excessive capacity to the competitor is ex post beneficial, it may reduce firm A's ex ante profitability. In the Appendix we present conditions for this counter-intuitive result to happen. As the proof implies, one necessary condition is that firm A is more aggressive than firm B in the case without capacity sharing and that the reverse is true under the ex post contracting scheme. It is only when the relative

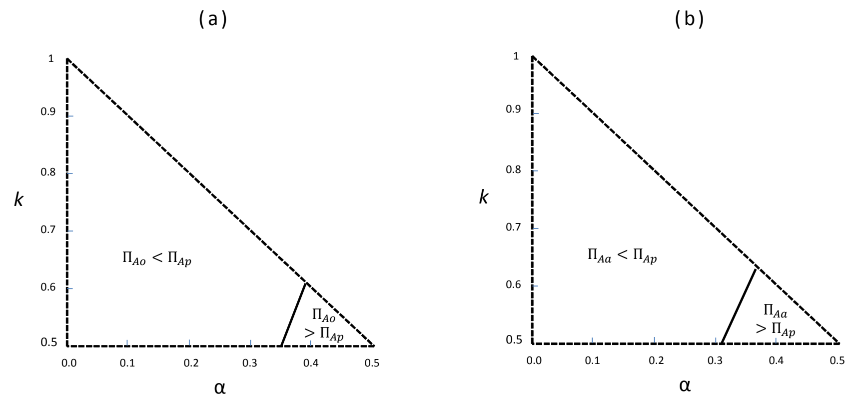

Figure 3: Comparison of Firm A's Equilibrium Profits under Asymmetric Capacity

pricing aggressiveness between the firms can be reversed that price competition can be intensified. Moreover, for the equilibrium lower bound of price support (and hence firm A's equilibrium payoff) to be lowered, it must be the case that firm B has a less compelling incentive for price cut in the case without capacity sharing than firm A does under the ex post contracting scheme. This may arise in the parameter range under which, as Figure 3 (a) demonstrates, α is sufficiently high. In contrast, firm B always benefits from capacity sharing. Intuitively, this is because the reversal in the firms' aggressiveness is only in one way but not the other. In other words, by becoming relatively more aggressive in price cut, a firm may hurt the rival but cannot hurt itself.

This differential impact of capacity sharing on the firms' pricing aggressiveness arises in the ex ante contracting scheme as well. It is also the driving force there for the result in Proposition 7 that capacity sharing can be harmful for firm A if the capacity transfer price is too low. However, when the firms can ex ante determine the capacity transfer price, no firm will accept an offer that makes it worse off than the outside option of no capacity sharing. This means that, under the equilibrium capacity transfer price, the harmful impact of capacity sharing can be shunned (i.e., $\Pi_{Aa} > \Pi_{Ao}$). Nevertheless, Proposition 9 suggests that, when the firms cannot commit to the capacity sharing price, the firm with excessive capacity may be hurt.

4.4 Ex Ante Versus Ex Post Contracting

Figure 3 presents the comparison of firm A's equilibrium profits. Recall that the ex ante contracting scheme always leads to a higher equilibrium profit than that without capacity sharing, whereas firm A may be worse off under ex post contracting than under no capacity sharing. Similar to the symmetric capacity case, firm A's preference for ex ante versus ex post contracting is determined by the difference in the ex ante expected capacity transfer prices. This is because in equilibrium capacity sharing always takes place under either contracting scheme. Similar forces also influence the difference in the ex ante expected capacity transfer prices. In particular, firm A desires to commit to the highest possible capacity transfer price that maximizes the chance of capacity sharing, whereas the capacity transfer price under ex post contracting is set conditional on the product prices. Therefore, firm A's optimal capacity sharing price under ex ante contracting is more sensitive to the market parameters than the expected capacity sharing price under ex post contracting. As a result, similar to Figure 2 (a), firm A is more likely to prefer to ex ante determine the capacity transfer price when the market is relatively less competitive (e.g., α is sufficiently high).

There are, however, some notable differences from the symmetric case. First, asymmetric capacity may change the firms' preferences for the capacity transfer price under ex ante contracting but not under ex post contracting. Note that, when the firms are symmetric, both of them can be the capacity borrower as well as the lender. Because at the time of pre-competition contracting the firms do not know yet which firm will be the borrower or the lender, they share the same preference to maintain the committed capacity transfer price. This differs from the asymmetric case, in which the role of the firms in capacity sharing is pre-determined and firm B can only be the capacity borrower but can never be the lender. This means that, firm B prefers to minimize the capacity sharing price as much as possible under the ex ante contracting scheme. Nevertheless, asymmetric capacity does not make this difference under ex post contracting. At the time the firms determine the capacity transfer price after the product prices are set, under either symmetric or asymmetric capacity, they know which firm is the seller or the buyer of excessive capacity. In other words, it is only under ex ante contracting, but not under ex post contracting, that asymmetric capacity can modify the firms' knowledge about their relative role in capacity sharing. This then constitutes a negative force to drive down the expected capacity transfer price under ex ante contracting relative to that under the ex post contracting scheme.

The second difference is that firm A has more capacity under the asymmetric capacity scenario. In comparison to the symmetric case, a higher capacity makes firm A more aggressive in price cut. This also increases firm B's aggressiveness, because it can sell only to its loyal buyers (i.e., no rationed switchers for firm A) if it charges a higher price than the rival. As a result, both firms are more aggressive under asymmetric capacity. Nevertheless, the increasing competitiveness has a stronger impact on ex ante contracting than on the ex post contracting scheme. This is, again, because the equilibrium capacity transfer price and the lower bound of price support under ex ante contracting are more responsive to changes in the competitiveness of the market.

Because of these two differences, firm A may have an increasing preference for the ex post contracting scheme. This can explain why, as the comparison between Figure 2 (a) and Figure

3 (b) shows, the parameter range under which firm A prefers ex post contracting is wider in the asymmetric case than under symmetric capacity. For example, when k converges to 1/2, ex ante contracting is always preferred for any α in Figure 2 (a), but ex post contracting may lead to a higher equilibrium payoff for small α in Figure 3 (b).

In contrast, firm B always earns a larger equilibrium profit under the ex post contracting scheme than under the ex ante contracting scheme (i.e., $\Pi_{Bp} > \Pi_{Ba}$). This can be explained by the more intense market competition under asymmetric capacity as well, which has a stronger impact on ex ante contracting. This result also suggests that the firms may have conflicting preferences for the timing of contracting over the capacity transfer price. Even though the capacity lender may prefer to pre-determine the capacity transfer price, the borrower may desire to postpone it until the market competition is cleared. The competing firms would need to be more cautious in aligning their interests about when they should engage in the determination of the capacity sharing price.

5 Concluding Remarks

5.1 Welfare Analysis

Collaborations among competitors, especially those that engage in agreements, are standard cases for Antitrust investigation. The general guideline is to gauge competitive collaborations under the rule of reason to determine the competitive and welfare implications. Therefore, a welfare analysis of competitive capacity sharing is useful to address legal issues that may potentially arise.

Our focus in this research is on the efficiency-improving role of capacity sharing. When capacity can be shared between competing firms, industry resources can be more efficiently allocated by matching excessive supply with unmet demand. In particular, stockout may lead some loyal buyers to be rationed and to leave the market without purchase, which would not arise if capacity sharing is feasible. As a result, in our setup capacity sharing is always socially beneficial. For example, in the symmetric capacity scenario, capacity sharing can increase social welfare from $r(1-w)$ to r.

However, capacity sharing may not always increase buyer surplus. This is because capacity sharing may mitigate price competition and thus increase firm profitability. Consider first the symmetric capacity scenario. Recall that, without capacity sharing, each firm's equilibrium profit is $r(\alpha+s)$. Under the ex ante contracting scheme, this is increased to $r(\alpha+s)+\lambda^* w$, resulting in a change in buyer surplus by $rw - 2\lambda^* w$, where $\lambda^* = \frac{r(\alpha+s)}{k-w}$. It can be readily verified that buyer surplus is improved under the ex ante contracting scheme if and only if the firms' endowed capacity k is sufficiently high. Intuitively, as shown in Proposition 3, if k is low enough and hence the market is not very competitive, the firms would be able to commit to a sufficiently high capacity transfer price λ^*. The firms' gain from capacity sharing would then exceed that of social welfare, thus leading to a lower buyer surplus. In contrast, under the ex post contracting scheme, even if k is very low, the expected capacity sharing price would not be overly high. We can numerically verify that, in this case buyer surplus is always greater than that without capacity sharing.

The impact of capacity sharing on buyer surplus is less unambiguous under the asymmetric capacity scenario. On one hand, the improvement in social welfare is lower than that under symmetric capacity. This is because now only the loyal buyers of firm B, but not those of firm A, may be rationed by stockout. On the other hand, as shown in Propositions 7 and 9, capacity sharing between asymmetric firms may intensify price competition and hurt the profitability of the firm with excessive capacity (i.e., firm A). Nevertheless, we can perform numerical analysis to show that, under either contracting scheme, capacity sharing can improve buyer surplus unless the number of loyal buyers is intermediate and the endowed capacity k is sufficiently low.

5.2 Summary and Future Research

Cooperation between competitors through capacity/inventory sharing is a prevalent phenomenon in many retail and manufacturing settings. However, the interaction between competition and capacity sharing is not well understood in the existing literature. In this paper we fill this gap by investigating capacity trade between two firms that engage in price competition. We examine optimal strategies and firm profitability of competitive capacity sharing under two alternative schemes, depending on whether the firms agree over the price of capacity sharing before or after product price setting. Our analysis yields some interesting results with substantial practical relevance.

When the firms have symmetric capacity, they share the same preference to commit to a capacity sharing price that is high enough while maximizing the likelihood of sharing. This means that the optimal capacity transfer price should take into account the influence on subsequent competition, and be equal to the lowest possible product price the firms anticipate to charge in equilibrium. We confirm that capacity sharing can indeed soften competition and lead to higher equilibrium profit under either contracting scheme. We also find that ex ante contracting leads to equilibrium outcomes that are more responsive to changes in market parameters, and thus tends to be more appealing than ex post contracting in less competitive markets (e.g., less capacity or higher buyer loyalty). Nevertheless, both the optimal sharing price and the equilibrium profit can be non-monotonically influenced by the number of loyal buyers.

Our analysis reveals that these insights on competitive capacity sharing can be altered by asymmetric capacity. It is not surprising that firms with different capacity may have differential incentives about what capacity sharing price to commit. However, despite the gain from improving capacity efficiency, capacity sharing can counter-intuitively intensify equilibrium competition and hurt the profitability of the firm without capacity constraint. This surprising result arises when capacity sharing reverses the relative pricing aggressiveness between the firms and makes the less competitive firm under no capacity sharing more competitive.

An important feature of our current model is that a firm's role in capacity sharing is endogenously determined by price competition. That is, depending on the relative prices between the competitors, a firm may become either the capacity borrower or the capacity lender. Nevertheless, the prospect of capacity sharing is certain in the sense that, conditional on the outcome of price competition, it is common knowledge which firm will be in stockout and which firm has excess capacity to lend. If demand is stochastic instead, there would exist (extra) uncertainty about the firms' roles in capacity sharing. That is, a firm charging relatively lower product price may not necessarily be short of capacity, and conversely, a firm with relatively higher price may not have excess capacity. Considering this extension can definitely make the model more realistic and expand the scope of research issues, while we believe that the main insights derived in current research would continue to hold.

We consider exogenous search behavior for the switchers in the case of stockout. That is, we assume that once the switchers are rationed and the inventory is not furnished immediately, they will turn to the competitor to make the purchase. This would be a reasonable assumption if the rationed switchers are unaware or ignorant of the prospect of capacity sharing between the competing firms. Nevertheless, in scenarios when capacity sharing between competitors is prevalent, rationed buyers may strategically decide whether and when to search for alternative firms. This endogenous search behavior may then influence the firms' optimal timing for the sharing of capacity. We leave this interesting but challenging problem for future research.

There are several extensions that can be investigated in future research. One interesting issue is regarding the role of firm differentiation in strategic capacity sharing. It is assumed in the current paper that the product transferred from the competitor has the same value as own product. New insights may arise if this assumption is relaxed and the firms are horizontally or vertically differentiated such that the bought capacity involves differential utility for the buyers.

Future research can investigate other aspects of capacity sharing that have not been addressed in the current paper. For example, firms can negotiate and commit to not only the price, but also the volume, of capacity sharing. It may also be interesting to consider non-linear contracts such as two-part tariff. Another interesting extension is to endogenize the timing of bargaining (Guo and Iyer 2013). For example, the outside option of breakdown for ex ante contracting is not the no-sharing case, but the ex post contracting case. Finally, future studies can consider virtual capacity sharing without the actual transfer of property rights. That is, firms can pool their capacity together that can be used to satisfy any participating firm's demand. This is particularly relevant for service industries (e.g., airlines) in which orders can be taken through advance reservation.

Appendix

Proof of Proposition 2: (i) Consider the first possible case $L \geq \lambda$. The firms' expected profit of setting the price p is given by

$$\begin{aligned} \pi(p) &= \int_L^p [p(\alpha+s)+\lambda w] f(p')dp' + \int_p^r [pk+(p-\lambda)w] f(p')dp' \\ &= pF(p)(\alpha+s-w-k) + 2\lambda wF(p) + p(w+k) - \lambda w. \end{aligned}$$

Taking the first order derivative with respect to p yields $\pi'(p) = [F(p)+pf(p)](\alpha+s-w-k) + 2\lambda wf(p)+w+k = 0$. Solving this differential equation by using the boundary condition $F(r) = 1$ leads to $F(p) = \frac{r(\alpha+s)+2\lambda w-(w+k)p}{2\lambda w+(\alpha+s-w-k)p}$.

Furthermore, we can impose the condition $F(L) = 0$ to solve for the lower bound of price support $L = \frac{r(\alpha+s)+2\lambda w}{w+k}$. It follows that the condition $L \geq \lambda$ is equivalent to $\lambda \leq \frac{r(\alpha+s)}{k-w}$.

The firms' equilibrium profit in this case is $\Pi(\lambda) = r(\alpha+s)+\lambda w$. It is straightforward that $\frac{d\Pi(\lambda)}{d\lambda} = w > 0$. Thus, $\Pi(\lambda)$ increases in λ for $\lambda \leq \frac{r(\alpha+s)}{k-w}$.

(ii) Consider then the alternative case $L \leq \lambda$. The firms' profit by setting the price $p \geq \lambda$ is:

$$\begin{aligned} \pi(p) &= \int_L^\lambda p(\alpha+s)f_1(p')dp' + \int_\lambda^p [p(\alpha+s)+\lambda w] f_2(p')dp' + \int_p^r [pk+(p-\lambda)w] f_2(p')dp' \\ &= pF_2(p)(\alpha+s-w-k) + 2\lambda wF_2(p) + p(w+k) - [1+F_1(\lambda)]\lambda w. \end{aligned}$$

Taking the first order derivative with respect to p yields $\pi'(p) = [F_2(p)+pf_2(p)](\alpha+s-w-k) + 2\lambda wf_2(p)+w+k = 0$. Solving this differential equation by using the boundary condition $F_2(r) = 1$ leads to $F_2(p) = \frac{r(\alpha+s)+2\lambda w-(w+k)p}{2\lambda w+(\alpha+s-w-k)p}$.

The firms' profit by setting the price $p \leq \lambda$ is:

$$\begin{aligned} \pi(p) &= \int_L^p p(\alpha+s)f_1(p')dp' + \int_p^\lambda pkf_1(p')dp' + \int_\lambda^r pkf_2(p')dp' \\ &= pF_1(p)(\alpha+s-k)+pk. \end{aligned}$$

Taking the first order derivative of $\pi(p)$ with respect to p leads to $[F_1(p)+pf_1(p)](\alpha+s-k)+k = 0$. Solving this differential equation by using the boundary condition $F_1(\lambda) = F_2(\lambda)$ leads to $F_1(p) = \frac{k}{k-\alpha-s} + \frac{(\alpha+s)[r(\alpha+s-k)+\lambda w]}{(k-\alpha-s)(k-\alpha-s-w)p}$.

We impose the condition $F_1(L) = 0$ to solve for the lower bound of price support $L = \frac{(\alpha+s)[r(\alpha+s-k)+\lambda w]}{k(\alpha+s+w-k)}$. It follows that the condition $L \leq \lambda$ is equivalent to $\lambda \geq \frac{r(\alpha+s)}{k-w}$.

The firms' equilibrium profit in this case is $\Pi(\lambda) = Lk = r(\alpha+s) + \frac{(\alpha+s)(r-\lambda)w}{k-\alpha-s-w}$. Therefore, we have $\frac{d\Pi(\lambda)}{d\lambda} = \frac{(\alpha+s)w}{\alpha+s+w-k} < 0$. Thus, $\Pi(\lambda)$ decreases in λ for $\lambda \geq \frac{r(\alpha+s)}{k-w}$.

Proof of Proposition 3: Note that $\lambda^* = \frac{r(\alpha+s)}{k-w}$, where $s = (1-2\alpha)\left(1-\frac{k}{1-\alpha}\right)$ and $w = \alpha\left(1-\frac{k}{1-\alpha}\right)$. Therefore, $\frac{d\lambda^*}{d\alpha} = \frac{r(2k-1)\left[k-(1-\alpha)^2\right]}{(k-\alpha+\alpha^2)^2}$. Thus, λ^* decreases in α for $\alpha < 1-\sqrt{k}$, and increases in α for $\alpha > 1-\sqrt{k}$. Moreover, $\frac{d\lambda^*}{dk} < 0$, because both s and w decrease in k.

Note that $\Pi_a = L_a k = \lambda^* k$. It follows that $\frac{d\Pi_a}{d\alpha} = k\frac{d\lambda^*}{d\alpha}$. Therefore, Π_a first decreases in α for $\alpha < 1-\sqrt{k}$ and then increases in α for $\alpha > 1-\sqrt{k}$. Note also that $\Pi_a \equiv \Pi(\lambda^*) = r(\alpha+s)+\frac{r(\alpha+s)}{k-w}w = \frac{r(\alpha+s)k}{k-w}$. Therefore, we have $\frac{d\Pi_a}{dk} = r\frac{-(1-2\alpha)k^2+2\alpha(1-\alpha)(1-2\alpha)k-\alpha(1-\alpha)^3}{(k-\alpha+\alpha^2)^2}$. Let $\Omega = -(1-2\alpha)k^2 + 2\alpha(1-\alpha)(1-2\alpha)k - \alpha(1-\alpha)^3$. It can be readily verified that Ω is a concave function of k, with the maximum value achieved at $k = \alpha(1-\alpha)$. Moreover, $\Omega|_{k=\alpha(1-\alpha)} = -\alpha(1-\alpha)^2(1-2\alpha+2\alpha^2) < 0$. Therefore, $\frac{d\Pi_a}{dk} < 0$ for all k.

Proof of Proposition 4: Note that $\Pi_a - \Pi_o = \frac{r(\alpha+s)}{k-w}w$. We have $\frac{d(\Pi_a-\Pi_o)}{d\alpha} = \frac{\left[(1-\alpha)^2-k\right]r\Gamma}{(1-\alpha)^2[k-\alpha(1-\alpha)]^2}$, where $\Gamma = (-2\alpha^2+4\alpha-1)k^2 + (3\alpha^2-4\alpha+1)k + \alpha^2(1-\alpha)^2$. Note that $\Gamma|_{k=1/2} = (1-2\alpha+2\alpha^2)^2/4 > 0$ and $\Gamma|_{k=1-\alpha} = \alpha(1-\alpha)^3 > 0$. The derivative of Γ with respect to k, evaluated at $k = 1/2$, is equal to $\alpha^2 > 0$. It follows that $\Gamma > 0$ for all $k \in (1/2, 1-\alpha)$, because the second order derivative of Γ with respective to k is independent of k. Therefore, $\frac{d(\Pi_a-\Pi_o)}{d\alpha} > 0$ for $\alpha < 1-\sqrt{k}$, and $\frac{d(\Pi_a-\Pi_o)}{d\alpha} < 0$ for $\alpha > 1-\sqrt{k}$. Moreover, $\frac{d(\Pi_a-\Pi_o)}{dk} < 0$, because both s and w decrease in k.

Proof of Proposition 5: To compare Π_a with Π_p, we just need to compare $I_a = \frac{k}{k-w}$ with $I_p = \left(\frac{\alpha+s}{k+w/2}\right)^{\frac{w/2}{\alpha+s-w/2-k}}$, where $s = (1-2\alpha)(1-k/(1-\alpha))$ and $w = \alpha(1-k/(1-\alpha))$.

Evaluating $I_a - I_p$ at $k = 1/2$ leads to $(I_a - I_p)|_{k=1/2} = \frac{1-\alpha}{1-2\alpha(1-\alpha)} - \left(\frac{2-4\alpha+4\alpha^2}{2-\alpha-2\alpha^2}\right)^{-\frac{1}{3}}$, which can be shown to be strictly positive for $\alpha \in (0, 1/2)$. By the continuity of $I_a - I_p$ in k, we then prove that $I_a - I_p > 0$ when k is sufficiently close to $1/2$.

We have $\frac{\partial(I_a-I_p)}{\partial k}|_{k=1-\alpha} = \frac{\alpha}{2(1-2\alpha)(1-\alpha)^2}\left[4\alpha - 2 - (1-\alpha)\log(\alpha/(1-\alpha))\right]$. It can be readily verified that $4\alpha - 2 - (1-\alpha)\log(\alpha/(1-\alpha))$ is convex, its first order derivative evaluated at $\alpha = 1/2$ is positive, and its value evaluated at $\alpha = 0$ is positive and evaluated at $\alpha = 1/2$ is zero. Therefore, there must exist an $\alpha' \in (0, 1/2)$ such that $\frac{\partial(I_a-I_p)}{\partial k}|_{k=1-\alpha} > 0$ for $0 < \alpha < \alpha'$ and $\frac{\partial(I_a-I_p)}{\partial k}|_{k=1-\alpha} < 0$ for $\alpha' < \alpha < 1/2$. It then follows from $(I_a - I_p)|_{k=1-\alpha} = 0$ that, for k sufficiently close to $1-\alpha$, $I_a - I_p < 0$ if $0 < \alpha < \alpha'$, and $I_a - I_p > 0$ if $\alpha' < \alpha < 1/2$.

Proof of Lemma 1: Following Narasimhan (1988), the equilibrium lower bound of price support is given by $L_o = \max\left\{\frac{r(\alpha+s)}{1-\alpha}, r\alpha/k\right\}$. Note that $\frac{r(\alpha+s)}{1-\alpha} > r\alpha/k$ if and only if $\Delta(k) = -\frac{1-2\alpha}{1-\alpha}k^2 + (1-\alpha)k - \alpha(1-\alpha) > 0$. $\Delta(k)$ is concave in k. Solving $\Delta(k) = 0$ leads to two solutions $k_1 = 1-\alpha$ and $k_2 = \frac{\alpha(1-\alpha)}{1-2\alpha}$. Therefore, $\Delta(k) < 0$ if and only if $1/2 < k < \min\left\{\frac{\alpha(1-\alpha)}{1-2\alpha}, 1-\alpha\right\}$. In this case we have $L_o = r\alpha/k$, and the firms' equilibrium profits are $\Pi_{Ao} = L_o(1-\alpha) = r\alpha(1-\alpha)/k > r(\alpha+s)$ and $\Pi_{Bo} = L_o k = r\alpha$. In addition, $\Delta(k) > 0$ if and only if $\max\left\{1/2, \frac{\alpha(1-\alpha)}{1-2\alpha}\right\} < k < 1-\alpha$, leading to $L_o = \frac{r(\alpha+s)}{1-\alpha}$, and the firms' equilibrium profits $\Pi_{Ao} = L_o(1-\alpha) = r(\alpha+s)$ and $\Pi_{Bo} = L_o k = \frac{r(\alpha+s)k}{1-\alpha} > r\alpha$.

Note that $\frac{\alpha(1-\alpha)}{1-2\alpha} < 1/2$ if and only if $\alpha < 1-\sqrt{2}/2$, and $\frac{\alpha(1-\alpha)}{1-2\alpha} < 1-\alpha$ if and only if $\alpha < 1/3$. Thus $1/2 < \frac{\alpha(1-\alpha)}{1-2\alpha} < 1-\alpha$ if and only if $\alpha \in \left(1-\sqrt{2}/2, 1/3\right)$.

Proof of Proposition 6: (i) Consider the first possible case $L \geq \lambda$. The firms' expected profits of setting the price p are given by

$$\begin{aligned}
\pi_A(p) &= \textstyle\int_L^p \left[p(\alpha+s)+\lambda w\right] dF_B(p_B) + \int_p^r p(1-\alpha) dF_B(p_B) \\
&= pF_B(p)(2\alpha+s-1) + \lambda w F_B(p) + p(1-\alpha), \\
\pi_B(p) &= \textstyle\int_L^p p\alpha dF_A(p_A) + \int_p^r \left[pk+(p-\lambda)w\right] dF_A(p_A) \\
&= pF_A(p)(\alpha-w-k) + \lambda w F_A(p) + pk + (p-\lambda)w.
\end{aligned}$$

Setting $F_A(L) = F_B(L) = 0$ leads to $\pi_A(L) = L(1-\alpha)$ and $\pi_B(L) = Lk+(L-\lambda)w$. Substituting into $\pi_i(p) = \pi_i(L)$, $i = A, B$, yields $F_B(p) = \frac{(p-L)(1-\alpha)}{p(1-2\alpha-s)-\lambda w}$ and $F_A(p) = \frac{(p-L)(w+k)}{p(w+k-\alpha)-\lambda w}$, respectively. As a result, $F_A(r) = 1$ leads to $L = \frac{r\alpha+\lambda w}{w+k}$, and $F_B(r) = 1$ leads to $L = \frac{r(\alpha+s)+\lambda w}{1-\alpha}$.

Suppose $F_A(r) = 1$. Then $L = \frac{r\alpha+\lambda w}{w+k} \geq \lambda$ is equivalent to $\lambda \leq r\alpha/k$, and $F_B(r) < 1$ is equivalent to $\lambda > \frac{r(1-2\alpha)k}{\alpha(1-\alpha-k)}$. But because $r\alpha/k < \frac{r(1-2\alpha)k}{\alpha(1-\alpha-k)}$, $F_A(r) = 1$ is impossible.

Therefore, we must have $F_B(r) = 1 > F_A(r)$ in equilibrium. This implies the equilibrium lower bound of price support is $L_a = \frac{r(\alpha+s)+\lambda w}{1-\alpha}$. The condition $L \geq \lambda$ leads to $\lambda \leq \frac{r(\alpha+s)}{1-\alpha-w}$, which can also guarantee that $F_A(r) < 1$ (i.e., $\lambda < \frac{r(1-2\alpha)k}{\alpha(1-\alpha-k)}$). The firms' equilibrium profits are $\Pi_A(\lambda) = L_a(1-\alpha)$ and $\Pi_B(\lambda) = L_a k + (L_a - \lambda)w$, respectively. It is straightforward that $\frac{d\Pi_A(\lambda)}{d\lambda} > 0$ and $\frac{d\Pi_B(\lambda)}{d\lambda} < 0$.

(ii) Consider then the alternative case $L \leq \lambda$. If the price is $p \leq \lambda$, the firms' expected profits are

$$\begin{aligned}
\pi_A(p) &= \textstyle\int_L^p p(\alpha+s)dF_{B1}(p_B) + \int_p^\lambda p(1-\alpha)dF_{B1}(p_B) + \int_\lambda^r p(1-\alpha)dF_{B2}(p_B) \\
&= pF_{B1}(p)(2\alpha+s-1) + p(1-\alpha), \\
\pi_B(p) &= \textstyle\int_L^p p\alpha dF_{A1}(p_A) + \int_p^\lambda pk dF_{A1}(p_A) + \int_\lambda^r pk dF_{A2}(p_A) \\
&= pF_{A1}(p)(\alpha-k) + pk.
\end{aligned}$$

Setting $F_{A1}(L) = F_{B1}(L) = 0$ leads to $\pi_A(L) = L(1-\alpha)$ and $\pi_B(L) = Lk$. Substituting into $\pi_i(p) = \pi_i(L)$, $i = A, B$, yields $F_{B1}(p) = \frac{(p-L)(1-\alpha)}{p(1-2\alpha-s)}$ and $F_{A1}(p) = \frac{(p-L)k}{p(k-\alpha)}$, respectively. As a result, $F_{A1}(\lambda) = \frac{(\lambda-L)k}{\lambda(k-\alpha)}$ and $F_{B1}(\lambda) = \frac{(\lambda-L)(1-\alpha)}{\lambda(1-2\alpha-s)}$.

If the price is $p \geq \lambda$, the firms' expected profits are

$$\begin{aligned}
\pi_A(p) &= \int_L^\lambda p(\alpha+s)dF_{B1}(p_B) + \int_\lambda^p \left[p(\alpha+s)+\lambda w\right] dF_{B2}(p_B) + \int_p^r p(1-\alpha)dF_{B2}(p_B) \\
&= pF_{B2}(p)(2\alpha+s-1) + \lambda w \left[F_{B2}(p) - F_{B2}(\lambda)\right] + p(1-\alpha), \\
\pi_B(p) &= \int_L^\lambda p\alpha dF_{A1}(p_A) + \int_\lambda^p p\alpha dF_{A2}(p_A) + \int_p^r \left[pk + (p-\lambda)w\right] dF_{A2}(p_A) \\
&= pF_{A2}(p)(\alpha-w-k) + \lambda w F_{A2}(p) + pk + (p-\lambda)w.
\end{aligned}$$

Evaluating the profit functions at $p = \lambda$ leads to $\pi_A(\lambda) = \lambda(2\alpha+s-1)F_{B2}(\lambda) + (1-\alpha)\lambda$ and $\pi_B(\lambda) = \lambda(\alpha-k)F_{A2}(\lambda) + \lambda k$. Substituting into $\pi_i(p) = \pi_i(\lambda)$, $i = A, B$, yields $F_{B2}(p) = \frac{(p-\lambda)(1-\alpha)+\lambda(1-2\alpha-s-w)F_{B2}(\lambda)}{p(1-2\alpha-s)-\lambda w}$ and $F_{A2}(p) = \frac{(p-\lambda)(w+k)+\lambda(k-\alpha)F_{A2}(\lambda)}{p(w+k-\alpha)-\lambda w}$, respectively. These can be simplified, by substituting $F_{B2}(\lambda) = F_{B1}(\lambda) = \frac{(\lambda-L)(1-\alpha)}{\lambda(1-2\alpha-s)}$ and $F_{A2}(\lambda) = F_{A1}(\lambda) = \frac{(\lambda-L)k}{\lambda(k-\alpha)}$. As a result, $F_{A2}(r) = 1$ leads to $L = r\alpha/k$, and $F_{B2}(r) = 1$ leads to $L = \frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)}$.

Suppose $F_{B2}(r) = 1$. Then $L = \frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)} \leq \lambda$ is equivalent to $\lambda \geq \frac{r(\alpha+s)}{1-\alpha-w}$, and $F_{A2}(r) < 1$ is equivalent to $\lambda < \tilde{\lambda} \equiv \frac{r\left[ks(1-s)-3\alpha ks+(1-k)\alpha^2-\alpha^3\right]}{k(\alpha+s)w}$. It can be shown that $\tilde{\lambda} > \frac{r(\alpha+s)}{1-\alpha-w}$. Suppose $F_{A2}(r) = 1$. Then $L = r\alpha/k$, and $F_{B2}(r) < 1$ is equivalent to $\lambda > \tilde{\lambda}$. It can be shown that $\tilde{\lambda} > r\alpha/k$. Therefore, if $\frac{r(\alpha+s)}{1-\alpha-w} \leq \lambda \leq \min\left\{\tilde{\lambda}, r\right\}$, the equilibrium lower bound of price support is $L_a = \frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)}$, which is decreasing in λ; if $\tilde{\lambda} \leq \lambda \leq r$, the equilibrium lower bound of price support is $L_a = r\alpha/k$.

Note that $\frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)} \geq r\alpha/k$ if and only if $\lambda \leq \tilde{\lambda}$, $\tilde{\lambda} \leq r$ if and only if $\frac{r(\alpha+s)}{1-\alpha} \leq r\alpha/k$, and $\frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)}$ converges to $\frac{r(\alpha+s)}{1-\alpha}$ when λ goes to r. This means that the equilibrium in the case when $\lambda \geq \frac{r(\alpha+s)}{1-\alpha-w}$ can be summarized as $L_a = \max\left\{\frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)}, r\alpha/k, \frac{r(\alpha+s)}{1-\alpha}\right\}$.

The firms' equilibrium profits are $\Pi_A(\lambda) = L_a(1-\alpha)$ and $\Pi_B(\lambda) = L_a k$, respectively. It is straightforward that $\frac{d\Pi_A(\lambda)}{d\lambda} \leq 0$ and $\frac{d\Pi_B(\lambda)}{d\lambda} \leq 0$, because L_a decreases (weakly) in λ.

Proof of Proposition 7: Note from Lemma 1 that firm A's equilibrium profit without capacity sharing is $\Pi_{Ao} = r\alpha(1-\alpha)/k > r(\alpha+s)$ if and only if $1/2 < k < \min\left\{\frac{\alpha(1-\alpha)}{1-2\alpha}, 1-\alpha\right\}$. From Proposition 6, if $\lambda \leq \frac{r(\alpha+s)}{1-\alpha-w}$, firm A's equilibrium profit under the ex ante contracting scheme is $\Pi_A(\lambda) = r(\alpha+s) + \lambda w$, which is less than $\Pi_{Ao} = r\alpha(1-\alpha)/k$ if λ is sufficiently low.

It is also straightforward that, if $\lambda \geq \frac{r(\alpha+s)}{1-\alpha-w}$, we always have $\Pi_A(\lambda) = L_a(1-\alpha) \geq \Pi_{Ao} = L_o(1-\alpha)$, because then $L_a = \max\left\{\frac{(\alpha+s)[r(1-2\alpha-s)-\lambda w]}{(1-\alpha)(k-\alpha)}, r\alpha/k, \frac{r(\alpha+s)}{1-\alpha}\right\} \geq L_o = \max\left\{\frac{r(\alpha+s)}{1-\alpha}, r\alpha/k\right\}$.

Proof of Proposition 8: The firms' expected profits of setting the price p are given by

$$\begin{aligned}
\pi_A(p) &= \textstyle\int_L^p \left[p(\alpha+s)+p_B w/2\right] dF_B(p_B) + \int_p^r p(1-\alpha) dF_B(p_B) \\
&= pF_B(p)(2\alpha+s+w/2-1) + p(1-\alpha) - \textstyle\int_L^p wF_B(p_B)/2dp_B, \\
\pi_B(p) &= \textstyle\int_L^p p\alpha dF_A(p_A) + \int_p^r (pk+pw/2) dF_A(p_A) \\
&= pF_A(p)(\alpha - w/2 - k) + p(k+w/2).
\end{aligned}$$

Taking the first order derivative with respect to p yields $\pi'_A(p) = F_B(p)(2\alpha+s-1)+pf_B(p)(2\alpha+s+w/2-1)+1-\alpha = 0$ and $\pi'_B(p) = F_A(p)(\alpha-w/2-k)+pf_A(p)(\alpha-w/2-k)+k+w/2 = 0$, respectively. Solving these differential equations, by using the boundary conditions $F_B(L)=0$ and $F_A(L)=0$, respectively, leads to $F_B(p) = \frac{1-\alpha}{2\alpha+s-1}(\frac{p}{L})^{-\frac{1-2\alpha-s}{1-2\alpha-s-w/2}} + \frac{1-\alpha}{1-2\alpha-s}$ and $F_A(p) = \frac{(p-L)(k+w/2)}{p(k+w/2-\alpha)}$. As a result, $F_A(r)=1$ leads to $L = \frac{r\alpha}{k+w/2}$, and $F_B(r)=1$ leads to $L = r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}}$.

Therefore, if $r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}} > \frac{r\alpha}{k+w/2}$, in equilibrium we have $F_B(r) = 1 > F_A(r)$ and $L = r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}}$. If $r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}} < \frac{r\alpha}{k+w/2}$, in equilibrium we have $F_A(r) = 1 > F_B(r)$ and $L = \frac{r\alpha}{k+w/2}$. In sum, the equilibrium lower bound of price support is given by $L_p = \max\left\{r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}}, \frac{r\alpha}{k+w/2}\right\}$. The firms' equilibrium profits follow immediately by substituting $F_A(L_p) = F_B(L_p) = 0$ into the profit function $\Pi_{ip} = \pi_i(L_p)$, $i = A, B$, respectively.

Proof of Proposition 9: Note that $\Pi_{Ap} = L_p(1-\alpha)$ and $\Pi_{Ao} = L_o(1-\alpha)$. As a result, $\Pi_{Ap} < \Pi_{Ao}$ if and only if $L_p = \max\left\{r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}}, \frac{r\alpha}{k+w/2}\right\} < L_o = \max\left\{\frac{r(\alpha+s)}{1-\alpha}, r\alpha/k\right\}$. It is straightforward that $r\left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}} > \frac{r(\alpha+s)}{1-\alpha}$ and $\frac{r\alpha}{k+w/2} < r\alpha/k$. This means that $L_p < L_o$ if and only if $\Delta = \left(\frac{\alpha+s}{1-\alpha}\right)^{\frac{1-2\alpha-s-w/2}{1-2\alpha-s}} - \alpha/k < 0$.

Evaluating Δ at $k = 1/2$ leads to $\Delta|_{k=1/2} = \left(\frac{1-2\alpha+2\alpha^2}{2(1-\alpha)^2}\right)^{1-\alpha/2} - 2\alpha$, which goes to $1/2$ as α converges to 0, and goes to 0 as α converges to $1/2$. The derivative of $\Delta|_{k=1/2}$ with respect to α, as α converges to $1/2$, is positive. It can be verified that the second order derivative of $\Delta|_{k=1/2}$ with respect to α is positive. Therefore, when $k \to 1/2$, $\Delta < 0$ if and only if α is sufficiently high.

Alternatively, evaluating Δ at $k = 1-\alpha$ leads to $\Delta|_{k=1-\alpha} = 0$. Taking the derivative of Δ with respect to k and evaluating it at $k = 1-\alpha$ leads to $\frac{d\Delta}{dk}|_{k=1-\alpha} = \frac{\alpha^2\log\left(\frac{\alpha}{1-\alpha}\right)-2+10\alpha-12\alpha^2}{2(1-2\alpha)(1-\alpha)^2}$. The denominator is always positive, and denote the numerator as $N = \alpha^2\log\left(\frac{\alpha}{1-\alpha}\right) - 2 + 10\alpha - 12\alpha^2$. It can be readily verified that $N|_{\alpha=0} < 0$, $N|_{\alpha=1/2} = 0$, $\frac{dN}{d\alpha}|_{\alpha=1/2} = -1$, and $\frac{d^2N}{d\alpha^2} = 2\log\left(\frac{\alpha}{1-\alpha}\right) - \frac{(3-4\alpha)(7-6\alpha)}{(1-\alpha)^2} < 0$. This proves that $N > 0$ if and only if α is above certain threshold. Therefore, when $k \to 1-\alpha$, $\Delta < 0$ if and only if α is sufficiently high.

References

[1] Anupindi, R., Y. Bassok, E. Zemel. 2001. A general framework for the study of decentralized distribution systems. *Manufacturing and Service Operations Management*, 3(4), 349-368.

[2] Apostolides, Z. 2014. Mazda officially opens Salamanca plant in Mexico. *Automotive Logistics.*

[3] Baake, P., J. Oechssler, C. Schenk. 1999. Explaining cross-supplies. *Journal of Economics*, 70(1), 37-60.

[4] Bidault, F., M. Schweinsberg. 1996. Fiat and Peugeot's SevelNord Venture (A). IMD-3-0644.

[5] Brandenburger A.M., B.J. Nalebuff. 1996. *Co-opetition*. New York: Currency Doubleday.

[6] Brandenburger A., H. Stuart. 2007. Biform games. *Management Science*, 53(4), 537-549.

[7] Caldieraro, F. 2016. The role of brand image and product characteristics on firms' entry and OEM decisions. *Management Science*, Forthcoming.

[8] Chen, Y. 2010. Strategic outsourcing between rivals. *Annals of Economics and Finance*, 11(2), 301-311.

[9] Comez, N., K. E. Stecke, M. Cakanyildirim 2012. In-season transshipments among competitive retailers. *Manufacturing and Service Operations Management*, 14(2), 290-300.

[10] Eisenstein, P.A. 2011. BMW, PSA Peugeot Citroen form battery car joint venture. *The Detroit Bureau*.

[11] Granot, D., G. Sošić. 2003. A three-stage model for a decentralized distribution system of retailers. *Operations Research*, 51(5), 771-784.

[12] Gurnani H., E. Murat, Y. Luo. 2007. Impact of product pricing and timing of investment decisions on supply chain co-opetition. *European Journal of Operational Research*, 180(1), 228-248.

[13] Guo, L., G. Iyer. 2013. Multilateral bargaining and downstream competition. *Marketing Science*, 32(3), 411430.

[14] Honeywell. 2010. Honeywell and DuPont announce joint venture to manufacture new automotive refrigerant. *Honeywell Press Release*.

[15] Hu, X., I. Duenyas, R. Kapuscinski. 2007. Existence of coordinating transshipment prices in a two-location inventory model. *Management Science*, 53(8), 1289-1302.

[16] Hu, X., R. Caldentey, G. Vulcano. 2013. Revenue sharing in airline alliances. *Management Science*, 59(5), 1177-1195.

[17] Huang, X., G. Sošić. 2011. Transshipment of inventories: Dual allocations vs. transshipment prices. *Manufacturing and Service Operations Management*, 12(2), 299-318.

[18] Ihlwan, M. 2006. Samsung and Sony's win-win LCD venture. *BloombergBusinessweek*.

[19] Jolly, D. 1997. Co-operation in a niche market: The case of fiat and PSA in multi purpose vehicles. *European Management Journal*, 15(1), 35-44.

[20] Li, L., R.Q. Zhang. 2015. Cooperation through capacity sharing between competing forwarders. *Transportation Research Part E*, 75, 115-131.

[21] Narasimhan, C. 1988. Competitive promotional strategies. *Journal of Business*, 61(4), 427-449.

[22] Reuters. 2012. UPDATE 1-Mazda to produce Toyota vehicles at its Mexico plant.

[23] Rudi, N., S. Kapur, D.F. Pyke. 2001. A two-location inventory model with transshipment and local decision making. *Management Science*, 47(12), 1668-1680.

[24] Shulman, J.D. 2014. Product diversion to a direct competitor. *Marketing Science*, 33(3), 422-436.

[25] Spiegel, Y. 1993. Horizontal subcontracting. *RAND Journal of Economics*, 24(4), 570-590.

[26] Tirole, J. 1988. *The Theory of Industrial Organization*. Cambridge: The MIT Press.

[27] Van Mieghem, J. A. 1999. Coordinating investment, production and subcontracting. *Management Science*, 45(7), 954-971.

[28] Varian, H.R. 1980. A model of sales. *American Economic Review*, 70(4), 651-659.

[29] Wassmer, U., P. Dussauge, M. Planellas. 2010. How to manage alliances better than one at a time. *MIT Sloan Management Review*, 51(3), 76-85.

[30] Wu, X., P. Kouvelis, H. Matsuo. 2013. Horizontal capacity coordination for risk management and flexibility: Pay ex ante or commit a fraction of ex post demand? *Manufacturing and Service Operations Management*, 15(3), 458-472.

[31] Wu, X., P. Kouvelis, H. Matsuo, H. Sano. 2014. Horizontal coordinating contracts in the semiconductor industry. *European Journal of Operational Research*, 237(3), 887-897.

[32] Yu, Y., S. Benjaafar, Y. Gerchak. 2015. Capacity sharing and cost allocation among independent firms with congestion. *Production and Operations Management*, 24(8), 1285-1310.