Modern Portfolio Theory with Private Equity[1]

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Investors are constantly looking for new profitable investment opportunities with the objective of seeking risk-adjusted excess returns (alphas) and diversifying their portfolios. Entrepreneurs actively seek new sources of capital and more efficient ways of financing their ventures. Globalization significantly enlarges the set of investable assets for investors and expands sources of capital for entrepreneurs. Private Equity (PE) emerged from a cottage industry in the U.S. in the 1970s and over the last several decades with increasing globalization, has become a very important financial intermediary bridging the demand and supply of capital by entrepreneurs and investors. It is important to recognize that there is substantial heterogeneity among investors (consider elite university endowments versus small family offices), as they may face very different investment opportunity sets, have very different existing asset-and-liability structures, and have very different investment horizons and demand for liquidity assets.

Finance as a modern academic field of business started with the Nobel-prize winning work by Harry Markowitz (1952) on asset allocation, which was later generalized to dynamic frameworks in 1969 and 1971 by Paul Samuelson and Robert Merton, two pioneering figures in Economics and Finance and Nobel laureates.[2] Markowitz, Merton, and Samuelson provide an elegant and operational paradigm that makes a simple recommendation to risk-averse investors on how to allocate wealth between the risky market portfolio and the risk-free asset. The industry's standard asset allocation, the 60-40 rule, which recommends a mix of 60% allocation in the stock market and 40% allocation in bonds, is premised on the Markowitz-Merton-Samuelson asset allocation paradigm.

However, modern portfolio theory in its original form faces serious criticisms. A simple one-

[1] This is the white paper for my Chazen grant proposal "Global Entrepreneurship and Private Equity." I acknowledge financial support by the Chazen Institute of International Business at Columbia Business School.

[2] Markowitz, Harry. "Portfolio selection." The journal of finance 7.1 (1952): 77-91. Samuelson, Paul A. "Lifetime portfolio selection by dynamic stochastic programming." The review of economics and statistics (1969): 239-246. Merton, Robert C. "Optimum consumption and portfolio rules in a continuous-time model." Journal of economic theory 3.4 (1971): 373-413.

size-fits-all asset allocation recommendation is undesirable and misleading with potentially significant damaging consequences for investors.

First, the strong demand for private equity by institutional investors is itself a very strong argument for us to revisit the standard 60-40 rule, which by its construction rules out illiquid assets in the asset allocation framework. Additionally, ignoring illiquid assets is inconsistent with equilibrium given the importance of the multi-trillion-dollar private equity market. Since 2013, PE funds raised $500 billion annually worldwide and uninvested dry powder today stands at a record $1.3 trillion.[3] Therefore, any asset allocation framework and recommendations have to seriously take into account the importance of private equity and other illiquid assets.

Some very successful institutional investors have aggressively deviated from the 60-40 rule. David Swensen, the Chief Investment Officer of Yale University since 1985, has consistently and successfully followed an asset allocation strategy that substantially overweighs equity, illiquid investments such as venture capital (VC) and private equity, and actively invests in less efficient markets seeking alpha. Swensen's investment strategy has become known as **the Endowment Model**. In the past three decades, Yale University's Endowment has returned 13.9% per annum significantly exceeding standard performance evaluation benchmarks by wide margins.[4] Yale's allocation is heavily tilted towards leveraged buyouts (LBOs), VC, hedge funds, real estate, and other illiquid assets. For example, Yale's actual portfolio allocation in June 2015 included 32.5% in private equity (with about half in LBOs and the other half in VC), 20.5% in hedge funds, 14% in real estate, and 6.7% in natural resources implying that 73.7% of the entire portfolio in 2015 was in illiquid or semi-illiquid assets.

While Yale Endowment is by no means a representative institutional investor, (e.g. its superior access to top-tier PE and VC managers and its early entry into the VC and other illiquid asset and less inefficient markets,) it is suggestive that we should develop models to better understand the economics of investing in illiquid assets.

Endowment Model was seriously challenged during the financial crisis. Harvard University's endowment fund, the largest university endowment in the world, lost 22% in value between July 1 and October 31, 2008.[5] A major criticism about Harvard Management Company during the financial crisis was its liquidity mismanagement caused by its large exposures to illiquid investments.

With these issues in mind, Jinqiang Yang and I ask the following research questions in our working paper:

1. What are the implied investors' beliefs about the risk-adjusted excess returns and

[3] See Bain and Company's Global Private Equity Report 2016.
[4] Yale Endowment 2015. See http://investments.yale.edu/images/documents/Yale_Endowment_15.pdf
[5] Ang, Andrew. "Liquidating Harvard." Columbia Case Works (2011).

diversification benefits that PE and other illiquid alternative assets are expected to deliver in order for investors to allocate a significant fraction of assets into these alternatives?

2. Is Endowment Model intellectually sound or has Yale University been lucky for thirty years?

3. What are the optimal spending rates for investors with different asset-liability structures, e.g., pension funds and university endowments?

4. How should investors optimally manage their liquidity risk, which is clearly important as we see from the recent financial crisis?

5. How do management and incentive fees influence investors' exposures to PE and other illiquid assets?

6. What are the implications for investors who do not have access to good investment opportunities? Should they follow the standard 60-40 allocation?

Building on our earlier work with M. Sorensen[6] and also our theoretical work with C. Wang,[7] Jinqiang and I develop an analytically tractable dynamic asset allocation model to answer these questions. Starting with the Markowitz-Samuelson-Merton framework, we incorporate PE as the third asset into the portfolio optimization problem. Unlike standard publicly traded liquid stocks and bonds, investors generally cannot dynamically rebalance their investments in illiquid assets without incurring transaction costs, which can often be substantial. Anticipating the *ex post* illiquidity exposures, investors *ex ante* prudently choose their allocation to PE. The standard investment strategy for PE assets is buy-and-hold. A typical PE fund has a life span of 10 years with a few potential one-year extension options. It is worth noting that markets for PE secondary investments have become much more active since the recent financial crisis. The emergence of this market provides some liquidity into the PE market.

Additionally, capital calls and distributions made by general partners (GPs), who manage PE investments on behalf of investors, who are also referred to as limited partners (LPs), are stochastic and tend to correlate with financing conditions, which generate additional risk exposures to investors' ability to manage their liquidity and market risks.

We show that an investment allocation such as the Yale Endowment's Target Portfolio may be

[6] Sorensen, Morten, Neng Wang, and Jinqiang Yang. "Valuing private equity." Review of Financial Studies 27.7 (2014): 1977-2021.
[7] Wang, Chong, Neng Wang, and Jinqiang Yang. "Optimal consumption and savings with stochastic income and recursive utility." Journal of Economic Theory, 165, (2016): 292-331.

potentially justified if Swensen and his team believe that their PE assets can deliver risk-adjusted excess returns, i.e., alphas, north of 4% per annum, which is roughly in line with the reported excess returns by Yale, Harvard, and other top-tier university endowment funds following the Endowment Model. To further compensate for the risks caused by stochastic distributions and calls, which further make liquidity management for the investment portfolio challenging, investors on average demand an additional 1%-2% risk premium per annum. Of course, investors who do not have good access to skilled managers and have large liquidity needs should not invest much in PE or other illiquid assets.

In addition to focusing on investors' investment opportunities, the existing liability structures and current and future spending needs also have first-order effects on asset allocation decisions. We show that pension funds should be much more conservative than Family Offices and High Net-worth Individuals when allocating to PE and other illiquid assets. This is due to the fact that pension funds have much less flexible liabilities (retirees' pension payments are not deferrable while luxurious consumption is discretionary and can be flexibly adjusted.) Pension funds should divide their portfolio into two sub-portfolios: the liability-hedging portfolio which is essentially invested in risk-free assets and the investment portfolio. Importantly, the liability-hedging portfolio and investment portfolio are interdependent.

Finally, investing in PE and illiquid assets often requires delegation to outside managers, who charge substantial management fees and performance-dependent carried interests. How should investors trade off the benefits of attracting and motivating talented managers with the costs of compensating them? How costly are these compensation structures? We leave these questions for future research.

In summary, we develop a dynamic asset allocation framework that builds on the Modern Portfolio Theory and incorporates Private Equity and other illiquid assets. Our research suggests that whether the Endowment Model, which significantly weighs PE and other illiquid assets in the portfolio, is appropriate or not for investors critically depends on investors' abilities to find risk-adjusted excess returns and their abilities to manage illiquidity risk.