On the Coordination Role of Stress Test Disclosure in Bank Risk

Taking

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Abstract

This paper identifies a cost of stress test disclosure that stems from the facilitating role that stress test information can potentially play in coordinating banks into taking excessive risk. We examine a model of risk taking by banks, the concurrent failure of which may induce a bailout decision by a regulator if the number of failing banks is large enough. We find a “if ain’t broken don’t fix it” result. In particular, we find that the disclosure of stress tests decreases the banks’ average risk taking if the regulator is prone to bail banks out to begin with. However, if the regulator has weak incentives to bail out banks, disclosing stress tests increases banks’ average risk taking.

1 Introduction

One of the measures adopted by the Federal Reserve in response to the financial crises of 2008 was to institute the public disclosure of stress tests to assess and publicly certify the stability and
resilience of the largest banks in the US. Stress tests are intended to expose the extent to which a bank is robust enough to endure a set of adverse macroeconomic scenarios and remain capable of performing its lending operations. The advocates of stress tests argue that the disclosure of stress test information allows markets to discipline banks’ behavior, thereby promoting financial stability. However, others have argued that such disclosure may have costly consequences. Goldstein and Sapra (2014), for instance, review potential problems that may be caused by the disclosure of stress tests, such as the distortion of the interbank market, the triggering of bank runs, etc. In this paper we argue that there is an additional cost of stress test disclosure that stems from the very nature of stress test information. In particular, stress test information may facilitate the coordination of banks into a concurrent failure forcing the regulator to bail them out to avoid the externalities associated with a massive bank failure.

In October of 2008, the Troubled Asset Relief Program (TARP) was signed into law, releasing $700 billion for the purpose of acquiring failing bank assets. In 2011, Bloomberg reported that by March 2009, adding up guarantees and lending limits, the Federal Reserve had committed $7.77 trillion to rescuing the financial system. The bailout of the financial system prevented a likely collapse of the economy into a major depression. The magnitude of the externalities associated with a massive bank failure left little choice to the FED. However, such inexorability raises the issue of whether banks had anticipated the bailout and that anticipation created the problem in the first place. Several papers have examined the perverse coordination incentives induced by the prospect of a bailout (Arya and Glover, 2006; Acharya and Yorulmazer, 2007; Farhi and Tirole, 2011). The prospect of a bailout upon the concurrent failure of multiple banks reduces the expected loss of failure for banks and, therefore, induces banks to coordinate their actions making such concurrent failure more likely. However, such coordination may not be an easy feat unless there is an external mechanism that facilitates it. In this paper, we argue that stress-test disclosure is precisely one
such possible mechanism.

We examine a setting with a continuum of banks taking risk decisions in their lending portfolio. They face the surveillance of a bank regulatory institution in the form of a stress test. All banks experience two shocks, a macroeconomic shock that affects all banks equally, and a bank specific shock. In addition each bank can either be vulnerable to the overall shock or not, which is tested by a stress test. The stress test result reveals whether the bank will be solvent in such a worst case scenario. The regulator decides ex ante whether to disclose stress tests for all banks to the public. After that, banks decide the risk of their investments. Once the outcomes of the investments are realized, some banks that obtain a bad outcome may fail. At that point, the regulator must decide whether to bail banks out. The cost of not bailing out a bank includes the externalities for the economy induced by the bank’s collapse. The social cost of bank failures is convex in the number of banks that fail simultaneously. Therefore, there is a threshold on the number of failing banks above which the regulator is compelled to bail out the banks.

We find that if the regulator does not disclose the stress tests, banks assess the probability of being bailed out according to their own information. They asses the probability that enough banks will fail simultaneously, and make their risk decisions accordingly. In a scenario without stress-test disclosure, banks take excessive risk with respect to what is socially optimal, but there is no multiplicity of equilibria since coordination between banks is hard. If stress tests are disclosed, however, they help banks assess the probability of a bail out with much more information about other banks. Indeed, stress-test disclosure becomes a coordination device that induces multiplicity of equilibria. In one of the equilibria, banks take a lower risk than without the stress-test disclosure, but in the other equilibrium banks take a lot more risk than without the stress-test disclosure. The reason is simple, if all banks expect the other banks to take excessive risk, the expected number of failing banks increases, and that increases the probability of a bailout. That, in turn, makes risk
much less costly for banks, making the riskier decision the optimal choice.

To obtain comparative statics results and regulatory insights we obtain a unique equilibrium resorting to a global games approach. With this approach, in the stress-test disclosure scenario, we obtain an equilibrium characterized by a threshold on the number of banks that fail the stress test. Below that threshold banks coordinate on low risk decisions, but above that threshold banks coordinate on high risk decisions. We take an ex-ante perspective and compare the average risk taking behavior of banks between the non-disclosure and the disclosure scenarios. We find a “if it ain’t broken don’t fix it” result. In particular, we find that the disclosure of stress tests decreases the banks’ average risk taking if the regulator is prone to bail banks out to begin with. That is, in a scenario in which the social cost of bank failures is very convex in the number of failing banks, or the cost of a bailout is low, disclosing stress tests curbs banks’ risk taking. However, if the regulator has weak incentives to bail out banks, disclosing stress tests increases banks’ average risk taking. The intuition behind this result can be grasped by gauging the relative impact of a bad stress-test outcome versus a good stress-test outcome on the coordination of banks’ risk decisions. If the regulator is already prone to bail banks out, disclosing that a large number of banks fail the stress test is not going to make much of a difference in inducing banks to take high risk. However, if the number of failing banks is small, this will deter banks from taking a high risk decision. In the opposite situation, if the regulator is quite reticent to bail banks out, a low amount of stress test failures does not make much of a difference in mitigating high risk bank decisions. However, if the number of failing banks is large it will induce banks to coordinate into taking a high risk decision.

The organization of the paper is the following. In Section 2 we review the related literature. In Section 3 we explain the model setup. In Section 4 we analyze the model by first considering the scenario without stress tests. Then we consider the scenario with stress tests, and apply the global games approach. Finally we compare the banks’ risk taking decision across scenarios. In Section 5
we conclude the paper.

2 Literature Review

Our paper contributes to the stress-test disclosure literature. Goldstein and Sapra (2014) provide a thorough review of the benefits and costs of disclosing stress-test examined in the extant literature. While stress-test disclosure is argued to have the benefits of improving market discipline and mitigating regulatory forbearance, Goldstein and Sapra also highlight four potential costs. The first cost is due to the “Hirshleifer effect,” that is, public disclosure may destroy risk-sharing opportunities among banks and impair the functioning of interbank markets (Hirshleifer, 1971; Goldstein and Leitner, 2013). Second, disclosure may lead to sub-optimal myopic decisions by banks through a “real-effect” channel of amplifying short-term price pressure (Gigler, Kanodia, Sapra and Venugopalan, 2014). Third, disclosure causes overweighting of the disclosed public information by banks’ investors in economies with strategic complementarity, which in turn may trigger inefficient panic-based bank runs (Morris and Shin, 2002). Lastly, through a “feedback effect” channel, disclosure may interfere with regulators’ learning from market prices for supervisory purposes, thus decreasing the efficiency of regulatory intervention (Bond and Goldstein, 2015). Our paper contributes to this literature by identifying a cost of stress-test disclosure that has not been previously studied. This cost stems from the very nature of stress test disclosure that disseminates information among not only investors in banking markets but also banks themselves. In particular, we show that this improvement in information sharing among banks may facilitate the coordination into an equilibrium of both excessive risk-taking by banks and excessive bailouts by regulators.

More broadly, our paper is related to the literature on information disclosure in the banking industry. Due to the size of this literature, as partly evidenced by the number of surveys, we refer
readers to three recent surveys by Goldstein and Sapra (2014), Beatty and Liao (2014) and Acharya and Ryan (2015). Besides the benefits and costs of disclosure discussed above, this literature also examines a number of other potential costs associated with disclosure, none of which is similar to the one identified in our paper. For instance, Plantin, Sapra and Shin (2008) argue that disclosure of assets’ fair value leads to fire-sale, which causes downward spirals in assets prices and solvency problems for otherwise-sound banks. Prescott (2008) shows that disclosing bank supervisory information publicly may reduce banks’ incentive to share the information with regulators in the first place. Corona, Nan and Zhang (2015) find that the disclosure of higher quality accounting information may encourage excessive risk-taking by banks through exacerbating competition in the deposit market.

Second, at the core of our mechanism is an interaction between stress-test disclosure, banks’ coordination incentives and regulators’ tendency to bail out. The literature of collective moral hazard has studied the perverse coordination incentives induced by the prospect of a bailout (Arya and Glover, 2006; Acharya and Yorulmazer, 2007; Farhi and Tirole, 2011). A key driving force in this literature is that bailouts occur more likely with the concurrent failure of multiple banks than of a single bank. As a result, in order to maximize the expected gains from bailouts, banks have incentives to coordinate to take high risk to make the concurrent failure more likely. However, such coordination may not be an easy feat unless there is an external mechanism that facilitates it. Our paper contributes to the literature by identifying one such mechanism through stress-test disclosure. We show that absent stress-test disclosure, risk-taking coordination is hindered by banks’ lack of information about each other such that banks take relatively small amounts of risk and the regulator bails out with a low frequency; with disclosure, however, coordination is greatly facilitated by the dissemination of bank-specific information across all banks. In particular, when bad stress-test results are disclosed, it produces an aggravating effect that coordinates banks into
taking maximal amounts of risk and forces the regulator to bail out with certainty.

3 The Model Setup

We consider a banking industry with a continuum of banks, indexed by \( i \in [0, 1] \). Each bank \( i \) is endowed with an investment project. The outcome of the project depends on the bank’s risk choice \( S_i \in [0, 1] \), the macroeconomic state \( \omega \in \{G, B\} \), and the idiosyncratic state of the bank \( \theta_i \in \{H, L\} \). One can think of the macroeconomic state as the macroeconomic conditions of the environment in which banks operate, such as the unemployment rate, the GDP growth, etc. The idiosyncratic state \( \theta_i \) represents a unique characteristic of each bank \( i \) that reflects how well the bank can endure bad macroeconomic conditions. For instance, some banks may have large liquidity buffers that allow them to withstand a crisis, while other banks with smaller liquidity buffers may not survive. We sometimes also refer to \( \theta_i \) as the bank \( i \)’s stress-test information, as each bank’s stress-test reflects its ability to survive a crisis. We assume that the probability of a good macroeconomic state is \( q, \Pr(\omega = G) = q \), and that \( \theta_i \) is independent across banks with \( \Pr(\theta_i = L) = \tilde{p} \sim U[0, 1] \). That is, the probability that bank \( i \) is in the idiosyncratic state \( L \) is a uniformly distributed random variable itself. Accordingly, a portion of banks \( \tilde{p} \) will be in state \( \theta_i = L \). For convenience, we call a bank with \( \theta_i = H \) a high-type bank and one with \( \theta_i = L \) a low-type bank.

For simplicity we assume that, if the macroeconomic state is good (i.e., \( \omega = G \)), any bank \( i \)’s project succeeds and generates a cash flow of \( S_i \), regardless of the bank’s idiosyncratic state. If the macroeconomic state is bad (i.e., \( \omega = B \)), the outcome of the project depends on the bank’s risk choice, \( S_i \), and the bank’s idiosyncratic state, \( \theta_i \). In particular, with probability \( 1 - S_i \), the project succeeds and generates a cash flow \( S_i \). With probability \( S_i \), the project fails and its outcome is contingent on the bank’s idiosyncratic state: if \( \theta_i = H \), the project generates a cash flow \( K > 0 \), which is the minimum capital that a bank needs to survive; if \( \theta_i = L \), the project generates a zero
cash flow, and that leads to a bank failure unless the bank obtains an external capital injection of $K$. There are several ways to justify the assumption that a bank cannot survive without an external capital injection. For example, as discussed in Goldstein and Leitner (2015), a bank may have a debt liability of $K$, and if the bank is unable to repay the debt, it will lead to bank failure. Alternatively, a bank may face a run if its cash holdings fall below some threshold $K$.

Bank failures are costly to the economy. We assume that bank failures have negative externalities on the economy, and capture the aggregation of such externalities with a function $C(n)$, where $n \in [0, 1]$ is the portion of failing banks. We often refer to $C(.)$ as the social cost of bank failures, and assume $C(0) = 0$, $C(1) = \infty$, $C'(0) = 0$, $C'(n) > 0$, and $C''(n) > 0$ for all $n \in [0, 1]$. Notice that the last assumption implies an increasing marginal cost of bank failures. This assumption captures the general idea that the failure of a bank is more costly the more banks fail concurrently.

To avoid the social cost of bank failures, the regulator can make an injection $K$ into each failing bank (henceforth, a “bailout”), but the bailing out a failing bank entails another social cost of $\lambda K$, with $\lambda > 0$, which we often refer to as the cost of a bailout. This social cost of a bailout can arise, for instance, as a consequence of the distortions produced by the increase in taxation required to collect the bailout funds. Alternatively, a bailout can increase government debt to the extent of potentially placing the government itself under financial distress. For simplicity, we assume that the bailout policy cannot be targeted, in the sense that, if the regulator decides to bail out banks, it has to bail out every failing bank. This assumption is descriptive of the set of bailout actions taken in the 2008-2009 crisis, such as the Term Auction Facility (TAF) which provides a liquidity backstop for all major banks, the Capital Purchase Program (CPP), with which the government injected capital into banks without solicitation, and the guarantee of short-term debt and transaction deposits of all insured banks by the Federal Deposit Insurance Corporation (FDIC). In our

\footnote{It is not necessary to make any assumption about the size of $K$ because, as it will be clear later on, in equilibrium the expected return upon success is always larger than that upon a failure.}
model, the regulator decides whether to bail out banks or not, minimizing the sum of the social cost of bank failures and the cost of bailouts.

Each bank observes its own idiosyncratic state $\theta_i$ but not the idiosyncratic states of other banks. In addition, each bank is required to communicate its idiosyncratic state $\theta_i$ to the regulator. In reality, the Federal Reserve works together with each bank to obtain and verify the bank’s stress-test information, which is represented by the idiosyncratic state $\theta_i$ in our model. Therefore, we assume that each bank’s report of its idiosyncratic state $\theta_i$ to the regulator is truthful. In our analysis, we focus on characterizing the equilibria in two scenarios. In the first scenario, the regulator does not disclose the idiosyncratic states to the public. That is, it does not disclose the stress tests. We call this scenario the “no-disclosure scenario.” In the second scenario, the regulator publicly discloses the idiosyncratic states of all banks, the stress tests, which we denote by $\{\theta_i\}_{i \in [0,1]}$. We call this second scenario the “disclosure scenario.” We further assume that, if the regulator discloses the stress-test results, it does so truthfully.

The time-line of the model is illustrated in Figure 1. At date 0, each bank communicates its stress-test information (that is, its idiosyncratic state $\theta_i$) to the regulator, and the stress-test results are disclosed publicly only in the disclosure scenario. At date 1, each bank chooses its risk level $S_i$. At date 2, the macroeconomic state and the banks’ project outcomes are realized and publicly observed, and the regulator decides whether to bail out failing banks.

4 The Analysis

We analyze the model by backward induction, starting from date 2. At date 2, upon the realization of the macroeconomic state as well as the project outcomes, the regulator makes the decision of whether to bail out failing banks by examining the trade off between the social cost of bank failures, $C(n)$, and the total cost of bailouts, $n\lambda K$. The regulator only bails out banks if the social cost
of bank failures is larger than the necessary bailout costs. Since the social cost of bank failures is convex in $n$, the regulator bails out failing banks if $n$ is sufficiently large. We summarize this result in the Lemma below.

**Lemma 1** *Conditional on the portion of failing banks $n$, the regulator bails out banks if and only if* $n \geq \hat{n}$, *where* $\hat{n}$ *is given by*

$$C(\hat{n}) = \hat{n}\lambda K.$$

Lemma 1 suggests that the regulator has a stronger incentive to bail out (i.e., has a lower $\hat{n}$) if the cost of bailing out a bank is small (i.e., $\lambda K$ is small) or the social cost of bank failure is very convex (i.e., $C''(.)$ is large). Henceforth, the functional form of the social cost function $C(.)$ is only important to the extent that it determines $\hat{n}$. This makes our analysis very robust to the specification and nature of the costs associated with bank failures and bailouts.

To avoid corner solutions in risk taking behavior we assume $\hat{n} < \frac{1}{2(1-\eta)} < 1 - \frac{K}{\eta}$. The first inequality ensures that the regulator always bails banks out with positive probability, and the second inequality ensures that risk decisions are interior solutions. If these inequalities are not satisfied, we may obtain that for some parameter regions the equilibrium degenerates into either
a scenario in which the regulator never bails out banks, or a scenario in which all banks take the maximum risk. In either case, these extreme equilibria are unrealistic and uninteresting, and we choose to omit their analysis for brevity.

4.1 The Bank’s First-Best Risk Decision Benchmark

Before characterizing the equilibrium risk decisions in the two scenarios previously introduced, we obtain the risk decisions that maximize social welfare in the absence of private information and, henceforth, refer to them as first-best risk decisions. In this benchmark, the idiosyncratic states of all banks are publicly observable and, thus, the realized portion of low-type banks, denoted by $p$, is also observable. The social welfare is defined to be the expected aggregate cash flows generated by all banks net of the social cost of bank failures as well as the cost of bailouts. The social welfare can be formally expressed as follows:

$$
(1 - p) \left\{ qS_H + (1 - q) \left[ (1 - S_H) S_H + S_H K \right] \right\} + p \left\{ qS_L + (1 - q) \left[ (1 - S_L) S_L + S_L I_{pS_L \geq \hat{n}} K \right] \right\} - (1 - q) \left\{ (1 - I_{pS_L \geq \hat{n}}) C(pS_L) + I_{pS_L \geq \hat{n}} pS_L (1 + \lambda)K \right\},
$$

where $S_H$ and $S_L$ represent the high-type and low-type banks’ risk choices respectively. The first term in braces in (1) represents the expected cash flow generated by high-type banks. Since by assumption high-type banks never fail, there are no bank failure costs nor bailout costs associated with these banks. A proportion $1 - p$ of banks are of high type. They obtain a cash flow of $S_H$ if the macroeconomic state turns out to be good (i.e., $\omega = G$), which happens with probability $q$. Otherwise, in a bad macroeconomic state, they obtain a cash flow of $S_H$ if their project succeeds, which happens with probability $1 - S_H$, and a cash flow of $K$ if their project fails. The last two terms
in braces in (1) represent the expected welfare impact of low-type banks. The indicator function in these terms, \( I_{n \geq \hat{n}} \), takes a value of 1 if \( n \geq \hat{n} \) and a value of 0 otherwise. The second term in braces reflects the expected cash flow obtained by low-type banks. If the macroeconomic state turns out to be good, all low-type banks obtain a cash flow of \( S_L \). Otherwise, in a bad macroeconomic state (i.e., \( \omega = B \)), their project succeeds with probability \( 1 - S_L \), obtaining a cash flow \( S_L \), and fails with probability \( S_L \), rendering a zero cash flow. Indeed, in this case, if the project fails, the bank also fails. Yet, if the proportion of failing banks, \( n = pS_L \), is larger than \( \hat{n} \), the regulator bails all failing banks out by injecting a capital \( K \) in each of them. The third term in braces reflects the social cost of bank failures and bailouts. If the macroeconomic state turns out to be good (i.e., \( \omega = G \)), banks never fail and the social cost of bank failures and bailouts are zero. If \( \omega = B \), low-type banks may fail and, the regulator must decide whether to bail them out or not. From Lemma 1, we know that it is socially optimal to bail out failed banks if and only if the portion of failed banks \( n \) is above the threshold \( \hat{n} \). Thus, if \( n = pS_L < \hat{n} \) the regulator does not bail banks out and incurs a social cost of bank failures of \( C(pS_L) \). Otherwise, the regulator bails banks out incurring a cost of \( pS_L (1 + \lambda)K \). The cost of bailing out banks comprises the aggregate capital injection \( pS_L K \) to failed banks, plus an additional cost \( pS_L \lambda K \). While the capital injection is simply a transfer and thus welfare neutral, the latter cost is a deadweight loss and is incurred because of the distortion costs associated with bailouts.

Welfare as expressed by (1) is maximized by the risk choices \( S_{FB}^H \) and \( S_{FB}^L \), which denote the first-best choices of the high-type and low-type banks respectively. Obtaining the first order conditions from expression (1), it is straightforward to verify that the high-type bank’s first-best risk choice has the expression,

\[
S_{FB}^H = \frac{1}{2(1 - q)} + \frac{K}{2},
\]
and the low-type bank’s first-best risk choice satisfies,

\[ S_{FB}^{L} < \frac{1}{2(1-q)}. \]

That is, the low-type bank’s first-best risk choice is always lower than \( \frac{1}{2(1-q)} \). To understand this, notice that if there were no bank failure costs nor bail-out costs, then the optimal risk level for the low-type bank would be \( \frac{1}{2(1-q)} \). However, once we consider the costs, because a higher risk level taken by low-type banks leads to more bank failures and greater social costs, the first-best risk \( S_{FB}^{L} \) that maximizes welfare must be lower than \( \frac{1}{2(1-q)} \).

We formally state the first-best risk choices in the Lemma below.

**Lemma 2** \( S_{FB}^{H} = \frac{1}{2(1-q)} + \frac{K}{2}, \quad S_{FB}^{L} < \frac{1}{2(1-q)} < S_{FB}^{H}. \)

Next, we examine banks’ equilibrium risk decisions in the no-disclosure and disclosure scenarios separately. We analyze each scenario and then compare banks’ risk decisions.

### 4.2 No-disclosure Scenario

We first consider the scenario in which the regulator does not disclose the banks’ stress-test information. Given the no-disclosure policy, at date 1, a bank \( i \) makes its risk decision, \( S_i \), to maximize its expected payoff. If the bank is in a high idiosyncratic state (i.e., \( \theta_i = H \)), its expected project payoff is

\[ qS_i + (1-q) \left[ (1-S_i)S_i + S_iK \right]. \]
By solving the first-order condition, we obtain a high-type bank’s optimal risk level, denoted by \( S^N_H \), where \( N \) stands for non-disclosure. The expression for \( S^N_H \) is,

\[
S^N_H = \frac{1}{2(1-q)} + \frac{K}{2}.
\]

In the Appendix, we verify that indeed \( S^N_H > K \), such that the bank earns a higher payoff when the project succeeds. Notice that the equilibrium risk-taking decision of the high-type bank is socially efficient and equal to the first-best level, i.e., \( S^N_H = S^{FB}_H \). Moreover, the high-type bank’s risk decision is independent of other banks’ decisions.

Next, we derive the equilibrium decision by the low-type bank (i.e., \( \theta_i = L \)), denoted by \( S^N_L \). Given other banks’ equilibrium risk choices \( \{ S^N_H, S^N_L \} \), the low-type bank chooses \( S_i \) to maximize its expected project payoff:

\[
qS_i + (1-q) \left[ (1-S_i)S_i + S_i \Pr (n \geq \hat{n} \mid S^N_H, S^N_L) K \right].
\]

The payoff for the low-type bank is the same as that for a high-type bank except that, in a bad macroeconomic state, if the low-type bank’s project is unsuccessful the bank obtains a cash flow of zero, unless the regulator decides to bail out the bank by injecting a capital of \( K \), which occurs with a probability \( \Pr (n \geq \hat{n} \mid S^N_H, S^N_L) \). The optimal risk level for the bank is,

\[
S^N_L = \frac{1}{2(1-q)} + \frac{K \Pr (n \geq \hat{n} \mid S^N_H, S^N_L)}{2}.
\]  

(2)

Notice that \( S^N_L < 1 \) because \( S^N_L < \frac{1}{2(1-q)} + \frac{K}{2} < 1 \). In the Appendix, we also verify that \( S^N_L > K \Pr (n \geq \hat{n} \mid S^N_H, S^N_L) \) such that the bank earns a higher expected payoff if the project succeeds. Equation (2) suggests that the low-type bank takes an excessive high level of risk, compared with
the first-best level, i.e., \( S^N_L > \frac{1}{2(1-q)} > S^{FB}_L \). A key economic force behind this result is that the low-type bank anticipates that it may be bailed out by the regulator with a probability \( \Pr(n \geq \hat{n}) \). Since a bailout improves the bank’s payoff when the project is unsuccessful, the bank responds by taking a higher risk than the first-best.

To solve for \( S^N_L \), we now derive the probability of a bailout given other banks’ equilibrium risk choices \( \Pr(n \geq \hat{n} | S^N_H, S^N_L) \). As previously discussed, given that \( \Pr(\theta_i = L) = \tilde{p} \) and that each low-type bank fails with probability \( S^N_L \), the proportion of banks that fail is given by \( n = \tilde{p}S^N_L \). Since the regulator does not disclose the banks’ stress-test information, banks do not observe other banks’ idiosyncratic states and, thus, hold a prior about the portion of low-type banks given by \( \tilde{p} \sim U[0, 1] \). The bailout probability is then given by the expression,

\[
\Pr(n \geq \hat{n}) = \Pr(\tilde{p}S^N_L \geq \hat{n}) = \Pr\left(\tilde{p} \geq \frac{\hat{n}}{S^N_L}\right).
\]

Notice that, since \( S^N_L > \frac{1}{2(1-q)} \), the assumption that \( \hat{n} < \frac{1}{2(1-q)} \) ensures that \( \frac{\hat{n}}{S^N_L} < 1 \) and, thereby, avoids the discussion of corner solutions. That is, the regulator always chooses to bail out with some strictly positive probability (bailouts always occur if \( \tilde{p} \) is sufficiently large). Therefore,

\[
\Pr(n \geq \hat{n}) = 1 - \frac{\hat{n}}{S^N_L},
\]

which leads to,

\[
S^N_L = \frac{1}{2(1-q)} + \frac{K\left(1 - \frac{\hat{n}}{S^N_L}\right)}{2}.
\]  

(3)

The implicit equation in (3) suggests that, because of the possibility of a bailout, low-type banks’ risk choices become strategic complements of each other. If other low-type banks choose a higher risk, \( S^N_L \), the probability that these banks fail increases. A higher probability of concurrent bank
failure leads to a higher bailout probability, and the higher bailout probability in turn encourages more risk-taking by every single low-type bank.

Solving equation (3) gives a unique solution for $S^N_L$, which we summarize in the proposition below.

**Proposition 1** If banks’ stress-test information is not publicly disclosed, there exists a unique equilibrium in which a high-type bank chooses a socially efficient level of risk

$$S^N_H = \frac{1}{2(1-q)} + \frac{K}{2} = S^{FB}_H,$$

and a low-type bank chooses a higher risk than the first-best

$$S^N_L = \frac{K}{4} + \frac{1}{4(1-q)} + \sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8K\hat{n}} > S^{FB}_L.$$

The regulator chooses to bail out with a probability $1 - \frac{\hat{n}}{S^N_L}$.

From Proposition 1, it is apparent that the risk choices of low-type banks are strictly decreasing in $\hat{n}$. This is because, for a lower $\hat{n}$, the expected bailout transfer in case of bank failure, $\Pr(n \geq \hat{n}|S^N_H, S^N_L) K$, is larger, and that encourages low-type banks to take more risk. In addition, since $S^N_L$ is decreasing in $\hat{n}$, the probability of a bailout, $1 - \frac{\hat{n}}{S^N_L}$, is strictly decreasing in $\hat{n}$ as well. In other words, a tougher regulator is able to induce lower risk decisions thanks to its lower propensity to bail out banks.

### 4.3 Disclosure Scenario

We now consider the scenario in which the regulator publicly discloses banks’ stress-test information. That is, the results of the stress-test, $\{\theta_i\}_{i \in [0,1]}$, are revealed publicly to all banks. As a result, all
banks learn the realized portion of low-type banks, \( p \), which helps banks to forecast the regulator’s bailout decision. The equilibrium risk choice of high-type banks is the same as in the non-disclosure scenario since these banks never receive capital injections from the regulator and thus observing \( p \) or not does not affect their risk choices. Let \( S_H^D \) denote the risk choice of a high-type bank in the disclosure scenario. Then we have \( S_H^D = S_H^N = \frac{1}{2(1-q)} + \frac{K}{2} = S_H^{FB} \). The risk-taking incentive for low-type banks, however, changes because of the release of the public information regarding \( \bar{p} \). In particular, the first-order condition gives the optimal risk chosen by low-type banks, denoted by \( S_L^D \), conditional on \( \bar{p} = p \) and other banks’ choices \( \{S_H^D, S_L^D\} \), is

\[
S_L^D(p) = \frac{1}{2(1-q)} + K \frac{\Pr(n \geq \hat{n}; p, S_H^D, S_L^D)}{2}.
\]

Notice that because of the anticipation of a possible bailout, banks respond by taking more risks than in the first-best scenario, i.e., \( S_L^D > S_L^{FB} \). We now derive the probability of a bailout, \( \Pr(n \geq \hat{n}|p, S_H^D, S_L^D) \). As discussed in the non-disclosure scenario, given \( \bar{p} = p \), each bank conjectures that the proportion of banks that fail is \( pS_L^D(p) \). If other banks choose a low risk level such that \( pS_L^D(p) < \hat{n} \), the regulator never bails out banks, i.e., \( \Pr(n \geq \hat{n}|p, S_H^D, S_L^D) = 0 \), and the low-type bank chooses a low risk level given by

\[
S_L^D(p) = \frac{1}{2(1-q)}.
\]

However, if other banks choose a high risk level such that \( pS_L^D(p) \geq \hat{n} \), the regulator bails out banks with certainty, i.e., \( \Pr(n \geq \hat{n}|p, S_H^D, S_L^D) = 1 \), and the low-type bank chooses a high risk level given by,

\[
S_L^D(p) = \frac{1}{2(1-q)} + \frac{K}{2}.
\]
As in the non-disclosure scenario, the possibility of a bailout renders the low-type banks’ risk choices strategic complements of each other. However, in contrast with the non-disclosure scenario, the stress-test disclosure by the regulator conveys information about the portion of low-type banks that may fail, and that helps banks to better forecast the regulator’s bailout decision, facilitating the coordination among banks in taking risk. This improvement in coordination facilitated by the stress-test disclosure can in turn lead to multiple equilibria, while the equilibrium is always unique in the non-disclosure scenario. In the following proposition, we show that when \( p \) is of an intermediate value, there always exist two equilibria, one in which the regulator always bails out failed banks thereby encouraging banks to take a high risk, and the other one in which the regulator never bails out banks thereby disciplining banks into taking a low risk.

**Proposition 2** If stress-test information is publicly disclosed, there are two equilibria such that:

1) In both equilibria high-type banks choose the socially efficient level of risk,

\[
S^D_H = \frac{1}{2(1-q)} + \frac{K}{2} = S^F_B.
\]

2) Low-type banks choose different levels of risk depending on the equilibrium.

- **Low-risk Equilibrium:** if \( p < \frac{n}{2(1-q)} + \frac{K}{2} \), the low-type bank risk choice is \( S^D_L^1 = \frac{1}{2(1-q)} > S^F_L \) and the regulator never bails out failed banks.

- **High-risk Equilibrium:** if \( p > \frac{n}{2(1-q)} + \frac{K}{2} \), the low-type bank risk choice is \( S^D_L^2 = \frac{1}{2(1-q)} + \frac{K}{2} > S^D_L^1 \) and the regulator always bails out all failed banks.

3) Since \( \frac{n}{2(1-q)} + \frac{K}{2} < \frac{n}{2(1-q)} \), for \( p \in \left[ \frac{n}{2(1-q)} + \frac{K}{2}, \frac{n}{2(1-q)} \right] \), the two equilibria coexist.

The key economic force behind the multiple equilibria result is that stress-test disclosure reveals the realized portion of low-type banks \( p \) publicly. On one hand, when \( p \) is not disclosed, the equilibrium is always unique in spite of the strategic complementarity between banks’ risk choices. The reason is that, if \( p \) is unknown, it is difficult for banks to forecast the regulator’s bailout decision,
which is driven by the portion of banks that fail, and ultimately depends on the realized portion of low-type banks, $p$. Even if a bank believes that all other banks choose the high risk $\frac{1}{2(1-q)} + \frac{K}{2}$, the bank is not willing to deviate and choose $\frac{1}{2(1-q)} + \frac{K}{2}$ because of the uncertainty regarding the realization of $\tilde{p}$. In fact, when the realization of $\tilde{p}$ is low, the regulator does not bail out banks even if all banks coordinate into taking a high risk. In other words, the imperfect information regarding $\tilde{p}$ weakens the strategic complementarity among banks’ risk choices and results in the uniqueness of the equilibrium.

On the other hand, the stress-test disclosure removes the uncertainty regarding $\tilde{p}$ by effectively revealing its realization, $p$, and reinforces the strategic complementarity among banks’ risk choices. In particular, when $p$ is revealed to be not too low, all banks know that, by coordinating with each other in choosing high risks, they could collectively force the regulator to bail them out with certainty, and that justifies banks’ risk-taking behavior to begin with. The strategic complementarity strengthened by the stress-test disclosure in turn serves as a source of multiple equilibria. More specifically, consider a case in which all banks choose a low risk, $S_{L}^{D1} = \frac{1}{2(1-q)}$, and the number of failing banks, $pS_{L}^{D1}$, is just below $\hat{n}$. This is an equilibrium because given that other banks choose a low risk, any individual bank rationally anticipates that the portion of banks that will fail is lower than the bail-out cutoff, $\hat{n}$. Therefore, the regulator never bails out. As a result, the individual bank also takes a low risk. However, there is another equilibrium. Suppose that all banks believe that other banks will deviate and choose a larger risk level such that $pS_{L}^{D} \geq \hat{n}$, i.e., the regulator always bails out failed banks. This encourages banks to take more risk, which makes the initial deviation self-fulfilling. As a result, banks can also coordinate in another equilibrium in which every bank chooses a high risk level $S_{L}^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$.

Proposition 2 highlights one potentially destabilizing consequence of disclosing stress-test results. The disclosure informs banks about each other’s idiosyncratic states. That facilitates the
coordination among banks in taking risk decisions, which ultimately fuels a self-fulfilling prophecy that may lead to excessive amount of risk in the banking system and an inexorable regulatory bailout. The analysis of the effect of stress-test disclosure on banks’ risk-taking decisions, however, is disrupted by the multiplicity of equilibria in the disclosure scenario. To obtain comparative statics results and generate better regulatory insights, we resort to a global game approach to obtain unique equilibrium outcome.

4.3.1 The Global Game

We now apply the global game technique to achieve a unique equilibrium outcome. The global game technique has been widely used in coordination games with multiple equilibria to obtain uniqueness. The equilibrium selection obtained by the global game approach has been supported by evidence in numerous experimental studies (Cabrales et al, 2004; Heinemann et al, 2004; Anctil et al, 2004; Anctil et al, 2010). In particular, we assume that upon the stress-test disclosure by the regulator, each bank observes a private noisy signal of the realized portion of low-type banks \( p \), \( \tilde{x}_i = p + \tilde{\epsilon}_i \). The noise \( \tilde{\epsilon}_i \) is distributed in the interval \([ -\eta, \eta] \), with a cumulative distribution function \( F(\cdot) \) and a density function \( f(\cdot) \). The noise terms, \( \tilde{\epsilon}_i \), are independent across banks. This heterogeneity in the information observed by different banks can be understood as, for instance, a difference in the interpretation of the regulator’s disclosures and/or a different random sampling of the stress tests to obtain an estimate of \( p \). In applying the global game technique, we consider the limiting case in which \( \eta \) goes to zero, such that the noise \( \tilde{\epsilon}_i \) becomes negligible and \( p \) is (almost) perfectly observable by all banks. Concurring with the results in the global game literature, the introduction of conditionally independent private signals in the disclosure scenario breaks common knowledge.

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2 The global game technique was first introduced by Carlsson and van Damme (1993) and later on popularized by Morris and Shin (1998). A majority of the extant global game literature focuses on models with binary actions. A notable exception is Guimaraes and Morris (2007) who examine a currency attack model with continuous actions. We apply the technique developed in Guimaraes and Morris (2007) to a collective-moral-hazard setting in which banks’ risk decisions are continuous, and obtain unique equilibrium.
and restores the uniqueness of the equilibrium. We summarize the results in the proposition below.

**Proposition 3** In the limit of \( \eta \to 0 \), there exists a unique equilibrium characterized by a threshold \( \hat{p} \) such that, conditional on \( \omega = B \), the regulator bails out failed banks if and only if \( p \geq \hat{p} \) and each low-type bank chooses \( S_{D2}^L = \frac{1}{2(1-q)} + \frac{K}{2} \). Otherwise, if \( p < \hat{p} \), the regulator does not bail out failing banks and each low-type bank chooses \( S_{D1}^L = \frac{1}{2(1-q)} \). The threshold \( \hat{p} \) is given by

\[
\hat{p} = \frac{\hat{n}}{2(1-q)} + \frac{K}{4}.
\]

Proposition 3 suggests that the information revealed by the stress-test disclosure plays an important role in determining the regulator’s bailout decision and the low-type banks’ risk choices. If the stress-test result is good, (i.e., the portion of low-type banks is small, \( p < \hat{p} \)), disclosing the stress-test result produces a disciplining effect. Knowing that most banks are of a high-type and will not fail, each low-type bank anticipates a small likelihood of a regulatory bailout and, thus, prefers to take a lower risk. This, in turn, induces other low-type banks to take a lower risk since low-type banks’ risk choices are strategic complements. As low-type banks are coordinated in taking low risks, this indeed weakens the regulator’s bailout incentive, which further discourages low-type banks from taking risks. Through this downward spiral, the equilibrium converges to a stable point in which low-type banks are discouraged from taking risks because the regulator decides never to bail out, which is in turn justified by low-type banks’ choices of low risk and the resulting low frequency of bank failure.

On the contrary, if the stress-test result is bad (\( p \geq \hat{p} \)), disclosing the stress-test result produces an aggravating effect. That is, given the looming risk of bank failure, low-type banks anticipate a high bailout likelihood and are thus coordinated into taking high risk, which then leads to an increase in bank failures and forces the regulator to bail out more often. Eventually, through an
upward spiral, the equilibrium converges to a stable point in which low-type banks take maximal amounts of risk and the regulator bails out failed banks with certainty.

In Proposition 3, we can see by inspection that the bail-out threshold $\hat{p}$ is strictly increasing in $\hat{n}$. The reason is that the higher the $\hat{n}$, the weaker the regulator’s incentive to bail out failed banks and, thus, the higher must be the threshold on $p$ for a bailout to occur. Notice also that $\hat{p}$ decreases in $q$. The effect of $q$ on $\hat{p}$ is intuitive. Keeping $p$ constant, the larger is $q$ the probability of a good macroeconomic shock, the larger is the risk that all banks take, and that in turn increases the number of bank failures. Therefore, the value of $p$ needed to reach a number of failures larger than $\hat{n}$, decreases with $q$. Finally, notice that the effect of $K$ on $\hat{p}$ is ambiguous. An increase in $K$ has two effects. On the one hand, it directly decreases $\hat{p}$ by increasing the risk taken by all banks, thereby increasing the number of bank failures keeping $p$ constant. On the other hand, it increases $\hat{n}$ by increasing the cost of a bailout for the regulator.

### 4.4 The Effect of Stress-Test Disclosure on Risk Taking

With the equilibrium in both the non-disclosure and the disclosure scenarios fully characterized, we now compare these scenarios in order to determine the effect of stress-test disclosure on banks’ risk-taking decisions. Since high-type banks always choose the socially efficient risk level in both scenarios, we focus on comparing the risk decisions taken by low-type banks.

The ex-post comparison of risk decisions between the two scenarios is straightforward. From Proposition 3, we can conclude that if the stress-test results turn out to be good (i.e., $p < \hat{p}$), then low-type banks choose a low level of risk, $S_{L}^{D1} = \frac{1}{2(1-q)}$, which is lower than the level of risk chosen under the non-disclosure scenario. However, if the stress-test results turn out to be bad (i.e., $p > \hat{p}$), low-type banks choose a high level of risk, $S_{L}^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$, which is higher than the level of risk chosen under the non-disclosure scenario. That is, $S_{L}^{D1} < S_{L}^{N} < S_{L}^{D2}$. 
In reality, however, the stress-test disclosure policy decision is made before the realization of the stress-test results. For instance, the Dodd-Frank act mandates the disclosure of banks’ stress-test results, regardless of whether the results are good or not. Therefore, in order to shed light on whether the disclosure of stress-tests information encourages or discourages risk-taking from an ex ante perspective, we compare the low-type banks’ expected risk choices between the two scenarios. Specifically, the expected risk in the disclosure scenario is,

\[ E[S^D_L] = \frac{1}{2(1-q)}\hat{p} + \left( \frac{1}{2(1-q)} + \frac{K}{2} \right)(1 - \hat{p}), \]

where the ex ante probability that the stress-test result is good is \( \Pr(p < \hat{p}) = \hat{p} \) and the probability that the stress-test result is bad is \( 1 - \hat{p} \). Comparing \( E[S^D_L] \) with the risk choice in the non-disclosure scenario \( S^N_L \) shows that there exists a single cutoff on \( \hat{n}, \hat{n}^T \), such that \( E[S^D_L] > S^N_L \) if and only if \( \hat{n} > \hat{n}^T \). We summarize this result in the proposition below.

**Proposition 4** There exists a cutoff \( \hat{n}^T = \frac{1}{4(1-q)} + \frac{K}{8} \), such that the average risk in the disclosure scenario \( E[S^D_L] \) is higher than the risk in the non-disclosure scenario if and only if \( \hat{n} > \hat{n}^T \).

Proposition 4 highlights the main result of this paper. It suggests that the stress-test disclosure reduces average risk-taking if and only if the regulator has a strong incentive to bail out failing banks (i.e., the bailout threshold \( \hat{n} \) is low). In situations in which the regulator does not bail out banks very often, disclosing stress-test can actually encourage banks to take more risk. To the extent that in reality the regulator’s bailout incentive is largely associated with a “too-big-to-fail” problem and the regulator’s lack of commitment, our findings suggest that one way to mitigate this commitment problem is to mandate the stress-test disclosure. This result seems consistent with the observation in the recent crisis during which time the Federal reserve implemented various “bail-out” policies to support the banking industry (e.g., the TAF, CPP programs discussed in the
model setup) while at the same time, the Dodd-Frank act mandates the disclosure of stress-test results of “systematically important” financial institutions which arguably are the ones that most likely will be bailed out in the case of failure.

The intuition behind Proposition 4 depends on the trade-off between the disciplining effect of disclosing good stress-test results and the aggravating effect of disclosing bad results. To see how \( \hat{n} \) affects this trade-off, consider two extreme examples. On the one hand, if \( \hat{n} \) is sufficiently small (i.e., \( \hat{n} \) is close to zero), the regulator bails out almost with certainty even without the disclosure of \( \tilde{p} \). Anticipating this strong bailout incentive, low-type banks coordinate to take the maximal risk, that is, \( S_L^N \) becomes close to \( \frac{1}{2(1-q)} + \frac{K}{2} \). In this case, disclosing the stress-test result is beneficial. This is because if the stress-test result is revealed to be bad, the aggravating effect of disclosure is minimal because low-type banks would have chosen the same maximal amounts of risk had the stress-test results not been disclosed. However, if the stress-test results turns out to be good, low-type banks are disciplined from taking risks excessively compared with the non-disclosure scenario.

On the other hand, consider the other extreme case in which \( \hat{n} \) is sufficiently large (i.e., \( \hat{n} \) is close to the assumed upper bound \( \frac{1}{2(1-q)} \)). In this case, the regulator bails out failed banks with almost zero probability. As a result, low-type banks have already coordinated to take the minimal amounts of risks without the disclosure of \( \tilde{p} \). Now disclosing the stress-test results actually encourages low-type banks to take risks. The reason is that if the stress-test result is good, the disciplining effect of disclosure is small since low-type banks would have chosen the same minimal amounts of risk without disclosure. However, if the result is bad, the aggravating effect of disclosure coordinates low-type banks to take the maximal amounts of risk.
5 Conclusions

The policy of disclosing stress tests publicly was initially adopted by the FED as a reaction to the financial crisis of 2008 with the expectation that a transparent regulatory oversight to control bank specific and systemic risk would discipline banks and reassure investors of the stability of the financial system. However, such a disclosure may potentially cause other problems, or potential worsen the problems that it intends to solve. The extant literature has examined several possible social costs that regulators should be aware of. This paper identifies an additional cost that is intimately related to the nature of stress test information. In particular, stress test information, by its very nature, provides information that can be used by banks to coordinate into taking higher risk with the expectation that such coordination will render a regulatory bailout unavoidable. We find that the effect of stress-test disclosure on banks’ average risk taking depends on the propensity of the regulator to bail out banks. If the regulator is already very inclined to bail out banks, then disclosing stress test may reduce banks’ risk taking. However, if the regulator is reticent to bail out banks, the disclosure of stress tests will only increase banks’ average risk taking. This “if it ain’t broken, don’t fix it” policy, seems to corroborate the use of stress-test disclosure as a disciplining mechanism, given the seemingly high predisposition to bail out banks.

A second issue, which this paper does not focus on, is the optimal use of information by the regulator (Tirole, 1994). A banking regulator can potentially use the information obtained through stress tests to take ex post actions to correct detected weaknesses and reduce systemic risk. Given the debate on whether stress test should be disclosed, we focus on the effects of disclosure instead. Moreover, the interaction between stress test disclosure and ex post corrective actions is an interesting topic that we leave for future research.
References


Appendix I: Proofs

Proof of Lemma 1

Proof. First, at \( n = 1 \), the difference between the social cost of bank failure and the social cost of bail out, i.e., \( C(n) - \lambda nK \), is

\[
C(1) - \lambda K = \infty > 0,
\]

since \( C(1) = \infty \). Second, at \( n = 0 \), \( C(n) - \lambda nK = C(0) = 0 \). In addition,

\[
\left. \frac{\partial (C(n) - \lambda nK)}{\partial n} \right|_{n=0} = C'(0) - \lambda K = -\lambda K < 0,
\]

therefore, for \( n = \varepsilon > 0 \), where \( \varepsilon \) is a small positive number, \( C(\varepsilon) - \lambda \varepsilon K < 0 \). Therefore, by the intermediate value theorem, there exists a \( \hat{n} \) such that \( C(\hat{n}) = \lambda \hat{n}K \).

We next prove that \( \hat{n} \) is unique. With some abuse of notation, denote the smallest root that solves \( C(n) = \lambda nK \) as \( \hat{n} \). Therefore, for \( n < \hat{n} \), \( C(n) - \lambda nK < 0 \). It must be the case that at \( n = \hat{n} \),

\[
\left. \frac{\partial (C(n) - \lambda nK)}{\partial n} \right|_{n=\hat{n}} > 0,
\]

since \( C(1) - \lambda K > 0 \) and \( C(\varepsilon) - \lambda \varepsilon K < 0 \). In addition, \( C(n) - \lambda nK \) is strictly convex in \( n \), because

\[
\frac{\partial^2 (C(n) - \lambda nK)}{\partial n^2} = C''(n) > 0.
\]

Combined with \( \left. \frac{\partial (C(n) - \lambda nK)}{\partial n} \right|_{n=0} < 0 \) and by the intermediate value theorem, there exists a unique \( n' \) such that \( C'(n') = \lambda K \). For \( n < (>) n' \), \( C'(n') < (>) \lambda K \). Recall that \( \left. \frac{\partial (C(n) - \lambda nK)}{\partial n} \right|_{n=\hat{n}} > 0 \) and thus \( \hat{n} > n' \). Therefore, \( C(n') - \lambda n'K < 0 \). In the region \( n \in [0, n'] \), \( C(n) - \lambda nK \) is strictly decreasing in \( n \) and \( C(n) < \lambda nK \). That is, there is no root in \([0, n']\) that solves \( C(n) = \lambda nK \).
In the region $n \in [n', 1]$, $C(n) - \lambda nK$ is strictly increasing in $n$. That is, there is a unique root in $[n', 1]$ that solves $C(n) = \lambda nK$, which is $n = \hat{n}$. Overall, the root that solves $C(n) = \lambda nK$ is unique. For $n < (>) \hat{n}$, $C(n) < (>) \lambda nK$. That is, the regulator bails out banks if and only if $n > \hat{n}$. ■

Proof of Lemma 2

**Proof.** The first-order condition of $S_{FB}^H$ is

$$q + (1 - q) \left(1 - 2S_{FB}^H + K\right) = 0,$$

which gives

$$S_{FB}^H = \frac{1}{2(1 - q)} + \frac{K}{2}.$$

To show that $S_{FB}^L < \frac{1}{2(1 - q)}$, it suffices to verify that for $S_i > \frac{1}{2(1 - q)}$, the welfare of low-type banks is strictly decreasing. To see this, notice that in the welfare function, the NPV is maximized at $S_i = \frac{1}{2(1 - q)}$. Therefore, for $S_i > \frac{1}{2(1 - q)}$, the NPV is decreasing in $S_i$. In addition, for the social costs term, when $\frac{1}{2(1 - q)} \geq \frac{\hat{n}}{p}$, $S_i > \frac{1}{2(1 - q)} \geq \frac{\hat{n}}{p}$, which gives $pS_i > \hat{n}$ and the social costs term becomes $-(1 - q) \lambda K pS_i$ which is decreasing in $S_i$. When $\frac{1}{2(1 - q)} < \frac{\hat{n}}{p}$, for $S_i \in \left(\frac{1}{2(1 - q)}, \frac{\hat{n}}{p}\right]$, $pS_i \leq \hat{n}$ and the social costs term becomes $-(1 - q) C(pS_i)$ which is decreasing in $S_i$. For $S_i > \frac{\hat{n}}{p}$, the social costs term becomes $-(1 - q) \lambda K pS_i$ which is decreasing in $S_i$. Overall, the welfare is decreasing in $S_i$ for $S_i > \frac{1}{2(1 - q)}$. As a result, the optimal $S_{FB}^L < \frac{1}{2(1 - q)}$. ■
Proof of Proposition 1

Proof. From the main text, $S^N_L$ is determined by the first-order condition

$$S^N_L = \frac{1}{2(1-q)} + \frac{K\left(1 - \frac{\hat{n}}{S^N_L}\right)}{2},$$

which reduces into a quadratic equation of $S^N_L$ and thus can have at most 2 roots. Recall that since we assume $\frac{\hat{n}}{2(1-q)} < 1$, $S^N_L > \hat{n}$. We now show that there exists a unique $S^N_L \in (\hat{n}, 1)$. First, at $S^N_L = \hat{n}$, the RHS of the equation is $\frac{1}{2(1-q)}$, which is larger than the LHS $\hat{n}$ given our assumption $\frac{\hat{n}}{2(1-q)} < 1$. At $S^N_L = 1$, the RHS is $\frac{1}{2(1-q)} + \frac{K(1-\hat{n})}{2}$ and smaller than the LHS 1 given our assumption $\frac{1}{2(1-q)} + \frac{K(1-\hat{n})}{2} < 1$. Therefore, by the intermediate value theorem, there exists a root in $(\hat{n}, 1)$ that solves the first-order condition. Moreover, there can only exist an odd number of solutions in $(\hat{n}, 1)$. Since the first-order condition is quadratic, there exists a single root, i.e., the equilibrium is unique and equal to

$$S^N_L = \frac{K}{4} + \frac{1 + \sqrt{[1 + K(1-q)]^2 - 8K\hat{n}(1-q)^2}}{4(1-q)}.$$

We now verify that $S^N_H > K$ and $S^N_L > K$ Pr $\left(n \geq \hat{n}; S^N_H, S^N_L\right)$. From the first-order condition of $S^N_H$, we have

$$q + (1-q)\left(1 - 2S^N_H + K\right)$$

$$= q + (1-q)\left(1 - S^N_H + K - S^N_H\right),$$

in order to make the first-order condition zero, it must be case that $S^N_H > K$. Otherwise, the first-order condition is always positive. Similarly, we verify that $S^N_L > K$ Pr $\left(n \geq \hat{n}; S^N_H, S^N_L\right)$. ■
Proof of Proposition 2

Proof. Notice that given the banks’ risk choices $S_L^D(p)$, there can only be two equilibria, either $pS_L^{D2}(p) \geq \hat{n}$ or $pS_L^{D1} < \hat{n}$. We first consider the equilibrium in which $pS_L^{D2}(p) \geq \hat{n}$. In this equilibrium, the regulator bails out banks with certainty and thus $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$. To have $p\left(\frac{1}{2(1-q)} + \frac{K}{2}\right) \geq \hat{n}$, it must be the case that $p \geq \frac{\hat{n}}{2(1-q) + \frac{K}{2}}$. Second, we consider the other equilibrium $pS_L^{D1} < \hat{n}$. In this equilibrium, the regulator never bails out banks and thus $S_L^{D1} = \frac{1}{2(1-q)}$. To have $p\frac{1}{2(1-q)} < \hat{n}$, it must be the case that $p \leq \frac{\hat{n}}{2(1-q)}$. To summarize, for $p < \frac{\hat{n}}{2(1-q)}$, the unique equilibrium is that $S_L^{D1} = \frac{1}{2(1-q)}$ and for $p > \frac{\hat{n}}{2(1-q)}$, the unique equilibrium is $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$. For $p \in \left[\frac{\hat{n}}{2(1-q)} + \frac{K}{2}, \frac{\hat{n}}{2(1-q)}\right]$, there are two equilibria, $S_L^{D1} = \frac{1}{2(1-q)}$ and $S_L^{D2} = \frac{1}{2(1-q)} + \frac{K}{2}$.

Proof of Proposition 3

Proof. The proof is similar to the proof in Guimaraes and Morris (2007). First, we denote by $H(p|x_i)$ the cumulative distribution over $\tilde{p}$ for a bank that observes $x_i$ and $H(p|x_i)$ is given by

$$H(p|x_i) = \frac{\int_0^p u(\tilde{p}) f(x_i - \tilde{p}) d\tilde{p}}{\int_0^1 u(\tilde{p}) f(x_i - \tilde{p}) d\tilde{p}} = \frac{\int_0^p f(x_i - \tilde{p}) d\tilde{p}}{\int_0^1 f(x_i - \tilde{p}) d\tilde{p}} = \frac{F(x_i) - F(x_i - p)}{F(x_i) - F(x_i - 1)},$$

where $u(\tilde{p}) = 1$ is the prior density function of $\tilde{p}$. In the limit of $\eta \to 0$, $H(p|x_i)$ becomes

$$\lim_{\eta \to 0} H(p|x_i) = \lim_{\eta \to 0} \frac{F(x_i) - F(x_i - p)}{F(x_i) - F(x_i - 1)} = 1 - F(x_i - p),$$

because as $\eta \to 0$, $x_i = p + \varepsilon > \eta$ which implies that $F(x_i) = 1$. Similarly, $x_i = p + \varepsilon < 1 - \eta$, which implies that $x_i - 1 < -\eta$ and thus $F(x_i - 1) = 0$.

Second, we show that there exists a threshold equilibrium in which the regulator bails out failed
banks if and only if \( p \geq \hat{p} \). To see this, notice that given the regulator bails out if \( p \geq \hat{p} \), a bank believes that the regulator bails out with a probability \( 1 - H(\hat{p}|x_i) = F(x_i - \hat{p}) \). As a result, the bank chooses a risk that is equal to

\[
S^*(\varepsilon_i; p) = \frac{1}{2(1-q)} + \frac{KF(x_i - \hat{p})}{2} = \frac{1}{2(1-q)} + \frac{KF(p + \varepsilon_i - \hat{p})}{2}.
\]

Given each bank chooses \( S^*(\varepsilon_i; p) \), the portion of banks that fail is

\[
n(S^*(\varepsilon_i; p); p) = \int_{-\eta}^{\eta} p S^*(\varepsilon_i; p) \, d\varepsilon_i = \int_{-\eta}^{\eta} \left[ \frac{1}{2(1-q)} + \frac{KF(p + \varepsilon_i - \hat{p})}{2} \right] \, d\varepsilon_i,
\]

where since \( F(\cdot) \) is strictly increasing in \( p \), \( n(S^*(\varepsilon_i; p); p) \) is strictly increasing in \( p \). Therefore, there exists a unique threshold \( \hat{p} \) that makes \( n(S^*(\varepsilon_i; \hat{p}); \hat{p}) = \hat{n} \). For \( p > (<) \hat{p} \), \( n(S^*(\varepsilon_i; p); p) > (<) \hat{n} \) and the regulator bails out (does not bail out). Therefore, the threshold equilibrium is indeed an equilibrium.

Lastly, we derive the threshold \( \hat{p} \). Recall that the probability of \( p \geq \hat{p} \) is \( 1 - H(\hat{p}|x_i) = F(x_i - \hat{p}) \). Thus any bank observing a signal

\[
\xi(\hat{p}, l) = \hat{p} + F^{-1}(l),
\]

attaches probability \( l \) to \( p \geq \hat{p} \). Moreover, since \( F(x_i - \hat{p}) \) increases in \( x_i \), any bank observing a signal less than \( \xi(\hat{p}, l) \) attaches a probability less than \( l \) to \( p \geq \hat{p} \). Thus if the true state is \( \hat{p} \), the
proportion of banks assigning probability $l$ or less to $p \geq \hat{p}$ is

$$\Gamma (l|\hat{p}) = \Pr (x_i \leq \xi (\hat{p}, l))$$

$$= \Pr (x_i \leq \hat{p} + F^{-1} (l))$$

$$= \Pr (\hat{p} + \varepsilon_i \leq \hat{p} + F^{-1} (l))$$

$$= \Pr (\varepsilon_i \leq F^{-1} (l))$$

$$= F (F^{-1} (l))$$

$$= l.$$

If a bank believes that $p \geq \hat{p}$ with a probability $l$, the bank chooses a risk level:

$$S^* (l) = \frac{1}{2 (1 - q)} + \frac{KL}{2}.$$

At $p = \hat{p}$, the proportion of banks that failed is

$$n = \int_0^1 S^* (l) \hat{p} \Gamma (l|\hat{p})$$

$$= \int_0^1 S^* (l) \hat{p} dl.$$

Since at $p = \hat{p}$, the regulator is indifferent between bail out and not to bail out, we must have

$$\int_0^1 S^* (l) \hat{p} dl = \hat{n},$$

which gives

$$\hat{p} = \frac{\hat{n}}{\int_0^1 S^* (l) dl} = \frac{\hat{n}}{\frac{1}{2 (1 - q)} + \frac{K}{4}}.$$
Therefore, there exists a threshold equilibrium such that the regulator bails out failed banks if and only if \( p \geq \frac{\hat{n}}{2(1-q) + \frac{K}{4}} \). Moreover, this equilibrium is also the unique equilibrium, following a general result from Frankel, Morris and Pauzner (2003) that show in games with strategic complementarity, arbitrary numbers of players and actions, and slightly noisy signals, the equilibrium is unique as the noise goes to zero. ■

**Proof of Proposition 4**

**Proof.** Recall that \( S^N_L \) is given by the first-order condition

\[
S^N_L = \frac{1}{2(1-q)} + \frac{K \left(1 - \frac{\hat{n}}{S^N_L}\right)}{2},
\]

and

\[
E \left[ S^D_L \right] = \frac{1}{2(1-q)} + \frac{K}{2} \left(1 - \hat{p}\right).
\]

Therefore, to show that \( E \left[ S^D_L \right] > S^N_L \), it is sufficient to verify that \( \hat{p} < \frac{\hat{n}}{S^N_L} \), which reduces into

\[
\frac{\frac{K}{4} + \frac{1}{4(1-q)} + \sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8Kh}}{2(1-q) + \frac{K}{4}} < 1.
\]

Notice that the LHS is strictly decreasing in \( \hat{n} \). Moreover, recall that we assume \( \hat{n} \in \left[0, \frac{1}{2(1-q)}\right] \).

At \( \hat{n} = 0 \), the RHS becomes \( \frac{\frac{K}{4} + \frac{1}{4(1-q)} + \sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8Kh}}{2(1-q) + \frac{K}{4}} > 1 \), the LHS. In addition, at \( \hat{n} = \frac{1}{2(1-q)} \), the RHS becomes \( \frac{\frac{1}{4(1-q)}}{2(1-q) + \frac{K}{4}} < 1 \), the LHS. Therefore, there exists a unique cutoff \( \hat{n}_T \) such that \( \hat{p} < \frac{\hat{n}}{S^N_L} \) if and only if \( \hat{n} > \hat{n}_T \). Solving \( \frac{\frac{K}{4} + \frac{1}{4(1-q)} + \sqrt{\left(\frac{1}{1-q} + K\right)^2 - 8Kh}}{2(1-q) + \frac{K}{4}} = 1 \) gives \( \hat{n}_T = \frac{1}{4(1-q)} + \frac{K}{8} \). ■