Effects of Increasing Enforcement on
Financial Reporting Quality and Firm Value

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Abstract

A widely held assumption in empirical research and policy making in capital markets is that increasing enforcement effectiveness improves financial reporting quality. In this paper, we show that it can be detrimental, even if enforcement is costless. We develop an agency model with a productive manager who can also engage in earnings management, an auditor, and an enforcement institution and establish the equilibrium strategies and the optimal management compensation. Financial reporting quality and firm value typically decrease if enforcement becomes too strong. Two effects are responsible for this result: First, while enforcement and auditing are complements under weak enforcement, they are substitutes under strong enforcement. Less auditing reduces reporting quality. Second, earnings management can be “good” if it corrects errors by an imprecise accounting system; mitigating earnings management reduces this corrective effect, which also lowers quality.

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1. Introduction

Enforcement assists in assuring the quality of financial reporting by listed companies through supervision of published audited financial reports. Effective enforcement has also been identified in many studies as being crucial for the efficiency of capital markets and perhaps more important than the quality of the accounting standards themselves (e.g., Ball, Kothari, and Robin 2000; Christensen, Hail, and Leuz 2013). Currently, the effectiveness of enforcement institutions differs widely around the world (Brown, Preiato, and Tarca 2014), and regulators strive to improve enforcement to foster capital market efficiency (e.g., SEC 2000, EU 2004).

A widely-held assumption in empirical research and policy making in capital markets is that increasing enforcement improves financial reporting quality, and several empirical studies support this assumption. This view suggests that solely the direct cost of enforcement prohibits full enforcement and supervision of financial reporting. Our paper challenges this assumption and shows that increasing enforcement, even if it is costless, can be detrimental for both financial reporting quality and firm value.

To establish this result, we develop an agency model with a manager who exerts productive effort, a strategic auditor, and an enforcement institution. The optimal contract that induces the manager to exert productive effort also creates incentives for earnings management. We derive equilibrium earnings management and audit effort and study the economic effects of a change in enforcement effectiveness on the equilibrium. The auditor strategically chooses the audit effort and corrects errors found in the preliminary financial report. After publication of the audited financial report, the enforcer supervises the report and identifies further errors. If the auditor is unable to provide evidence that the alleged error is in fact nonexistent, the enforcer takes an enforcement action, which imposes enforcement to the firm, to the auditor, and through claw-back of a bonus also to the manager. A key driver of

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our results is that auditing and enforcement are different activities. Auditing comprises the quality of the accounting system and internal controls as well as earnings management, whereas the scope of enforcement is more limited and geared towards detecting earnings management.

Our main findings are the following: We confirm the result that equilibrium earnings management decreases with stronger enforcement. However, increasing enforcement can either improve or reduce financial reporting quality, and we provide conditions in which one or the other happens. Further, we show that firm value is always higher if perfect enforcement is newly introduced, but varying existing enforcement can either increase or decrease firm value, contingent on key parameters of the economic situation. We find that financial reporting quality and firm value can move in parallel, but also in different directions; thus, increasing enforcement may improve financial reporting quality, but destroy firm value, and vice versa. Finally, we discuss empirical implications of our analyses.

Intuitively, two reasons are jointly responsible for our results that too strong enforcement becomes detrimental. First, introducing enforcement provides incentives for the auditor to increase audit effort, which increases financial reporting quality because it mitigates earnings management and corrects accounting system errors. If enforcement becomes sufficiently strong, enforcement becomes more prominent in deterring earnings management, and in equilibrium the auditor reduces audit effort. That is, auditing and enforcement become substitutes for strong enforcement. The eventual reduction in audit effort increases the accounting system errors, which can reduce financial reporting quality and firm value under the optimal contract.

Second, earnings management is not necessarily “bad” in that it reduces financial reporting quality and its usefulness for contracting. The optimal contract provides incentives to the manager to exert productive effort by paying a bonus for high reported earnings, thus generating an incentive to manage earnings upwards. This overstatement of earnings is “bad” if actual earnings are low because it disguises this fact, but it is “good” if it corrects an erroneous financial report that shows low earnings, although the actual outcome is high. The
latter effect becomes more likely if the accounting system is less precise and we give a condition for this. Because more effective enforcement reduces earnings management, it also reduces “good” earnings management, which reduces financial reporting quality if “good” earnings management prevails.

A change in enforcement effectiveness indirectly affects the optimal contract and production incentives because it affects the quality of the financial report on which the manager’s incentive compensation is based as well as audit fees and the risk of enforcement. Thus, the owner adjusts the incentive compensation in response to changes in enforcement, which again affects earnings management incentives and audit effort.

This paper contributes to better understanding the economic effects of enforcement on the main two objectives of financial reporting, decision usefulness and stewardship. In particular, it establishes that an increase of enforcement can have desirable and undesirable economic consequences (regardless of its direct cost). We are not aware of analytical papers that explicitly study economic effects of enforcement as a distinct assurance mechanism besides auditing. However, the paper is closely related to two strands of literature. First, many models study production effort and earnings management in multi-action agency models. For example, Feltham and Xie (1994) model productive effort and earnings management (“window dressing”), which are induced by the same information system, and provide insights into the optimal design of such an information system in a LEN setting. The present paper focuses on how auditing and enforcement shape the information system. Other models study earnings management in rational expectations equilibria, in which managers “jam” financial reports to increase the market price of the firm (see, e.g., Fischer and Verrecchia (2000); Ewert and Wagenhofer 2011 survey this literature). In these models, auditing and enforcement are implicit in the cost of earnings management. Königsgruber (2012) addresses enforcement in a model in which a manager decides on the investment in a risky project and is concerned about the market price of the firm after issuing a financial report. Enforcement in his paper is a technology that reveals the true outcome with a probability that is set \textit{ex ante} by a regulator and imposes a fine after detecting misreporting. Königsgruber finds that more
effective enforcement strictly increases reporting quality, but may reduce investment efficiency due to over-deterrence of viable projects. Different from that, our results show that both reporting quality and investment can decrease; the reason is that we explicitly model the interaction between auditing and enforcement.

A second set of literature are auditing models. One model type assumes that auditing is a technology (e.g., Ng and Stoeckenius 1979); other models assume a strategic auditor, who maximizes expected utility by the choice of audit effort (Antle 1982, Baiman, Evans, and Noel 1987), as we do in the present paper. Given that contingent audit fees are not allowed in most jurisdictions, the motivation for auditors to exert audit effort in these models usually results from the risk that the auditor is liable of malperformance if an error in the financial reports is uncovered later. The enforcement mechanism in the present paper is explicitly modeled based on its interaction with the audit results. Other papers assume that the liability arises from shareholder litigation. In that case, the cost to the auditor depends on decisions taken in a rational fashion by shareholders and on the liability regime (e.g., Ewert 1999, Hillegeist 1999). Related to the present paper is the audit literature that also considers internal controls, if one views internal controls as an assurance mechanism that steps in before auditing takes place (e.g., Smith, Tiras, and Vichitlekarn 2000, Pae and Yoo 2001). In the present model, we take as given internal controls by assuming the precision of the accounting system and focus on the interaction between auditing and enforcement.

The paper proceeds as follows. In Section 2, we set out the model and introduce the underlying production technology, the accounting system, the discretion for earnings management, auditing, and enforcement. Section 3 contains the analysis of the earnings management and auditing game, which depends on the enforcement. In this section, we characterize the reporting and auditing equilibrium and derive the effects of a variation in enforcement on financial reporting quality. Section 4 adds the production stage and derives the optimal compensation contract with the manager, which generates the incentives for

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2 Deng, Melumad, and Shibano (2012) find a related result for increased auditor liability.
earnings management that affect the subsequent reporting equilibrium. We show how enforcement affects the owner’s expected utility, firm value, and financial reporting quality with endogenous incentives. In Section 5, we point out empirical implications and conclude.

2. Model

We develop a one-period agency model with a representative owner of a firm, a manager, an auditor, and an enforcement institution (the “enforcer”). In the following, we describe these elements and their relation step by step. The notation is summarized in the appendix.

Production technology

The owners of the firm are represented by a risk neutral owner (or the board of directors to which the decision power is delegated). We abstract from potential conflicts of interest among different owners or among owners and board members. The firm owns a production technology and has an accounting system in place. The production technology requires the input of a manager (effort \( a \)), which, together with random events capturing other productive and environmental factors, determines the outcome. The output is represented by a monetary amount \( x \), where \( x \in \{ x_L, x_H \} \) and \( 0 < x_L < x_H \). We adopt the convention that \( x \) denotes the random variable and \( x_i (i = L, H) \) its discrete realizations. The owner receives the output of the production technology and pays the compensation to the manager \( s(\cdot) \).

The owner hires a manager, who is risk neutral and protected by limited liability. The manager chooses a productive effort \( a \in \{ a_L, a_H \} \) and incurs a private cost of 0 for \( a_L \) and \( V > 0 \) for \( a_H \). The effort determines the probability with which a low and a high output realize: \( x_H \) occurs with probability \( p \) upon high effort \( a_H \), and with probability \( q \) upon low effort, where \( p > q \) and each \( p \) and \( q \) are strictly within \((0, 1)\).

We focus on the case that the owner wants to induce the manager to exert high productive effort \( a_H \), because otherwise there is no agency problem. We assume that \( x \) is unobservable throughout the time period we examine; for example, the output can be the
expected net present value of future cash flows. The firm operates an accounting system and issues an audited financial report $r$. This report is contractible and is used in the manager’s compensation contract to elicit managerial effort.

The owner maximizes the expected utility that includes the following components: the expected productive outcome $(1 - p)x_L + px_H$ less expected compensation $\text{prob}(r_L)s(r_L) + \text{prob}(r_H)s(r_H)$, the audit fee $A$, and the expected costs due to an enforcement action.

**Accounting system**

The firm operates an accounting system that produces a signal $y \in \{y_L, y_H\}$, where $y_L < y_H$ (see Figure 1). We also refer to these signals as earnings. The accounting system is an imperfect “technology” subject to possible random errors and accounting standards that may produce biases. $\alpha$ is the “$\alpha$-error”, i.e., the probability that $y_L$ is reported although the output is $x_H$; and $\beta$ is the “$\beta$-error” with which $y_H$ is reported although the output is $x_L$. $\alpha$ and $\beta \in (0, \frac{1}{2})$ are exogenously determined by accounting standards and their implementation in the firm and are common knowledge. The manager privately observes the accounting signal $y$; hence, $y$ is not available for contracting.

After observing $y$, the manager can engage in earnings management and bias the signal to achieve a financial report $m$ that deviates from $y$. We refer to the report $m$ as the preliminary financial report because it is subject to auditing (see below). Earnings management includes the choice of probabilities $b_L \equiv b(y_L)$ and $b_H \equiv b(y_H)$ with which it is successful in diverting the accounting signal, i.e., reporting $m_i \neq y_i$, $i = L, H$. The cost of earnings management effort is increasing and convex in $b_i$, it is 0 at $b_i = 0$, and “very high” at $b_i = 1$. It captures disutility from, e.g., searching for earnings management opportunities, future disadvantages, reputation, or ethical behavior. For tractability reasons, we assume a

3 This assumption precludes writing a contract contingent on $x$. An alternative assumption is that the owner may sell the shares after the financial report has been issued and the manager was paid. To price the shares, capital market participants use the report about the future cash flow $x$. 

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quadratic cost function, $\frac{1}{2} \nu b_i^2$, where $\nu$ is a constant scaling factor. We assume that $\nu$ is sufficiently high that $b_i < 1$ (such a $\nu$ always exists)\(^4\) in order to avoid consideration of cases in which $b_i = 1$ and the financial report becomes uninformative.

![Diagram of Production and Accounting Structure](image)

**Production technology**
Manager’s productive effort

**Accounting system**

**Reporting stage**
Manager’s earnings management

Figure 1: Production and accounting structure

The manager receives compensation from the owner for exerting effort. We assume the manager has a reservation utility of zero and because of limited liability the compensation paid must be positive. Compensation $s(\cdot) \geq 0$ is written on the audited financial report $r \in \{r_L, r_H\}$, which is the contractible signal. Finally, the audited report is subject to enforcement. If the enforcer finds and publishes an error, we assume the owner invokes a claw-back of a bonus paid to the manager, thus penalizing the manager for identified earnings management. The claw-back imposes a contingent element in the otherwise simple bonus contract. We do not consider more complex compensation contracts.

**Auditing**

The firm is subject to mandatory auditing. The owner contracts with an auditor prior to the preparation of the preliminary report $m$ by the manager. The audit comprises tests of

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\(^4\) In the proof of Proposition 2, we derive the precise condition as $\nu > 2Vl((p - q)(1 - \alpha - \beta))$. 
controls and substantive procedures, including analytical procedures and tests of details, e.g., providing audit evidence of physical inventory, bank balances, loan quality, and the like, to identify material misstatements. After engagement, but before deciding on audit effort, the auditor receives the preliminary report $m$ from the manager, but no other information. The auditor knows the precision of the accounting system ($\alpha, \beta$) and uses it for risk assessment. Performing the audit, the auditor observes both the actual accounting signal $y$ that the manager observed and the true outcome $x$ with a probability that increases in audit effort. Let $g_i$ be the probability with which the auditor finds out $(x, y)$ given $m_i$, $i = L, H$. Providing audit effort $g_i$ is privately costly to the auditor; the cost is $\frac{1}{2}kg_i^2$, where $k > 0$ is a parameter that scales the quadratic cost.

The actual outcome $x$ is always more informative about the firm’s cash flows than the accounting signal $y$, and therefore we assume the auditor corrects the financial report based on $x$. That is, if the auditor finds out that $m_i$ has been reported but the outcome is $x_j$, $i \neq j$, ($i = L, H$) then he requires the manager to correct the financial report from $m_i$ to $r_j$; if $m_i = x_i$, no action is required and $r_i = m_i$. The audited financial report is as follows:

$$
\begin{align*}
    r_i &= \begin{cases} 
        x_i & \text{with probability } g_i \\
        m_i & \text{with probability } (1 - g_i)
    \end{cases}
\end{align*}
$$

The probabilities that the auditor finds and corrects an error, conditional on $m_i$, are $\text{prob}(x_i | m_i)g_i$ and $\text{prob}(x_i | m_i)g_i$. These are the probabilities that the audited report $r$ deviates from the preliminary report $m$. Therefore, $r$ is more informative in the terms of fineness than $m$ with regard to $x$ because $r$ is a combination of $m$ and $x$. In the extreme case, a perfect audit

\[ \frac{1}{2}kg_i^2, \quad k > 0 \]

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5 Given our assumptions, the auditor would be indifferent between correcting by $x$ or $y$ because the enforcer only observes $y$ (as we discuss below) and the auditor can provide evidence that the actual outcome is indeed $x$. We rule out these correction strategies by assuming that the auditor cares for higher-quality reports if indifferent and discuss this assumption in the Implications and Conclusions section.

6 We assume that if the manager does not correct the report the auditor issues a qualified audit opinion, which has the same informative effect.
(\(g_i \to 1\)) always reveals \(x\), making \(m\) useless; we rule out this case by assuming \(k\) that is sufficiently large to ensure that \(g_i < 1\) for \(i = L, H\).

The audit market comprises auditors with similar characteristics and is competitive. Capturing the requirements of typical audit regulations, we assume that the audit fee \(A > 0\) is constant (and not contingent on the auditor’s report) and determined through negotiation between the owners of the firm and the auditor. Under these conditions, \(A\) is the fee with which the auditor expects to break even on his engagement. After accepting the engagement, the auditor’s objective is the minimization of the expected cost of the audit and of costs resulting from any remaining uncorrected errors that are identified by enforcement. In case of an enforcement action, the auditor incurs a cost \(C^A > 0\). Assuming \(C^A/k \leq 1\) is sufficient to ensure \(g_i < 1\).\(^7\)

**Enforcement**

Enforcement is an institution that independently investigates published audited financial reports. The scope of enforcement is limited and the enforcer does not perform another audit. While the audit includes both tests of controls and substantive procedures, enforcement performs very limited investigations that often include few positions that are considered critical. In many environments, the enforcer even preannounces accounting issues that it focuses on, such as impairments, consolidation, deferred tax assets, and the like, which require significant judgment by management and are prone to earnings management. To model the difference between enforcement and auditing, we assume the enforcer focuses on the reporting stage with the manager’s earnings management.\(^8\) Specifically, after observing the audited report \(r_i\), the investigation by the enforcer uncovers the underlying signal \(y_j\) from

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\(^7\) Note that this assumption does not imply that the amount of the penalty is lower than the cost of effort. The effort cost depends on \(g_i\), which is 0 at \(g_i = 0\), but increases to a large amount if \(g_i \to 1\). In equilibrium, we show later that \(C^A\) is greater than the effort cost \(kg_i^2 / 2\).

\(^8\) In practice, the distinction between errors due to strategic earnings management and random errors is less clear-cut. We believe our results are qualitatively unchanged as long as enforcement is geared towards finding earnings management than on other audit procedures.
the accounting system with some probability \( f \) (also referred to as enforcement effectiveness), but not the actual outcome \( x \). As a consequence, auditing is always more comprehensive than enforcement and provides more information per unit of effort level. However, the activities uncover different errors because the probabilities of detecting errors are not correlated.

If the report \( r_i \) equals \( y_i \) \((i = L, H)\), the enforcer ends the investigation without a finding. If the report \( r_i \) deviates from \( y_j, i \neq j \), then it alleges an error has occurred. If the firm or the auditor can present evidence that \( r_i = x_i \), the enforcer accepts this and ends the investigation. However, if no such evidence is available, the enforcer declares an error in the financial report, the consequence of which is that the error is published and the parties involved are subject to penalties. We assume that presenting evidence is costless to the auditor because he already collected it during the audit, and there is no further search for evidence in case the enforcer alleged an error.

The firm’s costs of an enforcement action are a potential loss of reputation and credibility of its financial reports, penalties, and other costs of legal liability. We denote these costs by \( C^O > 0 \). We do not explicitly model shareholder litigation against the firm, the manager, or the auditor.\(^9\) The manager is protected by limited liability, and we assume there are no other costs, such as a loss of reputation, or personal sanctions imposed. Therefore, the sole consequence of an enforcement action is a claw-back of compensation paid from an erroneous report \( r \), which is paid back to the firm’s owners. Finally, the costs to the auditor \( C^A \) include penalties, fines, potential legal liability, but also indirect effects such as a reputation loss.

\(^9\) Litigation requires that there exists a mechanism that \( x \) becomes eventually observable. We believe that the introduction of a litigation stage does not materially affect our main results.
The enforcer operates on a fixed budget, which we assume as exogenously determined by a governmental institution. In our model, the budget determines the probability $f \in [0, 1]$ with which the enforcer detects $y$. A higher budget increases $f$. Without loss of generality, we cast our analysis in terms of $f$ directly.

- Owner offers contract to manager and engages auditor
- Manager provides productive effort $a$
- Manager observes accounting signal $y$ and engages in earnings management $b$
- Preliminary report $m$ is realized
- Auditor chooses audit effort $g$, learns $(x, y)$ and corrects errors $(m \neq x)$ in the preliminary report
- Audited report $r$ is publicly issued
- Manager receives contractual compensation $s(r)$
- Enforcer investigates audited report $r$, learns $y$ and alleges error $(r \neq y)$
- Auditor may provide evidence that no error occurred $(r = x$ although $r \neq y)$; otherwise publication of error and enforcement action
- Firm, manager, and auditor incur costs from enforcement action

Figure 2: Time line

Figure 2 summarizes the sequence of events. The subsequent analysis is by backward induction: We begin with an analysis of the effectiveness of enforcement and then turn to the reporting equilibrium that consists of the auditor’s decision problem and the manager’s earnings management decision. This analysis provides insights into the effects of enforcement on the reporting quality, keeping the compensation contract of the manager constant. Finally, we examine the productive effects of enforcement by analyzing the manager’s productive effort choice. Using the results, we then examine the owner’s problem of designing the

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10 We do not consider the possibility that firms directly or indirectly pay for the enforcement to isolate the strategic effects from direct cost effects. Taking direct costs of enforcement into account would reinforce our main result that more enforcement can be detrimental.

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manager’s compensation contract and determine the effects of enforcement on the owner’s expected expected utility.

3. Reporting equilibrium

3.1. Preliminary results

We start with a preliminary result on the structure of the compensation function and the manager’s earnings management decision, which simplifies the rest of the analysis.

The manager’s expected utility, given the high productive effort \( a_H \), is

\[
E[U^M | a_H] = \text{prob}(r_H)s(r_L) + \text{prob}(r_H)s(r_H) - V - n\left(\text{prob}(y_L)b_L + \text{prob}(y_H)b_H\right) - \text{(expected cost of claw-back)}
\]  

\( (1) \)

The owner wants to induce the manager to exert effort \( a_H \) through the contractual compensation \( s(r) \) promised to the manager.

Lemma 1: The optimal contract to induce \( a_H \) is characterized by \( s(r_H) > s(r_L) = 0 \). Furthermore, \( b_H = 0 \).

All proofs are in the appendix. This result is intuitive: First, to induce the manager to exert high effort at a personal cost \( V \), the compensation must be greater for the report that is more likely with \( a_H \) than with \( a_L \), which is \( r_H \) because \( \text{prob}(r_H | a_H) > \text{prob}(r_H | a_L) \). Therefore, \( s(r_H) > s(r_L) \). Second, there is no reason to pay the manager more than his reservation utility, therefore, \( s(r_L) = 0 \), the minimum payment in this case. We label \( s \equiv s(r_H) \) the bonus. Given this compensation structure, the manager has an incentive to engage in earnings management if she observes \( y_L \) to increase the probability of a report \( m_H \), but no incentive for earnings management if she observes \( y_H \), which is \( b_H = 0 \).

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3.2. Enforcement action

The enforcement affects all decisions taken prior to it because the parties consider the subsequent effects in their decisions. The two panels in Figure 3 depict the events evolving after the manager observes the accounting signal $y_L$ and $y_H$, respectively, and the conditional probabilities of the events.

![Diagram](attachment:image.png)

**Figure 3: Auditing and enforcement stages**

The first panel in Figure 3 depicts the events if $y = y_L$ is realized. In this case, the manager engages in earnings management $b_L \geq 0$. If it is unsuccessful (probability $1 - b_L$), the
preliminary report remains $m_L$. The auditor finds out $x$ with probability $g_L$; if $x = x_H$, the auditor requests that the preliminary report be corrected to $r_H$; otherwise, the audited report is $r_L$ and if enforcement does not unravel $y$, no error is detected. If the enforcer learns $y$, then it is $y_L$, hence again there is no error. If the audited report is $r_H$, it is not challenged if the enforcer does not learn $y$. If it finds out $y$ (probability $f$), it is $y = y_L$, and the enforcer alleges an error because $r_H \neq y_L$. However, this case can only occur under $y = y_L$ if the auditor corrected the preliminary report based on his observation of $x_H$; therefore, he will provide evidence to the enforcer that there is in fact no error.

If $y_L$ is realized and earnings management is successful (probability $b_L$), the preliminary report is $m_H$. Again, if the auditor learns $x$, he will request correction to $r_L$. (\prob(x_L | y_L) g_H). Because $r_L = y_L$, regardless of whether it observes $y$ or not, the enforcer will not find an error. If the auditor learns $x = x_H$, no correction is made because the enforcer finds out $y = y_L$ with probability $f$, but there is evidence that $r_H = x_H$ is correct. Finally, if the auditor did not find out $y$ (probability $1 - g_H$) and the enforcer finds out $y = y_L$, it alleges an error, which the auditor cannot object, and this is the only case in which an error is published and an enforcement action is triggered.

The second panel in Figure 3 shows the events for $y = y_H$. Because there is no earnings management ($b_H = 0$ by Lemma 1), the only situation in which $r = r_L$ results from the auditor learning $x$ and observing $x = x_L$, which occurs with $\prob(x_L | y_H) g_H$. In this case the auditor requests correction, and the audited report is $r_L$. If the enforcer does not learn $y$ it cannot find an error; if it learns $y$, it will allege an error because $y_L \neq r_H$. However, in this case the auditor will present evidence that the report $r_H = x_H$ is correct. That is, if $y_H$ is realized, enforcement never finds an error.

Taken together, an error found by enforcement can only occur in one particular constellation: the accounting system reports low earnings, the manager succeeds in managing

\footnote{Lemma 2 below establishes $g_L = 0$. Therefore, these events will not occur in equilibrium; this is shown by the dashed arrows.}
earnings upwards, the audit does not uncover this bias, and the enforcer observes the low accounting signal. Note, however, that even in this case, the resulting financial report is not free of error, because the enforcer does not observe the outcome $x$ that is ultimately relevant.

3.3. Audit effort

Given the auditor accepted the audit engagement, he determines the audit effort $g_i$ by maximizing the expected utility conditional on the preliminary report $m_i$,

$$U^A(m_i) = A - \frac{k}{2} g_i^2 - \text{prob(error|m_i)} C^A$$

(2)

where $A$ is a constant at this stage.

**Lemma 2:** The optimal audit effort levels are:

$$g_L = 0 \text{ and } g_H = \text{prob}(y_L|m_H) f C^A / k$$

where $g_H > 0$ if $\hat{b}_L > 0$ and $f > 0$.

The incentive of the auditor to provide audit effort results from the risk of an enforcement action, the cost of which is captured by the last term in his utility function (2), $\text{prob(error|m_i)} C^A$. Higher audit effort increases effort cost, but reduces the probability of an enforcement action that is costly.

As is apparent from Figure 3, there is no risk of an enforcement action if the preliminary report is $m_L$, because this case can only occur if accounting earnings are $y_L$ and the manager’s earnings management was unsuccessful (the manager never engages in earnings management if $y_H$ obtains because $b_H = 0$). Therefore, the auditor optimally chooses $g_L = 0$. In contrast, if the preliminary report is $m_H$, the auditor has an incentive to exert audit effort $g_H > 0$. The reason is that he faces the risk that the enforcer finds an (undisputed) error, that is, $\text{prob(error|m_H)} > 0$ if he conjectures that the manager engaged in earnings management ($\hat{b}_L > 0$) and if enforcement exists ($f > 0$). The error probability given $m_H$ is

$$\text{prob}(y_L|m_H) = \frac{\text{prob}(y_L) \hat{b}_L}{\text{prob}(y_L) \hat{b}_L + \text{prob}(y_H)}$$
which is 0 for \( \hat{b}_L = 0 \) and increases in \( \hat{b}_L \); therefore, \( g_H \) increases in \( \hat{b}_L \) as well. The audit effort also depends on the probability \( f \) that the enforcer finds out \( y \). If \( f = 0 \), the auditor anticipates that there is no enforcement and has no incentive to provide audit effort. For \( f > 0 \), audit effort increases in \( f \). Finally, the term \( C^H/k \) captures the relative cost of an enforcement action and audit effort.

Given the optimal audit effort, the auditor’s conditional utility equals

\[
U^A(m_H) = A - \frac{k}{2} g_H^2 - \text{prob}(y_L|m_H)(1 - g_H)fC^A = A - \frac{k}{2} g_H (2 - g_H)
\]

The auditor accepts the audit engagement if the expected utility is greater or equal to zero. In a competitive audit market with homogenous auditors the expected profit of the auditors is zero. If \( m = m_H \), \( A \) must at least equal \( A = k g_H (2 - g_H) / 2 \); if \( m = m_L \), the auditor exerts no effort and \( A = 0 \). Therefore, \textit{ex ante} the audit fee is

\[
A = \text{prob}(m_H) \left( \frac{k}{2} g_H (2 - g_H) \right).
\]

Note that \( A \) depends on the conjectured earnings management strategy \( \hat{b}_L \) directly through \( g_H \) and indirectly through \( \text{prob}(m_H) \).

### 3.4. Earnings management effort

The manager makes the earnings management decision based on the realized accounting signal \( y \) that she privately observes. In Lemma 1 we establish that \( s(r_H) = s > 0, s(r_L) = 0, \) and \( b_H = 0 \), that is, the manager never engages in earnings management after observing \( y_H \). In Lemma 2 we show that \( g_L = 0 \) and \( g_H \) increases in the auditor’s conjecture of earnings management \( \hat{b}_L \). To determine \( b_L \), the manager maximizes her expected utility conditional on \( y_L \) and the conjecture of the audit effort \( \hat{g}_H \):

\[
E[U^M|a_H, y_L] = \text{prob}(r_H|y_L)s - V - \frac{1}{2} vb_L^2 - b_L(1 - \hat{g}_H)fs
\]
where the last term, \( b_L (1 - \hat{g}_H) fs \), captures the cost of enforcement to the manager, which equals the probability that the enforcer finds an error given \( y_L \) multiplied by the bonus \( s \) that must be paid back.

The benefit of earnings management is that \( b_L \) increases the probability that the preliminary report is \( m_H \) if the accounting signal is \( y_L \), which increases the probability of receiving a bonus, which is

\[
\text{prob}(r_L \mid y_L) = b_L (1 - \hat{g}_H) + b_L \text{prob}(x_H \mid y_L) \hat{g}_H + (1 - b_L) \text{prob}(x_H \mid y_L) \hat{g}_L
\]

**Lemma 3**: Given some \( s \), earnings management decreases in the conjectured audit effort \((\partial b_L / \partial \hat{g}_H < 0)\) if and only if

\[
T \equiv \text{prob}(x_H \mid y_L) - (1 - f) < 0
\]

The lemma follows directly from the first-order condition of \( E[U^M \mid a_H, y_L] \) with respect to \( b_L \),

\[
b_L = \frac{s}{v} \left( (1 - \hat{g}_H)(1 - f) + \hat{g}_H \text{prob}(x_H \mid y_L) \right)
\]

\[
= \frac{s}{v} \left[ (1 - f) + \hat{g}_H \left( \text{prob}(x_H \mid y_L) - (1 - f) \right) \right]
\]

Intuitively, one would expect that earnings management always decreases if the conjectured audit effort \( \hat{g}_H \) increases. However, this relation holds only if the term \( T \equiv \text{prob}(x_H \mid y_L) - (1 - f) < 0 \). Ceteris paribus, earnings management decreases in audit effort only if enforcement \( f \) is “low”; whereas earnings management *increases* if \( f \) is “high”. To see why, note that a higher \( \hat{g}_H \) increases the probability that the auditor finds out the true \( x \), which has two opposing effects: (i) it reduces the probability of receiving a bonus because the auditor detects \( x \), including \( x_L \), more often and a bonus requires that the auditor does not find out \( x \) and enforcement is unsuccessful, which occurs with probability \((1 - f)\). (ii) However, if the auditor finds out \( x \), it can also be \( x_H \), which promises the manager a bonus regardless of enforcement. The probability of this second effect is

\[
\text{prob}(x_H \mid y_L) = \frac{p\alpha}{p\alpha + (1 - p)(1 - \beta)}
\]
That is, the manager intentionally increases earnings management to induce more auditing, which is beneficial in this case. The optimal \( b_L \) trades off these two effects, and this trade-off is captured in \( T \). An increase of \( b_L \) in \( \hat{g}_H \) is more likely if the enforcement level \( f \) is relatively high and/or the accounting system is less precise (i.e., \( \alpha \) is relatively high).

The next result establishes a unique equilibrium in this manager-auditor game, which includes both earnings management and audit effort.

**Proposition 1**: Given some \( s \) that induces \( a_H \) and \( f \in (0, 1) \), there exists a unique equilibrium with earnings management \( b_L^* > 0 \) and audit effort \( g_H^* > 0 \).

The equilibrium earnings management \( b_L^* \) and audit effort \( g_H^* \) depend in a complex way on all relevant parameters. The proof in the Appendix gives explicit expressions for \( b_L^* \) and \( g_H^* \). In the following subsection, we provide comparative statics results.

### 3.5. Effects of enforcement on the reporting equilibrium

Of particular importance are the effects of enforcement effectiveness \( f \) and the costs of enforcement actions \( C^A \). We also state the effects of variations in the bonus payment \( s \); we endogenize \( s \) in the subsequent section. Note that the owner’s cost of enforcement \( C^O \) has no effect on the reporting equilibrium because it affects neither the manager nor the auditor. Its only effect is that it raises the cost of motivating high productive effort \( a_H \), which ultimately may lead the owner to prefer the low effort \( a_L \).

**Corollary 1**: Assume some \( s \) that induces \( a_H \), Equilibrium earnings management and audit effort have the following properties:

(i) \( b_L^* \) strictly increases in \( s \), and \( g_H^* \) strictly increases in \( s \) for \( g_H^* > 0 \);

(ii) \( b_L^* \) strictly decreases in \( f \), and \( g_H^* \) strictly increases in \( f \) for \( f < f_0 \) and strictly decreases for \( f > f_0 \), where \( 1/2 < f_0 < 1 \);

(iii) \( b_L^* \) strictly decreases in \( C^A/k \) if and only if \( T < 0 \), and \( g_H^* \) strictly increases in \( C^A/k \).

Corollary 1 (i) establishes that both \( b_L^* \) and \( g_H^* \) strictly increase in the bonus payment. A greater \( s \) increases *ceteris paribus* the marginal benefit of earnings management, which provides stronger incentives to the manager to work hard and to engage in earnings
management. A higher conjecture of earnings management induces higher audit effort. However, the higher audit effort mitigates earnings management, which works against the direct increase through higher \( s \). Corollary 1 (i) shows that in equilibrium the net effect is still an increase in earnings management.

Corollary 1 (ii) confirms the intuitive result that earnings management strictly decreases in enforcement effectiveness \( f \). If enforcement becomes perfect \( (f \to 1) \), it eliminates earnings management altogether. In contrast, the effect of a change in the enforcement effectiveness on the equilibrium audit effort depends on the level of enforcement: Starting from \( f = 0 \), increasing \( f \) increases \( g^*_u \), which results from the increase in the expected cost of enforcement to the auditor. However, there is an enforcement level \( f_0 > 1/2 \) at which \( g^*_u \) achieves its maximum and increasing enforcement further reduces \( g^*_u \), until it approaches 0 for \( f \to 1 \), because perfect enforcement eliminates earnings management, which again takes away any enforcement risk and any audit incentives from the auditor. This result suggests a complementary relation between audit effectiveness and enforcement effectiveness if enforcement is weak, and a substitutive relation between the two if enforcement is strong.

Corollary 1 (iii) states the effect of a variation of the cost of an enforcement action \( C^a \) to the auditor and a variation of the audit effort cost parameter \( k \). The important parameter is the ratio \( C^a/k \), which captures the relative enforcement cost over the scaling parameter \( k \) on audit effort cost. The enforcement cost provides the incentive for the auditor to exert effort; a direct consequence of this is that audit effort increases in \( C^a \) (decreases in \( k \)). Given higher audit effort, one would expect a reduction of equilibrium earnings management. However, Corollary 1 (iii) states this holds only if

\[
T = \frac{1}{1 + \left( \frac{1 - p^a(1 - \rho)}{\rho p} \right)} - (1 - f) < 0 .
\]
Otherwise, $b^*_L$ strictly increases in $C^A$ (decreases in $k$). Recall that Lemma 3 establishes that $rac{\partial b_L}{\partial g_H} > 0$ if $T > 0$ and vice versa, and the reason for the result in Corollary 1 is similar. The manager’s optimal bias given $y_L$ is

$$b^*_L = \frac{S}{V} \left( (1 - g^*_H)(1 - f) + g^*_H \text{prob}(x_H | y_L) \right) \quad (5)$$

A greater $C^A$ (lower $k$) increases the audit effort, and this has two effects on the bias: (i) higher audit effort increases the probability that the auditor detects the true outcome, which is beneficial for the manager if the auditor finds $x_H$ because the manager receives the bonus without a risk of a clawback in case of effective enforcement. (ii) Higher audit effort reduces the probability of a bonus if the auditor is unsuccessful in identifying the true outcome. Here a clawback can arise after enforcement, thus only the net loss of the bonus is relevant. The term $T$ captures the trade-off between these two effects: If $T$ is positive, the positive effect dominates, thus leading to higher earnings management; and vice versa.

3.6. Effects of enforcement on the quality of the financial report

We examine the quality of the audited financial report ($r$) first and then the incremental effect on quality after the disclosure of an enforcement action. The reason to consider both measures is that the dominant effect of enforcement is its preventive or disciplinary role on earnings management and auditing, which results from the risk of an enforcement action. Results of the enforcement are published only after a lengthy investigation, so the information about a stated error arrives late. The incremental effect of enforcement actions is important because it imposes a cost on the owner and the auditor and leads to a claw-back of a bonus.

13 It is noteworthy that the equilibrium strategies behave differently to the more intuitive behavior of the reaction functions.
Quality of audited financial report

Our measure of the quality of the audited financial report is the probability that the report \( r \) anticipates the ultimate outcome \( x \), which captures the precision of the financial report. Financial reporting quality is

\[
FRQ = 1 - \text{prob}(\text{divergence}) \tag{5}
\]

A “divergence” occurs if the report differs from the final outcome, i.e., \( r_i \neq x_i \) (\( i = L, H \)), which occurs with a probability of

\[
\text{prob}(\text{divergence}) = \text{prob}(r_L)\text{prob}(x_H | r_L) + \text{prob}(r_H)\text{prob}(x_L | r_H) = \text{prob}(x_H, r_L) + \text{prob}(x_L, r_H)
\]

The first term is the probability that the report understates the actual outcome,

\[
\text{prob}(x_H, r_L) = p\alpha(1-b_L)
\]

and the second term is the probability that it overstates the outcome,

\[
\text{prob}(x_L, r_H) = (1-p)(1-\beta)b_L^*(1-g_H^*) + (1-p)\beta(1-g_H^*) = (1-p)(1-g_H^*)\left(\beta + b_L^*(1-\beta)\right)
\]

We focus our analysis on the unweighted sum of the two errors, but acknowledge that the cost of an under- or overstatement varies with the decision problem in which the financial report is used. In our subsequent analysis of the productive effect, the weights on different types of errors are determined endogenously for that purpose.

Rearranging terms, the total probability of a diverging report can be expressed through three terms, which facilitate to understand the sources for the errors:

\[
\text{prob}(\text{divergence}) = \underbrace{p\alpha}_{E_1} \underbrace{(1-p)\beta}_{E_2} \underbrace{b_L^* (1-\beta)}_{E_3} \geq 0 \tag{5}
\]

The first term, \( E_1 \), is the \textit{ex ante} probability of an \( \alpha \)- and \( \beta \)-error that define the precision of the accounting system. This error is independent of earnings management, auditing, and enforcement.
The second term, $E_2$, represents the direct effect of earnings management on the probability of divergence. The sign of $E_2$ depends on the parameters of the accounting system. Note that the *ex ante* probability of a report $y_L$,

$$\text{prob}(y_L) = p\alpha + (1 - p)(1 - \beta)$$

is the sum of two events: $(1 - p)(1 - \beta)$ is the probability that $x = x_L$ and $y = y_L$, which is a correct depiction of the outcome, and $p\alpha$ is the probability that $x = x_H$ and the accounting system wrongly reports $y = y_L$. If the manager engages in earnings management, $b^* L > 0$, then if successful, she reports $r_H$. If $x = x_L$, then earnings management disguises the originally correct signal $y_L$, which adds to the errors in the financial report. We refer to this situation as “bad” earnings management. Conversely, if $x = x_H$, then the accounting signal was wrong, and earnings management corrects this wrong signal, which is “good” earnings management because it lowers the errors in the financial report. If

$$p\alpha > (1 - p)(1 - \beta)$$

then earnings management is “good” on average, otherwise it is “bad.” Condition (5) is more likely to hold for greater $p$ and for greater $\alpha$ and $\beta$. That is, the less precise the accounting system is, the more does earnings management correct it. At the same time, a decrease in accounting precision implies an increase in $\text{prob}(x_H | y_L)$, the conditional probability that the high outcome actually obtains although the accounting system has produced the low signal. Considering the definition of $T$ in (5), it is apparent that the presence of “good” earnings management and a positive relation between earnings management and (anticipated) audit effort are closely related. Given an enforcement probability $f$, the less precise the accounting system, the higher is $\text{prob}(x_H | y_L)$ and the more likely it is that $T > 0$ holds, implying that a larger audit effort induces higher earnings management.

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14 Notice this condition does not imply that a high $\alpha$-error is desirable because $(E_1 + E_2)$ can increase or decrease in $\alpha$. It only says that if $E_2 < 0$, an increase in earnings management reduces (*ceteris paribus*) the probability of an error and increases financial reporting quality.
The third term in (5), $E_3$, captures the effect of auditing, which always leads to a (weak) reduction in the probability of divergence. It arises if the actual outcome is $x_L$ (probability $1-p$), but the accounting system produces a signal $y_H$ because of the $\beta$-error and earnings management. $E_3 = 0$ for the boundary cases of no enforcement ($f = 0$) and perfect enforcement ($f = 1$) because then $g_H = 0$.

Although the parameter $f$ does not directly appear in the probability term (5), it affects earnings management and the audit effort and thus has an impact on earnings quality. The following corollary provides some general insights.

**Corollary 2:** Assume some $s$ that induces $y_H$. Enforcement effectiveness $f$ has the following effects on financial reporting quality $FRQ$:

(i) If enforcement is perfect ($f = 1$), then $FRQ(f = 1) = 1 - (p\alpha + (1-p)\beta)$.

(ii) $FRQ$ is not necessarily monotonic in $f$. A necessary condition for $FRQ$ to increase everywhere is $p\alpha < (1-p)(1-\beta)$. If the accounting system is perfectly precise ($\alpha = \beta = 0$), $FRQ$ always strictly increases in $f$.

(iii) If $p\alpha > (1-p)(1-\beta)$, $FRQ$ strictly decreases in $f$ for $f > f_0$, where $1/2 < f_0 < 1$.

Corollary 2 (i) shows that if enforcement is perfect, $FRQ$ is determined solely by the precision of the accounting system. Perfect enforcement deters any earnings management, but at the same time induces no audit effort. Hence, financial reporting quality is then determined solely by the errors resulting from the accounting system, but not on any incentives. Note that $\left( p\alpha + (1-p)\beta \right)$ is the expected sum of the $\alpha$- and $\beta$-error.

Corollary 2 (ii) compares $FRQ$ under perfect enforcement with $FRQ$ under no enforcement ($f = 0$). It shows that perfect enforcement strictly improves $FRQ$ if and only if $p\alpha < (1-p)(1-\beta)$, otherwise it lowers $FRQ$. The reason is again that, on average, earnings management is “good” if $p\alpha > (1-p)(1-\beta)$ and is “bad” otherwise. Moreover, it states that an increase in enforcement effectiveness does not necessarily increase $FRQ$. This result can be easily seen if one considers the case $p\alpha > (1-p)(1-\beta)$, which implies $FRQ(f = 0) > FRQ(f = 1)$. Then $FRQ$ must decrease over some interval of $f$. Therefore, $p\alpha < (1-p)(1-\beta)$ is a necessary condition for a monotone increase of $FRQ$. A special case is a perfect
accounting system ($\alpha = \beta = 0$). Then $FRQ$ strictly increases in $f$ up to the maximum of $FRQ = 1$. In this case earnings management cannot be “good” ($p\alpha = 0 < (1 - p)(1 - \beta)$ holds). An increase in $f$ strictly lowers earnings management, which implies a strict increase in $FRQ$ even in the range $f > f_0$ (as defined in Corollary 1 (ii)) where the audit effort declines.

Corollary 2 (iii) describes the behavior of $FRQ$ for $p\alpha > (1 - p)(1 - \beta)$ in more detail: It states that $FRQ$ strictly decreases in $f$ for sufficiently large $f$ ($f > f_0$). The reason is that the equilibrium audit effort $g^*_\mu$ decreases in that range, which lowers the (negative) error component $E_3$ of $FRQ$ because the audit is less effective in reducing the errors in the reporting system.

Taken together, the conventional wisdom that greater enforcement effectiveness increases financial reporting quality does not generally hold (even if enforcement is costless). As long as earnings management is either “good” or only moderately “bad”, increasing enforcement can reduce financial reporting quality because it crowds out audit effort. Figure 4 gives an example in which financial reporting quality increases or decreases, contingent on the variation of a single parameter ($p = 0.3$ and 0.9, respectively). The other parameters are as follows: $s = 7$; $\alpha = \beta = 0.2$; $v = 20$; $C^A/k = 1$.

![Figure 4: Probability of erroneous audited financial report](image)

The left graph depicts “bad” earnings management ($p = 0.3$), the right graph “good” earnings management ($p = 0.9$)

$E_1$ = Effect of accounting system (ex ante probability of error)

$E_2$ = Effect of earnings management

$E_3$ = Effect of the audit
This result suggests that the magnitude of earnings management *per se* is no reliable indicator of FRQ. Empirical studies often use a measure of discretionary accruals as a proxy for earnings quality, assuming a monotonic negative relation between discretionary accruals and the quality of financial reports. The above results caution against this assumption.

*Quality of financial report after enforcement action*

If the enforcer alleges an error that is undisputed, an enforcement action will be taken against the firm and the auditor. The publication of such an action leads to a restatement of the financial report (or to the disclosure of the fact that an error has occurred, which we assume has the same informational consequences as a restatement). Since the process of enforcement investigations and actions starts after the publication of financial reports, restatements become public information considerable time after the fiscal year. In the meantime other, and more contemporaneous, information may have been published by the firm. Nevertheless, restatements contain new information and affect financial reporting quality. In our model, we abstract from subsequent information and only consider the incremental effect of a restatement on financial reporting quality.

The enforcer states an error only in case the report is \( r_H \), the auditor fails to learn the outcome \( x \), and the enforcer discovers \( y = y_L \), which occurs with a probability of

\[
\text{prob}(\text{error}) = \text{prob}(y_L) b_L^* \left(1 - g_H^* \right) f
\]  

(6)

This probability captures two distinct events: (i) An enforcement action leads to a correction of a deviation of the financial report if the report is \( r_H \), the enforcer observes \( y = y_L \), the auditor did not learn \( x \), and the outcome is in fact \( x_L \), which occurs with probability

\[
(1 - p)(1 - \beta) b_L^* \left(1 - g_H^* \right) f
\]

A restatement in this case unambiguously increases financial reporting quality. (ii) However, enforcement itself is not free of error because it does not uncover the outcome \( x \), but only the
accounting signal $y_j$ that provides imprecise information about $x$.\footnote{Another error occurs if the enforcer does not state an error, although there is in fact one. This occurs if the enforcer does not learn $y$, and the resulting error is embedded in the probability of a deviating report, which we analyze earlier.} In this case, the enforcer states an error even though the audited financial report was correct. This event occurs if the auditor did not learn $x$, but $x = x_H$, because then the enforcer’s alleged error cannot be challenged by audit evidence. The probability of this event is

$$\text{prob}(y_L b^*_L(1 - g^*_H)f \text{ prob}(x_H | y_L) = p\alpha b^*_L(1 - g^*_H)f$$

and a restatement decreases financial reporting quality.

The net decrease of $FRQ$ is

$$E_4 \equiv -(1-p)(1-\beta)b^*_L(1 - g^*_H)f + p\alpha b^*_L(1 - g^*_H)f$$

$$= f (1 - g^*_H)b^*_L(p\alpha - (1 - p)(1 - \beta))$$

which equals (6). Note that $E_4 = -f(1 - g^*_H)E_2 < -E_2$, so the net effect of the enforcement action mitigates the effect of $E_2$ on $FRQ$.

**Corollary 3**: Assume some $s$ that induces $a_H$. An enforcement action improves the quality of the issued audited financial report if and only if $p\alpha < (1 - p)(1 - \beta)$.

This result directly follows from the fact that $E_4$ is negative if $p\alpha < (1 - p)(1 - \beta)$. In particular, if earnings management is “good” on average, then restating the audited financial report as a result of the enforcement action reduces financial reporting quality.

Figure 5 plots the total error after an enforcement action for the same two examples that underlie Figure 4. The parameters are: $p = 0.3$ and $p = 0.9$, respectively; $s = 7$; $\alpha = \beta = 0.2$; $v = 20$; $C/k = 1$. The left graph ($p = 0.3$) shows that the total error decreases for relatively low $f$, but eventually increases again if $f$ grows large.
Figure 5: Probability of erroneous audited financial report after restatement
The left graph depicts “bad” earnings management ($p = 0.3$),
the right graph “good” earnings management ($p = 0.9$)
$E_1 =$ Effect of accounting system (ex ante probability of error)
$E_2 =$ Effect of earnings management
$E_3 =$ Effect of the audit
$E_4 =$ Effect of restatement

Finally, we consider the market price effect of a restatement.

**Corollary 4:** Assume some $s$ that induces $a_H$. The market price reaction to a restatement due to
an enforcement action is always negative. It is stronger the greater is $\frac{(1-p)(1-\beta)}{\alpha}$. An enforcement action occurs only if $r = r_H$, and it always consists of a downward
correction to $r = r_L$. Because $p > 0$ and $\alpha, \beta \in (0, 1/2)$, the price reaction must be negative,
i.e., the market price of the firm decreases. However, the enforcer can err, and the restatement
may be incorrect, which happens with probability $p a b_L (1 - g_H^*) f$, as we show above. A restatement is correct with probability $(1 - p)(1 - \beta)b_L (1 - g_H^*) f$. The probability that it is
correct increases in $\frac{(1-p)(1-\beta)}{\alpha}$, and the market price adjustment is stronger in this case.

4. **Optimal compensation contract**

4.1. **Owner’s decision problem**

We now turn to the first stage in the game, in which the owner hires the manager and
offers a compensation contract that induces the manager to exert high effort $a_H$. Our
preliminary results in Lemma 1 record basic properties of the optimal contract: it is a bonus
contract with \( s(r_h) = s > 0 \) and \( s(r_L) = 0 \). In determining the optimal compensation, the owner must consider that a higher bonus \( s \) increases the manager’s incentive to work hard, but also increases her incentive to engage in earnings management. Recall that Corollary 1 (i) establishes that equilibrium earnings management strictly increases in \( s \), which again affects the equilibrium audit effort and the cost of enforcement.

The owner maximizes the expected utility with regard to \( s \), taking into account the subsequent equilibrium strategies it triggers. The expected utility comprises the following components:

\[
E[U^o | a_H] = (1-p)x_L + px_H - \text{prob}(r_H)s - A - \text{prob(error)}C_o^o + \text{prob(error)}s
\]  

(7)

Because the expected outcome depends only on the production technology, the owner minimizes the expected compensation to the manager with respect to the bonus \( s \), considering the (endogenous) audit fee and the net cost of an error identified through enforcement. An enforcement action costs the firm \( C_o \), net of a claw-back of the manager’s bonus. The owner’s objective function becomes

\[
\min_s \left( \text{prob}(r_H)s + A + \text{prob(error)}(C_o^o - s) \right)
\]  

(8)

where

\[
\text{prob}(r_H) = (1-p)(1-\beta)b^*_L(1-g^*_H) + (1-p)\beta(1-g^*_H) + p\alpha b^*_L + p(1-\alpha)
\]

and

\[
\text{prob(error)} = \text{prob}(y_L)b^*_L(1-g^*_H)\beta
\]

Note that these probabilities indirectly depend on \( s \) through the equilibrium strategies \( b^*_L \) and \( g^*_H \).

The manager accepts the contract offered by the owner if it meets her reservation utility, which we normalized with 0. Because compensation is also bound by 0, any contract yields nonnegative expected compensation. It turns out that the crucial constraint is the manager’s incentive constraint that ensures she chooses the high effort \( a_H \). To see this, observe that the effort choice occurs before the accounting system reports the signal \( y \). The manager’s expected utility is
\[ E[U^M | a_H] = \text{prob}(r_H)s - V - \text{prob}(y_L)\frac{v}{2}b_{L}^{*2} - \text{prob}(y_L)b_{L}^{*}(1 - g_H^{*})fs \]  \quad (9)\]

where the first term is the expected bonus, the second term, \( V \), is the disutility of high effort, the third term is the expected cost of earnings management, and the fourth term is the expected claw-back of the bonus if the enforcer identifies an error. Substituting for \( \text{prob}(r_H) \) and \( b_{L}^{*} \), the expected utility becomes

\[ E[U^M | a_H] = s \text{prob}(y_H)(1 - g_H^{*} \text{prob}(x_L | y_H)) + \text{prob}(y_L)\frac{1}{2}vb_{L}^{*2} - V \]

The incentive compatibility constraint is

\[ E[U^M | a_H] \geq E[U^M | a_L] = s \text{prob}(y_H)[(1 - g_H^{*} \text{prob}(x_L | y_H)) + \text{prob}(y_L)a_L)\frac{1}{2}vb_{L}^{*2} \]  \quad (10)\]

where \( b_{L}^{*} = b_{L}(g_H^{*} | a_L) \) denotes the manager’s adjusted earnings management effort if she deviated from the equilibrium production effort \( a_H \). The auditor still conjectures \( a_H \) and \( b_{L}^{*} \); hence, he does not adjust the equilibrium audit strategy \( g_H^{*} \). Therefore, \( b_{L}^{*} \) is based on the reaction function \( b_L \), anticipating \( g_H^{*} \), which results in

\[ b_{L}^{*} = \frac{s}{v}[(1 - f) + g_H^{*} (\text{prob}(x_H | y_L, a_L) - (1 - f))] \]

The right-hand side of (10) is always positive for \( s > 0 \), implying that a contract that satisfies incentive compatibility induces rents to the manager and thus clearly meets her reservation utility of 0.

After deviating from \( a_H \) to \( a_L \), the manager would reduce earnings management because it becomes less likely that \( x = x_H \). The probabilities are:

\[ \text{prob}(x_H | y_L) = \frac{p\alpha}{p\alpha + (1 - p)(1 - \beta)} > \frac{q\alpha}{q\alpha + (1 - q)(1 - \beta)} = \text{prob}(x_H | y_L, a_L) \]

for \( p > q \), which results in \( b_{L}^{*} < b_{L}^{*} \). However, the probability \( y_L \) increases and so do the instances of earnings management. Denote the minimum \( s \) that satisfies the incentive compatibility constraint (10) by \( s > 0 \). The following proposition characterizes the optimal compensation contract.
Proposition 2: Under mild conditions, the optimal bonus is determined by the manager’s incentive compatibility constraint only, i.e., \( s^* = \xi \).

As shown in the appendix, \( \xi \) is implicitly defined by

\[
\xi = \frac{1}{(p-q)(1-\alpha - \beta (1-\xi^*))} \left[ V + \frac{\alpha}{2} \left( \text{prob}(y_L|a_L)b_{L}^{*2} - \text{prob}(y_L)b_{L}^{2} \right) \right]
\]

The proof examines each cost component included in the owner’s expected utility and establishes that the audit fee and the owner’s expected cost of enforcement unambiguously increase in \( s \). It also finds that the expected compensation (net of claw-back) increases in \( s \) under mild conditions. Together, these results imply that the owner chooses the bonus payment that just satisfies the incentive compatibility constraint, but does not pay more. The reason why formally mild conditions are required is subtle. Note that one would conjecture that an increase in \( s \) over \( \xi \) cannot be desirable to the owner, because it is not useful to increase productive effort but only increases the manager’s earnings management incentives. This intuition holds for all (direct and indirect) effects of increasing \( s \) over and above \( \xi \), except for one effect: The probability that the manager receives the bonus, \( \text{prob}(r_H) - \text{prob}(\text{error}) \), directly depends on the audit effort \( g_H^* \), which improves the quality of the financial report by reducing \( \text{prob}(r_H) \) through lowering the \( \beta \)-error. Ceteris paribus, an increase in \( s \) increases the audit effort, which reduces the probability of paying a bonus in a situation in which the productive outcome is \( x_L \), but the accounting system reports \( y_H \). The proof shows that this effect has a value of \( (1-p)\beta \frac{dg_H^*}{ds} s \). It is small and most likely outweighed by the other effects that increase the owner’s expected utility from increasing \( s^* \) over \( \xi \). Sufficient conditions, for example, are the following: \( \beta \) is „low,“ \( p \) is „high,“ or \( C^0 \) is “high.” Then the owner chooses the lowest \( s \) that implements \( a_H \), which is \( s^* = \xi \). But it is impossible to formally exclude a case that this effect might dominate. In the subsequent analysis, we assume that the mild conditions stated in Proposition 2 are satisfied.

To conclude the analysis of the owner’s decision problem, we briefly consider what happens if it becomes too costly to the owner to induce the manager to provide high productive effort \( a_H \). The next result provides the lower bound on the owner’s expected utility.
Lemma 4: The owner’s expected utility from inducing $a_L$ is

$$E[U^O | a_L] = (1 - q)x_L + qx_H$$  \hspace{1cm} (11)$$

Note that to induce $a_L$, the optimal contract pays the minimum compensation, which is $s(r_L) = s(r_H) = 0$. This compensation is independent of the financial report, which eliminates incentives of the manager to engage in earnings management – it would be costly, but of no benefit. The manager’s expected utility for low productive effort $a_L$ is 0. Enforcement will not find an error because there is no earnings management; hence, there is no cost of enforcement. Finally, the auditor has no incentive to provide audit effort either ($g = 0$). That is, $r_i = y_i$. In equilibrium, the auditor chooses $g = 0$ and expects no cost of enforcement. In a competitive market, the audit fee offered therefore is

$$A = \text{prob} \left( m_H \right) \left( \frac{k}{2} g_H \left( 2 - g_H \right) \right) = 0$$

The expected outcome from the production process is higher for $a_H$ than for $a_L$ because

$$\left( (1 - p)x_L + px_H \right) - \left( (1 - q)x_L + qx_H \right) > 0$$

holds because $p > q$. This benefit comes at a higher cost of inducing $a_H$. Clearly, if the financial reporting system (and the institutional safeguards) is not sufficiently informative to use it for compensation purposes, the expected cost of inducing $a_H$ can outweigh the expected benefit. For example, low (or no) enforcement may be such a case; increasing the level of enforcement then has a productive effect if it becomes beneficial to the owner to induce high effort. Our subsequent results show how the owner’s expected utility varies with a change in the enforcement effectiveness. If the expected utility decreases for a change in enforcement, production becomes more costly and perhaps even too costly to sustain high productive effort.

4.2. Effects of enforcement under the optimal contract

In this subsection, we examine how a change in enforcement effectiveness affects the management incentives provided by the owner and the expected utility of the owner, which is in our setting equal to the value of the firm.

The incentive compatibility constraint implicitly defines the minimum bonus,
The bonus $\xi$ must be set sufficiently high to cover the manager’s cost of effort $V$ and the difference in (net) utility arising from the fact that the manager chooses the conditionally optimal earnings management effort given $a_H$ and $a_L$, respectively (which is captured in the term $D$ in (12)). These two costs are scaled by the factor $\frac{1}{(p-q)(1-\alpha-\beta(1-g_{H*}^*)^2)}$, which captures the informativeness of the financial report $r$ about the productive effort. Note that higher audit effort $g_{H*}^*$ reduces the required $s$ because the auditor detects $x$ more often, and this reflects a direct benefit of auditing on incentives.

The functional behavior of the second term is complex because it depends on two different earnings management strategies, one played in equilibrium ($b_{L*}^*$) and the other out of equilibrium ($b_{L*}^{\perp}$). In general, equation (12) for $\xi$ cannot be explicitly solved. To gain some insight, we consider the boundary cases $f = 0$ (no enforcement) and $f = 1$ (perfect enforcement). If $f = 0$, then the audit effort $g_{H*}^* = 0$ and earnings management is equally high for both effort levels (i.e., $b_{L*}^* = b_{L*}^{\perp}$). The low signal $y_L$ occurs more frequently under $a_L$ than under $a_H$ because $\text{prob}(y_L | a_L) > \text{prob}(y_L | a_H)$, hence, the manager receives greater expected utility from earnings management if she chose the low effort. Therefore, $D(f = 0) > 0$. To be incentive compatible, the bonus must compensate the manager for the loss in expected benefits from earnings management if she decides to exert the high effort, but this increase in $s$ in turn increases the earnings management incentive further. If $f = 1$, there is no earnings management, in which case $D(f = 1) = 0$, and $D$ can be either positive or negative for $f$ somewhat below $f = 1$. The following result summarizes general properties of the minimum bonus $\xi$, which is the optimal bonus under the conditions described in Proposition 2.

\[
\xi = \frac{1}{(p-q)(1-\alpha-\beta(1-g_{H*}^*)^2)} \left[ V + \frac{V}{2} \left( \text{prob}(y_L | a_L)b_{L*}^{2} - \text{prob}(y_L | b_{L*}^{2}) \right) \right]
\]
**Proposition 3**: The minimum bonus $s$ has the following properties:

(i) If $f = 0$, $s > \frac{V}{(p-q)(1-\alpha - \beta)}$ and strictly decreases in $f$.

(ii) If $f = 1$, $s = \frac{V}{(p-q)(1-\alpha - \beta)}$ and increases if $f$ approaches 1 from below; the increase is strict if $\beta > 0$.

(iii) $s$ attains a minimum for $f = f_1 \in (0, 1)$ and $s(f_1) < \frac{V}{(p-q)(1-\alpha - \beta)}$ if $\beta > 0$.

The proof is in the appendix. Proposition 3 establishes that introducing enforcement has a non-monotonic effect on the optimal expected compensation: Increasing enforcement is beneficial for low levels of $f$, but becomes strictly detrimental for high levels of $f$ (except in the case of $\beta = 0$). We discuss the intuition for this result below.

The bonus to induce the manager to exert high effort under $f = 0$ is strictly higher than that under perfect enforcement ($f = 1$); the required bonus in the latter case is $s = \frac{V}{(p-q)(1-\alpha - \beta)}$, which is equal to the bonus that would result if the manager has no earnings management opportunity. In that case, enforcement would not identify any earnings management and the auditor would not exert audit effort because there is no risk of an enforcement action. This bonus is solely governed by the characteristics of the production technology and the accounting system. In particular, $s$ decreases the more precise the accounting signal is (lower $\alpha$ and $\beta$).

The optimal bonus in case of no enforcement is strictly greater because the manager engages in earnings management ($h_L^* > 0$), which is costly; and the differential between earnings management under productive effort levels $a_H$ relative to $a_L$ must be compensated by a higher bonus to continue to induce $a_H$. This increase in the bonus amplifies the earnings management incentive, which again pushes the required bonus further upwards. Moreover, earnings management strictly reduces the information content of the financial report, which is another reason for the increase of the bonus (recall that at $f = 0$ there is no benefit from auditing since the audit effort is zero).

Increasing $f$ from $f = 0$ has the following effects: It introduces a risk of an enforcement action, which mitigates the incentive of the manager to manage earnings (due to the risk of a
claw-back of bonus) and induces the auditor to exert positive audit effort – this audit effort further mitigates earnings management in equilibrium. Both effects together increase the information content of the accounting report, which allows the owner to reduce the bonus, which further alleviates earnings management and audit effort somewhat until an optimum is reached. Proposition 3 (i) establishes that the total effect from increasing $f$ from $f = 0$ strictly reduces the required bonus.

Proposition 3 (ii) shows that higher enforcement effectiveness increases the bonus $s$ if $f$ increases to a value close to 1. Statements in (i) and (ii) together imply that the bonus $s$ is minimal for a specific $f_1 \in (0, 1)$ and that this minimum is less than $s = \frac{\nu}{(\rho - q) \times (1 - a - \beta)}$ (except for the knife-edge case of $\beta = 0$, in which $f_1$ is a saddle point).

These characteristics suggest that the typical behavior of the optimal bonus (and the expected compensation cost) is u-shaped. The main reason that “too” strong enforcement is harmful for incentives is that enforcement substitutes audit effort if enforcement is strong, whereas it is a complement if enforcement is weak. Crowding out audit effort reduces the information content of reported earnings because it is the auditing function that uncovers and corrects errors that arise from the accounting system. Enforcement controls earnings management in the financial report (as does more auditing), but it does not perform a full audit. While we assume that enforcement is costless to the firm, factoring in a cost of enforcement amplifies this disadvantage.

The owner’s expected utility consists of the expected outcome less the expected bonus payment $s$ (net of a potential claw-back), the audit fee $A$, and the expected cost of an enforcement action. The equilibrium audit fee is

$$A = \text{prob}(m_H) \frac{k}{2} g^*_u (2 - g^*_u)$$

which is directly increasing in $k$ and equals 0 if $g^*_u = 0$, which is the case if $f = 0$ or 1. The expected enforcement cost is

$$\text{prob}(y_L) b^*_L (1 - g^*_u) f C^\alpha$$
which is linearly increasing in the cost of an enforcement action $C^O$ and is 0 if $b^*_L = 0$, which is again the case if $f = 0$ or 1. Therefore, in the boundary cases of $f = 0$ and $f = 1$ the owner’s expected utility equals the expected outcome minus the expected bonus payment, for which the relation in Proposition 3 holds. The following result summarizes the effects.

**Proposition 4:** The owner’s expected utility (firm value) is strictly greater under perfect enforcement ($f = 1$) than under no enforcement ($f = 0$). Varying enforcement effectiveness $f$ within 0 and 1 can increase or decrease the owner’s expected utility, depending on the parameters.

A reason for the indeterminate effects of varying $f \in (0, 1)$ is that the audit fee $A$ is directly related to the audit cost parameter $k$ (whereas the audit strategy and minimum bonus $g$ only depend on the auditor’s enforcement cost relative to the audit cost, $C^A/k$) and that the owner’s enforcement cost depend directly on $C^O$. Therefore, varying these parameters directly affects the owner’s expected utility. We illustrate the possible effects by an example using the following parameters: $p = 0.8$, $q = 0.2$, $\alpha = 0.2$, $V = 1$, $v = 40$, $C^A/k = 10$, $C^O = 1$; $\beta$ takes values between 0 and 0.3, and $k$ is either 1 or 5.\(^{17}\) Figure 6 depicts the equilibrium earnings management and audit effort for the full range of enforcement effectiveness for $\beta = 0.1$. Equilibrium earnings management $b^*_L$ always decreases for an increase in enforcement $f$, whereas equilibrium audit effort $g^*_{H}$ first increases and then decreases for higher $f$. This illustrates the crowding-out effect of stronger enforcement on audit effort.

\(^{17}\) We keep $C^A/k$ constant to ensure that equilibrium earnings management and audit effort are not affected by the change in $k$. That means that $C^A$ is 10 and 50, respectively. $C^A/k = 10$ does not satisfy the sufficient condition ($C^A/k \leq 1$) but is low enough to ensure $g^*_{H} < 1$. 

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Figure 6: Equilibrium strategies under the optimal contract ($\beta = 0.1$)

Figure 7 plots the required bonus $s$ for a variation of the enforcement for $\beta = 0, 0.1,$ and $0.2$. A lower $\beta$ is always beneficial to the owner because it makes the accounting system more precise (ceteris paribus), which allows the owner to reduce the required bonus. $\beta = 0$ is the special case in which the bonus decreases in $f$ over the full range of $f$, so that $f = 1$ minimizes the required bonus. For $\beta > 0$, the bonus minimizing enforcement effectiveness is strictly less than 1. If $\beta = 0.1$, the required bonus increases in $f$ in the range $(0.67, 1]$ and if $\beta = 0.2$, it increases in the range $(0.62, 1]$.
Figure 8: Owner’s expected cost

Figure 8 depicts the (negative) expected cost to the owner, which reflects the owner’s expected utility (firm value) before adding the constant expected outcome. Again, the owner’s expected utility is greater the more precise the accounting system is (lower $\beta$) and, as stated in Proposition 4, it is higher under perfect enforcement ($f = 1$) than under no enforcement ($f = 0$). The effect of increasing enforcement $f$ depends on the parameter constellations. In Figure 8, we vary $k$ and $C^A$ to show that for weak enforcement, increasing enforcement can either increase or decrease the owner’s expected utility, and a similar functional behavior occurs for strong enforcement. Notice that for $\beta = 0.3$, $k = 1$ and $C^A = 10$, the expected cost is minimal at an enforcement level that is strictly less than perfect enforcement, suggesting that “too” much enforcement destroys firm value. While not shown in the Figure, a higher cost of an enforcement action $C^O$ directly reduces the owner’s utility in the range of $f \in (0, 1)$.

Finally, enforcement can have an immediate productive effect if the cost to induce a high productive effort $a_H$ becomes so high that the owner is better off inducing the low productive effort $a_L$. In Figure 8 the latter option would introduce a constant line, $E[U^O|a_L] - E[U^O|a_H]$, which can be greater or less than the expected cost curves. For example, consider the case with $\beta = 0.1$, $k = 5$, and $C^A = 50$: If $E[U^O|a_L] - E[U^O|a_H] = -1.75$, then if enforcement effectiveness is between $[0, 0.12]$ or between $[0.73, 1]$ the owner implements $a_H$. 

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otherwise \( a_L \). Therefore, if enforcement effectiveness was 0.1 and increases to 0.2, there is a loss in productivity.

4.3. Financial reporting quality under the optimal contract

Our definition of financial reporting quality \( FRQ \) is 1 minus the probability of a divergence of the audited financial report from the true outcome, which is defined in equation (5). We examine \( FRQ \) in Corollary 2 earlier, assuming a constant \( s \) that induces the desired productive effort level \( a_\mu \). However, if the owner optimally chooses the manager’s compensation, the bonus affects equilibrium earnings management \( b_L^* \) and audit effort. In particular, according to Corollary 1 both earnings management and audit effort \textit{ceteris paribus} increase in \( s \). In the following, we show that the main results of Corollary 2 continue to hold with the endogenous bonus \( s \).

**Corollary 5:** Enforcement effectiveness \( f \) has the following effects on financial reporting quality \( FRQ \):

(i) If enforcement is perfect \((f = 1)\), then \( FRQ(f = 1) = 1 - (p\alpha + (1 - p)\beta) \).

(ii) \( FRQ \) is not necessarily monotonic in \( f \). A necessary condition for \( FRQ \) to increase everywhere is \( p\alpha < (1 - p)(1 - \beta) \).

The proof is obvious because if enforcement is perfect, \( FRQ \) is independent of incentives. The result for no enforcement holds for any feasible \( s \), so it must hold for the minimum bonus \( s \) as well. And the non-monotonicity result is a direct analogy to Corollary 2 (ii). It is difficult to provide more specific results on the functional form of \( FRQ \) over \( f \) because there is no explicit solution. The intuition is the same as that for constant \( s \), and also numerical examples show that the behavior of \( FRQ \) is similar.

Figure 9 depicts the equilibrium financial reporting quality for the same example as in Figure 7 for \( \beta = 0 \) and \( \beta = 0.1 \). The other parameters are: \( p = 0.8, q = 0.2, \alpha = 0.2, V = 1, v = 40, C_A/k = 10 \). \( FRQ \) is higher if the precision of the accounting system increases (\( \beta \) is lower). In both cases, \( p\alpha < (1 - p)(1 - \beta) \), so that \( FRQ(f = 0) < FRQ(f = 1) \). If \( \beta = 0.1 \), \( FRQ \) first increases and then declines for sufficiently high \( f \), and if \( \beta = 0 \), \( FRQ \) monotonically increases in \( f \).
The examples also suggest that a change in enforcement affects $FRQ$ differently to the expected utility of the owner (firm value). This result underscores the insight that different objectives of financial reporting require different characteristics of accounting standards and their enforcement. For example, in their conceptual frameworks the IASB (2010) and the FASB (2010) argue that accounting to inform capital providers in making investment decisions also covers the information demand for stewardship. Gjesdal (1982) shows that in a more general agency model the ranking of accounting systems designed for different purposes do not coincide. However, that does not necessarily imply that the differences in the respective optimal accounting systems are large.$^{18}$ Our findings are in line with these results. As noted earlier, in many decision situations an error from an overstatement of earnings has different economic consequences relative to an error from an understatement of earnings. We defined $FRQ$ as equally weighted errors, which is a specific case, but using reported earnings to provide incentives for productive effort (and to earnings management, which goes along with that) generates different economic consequences of the two types of errors and, therefore, the desired effectiveness of enforcement $f$ differs somewhat.

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$^{18}$ See, e.g., Drymiotes and Hemmer (2013).
5. Implications and conclusions

This paper challenges the conventional wisdom that increasing enforcement of financial reporting has positive economic effects. The assumption that increasing enforcement improves financial reporting quality ignores the fact that the strategies of the owner, managers, and auditors are interrelated and are determined in an equilibrium. We show that stronger enforcement, even if it is costless, can be detrimental for financial reporting quality and for firm value and we provide conditions when this result arises. Generally, increasing weak enforcement improves financial reporting quality and can improve or reduce firm value, whereas increasing enforcement that is already strong can decrease financial reporting quality and reduce or increase firm value.

Essentially, two related reasons are responsible for the result that better enforcement can be detrimental: First, introducing enforcement increases audit effort, but if enforcement becomes sufficiently strong, it crowds out auditing. Because enforcement is more limited in scope than auditing, this crowding out effect reduces financial reporting quality. Second, earnings management is not necessarily “bad” but can be “good” if the accounting system erroneously understates earnings. A manager with earnings-based compensation has an incentive to manage earnings upwards, which improves financial reporting quality if it corrects an understatement. If the expected error from an understatement is greater than that of an overstatement, earnings management has a positive effect on financial reporting quality. However, stronger enforcement mitigates earnings management and if it is “good,” financial reporting quality declines.

Our results provide several empirical predictions, such as:

- An increase of enforcement (unambiguously) mitigates earnings management.
- An increase of enforcement improves financial reporting quality if enforcement is weak; if enforcement is already strong, the effect is weaker or even reverses.
- An increase of enforcement can increase or decrease firm value, contingent on the situation.
− Audit fees increase in enforcement effectiveness if the enforcement is weak, but decrease if the enforcement is strong.

− The market price reaction to a restatement due to an enforcement action is always negative, but its magnitude is higher if earnings management is on average “bad.”

The results also suggest that a variety of parameters moderate the effect of an increase in enforcement. For example, changing audit effectiveness and the cost of enforcement actions to firms and auditors can reverse these effects.

Our model rests on a number of simplifying assumptions that facilitate tractability. We believe that relaxing most assumptions does not qualitatively affect the results we establish because the main strategic interactions between the players appear robust. Our fundamental assumption is that enforcement activities differ from audit services, that is, enforcement is not simply a second full audit. If the scope of enforcement were the same as that of auditing, enforcement would always be (gross) beneficial. We assume binary productive effort; allowing more effort levels can provide additional insights into the productive effects of enforcement. We also assume that the incentive for the auditor to perform a quality audit stems from the risk that enforcement identifies an error. This assumption has two consequences: (i) If enforcement is perfect, which eliminates earnings management totally, the auditor has no incentive to provide audit effort, and (ii) anticipating that the manager tends to overstate earnings, the auditor has no incentive to audit low earnings. Because auditing is a value-adding service, less auditing reduces financial reporting quality. In reality, there are other mechanisms that impose incentives to auditors, such as audit inspections by an audit oversight body or auditor liability from litigation by parties that relied on the audited report. Such mechanisms also provide a strict preference for correcting misstatements even if the enforcer would not find them, such as errors in the accounting system and internal controls.

We model the enforcement institution as a “technology” because we believe an enforcer is mainly driven by the budget it has available and not by profit maximization. This means the enforcer does not act strategically and does not anticipate particular strategies by the manager or the auditor. However, persons responsible for enforcement may be loss averse or have
other individual objectives, which then affect the enforcement strategy. Our model does not consider the threat of lawsuits by persons affected by financial reporting quality, which may affect the manager’s or the auditor’s strategies. These, as well as other, considerations provide avenues for future research.
References


Appendix

Summary of notation

\[ a \]  Productive effort by manager, \( a \in \{a_L, a_H\} \)
\[ A \]  Audit fee
\[ b_j \]  Earnings management: probability of report \( m \) given \( y_j \)
\[ C^O \]  Cost of enforcement action to owner
\[ C^A \]  Cost of enforcement action to auditor
\[ D \]  Term for earnings management part in manager’s expected utility
\[ E_i \]  Probability terms
\[ f \]  Probability of enforcer to detect \( y \)
\[ FRQ \]  Financial reporting quality \((1 – \text{probability of divergence of } r_i \neq x_i)\)
\[ g_j \]  Audit effort: probability of observing correct \( x \) given report \( m_j \)
\[ k \]  Scaling factor of cost of audit effort
\[ m \]  Preliminary report of manager, \( m \in \{m_L, m_H\} \)
\[ p \]  Probability of high outcome \( x_H \) given high effort \( a_H \)
\[ q \]  Probability of high outcome \( x_H \) given low effort \( a_L \)
\[ r \]  Audited financial report, \( r \in \{r_L, r_H\} \)
\[ s \]  Manager’s compensation \((s(r_j))\), bonus paid for high earnings
\[ T \]  Condition on probabilities
\[ U^A \]  Utility of auditor
\[ U^M \]  Utility of manager
\[ U^O \]  Utility of owner
\[ v \]  Scaling factor of disutility of earnings management \( b \)
\[ V \]  Disutility of manager for \( a_H \)
\[ x \]  Productive outcome, \( x \in \{x_L, x_H\} \)
\[ y \]  Signal from accounting system, \( y \in \{y_L, y_H\} \)
\[ \alpha \]  ”\( \alpha \)-error“, probability of report \( y_L \) given \( x_H \)
\[ \beta \]  ”\( \beta \)-error“, probability of report \( y_H \) given \( x_L \)
Proofs

Proof of Lemma 1

A compensation with both \( s(r_H) \) and \( s(r_L) > 0 \) cannot be optimal because the manager’s reservation utility is 0 and compensation can be reduced by \( \min\{ s(r_L), s(r_H) \} \) without changing the manager’s incentives, but increasing the owner’s utility. That is, at least one of the compensation payments must be zero.

If both \( s(r_H) \) and \( s(r_L) = 0 \) then the compensation does not depend on the financial report, which therefore becomes useless. The manager does not engage in earnings management because it is costly, and the enforcer will not find any error, hence, there is no cost of enforcement. The high effort \( a_H \) is not implementable because the manager’s disutility is \( V > 0 \), but the expected compensation is the same for \( a_H \) and \( a_L \). Therefore, two cases remain: \( s(r_H) > s(r_L) = 0 \) and \( s(r_H) > s(r_H) = 0 \).

Case 1: \( s(r_H) > s(r_L) = 0 \). The manager’s utility conditional on \( y_H \) (gross of effort and enforcement costs) becomes

\[
E[U^M | a_H, y_H] = \text{prob}(r_H | y_H) s(r_H) - \frac{1}{2} v b_H^2
\]

where \( ˆg \) denotes the conjectured audit effort and

\[
\text{prob}(r_H | y_H) = (1 - b_H) \left( (1 - ˆg_H) + \text{prob}(x_H | y_H) ˆg_H \right) + b_H \text{prob}(x_H | y_H) ˆg_L .
\]

Differentiating \( \text{prob}(r_H | y_H) \) with respect to \( b_H \) yields

\[
\frac{\partial}{\partial b_H} \text{prob}(r_H | y_H) = \text{prob}(x_H | y_H) ( ˆg_L - ˆg_H ) - (1 - ˆg_H)
\]

A necessary condition for \( b_H > 0 \) is that this derivative is positive. However, this cannot be the case because \( 0 < \text{prob}(x_H | y_H) < 1 \) and \( ˆg_L < 1 \) (recall the \( ˆg \)’s are probabilities).

Therefore, \( b_H = 0 \).

Case 2: \( s(r_L) > s(r_H) = 0 \). Due to symmetry, the same analysis applies for \( y = y_L \), with a change in the indexes \( L \) and \( H \). As a result, it must be the case that \( b_L = 0 \).

Next consider how the manager’s expected utility changes in \( p \). Recall that the audited financial report is as follows:
\[
r_i = \begin{cases} 
  x_i & \text{with probability } g_i \\
  m_i & \text{with probability } (1 - g_i)
\end{cases}
\]

Suppose the audit is ineffective, implying \( r_i = m_i \). Rewriting the manager’s expected gross utility yields

\[
E[U^M | a_H] = (1 - \text{prob}(m_H)) s(r_H) + \text{prob}(m_H) s(r_H) - \frac{v}{2} \left( \text{prob}(y_L)b_L^2 + \text{prob}(y_H)b_H^2 \right)
\]

\[
= s(r_H) + \left[ b_L + \text{prob}(y_H)(1 - b_L - b_H) \right] (s(r_H) - s(r_L)) - \frac{v}{2} \left( \text{prob}(y_L)b_L^2 + \text{prob}(y_H)b_H^2 \right)
\]

Because \( \text{prob}(y_H) > \text{prob}(y_H | a_L) \), the expected utility must \textit{ceteris paribus} increase in \( \text{prob}(y_H) \) to compensate for the higher disutility of \( a_H > a_L \), that is,

\[
\frac{\partial}{\partial \text{prob}(y_H)} E[U^M | a_H] = (1 - b_L - b_H) (s(r_H) - s(r_L)) - \frac{v}{2} (b_L^2 - b_H^2) > 0
\]

In case 1, \( s(r_H) > s(r_L) = 0 \) and \( b_H = 0 \), the derivative becomes

\[
(1 - b_L) s(r_H) + \frac{v}{2} b_L^2 > 0
\]

In case 2, \( s(r_L) > s(r_H) = 0 \) and \( b_L = 0 \) it is

\[
-(1 - b_H) s(r_L) - \frac{v}{2} b_H^2 < 0
\]

which contradicts the fact that \( E[U^M] \) must increase in \( \text{prob}(y_H) \). Therefore, case 2 cannot be a feasible solution to the problem, which leaves case 1.

If the audit is perfect (that is, \( r_i = x_i \)) the only difference to the analysis is that \( \text{prob}(x_i) = p \) replaces \( \text{prob}(m_i) \). The conclusion is the same. The same analysis holds for any combination of \( x_i \) and \( m_i \).

Finally, if the audit is perfect, there is no cost of enforcement. If \( g_i < 1 \), the manager incurs enforcement costs through a claw-back of a bonus only if \( r_H \) is reported. If the report is \( r_L \) and the enforcer finds out that \( y = y_H \), there is no consequence to the manager because he did not receive a bonus for \( r_L \). For a report of \( r_H \), enforcement is tied to the probability of the enforcer finding an error. As shown in subsequent analyses, this probability increases in
prob(y_L). Because prob(y_L) < prob(y_L|a_H) the cost of enforcement is smaller for a_H, which establishes the Lemma.

Proof of Lemma 2

If the auditor observes m_L, given his conjecture that the manager did not manage earnings (\(\hat{b}_H = 0\), where the “hat” indicates the conjecture), the auditor correctly anticipates that the enforcer will never find or allege an error because m_L = y_L. Therefore, prob(error|m_L) = 0 and the auditor faces no cost of enforcement. Consequently, g_L = 0.

If the auditor observes m_H, there is the chance that the enforcer identifies an error, which occurs if the auditor does not find out x (so that r = r_H = m_H) and the enforcer learned y = y_L.

The conditional probability of an error is (see again Figure 3)

\[
\text{prob(error} | m_H) = \text{prob}(y_L | m_H)(1 - g_H)f
\]

where \(\text{prob}(y_L | m_H) = \frac{\text{prob}(y_L|\hat{b}_L)}{\text{prob}(y_L|\hat{b}_L + \text{prob}(y_H))}\), which is greater 0 if \(\hat{b}_L > 0\). The auditor’s conditional expected utility is

\[
U^A(m_H) = A - \frac{k}{2} g_H^2 - \text{prob}(y_L | m_H)(1 - g_H)fC^A
\]

The first derivative with respect to g_H equals

\[
\frac{\partial}{\partial g_H} U^A(m_H) = -kg_H + \text{prob}(y_L | m_H)fC^A
\]

and setting it 0, the optimal audit effort is

\[
g_H = \text{prob}(y_L | m_H)f \frac{C^A}{k} > 0
\]

if f > 0. Our assumption that \(C^A/k < 1\) ensures \(g_H < 1\).

Proof of Proposition 1

The manager maximizes her expected utility with respect to b_L,

\[
E[U^M | a_H, y_L] = \left( \text{prob}(r_H | y_L) - b_L(1 - \hat{g}_H)f \right)s - V - \frac{1}{2} vb_L^2
\]

\[
= sb_L \left( (1 - \hat{g}_H)(1 - f) + \hat{g}_H \text{prob}(x_H | y_L) \right) - V - \frac{1}{2} vb_L^2
\]
The first order condition is
\[ \frac{\partial}{\partial b_L} E[U^M | a_H, y_L] = s \left( (1 - \hat{g}_H)(1 - f) + \hat{g}_H \text{prob}(x_H | y_L) \right) - vb_L = 0 \]

implying
\[ b_L = \frac{s}{v} \left( (1 - f) + \hat{g}_H \left( \text{prob}(x_H | y_L) - (1 - f) \right) \right) \geq 0 \]

We assume that \( v \) is sufficiently large to ensure \( b_L < 1 \). In the proof of Proposition 2, we show that the exact threshold we require is \( v \geq \frac{2V}{(p-q)(1-\alpha-\beta)} \).

Existence and uniqueness of an equilibrium in the feasible range for \( b_L \) and \( g_H \) follows from a fixed point argument. \( b_L \) is strictly positive and linearly increasing in \( \hat{g}_H \) if \( T > 0 \) and linearly decreasing otherwise. The boundaries are
\[ b_L = \frac{s}{v} \left( (1 - f) + \hat{g}_H \text{prob}(x_H | y_L) \right) = \begin{cases} \frac{s}{v} (1 - f) & \text{for } \hat{g}_H = 0 \\ \frac{s}{v} \text{prob}(x_H | y_L) & \text{for } \hat{g}_H = 1 \end{cases} \]

According to Lemma 2, \( g_H = \text{prob}(y_L | m_H) f \frac{C^A}{k} \) with boundaries
\[ g_H = \frac{\text{prob}(y_L \hat{b}_L)}{\text{prob}(y_L \hat{b}_L + \text{prob}(y_L))} \frac{fC^A}{k} = \begin{cases} 0 & \text{for } \hat{b}_L = 0 \\ \text{prob}(y_L) \frac{fC^A}{k} & \text{for } \hat{b}_L = 1 \end{cases} \]

Note that \( g_H \) is strictly concave in \( \hat{b}_L \) because
\[ \frac{\partial g_H}{\partial \hat{b}_L} = \frac{\text{prob}(y_L) \text{prob}(y_H)}{\left( \text{prob}(y_L \hat{b}_L + \text{prob}(y_H)) \right)^2} \frac{fC^A}{k} > 0 \]

and
\[ \frac{\partial^2 g_H}{\partial \hat{b}_L^2} = -2 \frac{\text{prob}(y_L)^2 \text{prob}(y_H)}{\left( \text{prob}(y_L \hat{b}_L + \text{prob}(y_H)) \right)^3} \frac{fC^A}{k} < 0 \]

The equilibrium conditions are \( \hat{b}_L = b_L \) and \( \hat{g}_H = g_H \). The two reaction functions \( b_L(\hat{g}_H) \) and \( g_H(\hat{b}_L) \) are monotonic and continuous, hence, the function \( g_H(b_L(\hat{g}_H)) \) is continuous, too. Furthermore, we have \( 0 \leq \hat{b}_L, \hat{g}_H \leq 1 \), \( b_L(\hat{g}_H) \in [0,1] \) and \( g_H(\hat{b}_L) \in [0,1] \). Therefore, Brouwer’s fixed point theorem implies that a fixed point of \( g_H(b_L(\hat{g}_H)) \) exists for \( g_H \in [0,1] \). This fixed point constitutes an equilibrium, proving existence. Figure A1 plots the
reaction functions for two cases, \( f = f_1 \) and \( f_2 \), where \( f_2 > (1 - \text{prob}(x_H|y_L)) > f_1 \), which implies \( T < 0 \) for \( f_1 \) and \( T > 0 \) for \( f_2 \). The equilibrium strategies are the intersections of the two reaction functions.

![Figure A1: Equilibrium earnings management and audit effort](image)

Uniqueness follows directly from the linearity of \( b_L \) and concavity of \( g_H \), which imply that there can be only a single crossing of the two functions over the feasible domains. A special case is \( f = 1 \). Here, \( g_H = \hat{g}_H = b_L = \hat{b}_L = 0 \) is a feasible fixed point. A second fixed point could exist if for a \( g_H \) around \( g_H = 0 \), the linear reaction function for \( b_L \) is larger than the reaction function for the audit effort. Inverting the first-order condition for \( g_H \) gives the value \( \hat{b}_L \) for the bias that makes a certain audit effort optimal for the auditor:

\[
\hat{b}_L = \frac{\text{prob}(y_H) g_H}{\text{prob}(y_L) f C^4 k - g_H}
\]

In Figure A1, this function is the auditor’s reaction function if \( g_H \) is assumed the independent variable. A necessary condition for a second fixed point is that
\[
\frac{\partial b_L}{\partial g_H} (g_H = 0, f = 1) \leq \frac{\partial b_L}{\partial g_H} (g_H = 0, f = 1)
\]

which implies

\[
\frac{\text{prob}(y_H)}{\text{prob}(y_L)} \left( \frac{1}{C_A} \right) < \frac{s}{v} \frac{\text{prob}(x_H | y_L)}{C_A/k}
\]

\[
p(1 - \alpha) + (1 - p) \beta < \frac{s}{v} \frac{C_A}{k} \alpha
\]

\[
\left( \frac{1}{\alpha} - 1 \right) + \frac{(1 - p) \beta}{\frac{p}{\alpha} < \frac{s}{v} \frac{C_A}{k}
\]

> 0 due to \( \alpha < \frac{1}{2} \)

The left-hand side of this inequality is greater 1, whereas the right-hand side is less than 1 because \( v \) is large and \( C_A/k < 1 \). Therefore, there does not exist a second equilibrium at \( f = 1 \) in the feasible domain.

Next, we derive explicit solutions for the equilibrium strategies \( b_L^* \) and \( g_H^* \):

\[
b_L = \frac{s}{v} \left[ (1 - f) + \hat{g}_H (\text{prob}(x_H | y_L) - (1 - f)) \right]
\]

Solving for \( \hat{g}_H \) implies \( \hat{g}_H = \frac{b_L (1 - f)}{\text{prob}(x_H | y_L) - (1 - f)} \). The optimal \( g_H \) given \( \hat{b}_L \) is

\[
g_H = \frac{1}{1 + \frac{1 - \text{prob}(y_L)}{\text{prob}(y_L) b_L}} \frac{fC_A}{k}
\]

Equating \( g_H = \hat{g}_H \) yields a quadratic equation

\[
\frac{v}{s} b_L^2 - \left[ \frac{fC_A}{k} T - \frac{v}{s} \left( \frac{1 - \text{prob}(y_L)}{\text{prob}(y_L)} \right) + (1 - f) \right] b_L - (1 - f) \left( \frac{1 - \text{prob}(y_L)}{\text{prob}(y_L)} \right) = 0
\]

The solution of \( T_2 b_L^2 - T_1 b_L - T_0 = 0 \) is \( b_L = \frac{T_1 \pm \sqrt{T_1^2 + 4T_0T_2}}{2T_2} \). The \( T_i \) are (exogenous) constants. \( T_0 > 0 \) and \( T_2 > 0 \), implying \( \sqrt{T_1^2 + 4T_0T_2} > |T_1| \). The sign of \( T_1 \) is indeterminate. If \( T_1 > 0 \), then the solution for \( b_L \) must be the positive root because otherwise \( b_L < 0 \), which is not feasible. If \( T_1 < 0 \), \( b_L \) must also be the positive root for the same reason. Therefore, the equilibrium earnings management is
$$b^*_L = \frac{T_4 + \sqrt{T_4^2 + 4T_4T_2}}{2T_2}$$

The explicit solution for $g^*_H$ follows from

$$g_H = \frac{\text{prob}(y_L)\hat{b}_L}{\text{prob}(y_L)\hat{b}_L + \text{prob}(y_H)} \frac{fC^A}{k} \frac{1}{T}$$

$$\text{prob}(y_L)\hat{b}_L \left( g_H - \frac{fC^A}{k} \right) + g_H \text{prob}(y_H) = 0$$

Inserting the equilibrium condition $\hat{b}_L = b_L$ yields

$$\text{prob}(y_L) s \left[ (1 - f) + g_H \left( \text{prob}(x_H | y_L) - (1 - f) \right) \right] g_H - \frac{fC^A}{k} + g_H \text{prob}(y_H) = 0$$

$$Tg_H^2 + \left( 1 - \frac{\text{prob}(y_L)}{s} \right)^{\frac{1}{T}} + (1 - f)\frac{C^A}{k} = 0$$

This is a quadratic equation $Tg_H^2 + T_4g_H - T_3 = 0$ with solutions $g_H = \frac{-T_4 \pm \sqrt{T_4^2 + 4TT_3}}{2T}$. $T_3 > 0$, and $T$ and $T_4$ can be positive or negative. Suppose $T > 0$. Then $\sqrt{T_4^2 + 4TT_3} > |T_4|$ and

regardless of the sign of $T_4$ the positive root is the only solution with $g_H > 0$. If $T < 0$, then $T_4 > 0$ and $\sqrt{T_4^2 + 4TT_3} < T_4$. A solution in real numbers requires that $T_4^2 + 4TT_3 \geq 0$, i.e., $T_4^2 \geq 4T_3 |T|$. This must hold because there exists a unique equilibrium $(b^*_L, g^*_H)$ in the feasible range. Denote the two roots $g^-_H > g^*_H$. The reaction function is

$$\tilde{b}_L = \frac{\text{prob}(y_H)}{\text{prob}(y_L)} \left( \frac{g_H}{\frac{fC^A}{k} - g_H} \right)$$

This function is a hyperbole that provides positive $b_L$ for small $g_H$ and negative $b_L$ for large $g_H$. Given this functional form, the positive root $g^*_H$ is the feasible solution, that is

$$g^*_H = \frac{-T_4 + \sqrt{T_4^2 + 4TT_3}}{2T}$$

Finally, consider the special case $T = 0$, where both numerator and denominator of $g^*_H$ are zero and the quotient is not properly defined. Applying de L’Hospital’s rule to $g^*_H$ yields
\[
\lim_{t \to 0} g^*_n = \lim_{t \to 0} \left( \frac{4T_t}{2\sqrt{T_t^2 + 4T_t}} \right) = \frac{4T_3}{4T_4} = \frac{T_3}{T_4} > 0 \text{ if } T = 0.
\]

The same solution obtains if \( Tg^*_n + T^*_4g^*_n - T_3 = 0 \) is solved for \( g^*_n \) at \( T = 0 \). \( \Box \)

**Proof of Corollary 1**

We prove first the results for \( b^*_L \). As shown in the proof of Proposition 1, \( b^*_L \) is implicitly defined by

\[
B \equiv T_2b^2_L - T_3b_L - T_0 = 0
\]

where \( T_0 = (1 - f) \frac{\text{prob}(y_H)}{\text{prob}(y_L)} \), \( T_1 = \frac{fC}{k} - \frac{T}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)} + (1 - f) \), \( T_2 = \frac{T}{s} \), and \( T = \text{prob}(x_H | y_L) - (1 - f) \). To save notation, we drop the asterisk on \( b^*_L \). The total differential with respect to parameters \( j = s, f, \) and \( C^j/k \) is

\[
\frac{\partial B}{\partial j} + \frac{\partial B}{\partial b_L} \frac{db_L}{dj} = 0 \Rightarrow \frac{db_L}{dj} = -\left( \frac{\partial B}{\partial j} \right) \left( \frac{\partial B}{\partial b_L} \right)^{-1}
\]

where \( \frac{\partial B}{\partial b_L} = 2T_2b_L - T_1 = 2T_2 \left( \frac{T_1 + \sqrt{T_1^2 + 4T_0T_2}}{2T_2} \right) - T_1 = \sqrt{T_1^2 + 4T_0T_2} > 0 \). Thus, \( \text{sign} \left( \frac{db_L}{dj} \right) = -\text{sign} \left( \frac{\partial B}{\partial j} \right) \) for each \( j \).

Part (i): \( \frac{\partial B}{\partial s} = -s^2b^2_L - b_L \frac{\text{prob}(y_H)}{\text{prob}(y_L)} < 0 \), which implies \( \frac{db_L}{ds} > 0 \).

Part (ii):

\[
\frac{\partial B}{\partial f} = b_L \left( -\frac{C^A}{k} - \frac{fC^A}{k} + 1 \right) + \frac{\text{prob}(y_H)}{\text{prob}(y_L)}
\]

\[
= b_L \left( 1 + (1 - 2f) \frac{C^A}{k} \right) + \frac{1}{\text{prob}(y_L)} \left( \text{prob}(y_H) - b_L \frac{C^A}{k} \right) p \alpha
\]

\[
= b_L \left( 1 + (1 - 2f) \frac{C^A}{k} \right) + \frac{1}{\text{prob}(y_L)} \left( p \left( 1 - \alpha \left( 1 + b_L \frac{C^A}{k} \right) \right) + (1 - p) \beta \right) > 0
\]

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The signs of the terms above follow from $\frac{C^A}{k} < 1, f \leq 1$ and $\alpha < 0.5$; and using $b_L < 1$ yields

$$\frac{\partial B}{\partial f} > 0.$$ This implies $\frac{db_L}{df} < 0$.

Part (iii): $\frac{\partial B}{\partial (C^A / k)} = -fTb_L$, implying $\frac{db_L}{d(C^A / k)} < 0$ if $T < 0$ and $\frac{db_L}{d(C^A / k)} > 0$ if $T > 0$. □

Next, we prove the results for $g_H^*$. As shown in the proof of Proposition 1, $g_H^*$ is implicitly defined by

$$G = Tg_H^2 + TAg_H - T_3 = 0$$

with $T_3 = (1-f) \frac{C^A}{k}$, $T_4 = \frac{\text{prob}(y_H)}{\text{prob}(y_L)} v + (1-f) - \frac{fC^A}{k} T$, and $T = \text{prob}(x_H | y_L) - (1-f)$.

To save notation, we drop the asterisk on $g_H^*$. The total differential with respect to parameters $j = s, f$, and $C^A/k$ is

$$\frac{\partial G}{\partial j} + \frac{\partial G}{\partial g_H} \frac{dg_H}{dj} = 0 \Rightarrow \frac{dg_H}{dj} = -\left(\frac{\partial G}{\partial j}\right) \left(\frac{\partial G}{\partial g_H}\right)^{-1}$$

where $\frac{\partial G}{\partial g_H} = 2Tg_H + T_4 = 2T \left(-\frac{T_4 + \sqrt{T_4^2 + 4TT_3}}{2T}\right) + T_4 = 2T \sqrt{T_4^2 + 4TT_3} > 0$. Therefore,

$$\text{sign}\left(\frac{dg_H}{dj}\right) = -\text{sign}\left(\frac{\partial G}{\partial j}\right) \text{ for each } j.$$  

Part (i): $\frac{\partial G}{\partial s} = -g_H \cdot \frac{\text{prob}(y_H)}{\text{prob}(y_L)} < 0$ implies $\frac{dg_H}{ds} \geq 0$ (strictly if $g_H > 0$).

Part (ii):

$$\frac{\partial G}{\partial f} = g_H^2 + g_H \left(\frac{1-C^A}{k} T - \frac{fC^A}{k}\right) - (1-2f) \frac{C^A}{k}$$

$$= g_H^2 - g_H \left(1 + \frac{C^A}{k} \text{prob}(x_H | y_L)\right) + (g_H - 1)(1-2f) \frac{C^A}{k}$$

$$= g_H \left(g_H - 1\right) - g_H^2 \frac{C^A}{k} \text{prob}(x_H | y_L) + (g_H - 1)(1-2f) \frac{C^A}{k}$$
If \( f \leq 1/2 \), \( \frac{\partial G}{\partial f} < 0 \) and \( \frac{dg_H}{df} > 0 \). If \( f > 1/2 \), the last term \( \frac{(g_H - 1)(1 - 2f)}{k} \) \( C_A > 0 \), and then the sign of \( \frac{\partial G}{\partial f} \) is indeterminate. Note that \( \frac{\partial G}{\partial f} \bigg|_{f=1} = \frac{C_A}{k} > 0 \) because at \( f = 1 \) we have \( g_H^* = 0 \). Due to continuity, \( \frac{\partial G}{\partial f} \) must be positive in a range of \( f < 1 \). In particular, there must exist an \( f_0 \in (1/2, 1) \) for which \( \frac{\partial G}{\partial f} \bigg|_{f=f_0} = 0 \). At this point, \( g_H^* \) attains a maximum over \( f \) and \( \frac{\partial g_H}{\partial f} \bigg|_{f=f_0} = 0 \) as well. This maximum is unique because

\[
\left. \frac{d^2 g_H}{df^2} \right|_{f=f_0} = -\left( \frac{\partial^2 G}{\partial f^2} \right)^{-1} \frac{\partial G}{\partial g_H} \left|_{f=f_0} \right. < 0
\]

\[
\frac{\partial^2 G}{\partial f^2} = -2 \left( g_H - 1 \right) \frac{C_A}{k} > 0,
\]

which implies \( \frac{d^2 g_H}{df^2} \bigg|_{f=f_0} < 0 \). Because this holds for each (local) extremum, \( f_0 \) must be the unique maximum; otherwise, there would exist a minimum over the range of \( f \), which is not the case.

Part (iii):

\[
\frac{\partial G}{\partial (C_A^k / k)} = -g_H f T - f \left( 1 - f \right)
\]

\[
= -g_H f \left( \text{prob}(x_H | y_L) - (1 - f) \right) - f \left( 1 - f \right)
\]

\[
= -g_H f \left( \text{prob}(x_H | y_L) + (1 - f) \right) f \left( g_H - 1 \right) < 0
\]

which implies \( \frac{dg_H}{d(C_A^k / k)} > 0 \).

Proof of Corollary 2

(i) \( f = 1 \): \( b_L^* = 0 \) and \( g_H^* = 0 \), hence, \( \text{prob(\text{divergence})} = E_1 \).

(ii) \( f = 0 \): \( b_L^* > 0 \) and \( g_H^* = 0 \). According to equation (5),

\[
\text{prob(\text{divergence})} = \frac{\alpha + (1 - p) \beta + b_L^* (1 - p)(1 - \beta) - p \alpha}{E_2} - (1 - p) g_H^* (\beta + b_L^* (1 - \beta))
\]

\[
\overset{=E_2}{=} E_2 > 0
\]
It follows that \( E_3 = 0 \). If \( p\alpha = (1 - p)(1 - \beta) \) then \( E_2 = 0 \), and \( \text{prob(divergence)} = E_1 \).

If \( p\alpha < (1 - p)(1 - \beta) \), \( E_2 > 0 \), and \( \text{FRQ}(f = 0) < \text{FRQ}(f = 1) \); and vice versa for \( p\alpha > (1 - p)(1 - \beta) \). Therefore, if \( f = 0 \), then

\[
\text{FRQ}(f = 0) = \begin{cases} 
> \text{FRQ}(f = 1) & \text{if } p\alpha > (1 - p)(1 - \beta) \\
= \text{FRQ}(f = 1) & \text{if } p\alpha = (1 - p)(1 - \beta) \\
< \text{FRQ}(f = 1) & \text{if } p\alpha < (1 - p)(1 - \beta)
\end{cases}
\]

Assume \( p\alpha = (1 - p)(1 - \beta) \). Then \( E_2 = 0 \), and \( \text{FRQ}(f = 0) = \text{FRQ}(f = 1) \). \( E_3 \) increases in \( f \) at \( f = 0 \) because \( g_H^* \) increases in \( f \), and \( E_3 \) decreases in \( f \) for \( f > f_0 \). \( \text{FRQ}(f = 0) = \text{FRQ}(f = 1) \) then implies that \( \text{FRQ} \) increases in \( f \) for low \( f \), and it decreases in \( f \) for high \( f \). The necessary condition that \( \text{FRQ} \) monotonically increases in \( f \) is that \( \text{FRQ}(f = 0) < \text{FRQ}(f = 1) \), which requires \( p\alpha < (1 - p)(1 - \beta) \).

If \( \alpha = \beta = 0 \), then

\[
\text{prob(divergence)} = p\alpha + (1 - p)\beta + b_L^* \left[ (1 - p)(1 - \beta) - p\alpha \right] - (1 - p)g_H^* \left[ \beta + b_L^*(1 - \beta) \right]
\]

\[
b_L^* \left[ (1 - g_H^*) \right] = b_L^* \left[ 1 - \frac{\text{prob}(y_L) b_L^*}{\text{prob}(y_H) + \text{prob}(y_L) b_L} f \frac{C^A}{k} \right] = b_L^* \left[ \frac{\text{prob}(y_H) + \text{prob}(y_L) b_L^* \left[ 1 - f \frac{C^A}{k} \right]}{\text{prob}(y_H) + \text{prob}(y_L) b_L^*} \right]
\]

\[
\frac{\partial}{\partial f} \left[ b_L^* \left( 1 - g_H^* \right) \right] = 
\left[ \frac{1}{\left( \frac{\text{prob}(y_H)}{b_L^*} + \text{prob}(y_L) \right)^2} \right] \text{prob}(y_L) \left[ \frac{db_L^*}{df} \left( 1 - f \frac{C^A}{k} \right) - b_L^* \frac{C^A}{k} \right] \left[ \frac{\text{prob}(y_H) + \text{prob}(y_L) b_L^*}{b_L^*} \right]_{>0} - 
\left[ \frac{\text{prob}(y_H) + \text{prob}(y_L) b_L^* \left[ 1 - f \frac{C^A}{k} \right]}{b_L^*} \right] \left[ \frac{\text{prob}(y_H) d b_L^*}{df} \right]_{>0} \left[ b_L^* \right]_{>0} < 0
\]
because \( \frac{db_*^L}{df} < 0 \) (Corollary 1 (ii)). Therefore, \( \text{prob(divergence)} \) strictly decreases in \( f \) and \( FRQ \) strictly increases in \( f \). If \( f = 1 \), then \( b_*^L = 0 \) and \( g_*^H = 0 \), hence, \( \text{prob(divergence)} = 0 \) and \( FRQ = 1 \).

(iii) The first derivative of the probability of a divergence with respect to \( f \) is

\[
\frac{\partial \text{prob(divergence)}}{\partial f} = \frac{db_*^L}{df}((1 - p)(1 - \beta) - p\alpha) - (1 - p)\frac{dg_*^H}{df}(\beta + b_*^L(1 - \beta)) - (1 - p)(1 - \beta)g_*^H \frac{db_*^L}{df}
\]

If \( f > f_0 \), then \( \frac{dg_*^H}{df} < 0 \); and since \( \frac{db_*^L}{df} < 0 \) always holds, the probability of a divergence must increase (and \( FRQ \) must decrease) if \( E_2 < 0 \) or, equivalently, \( p\alpha > (1 - p)(1 - \beta) \).

Finally, \( 1/2 < f_0 < 1 \) follows from Corollary 1 (ii).

\[ \square \]

**Proof of Proposition 2**

The proof proceeds by showing that each of the three cost terms in \( E[U^0 | d_H] \) in equation (8) increases in \( s \), which establishes the optimal bonus \( s^* = \frac{1}{2} \).

The first term is the expected compensation net of a claw-back

\[
E[\text{comp}] = (\text{prob}(r_H) - \text{prob}(error)) s = s \left[ (1 - p)(1 - \beta)b_*^L(1 - g_*^H)(1 - f) + (1 - p)\beta(1 - g_*^H) + pab_*^L(1 - f + g_*^H f) + p(1 - \alpha) \right]
\]

Differentiating with respect to \( s \) yields

\[
\frac{dE[\text{comp}]}{ds} = \left( \text{prob}(r_H) - \text{prob}(error) \right) + s \left( \frac{d(\text{prob}(r_H) - \text{prob}(error))}{ds} \right)
\]

The first term is strictly positive, and the second term on the RHS is

\[
s \left( 1 - p \right)(1 - \beta)(1 - f) \frac{db_*^L}{ds} \left( 1 - g_*^H \right) - (1 - p)\beta \frac{dg_*^H}{ds} + p\alpha \frac{db_*^L}{ds} \left( 1 - f + g_*^H f \right) + pab_*^L \frac{dg_*^H}{ds} f - (1 - p)(1 - \beta)g_*^H \frac{db_*^L}{ds}
\]

The signs of the last three terms follow because \( \frac{db_*^L}{ds} > 0 \) and \( \frac{dg_*^H}{ds} > 0 \) (see Corollaries 1 (i) and 2 (i)). The sign of the first term follows from the fact that
\[ b_L^*(1 - g_H^*) = \frac{\text{prob}(y_H) + \text{prob}(y_L) b_L^* \left(1 - f \frac{C^A}{k}\right)}{\text{prob}(y_H) + \text{prob}(y_L)} \]

depends on \( s \) only through \( b_L^* \), and \( \frac{db_L^*}{ds} > 0 \) implies \( \frac{d\left(b_L^*(1 - g_H^*)\right)}{ds} > 0. \)

Therefore, the only term that negatively enters the derivative is \((1 - p)\beta \frac{dg_H^*}{ds} \), and its magnitude depends on \( \beta \) and \( p \). The result that \( \frac{dE[\text{comp}]}{ds} > 0 \) requires that this term is “small” relative to the sum of the other terms. A “low” \( \beta \) or a high \( p \) are sufficient that the negative term is small. Moreover, because the other terms in the partial derivative of \((\text{prob}(r_H) - \text{prob(error)}) > 0 \) and the two other cost terms in \( E[U^O|a_H] \) also increase in \( s \) (see below), there are other conditions. An example is a sufficiently high cost of enforcement action \( C^O \) (see below).

The second term of \( E[U^O|a_H] \) is the audit fee \( A \),

\[ A = \frac{k}{2} g_H^*(2 - g_H^*) \]

The total derivative is

\[ \frac{dA}{ds} = \text{prob}(y_L) \frac{db_L^*}{ds} \left(\frac{k}{2} g_H^*(2 - g_H^*)\right) + \text{prob}(m_H) k (1 - g_H^*) \frac{dg_H^*}{ds} > 0 \]

The third term of \( E[U^O|a_H] \) is the expected cost of enforcement,

\[ \text{prob}(y_L) b_L^*(1 - g_H^*) fC^O \]

\[ \text{prob}(y_L) fC^O \frac{d\left(b_L^*(1 - g_H^*)\right)}{ds} > 0 \] follows from the fact that \( \frac{d\left(b_L^*(1 - g_H^*)\right)}{ds} > 0. \)

\[ \square \]

Proof of Proposition 3

Rewriting (10) yields
\[
E[U^M | a_H] - E[U^M | a_L] \\
= s \left[ \text{prob}(y_H)(1 - g_H^* \text{prob}(x_L | y_H)) - \text{prob}(y_H | a_L)(1 - g_H^* \text{prob}(x_L | y_H, a_L)) \right] \\
+ \frac{v}{2} \left[ \text{prob}(y_L)b_{LL}^2 - \text{prob}(y_L | a_L)b_{LL}^2 \right] - V \\
= s(p - q)(1 - \alpha - \beta(1 - g_H^*)) - V + \frac{v}{2} \left[ \text{prob}(y_L)b_{LL}^2 - \text{prob}(y_L | a_L)b_{LL}^2 \right] \geq 0
\]

The minimum bonus \(s\) is implicitly defined setting this inequality equal to zero:

\[
H = s(p - q)(1 - \alpha - \beta(1 - g_H^*)) - V - \frac{v}{2} \left( \text{prob}(y_L | a_L)b_{LL}^2 \left( \text{prob}(y_L | a_L) - \text{prob}(y_L) \right) \right) = 0
\]

(i): \(f = 0\). In this case \(g_H^* = 0\) and \(b_L^* = b_{LL}^* = \frac{s}{v}\), which yields

\[
H \bigg|_{f=0} = s(p - q)(1 - \alpha - \beta) - V - \frac{v}{2} \left( \text{prob}(y_L | a_L)b_{LL}^2 \left( \text{prob}(y_L | a_L) - \text{prob}(y_L) \right) \right) = 0
\]

which implies

\[
s(f = 0) = \frac{V}{(p - q)(1 - \alpha - \beta)} + \frac{s^2}{2v}
\]

and

\[
s(f = 0) = v \left( 1 - \sqrt{1 - \frac{2V}{v(p - q)(1 - \alpha - \beta)}} \right)
\]

because the smaller root is the solution. The equation has a solution in real numbers for \(s\) if

\[
v > \frac{2V}{(p - q)(1 - \alpha - \beta)},
\]

which is the precise condition for our assumption that \(v\) is “large.”

To prove that \(s(f = 0) > \frac{V}{(p - q)(1 - \alpha - \beta)} \equiv Z = s(f = 1)\), assume to the contrary that

\[
s(f = 0) = v \left( 1 - \sqrt{1 - \frac{Z}{v}} \right) < Z
\]

which implies

\[
s(f = 0) = v \left( 1 - \sqrt{1 - \frac{Z}{v}} \right) < Z \Rightarrow 1 - \frac{Z}{v} < \sqrt{1 - \frac{Z}{v}} \Rightarrow \left( 1 - \frac{Z}{v} \right)^2 < 1 - \frac{2Z}{v} \Rightarrow \left( \frac{Z}{v} \right)^2 < 0
\]

which is a contradiction. Furthermore, \(D(f = 0) = \frac{s^2}{2v}(p - q)(1 - \alpha - \beta) > 0\). 

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To prove $\left.\frac{ds}{df}\right|_{f=0} < 0$ apply the implicit function theorem to $H$:

$$\frac{\partial H}{\partial f} + \frac{\partial H}{\partial s} \frac{ds}{df} = 0 \Rightarrow \frac{ds}{df} = -\left(\frac{\partial H}{\partial f}\right)^{-1} \frac{\partial H}{\partial s}$$

$$\frac{\partial H}{\partial f} = s(p-q)\beta \frac{dg^*_H}{df} - \frac{\partial D}{\partial f}$$

and $\left.\frac{ds}{df}\right|_{f=0} > 0$ at $f=0$.

$$\left.\frac{\partial D}{\partial f}\right|_{f=0} = \frac{s}{v} \left\{ \begin{array}{c} \text{prob}(y_L|a_L) b^{*}_{tL} (f = 0) \left.\frac{db^*_t}{df}\right|_{f=0} - \text{prob}(y_L) b^*_L (f = 0) \left.\frac{db^*_L}{df}\right|_{f=0} \\ \text{prob}(y_L|a_L) \left.\frac{db^*_t}{df}\right|_{f=0} - \text{prob}(y_L) \left.\frac{db^*_L}{df}\right|_{f=0} \end{array} \right\}$$

Recall that $b^{*}_{tL} = \frac{s}{v} [(1-f) + g^*_H (\text{prob}(x_H|y_L|a_L) - (1-f))] =

$$b^{*}_{tL} = \frac{s}{v} g^*_H \left( \text{prob}(x_H|y_L) - \text{prob}(x_H|y_L, a_L) \right)$$. Thus, $\left.\frac{db^*_t}{df}\right|_{f=0} = \frac{db^*_t}{df} - \frac{dg^*_H}{df} \frac{s\Delta}{v}$, and inserting yields

$$\left.\frac{\partial D}{\partial f}\right|_{f=0} = \frac{s}{v} \left\{ \begin{array}{c} \text{prob}(y_L|a_L) \left.\frac{db^*_t}{df}\right|_{f=0} - \text{prob}(y_L) \left.\frac{db^*_L}{df}\right|_{f=0} \\ \text{prob}(y_L|a_L) \left.\frac{db^*_t}{df}\right|_{f=0} - \text{prob}(y_L) \left.\frac{db^*_L}{df}\right|_{f=0} \end{array} \right\} < 0$$

Therefore, $\left.\frac{\partial H}{\partial f}\right|_{f=0} > 0$.

To determine the sign of $\frac{\partial H}{\partial s} = (p-q)(1-\alpha - \beta) + s(p-q)\beta \frac{dg^*_H}{ds} - \frac{\partial D}{\partial s}$, recall that $g^*_H |_{f=0} = 0$, and from Corollary 1 (i) $\left.\frac{dg^*_H}{ds}\right|_{f=0} = 0$ implying

$$\left.\frac{\partial H}{\partial s}\right|_{f=0} = (p-q)(1-\alpha - \beta) - \left.\frac{\partial D}{\partial s}\right|_{f=0}$$

$$\left.\frac{\partial D}{\partial s}\right|_{f=0} = \frac{s}{v} \left\{ \begin{array}{c} \text{prob}(y_L|a_L) b^{*}_{tL} (f = 0) \left.\frac{db^*_t}{ds}\right|_{f=0} - \text{prob}(y_L) b^*_L (f = 0) \left.\frac{db^*_L}{ds}\right|_{f=0} \\ \text{prob}(y_L|a_L) \left.\frac{db^*_t}{ds}\right|_{f=0} - \text{prob}(y_L) \left.\frac{db^*_L}{ds}\right|_{f=0} \end{array} \right\}$$
Inserting \( \frac{db^*_l}{ds} \bigg|_{f=0} = \frac{db^*_l}{ds} \bigg|_{f=0} - \frac{1}{v} g^*_H \bigg|_{f=0} \Delta - \frac{s}{v} \frac{dg^*_H}{ds} \bigg|_{f=0} \Delta = \frac{db^*_l}{ds} \bigg|_{f=0} > 0 \) yields

\[
\frac{\partial D}{\partial s} \bigg|_{f=0} = s \left( \text{prob}(y_L | u_L) \frac{db^*_l}{ds} \bigg|_{f=0} - \text{prob}(y_L) \frac{db^*_l}{ds} \bigg|_{f=0} \right)
= s \frac{db^*_l}{ds} \bigg|_{f=0} (p-q)(1-\alpha-\beta)
\]

Collecting the results,

\[
\frac{\partial H}{\partial s} \bigg|_{f=0} = (p-q)(1-\alpha-\beta) \left[ 1 - \frac{db^*_l}{ds} \bigg|_{f=0} \right]
\]

That is, the sign of the term in square brackets determines the sign of the expression. Recall from the proof of Corollary 1 that \( \frac{db^*_l}{ds} = -\left( \frac{\partial B}{\partial s} \right) \left( \frac{\partial^2 B}{\partial b^*_l} \right)^{-1} \), where

\[
\frac{\partial B}{\partial s} = -\frac{v}{s^2} b^2_L - b_L \frac{v}{s} \left( \frac{\text{prob}(y_H)}{\text{prob}(y_L)} \right)
\]

Using \( f = 0 \) and \( b_L = s/v \) yields

\[
\frac{\partial B}{\partial b^*_l} = 2T_s b^*_l - T_i, \quad \text{which at } f = 0 \text{ leads to}
\]

\[
\frac{\partial B}{\partial b^*_l} \bigg|_{f=0} = 2T_s b^*_l (f = 0) - T_i (f = 0) = 2 \left( \frac{v}{s} \right) \left( \frac{s}{v} \right) - \left( \frac{v}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)} \right) = 1 + \frac{v}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)}
\]

Now it follows

\[
\frac{db^*_l}{ds} \bigg|_{f=0} = \left( \frac{1 + \frac{v}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)}}{1 + \frac{v}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)}} \right) = \frac{1}{v} \left( \frac{1 + \frac{v}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)}}{1 + \frac{v}{s} \frac{\text{prob}(y_H)}{\text{prob}(y_L)}} \right) = 1
\]
We show earlier that \( s(f=0) = v \left( 1 - \sqrt{1 - \frac{2Z}{v}} \right) < v \), hence, \( \frac{db_v^*}{ds} \bigg|_{f=0} = \frac{s}{v} < 1 \). Taken together, \( \frac{\partial H}{\partial s} \bigg|_{f=0} = (p-q)(1-\alpha-\beta) \left( 1 - \frac{s}{v} \frac{db_v^*}{ds} \bigg|_{f=0} \right) > 0 \). This proves \( \frac{ds}{df} \bigg|_{f=0} < 0 \).

(ii): \( f = 1 \). In this case, \( g_v^* = 0 \) and \( b_v^* = b_{vl}^* = 0 \), which implies \( D = 0 \) and

\[
\underline{s}(f = 1) = \frac{V}{(p-q)(1-\alpha-\beta)}.
\]

We have \( \left. \frac{\partial H}{\partial f} \right|_{f=1} = \underline{s}(p-q)\beta \frac{dg_v^*}{ds} \bigg|_{f=1} - \frac{\partial D}{\partial f} \bigg|_{f=1} \), where \( \frac{\partial D}{\partial f} \bigg|_{f=1} = 0 \) (because \( b_v^* = b_{vl}^* = 0 \)), and

\[
\left. \frac{dg_v^*}{df} \right|_{f=1} < 0.
\]

This implies \( \left. \frac{\partial H}{\partial f} \right|_{f=1} < 0 \) if \( \beta > 0 \), and \( \left. \frac{\partial H}{\partial f} \right|_{f=1} = 0 \) if \( \beta = 0 \). Furthermore,

\[
\left. \frac{\partial H}{\partial s} \right|_{f=1} = (p-q)(1-\alpha-\beta) + \underline{s}(p-q)\beta \frac{dg_v^*}{ds} \bigg|_{f=1} - \frac{\partial D}{\partial s} \bigg|_{f=1}.
\]

Since \( g_v^* \bigg|_{f=1} = 0 \), it follows from Corollary 1 (i) that \( \left. \frac{dg_v^*}{ds} \right|_{f=1} = 0 \). We also have \( \left. \frac{\partial D}{\partial s} \right|_{f=1} = 0 \) due to \( b_v^*(f=1) = b_{vl}^*(f=1) = 0 \). This yields \( \left. \frac{\partial H}{\partial s} \right|_{f=1} = (p-q)(1-\alpha-\beta) > 0 \).

Collecting terms yields

\[
\left. \frac{ds}{df} \right|_{f=1} = -\left( \left. \frac{\partial H}{\partial f} \right|_{f=1} \right) \left( \left. \frac{\partial H}{\partial s} \right|_{f=1} \right)^{-1} \geq 0
\]

with strict inequality if \( \beta > 0 \). This proves that \( s \) increases in \( f \) at \( f = 1 \). Due to continuity \( s \)

increases if \( f \) approaches \( f = 1 \) from below.

(iii) The existence of a minimum \( s(f_1) \) follows immediately from statements (i) and (ii) and continuity.