Information Disclosure and Real Investment in a Dynamic Setting

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Preliminary and Incomplete
1 Introduction

How does the quality of mandated public disclosures affect the welfare of a firm’s shareholders? This question is of central importance for both financial regulators and academics. Conventional wisdom from models with single round of trading is that more precise public information leads to investors demanding a lower risk premium for holding the firm’s stock (see, for instance, Easley and O’Hara, 2004, and Lambert et al., 2007). In a dynamic setting with overlapping generations of investors, Dutta and Nezlobin (2016) show that the firm’s current and future shareholders can have divergent preferences over mandated disclosure regimes. While the welfare of the firm’s current shareholders is maximized under the full disclosure regime, the welfare of future shareholders increases in the precision of public disclosures only if the expected growth rate in the firm’s operations during their period of ownership is above a certain threshold. All of these results are, however, obtained under the assumption of exogenous cash flows. In this paper, we study the relation between disclosure quality and investor welfare in a dynamic production economy, i.e., taking into account the effect of disclosure quality on the firm’s internal investment decisions.

The main findings of our paper are as follows. First, we show that the relation between the precision of public disclosures and the firm’s internal investment is unambiguous: the firm invests more when public disclosures are more precise. However, unlike in Dutta and Nezlobin (2016), the welfare of the firm’s current shareholders is not necessarily maximized under the full disclosure regime. If investment is sufficiently sensitive to the cost of equity capital, the firm’s current shareholders prefer a disclosure regime with imperfect precision. With regards to future shareholders, we show that their welfare is increasing in the quality of public information for a broader range of parameters than in the pure exchange setting studied in Dutta and Nezlobin (2016). More generally, we provide a complete characterization of how the quality of public disclosures affects the welfare of a firm’s current and future shareholders and the risk premia they demand for holding the firm’s stock.

Our model considers a firm whose stock is traded by overlapping generations of investors. Each generation holds the firm’s stock for one period of time, during which the firm makes one dividend payment and invests in a new project. At the end of each period, the firm releases a public report that informs investors about the next period’s cash flow. Once the report is released, the firm’s stock is sold in a competitive market to the next generation of investors. Therefore, each generation of shareholders is exposed to risk associated with
the forthcoming dividend, which we label \textit{dividend risk}, and risk associated with the future resale price of the stock, which we call \textit{price risk}. A higher quality reporting environment results in a lower dividend risk but also a higher price risk since each generation of investors anticipates that the resale price will be formed based on a more informative public report.

We first study the relation between information quality and the firm’s internal investment. For any given long-term project, when public disclosures are more precise, the resolution of risk shifts to earlier periods. As a consequence, such risk is borne by generations of investors that are further away from consuming the cash flows generated by the long-term project. We show that due to discounting, an earlier generation demands a lower premium for risk associated with a given cash flow than a later generation. Therefore, the total risk premium associated with a given project is unambiguously decreasing in the precision of public disclosures. This implies that the firm’s current shareholders prefer the firm to invest more when the required public disclosures are more precise.

Having established the monotonicity of the firm’s investment in the quality of public information, we turn to characterizing how the quality of public information affects the welfare of investors holding the firm’s equity over a particular period of time. To address this question, we need to account for several forces: the quality of public information affects the purchase price of the stock, the resale price of the stock, the uncertainty about the forthcoming dividend payment, and the firm’s past and future investment levels, i.e., the firm size. The overall effect of these forces will be summarized by the risk premium charged by investors for holding the stock over a given period of time and will determine their expected welfare over that period. Consistent with the earlier literature (e.g., Dye 1990 and Kurlat and Veldkamp 2015), we note that potential future shareholders prefer to have access to riskier investments, i.e., the welfare of investors who buy and sell the firm’s equity increases in the expected risk premium during their period of ownership.\footnote{As a special case of this observation, note that if investors only had access to the risk-free asset their expected utility would be less than if they also had access to a risky security.}

In a model with exogenous cash flows, Dutta and Nezlobin (2016) show that the expected welfare of future shareholders increases in the quality of public disclosures if the firm’s growth rate is above a certain threshold and decreases otherwise. The key to this result is that higher quality public disclosures reduce the firm’s dividend risk but simultaneously increase the resale price risk in every period. For fast-growing firms, resale price risk is relatively more important than dividend risk, and therefore the periodic risk premium for such firms
increases in the quality of public information. In our model, there is an additional effect of public information on the firm’s risk premium: the firm invests more when public disclosures are more precise, which translates into a higher price and dividend risk in every period.

We show that the threshold growth rate of the firm above which future investors’ welfare monotonically increases in the quality of public information is lower when one takes into account the effect of public information on the firm’s internal investment. For firms growing just below the threshold rate, the expected welfare of their future shareholders in increasing in the quality of public information when that quality is sufficiently low and decreasing afterwards. Lastly, for very slow-growing (or declining) firms, the periodic risk premia and the welfare of future investors monotonically decline in the quality of public information. Overall, in our production economy, the welfare of potential future shareholders is increasing in the quality of public information for a wider set of parameters than in a comparable pure exchange setting.

We next study the relation between the quality of public information disclosures and the welfare of the firm’s current shareholders. Financial reporting preferences of current shareholders are generally different from those of the potential future shareholders since the purchase price of the stock is a sunk cost for the former group yet a relevant cost for the latter group. We identify two effects of the quality of public information on the welfare of the current owners. First, holding the firm’s future investment levels fixed, current shareholders prefer more informative disclosure regimes since such regimes minimize the total risk premium associated with the firm’s future projects, and thus maximize the expected resale price of the stock for the current owners. Second, we show that when future generations of shareholders take control of the firm, they will make the firm invest at higher levels than what would be preferred by the current owners, i.e., the actual future investment levels will be higher than those that maximize the resale price of the stock for the current generation. As a consequence, the current owners can be better off under a less informative disclosure regime since it leads to less overinvestment by future generations. We show that the current owners will indeed prefer an imperfect disclosure regime if the firm’s investment is sensitive to the cost of capital and when the firm’s future investment opportunities are sufficiently large relative to its assets in place.

Comparing our results to those in Dutta and Nezlobin (2016), we find that when one endogenizes the firm’s internal investment decisions, the preferences of the firm’s current
shareholders for public information get weaker while the preferences of future shareholders get
stronger relative to a model with exogenous cash flows. These results might explain why the
current shareholders do not always lobby for the most informative public disclosure regime,
and, in fact, often oppose increasing the transparency of mandatory financial disclosures.
Our model allows us to make specific predictions about which shareholders are more likely
to prefer imperfect disclosure regimes – those are the current shareholders of firms whose
investment is more sensitive to the cost of capital and whose future investment opportunities
outweigh their assets in place. In contrast, the welfare of the potential future shareholders
decreases in disclosure quality only if the firm’s growth rate during their period of ownership
is sufficiently low. Since one of the stated objectives of the Financial Accounting Standards
Board is to provide information “useful to existing and potential investors, lenders, and
creditors,” it is important to shed additional light on the difference in disclosure preferences
between the current and potential future shareholders of the firm.

From the regulatory perspective, the need for increasing the informativeness of mandated
public disclosures is often justified by their effect on the economy-wide cost of capital.² Our
paper shows that in analyzing the effects of public information in a dynamic setting, it is
important to distinguish between two different concepts of the cost of capital: the cost of
financing new long-term projects and the equity risk premium that investors demand for
holding the firm’s stock for a period of time. The periodic equity risk premium reflects part
of the risk associated with assets in place at the purchase date as well as part of the risk
associated with the new projects undertaken since then. This measure determines investor
welfare over the given period, which, according to our results, is not necessarily monotonic in
the precision of public disclosures. In contrast, the cost of financing new projects is indeed
monotonically decreasing in the precision of public information. Though this cost is not
directly observable in the stock returns, it is reflected in the investment levels undertaken
by the firm.

Our paper follows the modeling framework used in Dutta and Nezlobin (2016), which is
closely related to the asset pricing literature based on infinite horizon overlapping generations
models with the CARA-Normal structure (e.g., De Long et al. 1990, Spiegel 1998, Suijs
2008, and Watanabe 2008). Christensen et al. (2010), Easley and O’Hara (2004), and

²e.g. SFAC #8: Reporting financial information that is relevant and faithfully represents what it purports
to represent helps users to make decisions with more confidence. This results in more efficient functioning
of capital markets and a lower cost of capital for the economy as a whole.
Hughes et al. (2007) also investigate the link between disclosure quality and the cost of capital. These studies show that higher quality disclosures reduce the \textit{ex post} cost of capital. Unlike our production setting, however, these studies consider pure exchange economies with exogenously specified distributions of future cash flows. Lambert et al. (2007) and Gao (2010) investigate production economies in static settings. However, these papers assume that public disclosure improves information available not only to the market but also to the decision-makers inside the firm. In contrast, our analysis focuses on a setting in which public disclosures do not alter the amount of information available to the firm when it makes its investment choices. Consequently, our analysis isolates the effect of information available to the stock market from the effect of the information available to the firm on its investment choices.

The real effects literature in accounting also investigates the equilibrium relationship between firms’ disclosure environments and their internal investment decisions (see, for instance, Kanodia and Sapra (2016) for a comprehensive survey of this literature.) However, unlike our model in which the firm’s investment choices are observed by the market, these real effects studies assume that the firm’s internal investment choices cannot be directly communicated to the market. Moreover, much of the work in the real effects literature assumes that investors are risk neutral, and hence does not focus on the link between disclosure quality and the equilibrium risk premium.

The rest of the paper is organized as follows. Section 2 describes the model setup. Section 3 first characterizes the equilibrium relationship between information disclosure and real investments. We then investigate the relationship among information disclosure, risk premium, and shareholder welfare. Section 4 concludes the paper.

2 Model Setup

Much of the earlier work on information disclosure, stock returns, and investor welfare has focused on pure exchange settings with exogenously specified cash flows (see, e.g., Christensen et al., 2010; Dutta and Nezlobin, 2016; Easley and O’Hara, 2004; Hughes et al., 2007; Lambert et. al., 2007; Suijs 2008). In contrast, we study a production setting in which the firm’s investment levels are endogenously determined.

Specifically, consider an infinitely lived firm that undertakes a sequence of overlapping
investment projects. Each project has a useful life of two periods, and the scale of each project is chosen irreversibly at its inception. Let $k_{t-2}$ denote the scale of the project started at date $t-2$, and let $v(k_{t-2})$ be the associated cost of investment. We assume that the cost of investment function, $v(\cdot)$, is increasing and convex in the project’s scale.

The project started at date $t-2$ generates a payoff of $c_t$ dollars at date $t$:

$$c_t \equiv x_t k_{t-2},$$

(1)

where the random variable $x_t$ models uncertain investment productivity in period $t$. The productivity parameters, $\{x_t\}$, are drawn from independent normal distributions with means $\{m_t\}$ and variance $\sigma^2$. The firm’s investment choices and realized cash flows are directly observed by all shareholders.

The firm’s stock is traded in a perfectly competitive market by overlapping generations of identical short-horizon shareholders with symmetric information. Specifically, the shareholders of generation $t$ buy all the firm’s shares from the previous generation at date $t-1$ and sell all the shares to the next generation at date $t$. In addition to the firm’s shares, investors can trade a risk-free asset, which is in unlimited supply and yields a rate of return of $r > 0$. It will be convenient to let $\gamma \equiv \frac{1}{1+r}$ denote the corresponding risk-free discount factor.

Since all shareholders of a given generation are identical and have the same information, we can, without loss of generality, model each generation as a single representative investor. The representative investor of generation $t$ chooses his portfolio at date $t-1$ so as to maximize the expected utility of consumption at date $t$. Let $\omega_t$ denote the investor’s terminal wealth (and also consumption) at date $t$. We assume that the preferences of the representative investor of generation $t$ are summarized by the following utility function:

$$U(\omega_t) = -\exp(-\rho \omega_t),$$

(2)

where $\rho$ is the coefficient of constant absolute risk aversion (CARA). It is well known that under CARA preferences, there is no loss of generality in normalizing the initial wealth of the generation-$t$ investor to zero.

We now turn to describing the firm’s mandated public disclosures. Prior to trading at date $t$, the firm must publish a financial report, $S_t$, that provides information about the next
period’s operating cash flow, $c_{t+1}$. Recall that the cash flow $c_{t+1}$ will be generated by the project commenced at date $t - 1$. Accordingly, we will sometimes refer to this project as the firm’s assets-in-place at date $t$. The report $S_t$ takes the following form:

$$S_t = k_{t-1} s_t,$$

where $s_t$ is a noisy measure of asset productivity in period $t + 1$. Specifically,

$$s_t = x_{t+1} + \varepsilon_t,$$  \hspace{1cm} (3)

where the error terms $\{\varepsilon_t\}$ are drawn from serially independent normal distributions with mean zero and variance $\sigma^2_{\varepsilon}$. We additionally assume that $\{\varepsilon_t\}$ are independent of $\{x_t\}$; i.e., the measurement error terms in the firm’s financial reports are independent of past, current, or future productivity parameters.

To measure the quality of the financial reporting system, we employ the following signal-to-noise ratio:

$$h \equiv \frac{\sigma^2}{\sigma^2 + \sigma^2_{\varepsilon}}.$$

A higher quality financial reporting system corresponds to a higher value of $h$. In particular, when $h$ is zero, $\sigma^2_{\varepsilon} = \infty$, and public financial reports provide no useful information about one-period-ahead cash flows. In contrast, when $h = 1$, each report $S_t$ perfectly reveals the forthcoming values of the productivity parameter, $x_{t+1}$, and operating cash flow, $c_{t+1}$.

For future reference, it is useful to note that conditional on the public signal $s_t$, the one-period ahead asset productivity parameter, $x_{t+1}$, will be distributed normally with mean $h s_t + (1 - h) m_{t+1}$ and variance $(1 - h) \sigma^2$. As one would expect, the firm’s shareholders will put more weight on the realized value of $s_t$ in updating their expectation of $x_{t+1}$ if the public reporting system is more precise. Also as expected, the conditional variance of the productivity parameter decreases in $h$. From the perspective of date $t - 1$, the date-$t$ conditional expectation of $x_{t+1}$, $E_t(x_{t+1})$, is also a normally distributed random variable with mean $m_{t+1}$ and variance $h \sigma^2$. The variance of the conditional expectation of $x_{t+1}$ is increasing in the precision of the public signals.
Figure 1: Sequence of events in period $t + 1$

The timeline of events during the ownership of generation-$t + 1$ shareholders is depicted in Figure 1 above. After the firm’s financial report, $S_t$, is released, the market for the firm’s shares opens and generation-$t + 1$ shareholders acquire the firm. Then, the firm’s new investment project, $k_t$, is commenced. We assume that at each point in time, the firm chooses the scale of its new project in the best interest of its current shareholders. Thus, $k_t$ is chosen so as to maximize the expected utility of the representative investor of generation $t$. 

In our setting with symmetric information, it is without loss of generality to assume that the firm retains only as much cash as necessary to fund the next investment project. Therefore, at the end of each period $t$, the firm retains enough cash, $v(k_t)$, to finance the investment level $k_t$ that would be chosen by the next generation of shareholders. Thus the net dividend distributed to generation-$t + 1$ shareholders at date $t + 1$ is equal to $c_{t + 1} - v(k_{t + 1})$. Once this dividend is paid out, the firm releases a new financial report $S_{t + 1}$, based on which the resale price of the stock, $p_{t + 1}$, will be formed in a perfectly competitive market.

Note that the public signal released at date $t$, $S_t$, does not provide any information about the payoff of the next project to be undertaken by the firm ($k_t$), which will be ultimately determined by $x_{t + 3}$. Furthermore, the scale of the project to which this signal relates ($k_{t - 1}$) cannot be changed at date $t$. A useful feature of this setup is that the quality of the financial reporting system, $h$, is not directly linked to the inherent riskiness of the firm’s operations. In other words, the uncertainty that the firm faces in making its investment decisions is the same regardless of the quality of the public reporting system. Therefore, the effect of information quality on the firm’s investment that we identify in this paper can indeed be

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3This assumption is made only for notational convenience. Our results would remain unchanged under an alternative financing structure in which the firm does not retain any cash (i.e., the current shareholders receive the entire amount of current cash, $c_t$, as dividends) and funds for the new investment project are instead directly provided by the next generations of shareholders.
attributed to the quality of information *disclosed* to the stock market at each trading date rather than the quality of information *available* to the firm at each decision-making time.

3 Information disclosure, real investment, and investor welfare

Our primary objective is to examine how public information affects the firm’s internal investment decisions and investor welfare. We will show that the firm’s growth trajectory plays a critical role in defining the nature of both these relations. Before considering the firm’s choice of optimal investments, we derive an expression for the equilibrium market price and risk premium for an exogenously given set of investment choices \((k_1, k_2, \ldots)\). The following result is due to Dutta and Nezlobin (2016):

**Lemma 1.** For any given sequence of investments \((k_1, k_2, \ldots)\), the equilibrium market price at date \(t\) is given by

\[
p_t = \sum_{\tau=1}^{\infty} \gamma^\tau [E_t(c_{t+\tau}) - v(k_{t+\tau}) - RP_{t+\tau}],
\]

where

\[
RP_{t+\tau} = \rho \sigma^2 \left[(1-h)k_{t+\tau-2} + \gamma^2 hk_{t+\tau-1} \right]
\]

is the risk premium in period \(t + \tau\).

**Proof.** All proofs are in the Appendix.

Equation (4) shows that the equilibrium market price at each date can be expressed as the sum of expected future cash flows net of periodic risk premia discounted at the risk-free interest rate \(r\). To understand the risk premium expression in (5), consider the portfolio choice problem of generation \(t+1\) investors. These investors’ gross return from buying the firm consists of the dividends that they receive during their period of ownership, \(c_{t+1} - v(k_{t+1})\), and the resale price at which they sell their shares to the next generation, \(p_{t+1}\). Given our assumptions on preferences and the distributions of cash flow, the expected excess return (i.e., the equilibrium risk premium) is given by

\[4\]In the special case of a stationary firm (i.e., \(k_t = k\) and \(m_t = m\)) and no information disclosure (i.e., \(h = 0\)), the expression for the price simplifies to \(\frac{1}{1} \cdot mk - v(k) - \rho k^2 \sigma^2\), which is consistent with the findings of DeLong et al. (1990).

\[5\]In our single risky asset setting, any risk is systematic and priced as such. Our results can be readily extended to multi-firm economies.
\[ RP_{t+1} = \rho Var_t(c_{t+1} - v(k_{t+1}) + p_{t+1}). \]

Note that \( v(k_{t+1}) \) is deterministic. Furthermore, since project cash flows are serially uncorrelated, price \( p_{t+1} \) and cash flow \( c_{t+1} \) are independently distributed. The equilibrium risk premium can thus be expressed as the sum of the investors’ compensation for bearing (i) the dividend risk as measured by \( Var_t(c_{t+1}) \), and (ii) the resale price risk as measured by \( Var_t(p_{t+1}) \). That is,

\[ RP_{t+1} = \rho Var_t(c_{t+1}) + \rho Var_t(p_{t+1}). \] (6)

The price risk reflects the current shareholders’ uncertainty about their payoffs from selling their shares to the next generation at date \( t + 1 \). Specifically, the current shareholders rationally perceive that potential future investors will have the opportunity to revise their estimates of the firm’s one-period ahead cash flows (i.e., \( c_{t+2} \)) in response to the public information that will be released at the end of the current period (i.e., \( s_{t+1} \)). Generation \( t + 1 \) investors demand compensation for bearing the risk associated with this ax ante uncertainty about future investors’ conditional expectations \( E_{t+1}(c_{t+2}) \), since they rightly anticipate that these expectations will affect price \( p_{t+1} \), and hence their gross return from buying the firm.

Equation (4) implies that \( Var_t(p_{t+1}) = \gamma^2 Var_t(E_{t+1}(c_{t+2})) = \gamma^2 h \sigma^2 k_t^2 \). On the other hand, dividend risk \( Var_t(c_{t+1}) \) is equal to \( \sigma^2 (1 - h) k_{t-1}^2 \). To summarize, equation (5) demonstrates that the investors buying the firm at date \( t \) are exposed to (i) uncertainty of the payoffs from project \( k_{t-1} \), since they receive these payoffs as dividends, and (ii) to uncertainty of the payoff from project \( k_t \) indirectly through these payoffs’ effect on date \( t + 1 \) resale price. The risk premium term corresponding to project \( k_t \) is discounted by \( \gamma^2 \) because \( p_{t+1} \) reflects the (one-period) discounted value of \( c_{t+2} \), and risk premium is proportional to variance in our mean-variance framework.

We now proceed to characterize the firm’s endogenous investment choices. Suppose the firm has retained \( v(k^*_t) \) cash for the investment at date \( t \). The firm chooses its investment level at date \( t \) so as to maximize the expected utility of its current (generation \( t + 1 \)) shareholders. Given the CARA-Normal framework, the firm would choose investment level \( k_t \) so as to maximize the certainty equivalent of its current shareholders’ date \( t + 1 \) consumption

\[ p_{t+1} - (1 + r)v(k_t) + \Gamma, \]
where $\Gamma \equiv c_{t+1} + (1 + r)(v(k_t^*) - p_t)$ does not depend on the firm’s actual choice of $k_t$. The proof of Proposition 1 shows that after dropping the terms unrelated to $k_t$, the firm’s optimization problem can be expressed as follows:\(^6\)

$$
\max_{k_t} V(k_t, h) \equiv \gamma m_{t+2} k_t - (1 + r)v(k_t) - \gamma \rho(1 - h)\sigma^2 k_t^2 - \frac{\gamma^2}{2} \rho h \sigma^2 k_t^2.
$$

(7)

The first term of objective function $V(k_t, h)$ reflects the present value of expected gross payoffs from the current investment. The second term in (7) captures the direct cost of investment $v(k_t)$. The third term in (7) reflects that a higher level of investment in the current period makes future cash flows riskier, which lowers the expected value of the selling price at date $t+1$. Lastly, a higher level of investment also makes $p_{t+1}$ more volatile lowering the current owners’ certainty equivalent by the amount of $\frac{\gamma^2}{2} \text{Var}_t(p_{t+1})$. This risk cost is captured by the last term of (7). We obtain the following result:

**Proposition 1.** The optimal investment level $k_t^*$ increases in the precision of public disclosure and is given by the following first-order condition:

$$
\gamma m_{t+2} = (1 + r)v'(k_t^*) + \gamma \rho \sigma^2 k_t^* [2(1 - h) + \gamma h] \tag{8}
$$

Proposition 1 shows that the optimal investment level $k_t^*$ increases in disclosure quality (i.e., $\frac{\partial k_t^*}{\partial h} > 0$). Intuitively, a more precise public disclosure lowers the risk-related marginal cost of investments as represented by the last two terms of the objective function in (7). Consistent with the standard intuition, it can be checked that the optimal investment level is more sensitive to the precision of public disclosure when $v''(\cdot)$ is small, or the expected marginal benefit $m_{t+2}$ is large.

For the remaining analysis, we assume that the cost of investment is quadratic; i.e.,

$$
v(k_t) = bk_t^2
$$

for all $t$. This assumption allows us to derive a closed form expression for the optimal

\(^6\)To ensure a finite market price for the firm, we assume that $\sum_{t=1}^{\infty} \gamma^t m_t^2 < \infty$ for each $t$. This condition will be satisfied, for example, when the asymptotic growth rate of the investment productivity parameters $\{m_t\}$ does not exceed $\sqrt{1+r}$.
investment level \( k_t^* \). Specifically, the first-order condition in (8) yields

\[
k_t^* = \frac{\gamma^2 m_{t+2}}{2b + \rho \gamma^2 \sigma^2 [2(1-h) + \gamma h]}
\] (9)

We now seek to characterize the impact of public information on investors’ welfare and risk premium. Proposition 2 below characterizes how the quality of public information affects welfare of the firm’s future prospective shareholders and the periodic risk premium. We investigate the effect of a change in disclosure quality on the firm’s current shareholders in Proposition 3. The distinction between existing and future shareholders is relevant for this welfare analysis, since existing shareholders already own the firm (i.e., the price they have paid for the firm is a sunk cost for them). Hence, they are primarily concerned with how the quality of public information affects the resale price of the stock. In contrast, any change to the disclosure regime will affect both the purchase and resale prices for future generations of shareholders.

**Proposition 2.** The risk premium in period \( t+\tau \) and welfare of future investors of generation \( t+\tau \) increases in the informativeness of public disclosure if

\[
\frac{m_{t+\tau+1}^2}{m_{t+\tau}^2} \geq (1 + r)^2 - l(h),
\] (10)

where \( l(h) \) is decreasing and positive for all \( h \). The risk premium in period \( t+\tau \) and welfare of generation \( t+\tau \) investors decreases in the informativeness of public disclosure if the opposite inequality holds.

The proof of Proposition 2 shows that the expected utility of future potential investors is positively related to the risk premium in the period during which they plan to hold the firm. Specifically, we find that \( CE_{t+\tau} = \frac{1}{2} RP_{t+\tau} \). The intuition for this result is that while higher risk premium means greater risk exposure, it also implies a lower asset price and hence higher expected return. With exponential utility and normally distributed payoffs, we find that the expected return effect always dominates.

Since the optimal investment level \( k_{t+\tau-1}^* \) is proportional to the productivity parameter \( m_{t+\tau} \), the inequality in the above result can be equivalently expressed in terms of the endogenous growth rate \( \mu_{t+\tau} \), where \( \mu_{t+\tau} \) is defined by \( k_{t+\tau}^* = (1 + \mu_{t+\tau})k_{t+\tau-1}^* \). Analogous to the finding in the exogenous investment setting of Dutta and Nezlobin (2016), this result
shows that the equilibrium relationship between risk premium and quality of public disclosure depends on the firm’s growth trajectory. For instance, it shows that the risk premium in period \( t + 1 \) increases in the informativeness of public information if the endogenous growth rate \( \mu_t \) exceeds a certain threshold. However, since \( l > 0 \), the threshold growth rate is lower than \( r \), the threshold for the exogenous investment setting. More generally, since \( l(h) \) is a decreasing function, Proposition 2 implies that the risk premium (and hence investor welfare) (i) monotonically increases in the quality of public information for relatively fast growing firms, (ii) first increases and then decreases in disclosure quality for medium growth firms, and (iii) monotonically decreases in the quality of public information for relatively slow or negative growth firms.

Note that the risk premium in period \( t + 1 \) is given by:

\[
RP_{t+1} = \rho \sigma^2 \left[ (1 - h)(k_{t-1}^*)^2 + \gamma^2 h(k_t^*)^2 \right],
\]

where the optimal investment levels \( k_{t-1}^* \) and \( k_t^* \) are as defined in (9). The above expression shows that the risk premium varies with the precision of public disclosure for two reasons. First, holding the investment levels fixed, Dutta and Nezlobin (2016) show that the risk premium would decrease (increase) in the informativeness of public disclosure when the investment growth rate is lower (higher) than \( r \). With endogenous investments, however, a more precise public disclosure also results in higher optimal investment levels \( k_{t-1}^* \) and \( k_t^* \), which leads to higher risk premium. It is because of this real effect of public disclosure that the threshold growth rate is lower in the endogenous investment setting.

Comparing our results in Propositions 1 and 2 reveals that two different notions of the cost of capital arise in our model. First, one can calculate the total risk premium associated with a given long-term project, i.e., the cost of raising equity capital for that particular project. Since in our model each project lasts for two periods, this risk premium will consist of two components charged by two different generations of investors. Our result in Proposition 1 shows that this “per project” cost of capital monotonically decreases in the quality of public information, and, as a consequence, the firm invests more when public disclosures are more precise. This result is largely consistent with the conventional wisdom that better disclosure regimes lead to a lower cost of capital.

In contrast, Proposition 2 speaks about a different notion of the cost of capital – the risk premium that investors of a given generation demand for holding the firm’s stock for one
period of time. This periodic risk premium originates, in our model, from two projects: the firm’s assets-in-place at the beginning of the period and the new project that was started during the period. Our result in Proposition 2 shows that the periodic risk premium can be increasing or decreasing in the quality of public information depending on the firm’s growth trajectory. It is important to note that it is this “periodic” cost of capital that determines the welfare of investors holding the stock over a given period and gets directly reflected in the firm’s stock returns. In contrast, the “per project” cost of capital partially affects the firm’s stock returns in two different periods of time and, similarly, enters the utility function of two different generations of shareholders.

It might appear from our result in Proposition 1 that the total risk premium charged by all future shareholders for holding all of the future projects is decreasing in the quality of public information. Indeed, holding the firm’s future investment levels fixed, the risk premium per each project decreases in $h$, and therefore the discounted sum of future risk premia should also be decreasing in $h$. Then, Lemma 1 suggests that the current shareholders of the firm will prefer the full disclosure regime since it maximizes their resale price of the stock. While this intuition would indeed hold in a model with exogenous investments, it does not apply in our setting. Recall that according to Proposition 1 the firm’s investment increases in $h$, and this increase in investment leads to increased risk premia for future projects. It turns out that under certain circumstances, this effect of increasing investment can dominate the effect of reduced “per-project” cost of capital.

Our next result characterizes the net effect of the quality of public information on welfare of current owners. To investigate this effect, suppose a new disclosure policy (i.e., a new value of precision for all future disclosures) takes effect when the firm is owned by generation $t$ investors between dates $t-1$ and date $t$. We assume that the disclosure policy change takes place after period $t$ investment decision is made by the firm.\footnote{Our results would remain unchanged if the disclosure policy change were to take place prior to the firm’s choice of investment $k_{t-1}$.}

**Proposition 3.** Welfare of the existing shareholders is maximized at an intermediate level of public disclosure if future investments are sufficiently sensitive to information disclosure and sufficiently large relative to the firm’s end of the period assets-in-place.\footnote{Specifically, this result obtains for low values of $b$ and high values of $\{m_{t+1}\}$. See the proof of Proposition 3 for a precise set of sufficient conditions.}

This finding contrasts with the result in Dutta and Nezlobin (2016) who show that the
current shareholders’ welfare unambiguously increases in the informativeness of public disclosure in pure exchange settings. In contrast, Proposition 4 shows that when investments are endogenously chosen, the current shareholders’ welfare is maximized at an intermediate level of disclosure if future investments are sufficiently sensitive to information disclosure. This result implies that even if the shareholders could increase the precision of public disclosures costlessly, they might still prefer financial disclosure regimes that require less than full disclosure.

The proof of Proposition 3 shows that the current shareholders’ expected utility can be represented by the following certainty equivalent expression:

\[ CE_t = V(k_{t-1}, h) + \sum_{\tau=1}^{\infty} \gamma^\tau \left[ V(k^*_t + \tau - 1, h) - \frac{\gamma^2}{2} \rho k^*_t + \tau - 1^2 h \sigma^2 \right], \]

where \( V(k^*_t, h) \) denotes the maximized value of the objective function in (7). The above expression makes clear that the current shareholders’ expected utility will vary with the amount of public information directly through its effect on the total risk premium (for fixed investment levels), as well as indirectly through the effect of public disclosures on the firm’s optimal investment choices. Differentiating the above expression for \( CE_t \) with respect to \( h \) and applying the Envelope Theorem gives

\[ \frac{dCE_t}{dh} = \frac{\partial CE_t}{\partial h} - \rho \gamma^2 h \sigma^2 \cdot \sum_{\tau=1}^{\infty} \gamma^\tau k^*_t + \tau - 1 \frac{\partial k^*_t + \tau - 1}{\partial h}. \]

The first term above, \( \frac{\partial CE_t}{\partial h} \), reflects the direct effect (i.e., holding investments fixed) of information disclosure on welfare and is always positive. This corresponds to the result in Dutta and Nezlobin (2016) that when investments are exogenously fixed, the current shareholders’ welfare increases in the informativeness of public disclosures. The intuition for this effect is as follows. Since the current shareholders already own the firm, any change in future disclosure policy can affect their welfare only through its impact on future prices. Consequently, holding the investment levels fixed, their expected utility can be represented by the following certainty equivalent expression:

\[ CE_t = E_{t-1}(p_t) - \frac{\rho}{2} Var_{t-1}(p_t) + \text{const.} \]

We omit the additive terms related to the certainty equivalents of \( c_t \) and \( p_{t-1} \), since they do not vary with the precision for future disclosures.
The expected value of the resale price increases in the quality of public information (i.e., $E_{t-1}(p_t)$ increases in $h$). Though a higher quality disclosure regime also makes the resale price more volatile (i.e., $Var_{t-1}(p_t)$ also increases in $h$), the expected price effect dominates. Hence, when investments are exogenously fixed, the current shareholders’ welfare monotonically increases in the precision of public information (i.e., $\frac{\partial CE_t}{\partial h} > 0$).

The second term in the right hand side of equation (11) captures the indirect effect of public disclosures on the shareholders’ welfare. Since the optimal investment increases in the quality of public information, this indirect effect is always detrimental to the original shareholders’ welfare. Intuitively, this effect arises because future generations of shareholders overinvest relative to the preferred amounts of investments from the perspective of the current shareholders, and the amount of overinvestment increases in the precision of public disclosure.

To see this, note from the expression for $CE_t$ that while future investors will choose $k_{t+\tau}$ to maximize $V(k_{t+\tau}, h)$, the current shareholders would prefer them to maximize $V(k_{t+\tau}, h) - \frac{x^2}{2} \rho k_{t+\tau}^2 \sigma^2$.

To further illustrate the intuition for this overinvestment result, it is instructive to consider a three period lived firm that has access to an investment opportunity. For notational simplicity, we will ignore discounting. The current owners sell the firm to generation 1 investors at date 1. These new shareholders invest $v(k)$ in the investment project which yields random payoffs of $xk$ at date 3, where $x \sim N(m, \sigma^2)$. At date 2, the firm releases public information about the forthcoming cash flow and generation 1 investors sell the firm to the next generation who receives a terminal dividend of $xk$ at date 3. If the firm makes full disclosure at date 2, all the uncertainty is resolved and date 2 price will be simply equal to $xk$. Consequently, generation 1 will choose $k$ to maximize the following certainty equivalent expression:

$$CE_1 = mk - v(k) - \frac{\rho}{2} k^2 \sigma^2.$$ 

On the other hand, the firm’s current owners will prefer a $k$ that maximizes the price at date 1,

$$p_1 = mk - v(k) - \rho k^2 \sigma^2.$$ 

A comparison of the two objective functions reveals that generation 1 will overinvest relative to the preferred amount of investment from the current owners’ point of view. On the other hand, if the firm made no disclosure, then date 2 price will be $p_2 = mk - \rho k^2 \sigma^2$. In this case, investment preferences of the two generations become perfectly congruent.
Since this indirect detrimental effect due to overinvestment vanishes for the limiting case of no disclosure (i.e., $h = 0$), the current shareholders’ welfare is always increasing in the precision of public disclosure for small values of $h$. For large values of cost parameter $b$, the optimal investment levels are relatively insensitive to the precision of public disclosure and hence the direct beneficial effect dominates, and the welfare of the current shareholders increases in the informativeness of public disclosures. When the marginal product of the current investment is large relative to the marginal productivities of future investments, the current shareholders’ welfare is primarily determined by their expected utility from the payoffs related to the current project; i.e., $V(k_{t-1}, h)$, which monotonically increases in the quality of public information. In all other cases, the current shareholders’ welfare is maximized at an intermediate level of disclosure.

For analytical tractability, we have assumed that cost function $v(\cdot)$ is quadratic, which allows us to derive closed form expressions for the optimal investment choices and the relevant thresholds in Propositions 2 and 3. Though closed form expressions for the optimal investments are not available under more general assumptions on the cost of investment, the qualitative nature of our results in Propositions 2 and 3 continue to hold. To see this, consider a general cost function $v(k_t)$ that is increasing and weakly convex. Since the optimal investment $k_t^*$ increases in the precision of public disclosures, the threshold growth rate in Proposition 2 will again exceed the threshold rate of $r$ in the exogenous investment setting. Equation (11) implies that there is again a tradeoff between a direct beneficial effect and an indirect adverse effect of increasing the precision of public disclosure on the current shareholders’ welfare. When $v''(\cdot)$ is relatively large and the optimal investment level is largely insensitive to the precision of public disclosure, the current shareholders’ welfare increases in the informativeness of public disclosure. On the other hand when $v''(\cdot)$ is relatively small, the welfare of the current shareholders will be maximized for some intermediate precision of public disclosure.
Appendix

Proof of Lemma 1. We want to prove that the equilibrium market price of the firm at date \( t \) is given by

\[
p_t = \sum_{\tau=1}^{\infty} \gamma^\tau \left[ E_t (c_{t+\tau}) - v(k_{t+\tau}) - RP_{t+\tau} \right],
\]

where

\[
RP_{t+\tau} = \rho \sigma^2 \left[ (1-h)k_{t+\tau-2}^2 + \gamma^2 h k_{t+\tau-1}^2 \right]
\]
denotes the equilibrium risk premium in period \( t + \tau \). Since signal \( s_t \) is uninformative about \( x_{t+\tau} \) for all \( \tau > 1 \), \( E_t(x_{t+\tau}) = m_{t+\tau+1} \) for all \( \tau > 1 \). The formula for the conditional expectations of normal random variables gives \( E_t(x_{t+1}) = hs_t + (1-h)m_{t+1} \), where \( h \equiv \frac{\sigma^2}{\sigma^2 + \epsilon} \).

Hence, the pricing function in (13) can be expressed as follows:

\[
p_t = \beta_t + \gamma h k_{t-1} s_t,
\]

where \( \beta_t \) is a constant. Since signal \( s_t \) is normally distributed, equation (14) implies that \( p_t \) is also normal from the perspective of date \( t - 1 \).

We will now verify that the pricing function in (13) satisfies the market clearing condition at each \( t \). Consider the portfolio choice problem of the representative investor of generation \( t \). Without loss of generality, we normalize the investor’s initial wealth to zero and assume that the investor pays for the purchase cost of shares by borrowing at the risk-free rate of \( r \). If the representative investor of generation \( t - 1 \) buys \( \alpha \) fraction of the firm’s shares outstanding at date \( t - 1 \), her date \( t \) wealth (consumption) is given by

\[
\omega_t = \alpha [c_t - v(k_t) + p_t - (1 + r)p_{t-1}].
\]

Taking price \( p_{t-1} \) as given, the investor chooses \( \alpha \) to maximize his expected utility of wealth \( \omega_t \). Since \( p_t \) as conjectured in (13) is normal, the investors’s terminal wealth \( \omega_t \) is also normal. Given the CARA-Normal framework, maximizing expected utility is equivalent to maximizing the following certainty equivalent expression:

\[
CE_{t-1}(\alpha) = \alpha [E_{t-1} (c_t + p_t) - v(k_t) - (1 + r)p_{t-1}] - \frac{\rho}{2} \alpha^2 Var_{t-1} (c_t + p_t).
\]
Therefore, the optimal $\alpha$ is determined by the following first-order condition:

$$E_{t-1} (c_t + p_t) - v(k_t) - (1 + r)p_{t-1} - \rho \alpha \text{Var}_{t-1} (c_t + p_t) = 0.$$ 

Imposing the market clearing condition $\alpha = 1$ gives

$$p_{t-1} = \gamma [E_{t-1} (c_t + p_t) - v(k_t) - \rho \text{Var}_{t-1} (c_t + p_t)].$$ (15)

Equation (15) implies that the equilibrium risk premium in period $t$, $RP_t$, is given by $\rho \text{Var}_{t-1}(c_t + p_t)$. We note that $\text{Var}_{t-1}(c_t) = k_{t-2}^2(1 - h)\sigma^2$ and equation (14) implies $\text{Var}_t(p_t) = \gamma^2 k_{t-1}^2 h \sigma^2$. Since $p_t$, as conjectured in equation (13), is independent of $c_t$, it follows that

$$RP_t = \rho [\text{Var}_{t-1}(c_t) + \text{Var}_{t-1}(p_t)]$$

$$= \rho \sigma^2 [(1 - h)k_{t-2}^2 + \gamma^2 h k_{t-1}^2].$$

We can now verify that if the prices are given by equation (13), the market clearing condition (15) holds at all dates. To show this, we note that equation (13) implies

$$p_{t-1} = \sum_{\tau=1}^{\infty} \gamma^\tau [E_{t-1}(c_{t+\tau-1}) - v(k_{t+\tau-1}) - RP_{t+\tau-1}],$$

which can be written as

$$p_{t-1} = \gamma [E_{t-1}(c_t) - v(k_t) - RP_t] + \gamma \sum_{\tau=1}^{\infty} \gamma^\tau [E_{t-1}(c_{t+\tau}) - v(k_{t+\tau}) - RP_{t+\tau}]$$

$$= \gamma [E_{t-1}(c_t + p_t) - v(k_t) - RP_t].$$

This proves that the pricing function in (13) satisfies the market clearing condition in (15).

\[\square\]

**Proof of Proposition 1.**

Taking the price process (13) as given, the representative investor of generation $t + 1$ chooses $k_t$ to maximize the expected utility of his date $t + 1$ consumption, $c_{t+1} - v(k_{t+1}) + p_{t+1} - (1 + r)[v(k_t) - v(k_t^*)] - (1 + r)p_t$, where $k_t^*$ denotes the amount of period $t$ investment anticipated by the firm and $v(k_t^*)$ is the corresponding amount of cash retained in the firm.
In equilibrium, the current shareholder’s optimal choice of investment will coincide with the conjectured amount $k_t^*$. Generation $t + 1$ shareholder’s expected utility of his date $t + 1$ consumption can be represented by the following certainty equivalent expression:

$$CE_{t+1} = E_t(p_{t+1}) - (1 + r)v(k_t) - \frac{\rho}{2}Var_t(p_{t+1}) + \Gamma,$$

where $\Gamma \equiv E_t(c_{t+1}) - \frac{\rho}{2}Var_t(c_{t+1}) - (1 + r)(p_t - v(k_t^*))$ does not depend on the investors’ choice of $k_t$.

Applying equation (13), we get:

$$E_t(p_{t+1}) - \frac{\rho}{2}Var_t(p_{t+1}) = \gamma m_{t+2}k_t - \rho\gamma(1 - h)\sigma^2k_t^2 - \frac{\rho}{2}\gamma h\sigma^2k_t^2 + A_{t+1},$$

where $A_{t+1}$ does not depend on $k_t$. Therefore, generation $t+1$ investor’s optimization problem can be written as:

$$\max_{k_t} V(k_t, h) \equiv \gamma m_{t+2}k_t - (1 + r)v(k_t) - \rho\gamma(1 - h)\sigma^2k_t^2 - \frac{\rho}{2}\gamma h\sigma^2k_t^2.$$

Equation (8) follows from the first-order condition of the above maximization problem.

Implicitly differentiating equation (8) with respect to $h$ yields

$$\frac{dk_t^*}{dh} = \frac{(2 - \gamma)\gamma \rho\sigma^2k_t^*}{(1 + r)v''(k_t^*) + \gamma \rho\sigma^2[2(1 - h) + \gamma h]}.$$

Since $v''(\cdot) > 0$, it follows that $\frac{dk_t^*}{dh} > 0$.

**Proof of Proposition 2:** To prove the result, we will first show that generation $t + \tau$ shareholders’ expected utility increases in the risk premium during the period in which they hold the firm; i.e., $RP_{t+\tau}$. The equilibrium expected utility of generation $t + \tau$ investor is monotonically increasing in following certainty equivalent expression:

$$CE_{t+\tau} = E_{t+\tau-1}(c_{t+\tau} + p_{t+\tau}) - v(k_{t+\tau}) - (1 + r)p_{t+\tau-1} - \frac{\rho}{2}Var_{t+\tau-1}(c_{t+\tau} + p_{t+\tau}).$$

Substituting for $p_t$ from (15) yields

$$CE_{t+\tau} = \frac{1}{2}\rho Var_{t+\tau-1}(c_{t+\tau} + p_{t+\tau}).$$
The proof of Lemma 1 shows that $RP_{t+\tau} = \rho Var_{t+\tau}(c_{t+\tau} + p_{t+\tau})$, and hence $CE_{t+\tau} = \frac{RP_{t+\tau}}{2}$. It thus follows that generation $t + \tau$ investor’s expected utility decreases (increases) in the precision of public disclosure if $RP_{t+\tau}$ increases (decreases) in $h$.

We now investigate how the risk premium varies with the quality of information. For given investment levels $k^*_t - 2$ and $k^*_t - 1$, the risk premium in period $t + \tau$ is given by $RP_{t+\tau} = \rho \sigma^2 [(k^*_t - 2)^2(1 - h) + \gamma^2 (k^*_t - 1)^2 h]$. Substituting for the optimal investments from (9) yields

$$RP_{t+\tau} = \frac{\rho \gamma^4 \sigma^2 [(1 - h)m^2_{t+\tau} + \gamma^2 hm^2_{t+\tau+1}]}{[2\rho \gamma^2 \sigma^2 (1 - h) + \rho \gamma^3 \sigma^2 h + 2b]^2}.$$  

Differentiating with respect to $h$ reveals that

$$\text{sgn}\left[\frac{\partial RP_{t+\tau}}{\partial h}\right] = \text{sgn}\left[\frac{m^2_{t+\tau+1}}{m^2_{t+\tau}} - \frac{2b - 2(1 - \gamma) \rho \gamma^2 \sigma^2 + (2 - \gamma) \gamma^2 \rho h \sigma^2}{\gamma^2 (2b + 2\rho \gamma^2 \sigma^2 + (2 - \gamma) \rho \gamma^3 h \sigma^2)}\right].$$

Therefore, $\frac{\partial RP_{t+\tau}}{\partial h} \geq 0$ if and only if

$$\frac{m^2_{t+\tau+1}}{m^2_{t+\tau}} \geq \frac{2b - 2(1 - \gamma) \rho \gamma^2 \sigma^2 + (2 - \gamma) \gamma^2 \rho h \sigma^2}{\gamma^2 (2b + 2\rho \gamma^2 \sigma^2 + (2 - \gamma) \rho \gamma^3 h \sigma^2)}.$$ 

The inequality above can be simplified as follows:

$$\frac{m^2_{t+\tau+1}}{m^2_{t+\tau}} \geq (1 + r)^2 - l(h),$$

where

$$l(h) = \frac{(4 - 2\gamma) \rho \sigma^2}{2b + \gamma^2 \rho \sigma^2 [2 + (2 - \gamma) h]}.$$ 

We note that $l(h)$ is decreasing in $h$ and positive for all $h \in [0, 1]$. ∎

**Proof of Proposition 3:** We will first show that holding investment amounts exogenously fixed, the expected utility of the existing shareholders of generation $t$ increases in the precision of public disclosure. The current shareholders’ expected utility can be represented by the following certainty equivalent expression:

$$CE_t = E_{t-1}(p_t) - v(k_t) - \frac{\rho}{2} Var_{t-1}(p_t) + \beta_t,$$

where $\beta_t \equiv E_{t-1}(c_t) - v(k_t) - (1 + r)p_t - \frac{\rho}{2} Var_{t-1}(c_t)$ does not depend on the precision of
future disclosures. Using the law of iterated expectations, equation (13) yields

\[ E_{t-1}(p_t) = \sum_{\tau=1}^{\infty} \gamma^\tau [m_{t+\tau} k_{t+\tau-2} - v(k_{t+\tau}) - RP_{t+\tau}] . \]

Moreover, we note that \( \text{Var}_{t-1}(p_t) = \gamma^2 h \sigma^2 k_{t-1}^2 \). Substituting these into (16) and denoting the terms independent of the precision of future disclosures by \( A_t \) yield

\[ CE_t = A_t - \frac{\rho}{2} \gamma^2 h \sigma^2 k_{t-1}^2 - \sum_{\tau=1}^{\infty} \gamma^\tau RP_{t+\tau}. \]

Since

\[
\sum_{\tau=1}^{\infty} \gamma^\tau RP_{t+\tau} = \rho \sigma^2 \sum_{\tau=1}^{\infty} \gamma^\tau ((1-h)k_{t+\tau-2}^2 + \gamma^2 h k_{t+\tau-1}^2) = \rho \gamma k_{t-1}^2 (1-h) \sigma^2 + \rho \sigma^2 \sum_{\tau=1}^{\infty} \gamma^{\tau+1} ((1-h) + \gamma h) k_{t+\tau-1}^2,
\]

it follows that

\[ CE_t = A_t - \rho \gamma^2 \left[ (1-h) + \frac{\gamma}{2} h \right] k_{t-1}^2 - \rho \sigma^2 \sum_{\tau=1}^{\infty} \gamma^{\tau+1} ((1-h) + \gamma h) k_{t+\tau-1}^2. \]

Differentiating with respect to \( h \) gives

\[
\frac{\partial CE_t}{\partial h} = \rho \gamma \sigma^2 \left( 1 - \frac{\gamma}{2} \right) k_{t-1}^2 + \rho \sigma^2 \gamma (1-\gamma) \sum_{\tau=1}^{\infty} \gamma^{\tau+1} k_{t+\tau-1}^2, \tag{17}
\]

which is positive.

Substituting for the equilibrium price at date \( t \) from equation (4) and rearranging terms, it can be verified that equation (16) yields

\[ CE_t = B + V(k_{t-1}, h) + \sum_{\tau=1}^{\infty} \gamma^\tau \left[ V(k_{t+\tau-1}^*, h) - \frac{\gamma^2}{2} \rho (k_{t+\tau-1}^*)^2 h \sigma^2 \right] , \]

where \( V(k_{t-1}^*, h) \equiv \gamma m_{t+2} k_{t-1}^* - (1+r) b(k_{t-1}^*)^2 - \gamma \rho (k_{t-1}^*)^2 (1-h) \sigma^2 - \frac{\gamma^2}{2} \rho (k_{t-1}^*)^2 h \sigma^2 \) denotes the maximized value of the firm’s period \( \tau \) objective function, as defined in (7). To emphasize that date \( t - 1 \) investment does not vary with the precision of future disclosures, we do not
use any superscript on $k_{t-1}$. Differentiating with respect to $h$ and applying the Envelope Theorem yield

$$
\frac{dCE_t}{dh} = \frac{\partial CE_t}{\partial h} - \rho \gamma^2 h \sigma^2 \cdot \sum_{\tau=1}^{\infty} \gamma^\tau k^*_{t+\tau-1} \frac{\partial k^*_{t+\tau-1}}{\partial h}.
$$

We note that $\frac{dCE_t}{dh} > 0$ at $h = 0$ because (i) equation (17) shows that $\frac{\partial CE_t}{\partial h} > 0$ for all $h \in [0, 1]$, and (ii) the second term on the right hand side of the above expression is zero for $h = 0$. It thus follows from continuity that there exists a $h_L \in (0, 1)$ such that the existing shareholders’ welfare increases in $h$ for all $h \in [0, h_L]$. Substituting $\frac{\partial CE_t}{\partial h} = \rho \gamma^2 (1 - \frac{\gamma}{2}) k^2_{t-1} + \rho \sigma^2 \gamma (1 - \gamma) \sum_{\tau=1}^{\infty} \gamma^{\tau+1} k^2_{t+\tau-1}$ from equation (17), the optimal investments $k^*_t$ from (9), and simplifying reveal that

$$
\left. \frac{dCE_t}{dh} \right|_{h=1} = \frac{\rho \gamma (2 - \gamma)}{2} k^2_{t-1} - \frac{\rho \gamma^5 [\rho \gamma^3 \sigma^2 - 2(1 - \gamma)b]}{[2b + \rho \gamma^3 \sigma^2]^3} \sum_{\tau=1}^{\infty} \gamma^\tau m^2_{t+\tau+1}.
$$

The above equation implies that $\left. \frac{dCE_t}{dh} \right|_{h=1} < 0$ if

$$
2(1 - \gamma)b < \rho \gamma^3 \sigma^2
$$

and

$$
\sum_{\tau=1}^{\infty} \gamma^\tau m^2_{t+\tau+1} > \frac{(2 - \gamma)[2b + \rho \gamma^3 \sigma^2]^3}{2 \gamma^4 [\rho \gamma^3 \sigma^2 - 2(1 - \gamma)b]} \cdot k^2_{t-1}.
$$

It then follows from continuity that if the inequalities in (18-19) hold, there exists a $h_H \in (h_L, 1)$ such that $CE_t$ decreases in $h$ for all $h \in [h_H, 1]$. This proves that when (18-19) hold, the existing shareholders’ welfare is maximized at some $h \in [h_L, h_H]$. 

$\square$
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