Equilibrium Voluntary Disclosures under Mean-Variance Pricing with Multiple Assets

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Abstract

We develop a model of a firm's manager's disclosure decisions in a mean-variance pricing framework with multiple assets that yields several new predictions concerning both the ex-ante (pre-disclosure) and the ex-post (post disclosure) economic effects of the manager's adoption of an equilibrium disclosure policy.

1 Introduction

In this paper we study a model of a firm's manager's voluntary disclosure decisions in a mean-variance pricing framework. We consider the natural situation where the manager sometimes privately receives information pertinent to valuing her firm which she may credibly disclose to or withhold from investors. The manager's objective is share price maximization, and hence the manager discloses the information she receives only when disclosure yields a higher market price for her firm's shares than does nondisclosure. Covariances between the future cash flows of the manager's firm and those of other firms/assets affects the manager's firm's market price and hence her disclosure decision. The joint distribution of the disclosing firm's future cash flows, the future cash flows of other firms/assets, and whatever information the manager receives is multivariate normal. In this framework, the paper generates many new predictions derived from the study of the manager's equilibrium disclosure policy, i.e., the disclosure policy which, if investors expect the manager to adopt that policy, is the policy that she has a self-interested incentive to adopt.

Our new findings can be broadly described as belonging to one of two categories, "ex ante" (or pre-disclosure) and "ex post" (or post-disclosure). Our ex post results include a wide array of empirically salient predictions, including predictions about the effects of "own" firm nondisclosure on investors' perceptions of each of: uncertainty about "own" firm's future cash flows; the covariance between "own" firm's and "other" firms' future cash flows; and information transfers between "own" and "other" firms. Our ex ante results include a rich set of comparative statics concerning the effects of changing exogenous parameters of the model on both "own" firm's manager's equilibrium disclosure policy and "own" firm's cost of capital.

We now present and discuss specific examples of our new findings. We start by describing some new ex post results. One set of new ex post results involves how, when a firm's manager adopts an equilibrium disclosure policy, the manager's disclosure decision affects investors' perceptions of the covariance between its future cash flows and the future cash flows of other firms. We show that if the unconditional (i.e., before the time the manager could have received or disclosed any new information) covariance between the given firm's future cash flows and another firm's future cash flows is positive, nondisclosure by the given firm results in investors' perception of the covariance between the two firms' future cash flows becoming more positive, whereas if the unconditional covariance between the two firms' future cash flows is negative, nondisclosure by the given firm results in investors' perception of the covariance between the two firms' future cash flows becoming more negative. In contrast, if the manager discloses her private information, then investors' perception of the covariance between firms' future cash flows shrinks toward zero. These results accord with intuition, as can be seen by considering a limiting case: if the manager were to disclose more and more information to investors, and investors condition their perceptions of the covariances between firms' future cash flows on this ever expanding set of disclosed information, then investors' perceptions of the covariances between firms' future cash flows must shrink toward zero, since if investors condition on enough information, the firms' future cash flows become determined by the information set investors are conditioning on and hence become constant (and so have no covariance with anything).

Another set of new ex post findings involve how investors' perceptions of the uncertainty about a firm's future cash flows change upon their learning that the firm's manager is not going to make a disclosure during the period. We obtain the robust second moment finding that nondisclosure strictly increases investors' uncertainty about the manager's firm's future cash flows relative to their prior beliefs about the firm's future cash flows. We characterize this result as "robust" because we show that it holds both when "increases in investors' uncertainty" is measured in terms of changes in investors' beliefs about the variance of the firm's future cash flows and also when "increases in investors' uncertainty" is measured in terms of changes in investors' beliefs about the total risk of the firm's future cash flows, where "total risk" is defined as a combination of the variance of the firm's future cash flows and the sum of the covariances between its and all other firms' future cash flows. Under the latter (total risk) measure of increased uncertainty, our new finding that nondisclosure is uncertainty-increasing is only subject to the qualification that the initial (unconditional) total risk of the firm is positive. Under the former (variance) measure of increased uncertainty, our claim holds without qualification.

We consider this last finding as unexpected. After all, there is news in a firm's manager's decision not to disclose information during in a period, and the news causes investors to revise their beliefs about the distribution of the manager's firm's future cash flows. As we discuss further in the paper, one of the implications of the "law of total variance,"a general theorem in statistics,[1] is that any new information (e.g., news of the manager's nondisclosure) about any random variable (e.g., a firm's future cash flows) is expected to reduce, rather than increase, decision makers' (e.g., investors') uncertainty about that random variable. At first blush, our finding appears inconsistent with this theorem. We discuss in the body of the paper why our result is not inconsistent with this theorem. Here we note only that when investors learn that a manager is not going to make a disclosure during a period, they recognize that they must then view the distribution of the firm's future cash flows as a mixture of two distributions, one involving their initial priors (associated with the manager not having received information) and one involving a truncated distribution (associated with the manager having received and decided to withhold information). When the manager's disclosure policy is such that these two component distributions of

[1] See, e.g., Weiss [2005].

the mixture are sufficiently different from each other, then investors' learning of the manager's nondisclosure results in their perceiving the firm's future cash flows as being more risky than they did at the start of the period.

One consequence of this new second moment result is that, when combined with the existing literature's first moment result that a manager's nondisclosure conveys negative information to investors about the firm's expected future cash flows (Dye [1985] and Jung and Kwon [1988]), we see that nondisclosure is unalloyed bad news about "own" firm under mean-variance pricing, since nondisclosure increases investors' perceptions of the variance of its future cash flows at the same time it reduces their perceptions of the mean of its future cash flows. An immediate and further consequence of this two-dimensional effect of nondisclosure is that the manager is induced to disclose the information she receives more often than when investors are risk-neutral so as to diminish the price penalty risk-averse investors impose on the firm for the uncertainty its manager's nondisclosure generates.

A third new set of ex post findings involves information transfers that take place between "own" firm and other firms induced by "own" firm's nondisclosure. We show that when investors' perceptions of the unconditional covariance between the two firms' future cash flows is positive, nondisclosure by "own" firm reduces investors' perceptions of the other firm's expected future cash flows, whereas if the unconditional covariance between the two firms' future cash flows is negative, we show that nondisclosure by "own" firm increases investors' perceptions of the other firm's future expected cash flows. We also show that, at the same time these first moment information transfers occur, second moment information transfers also occur. More precisely, we show that as long as the unconditional covariance between the future cash flows of "own" firm and "other" firm is nonzero, then when investors observe the manager of "own" firm making no disclosure during a period, this will cause investors' perceptions of the variance of "other" firm's future cash flows to increase. Thus, we show that the variance-increasing effects of "own" firm's nondisclosure extend to other firms too.

Next, we present examples of some of our new ex ante results. On example of our new ex ante findings is that we show, under mean-variance pricing, that the manager always discloses her private information (when received) less often as investors' prior beliefs about the variance of the firm's future cash flows declines. Since a primary reason investors' prior beliefs about the variance of the firm's future cash flows declines is because of their receipt of information from other sources - e.g., analysts, newspapers, disclosures of other firms, etc., - this result can be recast as asserting that: information from other sources and the manger's voluntary disclosures are partial substitutes. While this is a seemingly natural and intuitive result, we show that it *never* holds under risk-neutral pricing within this disclosure framework where investors are uncertain of the manager's receipt of information.

Other new ex ante results include: under mean-variance, the manager discloses information more often, and the firm's cost of capital decreases, as either the probability she receives information increases or as the precision of the information she receives increases, and the firm's cost of capital increases with increases in investors' aggregate risk aversion, even though the manager discloses her information more often as investors' aggregate risk-aversion increases. We also show that increases in voluntary disclosure always reduces a firm's cost of capital, ceteris paribus, but that One of our new ex ante results is motivated by the finding of Botosan [1997], Botosan and Plumlee [2002], and others that there is an inverse relationship between disclosure and cost of capital. This documented relationship invites the question: if enhanced disclosure always lowers a firm's cost of capital, then what countervailing force exists that prevents managers of firms from "going to the limit" and disclosing all of the information they receive so as to minimize their firm's cost of capital? Our analysis answers this question. Consistent with the empirical literature, we begin by showing [2] that a manager can always lower her firm's cost of capital under mean-variance pricing by disclosing the information she receives more often. But, we follow up on that observation by noting that this result does not imply that the manager will seek to engage in full disclosure. Since a manager's disclosure policy is private information to the manager, she cannot commit herself to a disclosure policy; rather, it must be in her self-interest to adopt the disclosure policy she adopts. That is to say, the only disclosure policies investors can expect her to

[2] See Theorem 11 below.

adopt are equilibrium disclosure policies. We show that if the manager's goal is share price maximization, then no *equilibrium* disclosure policy consists of full disclosure. In other words, the "countervailing force" that prevents managers from engaging in full disclosure is that a full disclosure policy predicated on minimizing a firm's cost of capital is inconsistent with a disclosure policy that maximizes the firm's share price.

While the current literature has displayed some of these ex ante results under risk-neutral pricing, as far as we are aware, all these results are new under mean-variance pricing.

In summary, this paper's contributions to the literature include: a series of new ex post findings concerning the variance-increasing, covariance-increasing, and total risk-increasing effects of nondisclosure; several new results concerning the information transfers that take place between firms as a consequence of one of the firm's nondisclosure; and a series of new ex ante findings under mean-variance pricing involving the effects of changing various exogenous parameters of the model on both the firm's manager's equilibrium disclosure policy and the firm's cost of capital when evaluated at the manager's equilibrium disclosure policy.

This paper is part of a growing literature in accounting, economics, and finance that studies formal models of voluntary disclosure. Surveys of this literature include Dranove and Jin [2010], Dye [2001], Fishman and Hagerty [1998], Gertner [1998], Milgrom [2008], and Verrecchia [2001]. Within this literature, there are several papers that study disclosure problems where investors are uncertain of a firm's manager's receipt of information, as in Dye [1985] and Jung and Kwon [1988]. Among these are papers by Goto, Watanabe, and Xu [2009], Hughes and Pae [2004], [2014], and Pae [1999], [2002], [2005]. Clinch and Verrecchia [2013] also consider the effects of risk aversion on a firm's disclosure decisions within this framework; they arrive at results distinctly different from ours.[3] There are also other papers in the accounting literature that also examine disclosures with risk averse investors. For example, Cheynel [2015] considers voluntary disclosure in a large economy setting with risk averse investors, but

[3] E.g., they do not obtain an inverse relationship between disclosure and a firm's cost of capital, and they fail to find any effect of changes in exogenous parameters on the disclosing firm's cost of capital (see, e.g., their page 202).

her model assumes private information only pertains to diversifiable risks. Absent constraints emerging from under investment in cash generating projects, she does not characterize any voluntary disclosure effects on risk premiums.

The paper proceeds as follows. The next section, Section 2, contains the model setup. Section 3 contains the definition of an equilibrium disclosure strategy along with some other preparatory material. Section 4 contains some initial results followed by Section 5 which has additional empirical implications. Section 6 discusses information transfers to other firms. Section 7 contains conclusions and suggestions for future research. This is followed by the Appendix, which contains statements of additional supporting lemmas, as well as several of the proofs of the theorems, lemmas, and corollaries stated in either the text or the Appendix itself.[4]

2 Model Setup

Our goal is to set up and study the most parsimonious model possible of voluntary disclosure by the manager of a single firm when that firm is subject to mean-variance pricing, that is, when the market price of the firm is determined by investors' beliefs about each of: the mean of the firm's future cash flows, the variance of its future cash flows, and the covariances between its future cash flows and the future cash flows of other firms.

We refer to the firm under scrutiny as firm i, and we suppose there are $n-1$ firms other than firm i, indexed by $j \in \{1, 2, ..., n\} \backslash \{i\}$. The model begins with the manager of firm i either receiving or not receiving some private information pertinent to valuing her firm, which she may disclose to or withhold from investors. After the manager has made her disclosure decision, a securities market opens on which all n firms' shares can be traded. On this securities market, investors exchange rights to the future cash flows of the n firms. Firm $i's$ (resp., $j's$) future cash flows are the realization x_i (resp., x_j) of the random variable $\tilde{x}_i$ (resp., $\tilde{x}_j$). The joint distribution of all n firms' future cash flows $(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$ is multivariate normal, with $cov(\tilde{x}_i, \tilde{x}_j)$ denoting the (unconditional) prior covariance between $\tilde{x}_i$ and $\tilde{x}_j$ and $var(\tilde{x}_j)$ denoting the

[4] To keep the paper to a reasonable length, we only sketch the proofs of some results. Complete proofs of all results are available on request.

(unconditional) prior variance of $\tilde{x}_j$. We sometimes write $var(\tilde{x}_i)$ as $\sigma^2_{x_i}$ or as $\frac{1}{\tau}$, where τ is the prior precision of $\tilde{x}_i$. The prior (unconditional) mean of $\tilde{x}_i$ is denoted by m_i.

The manager of firm i receives the private value-relevant information about her firm with probability $p \in (0,1)$, as in Dye [1985] or Jung and Kwon [1988]. This information is an imperfect, but unbiased, normally distributed estimate of the realization of $\tilde{x}_i$. Specifically, it is the realization v_i of the random variable $\tilde{v}_i$ where $\tilde{v}_i|_{x_i} = x_i + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i$ is a normally distributed zero mean noise term independent of all other variables in the model with variance $\sigma^2_{\varepsilon} < \infty$. We write this as $\tilde{\varepsilon}_i\tilde{\ }N(0,r)$ where $r \equiv \frac{1}{\sigma^2_{\varepsilon}}$ is the precision of the estimate. We denote the ex ante variance of the estimate $\tilde{v}_i$ by σ^2_v. Since $\tilde{\varepsilon}_i$ is independent of $\tilde{x}_i$, it follows that $\sigma^2_v = \sigma^2_{x_i} + \sigma^2_{\varepsilon}$.

When the manager of firm i receives the estimate v_i, either the manager discloses the estimate or discloses nothing. As is typical in this literature, we assume: all disclosures are confined to be truthful; the manager cannot partially disclose v_i; the manager cannot credibly disclose that she did not receive information; the manager makes no disclosure when she does not receive v_i (so she has a disclosure decision to make only when she receives information). We posit that the manager's goal is to maximize the market value of her firm, so in the event the manager receives v_i, she discloses v_i to investors only if disclosure produces a higher market price for the firm than does no disclosure.[5]

All investors are posited to be homogeneously informed about the distribution of $\tilde{x}_i$ at all points in time, so there is no need to index information sets by individual investors. We let Ω_i denote the public information set associated with the manager of firm $i's$ disclosure. Ω_i consists of v_i when the manager discloses v_i, and consists of the fact of the manager's nondisclosure when the manager makes no disclosure. That is, either $\Omega_i = \{v_i\}$ or $\Omega_i = \{\text{no disclosure}\}$.

We let $E[\tilde{x}_i|\Omega_i]$, $var(\tilde{x}_i|\Omega_i)$, and $cov(\tilde{x}_i, \tilde{x}_j|\Omega_i)$ respectively denote investors' perceptions of the mean and variance of $\tilde{x}_i$ and the covariance between $\tilde{x}_i$ and

[5] As will become evident below, the manager's risk preferences are irrelevant to the manager's disclosure decision, and so we do not specify her risk preferences in what follows. Also, to be consistent with both the exiting literature and the previously assumed objective function for the manager, we ignore potential inside-trading issues and presume the manager does not trade on the securities market.

$\tilde{x}_j$ conditional on information set Ω_i. Also, we suppose that there are constants γ and η such that the price of firm i takes the following form:

$$P_i(\Omega_i) = E[\tilde{x}_i|\Omega_i] - \gamma(var(\tilde{x}_i|\Omega_i) + \eta \times \sum_{j \neq i} cov(\tilde{x}_i, \tilde{x}_j|\Omega_i)). \tag{1}$$

Thus, the price of firm i depends on investors' perceptions of the mean and, if $\gamma > 0$, variance of firm $i's$ future cash flows and also, if $\eta \neq 0$, the covariance between firm $i's$ and firm $j's$ future cash flows for every $j \neq i$, conditional on information set Ω_i. When $\gamma >$ (resp., $=$)0, we say there is mean-variance (resp., risk-neutral) pricing. We ignore discounting in what follows.

Throughout the following, we take mean-variance/risk-neutral pricing, i.e., equation (1), as given, and do not derive it. This is in conformity with the pricing assumptions of other models of voluntary disclosure in the literature, for example, in Verrecchia [1983], [1990], who also takes mean-variance pricing, albeit in a single firm context, as given and does not derive it. An additional advantage of postulating mean-variance pricing coupled with normally distributed future cash flows and information is that, taken together, these specifications ensure that the assumptions underlying all of our findings can be described simply, in terms of only the first and second moments of the distributions of the model's random variables coupled with a specification of investors' aggregate risk-aversion. We provide some additional commentary on this pricing equation in the accompanying footnote.[6]

[6] When distributions of all firms' future cash flows are jointly normally distributed and all investors have constant absolute risk averse (CARA) preferences, it is well known that the equilibrium price of each traded security is exactly as given in (1) above with η set to 1 and γ set equal to the aggregate risk-aversion parameter of investors. When the manager of firm i makes no disclosure, investors cannot tell whether the reason the manager did not make a disclosure was that he did not receive information or withheld information, in which case investors perceive the firm's future cash flows to be a mixture of two distributions, one the prior distribution of the firm's future cash flows applicable when the manager did not receive information and a second truncated distribution applicable when the manager received and withheld information. Mixtures of such distributions are not normally distributed, and so pricing equations of the form (1) must be considered as only approximations to equilibrium prices.

We justify our adoption of this pricing approximation by: first, noting that recourse to the approximation is common in the disclosure literature (see, e.g., the Verrecchia articles cited in the text); second, noting that the approximation has been found by Hanson and Ladd [1991] to be a good approximation to equilibrium prices in other settings where random variables being priced have truncated normal distributions; third, recalling Friedman's [1953] oft-cited observation that an assessment of modeling assumptions rests with the quality of the insights that follow from results derived from those assumptions; and related, fourth, appealing to

3 The Definition of a Manager's Equilibrium Disclosure Policy

We start the analysis by developing an explicit expression for the price of firm i when its manager discloses the estimate $\tilde{v}_i = v_i$. That requires calculating the mean and variance of $\tilde{x}_i$ conditional on v_i, as well as the covariance between $\tilde{x}_i$ and $\tilde{x}_j$ for each $j \neq i$, also conditional on v_i. It is well known that for normal distributions conditional means are linear. A useful way of writing the linear conditional mean $E[\tilde{x}_i|v_i]$ is:

$$E[\tilde{x}_i|v_i] = m_i + \frac{\sigma^2_{x_i}}{\sigma_v} \times \frac{v_i - m_i}{\sigma_v}.$$

It is also well known that the variance of firm i's future cash flows conditional on the manager's disclosure of the estimate v_i is independent of v_i and given by:

$$var(\tilde{x}_i|v_i) = \alpha \times var(\tilde{x}_i), \text{ where } \alpha \equiv \frac{\sigma^2_\varepsilon}{\sigma^2_v} < 1. \tag{2}$$

Thus, the manager's disclosure of v_i shrinks investors' perceptions of the variance of $\tilde{x}_i$ by fraction α. It is easy to confirm that the same is true for covariances: upon observing the manager's disclosure of the estimate v_i, investors' perceptions of covariance between $\tilde{x}_i$ and $\tilde{x}_j$ for any $j \neq i$, also shrinks by the same constant fraction α :[7]

$$cov(\tilde{x}_i, \tilde{x}_j|v_i) = \alpha \times cov(\tilde{x}_i, \tilde{x}_j). \tag{3}$$

Next we define firm $i's$ *total unconditional risk* R_i as:

$$R_i \equiv var(\tilde{x}_i) + \eta \times \sum_{j \neq 1} cov(\tilde{x}_i, \tilde{x}_j). \tag{4}$$

Smolin [2007], who observed that analyses based on approxmiations may lead to more insight than those achievable from analyses that avoid the use of approximations.

[7] This is simply a specific application of the well known result (see, e.g., Anderson [2003]) that if the two random vectors $\tilde{\mathbf{X}}_1$ and $\tilde{\mathbf{X}}_2$ are jointly multivariate normal, where the variance-covariance matrix of $\tilde{\mathbf{X}}_1$ (resp., $\tilde{\mathbf{X}}_2$) is denoted by $\mathbf{\Sigma}_{11}$ (resp., $\mathbf{\Sigma}_{22}$) and $\mathbf{\Sigma}_{12}$ denotes the covariance matrix between $\tilde{\mathbf{X}}_1$ and $\tilde{\mathbf{X}}_2$, then the variance-covariance matrix of $\tilde{\mathbf{X}}_1$ given $\tilde{\mathbf{X}}_2 = \mathbf{X}_2$ is $\mathbf{\Sigma}_{11} - \mathbf{\Sigma}_{12}\mathbf{\Sigma}_{22}^{-1}\mathbf{\Sigma}_{21}$. Applying this result with $\tilde{\mathbf{X}}_1 = \begin{bmatrix} \tilde{x}_i \\ \tilde{x}_j \end{bmatrix}$ and $\tilde{\mathbf{X}}_2 = \tilde{v}_i$, so $\mathbf{\Sigma}_{11} = \begin{bmatrix} var(\tilde{x}_i) & cov(\tilde{x}_i, \tilde{x}_j) \\ cov(\tilde{x}_i, \tilde{x}_j) & var(\tilde{x}_j) \end{bmatrix}$, $\mathbf{\Sigma}_{12} = \begin{bmatrix} cov(\tilde{x}_i, \tilde{v}_i) \\ cov(\tilde{x}_j, \tilde{v}_i) \end{bmatrix}$, and $\mathbf{\Sigma}_{22} = \sigma^2_v$, immediately yields the assertion in (3).

We denote firm $i's$ total risk conditional on the manager's disclosure of v_i by $R_i^d(v_i)$. It follows from the above discussion that firm $i's$ total risk also shrinks by fraction α upon the manager's disclosure of v_i :

$$R_i^d(v_i) = \alpha \times R_i. \tag{5}$$

Thus, in accordance with the mean-variance pricing equation (1) above, the price of firm i when the manager discloses v_i is given by:

$$P_i^d(v_i) = m_i + \frac{\sigma_{x_i}^2}{\sigma_v} \times \frac{v_i - m_i}{\sigma_v} - \gamma \times \alpha \times R_i. \tag{6}$$

Among other things, this shows that $P_i^d(v_i)$ is strictly increasing. Since the price of the firm with no disclosure is constant, we conclude that if the manager of firm i discloses some estimate v_i she receives, then the manager will also disclose any estimate v_i' she receives for which $v_i' > v_i$. Thus, the manager's disclosure policy is described by some cutoff v^c. When investors expect the manager to use the cutoff v^c in deciding whether to disclose the information she receives, we refine the notation for the manager's no disclosure price by writing it as $P_i^{nd}(v^c)$.

In analogy with the definition of firm $i's$ total unconditional risk R_i in (4) above, we define the total risk of firm i conditional on nondisclosure and use of the cutoff v^c to be:

$$R_i^{nd}(v^c) \equiv var(\tilde{x}_i|nd, v^c) + \eta \times \sum_{j \neq i} cov(\tilde{x}_i, \tilde{x}_j|nd, v^c).$$

Here, the notation $var(\tilde{x}_i|nd, v^c)$ (resp., the notation $cov(\tilde{x}_i, \tilde{x}_j|nd, v^c)$) for the conditional variance of $\tilde{x}_i$ (resp., conditional covariance between $\tilde{x}_i$ and $\tilde{x}_j$) represents the variance of $\tilde{x}_i$ (resp., covariance between $\tilde{x}_i$ and $\tilde{x}_j$), given the manager makes no disclosure and investors believe she uses the disclosure strategy defined by the cutoff v^c. The expression for the conditional mean of $\tilde{x}_i$, $E[\tilde{x}_i|nd, v^c]$, has the obvious analogous interpretation.

Since mean-variance pricing is also assumed to apply when the manager makes no disclosure, we have:

$$P_i^{nd}(v^c) = E[\tilde{x}_i|nd, v^c] - \gamma \times R_i^{nd}(v^c). \tag{7}$$

We say that the manager adopts an equilibrium disclosure policy when the disclosure policy she uses is defined by an equilibrium cutoff, defined as:

Definition 1 *A cutoff v^c is an equilibrium cutoff if it satisfies the equation:*

$$P_i^d(v^c) = P_i^{nd}(v^c). \tag{8}$$

Under an equilibrium cutoff, the manager discloses information exactly when investors expected her to, since a manager who knows $\tilde{v}_i = v_i$ will get a higher price by disclosing v_i if and only if v_i exceeds v^c, that is if and only if the price of the firm with the disclosure of v_i is higher than the price of the firm with no disclosure.[8]

In the following, we sometimes describe the manager's cutoff disclosure policy v^c - equilibrium or not - in terms of its "standardized" value, defined as $z^c \equiv \frac{v^c - m_i}{\sigma_v}$. Slightly abusing notation, we use the standardized cutoff z^c interchangeably with v^c. For example, we consider z^c to be an equilibrium cutoff if it satisfies the equation $P_i^d(z^c) = P_i^{nd}(z^c)$.

3.1 Initial Results

In this section, we start by obtaining explicit expressions for $var(\tilde{x}_i|nd, v^c)$, $cov(\tilde{x}_i, \tilde{x}_j|nd, v^c)$, $R_i^{nd}(v^c)$, and $E[\tilde{x}_i|nd, v^c]$ for a fixed but arbitrary cutoff v^c (or equivalently z^c). Then, we use those expressions both to characterize the manager's equilibrium disclosure policy and to derive some empirical predictions that follow from the manager having adopted an equilibrium disclosure policy.

We introduce the following additional notation. The symbols ϕ and Φ respectively refer to the density and cumulative distribution function (cdf) of the standard normal random variable. One interpretation of the function $f(z^c)$ defined by:

$$f(z^c) \equiv \frac{p\phi(z^c)}{1 - p + p\Phi(z^c)}, \tag{9}$$

is that it is the probability (density) describing investors' assessment of the likelihood the manager received the specific estimate $\tilde{v}_i = v_i$, where v_i satisfies

[8] When in the knife edge cases where the manager is indifferent between disclosing v_i and making no disclosure, it makes no difference in the following which disclosure decision the manager is presumed to adopt. For the sake of specificity only, we adopt the convention that the manager discloses v_i in these knife edge cases.

$z^c = \frac{v_i - m_i}{\sigma_v}$, given that she makes no disclosure and investors believe she uses the cutoff z^c in deciding whether to disclose her information. (We give another interpretation of the function $f(z^c)$ later in this section.)

In the next lemma, we obtain explicit expressions for investors' perceptions of: the variance of firm i's future cash flows, the covariance between firm $i's$ and firm $j's$ future cash flows, and the total risk of firm i, all conditional on no disclosure by the manager of firm i when investors believe the manager uses a disclosure policy described by the cutoff v^c (or its standardized value z^c).

Lemma 2 *Given a fixed, but arbitrary, cutoff v^c along with its standardized value $z^c = \frac{v^c - m_i}{\sigma_v}$, investors' perceptions of variance of firm $i's$ firm's future cash flows, the covariance between its future cash flows and firm $j's$ future cash flows, and the total risk associated with firm i, all conditional on no disclosure, are respectively given by:*

$$var(\tilde{x}_i|nd, v^c) = \theta(z^c) \times var(\tilde{x}_i); \tag{10}$$

$$cov(\tilde{x}_i, \tilde{x}_j|nd, v^c) = \theta(z^c) \times cov(\tilde{x}_i, \tilde{x}_j), \tag{11}$$

$$R_i^{nd}(z^c) = \theta(z^c) \times R_i; \tag{12}$$

where:

$$\theta(z^c) \equiv 1 - \frac{\sigma_{x_i}^2}{\sigma_v^2} \times f(z^c) \times (z^c + f(z^c)). \tag{13}$$

According to the lemma, the same function $\theta(z^c)$ in (13) scales each of: investors' perceptions of the ex ante variance of $\tilde{x}_i$ in (10), investors' perceptions of the ex ante covariance between $\tilde{x}_i$ and $\tilde{x}_j$ in (11), and investors' perceptions of firm $i's$ total risk in (12). It follows that nondisclosure has the same proportionate effect on investors' perceptions of each of these three risk measures. This scaling of these risk measures by $\theta(z^c)$ when the manager makes no disclosure is similar to the scaling of these risk measures by the constant α in the previous section when the manager discloses her private information: recall (2), (3), and (5).

But, there is a major difference between these two scalings of the risk measures: α is obviously a constant, whereas $\theta(z^c)$ is a function of the manager's cutoff, and so the effect of scaling these risk measures by $\theta(z^c)$ will depend on what the value of the equilibrium cutoff z^c is. Moreover, α is always strictly less than one whereas $\theta(z^c)$ can be bigger or smaller than one. Since $f(z^c)$ is always positive, inspecting expression (13) reveals that $\theta(z^c) > 1$ if and only if

$$z^c + f(z^c) < 0. \tag{14}$$

Combining this observation with (10), we see that investors' perceptions of the variance of the firm i's future cash flows given no disclosure are higher than their initial (unconditional) perceptions of this variance if and only if inequality (14) holds. That is, satisfaction of inequality (14) is necessary and sufficient for:

$$var(\tilde{x}_i|nd, v^c) > var(\tilde{x}_i).$$

When this last inequality holds, we say that "nondisclosure is variance-increasing."

Similarly, it follows from inspection of expression (11) that investors' perceptions of the covariance between firm i's and firm $j's$ future cash flows given no disclosure are higher (resp., lower) in absolute value than their initial (unconditional) perceptions of this covariance if (14) holds (resp., is reversed). That is, when inequality (14) holds, we have:

$$cov(\tilde{x}_i, \tilde{x}_j|nd, v^c) \quad > \quad cov(\tilde{x}_i, \tilde{x}_j) \text{ if } cov(\tilde{x}_i, \tilde{x}_j) > 0 \text{ and} \tag{15}$$

$$cov(\tilde{x}_i, \tilde{x}_j|nd, v^c) \quad < \quad cov(\tilde{x}_i, \tilde{x}_j) \text{ if } cov(\tilde{x}_i, \tilde{x}_j) < 0. \tag{16}$$

If this last pair of inequalities holds, we say (somewhat imprecisely) that "nondisclosure is covariance-increasing."

We can similarly conclude that nondisclosure is "total-risk increasing" if $R_i^{nd}(z^c) > (resp., <) R_i$, and, when $R_i > 0$, this happens when inequality (14) is satisfied (resp., reversed).

Analogous to the above, it is clear that nondisclosure is variance-decreasing, covariance-decreasing, and total risk-decreasing when $\theta(z^c) < 1$, and that this last inequality holds when inequality (14) is reversed.

To make predictions about whether we expect nondisclosure to be variance-increasing, covariance-increasing, and total risk-increasing or, instead variance-decreasing, covariance-decreasing, and total risk-decreasing, we must determine whether $\theta(\cdot)$ is above or below one when evaluated at the *equilibrium* cutoff.

We do this in two steps: first, we determine for which regions of cutoffs z^c the function $\theta(z^c)$ above or below one. Second, we identify in which of these two regions the equilibrium cutoff resides. We can completely resolve the first issue by defining the critical cutoff z^{*c} to be the solution to the equation

$$z^{*c} + f(z^{*c}) = 0. \tag{17}$$

It will become apparent in the following that z^{*c} is the manager's unique equilibrium disclosure cutoff when risk-neutral pricing prevails. We show in Lemma 5 in the Appendix that this critical cutoff z^{*c} exists, is unique, is negative, and most importantly for the following:

$$\text{if } z^c < (resp., >) z^{*c}, \text{ then } z^c + f(z^c) < (resp., >) 0. \tag{18}$$

From this discussion, it follows immediately that:

Lemma 3 *Nondisclosure is variance-increasing, covariance-increasing, and total risk-increasing (resp., variance-decreasing, covariance-decreasing, and total risk-decreasing) if and only if the standardized cutoff z^c defining the manager's disclosure policy satisfies $z^c < z^{*c}$ (resp., $z^c > z^{*c}$).*

To obtain some insight about how the location of the cutoff z^c (above or below z^{*c}) affects investors' perceptions of the second moments of the firm's future cash flows given no disclosure, we begin by recalling from the Introduction that when the manager makes no disclosure, investors perceive the distribution of the firm's future cash flows as a mixture of two distributions, one based on the manager's estimate being drawn from a truncated-from-above-by-the-cutoff v^c (and hence skewed left) normal distribution, and one based on the manager not receiving any information and hence being a symmetric normal distribution corresponding to investors' priors.[9] Investors must also assess the relative probability of these two distributions occurring in order to assess the value-implications of this mixture. Since the standardized cutoff z^c equals $\frac{v^c - m_i}{\sigma_v}$, and since z^{*c} is negative, it follows according to the lemma that investors'

[9] The importance of this discussion concerning the distribution of the firm's value conditional on no disclosure being a mixture of a skewed left and a symmetric distribution will be clear once we present and discuss Lemma 3 below.

perceptions of the variance of the firm's future cash flows conditional on no disclosure is above their prior perceptions of this variance if and only if the cutoff v^c defining the truncated component of this mixture is "sufficiently far below" investors' assessment of the prior mean of the firm's future cash flows, that is, if and only if the standardized cutoff $\frac{v^c - m_i}{\sigma_v} = z^c$ is below z^{*c}.

Lemma 3 also establishes that without specifying exactly where the manager's cutoff resides, the mere fact that the manager adopts a cutoff disclosure policy does not by itself determine whether investors have either a higher or a lower assessment of the variance of $\tilde{x}_i$ (resp., the covariance between $\tilde{x}_i$ and $\tilde{x}_j$) given no disclosure by the manager than their initial prior (unconditional) perceptions of this variance (resp., covariance). What matters is whether the cutoff is below or above the critical cutoff value z^{*c}. To give some intuition for the "variance" portion of this result, we note that, obviously, the truncated-from-above component of the mixture distribution becomes more and more concentrated in its left tail as z^c becomes more negative. This left-tailed "concentratedness" of the truncated component makes that component less and less similar to the unbiased symmetric (prior distribution) component of the mixture as z^c becomes more negative. So, as the cutoff z^c gets far enough "out" in the left tail of $\tilde{v}_i^\prime s$ distribution, it is natural that the variance of $\tilde{x}_i$ conditional on no disclosure exceeds the (prior) unconditional variance of $\tilde{x}_i$.

We next address the issue of whether the equilibrium cutoff is bigger or smaller than z^{*c}. This requires that we more fully characterize the equilibrium cutoff, which among other things requires that we learn more about the first-moment effects of nondisclosure. The next lemma tells us about this.

Lemma 4 *Given a fixed, but arbitrary, value for the cutoff v^c along with its standardized value $z^c = \frac{v^c - m_i}{\sigma_v}$, investors' perceptions of the mean of firm i's firm's future cash flows conditional on no disclosure are given by:*

$$E[\tilde{x}_i | nd, v^c] = m_i - \frac{\sigma_{x_i}^2}{\sigma_v} \times f(z^c). \tag{19}$$

(19) confirms in the present context the well-known (Dye [1985], Jung and Kwon [1988]) result that investors regard no disclosure as negative news regarding the firm's expected future cash flows, since it shifts investors' expectations

about $\tilde{x}_i$ to be below their prior perceptions $E[\tilde{x}_i] = m_i$. (19) also gives us another interpretation of the function $f(z^c)$. Notice that if the manager's estimate is perfect ($r = \infty$) and investors' initial priors on firm i's future cash flows are standard normal, then $\sigma_v = \sigma_{x_i} = 1$ and $m_i = 0$. In this special case, (19) simplifies to $-f(z^c)$. Thus, $-f(z^c)$ is investors' assessment of the expected value of firm i's future cash flows given no disclosure and use of the standardized cutoff z^c in this special case. From (19), we further see that the calculation of investors' perception of the mean of firm $i's$ future cash flows conditional on no disclosure in general, i.e., for whatever normally distributed priors investors have about the distribution of firm $i's$ future cash flows and for whatever precision of the estimate firm $i's$ manager sometimes receives, can be obtained from a simple transformation of investors' perceptions of the firm's expected future cash flows conditional on no disclosure in this special case.

Now, we are in a position to say more about the equilibrium value of the cutoff. From Lemmas 2 and 4, we see that, for a fixed cutoff v^c and its standardized value z^c, the price of firm i conditional on no disclosure is given by:

$$P_i^{nd}(v^c) = m_i - \frac{\sigma_{x_i}^2}{\sigma_v} \times f(z^c) - \gamma \times \theta(z^c) \times R_i. \tag{20}$$

Recall that we previously computed the price of firm i when its manager disclosed the estimate v_i in (6) above. Putting those two prices together, we see that the equation determining the equilibrium cutoff, $P_i^d(v^c) = P_i^{nd}(v^c)$, can be written explicitly, expressed in terms of the standardized cutoff $z^c = \frac{v^c - m_i}{\sigma_v}$, as:

$$m_i + \frac{\sigma_{x_i}^2}{\sigma_v} \times z^c - \gamma \times \alpha \times R_i = m_i - \frac{\sigma_{x_i}^2}{\sigma_v} \times f(z^c) - \gamma \times \theta(z^c) \times R_i. \tag{21}$$

Deleting m_i from both sides of this last equation, we see immediately that the standardized value of the equilibrium cutoff is independent of the prior mean m_i of firm $i's$ future cash flows $\tilde{x}_i$. When γ and R_i are both positive, a bit more algebraic manipulation yields the alternative expression of this equation appearing in equation (22) below. The study of this equation yields several other useful results, which are reported in the next lemma.

Lemma 5 *The equilibrium equation (21):*

(a) for all $\gamma \geq 0$ and all R_i, always has a solution;

(b) for all $\gamma \geq 0$ and all R_i, has every solution finite;

(c) for all $\gamma > 0$ and $R_i > 0$, can be rearranged to:

$$(z^c + f(z^c))(\frac{1}{\gamma \frac{R_i}{\sigma_v}} - f(z^c)) = -1; \tag{22}$$

(d) for all $\gamma > 0$ and $R_i > 0$, has every solution z^c satisfy $z^c < z^{*c}$;

(e) for all $\gamma \geq 0$ and $R_i \geq 0$, has a unique solution;

(f) when $\gamma = 0$ or $R_i = 0$, has unique solution $z^c = z^{*c}$.

There are three conclusions of this lemma that we wish to emphasize. First, part (b) might be considered to be a technical result, but it implies the important substantive conclusion that full disclosure ($z^c = -\infty$) is not an equilibrium. This conclusion is easy to understand: if investors believed that the manager engages in full disclosure, then the price they would attach to the firm given no disclosure would be the (finite) price $P^{nd} = E[\tilde{x}_i] - \gamma R_i$, since investors would infer that the only time the manager made no disclosure was when she received no information. But, as we displayed in (6) above, the price of the firm were the manager to v_i is linear in v_i and hence there are very low realizations of the manager's estimate v_i for which $P_i^d(v_i)$ is below P^{nd}. A manager interested in maximizing her firms' share price would not disclose such v_i, which is inconsistent with investors' beliefs that the manager engages in full disclsoure. Second, parts (d) and (e) show that when $\gamma > 0$ and $R_i > 0$, the equilibrium cutoff z^c is unique and is strictly below the critical threshold z^{*c}. Third, parts (e) and (f) show that when either of γ or R_i is zero, the (again unique) equilibrium cutoff is the critical cutoff z^{*c}. These last two findings lead to several testable implications, the first of which is described in the following theorem.

Theorem 6 *When γ and R_i are both positive and the manager of firm i adopts an equilibrium disclosure policy, observing the manager's nondisclosure during a period strictly increases investors' perceptions of each of: (a) the variance of $\tilde{x}_i$; (b) the absolute value of the covariance between firm i's and firm j's future cash flows, for any firm $j \neq i$; and (c) the total risk of firm i.*

Part (a) of this theorem might be considered unintuitive for two reasons. First, there is information conveyed to investors by the manager's nondisclosure, and this information would seem to reduce investors' uncertainty about the firm's future cash flows (relative to investors' uncertainty about those cash flows prior to learning of the manager's nondisclosure). Second and related, as we mentioned in the Introduction, there is a very general theorem in statistics that, at first blush, seems to run counter to this finding. That theorem, a consequence of the so-called "law of total variance,"[10] asserts that for any unknown random variable with finite variance (such as the firm's future cash flows), accumulating information about the random variable (such as learning that the manager is not going to make a disclosure) must reduce the expected value of the variance of the random variable or, put more casually, information always reduces uncertainty in expectation.[11] It turns out that this finding of part (a) does not contradict this theorem from statistics for reasons we explain in the accompanying footnote.[12]

Also, following up on some of remarks we made concerning Lemma 3 above, we see that part (a) is not a statistical result that holds for all cutoff disclosure policies; it is a result that is guaranteed to hold only for equilibrium cutoff disclosure policies.

The conclusions of Theorem 6 regarding the effects of the manager of firm $i's$ *non*disclosure on investors' second moment perceptions of firm $i's$ future cash flows stand in marked contrast to the conclusions regarding the effects of the manager's disclosure of her estimate on investors' perceptions of these same second moments of the distribution of firm $i's$ future cash flows, as described above in (2), (3), and (5). Since the "shrinkage factor" α applicable there is less than one, the latter conclusions can be cast as asserting:

Theorem 7 *Observing the manager's disclosure of v_i for any $v_i \in \mathbb{R}$ during a period strictly decreases investors' perceptions of each of: (a) the variance of $\tilde{x}_i$;*

[10] See, e.g., Weiss [2005].

[11] Formally, suppose the random variable is $\tilde{x}_i$ which realizes its value at some future time $T > 0$, when today is time 0. Let Ω_t and $var(\tilde{x}_i|\Omega_t)$ respectively denote the information known about this random variable and the variance of the random variable as of time $t < T$. Assuming $E[\tilde{x}_i^2|\Omega_0]$ is finite and $\Omega_{t'} \subset \Omega_t$ for all $t' < t \leq T$ (i.e., information accumulates over time), then the theorem is that the inequality $var(\tilde{x}_i|\Omega_{t'}) \geq E[var(\tilde{x}_i|\tilde{\Omega}_t)|\Omega_{t'}]$ always holds. The preceding result follows directly from the law of total variance.

[12] There is no contradiction to the general theorem in statistics that with the passage of time the expected value of the variance of any random variable conditional on the information available about the random variable always declines in expectation because Theorem 6 is predicated on the realization of the particular event that the firm did not make a disclosure, and hence it makes no claim about *the expected value* of the conditional variance.

(b) the absolute value of the covariance between firm $i's$ and firm $j's$ future cash flows, for any firm $j \neq i$; and (c) when R_i is positive, the total risk of firm i.

Juxtaposing these last two theorems, we see that observing the manager's disclosure or (equilibrium) nondisclosure causes investors to move their perceptions of all three risk measures (variances, covariances, total risk) in opposite directions from their prior beliefs.

We next note that the risk-increasing effects of nondisclosure depend strongly on mean-variance pricing, that is, on $\gamma > 0$. If risk-neutral pricing prevails, i.e., $\gamma = 0$, then - based on Lemma 3 and Lemma 5(e) - we can conclude that investors' perceptions of: (a) the variance of firm $i's$ future cash flows; (b) the absolute value of the covariance between firm $i's$ and firm $j's$ future cash flows, for any firm $j \neq i$; and (c) the total risk of investing in firm i, all conditional on no disclosure, are exactly the same as their prior perceptions of, respectively, this variance, covariance, and total risk. Thus, among all $\gamma \geq 0$, we have:

Corollary 8 *When $R_i > 0$, mean-variance pricing, i.e., $\gamma > 0$, is both necessary and sufficient for the manager's equilibrium no disclosure to be variance-increasing, covariance-increasing, and total risk-increasing.*

Next, we investigate in more detail factors affecting the "jump" in investors' uncertainty about firm $i's$ future cash flows brought about by their observing the manager's nondisclosure, as described in Theorem 6. To aid in that investigation, we introduce the notion of firm i's "jump variance induced by nondisclosure:"

$$\Delta var(z^c) \equiv var(\tilde{x}_i|nd, v^c) - var(\tilde{x}_i).$$

From Theorem 6(a), we know this jump variance is always positive in equilibrium under mean-variance pricing.

Theorem 9 *If $R_i > 0$ and $\gamma \times \frac{\sigma^2_{x_i}}{\sigma_v} > 1$, then firm $i's$ jump variance induced by nondisclosure, $\Delta var(z^c)$, is strictly increasing in the cutoff z^c when evaluated at the equilibrium cutoff.*

This theorem can be rephrased in terms of the consequences of the manager ex ante choosing to disclose more, i.e., choosing a smaller value for the disclosure cutoff z^c. In this rephrasing, the manager's ex ante choosing to disclosure more mitigates the jump in uncertainty attending investors' learning of the manager's nondisclosure, because additional ex ante disclosure (i.e., a smaller cutoff z^c) reduces the jump variance induced by nondisclosure.

Theorem 9 is predicated on the manager's disclosure policy being evaluated at the equilibrium cutoff. It is possible to show (the details are available on request) that there are regions of (nonequilibrium) values for the cutoff z^c for which the the conclusion of the theorem does not hold, i.e., for which $\Delta var(z^c)$ is strictly decreasing in z^c. This observation underscores that the empirical implications of voluntary disclosure policies here (and often elsewhere too) are confined to their evaluation only at equilibrium policies.

Since investors' aggregate risk-aversion, i.e., $\gamma > 0$, is responsible for firm $i's$ jump variance being positive, in the next corollary, we examine how the manager's equilibrium disclosure policy varies with γ. (Here, we write $z^c(\gamma)$ to reflect how the standardized equilibrium cutoff depends on γ; later, when we consider how the standardized equilibrium cutoff depends on other parameters, we do the same with those other parameters.)

Corollary 10 *If $\gamma > 0$ and $R_i > 0$ and the manager adopts an equilibrium disclosure policy, then:*

(a) the manager discloses the information she receives more often as the aggregate risk aversion parameter γ increases, i.e., $\frac{dz^c(\gamma)}{d\gamma} < 0$, and

(b) the jump variance, $\Delta var(z^c(\gamma))$, strictly (locally) decreases in γ when $\gamma \times \frac{\sigma_{x_i}^2}{\sigma_v} > 1$.

Part (a) is intuitive. γ increases only if investors in the aggregate become more risk-averse. If investors become more risk-averse, they will penalize the firm more for nondisclosure because, as we noted in Theorem 6, nondisclosure increases investors' uncertainty about the distribution of the firm's future cash flows. The manager can offset that increased uncertainty by disclosing the information she receives more frequently. The conclusion of part (b) is natural in view of part (a) and Theorem 9. Part (a) asserts that the manager optimally will react to an increase in the aggregate risk aversion parameter by disclosing the information she receives more often, and Theorem 9 asserts that (at least when γ is sufficiently large), the jump variance decreases as the nondisclosure region decreases. Combining these two results, it follows that the jump variance must decline as the aggregate risk-aversion parameter increases.

In the final results of this section, we examine the difference between the expected price the firm trades at and the expected value of the firm's future cash flows investors ultimately are entitled to collect. We refer to this difference, as does much of the literature, as the firm's "cost of capital."[13] To calculate the firm's cost of capital, we note first that, calculated ex ante, the probability of no disclosure by the firm is $1 - p + p\Phi(z^c)$ when the manager uses cutoff $z^c = \frac{v^c - m_i}{\sigma_v}$. The discount from expected value conditional on no disclosure is $\gamma R_i^{nd}(v^c)$. Also, the probability of disclosure is $p(1 - \Phi(z^c))$, and the discount from expected value conditional on disclosure is $\gamma\alpha var(\tilde{x}_i)$. Accordingly, firm $i's$ cost of capital when it uses the cutoff v^c (or its standardized value z^c) is given by:

$$CC(z^c, \gamma) = (1 - p + p\Phi(z^c)) \times \gamma R_i^{nd}(z^c) + p(1 - \Phi(z^c)) \times \gamma\alpha R_i. \qquad (23)$$

Here, we are interested in two separate but related phenomena. We are interested in learning how exogenous changes in a firm's disclosure policy - in the form of changes in z^c - change its cost of capital, that is, we seek to learn the partial derivative $\frac{\partial CC(z^c,\gamma)}{\partial z^c}$, and we also are interested in learning how a firm $i's$ cost of capital changes in equilibrium as γ changes, i.e., when we evaluate the cost of capital z^c at the manager's equilibrium disclosure policy $z^c(\gamma)$, that is, we seek to learn the total derivative $\frac{dCC(z^c(\gamma),\gamma)}{d\gamma}$.

The next theorem might be regarded as *the* fundamental result on the relationship between a firm's voluntary disclosure policy and its cost of capital.

Theorem 11 *If $\gamma > 0$ and $R_i > 0$, then the firm's cost of capital (24) is always strictly increasing in the cutoff z^c for any value of that cutoff, i.e.,* $\frac{\partial CC(z^c)}{\partial z^c} > 0$ *for all $z^c \in \mathbb{R}$.*

This theorem can be recast as asserting that enhanced disclosure, i.e., a smaller value for z^c, *always* reduces a firm's cost of capital. This result is consistent with the empirical findings of Botosan [1997], Botosan and Plumlee

[13] The literature sometimes describes this alternatively as a firm's "risk premium."

[2002], and others who have shown that there is an inverse relationship between disclosure and cost of capital. As the statement of the theorem asserts, this result holds for all cutoffs z^c. Based on this result and the related empirical findings, one might ask: if, as the theorem asserts, no matter what the firm's manager's current disclosure policy is (as summarized by the cutoff z^c), the manager can always further lower her firm's cost of capital by disclosing more (i.e., by reducing z^c), then why doesn't the manager opt for a disclosure policy entailing that she discloses all the information she receives (i.e., set $z^c = -\infty$)? The answer the present paper offers is: the manager's disclosure policy is private information to the manager, and so the only disclosure policy we can expect the manager to implement is an equilibrium disclosure policy. As we have already shown, no equilibrium disclosure policy entails full disclosure.[14] In other words, a disclosure policy that minimizes the firm's cost of capital is not an equilibrium disclosure policy.

Our final comment on this theorem is that while both intuitive and fundamental, this theorem is inconsistent with some other published claims in the voluntary disclosure literature, as we noted in the Introduction.

Corollary 12 *If $\gamma > 0$ and $R_i > 0$, then the firm's cost of capital evaluated at the equilibrium disclosure policy is strictly increasing in γ, i.e.,* $\frac{dCC(z^c(\gamma),\gamma)}{d\gamma} > 0$.

Corollary 12 is the combined result of two countervailing effects: the direct effect that naturally results in the cost of capital rising as investors become more risk averse in aggregate, holding the manager's disclosure strategy fixed, and the indirect effect associated with the change in the manager's equilibrium disclosure cutoff induced by investors' increased risk aversion. The indirect effect by itself causes the cost of capital to fall, since we know from Corollary 10(a) that the manager's equilibrium cutoff falls as investors' aggregate risk aversion rises, and also since, as reported in Lemma 11 above, the firm's cost of capital falls as its cutoff falls. What Corollary 12 shows is that the direct effect always dominates when the cost of capital is evaluated at the manager's equilibrium disclosure cutoff.

[14] See Lemma 5 part (b).

3.2 Additional "Own" Firm Findings

Having determined the effects of changing γ on the manager's equilibrium disclosure policy, we now address some additional empirically testable implications of the model that derive from examining how the manager's equilibrium disclosure policy varies with other exogenous parameters of the model.

Corollary 13 *If $\gamma > 0$ and $R_i > 0$ and the manager adopts an equilibrium disclosure policy, then the manager discloses the information she receives more often:*

(a) as the probability p the manager gets information increases, i.e., $\frac{dz^c(p)}{dp} < 0$;

(b) as the precision r of the estimate the manager sometimes receives increases, i.e., $\frac{dz^c(r)}{dr} < 0$; and

(c) as the prior variance of $\tilde{x}_i$ increases, i.e., $\frac{dz^c(\sigma^2_{x_i})}{d\sigma^2_{x_i}} < 0$.

Part (a) extends the result of Dye [1985] and Jung and Kwon [1988] to the setting of a risk-averse investors. When the probability p the manager receives information increases, a price-maximizing manager will disclose the information she receives more often, since investors will assign a lower price to her firm if she makes no disclosure (because they are more inclined to conclude that the reason for the manager's nondisclosure is that she received, and withheld, unfavorable information). In part (b), as the manager's information becomes more precise, there is greater price response to disclosure, which induces the manager to disclose her information more often. Part (c) is also intuitive: if there is a lot of uncertainty ex ante about the firm's future cash flows, there is a lot of uncertainty that can be resolved by disclosure, which results in disclosure occurring more often.

It is worthwhile to discuss another implication of part (c). Suppose, before the start of the current model, some other information about firm $i's$ future cash flows is released. The source of this other information might be firm i itself, some other firm, news organization, analyst, etc. Represent this other information by the realization of some random variable $\tilde{y}$, and suppose that the triple $(\tilde{y}, \tilde{x}_i, \tilde{v}_i)$ is jointly normally distributed. The information release

$\tilde{y} = y$ will change investors' perceptions about both the mean and the variance of firm $i's$ future cash flows. If, somehow, one could confine the effect of having investors observe $\tilde{y}'s$ realization so that it altered only their beliefs about the mean of $\tilde{x}_i$, and not their perceptions of the variance of $\tilde{x}_i$, then it is clear from (21) above, since the prior mean m_i there "washes out" of the equation defining the equilibrium cutoff, that this mean shift would have no impact on the equilibrium probability the manager of firm i discloses her private information. The equilibrium cutoff in this special case would just shift one–for-one in whatever amount the disclosure of $\tilde{y}$ caused investors to revise their beliefs about the expected value of $\tilde{x}_i$. But, of course seeing $\tilde{y}'s$ realization does alter - it lowers - investors' perceptions of the variance of $\tilde{x}_i$. Corollary 13 (c) shows that, as a consequence of this latter variance effect, the probability the manager subsequently discloses whatever information she receives declines, because according to this theorem, she shifts upward her preferred standardized disclosure threshold as this variance declines. Thus, in this model, observing $\tilde{y}$ acts as a partial substitute for the manager's disclosure of $\tilde{v}_i$, as it causes the manager to disclose $\tilde{v}_i$ less often (than she would were $\tilde{y}$ not disclosed). This result has some empirical support (e.g., Breuer, Hombah, and Muller [2016]).

While this "substitution effect" might not seem remarkable, what is worth noting is that no such substitution effect *ever occurs* under risk-neutral pricing when the three random variables $\tilde{y}$, $\tilde{x}_i$, and $\tilde{v}$ are multi-variate normal, as we report in the following corollary.[15]

Corollary 14 *If risk-neutral pricing prevails ($\gamma = 0$), then the equilibrium probability that the manager discloses $\tilde{v}_i$ is the same for all joint distributions of the three random variables $\tilde{y}$, $\tilde{x}_i$, and $\tilde{v}_i$, provided ($\tilde{y}$, $\tilde{x}_i$, $\tilde{v}_i$) are trivariate normal and $\tilde{v}_i$ is marginally informative about $\tilde{x}_i$ in the presence of $\tilde{y}$ in the sense that, given y, $E[\tilde{x}_i|y, v_i]$ varies nontrivially with v_i, i.e., $\frac{dz^c(r)}{dr} = \frac{dz^c(\sigma^2_{x_i})}{d\sigma^2_{x_i}} = 0$.*

We now examine how a firm's cost of capital changes in equilibrium as exogenous parameters of the model other than investors' aggregate risk aversion change. We modify the notation in the natural way. For example, if we want

[15] This result is a minor extension of results reported in Acharya et al [2011], and so we do not prove this result here.

to examine how changing the probability p the manager receives information alters her firm's cost of capital, we write the firm's cost of capital as:

$$CC(z^c(p), p) = (1-p+p\Phi(z^c(p))) \times \gamma R_i^{nd}(z^c(p)) + p(1-\Phi(z^c(p))) \times \gamma\alpha R_i. \quad (24)$$

Theorem 15 *If $\gamma > 0$ and $R_i > 0$, firm $i's$ cost of capital evaluated at the equilibrium disclosure policy is strictly decreasing in:*

(a) the probability p the firm receives information, i.e., $\frac{dCC(z^c(p),p)}{dp} < 0$;

(b) the precision r of the estimate the firm sometimes receives, i.e., $\frac{dCC(z^c(r),r)}{dr} < 0$;

(c) the prior precision τ of the distribution of the firm's future cash flows, i.e., $\frac{dCC(z^c(\tau),\tau)}{d\tau} < 0$.

All of these results are intuitive. Parts (a) and (b) are clear: the cost of capital declines if the manager acquires information more often or if the information she sometimes receives is more precise. Part (c) follows because as investors' prior beliefs become more precise, there is less remaining information for investors to learn about the firm's future cash flows, and so the gap between the expected selling price of the firm and its expected cash flows naturally decreases.

Next, we define $\tilde{x}_{-i} \equiv \sum_{j \neq i} \tilde{x}_j$ to be the sum of the future cash flows of all "other" firms. We let $cov(\tilde{x}_i, \tilde{x}_{-i})$ denote the unconditional covariance between firm $i's$ future cash flows and the aggregate future cash flows of all other firms. We have:

Corollary 16 [16] *If $\gamma > 0$ and $R_i > 0$, then:*

(a) the equilibrium cutoff z^c of firm i is strictly decreasing in $cov(\tilde{x}_i, \tilde{x}_{-i})$;

(b) the firm's cost of capital is strictly increasing in $cov(\tilde{x}_i, \tilde{x}_{-i})$.

Part (a) asserts that the manager discloses her information more often as the covariance between her firm's future cash flows and the sum of all other firms' future cash flows rises. This is intuitive insofar as such increases in "aggregate" covariances reflect an overall increase in investors' perception of the risk of owning firm i's shares, and so it is natural for the manager of firm i to respond

[16] The proofs of these results are straightforward and omitted.

by dampening this increase in perceived riskiness in the only way she can in the model: by disclosing the information she receives more often. Part (b) indicates that, despite the adjustments in the manager's equilibrium disclosure policy described in part (a) due to increases in $cov(\tilde{x}_i, \tilde{x}_{-i})$, the firm's cost of capital increases as $cov(\tilde{x}_i, \tilde{x}_{-i})$ increases. This is not surprising: adjustments in her disclosure policy cannot mask that when $cov(\tilde{x}_i, \tilde{x}_{-i})$ increases, firm i is a riskier firm than before $cov(\tilde{x}_i, \tilde{x}_{-i})$ increased, and this increased riskiness is reflected in an increase in the firm's cost of capital.

3.3 Information Transfers Between Firms Arising from Nondisclosure

In this section, we round out the analysis of "own" firm nondisclosure under mean-variance pricing by considering its effects on investors' perceptions of the distribution of future cash flows of "other" firms. This analysis is motivated by one of the traditional concerns in accounting, that of "information transfer" between firms (see, e.g., Olsen and Dietrich [1985]), here extended to the informational impact on other firms arising from "own" firm's nondisclosure.

We start by extending our notation in the obvious fashion to firm $j \neq i$ by letting $E[\tilde{x}_j|nd, v^c]$ and $var(\tilde{x}_j|nd, v^c)$ respectively denote investors' perceptions of the mean and variance of firm $j's$ future cash flows conditional on no disclosure by firm i, the manager's receipt of the estimate $\tilde{v}_i$ with probability p, and investors' perception that the manager of firm i uses the disclosure policy defined by the cutoff v^c.

Lemma 17 [17] *(a) Given any cutoff v^c (or its standardized value $z^c = \frac{v^c - m_i}{\sigma_v}$) used by the manager of firm i, then for any firm $j \neq i$:*

(a)

$$E[\tilde{x}_j|nd, v^c] = E[\tilde{x}_j] - \frac{cov(\tilde{x}_i, \tilde{x}_j)}{\sigma_v} \times f(z^c); \tag{25}$$

(b)

$$var(\tilde{x}_j|nd, v^c) = var(\tilde{x}_j) - \frac{(cov(\tilde{x}_i, \tilde{x}_j))^2}{\sigma_v^2} \times f(z^c)(z^c + f(z^c)).$$

[17] For the proof of part (a), see 28 in the Appendix with $\tilde{x}_j$ replacing $\tilde{y}$. The proof of part (b) is similar to that of the proof of Lemma 2 and so is not repeated here.

Part (a) of the preceding lemma is the foundation for the following theorem.

Theorem 18 [18] *For any disclosure cutoff v^c by the manager of firm i - equilibrium or otherwise - nondisclosure by firm i reduces (resp., increases) investors' perceptions of the expected value of firm j's cash flows when the unconditional covariance between firm i's and firm j's future cash flows is positive (resp., negative).*

This result is intuitive: we previously documented that nondisclosure has a negative effect on investors' perceptions of the first moment of "own" firm's future cash flows. If "own" firm's and "other" firm's future cash flows positively covary, nondisclosure by "own" firm adversely affects investors' perceptions of the expected future cash flows of "other" firms too. Conversely, if the two firms' future cash flows negatively covary, then what investors perceive of as bad news regarding the expected future cash flows of "own" firm will be perceived of as good news regarding "other" firm's future cash flows.

Part (b) of Lemma 17 is the basis for the following theorem.

Theorem 19 [19] *When the unconditional covariance $cov(\tilde{x}_i, \tilde{x}_j)$ between firm i's and firm j's future cash flows is nonzero:*

(a) investors' perceptions of the variance of firm j's cash flows conditional on no disclosure by firm i are higher (resp., lower) than their prior (unconditional) perceptions of that variance if and only if the standardized value of the cutoff v^c, $z^c = \frac{v^c - m_i}{\sigma_v^2}$, used by the manager of firm i is below (resp., above) z^{*c};

(b) if $\gamma > 0$ and $R_i > 0$ and the manager of firm i adopts an equilibrium disclosure policy, then investors' perceptions of the variance of firm j's cash flows conditional on no disclosure by firm i are always strictly higher than their prior (unconditional) perceptions of that variance.

Theorem 19 clearly demonstrates that nondisclosure is variance-increasing for "other" firms when "other" firms' future cash flows have nonzero correlation with "own" firm's cash flows. In part (a), it is noteworthy that the critical value of the standardized cutoff that determines whether the prior unconditional

[18] The proof is obvious given Lemma 17.

[19] The proof is also obvious given Lemma 17.

variance of $\tilde{x}_j$ is above or below investors' perception of the variance of $\tilde{x}_j$ given no disclosure by firm i is the exact same value as the critical value of that cutoff for "own" firm's nondisclosure to be variance-increasing as originally reported in Lemma 3 above. In part (b), we see that, when attention is confined to equilibrium disclosure policies, we get the strong conclusion that investors' perceptions of the variance of firm j's future cash flows conditional on nondisclosure by firm i are always strictly higher than their prior perceptions of the variance firm $j's$ future cash flows, as long as the unconditional covariance between the two firms' future cash flows is nonzero.

Adding the conclusion of part (b) of Theorem 19 to the conclusions of Theorem 6 parts (a), (b), and (c), we now know that, in equilibrium when γ and R_i are both positive, nondisclosure increases investors' perceptions of each of: 1. the variance of "own" firm's future cash flows; 2. the absolute value of the covariance between "own" and "other" firms' future cash flows; 3. the total risk of "own" firm's future cash flow; and 4. the variance of "other" firms' future cash flows, when the unconditional covariance between "own" firm's and "other" firms' future cash flows is nonzero.

4 Conclusions

We have studied the economic consequences of the equilibrium voluntary disclosure decisions of the manager of a single firm when multiple firms are present and mean-variance pricing prevails. We have uncovered several new empirically testable findings. Our new ex post (or post-disclosure) findings include predictions that nondisclosure is "uncertainty-increasing" in manifold ways (as summarized in the last paragraph of the previous section), along with predictions concerning both the first- and second- moment information transfers that occur between firms when one firm makes no disclosure. Our new ex ante (or pre-disclosure) findings include predictions involving the effects on the disclosing firm's cost of capital induced by changes in: the precision of the private information the firm's manager sometimes receives, the frequency with which the manager receives new information, with improvements in the information environment of the disclosing firm, with expansions in the manager's disclosure set, and with alterations in investors' aggregate risk aversion. Additionally, we have obtained predictions about how the manager's equilibrium disclosure policy changes with each of: investors' aggregate risk aversion, the precision of the manager's private information, the probability the manager receives information increases, and the amount of ex ante uncertainty that exists about the firm's future cash flows.

We have also identified the extent to which our findings are sensitive to mean-variance pricing. We have shown, for example, that information from other sources serves as a partial substitute for the manager's voluntary disclosures when a firm is subject to mean-variance pricing, but not when it is subject to risk-neutral pricing. Related, we have shown that *none* of the manifold ways in which nondisclosure is "uncertainty-increasing" under mean-variance pricing extend to risk-neutral pricing.

As far as we aware, all of these results are new to the disclosure literature.

These results are robust: they holds for all parameterizations of the prior (normal) distributions describing firms' future cash flows, for all precisions of the estimate the firm's manager sometimes receives, and for all probabilities describing the likelihood that the given firm's manager will receive private information. The results are also robust in another sense: all the results are obtained when the prior distributions of firms' future cash flows are jointly normal. Since each of the many versions of the central limit theorem demonstrates that normally distributed random variables characterize well the distributions of processes involving the sum of many independent random variables, and hence are likely to be appropriate for characterizing the aggregate cash flows of most firms since those aggregate cash flows are often the sum of many independent random shocks, we expect our results to hold broadly when subjected to empirical examination.

5 References

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6 Appendix: Select Proofs

Proof of Lemma 2 We temporarily drop the index i on $\tilde{x}_i$, $\tilde{v}_i$, m_i and $\tilde{\varepsilon}_i$, and instead write $\tilde{x}$, $\tilde{v}$, m, and $\tilde{\varepsilon}$. We also drop the i from variances and write σ_x^2 for the prior variance of $\tilde{x}$. Recall $\tilde{x}\tilde{\ }N(m_i, \tau)$ are the priors on $\tilde{x}$, and given $\tilde{x} = x$, $\tilde{v} = x + \tilde{\varepsilon}$, where $\tilde{\varepsilon}\tilde{\ }N(0, r)$. σ_v^2 denotes the prior variance of $\tilde{v}$. Based on the preceding, we have $E[\tilde{v}] = m$. The cumulative distribution function and density of $\tilde{v}$ are denoted by $G(v)$ and $g(v)$ respectively.

Anticipating some results that occur later in the paper, we start by computing an expectation slightly more general than that appearing in (19), namely: for any random variable $\tilde{y}$ such that $(\tilde{x}, \tilde{y})$ is multivariate normal, we first seek to compute $E[\tilde{y}|nd, v^c, m]$.[20] Once we obtain obtain this expectation, then (19) will "fall out" immediately when we substitute $\tilde{x}_i$ for $\tilde{y}$.

Let the realization of the random variable $\tilde{\omega}$ indicate the reason, in the case the manager makes no disclosure, why she made no disclosure: $\omega = no\ info$ indicates that the manager did not receive information, and $\omega = withheld$ indicates the manager withheld information. (Of course, since the manager receives information privately, investors do not know the realization of $\tilde{\omega}$ when the manager makes no disclosure.) Let "nd" indicate the public event that the manager made no disclosure, but not the reason for the manager's nondisclosure. We fix the disclosure cutoff at some constant v^c.

We begin by observing:

$$\begin{aligned} E[\tilde{y}|\tilde{\omega} &= withheld] = E[E[\tilde{y}|\tilde{v}]|\tilde{v} < v^c] = \frac{1}{G(v^c)} \int_{-\infty}^{v^c} E[\tilde{y}|v]g(v)dv \quad (26) \\ \text{and } E[\tilde{y}|\tilde{\omega} &= noinfo] = E[\tilde{y}]. \end{aligned}$$

When the manager's disclosure strategy is defined by the cutoff v^c, the manager makes no disclosure either because she did not receive information (which occurs with probability $1 - p$) or because she receives information that she withholds (which occurs with probability $pG(v^c)$), so $\Pr(nd) = 1-p+pG(v^c)$, so by Bayes' rule, $\Pr(no\inf o|nd) = \frac{1-p}{1-p+pG(v^c)}$ and so:

$$E[\tilde{y}|nd, v^c, m] = \frac{(1-p)E[\tilde{y}] + pG(v^c) \times \frac{1}{G(v^c)} \int^{v^c} E[\tilde{y}|v]g(v)dv}{1 - p + pG(v^c)}.$$

Recall the elementary fact concerning bivariate normal random variables that,

[20] Notational comment: expressions such as $E[\tilde{x}_i|nd, v^c, m]$ appear in this appendix. Such expressions have the same meanings as similar expressions in the text, such as $E[\tilde{x}_i|nd, v^c]$. The only difference between the expressions is the reminder that $E[\tilde{x}_i] = m$ in the expressions here. We introduce these reminders in this appendix, because here we sometimes shift the prior mean m of $\tilde{x}_i$ within a proof, and we want to be clear when such shifts occur. (In the text, such expansive notation is unnecessary, since we never shift the prior mean of $\tilde{x}_i$ in the text.) Related, when calculating other expressions in this appendix, such as $E[\tilde{y}|nd, v^c, m]$ for some random variable $\tilde{y}$, the quantity in the last position among the conditioning variables always refers to $E[\tilde{x}_i] = m$. (For example, $E[\tilde{y}|nd, v^c, 0]$ refers to the conditional expectation of $\tilde{y}$ when (among other things) $E[\tilde{x}_i] = 0$.)

with $\beta_y \equiv \frac{cov(\tilde{y},\tilde{v})}{\sigma_v^2}$, we have $E[\tilde{y}|v] = E[\tilde{y}] + \beta_y(v - m_i)$ and so:

$$\begin{aligned} E[\tilde{y}|nd, v^c, m] &= \frac{(1-p)E[\tilde{y}] + p\int^{v^c}(E[\tilde{y}] + \beta_y(v-m))g(v)dv}{1-p+pG(v^c)} \\ &= \frac{(1-p)E[\tilde{y}] + pE[\tilde{y}]G(v^c) + p\beta_y\int^{v^c} vg(v)dv - p\beta_y mG(v^c)}{1-p+pG(v^c)} \\ &= E[\tilde{y}] + \frac{p\beta_y\int^{v^c} vg(v)dv - p\beta_y mG(v^c)}{1-p+pG(v^c)} \end{aligned}$$

Next, we substitute the easily confirmed fact

$$\int_{-\infty}^{v^c} vg(v)dv = mG(v^c) - \sigma_v^2 g(v^c) \tag{27}$$

into the last expression to conclude:

$$\begin{aligned} E[\tilde{y}|nd, v^c, m] &= E[\tilde{y}] + \frac{p\beta_y(mG(v^c) - \sigma_v^2 g(v^c)) - p\beta_y mG(v^c)}{1-p+pG(v^c)} \\ &= E[\tilde{y}] - \frac{p\beta_y\sigma_v^2 g(v^c)}{1-p+pG(v^c)} \\ &= E[\tilde{y}] - \frac{pcov(\tilde{y},\tilde{v})g(v^c)}{1-p+pG(v^c)}. \end{aligned}$$

Recall from the text that ϕ and Φ denote the density and cdf of the standard normal. With $z^c \equiv \frac{v^c - m}{\sigma_v}$, it is clear that $\sigma_v g(v^c) = \phi(z^c)$ and $G(v^c) = \Phi(z^c)$, so $E[\tilde{y}|nd, v^c, m] = E[\tilde{y}] - \frac{p\frac{cov(\tilde{y},\tilde{v})}{\sigma_v}\phi(z^c)}{1-p+p\Phi(z^c)}$, and thus, recalling the definition of $f(z^c)$ in (9):

$$E[\tilde{y}|nd, v^c, m] = E[\tilde{y}] - \frac{cov(\tilde{y},\tilde{v})}{\sigma_v} f(z^c). \tag{28}$$

In the special case where $\tilde{y} \equiv \tilde{x}$, then $cov(\tilde{y},\tilde{v}) = \sigma_x^2$, so:

$$E[\tilde{x}|nd, v^c, m] = m - \frac{\sigma_x^2}{\sigma_v} f(z^c). \tag{29}$$

This last expression confirms (19).

Next, we appeal to the following lemma, which tells us how to convert investors' perceptions of the covariance betwen $\tilde{x}$ and $\tilde{y}$ or the variance of $\tilde{x}$ both conditional on no disclosure and calculated when $E[\tilde{x}] = m \neq 0$ to a corresponding conditional covariance or variance when $E[\tilde{x}] = 0$.

Lemma 20 *Let $\tilde{x}\tilde{\ }N(m,\tau)$, $\tilde{v} = \tilde{x} + \tilde{\varepsilon}$, and let v^c denote the disclosure cutoff applicable to the estimate $\tilde{v}$. Suppose $\tilde{y}$ is a random variable such that $(\tilde{x},\tilde{y})$ is* *jointly normally distributed. Then:* $cov(\tilde{x}, \tilde{y}|nd, v^c, m) = cov(\tilde{x}, \tilde{y}|nd, v^c - m, 0)$ *and* $var(\tilde{x}|nd, v^c, m) = var(\tilde{x}|nd, v^c - m, 0)$.

The proof is not difficult, but it takes quite a bit of space, so we do not present it here.

Now, continuing with the proof of Lemma 2: as above, we start by developing an expression more general than that demanded to prove Lemma 2. We are going to compute $cov(\tilde{x}, \tilde{y}|nd, v^c, m)$, where $\tilde{y}$ is any arbitrary random variable such that $(\tilde{x}, \tilde{y})$ is bivariate normal. Then, as a by-product, we will obtain the expression for $var(\tilde{x}|nd, v^c, m)$ by setting $\tilde{y} \equiv \tilde{x}$.
We begin by studying the special case where $E[\tilde{x}] = m = 0$. Since $\sigma_v^2 = \sigma_x^2 + \sigma_\varepsilon^2$, it follows (see (3)) that $1 - \alpha = \frac{\sigma_x^2}{\sigma_v^2}$. In the case currently being studied ($m = 0$), all of the following are true: $E[\tilde{v}] = 0$; $E[\tilde{x}\tilde{y}] = cov(\tilde{x}, \tilde{y})$; the regression $E[\tilde{y}|v] = E[\tilde{y}] + \beta_y(v - m)$ simplifies to $E[\tilde{y}|v] = E[\tilde{y}] + \beta_y v$; and $E[\tilde{x}|v] = (1-\alpha)v$. Combining the preceding with the fact from (3) that $cov(\tilde{x}, \tilde{y}|v) = \alpha cov(\tilde{x}, \tilde{y})$, we conclude:

$$\begin{aligned} E[\tilde{x}\tilde{y}|v] &= cov(\tilde{x}, \tilde{y}|v) + E[\tilde{x}|v]E[\tilde{y}|v] = \alpha cov(\tilde{x}, \tilde{y}) + (1-\alpha)v \times (E[\tilde{y}] + \beta_y v) \\ &= \alpha cov(\tilde{x}, \tilde{y}) + (1-\alpha)E[\tilde{y}]v + (1-\alpha)\beta_y v^2. \end{aligned}$$

Now we use these just-made observations, along with (27) and the easily confirmed fact that for $\tilde{v} \,\tilde{}\, N(0, \sigma_v^2)$:

$$\int_{-\infty}^{v^c} v^2 g(v)dv = \sigma_v^2(G(v^c) - v^c g(v^c)), \tag{30}$$

to compute:

$$\begin{aligned} \int^{v^c} E[\tilde{x}\tilde{y}|v]g(v)dv &= \int^{v^c} \{\alpha cov(\tilde{x}, \tilde{y}) + (1-\alpha)E[\tilde{y}]v + (1-\alpha)\beta_y v^2\}g(v)dv \\ &= \alpha cov(\tilde{x}, \tilde{y})G(v^c) - (1-\alpha)E[\tilde{y}]\sigma_v^2 g(v^c) + (1-\alpha)\beta_y\sigma_v^2 G(v^c) - (1-\alpha)\beta_y\sigma_v^2 v^c g(v^c) \\ &= \alpha cov(\tilde{x}, \tilde{y})G(v^c) - (1-\alpha)(E[\tilde{y}]\sigma_v^2 + cov(\tilde{y}, \tilde{v})v^c)g(v^c) + (1-\alpha)cov(\tilde{y}, \tilde{v})G(v^c) \\ &= \alpha cov(\tilde{x}, \tilde{y})G(v^c) - (1-\alpha)(E[\tilde{y}]\sigma_v^2 + cov(\tilde{y}, \tilde{x})v^c)g(v^c) + (1-\alpha)cov(\tilde{y}, \tilde{x})G(v^c) \\ &= cov(\tilde{x}, \tilde{y})G(v^c) - (1-\alpha)(E[\tilde{y}]\sigma_v^2 + cov(\tilde{y}, \tilde{x})v^c)g(v^c). \end{aligned}$$

(We used $cov(\tilde{y}, \tilde{v}) = cov(\tilde{y}, \tilde{x})$ in obtaining the fourth equality above.) Hence, when $z^c = \frac{v^c}{\sigma_v}(= \frac{v^c - m}{\sigma_v}$, presently):

$$\begin{aligned}
E[\tilde{x}\tilde{y}|nd, v^c, 0] &= \frac{(1-p)E[\tilde{x}\tilde{y}] + p\int^{v^c} E[\tilde{x}\tilde{y}|v]g(v)dv}{1-p+pG(v^c)} \\
&= \frac{(1-p)cov(\tilde{x},\tilde{y})}{1-p+pG(v^c)} \\
&\quad + \frac{p(cov(\tilde{x},\tilde{y})G(v^c) - (1-\alpha)(E[\tilde{y}]\sigma_v^2 + cov(\tilde{x},\tilde{y})v^c)g(v^c))}{1-p+pG(v^c)} \\
&= cov(\tilde{x},\tilde{y}) - p\frac{(1-\alpha)(E[\tilde{y}]\sigma_v^2 + cov(\tilde{x},\tilde{y})v^c)g(v^c)}{1-p+pG(v^c)} \\
&= cov(\tilde{x},\tilde{y}) - p\frac{(E[\tilde{y}]\sigma_x^2 + cov(\tilde{x},\tilde{y})\frac{\sigma_x^2}{\sigma_v^2}v^c)g(v^c)}{1-p+pG(v^c)} \\
&= cov(\tilde{x},\tilde{y}) - p\frac{(E[\tilde{y}]\frac{\sigma_x^2}{\sigma_v} + cov(\tilde{x},\tilde{y})\frac{\sigma_x^2}{\sigma_v^2}\frac{v^c}{\sigma_v})\phi(z^c)}{1-p+pG(v^c)} \\
&= cov(\tilde{x},\tilde{y}) - (\frac{\sigma_x^2}{\sigma_v}E[\tilde{y}] + cov(\tilde{x},\tilde{y})\frac{\sigma_x^2}{\sigma_v^2}z^c)f(z^c).
\end{aligned}$$

Recalling from (28) and (29) that $E[\tilde{x}|nd, v^c, 0] = -\frac{\sigma_x^2}{\sigma_v}f(z^c)$ and $E[\tilde{y}|nd, v^c, m] = E[\tilde{y}] - \frac{cov(\tilde{x},\tilde{y})}{\sigma_v}f(z^c)$, we conclude:

$$\begin{aligned}
cov(\tilde{x},\tilde{y}|nd, v^c, 0) &= E[\tilde{x}\tilde{y}|nd, v^c, 0] - E[\tilde{x}|nd, v^c, 0] \times E[\tilde{y}|nd, v^c, 0] \\
&= cov(\tilde{x},\tilde{y}) - (\frac{\sigma_x^2}{\sigma_v}E[\tilde{y}] + cov(\tilde{x},\tilde{y})\frac{\sigma_x^2}{\sigma_v^2}z^c)f(z^c) + \frac{\sigma_x^2}{\sigma_v}f(z^c) \times (E[\tilde{y}] - \frac{cov(\tilde{x},\tilde{y})}{\sigma_v}f(z^c)) \\
&= cov(\tilde{x},\tilde{y}) - cov(\tilde{x},\tilde{y})\frac{\sigma_x^2}{\sigma_v^2}z^c f(z^c) - \frac{\sigma_x^2}{\sigma_v}f(z^c)\frac{cov(\tilde{x},\tilde{y})}{\sigma_v}f(z^c) \\
&= cov(\tilde{x},\tilde{y}) \times (1 - \frac{\sigma_x^2}{\sigma_v^2}f(z^c)(z^c + f(z^c))). \qquad (31)
\end{aligned}$$

In the special case where $\tilde{y} \equiv \tilde{x}$, we get

$$var(\tilde{x}|nd, v^c, 0) = \sigma_x^2 - \frac{\sigma_x^4}{\sigma_v^2}f(z^c)(z^c + f(z^c)) = \sigma_x^2(1 - \frac{\sigma_x^2}{\sigma_v^2}f(z^c)(z^c + f(z^c))).$$

This last observation proves (10) in the statement of the lemma for the case $m = 0$. To compute the conditional covariance or conditional variance for the general m case, all we have to do is apply the transformation in Lemma 20 to the preceding. That is, (31) still expresses the conditional variance in the case $m \neq 0$, as long as we now assign z^c the value $z^c = \frac{v^c - m}{\sigma_v}$ (rather than $z^c = \frac{v^c}{\sigma_v}$).■

Lemma 21 *For all $p \in [0,1]$, the point z^{*c} defined by the solution to equation (17) in the text exists, is unique, is negative, and LHS(17) is positive if $z^c > z^{*c}$ and LHS(17) is negative if $z^c < z^{*c}$.*

Proof of Lemma 21 Define

$$\Gamma(z^c) \equiv z^c\Phi(z^c) + \phi(z^c), \tag{32}$$

and notice that

$$z^c(1-p) + p\Gamma(z^c) = z^c(1-p+p\Phi(z^c)) + p\phi(z^c), \tag{33}$$

and also notice that this last expression has the same sign as LHS(17) (NB: when referring to LHS(17) here and in the remainder of this proof, we mean for a generic value of the cutoff z^c to replace z^{c*} in LHS(17)). It is easy to check that: 1. the function $\Gamma(z^c)$ is continuous; 2. strictly increasing, and 3. $\lim_{z^c\to\infty}\Gamma(z^c) = \infty$. An application of L'Hospital's rule further shows that $\lim_{z^c\to-\infty}\Gamma(z^c) = 0$, so $\Gamma(z^c)$ is positive for all (finite) z^c. Hence, (33) is strictly continuously increasing on all of $\mathbb{R}$ and ranges over $-\infty$ to $+\infty$. Accordingly, as claimed, there is a unique value z^c, call it z^{*c}, such that the equation (17) has a solution. Clearly, z^{*c} is negative. The preceding also shows that LHS(17) is positive when $z^c > z^{*c}$ and LHS(17) is negative when $z^c < z^{*c}$.■

Proof of Lemma 5 parts (a) and (b). Clearly, both $f(z^c)$ defined in (9) and $f(z^c)\times(z^c+f(z^c))$ go to 0 as $z^c \to \pm\infty$. Thus, $\theta(z^c)$ goes to 1 as $z^c \to \pm\infty$. Thus LHS(21) $\to \pm\infty$ as $z^c \to \pm\infty$, whereas RHS(21)$\to m_i - \gamma R_i$ as $z^c \to \pm\infty$. Since both sides of (21) are continuous in z^c, it follows that (21) always has a solution, and this solution cannot be $z^c = \pm\infty$. This proves both parts (a) and (b).

Part (c) When $\gamma > 0$ and $R_i > 0$, each of the following equations is a rearrangement of equation (21) in the text:

$$\begin{aligned} \frac{\sigma^2_{x_i}}{\sigma_v}(z^c+f(z^c)) &= \gamma R_i(\alpha-1) + \gamma \times \left[R_i\frac{\sigma^2_{x_i}}{\sigma^2_v}f(z^c)(z^c+f(z^c))\right] \\ (z^c+f(z^c))\times(\frac{\sigma^2_{x_i}}{\sigma_v} - \gamma R_i\frac{\sigma^2_{x_i}}{\sigma^2_v}f(z^c)) &= -\gamma R_i\frac{\sigma^2_{x_i}}{\sigma^2_v} \\ (z^c+f(z^c))\times(\frac{1}{\frac{\gamma R_i}{\sigma_v}} - f(z^c)) &= -1. \end{aligned} \tag{34}$$

This last equation is equation (22) in the text.

part (d) is proven next first for small $\gamma > 0$ and then for all $\gamma > 0$.

First observe that the value z^{*c} defined by (17) maximizes the function $f(z^c)$. To see this, just note that for all z^c :

$$f'(z^c) = -f(z^c)(z^c + f(z^c)). \tag{35}$$

Then apply the observation (18).

Next, define γ_0 by $\gamma_0 \equiv \frac{1}{\frac{R_i}{\sigma_v} f(z^{*c})}$. Clearly, $\gamma_0 > 0$ and since z^{*c} is now known to maximize $f(z^c)$, it is also clear that: if $\gamma < \gamma_0$, then for all z^c in $\mathbb{R}$, we have, since R_i is assumed to be positive: $\frac{1}{\gamma \frac{R_i}{\sigma_v}} - f(z^c) > \frac{1}{\gamma_0 \frac{R_i}{\sigma_v}} - f(z^{*c}) = 0$.

Now, suppose for some $\gamma \in (0, \gamma_0)$ there is a solution $z^c = z^c(\gamma)$ to (22) with $z^c(\gamma) > z^{*c}$. Recall from Lemma 21 that LHS(17) is positive for all $z^c > z^{*c}$. Combining this recollection with the observations in the previous paragraph, we conclude that for $z^c = z^c(\gamma)$, both of the two factors in LHS(22) are positive. Clearly, this is impossible, because RHS(22) is negative. By a similar argument it is also clear that for no $\gamma < \gamma_0$ can $z^c(\gamma) = z^{*c}$ either (since, were there such a γ, then LHS(22) would be zero while RHS(22) is negative, another contradiction). Thus, we conclude that for any $\gamma < \gamma_0$, we must have $z^c(\gamma) < z^{*c}$.

We have just shown for all $\gamma < \gamma_0$ that the equilibrium cutoff $z^c(\gamma)$ is strictly less than z^{*c}. It is not hard to show that the equilibrium cutoff $z^c(\gamma)$ is continuous in γ. It follows that if there were some (large) $\gamma > \gamma_0$ for which $z^c(\gamma) > z^{*c}$, then by the intermediate value theorem there would have to be some other value of γ, say $\hat{\gamma}$, with $z^c(\gamma) = z^{*c}$. But, were such a $\hat{\gamma}$ to exist, it could not possibly satisfy the equilibrium equation (22), because LHS(22) would be zero at that $\hat{\gamma}$. This contradiction proves there can be no such $\hat{\gamma}$, and hence there can be no γ with $z^c(\gamma) > z^{*c}$ either.

part (e) For uniqueness of the cutoff for $\gamma > 0$ first recall (35). Then it follows that the derivative of LHS(22) is given by

$$(z^c + f(z^c))^2 f(z^c) + (1 - f(z^c)(z^c + f(z^c)))(\frac{1}{\gamma \frac{R_i}{\sigma_v}} - f(z^c)). \tag{36}$$

If $R_i > 0$ and z^c is an equilibrium cutoff, then we know $z^c + f(z^c)$ is negative from part (c). The equilibrium equation (22) then ensures that $\frac{1}{\gamma \frac{\sigma_{x_i}^2}{\sigma_v}} - f(z^c)$ is positive. It follows that (36) is positive, i.e., that LHS(22) is strictly increasing when evaluated at any equilibrium cutoff. This is sufficient to establish the uniqueness of the cutoff, since if (1) the LHS(22) is positive at every equilibrium cutoff and (2) LHS(22) is continuous in z^c (which it is), then there cannot be more than one equilibrium cutoff.

part (f) This is obvious.■

Proof of Theorem 9 In the following we write f in place of $f(z^c)$ and f' in place of $\frac{df(z^c)}{dz^c}$. Since Lemma 2 shows $\Delta var(z^c) = -\frac{\sigma_{x_i}^4}{\sigma_v^2} f(z^c + f)$, it follows that $sgn\frac{d}{dz^c}\Delta var(z^c) = -sgn\frac{d}{dz^c}f(z^c + f)$. Recall (35) to conclude: $\frac{d}{dz^c}f(z^c + f) = -f(z^c + f)^2 + f(1 - f(z^c + f))$, and hence

$$sgn\frac{d}{dz^c}\Delta var(z^c) = sgn[(z^c+f)^2+f(z^c+f)-1] = sgn[(z^c+f)(z^c+2f)-1]. \quad (37)$$

Equation (22) can be written as:

$$z^c + f = \frac{1}{f - \frac{1}{\gamma\frac{R_i}{\sigma_v}}}. \quad (38)$$

The preceding shows that $\frac{1}{f-\frac{1}{\gamma\frac{R_i}{\sigma_v}}} < 0$ in equilibrium. (22) also can be written as

$$\frac{z^c + f + \gamma\frac{R_i}{\sigma_v}}{\gamma\frac{R_i}{\sigma_v}} = f(z^c + f). \quad (39)$$

Thus, in equilibrium we also get:

$$z^c + f + \gamma\frac{R_i}{\sigma_v} < 0. \quad (40)$$

Now, we invoke the parameter restriction in the statement of Theorem 9 that $\gamma\frac{R_i}{\sigma_v} > 1$. Of course, this parameter restriction is equivalent to $\gamma\frac{R_i}{\sigma_v} > \frac{1}{\gamma\frac{R_i}{\sigma_v}}$, which yields $z^c + f + \frac{1}{\gamma\frac{R_i}{\sigma_v}} < z^c + f + \gamma\frac{R_i}{\sigma_v}$, and so by (40) we conclude:

$$z^c + f + \frac{1}{\gamma\frac{R_i}{\sigma_v}} < 0 \quad (41)$$

too. Now, use (38) to conclude that:

$$(z^c + f)(z^c + 2f) - 1 = \frac{z^c + 2f}{f - \frac{1}{\gamma\frac{R_i}{\sigma_v}}} - 1 = \frac{z^c + f + \frac{1}{\gamma\frac{R_i}{\sigma_v}}}{f - \frac{1}{\gamma\frac{R_i}{\sigma_v}}}. \quad (42)$$

So, in view of the observations above about the signs of RHS(38) and LHS(41), it follows that $(z^c + f)(z^c + 2f) - 1$ is positive. Thus, (37) is positive.$\blacksquare$

Proof of Corollary 13 and Proof of Corollary 10(a) We do not present the proofs, but the proofs proceed in the usual way comparative statics results are generated, by totally differentiating (22) with respect to each parameter, and then signing the components of the total derivative.

Proof of Corollary 10(b) Since γ does not appear directly in any of the components of $\Delta var(z^c)$, it follows that $\frac{d}{d\gamma}\Delta var(z^c) = \frac{d}{dz^c}\Delta var(z^c) \times \frac{dz^c}{d\gamma}$. By Theorem 9, $\frac{d}{dz^c}\Delta var(z^c)$ is positive, and by Corollary 10(a), $\frac{dz^c}{d\gamma}$ is negative. This proves part (b) of the corollary.$\blacksquare$

Proof of Theorem 11 Write ϕ as shorthand for $\phi(z^c)$, Φ as shorthand for $\Phi(z^c)$, f as shorthand for $f(z^c)$, and introduce the notation $D \equiv 1 - p + p\Phi$. In this notation, CC can be written as (since $Df = p\phi$):

$$\begin{aligned} CC &= \gamma D R_i \theta(z^c) + p(1-\Phi)\alpha R_i = \gamma D R_i (1 - \frac{\sigma^2_{x_i}}{\sigma^2_v} f(z^c + f)) + p(1-\Phi)\alpha R_i \\ &= \gamma R_i (D - \frac{\sigma^2_{x_i}}{\sigma^2_v} p\phi(z^c + f)) + p(1-\Phi)\alpha R_i. \end{aligned}$$

So, recalling (35), $\phi' = -z^c\phi$, and $1 - \alpha = \frac{\sigma^2_{x_i}}{\sigma^2_v}$, we have:

$$\begin{aligned} sgn \frac{1}{\gamma R_i}\frac{dCC}{dz^c} &= sgn \frac{dCC}{dz^c} \\ &= sgn \left[p\phi + \frac{\sigma^2_{x_i}}{\sigma^2_v} p z^c \phi(z^c + f) - \frac{\sigma^2_{x_i}}{\sigma^2_v} p\phi(1 - f(z^c + f)) - p\phi\alpha \right] \\ &= sgn \left[1 - \alpha + \frac{\sigma^2_{x_i}}{\sigma^2_v} z^c(z^c + f) - \frac{\sigma^2_{x_i}}{\sigma^2_v}(1 - f(z^c + f)) \right] \\ &= sgn \left[\frac{\sigma^2_{x_i}}{\sigma^2_v} + \frac{\sigma^2_{x_i}}{\sigma^2_v} z^c(z^c + f) - \frac{\sigma^2_{x_i}}{\sigma^2_v}(1 - f(z^c + f)) \right] \\ &= sgn \left[1 + z^c(z^c + f) - (1 - f(z^c + f) \right] = sgn \left[(z^c + f)^2 \right] > 0. \end{aligned}$$

$\blacksquare$

Proof of Theorem 15 This proof is rather lengthy and so is not presented here, but the idea underlying it is simple: totally differentiate the expression for the cost of capital (24) with respect to each parameter, while employing calculations derived from the proof of Corollary 13.$\blacksquare$