Market Interactions between Audit and Non-audit Services: Bundling, Bans, and Competition*

Henry L. Friedman        Lucas Mahieux

November 21, 2018

Abstract

We model a setting in which an auditor offers both audit and non-audit services (NAS) to a set of heterogeneous clients. We examine how the potential to bundle audit and NAS, competition in the market for NAS, and regulatory bans on the provision of NAS both independently and jointly affect audit quality and economic surplus generated by the auditor and NAS competitor. We provide several interesting results. First, bundling auditing and NAS may decrease audit quality because it changes the types of clients who buy audits. Second, absent competition, a ban on NAS provision to audit clients can improve audit quality even without negative effects on auditor independence. Third, with NAS competition, bans on the provision of NAS to audit clients can decrease audit quality. These contrasting effects suggest a more nuanced view of how regulating an auditor’s offering of NAS might affect audit quality, and operate through competitive and pricing-related effects rather than auditor independence or knowledge spillover channels.

Keywords: Audit quality; Auditing services; Non-audit services; Competition.

JEL codes: L10; M41; M42.

*Henry Friedman is from the Anderson School of Management at the University of California, Los Angeles. Lucas Mahieux is from the School of Economics and Management at Tilburg University. Contact information: henry.friedman@anderson.ucla.edu; l.mahieux@tilburguniversity.edu. The authors thank Jenny Chu, Gilad Livne, Ilan Guttman, Xiaojing Meng, Michael Minnis, Joshua Ronen, Brett Trueman, Raghu Venugopalan (discussant) and conference/workshop participants at New York University and the 2018 CFEA at Tulane University for helpful comments. Mahieux gratefully acknowledges funding from the European Research Council (grant agreement No. 669217).
1 Introduction

There is a long-standing concern about the negative effects of non-audit services on audit quality (see, e.g., Che et al., 2018; DeAngelo, 1981; Kowaleski et al., 2018). For instance, payments for NAS by client firms can impair auditor independence and motivate auditors to ignore client deficiencies. Simunic (1984) and Bouwens (2018), in contrast, suggest positive spillovers from NAS provision to audit quality, due to, for instance, knowledge transfer across audit and NAS employees at the same firm. These arguments are generally constructed in a single-client setting, in which one auditor provides services to one client. Such a setting precludes a number of interesting economic forces from operating. In this paper, we examine a market of heterogeneous clients and introduce competition in the NAS market. This allows us to consider the individual and joint effects of audit and NAS bundling, competition in the NAS market, and NAS restrictions on various features of the audit market, including audit quality, pricing, and an auditor’s producer surplus. Analysis of the model provides an alternative perspective on how economic forces related to NAS might affect the market for audit services.

Our model features a continuum of client firms who can purchase auditing services and NAS from an auditor and NAS from a consultant. Because our focus is on the auditor, we refer to the operating firms as clients and the consultant as a competitor. Each client has a project that can be either good or bad. A good project succeeds, yielding a positive cash flow, with positive probability and fails, yielding no cash flow, otherwise. A bad project always fails. The business risk of a client firm is the probability that its project is bad. This risk varies across client firms and is observable only to that client. Each client firm can hire the auditor to provide an opinion on whether the project is good or bad, as in Dye (1995) and Laux and Newman (2010), and can simultaneously hire either the auditor or the competitor to provide consulting services that increase the expected profitability of good projects. Clients with high business risk therefore place a high value on auditing, while clients with low business risk place a high value on NAS.\footnote{We explore an extension in which consulting services increase the expected profitability of all projects equally.}

Before offering its services to clients, the auditor invests in firm-wide audit quality, with higher audit quality increasing the probability that the auditor successfully detects a bad project that the client can optimally avoid investing in. This and the inability for the auditor to observe client
business risk imply that the auditor offers the same auditing services for the same price to clients.\textsuperscript{2} Both the auditor and competitor offer equivalent-quality NAS and cannot use NAS quality or client business risk to price discriminate and segment the market. However, both the auditor and competitor choose how to price the services they offer, and clients choose whom to purchase services from. Differentiation in the NAS market comes only from the auditor’s ability to offer audit services.

We begin our analysis with a setting in which the auditor is a monopolist in both the auditing and NAS markets. First, we assume that the auditor offers auditing and NAS separately to clients. Because client preferences for auditing and NAS are negatively correlated, different segments of the market purchase different services. The auditor chooses maximal audit quality and provides auditing and NAS to the clients who value each service the most. Monopolist rents are earned in both markets. Next, we allow the auditor to offer a bundle of both audit and NAS. We assume, consistent with recent evidence, that NAS are more valuable (\textit{The Economist}, May 24, 2018).\textsuperscript{3} The auditor therefore targets the bundle at low-risk clients who view NAS as the primary product, making auditing services essentially an add-on. The auditor optimally chooses a lower level of audit quality, as audits are now purchased in the bundle by low-risk clients who value them the least. A restriction on NAS to audit clients eliminates the potential for the auditor to offer the bundle. When the bundle is optimal (which would occur with low audit costs), this leads to an increase in audit quality and a shift in audit clients from low-risk to high-risk. This result suggests that we could observe a positive association between audit quality and NAS restrictions to audit clients, even in the absence of a client-level audit independence effect, because NAS restrictions to audit clients change the benefit to the auditor’s ex ante investment in audit quality via market forces.

We next introduce the competitor, who can offer NAS but is not an auditor. With separate contracts, the auditor and competitor engage in full Bertrand competition for NAS clients. This pushes the auditor’s rents from NAS to zero and reduces the auditor’s supplier surplus. However, client surplus is greater in this setting because the price of consulting services is competed down. Due to market segmentation, the auditor continues to choose maximal audit quality, implying that with separate service offerings, audit quality is not directly affected by NAS competition. When

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{2}Prices can be interpreted as hourly rates, given our market where firms differ only in business risk.
\item \textsuperscript{3}We examine a setting in which audits are more valuable as an extension in section 5.3.
\end{itemize}
\end{footnotesize}
NAS competition is added to a setting in which the auditor offers bundled services, the competitor essentially can pick off clients who place the highest value on NAS and the lowest value on audits. This pushes the auditor to offer the bundle to more intermediate-risk clients, who place a relatively higher value on auditing. Therefore, adding NAS competition to the bundling setting yields higher audit quality. Here, competition on NAS increases audit quality.

An interesting effect emerges with the combination of NAS competition and the restriction of auditor provision of NAS to audit clients: a restriction on NAS to audit clients can lead to lower audit quality. Relative to a case where the auditor offers standalone services even without NAS restrictions or where the auditor faces NAS restrictions but acts as a monopolist in both markets, NAS restrictions to audit clients combined with competition unambiguously lead to weakly lower audit quality. This occurs solely through the interaction between NAS restrictions and competitive effects. Specifically, the NAS restriction carves out a niche in which the competitor is a monopolist, i.e., in offering NAS to audited clients. If the competitor cannot price discriminate between audited and unaudited clients, a larger equilibrium set of audited clients increases the niche in which the competitor is a monopolist, and causes the competitor to raise prices. This allows the auditor to raise prices for the NAS it offers to non-audit clients. By lowering audit quality and expanding audit coverage (which are linked in our model via demand heterogeneity), the auditor indirectly increases the rents it earns in the NAS market.

Finally, some regulators and practitioners have proposed breaking up integrated audit-consulting firms, i.e., prohibiting audit firms from offering NAS to any clients (see, e.g., Kowaleski et al., 2018; The Economist, May 24, 2018). The main concern is that providing both services may create conflicts and inherent biases across services, even if not provided to common clients. Our analysis shows that a regulatory ban on the provision of NAS to non-audit clients may increase audit quality relative to a prohibition on providing NAS solely to audit clients, because this removes the potential for the auditor to benefit from competition-related price increases in the NAS market.

1.1 Contribution and related literature

This paper contributes primarily to auditing literatures that have independently studied the effects of NAS provision by auditors and the effects of competition between auditors, primarily on auditing services. Simunic (1984) shows that auditors should provide NAS to audit clients because of
potential knowledge spillovers. On the contrary, DeAngelo (1981) argues that NAS may threaten auditors’ independence by creating an economic bond between auditors and client firms.\(^4\) In a recent study, Mahieux (2018) highlights a positive externality of NAS in that the potential to hire an auditor for NAS can give the auditor stronger motives to exert audit effort. Bar-Yosef and Sarath (2005) model competing auditors, providing a rationale for well-capitalized auditors to capture a greater share of the audit market when audit failure can result in litigation recoveries from auditors. Gerakos and Syverson (2015) estimate public firms’ demand for differentiated audit services to provide numerical estimates of the potential effects of mandatory auditor rotation or the exit of one of the Big 4 auditors. To our knowledge, no study has examined how competition in the NAS market might affect audit quality.

Empirical evidence on the effects of NAS and NAS restrictions to audit clients on audit quality is mixed.\(^5\) For a focused review of such studies, see Bouwens (2018). While anecdotes and investors and regulatory perceptions of consulting fees eroding audit quality provide support for restrictions on the provision of NAS to audit clients, most studies examining measures of audit quality in large samples fail to find a negative association between consulting fees and audit quality (e.g., DeFond et al., 2002). While some studies find a negative impact of NAS on audit quality, (e.g., Bell et al., 2015; Causholli et al., 2014), others find positive effects, at least for some clients (Kinney et al., 2004; Nam and Ronen, 2012). Our study suggests that the relations between NAS and audit quality can depend on competition in the NAS market, a potential reason why tax-related NAS, for which auditors might face weaker competition, can have different implications from more implementation-related NAS that face greater competition.

Beyond NAS, several theoretical studies have examined the incentives of auditors to deliver high quality audits. Dye (1993) and Ye and Simunic (2013) study the effects of auditing standards on audit quality. Laux and Newman (2010) analyze the incentive effects of legal penalties. Absent contingent contracts, which are generally restricted, incentives for auditors to deliver high audit quality in models where audit quality results from unobservable effort typically either come from reputation benefits or legal liability after an audit failure. In our study, audit quality is an initial investment (e.g., hiring high-quality auditors) observable to clients, and the incentive for higher au-

\(^4\)Chan and Liu (2018) shows that while restricting NAS can restore faithful reporting, it also adversely influences audit effort.

\(^5\)For a recent survey of the related archival auditing literature, see DeFond and Zhang (2014).
dit quality comes from higher audit-related revenues. Our paper adds to the literature by studying how these incentives interact with market features such as client heterogeneity and the potential to provide NAS to both audit and non-audit clients.

Finally, there is a broad literature in industrial organization (IO) studying the costs and benefits of bundling. Bundling is important in our setting because it both provides a mechanism for the auditor to extract additional supplier surplus and is by definition restricted by prohibitions on the provision of NAS to non-audit clients. The traditional explanation for bundling is that it is a tool of price discrimination for a monopolist (Adams and Yellen, 1976), which is true in our setting. Carlton and Waldman (2002) and Nalebuff (2004) show that bundling is a particularly effective entry-deterrent strategy. Although we do not explicitly examine whether NAS and audit bundling affects the competitor’s entry, we complement this literature by studying the effects of bundling on a monopolist’s incentives to invest in the quality of the service that does not face competition. Further, we show that bundling may be a way to commit not to enter into Bertrand competition with a single-product competitor. We think that the audit setting, with its particular institutional detail, may offer an interesting venue for studying questions relevant to IO economists, who often focus on specific industries due to their unique features (e.g., Syverson (2004) in the ready-mix concrete market).

The rest of the paper proceeds as follows. Section 2 describes the baseline model. Sections 3 and 4 provide the main analysis. In section 3, the auditor is a monopolist in both markets. In section 4, the auditor faces competition for NAS. Section 5 provides four extensions to the main model, and section 6 concludes. The extensions examine the implications of additional contractual offerings (mixed bundling), whether the NAS competitor can tell which firms are audited (by our auditor of interest), changing the relative value of audit services and NAS, and allowing NAS to improve both good and bad projects. Proofs are relegated to the appendix.

2 Baseline model

This section describes our model setup and timing. Key assumptions are discussed in Section 2.1. We start from a standard one-firm audit model following Dye (1995) and Laux and Newman (2010) expanded to a continuum of client firms of mass $N > 0$. The board of each client firm
hires a manager to run a project that can be either good \((G)\) or bad \((B)\). A good project yields cash flow \(R\) with probability \(q\) and 0 with probability \(1 - q\). A bad project yields cash flow 0 with probability 1. The prior probability that a project is bad is \(1 - p\) and this probability is also each manager’s private type. We assume that \(1 - p\) is uniformly distributed according to the density \(h\) on \([0, 1 - p]\) and i.i.d. across managers, where the upper bound \(1 - \frac{p}{q}\) is defined below. After hiring the manager, the board observes the manager’s type. Thus, the manager’s type becomes the client firm’s type, which, consistent with Lu and Sapra (2009), we refer to as the “client business risk” or “risk.” The board, after observing the manager’s type, chooses whether to hire the auditor and consultant.

Each manager proposes the project to the board. Each project requires an initial investment \(I\). The board chooses whether to approve the manager’s proposed investment. The manager does not have private information about project quality and always sends the board a favorable report. The auditor’s job, if hired, is to verify the report.\(^6\) It is straightforward to view the project as an “expansion.” The manager proposes the expansion to the board by showing the success of a pilot project summarized in financial reports. The auditor’s job is to assess the validity of the pilot project’s financial reports.

We assume that the upper bound on client risk is \(1 - p = 1 - I / qR\), which implies that \(pqR > I\) is satisfied for all client firms.\(^7\) Hence, as is common in theoretical auditing studies, any client firm prefers to invest in a project in the absence of audit, but would not invest if the audit revealed a bad project. We define

\[
t = \frac{N}{1 - I / (qR)}
\]

such that, for all \(u \in [0, 1 - p]\), \(h(u) = \frac{t}{N}\).

In return for an audit fee \(F_A\), the auditor issues an audit report \(r \in \{b, g\}\) to the board of the client firm. Audit quality is \(a \in [0, 1]\), and is the same for all audited firms. That is, while audit reports are conditionally independent across firms, \(a\) is the same for all firms audited by our

---

\(^6\)This is a standard assumption in the audit literature that helps to focus the analysis on the role of the auditor. For a similar assumption, see, e.g., Lu and Sapra (2009). In many audit models the audit affects the price at which the client firm’s owners might raise capital or sell (a fraction of) the client firm. Because the client firm benefits from an audit even absent these incentives, we suppress them for simplicity. Bae et al. (2017) provide empirical evidence that auditors improve investment efficiency.

\(^7\)A uniform distribution on \([0, 1 - p]\) implies that the CDF is \(H(u) = u / (1 - p)\). We use the strict inequality in \(pqR > I\) because \(P(1 - p = I / qR) = 0\).
The auditor chooses audit quality. Client firms decide to buy auditing services and consulting services. The auditor makes an audit report to client firms that have bought audit services. Client firms decide to invest. In case of investment, consulting services are provided to client firms that have bought NAS. The cash flows of the projects are realized.

Figure 1: Timeline of the baseline model

monopolist auditor (discussed further below). Audit quality determines the probability of detecting a bad project; the audit technology is as follows:

\[ P(r = g \mid G) = 1, \]  
\[ P(r = g \mid B) = 1 - a = 1 - P(r = b \mid B). \]

The cost to the auditor of choosing audit quality \(a\) and providing this audit quality to a measure \(Q_A\) of client firms is \(k(aQ_A)^2/2\), where \(k > 0\) is large enough to maintain \(a \in (0, 1)\). The absolute and marginal costs are increasing in audit quality and in the measure of audit clients served. The cost function implies that the auditor makes an ex ante investment in audit quality and capacity, which we discuss further in Section 2.1.

The auditor may also provide consulting services for a fee \(F_C\). These non-audit services (NAS) increase by \(\tau \in (0, 1 - q)\) the probability that the cash flow of a good firm is \(R\). In particular, a good project with a consultant yields cash flow \(R\) with probability \(q + \tau\) and 0 with probability \(1 - q - \tau\). The parameter \(\tau\) captures consulting quality and the value of NAS to client firms. We denote by \(Q_C\) the measure of NAS clients of the auditor. For simplicity and because our interests lie on the audit side of the market, we assume that the auditor’s cost of providing consulting services is 0.

The timing of the model is summarized in Figure 1. We assume that each firm must choose whether to purchase audit services and NAS simultaneously. This eliminates the possibility of purchasing consulting services conditional on a positive audit outcome. With such a sequence, there would be a clear complementarity between audit and NAS. While such a spillover may be interesting, we do not explore it here.

The utility function of a client firm hiring the auditor with audit quality \(a\) for a fee \(F_A\) and a

---

8 We use the terms “consulting services” and “non-audit services” interchangeably.
9 Mahieux (2018) highlights an audit incentive benefit to having auditors providing NAS contingent on the audit outcome.
consultant for a fee $F_C$ is

$$U_{\text{client}}(1-p) = \underbrace{pqR - I}_{\text{investment return}} + \underbrace{a(1-p)I}_{\text{audit benefit}} + \underbrace{p\tau R - F_A}_{\text{NAS value}} - \underbrace{F_C}_{\text{audit fee}}. \quad (3)$$

Client firms with lower business risk (i.e., higher $p$) are more willing to pay for consulting services but less willing to pay for auditing services. Each client firm, i.e., each board, privately observes its type $1 - p$ prior to buying services. The auditor does not observe $1 - p$ for client firms, and thus cannot price discriminate. Combined with the auditor setting firm-wide quality rather than client-specific quality, this implies that the auditor will have limited ability to price discriminate or screen clients. Similarly, the fixed firm-wide NAS quality precludes price discrimination and screening for NAS.

As the auditor may provide two services, we also analyze the possibility of bundling in which the auditor provides both auditing services of quality $a$ and consulting services in exchange for a fee $F_B$. We defer to the extensions analysis of a setting in which the auditor can offer both a bundle and standalone services. Nalebuff (2004), in an influential paper on bundling and competition, also relegates this “mixed bundling” scenario to an extension due to the limited potential for mixed bundling to improve on pure bundling in the face of competition.\(^\text{10}\)

Finally, we make the following assumptions concerning parameter values:

\[
\begin{align*}
t_k &> I, \quad (4) \\
t_k &> \frac{2pI}{1 - 2p} = \frac{2I^2/(qR)}{1 - 2I/(qR)} > 0 \quad (5) \\
\text{and } \tau R &> I. \quad (6)
\end{align*}
\]

The first assumption in (4) helps keep $a \in [0, 1]$. The second assumption in (5) implies that $p = I/(qR) < 1/2$ and the cost of audit is large enough such that there is no interaction between the audit market and the NAS market in the absence of bundling and competition (see section 3.1). The third assumption in (6) implies the value added of NAS, $\tau R$, is larger than the value added of auditing services for any level of audit quality, i.e., $\tau R > aI$ for all $a \in [0, 1]$. This assumption

\(^{10}\)Indeed, the competitor may use the auditor’s audit contract to create a rival bundle and thereby steal the auditor’s bundle sales. Thus, the audit- and NAS-only contracts need to be priced high relative to the bundle, and generate few additional sales themselves under mixed bundling.
is consistent with the growing concern that consulting is more important than auditing, even to auditors’ revenue streams, as the Big 4 auditors earned nearly three times as much revenue from consulting and tax advice than from providing auditing services (*The Economist*, May 24, 2018).

We relax this third assumption in section 5.3.

2.1 Discussion of the main assumptions

**Audit quality:** Audit quality is an ex-ante investment by the auditor and, therefore, audit quality is the same for all audit clients. We abstract away from ex-post adjustments in audit effort/quality. In some sense, we are capturing the ex ante skill and capacity of auditors, who will adjust optimally to the information they get on the job.\(^{11}\)

One interpretation of the cost of audit quality and capacity, \(k(aQ_A)^2/2\), is that an audit firm needs to hire and train enough high-quality auditors to service the market and it needs to do this before offering to audit client firms. Conditional on a desired level of market-wide quality, hiring and training each auditor is more expensive than the previous one. Similarly, conditional on the size of the firm, quality improvements are increasingly costly as audit quality increases. While we do not model a labor market explicitly, our cost function captures these two important features of the scarcity of human capital or labor inputs necessary to carry out high quality audits. Duguay et al. (2017) provide evidence consistent with labor input scarcity being an important factor in audit markets.

This ex-ante choice of audit quality can also be interpreted as an investment in internal controls. Aobdia (2018) finds a negative association between audit firms’ quality control system deficiencies, mainly performance-related, and audit quality. Further, several studies find that PCAOB inspections improve perceived or actual firm-wide audit quality (see, e.g., DeFond and Lennox, 2017; Fung et al., 2017; Gipper et al., 2017).

Audit quality is observable in our setup, while in practice audit quality for a given engagement may not be observable even after the audit is done. In our setting, firm-wide audit quality, \(a\), can represent reputation and skill based on prior engagements, the capacity to deliver high quality audits, and/or the average quality of auditors assigned to audits, who then make optimizing decisions.

\(^{11}\)This is in line with KPMG UK’s 2017 Annual Report that states, “We invest heavily in delivering and developing our audit offering and this includes significant investment in training and research and development on audit and assurance” (p. 62).
to adjust audit quality. Note that the idea that an auditor can adjust audit quality ex post implies both that the auditor is of sufficient quality (i.e., human capital, prior training) to provide valuable effort, and that she possesses the ability, based on prior ability or training, to make judgments regarding audit scope, client risk factors, and her team’s capacity and skill. In any model involving ex post costly audit effort, a certain measure of ex ante auditor quality is therefore implicit. Our focus is on this ex ante quality.

Technically, client firms in our model are not required to buy audit services from our modeled audit firm. In practice, annual audits are compulsory for publicly traded firms. It is straightforward, however, to interpret unaudited clients in our model as being audited by a low-quality auditor with \((a, F_A) = (0, 0)\). Low quality audits could come from non-Big 4 auditors, in contrast to our focal, higher quality, Big 4 auditor. Empirical evidence that audit quality is higher for the Big 4 audit firms supports this interpretation (Francis and Yu, 2009).

**Price discrimination:** The auditor does not observe the client business risk and offers the same contracts to all the clients. The inability to price discriminate is consistent with Liu and Simunic (2005), who argue that, “client type is unlikely to be contractable because a client’s operating characteristics that determine its type are generally difficult to verify.” In practice, when performing an audit, an auditor uses risk assessment procedures to assess the risk that material misstatement exists. However, this risk assessment is not perfect. Our model applies to a set of client firms that have received the same risk assessment and for which the auditor is not able to detect the business risks. In other words, our model may be of one segment of audit clients, amongst whom audit risk varies but is difficult to gauge.

**Consulting services:** In our main model, low-risk clients place a higher value on NAS, while high-risk clients place a higher value on audit services. NAS add value to good projects but not to bad projects. This is a simplifying assumption to capture the idea that the value added of NAS to good projects is greater than the value added to bad projects. Indeed, the nature of a project is innate and, therefore, a consultant can only marginally (negligibly) improve the value of a project that is bad in nature. In practice, consultants often provide NAS to distressed firms, and NAS might add value to inherently bad projects, too. We analyze a setting in which NAS add value to bad projects in section 5.4.
3 Monopoly in the NAS market

Our analysis begins with a setting in which the auditor is a monopolist in both the audit and NAS markets. We first derive audit quality, audit coverage, and NAS coverage assuming that the auditor offers separate contracts for auditing and NAS. We subsequently derive the same in a scenario where the auditor offers a bundle of both audit services and NAS and discuss the implications of restricting the provision of NAS to audit clients.

3.1 Separate contracts

We start by solving for the equilibrium in which the auditor offers separate contracts for auditing, \((a, F_A)\), and consulting, \(F_C\), i.e., without bundling. We initially assume that there is no restriction on the provision of NAS to audit clients. Hence, a client firm can buy both audit services and NAS. In particular, a client firm with business risk \(1 - p\) buys auditing services if and only if

\[
pqR - I - F_A + a(1-p)I > pqR - I \iff 1 - p > \frac{F_A}{aI}. \tag{7}
\]

Therefore, only client firms with high business risk buy auditing services. The measure of audit clients, conditional on price and quality, is

\[
Q_A = N \left[ H \left(1 - p \right) - H \left(\frac{F_A}{aI}\right) \right] = t \left(1 - p - \frac{F_A}{aI}\right). \tag{8}
\]

A client firm with business risk \(1 - p\) buys consulting services if and only if

\[
p(q + \tau)R - I - F_C > pqR - I \iff 1 - p < 1 - \frac{F_C}{\tau R}. \tag{9}
\]

Hence, only client firms with low business risk buy consulting services.\(^{12}\) The measure of NAS

\[\text{\textsuperscript{12}}\text{Note that a client firm with business risk } 1 - p \text{ buying auditing services buys consulting services if and only if}
\]

\[
p(q + \tau)R - I + a(1-p)I - F_A - F_C > pqR - I + a(1-p)I - F_A. \tag{10}
\]

Condition (10) is equivalent to condition (9). Similarly, a client firm with business risk \(1 - p\) buying consulting services buys auditing services if and only if

\[
p(q + \tau)R - I + a(1-p)I - F_A - F_C > p(q + \tau)R - I - F_C. \tag{11}
\]

Condition (11) is equivalent to condition (7).
clients is given by

\[ Q_C = N \left( H \left( 1 - \frac{F_C}{\tau R} \right) - H(0) \right) = t \left( 1 - \frac{F_C}{\tau R} \right). \] (12)

The maximization problem of the auditor is

\[
\max_{F_C, F_A, a} U_{\text{auditor}} = Q_A F_A \text{audit revenue} + Q_C F_C \text{NAS revenue} - \frac{k(Q_A a)^2}{2} \text{cost of audit}. \] (13)

It is instructive to derive the optimal audit fee, \( F_A \), and the audit market coverage, \( Q_A \), for a given level of audit quality \( a \):

\[
F_A = \frac{1}{2} a I(1 - p) \left( 1 + \frac{tka^2/2}{I + tk} \right), \] (14)

\[
Q_A = t \left( 1 - p - \frac{F_A}{aI} \right) = t \frac{(1 - p)I}{2I + tk}. \] (15)

When choosing audit quality, there is a tradeoff for the auditor between audit fee and audit market coverage as indicated by \( dF_A/da > 0 \) in (14) and \( dQ_A/da < 0 \) in (15). With higher audit quality, the auditor can charge a higher fee for all clients but may lose marginal clients. In contrast, with lower audit quality and greater coverage, the auditor must lower its audit fee and gains extra revenues only from additional audit clients at the margin. Solving for the optima in (13) yields the following proposition:

**Proposition 1** With separate contracts and a monopoly in the NAS market, the auditor offers the auditing contract

\[ (a_{NB}^*, F_A^*) = \left( 1, \frac{1}{2} I(1 - p) \left( 1 + \frac{tk/2}{I + tk/2} \right) \right) \]

and the consulting contract \( F_C^* = \tau R/2 \).

In equilibrium, the auditor chooses the maximum audit quality, i.e., \( a_{NB}^* = 1 \). Hence, with separate contracts and a monopoly in the NAS market, the auditor is better off with a higher audit quality and a lower audit market coverage, given the tradeoff between the two discussed above.

Given those two optimal contracts, the utility of the auditor is

\[
U_{\text{auditor}}^{NB} = t \underbrace{(1 - p)^2 I^2}_{\text{rent from audit clients}} + \underbrace{t \frac{\tau R}{4}}_{\text{rent from NAS clients}}. \] (16)
The utility of the auditor consists of two terms. The first term is the monopoly rent that the auditor earns with the audit contract, and the second term is the rent that the auditor earns with the NAS contract.

The measure of audit and NAS clients are, respectively,

\[
Q_{NB}^A = t \left( \frac{1 - p}{2I + tk} \right) I_2 + tk, \quad \text{and} \quad Q_{NB}^C = t \left( 1 - \frac{F_C^*}{\tau R} \right) = \frac{t}{2}. \quad (17)
\]

What is the impact of a restriction on the provision of NAS to audit clients in this case of separate contracts and a monopoly in the NAS market? We know that

\[
\frac{F_A^*}{a_{NB}^*} = \frac{1 + tk}{2I + tk} (1 - p) > 1 - \frac{F_C^*}{\tau R} = \frac{1}{2} \iff p < \frac{tk}{2(I + tk)},
\]

which is satisfied by the assumption in (5). Hence, there is no overlap in the demands for auditing services and NAS with a monopoly auditor in the NAS market offering separate contracts. Thus, restricting NAS provision to audit clients would have no effect, and we can attribute later results in which NAS restrictions affect audit quality and coverage either to bundled audit-NAS offerings or to competition in the NAS market. The lack of effects of NAS restriction on audit quality, audit fees and consulting fees is summarized in Lemma 1. The equilibrium is represented in Figure 2.

**Figure 2:** Equilibrium with a monopoly in the NAS market and separate contracts

**Lemma 1** If the auditor is a monopolist in the NAS market and offers separate auditing and NAS contracts, a ban on the provision of NAS to audit clients has no impact on audit quality, audit and NAS coverage, and audit and NAS fees.

### 3.2 Pure bundling

We now study a setting in which the auditor offers a bundle of audit services and NAS. As in Nalebuff (2004), we focus our main analysis on pure bundling, i.e., the monopoly auditor offers only one bundling contract \((a, F_B)\) to client firms, rather than offering both the bundle and standalone services. This contract specifies that the auditor provides auditing services of quality \(a\) and NAS
in exchange for a fee $F_B$. We discuss the case of mixed bundling in section 5.1. A client firm with business risk $1 - p$ buys the bundle of services if and only if

$$p(q + \tau)R - I + a(1 - p)I - F_B > pqR - I.$$  \hspace{1cm} (18)

Note that we have assumed $\tau R > I$. Hence, the value added of NAS, $\tau R$, is larger than the value added of auditing services, $aI$, and client firms with low business risk find the bundle more attractive than those with high risk, and are thus willing to pay more. The auditor therefore targets the bundle at these clients. The equilibrium is represented in Figure 3 and the measure of client firms for the bundle is

$$Q_B = \frac{\tau R - F_B}{\tau R - aI}.$$  \hspace{1cm} (19)

Figure 3: Equilibrium with a monopoly in the NAS market and pure bundling

The following proposition gives the optimal bundling contract offered by the monopoly auditor in the NAS market. The corollary immediately following compares audit quality and fees across the separate contracts and pure bundling scenarios.

**Proposition 2** With pure bundling and a monopoly in the NAS market, the optimal bundling contract offered by the auditor is

$$(a_{PB}^*, F_B^*) = \left( \frac{I}{tk}, \frac{\tau^2 R^2 tk}{2\tau R tk - I^2} \right).$$

**Corollary 1** If the auditor is a monopolist in the NAS market, audit quality is smaller with pure bundling than with separate contracts. The bundling fee is lower than the sum of the audit fee and the NAS fee, i.e., $F_B^* < F_A^* + F_C^*$.

Our assumption about the relative value of NAS and audit services implies that it is optimal for the auditor to offer auditing services as an add-on to NAS and target the bundle at client firms who value NAS the most.\(^{13}\) We compare audit coverage under pure bundling to audit coverage

\(^{14}\text{In section 5.3, we examine a case where audits are more valuable than NAS and find that there the auditor who offers a bundled set of services targets client firms who value audit the most.}\)
under separate contracts. With pure bundling, the measure of audit clients is

$$Q_{PB}^B = \frac{t \tau R k}{2 \tau R k - I^2}. \quad (20)$$

**Corollary 2** If the auditor is a monopolist, the measure of audit clients is larger with pure bundling than with separate contracts.

Bundling increases the number (technically, the measure) of audit clients. However, with pure bundling, the audit clients are firms with low business risk whereas with separate contracts, the audit clients are firms with high business risk. Interestingly, bundling leads not only to a change in the measure of client firms procuring audits, but to a shift in the business risk of the client firms buying auditing services.

**Corollary 3** If the auditor is a monopolist in the NAS market, then the measure of NAS clients is larger with pure bundling than with separate contracts.

Corollary 3 follows from the ability of the auditor to attract medium business risk clients with the bundle. These clients place a lower value on NAS, but purchase the bundle because they have intermediate valuations of both auditing and NAS. Finally, the utility of the auditor with pure bundling is

$$U_{auditor}^{PB} = \frac{tkR^2}{2tkR - I^2}. \quad (21)$$

**Corollary 4** Let $tk^\diamond \equiv \left(\frac{2I(\tau R + (1 - p)^2I)}{(4(1 - p)^2 - 1)\tau R}\right)$. If the cost of audit is high, i.e., $tk > tk^\diamond$, a monopoly auditor in the NAS market is worse off with pure bundling than with separate contracts. Otherwise, if the cost of audit is low, i.e., $tk < tk^\diamond$, a monopoly auditor in the NAS market is better off with pure bundling than with separate contracts.

Under pure bundling, the monopoly auditor is not able to charge high audit fees to client firms with high business risk and high consulting fees to client firms with low business risk. Indeed, the monopoly auditor sells the bundling contract to a larger number of client firms. Hence, with auditing costs that are convex in audit coverage, if the cost of audit is high, the monopoly auditor is better off with separate contracts. However, bundling is also a tool that helps reduce heterogeneity in valuations and helps a monopolist earn greater profits (Adams and Yellen, 1976; McAfee et al., 1989). Hence, if the cost of audit is low, the monopoly auditor is better off with pure bundling. In
the next section, we show that there exists another role for bundling in the presence of a competitor in the NAS market.

The main interesting results of this section are that bundling auditing and consulting services may lead to a decrease in audit quality while expanding audit coverage and shifting the type of client firms that procure audits. This result holds in the absence of a NAS-driven conflict of interests, whereby an auditor provides more favorable opinions to firms who also buy NAS from the auditor. The main driver of this result is that bundling causes the auditor to target client firms who place a high value of NAS, i.e., client firms with low business risk, which in turn decreases the incentives for the auditor to choose a high audit quality. Furthermore, a restriction on providing NAS to audit clients can lead to higher audit quality by eliminating the auditor’s ability to offer the audit-NAS bundle targeted at low-risk clients who value NAS more than audits.

4 Competition in the NAS market

We now introduce a competitor in the NAS market and study the impact of NAS competition on both the audit market and the NAS market. We assume that the competitor offers its services at the same time as the auditor. The fee charged by the NAS competitor is denoted by $F'_C$. Note that the competitor only provides NAS and cannot provide auditing services, consistent with the plethora of consulting firms that do not offer audits. For simplicity, the value of NAS provided by the competitor is the same as the value of NAS provided by the auditor, i.e., there is no product differentiation in the NAS market. This and the assumption that the competitor offers its services at the same time as the auditor allow us to focus on how competition in the NAS market per se affects the auditor and audit provision in the economy when the auditor and the competitor compete on equal footing in the NAS market absent NAS restrictions and bundling considerations. As noted above, it also precludes audits from providing value in helping clients determine whether to hire a consultant conditional on audit-based information about project quality.

4.1 Separate contracts

First, we solve the model with the auditor offering separate contracts, i.e., an audit contract $(a, F_A)$ and a NAS contract $F_C$. The competitor offers a NAS contract $F'_C$. There is no restriction on the
provision of NAS to audit clients.

Bertrand competition on the NAS market yields $F_C^* = F_C' = 0$. However, the NAS market and the audit market are separate markets and therefore, audit quality and audit fees are not affected by the competition on the NAS market.

**Lemma 2** With separate contracts and NAS competition, the auditor offers the auditing contract

$$(a^*_{NB}, F_A^*) = \left( \frac{1}{2} I(1 - p) \left( 1 + \frac{tk/2}{I + tk/2} \right) \right).$$

The consulting contract offered by the auditor and the NAS competitor is such that $F_C^* = F_C' = 0$.

With separate contracts and NAS competition, the utility of the auditor is

$$U_{auditor} = t \frac{(1-p)^2 I^2}{2(2I + tk)}$$

and the utility of the competitor is equal to zero, i.e., $U_{competitor} = 0$.

The auditor still earns a positive rent via auditing services. However, the rent in the consulting market is dissipated by competition. The measure of audit clients is equivalent to the case of separate contracts without NAS competition. However, all client firms buy consulting services when the consulting market is competitive because the price is driven to zero. As a result, client firms strictly benefit from NAS competition in the separate contracts scenario.

Assume next that there is a restriction on the provision of NAS to audit clients and that the competitor does not observe whether a client firm buys auditing services. As a result, the competitor cannot price discriminate and offers the same NAS contract to audit clients and non-audit clients. In equilibrium, it should be the case that $F_C' \geq F_C$. Indeed, if $F_C' < F_C$, then all client firms buy NAS from the competitor but the auditor can reduce $F_C$ to attract some client firms who are not also being audited. In contrast, if $F_C' > F_C > 0$, the competitor and the auditor both earn rents from NAS. If the competitor lowered its price below $F_C$, it could capture non-audit clients, but would reduce the rents it earns from audit clients. As a result, client firms have the choice between buying NAS for a fee $F_C$ or a fee $F_C'$, auditing services of quality $a$ for a fee $F_A$ or both NAS for a fee $F_C'$ and auditing services of quality $a$ for a fee $F_A$.\footnote{Client firms may also buy no services but this does not happen in equilibrium (see proof of Proposition 3).}

A client firm with business risk $1 - p$ buys NAS from the competitor and auditing services from
the auditor rather than NAS only from the auditor if and only if

\[ p(q + \tau)R - I + a(1 - p)I - F_A - F'_C > p(q + \tau)R - I - F_C, \]  

which is equivalent to

\[ \frac{F_A + F'_C - F_C}{aI} < 1 - p. \]  

Similarly, a client firm with business risk \( 1 - p \) buys NAS from the competitor and auditing services from the auditor rather than auditing services only from the auditor if and only if

\[ p(q + \tau)R - I + a(1 - p)I - F_A - F'_C > pqR - I + a(1 - p)I - F_A, \]  

which is equivalent to

\[ 1 - p < 1 - \frac{F'_C}{\tau R}. \]  

The equilibrium is represented in Figure 4. In particular, note that we must have \( F_C \) lower than \( F'_C \) but the difference can be infinitesimal. So, we approximate via \( F_C = F'_C \) but the infinitesimal difference will cause the non-audit clients to buy NAS from the auditor.

\[ \text{Figure 4: Equilibrium with competition in the NAS market and NAS restrictions to audit clients} \]

**Proposition 3** With NAS competition and separate contracts, if the regulator bans the provision of NAS to audit clients, there exists \( tk^\dagger \) such that, if the cost of audit is high, i.e., \( tk > tk^\dagger \), the auditor offers the auditing contract

\[ (a^\ast_{NBC}, F^\ast_A) = \left( a^\ast_{NBC} \in (0, 1), a^\ast_{NBC}I \left( 1 - p \right) (a^\ast_{NBC}I + tk(a^\ast_{NBC})^2) + \tau R/2 \right) \times \left( 2a^\ast_{NBC}I + \tau R + tk(a^\ast_{NBC})^2 \right); \]

otherwise, if the cost of audit is low, i.e., \( tk < tk^\dagger \), the auditor offers the auditing contract

\[ (a^\ast_{NBC}, F^\ast_A) = \left( 1, I \left( 1 - p \right) (I + tk) + \tau R/2 \right) \times \left( 2I + \tau R + tk \right). \]
The consulting contract offered by the auditor and the NAS competitor is such that
\[ F_C^* = F_C^{*t} = \frac{1}{2} \tau R \left( 1 - \frac{F_A}{a_{NBC}^* I} \right). \]

The auditor and the consultant engage in Bertrand-like competition, with the consultant’s monopoly for audited client firms sustaining a strictly positive price. We first note that the ban on NAS to audit clients yields a strictly lower audit quality relative to the case in which the auditor offers separate contracts and is free to provide NAS to audit clients. Note that as \( p = I/(qR) \) goes to zero, the optimal audit quality \( a_{NBC}^* \) converges towards \( \tau R I/(\tau R I k - 2 I^2) \).

**Corollary 5** With NAS competition and separate contracts, if the regulator bans the provision of NAS to audit clients, there is a decrease in audit quality.

This result is consistent with the argued positive effect of knowledge spillovers, but results from competitive effects described in further detail after the next corollary.

**Corollary 6** With NAS competition and separate contracts, the measure of audit clients is larger with NAS restrictions to audit clients than without NAS restrictions.

If the regulator bans the provision of NAS to audit clients, the auditor decreases audit quality and increases the measure of audit clients. This expands the set of client firms where the competitor is effectively a NAS monopolist. Furthermore, the set of client firms expands to include lower business risk firms, which causes the competitor to set a higher price. This increases the price that the auditor can charge to the lowest business risk firms. Although it serves fewer, it gains from the higher price it can charge to all.

**Corollary 7** With NAS competition and separate contracts, the auditor is better off with NAS restrictions to audit clients than without NAS restrictions. The competitor is also better off with NAS restrictions to audit clients.

This is an interesting result driven by the fact that NAS restrictions to audit clients allow for positive pricing in the NAS market. Such NAS restrictions allow the competitor to become a monopolist in a subset of the market, i.e., audited client firms. But, since the competitor cannot price discriminate or screen, it must charge the same price to all client firms. This price sets a ceiling on the price the auditor can charge for NAS. Crucially, this ceiling is greater than the Bertrand ceiling of zero, and thus allows the auditor to earn rents from providing NAS to non-audit clients.
Note that with NAS restrictions to audit clients, the auditor faces an additional cost that more audit clients means fewer consulting clients from which the auditor also earns rents. This cost would go away if the audits were public and the competitor could price discriminate based on whether client firms were audited or not. In this case, the auditor and competitor would engage in full Bertrand competition to provide NAS to unaudited clients, driving the NAS fee for these clients down to zero. This would also discourage clients from buying audit services, because being audited indirectly makes NAS more expensive in such a setting. We explicitly analyze a variant of the model with publicly observable audit purchase and competitor price discrimination in section 5.2.

The effects in Corollaries 5, 6, and 7 operate through the effects of competition between the auditor and competitor in the market to provide NAS to non-audit clients. Corollary 8 considers the effect of shutting down this channel via a ban on auditors providing NAS to any clients.

**Corollary 8** With NAS competition and separate contracts, if a ban on the provision of NAS to audit clients is already in place, banning the provision of NAS to non-audit clients results in an increase in audit quality.

This result is particularly interesting because it provides a rationale for the current regulatory concerns regarding the provision of NAS to non-audit clients (Kowaleski et al., 2018; Lisic et al., 2014; *The Economist*, May 24, 2018).

### 4.2 Bundling

Note first that bundling would be prohibited by rules that restrict auditors from providing NAS to audit clients. As before, we restrict our attention to pure bundling and defer mixed offerings to an extension. With the aforementioned limits to price discrimination, the auditor offers a single bundle described by \((a,F_B)\), and the competitor offers a NAS contract \(F'_C\). A client firm with business risk \(1 - p\) buys the bundle of services rather than NAS if and only if

\[
p(q + \tau)R - I + a(1 - p)I - F_B > p(q + \tau)R - I - F'_C. \tag{26}
\]

Recall that the value of auditing services is lower than the value of consulting services, i.e.,
\(aI < \tau R\) for all \(a \in [0, 1]\). Therefore, the maximization problem of the auditor is

\[
\max_{F_B, a} U_{\text{auditor}} = t \left( \frac{\tau R - F_B}{\tau R - aI} - \frac{F_B - F_C'}{aI} \right) F_B - k \frac{a^2}{2} \left( \frac{\tau R - F_B}{\tau R - aI} - \frac{F_B - F_C'}{aI} \right)^2
\]

such that the competitor maximizes its own utility function, i.e.,

\[
F_C' \in \arg \max_{F_C} U_{\text{competitor}} = t \left( \frac{F_B - F_C'}{aI} \right) \left( \text{measure of clients buying NAS from the competitor} \right)
\]

Taking the first-order condition of the maximization problem of the NAS competitor yields

\[
F_C' = F_B / 2.
\]  

(27)

The intuition for this result is that the NAS competitor acts as a monopolist for the client firms that do not buy the bundle. The equilibrium is represented in Figure 5

![Figure 5: Equilibrium with competition in the NAS market and pure bundling](image)

**Figure 5:** Equilibrium with competition in the NAS market and pure bundling

**Proposition 4** Let \(tk^\dagger = \frac{2(I^2 + (\tau R)^2)}{I + \tau R}\). With NAS competition and pure bundling, if the cost of audit is high, i.e., \(tk > tk^\dagger\), the auditor offers the contract

\[
(a^*_{PBC}, F_B^*) = \left( \frac{I}{tk^\dagger}, 1, \frac{\tau R a^*_{PBC}}{\tau R + a^*_{PBC}} \right) = \left( 1, \frac{I}{\tau R + I}, \frac{4I(\tau R - I)}{4I(\tau R - I) + tk(\tau R + I)} \right),
\]

where \(a^*_{PBC}\) is defined by

\[-tkI(a^*_{PBC})^3 + (2I^2 - \tau R tk)(a^*_{PBC})^2 + 2(\tau R)^2 = 0;\]

otherwise, if the cost of audit is low, i.e., \(tk < tk^\dagger\), the auditor offers the contract

\[
(a^*_{PBC}, F_B^*) = \left( 1, \frac{\tau R I}{\tau R + I} \right) = \left( 1, \frac{4I(\tau R - I)}{4I(\tau R - I) + tk(\tau R + I)} \right).
\]

The consultant offers the contract \(F_C^* = F_B^* / 2\).
**Corollary 9** With pure bundling, audit quality is higher, the measure of audit clients is lower, and the auditor is worse off in the presence of a NAS competitor.

Corollary 9 results from the competitor picking off the auditor’s high-value customers. When it faces NAS competition, the auditor serves higher business risk firms, but this clientele expansion does not make up for the lost of low business risk clients. We show next that the auditor would not gain from offering a NAS contract $F_C$ on top of the bundling contract $(a, F_B)$.

**Proposition 5** With NAS competition, if the auditor can offer both a NAS-only contract and a bundling contract, the auditor never offers a NAS-only contract and only offers the bundle.

This result contrasts with the potential for the auditor to gain from offering a NAS-only contract when it acts as a monopolist in both audit and NAS markets, shown explicitly in section 5.1. Indeed, bundling is a way for the auditor to commit not to enter into Bertrand competition with the NAS competitor. However, we show in section 5.1 that the auditor can benefit from being able to offer a standalone auditing contract in addition to the bundle.

Our next result concerns the preference of the auditor between pure bundling and separate contracts when there is no NAS restriction.

**Corollary 10** With NAS competition and without NAS restrictions, the auditor is better off with pure bundling than with separate contracts.

With separate contracts and without NAS restrictions, there is Bertrand competition in the NAS market which implies that the auditor does not get NAS-related rents from any clients. Hence, the auditor is better off with pure bundling because bundling reduces competition in the NAS market.

Finally, we analyze the impact on audit quality of restricting the provision of NAS to audit clients, when the auditor faces NAS competition and would offer a bundle absent these NAS restrictions.

**Corollary 11** With NAS competition, there exists $tk^*>0$, such that, if the cost of audit is high, i.e., $tk>tk^*$, NAS restrictions to audit clients imply a decrease in audit quality, i.e., $a_{PBC}^* > a_{NBC}^*$.

Interestingly, we show that the effect of NAS restrictions to audit clients on audit quality depends on NAS competition. In section 3 we show that a regulatory ban on the provision of NAS to audit clients increases audit quality. In contrast, with NAS competition, such a regulatory ban
Separate contracts | Pure bundling | NAS restrictions to audit clients
---|---|---
NAS monopoly | $a_{NB}^* = 1$ | $a_{PB}^* = I/tk$ | $a_{NB}^* = 1$
NAS competition | $a_{NB}^* = 1$ | $a_{PBC}^* \in (I/tk, 1)$ if $tk > tk^\dagger$ | $a_{NBC}^* \in (0, 1)$ if $tk > tk^\dagger$

Table 1: Audit quality

may decrease audit quality. Table 1 summarizes the equilibrium audit qualities derived across the settings studied thus far.

## 5 Extensions

### 5.1 Mixed bundling

In this extension, we assume that the monopoly auditor offers the following three contracts: $F_C$, $(a, F_A)$ and $(a, F_B)$. In the first contract, the auditor provides NAS in exchange for a fee $F_C$. In the second contract, the auditor provides auditing services of quality $a$ in exchange for a fee $F_A$. Finally, in the third contract, the auditor provides NAS and auditing services of quality $a$ in exchange for a fee $F_B$. Note that audit quality is the same in the auditing contract and in the bundling contract because the auditor chooses a firm-wide audit quality. The equilibrium is represented in Figure 6.

![Figure 6: Equilibrium with a monopoly in the NAS market and mixed bundling](image)

**Proposition 6** With mixed bundling and a monopoly in the NAS market, the auditor offers the consulting contract

$$F_C^* = \frac{\tau R}{2} - \frac{1 - p}{2} \frac{Itk}{2(I + tk)},$$

the auditing contract

$$(a^*, F_A^*) = \left(1, \frac{I}{2} \left(1 - p + \frac{(1 - p)tk(\tau R - I) + tk\tau R}{2(I + tk)(\tau R - I) + Itk}\right)\right)$$
and the bundling contract

\[(a^*, F_B^*) = \left(1, \frac{2\tau R(\tau R - I)(1 + tk) + 2I(\tau R + (1 - p)(\tau R - I))}{4(\tau R - I)(1 + tk) + 2I} \right)\].

The auditor is better off with mixed bundling than with pure bundling or separate contracts. We next analyze the mixed bundling equilibrium in the presence of a NAS competitor. The equilibrium is the same as in the case of pure bundling with a NAS competitor except that the auditor also offers an audit contract \((a, F_A)\). In particular, the auditor does not offer a consulting contract and lets some clients buy NAS at price \(F_C' = F_B/2 > 0\) from the competitor. The equilibrium is represented in Figure 7.

![Figure 7: Equilibrium with NAS competition and mixed bundling](image)

**Proposition 7** With mixed bundling and NAS competition, the auditor offers the auditing contract

\[(a^*, F_A^*) = \frac{1}{2} a^* I \left(1 - p + tk a^* \right) \left(\frac{1 - p}{2I + tk} + \frac{\tau R - a^* I}{\tau R - a^* I} \right) + \frac{\tau R}{\tau R - a^* I} \left(\frac{1 - p}{2I + tk} + \frac{\tau R - a^* I}{\tau R - a^* I} \right) \]

and the bundling contract

\[(a^*, F_B^*) = \frac{2a^* I \tau R(\tau R - a^* I)(2I + tk a^*) + tk a^* I(\tau R + a^* I)(\tau R + (1 - p)(2\tau R - a^* I))}{2(\tau R - a^* I)(2I + tk a^*)(\tau R + a^* I) + tk a^* I(\tau R + a^* I)^2} \].

The auditor does not offer a consulting contract and the competitor offers the consulting contract \(F_C' = F_B/2\). Further, there exists \(tk^\delta\) such that, if \(tk > tk^\delta\), \(a^* \in (0,1)\), and if \(tk < tk^\delta\), \(a^* = 1\).

As previously emphasized, offering a standalone NAS contract would lead to Bertrand competition in the NAS market, which would drive the price of NAS down to zero and reduce rents across the auditor’s services. As discussed in section 2, adding the possibility for the auditor to offer a standalone audit contract does not provide additional insights because the audit contract generates few additional sales.

### 5.2 NAS restrictions when the competitor observes which firms are audited

In this extension, we analyze the impact of a restriction on the provision of NAS to audit clients when the competitor observes which client firms are audited, i.e., audits are “public.” As a result,
the competitor can price discriminate between audit clients and non-audit clients. Note that this requires slightly different timing than in the baseline model because the competitor makes its offer after the client firms have decided whether to buy auditing services. There is Bertrand competition between the auditor and the competitor for the non-audit clients. Hence, the consulting fee for non-audit clients is zero. However, the competitor behaves like a monopolist for audit clients because the auditor cannot provide NAS to audit clients.

A client firm has the choice between buying NAS for a fee of 0, auditing services of quality \( a \) for a fee \( F_A \), or both NAS for a fee \( F'_C \) and auditing services of quality \( a \) for a fee \( F'_A \).\(^{15}\) In particular, a client firm buys NAS at price 0 rather than both services if and only if

\[
p(q + \tau)R - I > p(q + \tau)R - I - F_A - F'_C + a(1 - p)I,
\]

which is equivalent to

\[
\frac{F_A + F'_C}{aI} > 1 - p.
\]

The equilibrium is represented in Figure 8.

![Figure 8: Equilibrium with NAS competition, NAS restrictions to audit clients and public audits](image)

**Proposition 8** With separate contracts and public audit, the auditor offers the auditing contract

\[
(a^*, F_A^*) = \left(1, \frac{\tau R + 2I - 2p(\tau R + I)}{(\tau R + 2I)(4I(\tau R + I) + tk(\tau R + 2I))} \right).
\]

The consulting fee set by the auditor and the competitor for non-audit clients is 0. Further, the competitor offers the NAS contract

\[
F'_C = \tau R I (\tau R + 2I) - p(2I(\tau R + I) + tk(\tau R + 2I))
\]

\[
(\tau R + 2I)(4I(\tau R + I) + tk(\tau R + 2I))
\]

to audit clients.

\(^{15}\)Client firms may also buy no services but this does not happen because any client firm would rather buy NAS for a fee of 0.
Corollary 12 If the auditor offers separate contracts, audits are public and there is a NAS competitor, a restriction on the provision of NAS to audit clients has no impact on audit quality.

With separate contracts, there is Bertrand competition for NAS. With an audit restriction, the auditor can no longer offer NAS to its audit clients, but continues to earn zero rents there. The competitor can now earn rents as a monopolist providing NAS to the audited firms, but this does not affect the auditor.

The utility of the auditor is

\[ U_{auditor} = t \frac{I^2 (2(\tau R + I)(1 - p) - \tau R)^2}{2(\tau R + 2I)(4I(\tau R + I) + tk(\tau R + 2I))} \]

and the utility of the NAS competitor is

\[ U_{competitor} = t \frac{I(\tau R + 2I)(2\tau R + I - \tau RI) + p(3\tau R + 2I)(2I(\tau R + I) + tk(\tau R + 2I))}{(\tau R + 2I)(4I(\tau R + I) + tk(\tau R + 2I))} F_C^* . \]

Further, the measure of audit clients is

\[ Q_A = t \left( 1 - p - \frac{F_A + F_C^*}{aI} \right) = tI \left( \frac{\tau R + 2I - 2p(\tau R + I)}{4I(\tau R + I) + tk(2I + \tau R)} \right) . \]

Corollary 13 With public audits, a restriction on NAS to audit clients lowers audit coverage. The auditor is worse off whereas the competitor is better off.

Buying auditing services is costlier for client firms because an audit client has to pay a higher fee to buy NAS. Hence, the demand for auditing services is smaller than without NAS restrictions. This hurts the auditor that earns rents only from audit clients.

5.3 Low-value NAS

In this extension, we relax the assumption that the value of NAS, \( \tau R \), is larger than the initial investment \( I \). Hence, it may be the case that \( aI > \tau R \) in equilibrium, i.e., the value of auditing services may be larger than the value of consulting services. This has no effects with separate contracts, but has implications for equilibria with bundling.

We begin with the case of pure bundling with no NAS competitor. There are two possible cases
in equilibrium. First, if audit quality is low enough such that $\tau R > aI$, i.e., the value of NAS is larger than the value of auditing services, then client firms with low business risk find the bundle more attractive. This case is similar to the one analyzed in sections 3 and 4.

Otherwise, if audit quality is large enough such that $\tau R < aI$, the value of auditing services is larger than the value of NAS, and client firms with high business risk find the bundle more attractive. The measure of client firms for the bundle is

$$Q_B = t \left(1 - p - \frac{F_B - \tau R}{aI - \tau R}\right).$$

(31)

Figure 9: Equilibrium with a monopoly in the NAS market, pure bundling and $aI > \tau R$

The equilibrium is represented in Figure 9. The following proposition gives the contracts offered by the auditor in those two cases.

**Proposition 9** With pure bundling, the optimal bundling contract offered by the monopoly auditor is as follows.

- If the value of auditing services is lower than the value of NAS, i.e., $tk > \frac{I^2}{(\tau R)}$, then the contract is similar to the one in Proposition 2;
- otherwise, if the value of auditing services is larger than the value of NAS, i.e., $tk < \frac{I^2}{(\tau R)}$, then

$$(a^*_PB, F^*_B) = \left(1, \frac{1}{2} \left(\tau R + (1 - p)(I - \tau R)\right) \left(1 + \frac{tk/2}{I - \tau R + tk/2}\right)\right).$$

Next, we introduce NAS competition. If the value of auditing services is higher than the value of NAS, i.e., $aI > \tau R$, the equilibrium is represented in Figure 10. In particular, note that the NAS competitor acts as a monopolist, i.e., $F'_C = \tau R/2$. The maximization problem of the auditor is similar to the case of pure bundling absent NAS competition.

Figure 10: Equilibrium with competition in the NAS market, pure bundling and $aI > \tau R$
Proposition 10 With NAS competition and pure bundling:

- if \( tk > 2I^2/(\tau R) \), i.e., the value of auditing services is lower than the value of NAS, then the auditor and the consultant offer the same contracts as in Proposition 4;

- otherwise, if \( tk < 2I^2/(\tau R) \), i.e., the value of auditing services is higher than the value of NAS, then the auditor offers the contract

\[
(a_{PB2C}^*, F_B^*) = \left( 1; \frac{1}{2} \left( \tau R + (1 - p)(I - \tau R) \right) \left( 1 + \frac{tk/2}{I - \tau R + tk/2} \right) \right)
\]

and the consultant offers the contract \( F_C^* = \tau R/2 \).

Note that the region \( 1 - p \in [1 - F_C^*/(\tau R); (F_B - \tau R)/(aI - \tau R)] \) exists if and only if \( 1 - p > (2I + tk)/(4I + tk) \), which we assume is satisfied.\(^{16}\) Hence, our results in Sections 3 and 4 are robust to the case \( I > \tau R \) as long as the cost of audit is high enough.

5.4 NAS also add value to bad projects

We analyze a setting in which NAS add value to both good and bad projects. In particular, a bad project with a consultant yields cash flow \( R \) with probability \( \tau \) and 0 with probability \( 1 - \tau \). We assume that \( \tau R < I \).\(^{17}\) As in the main text, we start by solving for the equilibrium in which the auditor offers separate contracts for auditing and consulting. A client firm with business risk \( 1 - p \) buys consulting services if and only if

\[
pqR + \tau R - I - F_C \geq pqR - I \iff \tau R \geq F_C.
\]

Proposition 11 With separate contracts and a monopoly in the NAS market, the auditor offers the auditing contract

\[
(a_{NB}^*, F_A^*) = \left( 1, \frac{1}{2}I(1 - p) \left( 1 + \frac{tk/2}{I + tk/2} \right) \right)
\]

and the consulting contract \( F_C^* = \tau R \).

As in the main text, in equilibrium, the auditor chooses the maximum audit quality, i.e., \( a_{NB}^* = 1 \). Further, all the client firms buy NAS. Next, we derive audit quality, audit coverage, and NAS coverage assuming that the auditor offers a bundle of both audit services and NAS.

\(^{16}\)If this is not the case, the equilibrium is such that \( a^* = 1, F_B^* = (1 - p)(4I + 2tk)/(4I + tk) \) and \( F_C^* = F_B^*/2 = (2I + tk)/(4I + tk) \). We rule out this case for simplicity because it does not provide additional interesting insights.

\(^{17}\)Otherwise, client firms may find it optimal to invest regardless of the audit outcome.
A client firm with business risk $1 - p$ buys the bundle of services if and only if

$$pqR + \tau R - I + a(1 - p)I - F_B > pqR - I,$$

which is equivalent to

$$1 - p > \frac{F_B - \tau R}{aI}.$$ (34)

Therefore, client firms with high business risk buy the bundle.

**Proposition 12** With pure bundling and a monopoly in the NAS market, the optimal bundling contract offered by the auditor is

$$(a^*_P, F_B^*) = \left(1, \frac{1}{2} \left( (1 - p)I + \tau R \right) \frac{I + tk}{I + tk/2} \right).$$

We now introduce a competitor in the NAS market and study the impact of NAS competition on both the audit market and the NAS market. First, we solve the model with the auditor offering separate contracts, i.e., an audit contract $(a, F_A)$ and a NAS contract $F_C$. The competitor offers a NAS contract $F'_C$. There is no restriction on the provision of NAS to audit clients.

Bertrand competition on the NAS market yields $F_C^* = F'_C = 0$. However, the NAS market and the audit market are separate markets and therefore, audit quality and audit fees are not affected by the competition on the NAS market.

**Lemma 3** With separate contracts and NAS competition, the auditor offers the auditing contract

$$(a^*_{NB}, F_A^*) = \left(1, \frac{1}{2} (1 - p) \left(1 + \frac{tk/2}{I + tk/2} \right) \right).$$

The consulting contract offered by the auditor and the NAS competitor is such that $F_C^* = F'_C = 0$.

---

**Figure 11** Equilibrium with competition in the NAS market and NAS restrictions to audit clients

**Proposition 13** With NAS competition and separate contracts, if the regulator bans the provision of NAS to audit clients, the auditor offers the auditing contract

$$(a^*_{NBC}, F_A^*) = \left(1, I(1 - p) \frac{2I + tk}{4I + tk} \right).$$
The consulting contract offered by the auditor and the NAS competitor is such that

\[ F_C^* = F_C^* = (1 - p) \frac{2I^2}{4I + tk}. \]

Lastly, we analyze a setting with NAS competition and in which the auditor offers a bundle of both audit services and NAS.

\[ \begin{align*}
0 & \quad \text{Client firms buy NAS at price } F_C^* \\
\frac{F_B - F_C^*}{a & } & \quad \text{Client firms buy the bundle of services } (a, F_B)
\end{align*} \]

\begin{align*}
\text{Client business} & \quad \text{risk } 1 - p
\end{align*}

Figure 12: Equilibrium with competition in the NAS market and pure bundling

**Proposition 14** With NAS competition and pure bundling, the auditor offers the contract

\[ \left(a_{PBC}^* = 1, F_B^* = 2(1 - p) \frac{2I^2 + tkI}{4I + tk}\right). \]

The consultant offers the contract \( F_C^* = F_B^*/2. \)

This extension shows that for effects on audit pricing, it is important to have some correlation in demand for audit and NAS. Specifically, note that when NAS provides value to good projects and bad projects equally, the value of NAS is independent of client business risk, so it is uncorrelated with the value of audit services. As shown in this section, audit quality remains at 1 regardless of bundling, competition, and bans.

### 6 Conclusion

We study the impact of non-audit services (NAS) on audit quality, audit coverage, audit fees, and NAS pricing. We consider three potential features of the NAS setting relevant to auditors: 1) competition for NAS clients from non-auditors; 2) the potential to bundle audits and NAS; and 3) restrictions on the provision of NAS to audit clients. Our study yields several interesting results, which contribute to our understanding of the interplay between audit and NAS beyond the potential for NAS to impair auditor independence and the countervailing possibility that NAS provides knowledge spillovers that improve audit quality. Instead, our results follow from economic forces related to competition, pricing, and investments in quality.
First, we show that bundling auditing and consulting services may decrease audit quality because bundling increases the measure of audit clients and changes the types of client firms that buy audits in equilibrium. This in turn implies that the monopoly auditor has weaker incentives to deliver high audit quality when audit and NAS are offered in a bundle rather than separately.

Second, we show how regulatory bans on the provision of NAS to audit clients can make the auditor and a NAS competitor better off. Essentially, such restrictions create niches in which each supplier earns monopoly rents where before the rents were dissipated by Bertrand competition.

Third, and perhaps most interesting, we find that the effects of NAS restrictions on audit quality depend on whether the auditor faces competition in the NAS market, the cost of audit quality, and whether NAS for non-audit clients are also restricted. Absent competition, NAS restrictions to audit clients can improve audit quality by eliminating bundling. With competition, NAS restrictions to audit clients change the auditor’s incentives and can encourage it to supply lower-quality audits more broadly. This increases the market niche in which the competitor is a monopolist supplier of NAS, which increases the price of the competitor’s offering. This price in turn sets a ceiling on the price that the auditor can charge for NAS to non-audit clients, such that the auditor benefits from the competitor’s price increase.

Fourth, we show that bundling auditing services and NAS is a way for the auditor to commit not to enter into Bertrand competition with the NAS competitor and to earn NAS rents. Interestingly, NAS restrictions to audit clients provides a similar economic force. Additionally, we develop several setting-dependent equilibrium characterizations that could yield comparative statics applicable in empirical studies.

There are many interesting routes for future research. The model could be extended via the addition of competition in the audit market, multiple NAS competitors, or an explicit labor market in which both auditors and NAS firms compete explicitly to hire talented workers. Features related to auditor independence, knowledge spillovers, and legal liability could be added to examine how these interact with competition in the audit and NAS markets. Additionally, empirical studies could examine how NAS competition manifests in U.S. and non-U.S. capital markets.
Bibliography


Aobdia, D. (2018), ‘Does the Culture of the Largest Audit Firms Influence Their Audit Quality and Efficiency?’. 


Mahieux, L. (2018), ‘Non-Audit Services, Incentives and Audit Quality’.


The Economist (May 24, 2018), Reforming the Big Four.

Appendix: Proofs

Proof of Proposition 1:

The first order condition with respect to $F_C$ yields

$$F_C^* = \frac{\tau R}{2}.$$ \hspace{1cm} (35)

Hence, the new maximization problem of the auditor is

$$\max_{F_A, a} \left( 1 - p - \frac{F_A}{aI} \right) \left( F_A - tk \frac{a^2}{2} \left( 1 - p - \frac{F_A}{aI} \right) \right) + \left( \frac{1}{2} \right) \left( \frac{\tau R}{2} \right).$$ \hspace{1cm} (36)

The first order condition with respect to $F_A$ yields

$$F_A = \frac{1}{2} a I (1 - p) \left( \frac{a I + tk a^2}{a I + tk a^2/2} \right) = \frac{1}{2} a I (1 - p) \left( 1 + \frac{tk a^2/2}{I + tk a^2/2} \right).$$ \hspace{1cm} (37)

Reinjecting into the maximization problem of the auditor yields

$$\max_a \frac{a}{I + tk a/2}.$$ \hspace{1cm} (38)

The objective function is increasing in $a$, which implies that

$$a_{NB}^* = 1.$$ \hspace{1cm} (39)

Taking the optimal audit quality into account, we get

$$F_A^* = \frac{1}{2} I (1 - p) \left( 1 + \frac{tk/2}{I + tk/2} \right).$$ \hspace{1cm} (40)

Proof of Proposition 2:

The value added of NAS is larger than the value added of auditing services. Hence, client firms with low business risk find the bundle more attractive.

The condition (18) is equivalent to

$$1 - p < \frac{\tau R - F_B}{\tau R - aI}.$$ \hspace{1cm} (41)

The measure of audit and NAS clients is

$$Q_B = I \frac{\tau R - F_B}{\tau R - aI}.$$ \hspace{1cm} (42)

Hence, the maximization problem of the auditor is

$$\max_{a, F_B} \frac{\tau R - F_B}{\tau R - aI} \left( F_B - tk \frac{a^2}{2} \frac{\tau R - F_B}{\tau R - aI} \right).$$ \hspace{1cm} (43)
Taking the first order condition with respect to \( F_B \) yields

\[
F_B = \frac{1}{2} \tau R \left( \frac{\tau R - aI + tka^2}{\tau R - aI + tka^2/2} \right).
\]

The maximization problem of the auditor is

\[
\max_a \frac{1}{2 \tau R + tka^2 - 2aI}. \tag{44}
\]

Hence the optimal audit quality in this case is

\[
a^*_P B = \frac{I}{tk}. \tag{45}
\]

Taking this into account, we derive the optimal bundling fee:

\[
F^*_B = \frac{1}{2} \tau R \frac{\tau R}{\tau R - I^2/(2tk)} = \frac{\tau^2 R^2 tk}{2 \tau R tk - I^2}. \tag{45}
\]

**Proof of Corollary 1:**

The inequality \( F^*_B \leq F^*_A + F^*_C \) is equivalent to

\[
\frac{(\tau R)^2 tk}{2 \tau R tk - I^2} \leq \frac{(1-p)I(I+tk)}{2I+tk} + \frac{\tau R}{2},
\]

which is equivalent to

\[
2(\tau R)^2 tk (2I+tk) \leq 2(1-p)I(I+tk)(2\tau R tk - I^2) + \tau R(2I+tk)(2\tau R tk - I^2),
\]

which is equivalent to

\[
I \tau R(2I+tk) \leq 2(1-p)(I+tk)(2\tau R tk - I^2). \tag{46}
\]

Furthermore, we have

\[
2(1-p)(I+tk)(2\tau R tk - I^2) > 2 \frac{2I + tk}{2(I+tk)} (I + tk)(2\tau R tk - I^2).
\]

Hence, (46) is satisfied. □

**Proof of Corollary 2:**

The inequality \( Q^*_B \geq Q^*_A \) is equivalent to

\[
t \frac{\tau R tk}{2 \tau R tk - I^2} \geq t \frac{(1-p)I}{2I+tk},
\]

which is equivalent to

\[
(\tau R tk)(2I+tk) \geq (1-p)I(2\tau R tk - I^2),
\]

36
which is equivalent to
\[ p2I\tau Rtk + \tau R(tk)^2 \geq -(1 - p)I^3, \]
which is satisfied. □

**Proof of Corollary 3:**

The inequality
\[ Q_{PB}^P \geq Q_{NA}^P \]
is equivalent to
\[ \frac{\tau Rtk}{2\tau Rtk - I^2} \geq \frac{1}{2}, \]
which is equivalent to
\[ 2\tau Rtk \geq 2\tau Rtk - I^2, \]
which is satisfied. □

**Proof of Corollary 4:**

The inequality
\[ U_{PB}^E \leq U_{NA}^E \]
is equivalent to
\[ \frac{t}{2} (\tau R)^2 tk \leq \frac{t}{2} (1 - p)^2 I^2 + \frac{\tau R}{4}, \]
which is equivalent to
\[ 2(\tau R)^2 tk(2I + tk) \leq 2(1 - p)^2 I^2 (2\tau Rtk - I^2) + \tau R(2I + tk)(2\tau Rtk - I^2), \]
which is equivalent to
\[ \tau R(2I + tk) \leq 2(1 - p)^2 (2\tau Rtk - I^2), \]
which is equivalent to
\[ 2I(\tau R + (1 - p)^2 I) \leq (4(1 - p)^2 - 1)\tau Rtk, \]
which is equivalent to
\[ tk \geq t\tau^3 \equiv \frac{2I(\tau R + (1 - p)^2 I)}{(4(1 - p)^2 - 1)\tau R}. \]
□

**Proof of Proposition 3:**

There are two possible cases in equilibrium:

- Case (1)
  - \( 0 < 1 - p < \frac{F_A^E - F_C}{a/t} \) buys NAS at price \( F_C \);
  - \( \frac{F_A^E - F_C}{a/t} \leq 1 - p < 1 - \frac{\psi}{\tau R} \) buys NAS at price \( F_C^E \) and audit services \((a, F_A)\);
  - \( 1 - \frac{\psi}{\tau R} < 1 - p < 1 - \frac{\psi}{\tau R} \) buys audit services \((a, F_A)\).
- Case (2)
Let us analyze the two cases separately.

Case (1)

The maximization problem of the auditor is

\[
\max_{F_A, F_C, a} \left( 1 - p - \frac{F_A + F_C' - F_C}{aI} \right) \left( F_A - \left( 1 - p - \frac{F_A + F_C' - F_C}{aI} \right) \frac{ta^2}{2} \right) + \frac{F_A + F_C' - F_C}{aI} F_C
\]

such that the competitor maximizes

\[
F_C' \in \arg \max_{F_C'} \left( 1 - \frac{F_C'}{\tau R} - \frac{F_A + F_C' - F_C}{aI} \right) F_C' \tag{48}
\]

and such that

\[
F_C \leq F_C'.
\]

The solution to this optimization problem is a corner solution:

\[
F_C = F_C'.
\]

Hence, taking the first-order condition with respect to \(F_C'\) yields

\[
F_C = F_C' = \frac{1}{2} \tau R \left( 1 - \frac{F_A}{aI} \right). \tag{49}
\]

Then, the first-order condition with respect to \(F_A\) yields

\[
F_A = \frac{1}{2} aI \left( 1 - p \right) \left( aI + tka^2/2 \right) + \tau R/2 \tag{50}
\]

The maximization problem of the auditor becomes

\[
\max_{a} \frac{(aI + \tau R/2)^2 + p(\tau R tka^2 - aI \tau R - 2(aI)^2) + p^2((aI)^2 - \tau R tka^2)}{2aI + \tau R + tka^2}.
\]

The optimal audit quality is defined by the following first order condition:

\[
H'(a) \equiv (aI + \tau R/2)(2aI^2 + I \tau R - tka \tau R) + p^2(aI + \tau R)(2aI^2 - 2 \tau R tka)
\]

\[
+ p ((aI + \tau R)(2tka \tau R - I \tau R - 4a^2) + (I + tka)aI \tau R) = 0. \tag{51}
\]
We have $H'(0) = I(\tau R)^2/2 - pR(I(\tau R))^2 = I(\tau R)^2(1/2 - p) > 0$. Further, we define the threshold $t_k^\dagger$ such that

$$tk^\dagger = \left(\frac{1}{\tau R/2} + \frac{p(4I^3 + I\tau R + 4I^2) + p^2(I + \tau R)2I^2}{\tau R(I + \tau R/2 + 2p^2(I + \tau R) - 3p - 2p\tau R)}\right).$$

Finally, note that the region $\frac{F_A + F_C'}{aI} < 1 - p < 1 - \frac{F_C'}{\tau R}$ always exists. Indeed, the inequality

$$\frac{F_A + F_C' - F_C}{aI} < 1 - \frac{F_C'}{\tau R}$$

is equivalent to

$$2\frac{F_A}{aI} < 1 + \frac{F_A}{aI},$$

which is always satisfied.

Case (2)

The maximization problem of the auditor is

$$\max_{F_A, F_C, a} \left(1 - p - \frac{F_A}{aI} + \frac{\tau R - F_A - F_C'}{\tau R - aI} - \frac{F_A + F_C' - F_C}{aI}\right) (F_A - tk^\dagger aI^2)\left(1 - p - \frac{F_A}{aI} + \frac{\tau R - F_A - F_C'}{\tau R - aI} - \frac{F_A + F_C' - F_C}{aI}\right) + \frac{F_A + F_C' - F_C}{aI} F_C$$

such that the competitor maximizes

$$F_C' \in \arg \max_{F_C'} \left(\frac{\tau R - F_A - F_C'}{\tau R - aI} - \frac{F_A + F_C' - F_C}{aI}\right) F_C'$$

and such that

$$F_C \leq F_C'.$$

In this case, it is optimal for the auditor to set $F_C = F_C'$. This implies that case (2) is not a possible equilibrium because no client firm has business risk $1 - p$ such that $(F_A + F_C' - F_C)/(aI) < 1 - p < F_A/(aI)$. Hence, the equilibrium is always as in case (1). □

Proof of Corollary 6:

If there is no NAS restrictions, we know that

$$Q_{A}^{NB} = t \frac{(1 - p)I}{2(2I + tk^\dagger)}.$$

Furthermore, if there is NAS restrictions, we have

$$Q_{A}^{NBC} = t \frac{(1 - p)(aI + \tau R) - \tau R/2}{2aI + \tau R + tk^\dagger}.$$
Further, the condition
\[ t \frac{(1-p)(I + \tau R) - \tau R/2}{2I + \tau R + tk} \geq \frac{t}{2} \frac{(1-p)I}{2(2I + tk)} \]
is equivalent to
\[ \frac{tk}{2I + tk} \geq \frac{p}{2}, \]
which is satisfied by assumption. Hence, it is the case that
\[ Q_A^{NBC} \geq Q_A^{NB}. \]

Proof of Corollary 7:
If there is no NAS restrictions, we know that
\[ U_{auditor} = t \frac{(1-p)^2I^2}{2(2I + tk)}. \]
Furthermore, with NAS restrictions, we have
\[ U_{auditor}^{NBC}(a^*_{NBC}) \geq U_{auditor}(a = 1) = t \frac{(1-p)^2I^2}{2(2I + tk)}. \]

Further,
\[ \frac{t}{2} \frac{(1+\tau R/2)^2 + p(\tau Rtk - I\tau R - 2I^2) + p^2(I^2 - \tau Rtk)}{2I + \tau R + tk} \geq t \frac{(1-p)^2I^2}{2(2I + tk)} \]
is equivalent to
\[ (2I + tk)((\tau R/2)^2 + p(1-p)\tau Rtk) + tkI(1-p)\tau R \geq (1-p)^2I^2(2I + tk), \]
which is always satisfied. Therefore,
\[ U_{auditor}^{NBC}(a^*_{NBC}) \geq U_{auditor} = t \frac{(1-p)^2I^2}{2(2I + tk)}. \]

Proof of Proposition 4:
As discussed in the main text, the first-order condition of the maximization problem of the NAS competitor yields
\[ F_C = \frac{F_B}{2}. \] (54)
Reinjecting into the maximization problem of the auditor yields
\[ \max_{F_B, a} \left( \frac{\tau R - F_B}{\tau R - aI} - \frac{F_B}{2aI} \right) \left( \frac{F_B}{2aI} \right) \left( \frac{\tau R - F_B - 1}{\tau R - aI} \right) \] (55)
Taking the FOC with respect to \( F_B \) yields
\[ F_B = \frac{\tau R + aI}{\tau R + aI} \left( 1 + \frac{tka^2}{2aI} \right) \frac{\tau R + aI}{2aI(\tau R - aI)}. \] (56)
Reinjecting into the maximization problem of the auditor yields

$$\max_a \frac{a}{4(\tau R - aI)(\tau R + aI) + tka(\tau R + aI)^2}. \quad (57)$$

The first-order condition of the maximization problem with respect to $a$ yields

$$G'(a) \equiv -tkIa^3 + (2I^2 - \tau Rtk)a^2 + 2(\tau R)^2 = 0. \quad (58)$$

Further, $G''(a) = -3tkIa^2 - 2a(\tau Rtk - 2I^2)$, which is negative for $tk > 2I^2/(\tau R)$. This implies that $G'$ is decreasing for $tk > 2I^2/(\tau R)$.

We have

$$G'(0) = 2(\tau R)^2 > 0$$

and

$$G'(I/(tk)) = -\frac{I^4}{(tk)^2} + \frac{I^2}{(tk)^2}(2I^2 - \tau Rtk) + 2(\tau R)^2.$$  

Hence, if $tk > 2I^2/(\tau R)$, $G'(I/(tk)) > 0$. Furthermore,

$$G'(1) = 0$$

is equivalent to

$$tk = 2\frac{I^2 + (\tau R)^2}{1 + \tau R}.$$

Taking the partial derivative of (58) with respect to $k$ yields

$$\frac{\partial a}{\partial k} = \frac{a^2(\tau R) + tIa^3}{a^2(2I^2 - \tau Rtk) - 3tkIa^2}. \quad (59)$$

Hence $a^*$ is decreasing in $k$ for $tk \geq 2\frac{I^2 + (\tau R)^2}{1 + \tau R}$, and $a^* = 1$ if $tk \leq 2\frac{I^2 + (\tau R)^2}{1 + \tau R}$.

Finally, note that $\frac{\tau R - F_B}{\tau R - a_I} > \frac{F_B - F_C}{a_I}$ is always verified. □

**Proof of Corollary 9:**

The utility of the auditor with pure bundling and no NAS competition is larger than the utility of the auditor with pure bundling and NAS competition if and only if

$$\frac{t}{2} \frac{tk(\tau R)^2}{2tk\tau R - t^2} \geq \frac{2(\tau R)^2I^2a_{PB}^*}{4I((\tau R)^2 - (a_{PB}^*I)^2)} + tk\frac{a_{PB}^*(\tau R + a_{PB}^*I)^2}{2I}.$$  

This latter inequality is always satisfied.

Similarly, the measure of audit clients with pure bundling and no NAS competition is larger than the measure of audit clients with pure bundling and NAS competition if and only if

$$\frac{t}{2} \frac{tk\tau R}{2tk\tau R - t^2} \geq \frac{\tau R}{2(\tau R - a_{PB}^*I) + tk\frac{a_{PB}^*(\tau R + a_{PB}^*I)}{2I}}.$$  

This latter inequality is always satisfied. □
Proof of Corollary 10:

With pure bundling and NAS competition, the utility function of the auditor is

\[ U_{\text{auditor}}(a_{PBC}) = \frac{2(\tau R)^2 I^2 a_{PBC}}{4I((\tau R)^2 - (a_{PBC}I)^2)} + tka_{PBC}(\tau R + a_{PBC}I)^2. \]

Further, we have

\[ U_{\text{auditor}}(a_{PBC}) \geq U_{\text{auditor}}(a = 1) = \frac{2(\tau R)^2 I^2}{4I((\tau R)^2 - I^2)} + tk(\tau R + I)^2. \]

With NAS competition, separate contracts and without NAS restrictions,

\[ U_{\text{auditor}} = \frac{(1 - p)^2 I^2}{2(2I + tk)}. \]

Finally, the inequality

\[ t \frac{2(\tau R)^2 I^2}{4I((\tau R)^2 - I^2) + tk(\tau R + I)^2} \geq \frac{(1 - p)^2 I^2}{2(2I + tk)} \]

is always satisfied. □

Proof of Corollary 11:

First, we have \( G'(\sqrt{\tau R/(tk)}) > 0 \) is equivalent to

\[ (\tau R)^2 + 2I^2 \tau R/(tk) > I(\tau R)^{3/2}/\sqrt{tk}, \]

which is equivalent to

\[ \sqrt{tk}(\sqrt{tk} \tau R - I \sqrt{\tau R}) > -2I^2. \]

This latter inequality is satisfied if \( tk > I^2/(\tau R) \).

Furthermore, \( G'(\sqrt{2\tau R/(tk)}) < 0 \) is equivalent to

\[ 2I^2 \tau R(2 - \sqrt{2\tau Rtk}) < 0. \]

Therefore, if \( tk > 2(I^2 + (\tau R)^2)/(I + \tau R) \), then

\[ a^*_{PBC} \in \left( \sqrt{\frac{\tau R}{tk}}, \sqrt{\frac{2\tau R}{tk}} \right). \]

Further,

\[ H' \left( \sqrt{\frac{\tau R}{tk}} \right) = \left( I \sqrt{\frac{\tau R}{tk}} + \tau R \right) \left( 2I^2 \sqrt{\frac{\tau R}{tk}} + I\tau R - \sqrt{tk(\tau R)^{3/2}} \right) \]

\[ + p \left( \left( I \sqrt{\frac{\tau R}{tk}} + \tau R \right) \left( 2\sqrt{tk(\tau R)^{3/2}} - I\tau R - 4I^2 \sqrt{\frac{\tau R}{tk}} \right) + I\tau R(I + \sqrt{tk\tau R}) \sqrt{\frac{\tau R}{tk}} \right) \]

\[ + p^2 \left( I \sqrt{\frac{\tau R}{tk}} + \tau R \right) \left( \frac{\tau R}{tk} (2I^2 - 2\tau Rtk) \right). \]
Therefore, there exists $tk^*$ such that, for $tk > tk^*$,

$$H' \left( \sqrt{\frac{\tau R}{tk}} \right) < 0.$$ 

Hence, for $tk > tk^*$,

$$a_{NBC}^* < \sqrt{\frac{\tau R}{tk}}.$$ 

As a result, for $tk > tk^*$,

$$a_{NBC}^* < \sqrt{\frac{\tau R}{tk}} < a_{PBC}^*. \square$$

Proof of Proposition 6:

There are two possible cases in equilibrium:

- Case (1)
  - $0 < 1 - p < \frac{F_B - F_C}{aI}$ buys NAS at price $F_C$;
  - $\frac{F_B - F_C}{aI} < 1 - p < 1 - \frac{F_B - F_A}{\tau R}$ buys the bundle of services;
  - $1 - \frac{F_B - F_A}{\tau R} < 1 - p < 1 - p$ buys auditing services.

- Case (2)
  - $0 < 1 - p < \frac{F_B - F_C}{aI}$ buys NAS at price $F_C$;
  - $\frac{F_B - F_C}{aI} < 1 - p < \frac{\tau R - F_B}{\tau R - aI}$ buys the bundle of services;
  - $\frac{\tau R - F_B}{\tau R - aI} < 1 - p < \frac{F_A}{aI}$ buys no services;
  - $\frac{F_A}{aI} < 1 - p < 1 - p$ buys auditing services.

Let us analyze the two cases separately.

Case (1)

The measure of client firms buying the NAS contract is

$$N \left( H \left( \frac{F_B - F_C}{aI} \right) - H(0) \right),$$

the measure of client firms buying the bundling contract is

$$N \left( H \left( 1 - \frac{F_B - F_A}{\tau R} \right) - H \left( \frac{F_B - F_C}{aI} \right) \right),$$

and the measure of client firms buying only the auditing contract is

$$N \left( H \left( 1 - p \right) - H \left( 1 - \frac{F_B - F_A}{\tau R} \right) \right).$$
Therefore, the maximization problem of the monopoly auditor is

$$\max_{F_B, F_A, F_C, a} \quad \frac{F_B - F_C}{aI} F_C + \left( 1 - \frac{F_B - F_A}{\tau R} \right) F_B + \left( 1 - \frac{F_B - F_A}{\tau R} \right) F_A \quad - tk \frac{a^2}{2} \left( 1 - \frac{F_B - F_C}{aI} \right)^2. \quad (64)$$

The first order condition with respect to $F_A$ yields

$$F_A = F_B - \frac{\tau R}{2}. \quad (65)$$

The optimal value of $F_C$ is such that

$$F_C = F_B - \frac{(1-p)tka^2}{1 + tk a^2 / (2aI)}. \quad (70)$$

Reinjecting into the objective function and taking the first order condition with respect to $F_A$ yields

$$F_A = \frac{1}{2} a I \left( 1 - p + tk a + \frac{1}{2} \left( \frac{\tau R - F_B}{\tau R - aI} \right) \right). \quad (71)$$
Reinjecting into the objective function and taking the first order condition with respect to $F_B$ yields

$$F_B = \frac{\tau R(\tau R - aI) + (2\tau R - aI) \frac{aI\text{tk}a}{\tau R + \text{tk}a}}{2(\tau R - aI) + \frac{aI\text{tk}a}{\tau R + \text{tk}a}}. \quad (72)$$

Reinjecting into the objective function yields

$$\max_a 2(\tau R)^2(I + \text{tk}a) - 2a^2 I tk(I - p) + (1 - p)^2 (a^2 I tk(2aI - \tau R) + 2aI(\tau R - aI)(I + \text{tk}a))$$

$$\frac{2(\tau R - aI)(I + \text{tk}a) + \text{tk}a^2}{2(\tau R - aI)(I + \text{tk}a) + \text{tk}a^2}. \quad (73)$$

The objective function is increasing in $a$, which implies that $a^* = 1$. □

Proof of Proposition 7:

The maximization problem of the auditor is

$$\max_{F_A, F_B, a} \left(1 - p - \frac{F_A}{aI}\right) F_A + \left(\frac{\tau R - F_B}{\tau R - aI} - \frac{F_B - F_C}{\tau R - aI}\right) F_B - tk\frac{a^2}{2} \left(1 - p - \frac{F_A}{aI} + \frac{\tau R - F_B}{\tau R - aI} - \frac{F_B - F_C}{\tau R - aI}\right)$$

such that the NAS competitor maximizes

$$F_C' \in \arg \max_{F_C} \frac{F_B - F_C'}{aI} F_C'. \quad (75)$$

The first-order condition with respect to $F_C'$ yields $F_C' = F_B/aI$. Reinjecting into (74) and taking the first order condition with respect to $F_A$ yields

$$F_A = aI \frac{(1 - p)(I + \text{tk}a) + \text{tk}a \left(\frac{\tau R - F_B}{\tau R - aI} - \frac{F_B}{2aI}\right)}{2I + \text{tk}a}. \quad (76)$$

Reinjecting into the objective function and taking the first-order condition with respect to $F_B$ yields

$$F_B = \frac{2aI\tau R(\tau R - aI)(2I + \text{tk}a) + \text{tk}a^2 I (\tau R + aI)(\tau R + (1 - p)(2\tau R - aI))}{2(\tau R - aI)(2I + \text{tk}a)(\tau R + aI) + \text{tk}a(\tau R + aI)^2}. \quad (77)$$

Reinjecting into the objective function, there exists $tk^\delta$ such that, if $tk > tk^\delta$, $a^* \in (0, 1)$, and if $tk < tk^\delta$, $a^* = 1$. □

Proof of Proposition 8:

The maximization problem of the auditor is

$$\max_{F_A, a} \left(1 - p - \frac{F_A + F_C'}{aI}\right) \left(F_A - tk\frac{a^2}{2} \left(1 - p - \frac{F_A + F_C'}{aI}\right)\right)$$

such that the NAS competitor maximizes

$$F_C' \in \arg \max_{F_C} \left(1 - \frac{F_C'}{\tau R} - \frac{F_A + F_C'}{aI}\right) F_C'. \quad (79)$$

Taking the first order condition with respect to $F_C'$ yields

$$F_C' = \frac{1}{2} \left(1 - \frac{F_A}{aI}\right) \left(\frac{\tau RaI}{\tau R + aI}\right). \quad (80)$$

45
The first order condition with respect to $F_A$ yields

$$F_A = \frac{1}{2} a I \left(1 - \frac{p}{2(\tau R + a I)}\right) \left(a I + \left(1 - \frac{\tau R}{2(\tau R + a I)}\right) t k a^2\right). \tag{81}$$

Therefore the maximization problem of the auditor is

$$\max_a \frac{a (2(\tau R + a I)(1 - p) - \tau R)^2}{(\tau R + 2a I)(4I(\tau R + a I) + t k a(\tau R + 2a I))}. \tag{82}$$

The objective function is increasing in $a$. Hence, the optimal audit quality is $a^* = 1$. Finally, note that the region $\frac{F_A + F_C'}{a I} < 1 - \frac{F_C'}{\tau R}$ always exists. Indeed, the inequality

$$\frac{F_A + F_C'}{a I} < 1 - \frac{F_C'}{\tau R}$$

is equivalent to

$$\frac{F_A}{a I} + \frac{1}{2} \left(1 - \frac{F_A}{a I}\right) \frac{\tau R}{\tau R + a I} < 1 - \frac{1}{2} \left(1 - \frac{F_A}{a I}\right) \frac{a I}{\tau R + a I},$$

which is equivalent to

$$\frac{F_A}{a I} < 1,$$

which is always satisfied. □

**Proof of Corollary 13:**

Without NAS restrictions, we have

$$Q_A = t \frac{(1 - p)I}{2I + tk}$$

and, with NAS restrictions to audit clients and public audits,

$$Q_A = t I \frac{\tau R + 2I - 2p(\tau R + I)}{4I(\tau R + I) + tk(\tau R + 2I)}.$$

The condition

$$t \frac{(1 - p)I}{2I + tk} \leq t I \frac{\tau R + 2I - 2p(\tau R + I)}{4I(\tau R + I) + tk(\tau R + 2I)}$$

is never satisfied.

Furthermore, without NAS restrictions, we have

$$U_{auditor} = t \frac{(1 - p)^2 I^2}{2I + tk}$$

and, with NAS restrictions and public audits,

$$U_{auditor} = t \frac{(2(\tau R + I)(1 - p) - \tau R)^2 I^2}{2(\tau R + 2I)(4I(\tau R + I) + tk(\tau R + 2I))}.$$
The condition
\[
\frac{t}{2} \frac{(1-p)^2 I^2}{2I + tk} \geq \frac{t}{2} \frac{(2(\tau R + I)(1-p) - \tau R)^2 I^2}{2(\tau R + 2I)(4I(\tau R + I) + tk(\tau R + 2I))}
\]
is always satisfied. □

**Proof of Proposition 9:**

*First case \( \tau R > aI \)*

Similar to the proof of Proposition 2.

*Second case \( \tau R < aI \)*

The value added of auditing services is larger than the value added of NAS. Hence, client firms with high business risk find the bundling contract more attractive.

In this second case, condition (18) is equivalent to

\[
1 - p > \frac{F_B - \tau R}{aI - \tau R}.
\]  \hspace{1cm} (83)

The measure of audit and NAS clients is

\[
Q_B = t \left( 1 - p - \frac{F_B - \tau R}{aI - \tau R} \right).
\]  \hspace{1cm} (84)

Hence, the maximization problem of the auditor is

\[
\max_{a, F_B} \left( 1 - p - \frac{F_B - \tau R}{aI - \tau R} \right) \left( F_B - \frac{tka^2}{2} \left( 1 - p - \frac{F_B - \tau R}{aI - \tau R} \right) \right)
\]  \hspace{1cm} (85)

such that

\[
a > \tau R/I.
\]

The first order condition with respect to \( F_B \) yields

\[
F_B = \frac{1}{2} \left( \tau R + (1-p)(aI - \tau R) \right) \left( \frac{aI - \tau R + tka^2}{aI - \tau R + tka^2/2} \right).
\]  \hspace{1cm} (86)

Hence, reinjecting this expression into the objective function yields

\[
\max_a \left( \frac{\tau R + (1-p)(aI - \tau R)}{aI - \tau R + tka^2/2} \right)^2
\]  \hspace{1cm} (87)

such that

\[
a > \tau R/I.
\]

Solving for the optimal audit quality yields the corner solution

\[
a_{PB2}^* = 1.
\]  \hspace{1cm} (88)
Taking this into account, we derive the optimal bundling fee:

\[
F_B = \frac{1}{2} \left( \tau R + (1 - p)(I - \tau R) \right) \left( 1 + \frac{tk/2}{I - \tau R + tk/2} \right). \tag{89}
\]

Finally,

\[
U_{\text{auditor}}^{PB} \geq \frac{t}{2} \frac{I^2}{2I - 2\tau R + tk} \geq \frac{t}{2} \frac{\left( \tau R + (1 - p)(I - \tau R) \right)^2}{2I - 2\tau R + tk} = U_{\text{auditor}}^{PB2},
\]

Therefore, the equilibrium is \( aI < \tau R \) if and only if \( tk > I^2/(\tau R) \). □

**Proof of Proposition 10:**

**First case \( \tau R > aI \)**

As in the proof of Proposition 4, the first-order condition of the maximization problem with respect to \( a \) is

\[
G'(a) = -tkIa^3 + (2I^2 - \tau Rtk)a^2 + 2(\tau R)^2 = 0. \tag{90}
\]

We have

\[
G'(0) = 2(\tau R)^2 > 0
\]

and

\[
G'(I/(tk)) = -I^4 (tk)^2 + I^2 (tk)^2 (2I^2 - \tau Rtk) + 2(\tau R)^2.
\]

Hence, if \( tk > 2I^2/(\tau R) \), \( G'(I/(tk)) > 0 \). Furthermore,

\[
G'(\tau R/I) = -tk \frac{(\tau R)^3}{I^2} - (\tau R)^2 \frac{tk}{I^2} + 4(\tau R)^2.
\]

Therefore, \( G'(\tau R/I) < 0 \) if and only if \( tk > 2I^2/(\tau R) \). Finally, we have

\[
G''(a) = -3Ika^2 + 2(2I^2 - \tau Rtk)a,
\]

which is negative if \( tk > 2I^2/(\tau R) \). This implies that the function \( G' \) is decreasing in \([0, \tau R/I]\) if \( tk > 2I^2/(\tau R) \). Hence, if \( tk > 2I^2/(\tau R) \), the optimal audit quality \( a^* \in (I/tk, \tau R/I) \).

**Second case \( \tau R < aI \)**

As discussed in the main text, the equilibrium contract of the auditor is similar to the one in the case of pure bundling without NAS competition. The NAS competitor acts as a monopolist on the NAS market and chooses \( F_C' = \tau R/2 \).

Finally, we have

\[
U_{\text{auditor}}^{PB} (a) = \frac{t}{4I(\tau R + aI)(\tau R - aI) + tk(\tau R + aI)^2},
\]

and

\[
U_{\text{auditor}}^{PB} (a^*_{PB}) \geq U_{\text{auditor}}^{PB} (a = \tau R/I) = \frac{tt^2}{2tk}.
\]

Furthermore,

\[
\frac{t^2}{2tk} \geq U_{\text{auditor}}^{PB2} = \frac{t}{2} \frac{\left( \tau R + (1 - p)(I - \tau R) \right)^2}{2I - 2\tau R + tk}
\]
Proof of Proposition 11:
The proof to derive the equilibrium audit contract is similar to the proof of Proposition 1. Further, a client firm with business risk $1 - p$ buys consulting services if and only if $F_C \leq \tau R$. Hence, the auditor offers the NAS contract $F_C^* = \tau R$ and all the client firms buy NAS. □

Proof of Proposition 12:
The maximization problem of the auditor is

$$\max_{a, F_B} \left( 1 - p - \frac{F_B - \tau R}{aI} \right) \left( F_B - \frac{tka^2}{2} \left( 1 - p - \frac{F_B - \tau R}{aI} \right) \right).$$

The first order condition with respect to $F_B$ yields

$$F_B = \frac{1}{2} \left( aI + tka^2 \right) \left( \frac{(1 - p)aI + \tau R}{aI + tka^2/2} \right).$$

Hence, reinjecting this expression into the objective function yields

$$\max_a \left( \frac{(1 - p)aI + \tau R)^2}{2aI + tk^2} \right).$$

Solving for the optimal audit quality yields the corner solution

$$a_{PB}^* = 1.$$  □

Proof of Proposition 13:
The maximization problem of the auditor is

$$\max_{a, F_A, F_C, F'_C} \left( F_A + F'_C - F_C \right) \left( 1 - p - \frac{F_A + F'_C - F_C}{aI} \right) \left( F_A - \frac{tka^2}{2} \left( 1 - p - \frac{F_A + F'_C - F_C}{aI} \right) \right).$$

such that

$$F'_C \in \arg \max_{F'_C} \left( 1 - p - \frac{F_A + F'_C - F_C}{aI} \right).$$

As discussed in the main text, in equilibrium, it must be the case that $F_C = F'_C$. The first order condition with respect to $F'_C$ yields

$$F'_C = aI \left( 1 - p - \frac{F_A}{aI} \right).$$

Hence, reinjecting this expression into the objective function and taking the first-order condition with respect to $F_A$ yields

$$F_A = (1 - p)aI \frac{2I + tk^2}{4I + tk^2}.$$ 

Solving for the optimal audit quality yields the corner solution

$$a_{NBC}^* = 1.$$ □
**Proof of Proposition 14:**

The maximization problem of the auditor is

\[
\max_{a,F_B} \left( 1 - p - \frac{F_B - F_C^*}{aI} \right) \left( F_B - \frac{tka^2}{2} \left( 1 - p - \frac{F_B - F_C^*}{aI} \right) \right)
\]  

such that

\[
F_C^* \in \arg \max_{F_C} \frac{F_B - F_C^*}{aI} F_C^*
\]

The first order condition with respect to \( F_C^* \) yields

\[
F_C^* = F_B/2.
\]

Hence, reinjecting this expression into the objective function and taking the first-order condition with respect to \( F_B \) yields

\[
F_B = 2(1 - p)aI + tka.
\]

Solving for the optimal audit quality yields the corner solution

\[
a_{PBC}^* = 1.\Box
\]