The Effect of Analyst Coverage on Corporate Voluntary Disclosure, Price Efficiency and Liquidity

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Very Preliminary and Incomplete

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1 Introduction

Voluntary corporate disclosure is one of the major sources of information in capital markets. The vast theoretical literature on voluntary disclosure has focused on settings that consider a single information provider. However, in practice, corporate disclosure environment is complex and often characterized by additional agents who may discover the firm's private information and disclose it to the public. Financial analysts are one example of such agents. The first question we address in this paper is how the presence of an additional potential source of information that can discover and disclose the firm's private information, such as financial analysts, affects the firm's voluntary disclosure policy? Our model demonstrates that analyst coverage crowds-out corporate voluntary disclosure, i.e., firms respond to an increase in analyst coverage by decreasing the amount of information they disclose.

The second and more challenging question that we address in this paper regards the overall amount of information available to the market and the informational content of market prices, which are among the most important and studied topics in Finance and Accounting. In particular, we study how the extent of analyst coverage (or more generally, the probability that the firm's private information is discovered and publicly disclosed by any source of information) affects the overall information available to the market. Given our finding that analyst coverage crows-out firm's voluntary disclosure, the overall effect of an increase in analyst coverage on the overall amount of publicly available information is not clear. We use two separate measures to capture the overall information available to the market. First, we suggest a theoretical measure of price efficiency that is increasing in the expected deviation of the realized price from the true fundamental value. Then, to get an observable measure that reflects the extent of information asymmetry in the market, we use the expected bid-ask spread. We do so by introducing a trading stage a la Glosten and Milgrom (1985), that follows the disclosure stage by the manager and the analyst. The trade is affected by the realization of the disclosure stage. Our measure of illiquidity is the expected bid-ask spread of the firm's stock.

Our model demonstrates that both of our measures, price efficiency and the liquidity, are affected in the same direction following an increase in analyst coverage. In particular, an increase in analyst coverage increases price efficiency and decreases the illiquidity the firm's stock.

Since most of the theoretical literature (with a few notable exceptions that we discuss below) focuses on the effect of a single informed agent on public information and prices, our model contributes to better understanding of informational environment in more complex and realistic settings. Better understanding of the overall informational effect of changes in one agent's behavior, may provide insight that are useful for regulatory decisions. For example, given that analyst coverage crowds-out corporate voluntary disclosure, it is ex-ante not clear whether regulations, or other incentives that are designed to increase analyst coverage (or increase other sources of public information) are beneficial or detrimental to the overall information that will become available to the public. Our paper contributes to this question by showing that in our setting an increase in analyst coverage has a positive overall effect.

Our model's starting point is a standard voluntary disclosure setting with uncertainty about information endowment (a la Dye 1985 and Jung and Kwon 1988). In particular, a firm's manager who wishes to maximize the market's beliefs about the firm's value, may learn a private signal about the firm's value. If an information event occurs, and the manager learns the realization of the private signal, she can voluntarily disclose it to the market. As standard in this literature, disclosure is assumed to be costless and truthful/verifiable, the probability that the manager is informed is independent of the signal's value, and an uninformed manager cannot credibly convey its lack of information to the market.

Our model's novelty relative to the previous literature is in introducing an additional agent, say a financial analyst, who may learn the firm's private information, even if the firm does not disclose this information. Financial analysts constantly search for information about the firms they cover, and with some probability they succeed in discovering the firm's private information. When discovering such private information financial analysts disclose it to the market. Our model is not constrained to financial analysts and can be applied, without affecting the results of our model, to any alternative mechanism that induces stochastic public supply of the firm's private information. Such alternative mechanisms could be through media, competitors, suppliers, social media, regulators etc.

If the manager obtains private information, analysts learn this information with some positive probability. If an analyst discovers the manager's private information, she publicly report it. Neither the analyst nor the market know for certain whether an information event occurred or not, unless the analyst is informed. The probability with which the analyst discovers the manager's private information captures the quality and/or the quantity of analysts that cover the firm, and we refer to it as "analyst coverage." The manager seeks to maximize the expectation of the public beliefs of the firm value, which will be the firm's price, at the end of the reporting game - following the potential disclosure by both the manager and the analysts.

We first show, as expected, that there exists a unique equilibrium to our disclosure game. The manager's equilibrium strategy is characterized by a disclosure threshold, such that the manager discloses her private signal only when it is greater than this threshold. We then show that greater analyst coverage (higher probability that the analyst discovers the manager's private information) is associated with a higher equilibrium threshold, that is, with less disclosure. In other words, analyst coverage crowds-out managerial voluntary disclosure.

The intuition for this result is as follows. Following an increase in analyst coverage, no disclosure by the manager is less of a bad signal, since now the likelihood that the manager is informed and actively withholding bad news is lower. As such, the price given no disclosure is higher for greater analyst coverage, which gives the manager a greater incentive to withhold information, yielding a lower disclosure threshold. This result, which is new to the theoretical literature, is consistent with the empirical evidence in Balakrishnan et al. (2014) who document an increase in corporate voluntary disclosure following an exogenous decline in analyst coverage.

Similar to the standard Dye (1985) setting, an increase in the likelihood that the manager is informed decreases the equilibrium disclosure threshold and the price given no disclosure.

An increase in analyst coverage has a direct effect of increasing the probability that the manager's private information will be disclosed by analysts, but also has an indirect effect of decreasing the probability that the manager will voluntarily disclose her information – crowding out managerial disclosure. We want to analyze the overall effect such an increase on informational environment and on the overall information that becomes publicly available. We first show that the probability that the manager's signal gets publicly disclosed, either by the manager or the analyst, is increasing in analyst coverage. This probability however, captures only part of the informational effect of analyst coverage. The reason is that changes in analyst coverage also affect the precision of the public beliefs about the firm value when no disclosure is made. To capture the entire informational effect of analyst coverage, we measure the quality of public information using the expected squared deviation between actual price (which is the best estimate given public information) and the true fundamental value. This measure, which we refer to as Price Efficiency (PEF), captures the expected squared "pricing error." We show that an increase in analyst coverage always increases price efficiency, i.e., the crowding-out effect is only a second order effect. Our price efficiency measure is very appealing from a theoretical stand point, however, it is not trivial to estimate it empirically.

Finally, to further study the informational implication of analyst coverage, and to use an observable measure that reflects the extent of information asymmetry in the market, we want to study the effect of analyst coverage on the expected bid-ask spread. To do that,we extend our model by introducing an additional stage following the disclosure stage, in which the firm's stock is traded in a Glosten and Milgrom (1985) type of model. In this model, the trader can be either an informed trader who knows the firm's true value, or a liquidity trader whose trade is unrelated to the firm value or price. The market maker is not informed, and only has the public information available at the end of the disclosure game. We derive the bid-ask spread of this trading game and use the expected bid-ask spread as our measure of illiquidity of the firm's stock. We show that an increase in analyst coverage increases the liquidity (decreases illiquidity) of the firm's stock.

Unlike the theoretical literature, the empirical literature has studied the effect of analyst coverage on firm's voluntary disclosure and on the liquidity of the firm's stock. For example, Kelly and Ljungqvist (2012) show that following an exogenous decrease in the supply of public information, due to closing of brokerage houses that resulted in decreased analyst coverage, the affected firms' information asymmetry increased and their stocks' liquidity decreased.

Balakrishnan et al. (2014) use the exogenous shock to analyst coverage that was identified in Kelly and Ljungqvist (2012) to establish a causal effect of decrease in analyst coverage on firms' voluntary disclosure. They show that one quarter following the decrease in analyst coverage, the effected firms increase their voluntary disclosure (earning guidance) to mitigate the increased information asymmetry and the decreased liquidity. This increased disclosure partially reverses the decrease in liquidity, although the overall effect remains negative, consistent with our model's prediction.

There is a more extensive empirical literature that studies how disclosure and transparency affect informational environment in general and in particular the bid-ask spread. While the results are mixed, many papers find that increased disclosure increases informativeness and decreases bid-ask spread (e.g., Welker 1995; Healy, Hutton, and Palepu, 1999; Leuz and Verrecchia, 2000; Heflin, Shaw, and Wild, 2005).

We believe that by studying voluntary disclosure in the presence of potentially informed trader, our paper contributes to two separate streams of the theoretical literatures. The first stream is the voluntary disclosure literature. To the best of our knowledge, there are only few theoretical papers that study voluntary disclosure in the presence of potentially informed trader/receiver. Langberg and Sivaramakrishnan (2008, 2010) offer two models with a firm that can voluntary disclose information and strategic analysts that scrutinize the quality of the firm's disclosure. In their equilibrium, managers voluntarily disclose unfavorable information only if it is sufficiently precise, and disclose favorable news even when it is less accurate. In these papers, by construction, greater firm disclosure encourages the analysts to obtain more information. Dutta and Trueman (2002) study a setting in which firm's manager can credibly disclose verifiable private information, but not how to interpret this information. Since the manager is uncertain whether the market will interpret the disclosed information as good or bad news, the manager faces uncertainty regarding the market reaction. In this setting, Dutta and Trueman show that the equilibrium disclosure strategy is not necessarily a threshold strategy.

The second related stream of literature studies how changes in one source of information affects information acquisition incentives by other parties, and the resulting overall effect on information available. Goldstein and Yang (2017) present a noisy REE model with a public signal that can be interpreted either as corporate mandatory disclosure or as disclosure by a third party, such as an analyst. They show that when keeping traders' information constant, a more precise public signal improves market liquidity and price efficiency. However, better public information undermines the incentives of traders to acquire information, and thus the overall effect is ambiguous and depends on the measure used to identify market quality. By contrast, we endogenize the corporate disclosure decision and allow for voluntary rather than mandatory disclosure. Another related paper is Gao and Liang (2013), which studies how firm's commitment to disclosure policy affects investors incentives to acquire information. Their focus is on the feedback effect, by which the firm can benefit from the information provided from prices.

The remainder of the paper is organized as follows. In the next section, we describe the setting of our model. Section 3 derives the equilibrium of our disclosure game. In Section 4 we introduce our measure of price efficiency and study how analyst coverage affects the firm's voluntary disclosure and the overall price efficiency. In Section 5 we study how the liquidity of the firm, in a Glosten Milgrom (1985) setting is affected by analyst coverage.

2 Setting

Corporate disclosure environment is complex and often involves multiple players. In this paper, we study how an introduction of an analysts to a standard corporate/managerial voluntary disclosure setting affects the manager's disclosure strategy, the overall information available to the market and the liquidity of the firm's stock. Our starting point is a standard voluntary disclosure setting with uncertainty about information endowment (a la Dye 1985 and Jung and Kwon 1988) that has been studied extensively. In particular, consider a firm that is involved in a project, e.g., drug development, which may eventually succeed or fail. Denote the ultimate binary outcome of the project by $\tilde{x}$, and without loss of generality assume that $\tilde{x} \in \{0, 1\}$. The ex-ante probability of success is $p_0 \equiv \Pr(x = 1)$ and the probability of failure is $(1 - p_0) \equiv \Pr(x = 0)$.[1]

Further assume that the actual probability of success of the project is given by s, where s is the realization of a random variable $\tilde{s} \in [0, 1]$, with a pdf $f(s)$, a cdf $F(s)$, and $E[\tilde{s}] = p_0$. The firm's manager is not always endowed with the information about s. The manager may privately learn s with probability $q \in [0, 1]$ and with probability $1 - q$ the manager does not learn the realization of s. The event of information endowment to the manager is independent of the realization of s. We sometime refer to the manager being privately informed about s as an information event. Such an event could be, for example, the results of a clinical trial or of oil exploration. If the manager obtains a private signal, she learns the

[1] Note that the results of our disclosure game hold for any distribution of the firm value, both bounded and unbounded support. We use this particular structure only for simplicity of the trading stage, which uses a Glosten and Milgrom (1985) setting.

updated probability of success. The expected value of s (or x) conditional on an information event equals the expected value conditional on no information event and equals p_0.

If the manager learns the realization of the private signal s, she can voluntarily disclose it to the market. Disclosure is assumed to be costless and the disclosed value is truthful (verifiable at no cost). As standard in the voluntary disclosure literature, if the manager does not obtain the private signal, she cannot credibly convey that she is not informed. The manager wants to maximize the market's expectation about the firm's value, $E[x]$. The above is a standard voluntary disclosure setting with uncertainty about information endowment that has been studied extensively.

Corporate disclosure environment is often characterized by additional agents who may learn the firm's private information even if the firm does not disclose this information. Financial analysts, which are one of the major sources of information in capital markets, are such agents. They constantly search for information about the firms they cover, and with some probability they succeed in discovering the private information that firms obtain. When discovering such private information financial analysts disclose it to the market. Any alternative mechanism that induces stochastic public supply of the firm's private information, such as media, competitors, suppliers, social media, regulators etc., will have a similar effect in our model.

We add to the above standard setting of voluntary disclosure the presence of financial analysts. The manager seeks to maximize the beliefs about the firm value (price) at the end of the reporting game - following the potential disclosure by both the manager and the analysts. Following an information event, analysts may learn the outcome of the information event (the manager's private signal), s, with probability r. If an analyst discovers the manager's private information he publicly reports it. We abstract from potential incentives by the analyst to bias his forecast and assume truthful disclosure by the analyst.[2] When no information event occurs, the analyst does not discover any information, and hence does not issue a report. The analyst does not know whether an information event occurred or not, unless he is informed.

We assume that the probability that the analyst discovers the manager's signal is independent of the realization of the signal. The parameter r represents the quality and/or the

[2] It is immediate to see that all of our results are robust to an analyst's reporting strategy that is potentially biased, as long as the analyst always issues a report when obtaining information and the analyst's forecast follows a separating strategy. For an example and additional references see Beyer and Guttman (2011).

quantity of analysts that cover the firm. Henceforth we refer to r as "analyst coverage."

To summarize our disclosure game, the timeline is as follows.

1. With probability q an information event occurs and the manager privately learns the signal s.

2. If the manager is informed, she decides whether to voluntarily disclose the signal s to the market.

3. If an information event occurred, the analyst learns the signal s with probability r, in which case the analyst discloses it to the market.[3]

4. Following the disclosure or lack of disclosure by both the manager and the analyst, investors update their beliefs about the expected value of the project, $E[\tilde{x}]$, which we will refer to as the price, and denote it by P.

The setting and all the parameters of the model are common knowledge.

While the presence of analysts brings additional information to the market through analysts disclosure in some states of nature, we show that the presence of analysts also crowds out voluntary disclosure by the firm. Which of the above opposite effects on the informational environment dominates, and under what conditions, is a harder question to answer. Our main research question is the impact of the presence of analysts, as captured by r, on the overall amount of information that will be available to the market. We measure the overall amount of information available to the market using two different measure. We first study the effect of analyst coverage on a measure of price efficiency that is decreasing in the expectation of the squared difference between the price, P, and the fundamentals, s (Section 4). In Section 5 we study how our measure of liquidity of the firm's stock price is affected by analyst coverage.

Unlike the theoretical literature, the empirical literature has studied the effect of firm's voluntary disclosure and analyst's following on the liquidity of the firm's stock. For example, Kelly and Ljungqvist (2012) show that following an exogenous decrease in supply of public

[3] Whether stage 3 is after, before, or contemporaneous to stage 2 does not affect our results. Of course, if the analyst publishes a report before the manager makes her disclosure decision, the manager will be indifferent whether to disclose or not. However, the manager's strategy in case of no-report by the analyst will be similar to her strategy in a model where corporate disclosure precedes the analyst report.

information, due to closing of brokerage houses that resulted in decreased analyst's coverage, affected firms' information asymmetry increased and their stocks' liquidity decreased. Balakrishnan et al. (2014) show that following decrease in supply of public information, due to the same exogenous shock to analyst coverage identified in Kelly and Ljungqvist (2012), the effected firms increase their voluntary disclosure to mitigate the increased information asymmetry and the decreased liquidity.

In order to provide a theoretical foundation to, and prediction for, the effect of changes in the supply of public information on liquidity measures, and to further check the robustness of our price efficiency measure, we also study a market in which the firm's stock is traded. In particular, we study a stylized Glosten Milgrom (1985) microstructure model in which the trader in the firm's stock may be either an uninformed liquidity trader or an informed trader that perfectly knows the firm value, x. The details of this setting are deferred to Section 5.

We start our analysis by deriving the equilibrium of the disclosure game and characterizing the manager's reporting strategy as a function of the model's parameters. We then show that the price efficiency increases in the probability of the analyst being informed, r (Section 4). Finally, in Section 5 we analyze the trading game to study how liquidity is affected by the probability of the analyst being informed, r.

3 Equilibrium of the Disclosure Game

The manager seeks to maximize the expected price $P = E\left[\tilde{x} \mid I\right]$, where I is all the information available to the market at the end of the disclosure game. In case the manager and or the analyst disclose s, the price of the firm equals its expected value

$$P\left(D = s\right) = E\left[\tilde{x}|\right] = s,$$

where D indicates that there was a disclosure.

Denote the event that no disclosure was made by either the manager or the analyst by ND, and the price given no disclosure by P^{ND}. The price given no disclosure is determined by investors' beliefs about the manager's disclosure strategy. The equilibrium price given no disclosure is a fixed point, such that the manager's optimal disclosure strategy is consistent with the market pricing given no disclosure. Note that if the manager is informed and also the analyst obtains the private signal, the manager's decision whether to disclose or not has no effect on the ultimate price. As such, an informed manager's decision matters only in the cases where the analyst does not become informed. A manager's optimal strategy is to disclose if and only if the realized signal s (and the resulting price if the manager discloses, $P(D = s)$) is higher than the price given no disclosure, P^{ND}. Hence, for any price given no disclosure, the manager's equilibrium strategy is characterized by a threshold signal s^*, such that the manager discloses if and only if the realized signal is higher than the threshold. The reason that any equilibrium strategy is a threshold strategy is that when the analyst is uninformed, the manager's payoff given no disclosure, P^{ND}, is independent of her type s, where if the manager does disclose her payoff, $P(D = s) = s$, is increasing in her type s. When the analyst is informed, the manager's payoff is independent of her disclosure decision.

In order to determine the price given no disclosure, investors need to estimate the probability that an information event occurred given that neither the manager nor the analyst disclosed s. This probability is lower than the prior probability of an information event, q. For any exogenously given market beliefs about the manager's disclosure threshold, which we denote by π, the price given no disclosure, P^{ND}, can be written as:

$$P^{ND}(\pi) \equiv E[\tilde{x} \mid ND, \pi] \frac{(1-q)E[\tilde{s}] + qF(\pi)(1-r) \cdot E[\tilde{s} \mid s < \pi]}{(1-q) + q(1-r)F(\pi)}. \tag{1}$$

Note that for any disclosure threshold π the price given no disclosure is lower than the prior mean, i.e., $P^{ND}(\pi) < E[\tilde{x}] = p_0$. The reason is that low types withhold information and high informed type disclose, and therefore no-disclosure is a bad sign. Furthermore, the price given no disclosure is increasing in r, i.e., $\frac{\partial P^{ND}(\pi)}{\partial r} > 0$. No-disclosure is less of a bad sign when the likelihood of the analyst being informed is high, because in such a case, an uninformed analyst suggests that an information event is less likely. In the extreme case where r goes to one, no disclosure implies that the manager is uninformed and hence the price equals the prior mean.

Lemma 1 *There exists a unique disclosure equilibrium to the disclosure game. The equilibrium is characterized by a disclosure threshold s^* such that the manager discloses her private information s if and only if s is greater than the threshold s^*, where s^* is the unique solution to*

$$s^* = P^{ND}(s^*).$$

While our setting is not the standard voluntary disclosure with uncertainty about information endowment, the proof of the existence and uniqueness of the disclosure threshold, can be shown in a similar way to the standard models (a la Dye 1985). Our proof relies on the "minimum principle", described in Acharya et. al. (2010). This proof also show that the equilibrium disclosure threshold s^* equals to the exogenously determined disclosure threshold (not necessarily the equilibrium threshold) that minimizes the price given no disclosure. More formally, the unique equilibrium disclosure threshold satisfies: $s^* = \min_\pi P^{\mathrm{ND}}(\pi)$.

The following corollary provide our main comparative statics for the equilibrium disclosure threshold, s^*.

Corollary 1 *The manager's equilibrium disclosure threshold s^*,which equals the equilibrium price given no disclosure P^{ND}, is strictly increasing in r, i.e.,*

$$\frac{ds^*}{dr} > 0.$$

The Corollary demonstrates that analyst coverage crowds out corporate disclosure. As we indicated earlier, the reason is that following an increase in analyst coverage, r, no disclosure is less of a bad signal, since now the likelihood that the manager is uninformed given no discloser is higher. As such, the price given no disclosure is higher for higher values of r, and hence the manager's payoff from non-disclosure is higher. This induces less managerial voluntary disclosure, i.e., higher disclosure threshold. This result is consistent with the empirical evidence in Balakrishnan et al. (2014) who document an increase in corporate voluntary disclosure following an exogenous decline in analyst coverage.

The effect of the likelihood that the manager is informed, q, on the equilibrium disclosure threshold and the price given no disclosure, P^{ND}, is similar to the standard Dye (1985) setting In particular, investors beliefs that the manager is informed but conceals his information is higher following an increase in q (keeping all else equal), i.e., $\frac{ds^*}{sq} < 0$. As such, the price given no disclosure will be lower, which provides incentive for more disclosure and hence lowering the disclosure threshold.

4 Price Efficiency

An increase in analyst coverage, r, has the immediate direct effect of increasing disclosure by analysts, which provides more public information and increases expected price informativeness. However, in the previous section we have established that an increase in analyst coverage, r, also has an indirect effect on overall disclosure, which decreases corporate voluntary disclosure. As such, it is not clear what is the overall effect of changes in analyst coverage on investors' information, or price informativeness. In this section we analyze and answer this question.

To demonstrate the question more formally, note that the overall probability that the signal s becomes public, either by the manager and/or by the analyst, is $q - q(1-r)F(s^*)$. Given that the manager's equilibrium disclosure threshold, s^*, is increasing in analyst coverage r, it is not clear whether the probability that the signal s becomes publicly available increases or decreases following an increase in r.

The probability that s becomes publicly available captures only part of the informational effect of analyst coverage. Changes in r also affect the precision of the pubic information about the expected output when s is not disclosed. The reason is that changes in r affect the manager's disclosure threshold, the price given no disclosure, and also the uncertainty in investors beliefs given no disclosure.

In order to get at the overall information available to the market, which includes the states of nature under which the signal s is revealed, as well as the information the market can infer about the firm value if no disclosure is made, we develop in the next section a measure for price efficiency. This measure captures the expectation of the extent to which prices are "close" to the fundamental value of the firm. We show that an increase in analyst coverage always increases price efficiency according to this measure.

4.1 A Measure of Price Efficiency

The model above generates an equilibrium distribution of prices P. In case information becomes public, either by the manager or by the analyst, the price perfectly reflects the underlying value, which is the signal s about the future cash flow, i.e., the price $P = E[x|s] = s$. In case information does not become public the price is $P = P^{ND}$. In this case, although the price is on average correct, it is a noisy measure of the of the signal s (that was either obtained by the manager or was not).

In order to measure how efficiently prices reflect information about future cash flows, we adopt the commonly used expected squared deviation between the market price and the signal s. We refer to the price efficiency measure as PEF, and is given by

$$PEF \equiv -E\left[(s-P)^2\right]. \quad (2)$$

One can think of PEF as representing the "social" benefit from having a price that is close to the fundamental, or the externalities and gains that are obtained from the informativeness of prices.

PEF is the variance of the noise in the price relative to the true underlying value s. Moreover, a higher price efficiency means a decrease in the future volatility of prices (the future movement of prices when the future cash flows x are revealed).

4.2 Analyst Coverage and Price Efficiency

As indicated earlier, it is hard to determine even the directional effect of changes in analyst cover, r, on price efficiency. One of our main results, is that an increase in analyst coverage always increase price efficiency. The following Proposition formalizes this result.

Proposition 1 *Price efficiency increases in analyst coverage, i.e.,*

$$\frac{dPEF(r)}{dr} > 0.$$

The formal proof of the Proposition, which is quite involved, and hence is delegated to the appendix. Our intuition for the result in the Proposition is as follows. In equilibrium, whenever the manager obtains a signal below the disclosure threshold, $s < s^*$, she does not disclose and the resulting price is $P^{ND} = s^*$. Consider a small increase in analyst coverage, r.

The first effect of an increase in analyst coverage is that it increases the probability that such types will be revealed by the analysts, which will result in a price that reflects their true type. Thus this effect increases price efficiency.

The second effect is a decrease in managerial disclosure, or equivalently an increase in the manager's disclosure threshold. This increase in the threshold means that for signals s that are just above the original threshold s^*, which originally were priced correctly, the manager will now not disclose and the resulting price (which equals to the new threshold), will be just above s^*. Note that this effect, while decreasing price efficiency in the sense that the types just above s^* are no longer disclosed by the manager, does not have a major negative effect on price efficiency. The reason is that the new types who stopped disclosing following the increase in r are close to the new disclosure threshold, and thus obtain a price that is still close to their fundamental value.

The third effect is that uninformed managers are now being priced higher, as the price given no disclosure increases. Since the price given no disclosure, P^{ND}, is always lower than the prior mean, such an increase in P^{ND} increases the price efficiency for uninformed types. Thus, the only effect that decreases price efficiency, is the second effect. Since this effect is small, as indicates above, one can show that it is only a second order effect. Hence the overall effect of increase in r is to increase price efficiency.

The result of Proposition 1 implies that although analyst coverage has adverse effect on corporate voluntary disclosure, the overall informational effect of analyst coverage is positive. This result might have regulatory implications, when it comes to regulatory intervention in order to increase or decrease the extent of mechanisms that could reveal corporate public information. While financial analyst are a major mechanism in the capital market to discover firm's private information, such revelation can arrive from other sources, such as media, social media, competitors, suppliers or governmental bodies. Our model indicates that to the extent that increasing such sources is not too costly, it is beneficial in terms of price efficiency.

5 Analyst Coverage, Informed Trading and Liquidity

To further study the informational implication of analyst coverage, we extend our disclosure model by adding a stylized trading model. The trading stage takes place after the manager's potential voluntary disclosure decision and after the potential release of the analyst's report. The trading stage is a static version of the Glosten and Milgrom (1985) model (henceforth GM). The trading stage consists of two players: a competitive market maker and a single trader. The trader may either be informed or uninformed about the firm's final value, x.

Similar to GM, we assume that a trader can either buy or sell one unit (share) of the firm's stock. With probability γ the trader is an informed trader that knows the fundamental value of the asset. With probability $1-\gamma$ the trader is a "liquidity trader", who sells or buys due to a liquidity shock that is not correlated with value. As common in this literature, we assume that the liquidity trader chooses to sell or to buy one unit with equal probabilities.[4]

As common in these microstructure models, the risk neutral market maker does not have private information about the firm value, he does not know whether the trader is informed or not, and he operates in a competitive market for market making and hence sets prices such that his expected profit is zero. The market maker sets a bid price, b, for a trader that wants to sell a share and an ask price, a, for a trader that wants to buy a share from the market maker.

5.1 Prices and the Bid-Ask Spread

We start this section by providing a short derivation of the bid and ask prices and the bid-ask spread in a standard static GM setting. Readers who are familiar with this derivation can skip directly to Lemma 2. The appendix provides a more detailed derivation.

Let the public belief about the firm terminal value at the beginning of the trading stage, after the disclosure stage, be $p = \Pr(x = 1)$.

The market maker sets the stock prices, following either a buy or a sell order by the trader, such that the market maker breaks even on expectation. When the trader wants to buy a share, the market maker sets a price that equals the expected value of the share given a buy order. To set such a price given a buy order, the conditional probability of trading against an informed trader times the loss from this event should equal the conditional probability of trading against an uninformed liquidity trader times the expected gain from this event. The loss of the market maker if he trades against an informed buyer (in which case the firm value is 1 where the price is a) is $1 - a$. The ex-ante probability of this event is the probability that the trader is informed, γ, times the probability that the value is one, p. The conditional probability that the trader is informed given that he buys is given by applying Bayes - $\Pr(Informed|buyer) = \frac{\gamma p}{\gamma p + (1-\gamma)\frac{1}{2}}$.

The expected profit of the market maker if he trades against an uninformed buyer is $a - p$. The ex-ante probability of this event is the probability that the trader is uninformed, $1-\gamma$, times the probability that an uninformed trader buys a share, which is $\frac{1}{2}$. Using Bayes rule, the conditional probability that the trader is uninformed given that he buys a share is

[4] Allowing the probability that a liquidity trader buys a share to be different than 0.5 has no qualitative effect on our results.

$\Pr\left(UnInformed|buyer\right) = \frac{(1-\gamma)\frac{1}{2}}{\gamma p+(1-\gamma)\frac{1}{2}}$.

The ask price is the price that yields a zero expected profit to the market maker when the trader is a buyer. That is, the ask price is the price that equates sets to zero the weighted average of loss from trading against an informed buyer to the expected gain from trading against an uninformed buyer, where the weights are the conditional probabilities of each event respectively. As such, the ask price is the solution to:

$$\frac{\gamma p}{\gamma p+(1-\gamma)\frac{1}{2}}(1-a) = \frac{(1-\gamma)\frac{1}{2}}{\gamma p+(1-\gamma)\frac{1}{2}}(a-p).$$

Solving the above equation for a, yields the ask price:

$$a = p\frac{1+\gamma}{1-\gamma(1-2p)}.$$

Deriving the bid price in a similar way (see the appendix for more details) yields

$$b = \frac{p(1-\gamma)}{1-\gamma(2p-1)}.$$

In the appendix, we formally derive the bid and the ask prices and show that the ask price is an increasing concave function of p and that the bid price is an increasing convex function of p. Moreover, for any $p \in (0,1)$ we have $a > p > b$. Finally, it is easy to see that $b(p=0) = a(p=0) = 0$ and $b(p=1) = a(p=1) = 1$.

While we know how to theoretically measure price efficiently, e.g., using our measure PEF, empirically measuring or estimating price efficiency is not an easy task. One common measure of information asymmetry, that is relatively easy to estimate and directly reflects the uncertainty of the market maker, is the bid-ask spread. In order to study how the information asymmetry that remains after the disclosure game manifests itself in the market for the firm's stock, we analyze how the bid-ask spread is affected by the parameters of our disclosure game. Our main focus is on the effect of analyst coverage on the expected bid-ask spread. The bid-ask spread, which we denote by $\Psi(p)$, is the difference between the ask and the bid prices above, and is given by

$$\Psi(p) \equiv a-b = 4p\gamma\frac{1-p}{1-\gamma^2(1-2p)^2}.$$

The following Lemma provides some properties of the bid-ask spread.

Lemma 2 *The bid-ask spread, $\Psi(p)$, is characterized as follows*

1. *It is a strictly concave inverse U-shape function of p.*

2. *for any $\gamma \in (0,1)$, the spread is maximized at $p = 0.5$.*

3. *$\Psi(p = 0) = \Psi(p = 1) = 0$ (and the spread is zero also for any p if $\gamma = 0$ or $\gamma = 1$).*

We provide the formal proof to the Lemma in the appendix. The main characteristic of the bid-ask spread that we will be using is the concavity in the beliefs, p. [5]

In the next section, we study how the disclosure game affects the expected bid-ask spread, which is our measure of illiquidity.

5.2 Disclosure and Liquidity

The information available to the market after the disclosure game, which is the information available at the trading stage, depends on whether the manager is informed or not, if so, what is the realized signal and whether the analyst is informed or not. In order to obtain an additional perspective on the informational environment following the disclosure game, to supplement our PEF measure, we study how the parameters of the disclosure game affect the expected bid-ask spread, which is a common measure of information asymmetry and illiquidity.

Our measure of illiquidity, $IL(q,r)$, which depends on the parameters of the disclosure game, q and r, is given by

$$IL(q,r) \equiv E[\Psi(p)|q,r].$$

When we refer to liquidity we refer to $L(q,r) = -IL(q,r)$.

The market's expectation of the realization of $\tilde{x}$ at the beginning of the trading stage, which equals the expectation at the beginning of the trading game, p, depend on the outcome of the disclosure game. If neither the manager nor the analyst disclose s, the market's expectation of $\tilde{x}$ is given by $p = \pi^* = E[x|ND]$. If the manager and/or the analyst disclosed

[5] For simplicity, and as common in this literature, we assume that the probability that a liquidity trader buys a share is half. Relaxing this assumption and allowing for this probability to be anywhere between zero and one, does not affect our main results. In particular, the bid-ask spread remains a concave inverse U-shape function of p.

the signal s, then the market's expectation is $p = s$. The probability of such an event, where s is publicly revealed, is given by $(qr + q(1-r)(1 - F(\pi^*)))$. This event is comprised of two events. The event where an information event has occurred and the analyst is informed - which occurs with probability qr. An the event where an information event has occurred, the analyst is uninformed and the signal is higher than the disclosure threshold π^*- which occurs with probability $q(1-r)(1 - F(\pi^*))$.

Using the above probabilities of public revelation and no revelation of s, we can write the expected bid-ask spread as follows

$$\begin{aligned} IL(q,r) = {} & [1 - q + q(1-r)F(\pi^*)] \cdot \Psi(\pi^*) + q \cdot r \cdot E[\Psi(s)] \\ & + q \cdot (1-r)(1 - F(\pi^*)) \cdot E[\Psi(s) \mid s \geq \pi^*] . \end{aligned} \tag{3}$$

The first term reflects the case that no disclosure is made, following which the expectation of p equals the disclosure threshold π^*. The second term reflects the case that the analyst is informed. In this case the analyst always discloses s, and hence the expectation of the spread is taken over the initial distribution of s. The third term reflects the case that only the manager is informed and discloses s. This occurs only for $s > \pi^*$, and hence the expectation is taken over $s > \pi^*$.

When the manager's probability of being informed, q, increase (keeping r constant) it decreases the disclosure threshold (see Corollary 1) and increases the likelihood of disclosure by the manager. An increase in the probability of information event, q, also increase the likelihood that the analyst is informed and discloses s. The overall effect of changes in the probability of an information event on liquidity, is hence hard to determine. The following Lemma shows that an increase in the probability of information event, q, always decreases the expected bid-ask spread and hence increases our measure of liquidity.

Lemma 3 *$IL(q,r)$ is decreasing in q for any $r \in (0,1)$.*

The proof of the Lemma is a particular case of the proof of Proposition 2 (which we provide in the appendix).

While the above Lemma presents an interesting result, our main research question is how changes in analyst coverage, r, affect the information environment in general, and in particular the liquidity of the stock. Answering this question involves two levels of complexity. The first challenge is similar to the difficulty of determining the effect of analyst coverage, r, on PEF due to the two opposing effects. The second challenge, is that even though Proposition 1 demonstrates that price efficiently always increases in r, the effect of r on expected liquidity, which is not a linear function of PEF, is still not clear.

The following Proposition show that an increase in analyst coverage, r, always increases the liquidity of the firm's stock, despite of the decrease in corporate disclosure.

Proposition 2 *$IL(q,r)$, the expected bid-ask spread, is decreasing in r for any $q \in (0,1)$, that is*

$$\frac{dIL\left(q,r\right)}{dr} < 0.$$

.

Proof. Let q be constant. Define a function $H\left(r,\pi\right)$ that equals the expected spread conditional on analyst coverage r and any exogenously given disclosure threshold π (which may not be the equilibrium disclosure strategy), as follows:

$$\begin{aligned} H\left(\pi,r\right) \equiv & \left[1-q+q(1-r)F(\pi)\right]\Psi\left(p^{ND}(\pi,r)\right)+q\cdot r\cdot E\left[\Psi(s)\right] \\ & +q\cdot(1-r)\int_{\pi}^{1}\Psi(s)\cdot f(s)\,ds, \end{aligned} \tag{4}$$

where

$$p^{ND}\left(\pi,r\right) \equiv \frac{(1-q)E\left[s\right]+q(1-r)F(\pi)E\left[\tilde{s}\mid s\leq\pi\right]}{1-q+q(1-r)F(\pi)} \tag{5}$$

is the market expectation of $\tilde{x}$ following no-disclosure by the manager or the analyst.

When evaluated at the equilibrium disclosure threshold $\pi=\pi^{*}(r)$, $H\left(\pi,r\right)$ if our measure of Illiquidity, $IL\left(q,r\right)$. The total derivative of $IL\left(q,r\right)$ with respect to r is:

$$\frac{dIL\left(q,r\right)}{dr} = \frac{\partial H\left(\pi,r\right)}{\partial r}\mid_{\pi=\pi^{*}(r)} + \frac{\partial H\left(\pi,r\right)}{\partial\pi}\mid_{\pi=\pi^{*}(r)}\frac{d\pi^{*}\left(r\right)}{dr}. \tag{6}$$

A sufficient condition for $\frac{dIL(q,r)}{dr}<0$ is that both $\frac{\partial H(\pi,r)}{\partial r}\mid_{\pi=\pi^{*}(r)}<0$ and $\frac{\partial H(\pi,r)}{\partial \pi}\mid_{\pi=\pi^{*}(r)}=0$. We establish these sufficient conditions in the two Lemmas below.

Lemma 4 $\frac{\partial H(\pi,r)}{\partial r}\mid_{\pi=\pi^{*}(r)}<0.$

The proof of Lemma 4 is quite involved, and hence is delegated to the appendix.

Lemma 5 $\frac{\partial H}{\partial \pi} \mid_{\pi=\pi^*(r)}= 0.$

Proof. *Differentiating (4) with respect to* π *we obtain*

$$\frac{\partial H}{\partial \pi} = q(1-r)f(\pi)\left[\Psi\left(p^{ND}(\pi,r)\right) - \Psi(\pi)\right] + \left[1-q+q(1-r)F(\pi)\right]\Psi'(\cdot)\frac{\partial p^{ND}}{\partial \pi}. \qquad (7)$$

From Section 3 we know that the equilibrium threshold $\pi^*(r)$ *satisfies:* (i) $\pi^* = p^{ND}(\pi^*, r)$, *and* (ii) *the minimum principle, that is* $\frac{\partial p^{ND}}{\partial \pi} \mid_{\pi=\pi^*(r)}= 0$. (i) *implies that the first term in* (7) *is zero where* (ii) *implies that the second term is zero. Thus ,*$\frac{\partial H}{\partial \pi} \mid_{\pi=\pi^*(r)}= 0.$ ■

This completes the proof of Proposition 2 ■

The result of Proposition 2, which provides additional motivation for the informational benefit of analyst coverage, is consistent with the empirical findings of Kelly and Ljungqvist (2012). Kelly and Ljungqvist (2012) finds that following an exogenous negative shock to analyst coverage, there is a decrease in the liquidity of the firms that were affected by the decreased in analyst coverage.

6 Appendix

Proof of Proposition 1. Denote by $v^{ND}(\pi, r)$ the price given no disclosure for an exogenously given disclosure threshold, π, and a given analyst coverage r. $v^{ND}(\pi, r)$ is given by

$$v^{ND}(\pi, r) \equiv \frac{(1-q)E[s] + q(1-r)F(\pi)E[s \mid s \leq \pi]}{1-q+q(1-r)F(\pi)}.$$

Define the function $G(r, \pi)$, which is the PEF for an exogenously given disclosure threshold π and analyst coverage r, as

$$\begin{aligned} G(r,\pi) &= E[h(s-v|r,\pi)] \\ &= ((1-q)+q(1-r)F(\pi))\,E\left[h\left(s-v^{ND}(\pi,r)\,|ND\right)\right] + q\left(r+(1-r)(1-F(\pi))\right)h(0) \end{aligned}$$

where v denotes the price, such that $v = \begin{pmatrix} s & \text{if a disclosure of } s \text{ is made} \\ v^{ND}(\pi, r) & \text{if no disclosure is made} \end{pmatrix}$.

Note that in equilibrium the manager's disclosure threshold is $\pi = \pi^*(r)$ and hence, $PEF(r) = G(r, \pi^*(r))$.

We need to show that in equilibrium, PEF is increasing in r, that is $\frac{dPEF}{dr}|_{\pi=\pi^*(r)} > 0$. The total derivative of PEF with respect to r is

$$\frac{dPEF}{dr} = \frac{dG(r,\pi)}{dr}|_{\pi=\pi^*(r)} = \frac{\partial G(r,\pi)}{\partial r}|_{\pi=\pi^*(r)} + \frac{\partial G(r,\pi)}{\partial \pi}|_{\pi=\pi^*(r)} \frac{d\pi^*(r)}{dr}.$$

A sufficient condition for $\frac{\partial PEF}{\partial r} > 0$ it that (1) $\frac{\partial G}{\partial r}|_{\pi=\pi^*(r)} > 0$ and (2) $\frac{\partial G}{\partial \pi}|_{\pi=\pi^*(r)} = 0$.

- We next show that $\frac{\partial G}{\partial r}|_{\pi=\pi^*(r)} > 0$.

$\frac{\partial G(r,\pi)}{\partial r}$ is given by

$$\begin{aligned} \frac{\partial G(r,\pi)}{\partial r} &= (qF(\pi))\,E\left[h\left(s-v^{ND}(\pi,r)\,|ND\right)\right] \\ &\quad + ((1-q)+q(1-r)F(\pi))\frac{\partial}{\partial r}E\left[h\left(s-v^{ND}(\pi,r)\,|ND\right)\right] \\ &\quad + qF(\pi)\,h(0) \end{aligned}$$

Using the assumption that $h(\cdot)$ is the quadratic loss function

$$h(y) = -y^2,$$

and rewriting $\frac{\partial G(r,\pi)}{\partial r}$ at $\pi = \pi^*(r)$, we get

$$\begin{aligned}\frac{\partial G\left(r,\pi\right)}{\partial r}|_{\pi=\pi^*(r)} &= \left(qF\left(\pi\right)\right)E\left[\left(s-v^{ND}\left(\pi,r\right)|ND\right)^2\right]|_{\pi=\pi^*(r)} \\ &+2\left(\left(1-q\right)+q(1-r)F\left(\pi\right)\right)E\left[\left(s-v^{ND}\left(\pi,r\right)|ND\right)\right] \quad | \quad _{\pi=\pi^*(r)}+0.\end{aligned}$$

Since, for $\pi = \pi^*(r)$ the price given no disclosure equals the expected type given no disclosure, i.e., $v^{ND}\left(\pi^*(r), r\right) = E\left[(s|ND)\right] (= \pi^*(r))$, we have

$$E\left[\left(s-v^{ND}\left(\pi,r\right)|ND\right)\right]=0.$$

Therefore

$$\frac{\partial G\left(r,\pi\right)}{\partial r}|_{\pi=\pi^*(r)}=\left(qF\left(\pi\right)\right)E\left[\left(s-v^{ND}\left(\pi,r\right)|ND\right)^2\right]|_{\pi=\pi^*(r)}>0.$$

- We now show that $\frac{\partial G}{\partial \pi}|_{\pi=\pi^*(r)}=0$.

We can rewrite $G\left(r,\pi\right)$ as

$$\begin{aligned}G\left(r,\pi\right) &= (1-q)\int_0^1 h\left(s-v^{ND}\left(\pi,r\right)\right)f(s)\,ds \\ &+q\left[(1-r)F\left(\pi\right)\int_0^{\pi} h\left(s-v^{ND}\left(\pi,r\right)\right)\frac{f(s)}{F\left(\pi\right)}ds+q\left(r+(1-r)\left(1-F\left(\pi\right)\right)\right)h\left(0\right)\right].\end{aligned}$$

$$\begin{aligned}\frac{\partial G}{\partial \pi}=&(1-q)\int_0^1 h'\left(\cdot\right)\left(-\frac{\partial v^{ND}\left(\pi,r\right)}{\partial \pi}\right)f(s)\,\mathrm{d}s+q(1-r)\int_0^{\pi} h'\left(\cdot\right)\left(-\frac{\partial v^{ND}\left(\pi,r\right)}{\partial \pi}\right)f(s)\,\mathrm{d}s \\ &+q(1-r)h\left(\pi-v^{ND}\left(\pi,r\right)\right)-\frac{\partial F\left(\pi\right)}{\partial \pi}q\left(1-r\right)h\left(0\right)\end{aligned} \tag{8}$$

When computing $\frac{\partial G}{\partial \pi}_{\pi=\pi^*(r)}$, we have $\pi^*\left(r\right) = v^{ND}(\pi^*\left(r\right),r)$ and hence the third term in 8 is zero. Also, since $h\left(0\right)=0$, the last term in 8 is also zero. Finally, since in the disclosure game the minimum principle holds, we have $\frac{\partial v^{ND}(\pi,r)}{\partial \pi}|_{\pi=\pi^*(r)}=0$. Therefore, also the first two terms in 8 are zero. Thus $\frac{\partial G}{\partial \pi}|_{\pi=\pi^*(r)}=0$. ■

Proof of Lemma 2 and Derivation of the Bid-Ask Spread. We first derive and characterize the ask and the bid prices and then characterize the bid-ask spread.

Ask Price

When the trader wants to buy a share, the market maker sets a price that equals the expected value of the share. In particular, upon observing a buy request, the conditional probability of trading against an informed trader times the loss from this event should equal the conditional probability from trading against an uninformed liquidity trader times the expected gain from this event.

The loss of the market maker given that he trades against an informed trader that buys, i.e., when the firm value is 1 and the trade price is a, is $1-a$. The ex-ante probability of this event is the probability that the trader is informed, γ, times the probability that the value is one and the informed trader buys, p. The conditional probability that the trader is informed given that he buys a share is

$$\Pr\left(Informed|buyer\right)=\frac{\gamma p}{\gamma p+\left(1-\gamma\right)\frac{1}{2}}$$

The expected profit of the market maker given that he trades against an uninformed trader that buys is $a-p$.

The ex-ante probability of this event is the probability that the trader is uninformed, $1-\gamma$, times the probability that an uninformed trader buys a share, which is $\frac{1}{2}$. The conditional probability that the trader is uninformed given that he buys a share is

$$\Pr\left(UnInformed|buyer\right)=\frac{\left(1-\gamma\right)\frac{1}{2}}{\gamma p+\left(1-\gamma\right)\frac{1}{2}}$$

■

Proof. The ask price is the price that yields a zero expected profit to the market maker when the trader wishes to buy a share, and is the solution to:

$$\frac{\gamma p}{\gamma p+\left(1-\gamma\right)\frac{1}{2}}\left(1-a\right)=\frac{\left(1-\gamma\right)\frac{1}{2}}{\gamma p+\left(1-\gamma\right)\frac{1}{2}}\left(a-p\right).$$

The ask price is given by

$$a=\frac{p+p\gamma}{2p\gamma+1-\gamma}.$$

Lemma 6 *The ask price is:*

1. *An increasing concave function of the prior beliefs p.*

2. *An increasing function of the prior probability of informed trading γ. It is convex (concave) in γ for $p<0.5$ ($p>0.5$).*

3. $a\,(p=0)=0$ *and* $a\,(p=1)=1$.

To characterize how the ask price varies with the prior, p, for any γ, note that

$$\begin{aligned}
\frac{da\,(p)}{dp} &= \frac{d}{dp}\frac{p+p\gamma}{2p\gamma+1-\gamma}=\frac{1-\gamma^2}{(2p\gamma-\gamma+1)^2}>0, \\
\frac{d^2a\,(p)}{dp^2} &= \frac{d}{dp}\frac{1-\gamma^2}{(2p\gamma-\gamma+1)^2}=-4\gamma\frac{1-\gamma^2}{(2p\gamma-\gamma+1)^3}<0, \\
\frac{da\,(\gamma)}{d\gamma} &= \frac{d}{d\gamma}\frac{p+p\gamma}{2p\gamma+1-\gamma}=2p\frac{1-p}{(2p\gamma-\gamma+1)^2}>0 \\
\frac{d^2a\,(\gamma)}{d\gamma^2} &= \frac{d}{d\gamma}2p\frac{1-p}{(2p\gamma-\gamma+1)^2}=4p\,(1-2p)\,\frac{1-p}{(2p\gamma-\gamma+1)^3}
\end{aligned}$$

Which demonstrates that the ask price is an increasing concave function of the prior p. Moreover, $a\,(p=0)=0$ and $a\,(p=1)=1$.

Bid Price

We derive the bid price in a parallel way.

The loss of the market maker given that he trades against an informed trader that sells, i.e., when the firm value is 0 and the trade price is b, is b. The ex-ante probability of this event is the probability that the trader is informed, γ, times the probability that the value is zero and the informed trader sells, $(1-p)$. he conditional probability that the trader is informed given that he sells a share is $\Pr\left(Informed|seller\right)=\frac{\gamma(1-p)}{\gamma(1-p)+(1-\gamma)\frac{1}{2}}$.

The expected profit of the market maker given that he trades against an uninformed trader that sells is $p-b$.

The ex-ante probability of this event is the probability that the trader is uninformed, $1-\gamma$, times the probability that an uninformed trader sells a share, which is $\frac{1}{2}$. The conditional probability that the trader is uninformed given that he sells a share is $\Pr\left(UnInformed|buyer\right)=\frac{(1-\gamma)\frac{1}{2}}{\gamma(1-p)+(1-\gamma)\frac{1}{2}}$.

The bid price is the price that yields a zero expected profit to the market maker when the trader wishes to sell a share, and is the solution to:

$$\frac{\gamma\,(1-p)}{\gamma\,(1-p)+(1-\gamma)\,\frac{1}{2}}b=\frac{(1-\gamma)\,\frac{1}{2}}{\gamma\,(1-p)+(1-\gamma)\,\frac{1}{2}}\,(p-b)\,.$$

The bid price is given by

$$b=\frac{p\,(1-\gamma)}{1-\gamma\,(2p-1)}.$$

Lemma 7 *The bid price is:*

1. *An increasing convex function of the prior beliefs p.*

2. *A decreasing function of the prior probability of informed trading γ. It is convex (concave) in γ for $p < 0.5$ ($p > 0.5$).*

3. $b(p = 0) = 0$ *and* $b(p = 1) = 1$.

■

Proof. To see how the ask price varies with the priors, p, and γ, note that

$$
\begin{aligned}
\frac{db(p)}{dp} &= \frac{d}{dp}\frac{p(1-\gamma)}{1-\gamma(2p-1)} = \frac{1-\gamma^2}{(1+\gamma(1-2p))^2} > 0, \\
\frac{d^2b(p)}{dp^2} &= \frac{d}{dp}\frac{1-\gamma^2}{(\gamma-2p\gamma+1)^2} = 4\gamma\frac{1-\gamma^2}{(1+\gamma(1-2p))^3} > 0, \\
\frac{db(\gamma)}{d\gamma} &= \frac{d}{d\gamma}\frac{p(1-\gamma)}{1-\gamma(2p-1)} = -2p\frac{1-p}{(1+\gamma(1-2p))^2} < 0, \\
\frac{d^2b(\gamma)}{d\gamma^2} &= \frac{d}{d\gamma}\left(-2p\frac{1-p}{(\gamma-2p\gamma+1)^2}\right) = 4p(1-2p)\frac{1-p}{(1+\gamma(1-2p))^3} \\
b(p=0) &= b(p=1) = 1
\end{aligned}
$$

Which demonstrates that the bid price is an increasing convex function of the prior p. Moreover, $b(p = 0) = 0$ and $b(p = 1) = 1$.

Bid-Ask Spread

The spread, $\Psi(p) = a - b$, is given by

$$\Psi(p) = \frac{p + p\gamma}{2p\gamma + 1 - \gamma} - \frac{p(1-\gamma)}{1-\gamma(2p-1)} = 4p\gamma\frac{p-1}{4p^2\gamma^2 - 4p\gamma^2 + \gamma^2 - 1}.$$

Since the ask is concave and the bid is convex in p, and $a \geq b$, the bid-ask spread is a concave inverse U-shape of p (alternatively, one can take the second derivative of $\Psi(p)$ and show it is negative). Showing 2 is straight forward using by taking the first derivative with respect to p and equate it to zero. Showing 3 is immediate. ■

Proof of Lemma 4. We show that the Lemma holds for any exogenously given π, and hence it also holds for $\pi = \pi^*(r)$. Given the continuity of $H(r, \pi)$ in r, it is sufficient to show that $H(r_h, \pi) < H(r_l, \pi)$ for any $r_h > r_l$ and any π. First note that we can rewrite $H(r, \pi)$ from (4) in the following way.

$$H(r, \pi) = [1 - q + q(1 - r)F(\pi)]\,\Psi\left(p^{ND}(\pi, r)\right) + q \cdot r \cdot \int_0^1 \Psi(s) \cdot f(s)ds$$
$$q \cdot (1 - r) \int_\pi^1 \Psi(s) \cdot f(s)\,ds.$$

Substituting $q \cdot r \cdot \int_0^1 \Psi(s) \cdot f(s)ds = q \cdot r \cdot \int_0^\pi \Psi(s) \cdot f(s)ds + q \int_\pi^1 \Psi(s) \cdot f(s)ds$ into $H(r, \pi)$ and rearranging terms yield

$$H(r, \pi) = [1 - q + q(1 - r)F(\pi)]\,\Psi\left(p^{ND}(\pi, r)\right) + q \cdot r \cdot \int_0^\pi \Psi(s) \cdot f(s)ds$$
$$+\, q \int_\pi^1 \Psi(s) \cdot f(s)ds.$$

Using the above representation of $H(r, \pi)$, noting that $\int_0^\pi \Psi(s) \cdot f(s)ds$ and $\int_\pi^1 \Psi(s) \cdot f(s)ds$ are independent of r, we compute $H(r_l, \pi) - H(r_h, \pi)$.

$$\begin{aligned} & H(r_l, \pi) - H(r_h, \pi) \qquad (9) \\ = \; & [1 - q + q(1 - r_l)F(\pi)]\,\Psi\left(p^{ND}(\pi, r_l)\right) - [1 - q + q(1 - r_h)F(\pi)]\,\Psi\left(p^{ND}(\pi, r_h)\right) \\ & -q \cdot (r_h - r_l) \cdot \int_0^\pi \Psi(s) \cdot f(s)\,ds. \end{aligned}$$

We now digress to establish an equality that will be instrumental in the final stage of proving the Lemma. Using the definition of $p^{ND}(\pi, r_l)$ (Equation (5)) (recall that we keep π constant and only change r), it is easy to see that the following equations hold:

$$[1 - q + q(1 - r_l)F(\pi)]\,p^{ND}(\pi, r_l) = [1 - q + q(1 - r_l)F(\pi)]\,\frac{(1 - q)E\,[s] + q(1 - r_l)F(\pi)E\,[s \mid s \leq \pi]}{[1 - q + q(1 - r_l)F(\pi)]}$$
$$= (1 - q)E\,[s] + q(1 - r_l) \int_0^\pi s \cdot f(s)\,ds.$$

We next use the above equation to get an expression that resembles $H(r_l, \pi) - H(r_h, \pi)$ (equation 9). We use a similar equation to the one above, which is applied to r_h instead of r_l, and perform some simple algebra to get the following equation:

$$\begin{aligned} & [1 - q + q(1 - r_l)F(\pi)]\,p^{ND}(\pi, r_l) \qquad (10) \\ & = [1 - q + q(1 - r_h)F(\pi)] \cdot p^{ND}(\pi, r_h) + q \cdot (r_h - r_l) \cdot \int_0^\pi s \cdot f(s)\,ds \end{aligned}$$

This difference between the RHS and LHS of equation 10 resembles the RHS of equation $H(r_l,\pi)-H(r_h,\pi)$ (equation 9), where the difference is that in $H(r_l,\pi)-H(r_h,\pi)$ (equation 9) we have $\Psi\left(p^{ND}(\pi,r)\right)$ instead of $p^{ND}(\pi,r)$ in (10).

Note that if $\Psi(s)$ was a linear function of s then it is immediate to see that $H(r_l,\pi)-H(r_h,\pi)$ would have been zero (see equation 9).

The LHS of (10) is the probability of no disclosure for $r=r_l$ times the price given no disclosure for $r=r_l$. The RHS of (10) reflects the probability of no disclosure for $r=r_h$ times the price given no disclosure for $r=r_h$ plus the term $q\cdot(r_h-r_l)\cdot\int_0^{\pi} s\cdot f(s)\,ds$, which is the increase in probability of disclosure by the analyst when increasing r from r_l to r_h. In other words, the LHS reflects the types that are not disclosed by the manager or the analyst when $r=r_l$ where the RHS can be viewed as performing a mean-preserving-spread over these types.

The average beliefs given no disclosure under r_l is given by $p^{ND}(\pi,r_l)$. Increasing r to r_h has the following effect on the types that were not disclosed by either the analyst or the manager under r_l. Some types that were not disclosed because no information event took place are now being disclosed under r_h (either by the manager for $s\geq\pi$ or by the analyst for $s\in[0,1]$). Note that since we keep the disclosure threshold constant, there are no types that were disclosed by the manager under r_l and are concealed under r_h. As such the increase in r (keeping π constant), induces some types that were not disclosed (and were priced at the average type given no disclosure) to be disclosed following the increase in r, generating more dispersion without. While the dispersion of prior beliefs increases, the expected beliefs given no disclosure remains the same (as we keep π constant).

A similar intuition holds for the expression $H(r_l,\pi)-H(r_h,\pi)$ (equation 9), only that now it is applied to the bid-ask spread $\Psi(\cdot)$ given the beliefs about x rather than to the beliefs themselves. Since $\Psi(s)$ is a concave function of s, introducing more dispersion (mean-preserving-spread on the beliefs about s) while keeping the mean constant decreases the expectation of $\Psi(s)$. As such

$$
\begin{aligned}
[1-q+q(1-r_l)F(\pi)]\,\Psi\left(p^{ND}(\pi,r_l)\right) > {} & [1-q+q(1-r_h)F(\pi)]\,\Psi\left(p^{ND}(\pi,r_h)\right) \\
& + q\cdot(r_h-r_l)\cdot\int_0^{\pi}\Psi(s)\cdot f(s)\,ds,
\end{aligned}
$$

which implies that $H(r_l,\pi)>H(r_h,\pi)$. $\blacksquare$

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