Accrual Accounting and Periodicity in Performance Measurement

Moritz Hiemann*

November 2017†

* Columbia Business School, Columbia University. 617 Uris Hall, 3022 Broadway, New York, NY, 10027. Tel. 212-854-8659. Email: mh3338@columbia.edu.

† Preliminary draft. Please do not circulate without permission.

Accrual Accounting and Periodicity in Performance Measurement

Abstract. The paper develops an accrual-basis performance measurement rule that enables incentive-compatible compensation contracts in an agency setting with hidden actions and information, non-time-separable investment payoffs, and actors with limited horizons. Although interdependence of investments across time makes the value contribution by managerial decisions in a specific period ambiguous, there exist targeted accrual adjustments to realized cash flows that, when employed as a performance metric for the manager, prevent over- or underinvestment. The magnitude of these adjustments depends on the degree to which the information the manager observes on the job resolves uncertainty about future outcomes. Due to the actors' limited horizons, the resulting incentive contract attains better outcomes than transferring ownership to the manager. The optimal organizational design in this model therefore involves the separation of ownership and control.

1. Introduction

The primary function of accrual-basis accounting lies in measuring an entity's performance, specifically, the profitability of its activities over successive, discrete time periods. This performance measurement over finite time intervals is both necessary and problematic at the same time. The necessity arises because investors, managers and other stakeholders require up-to-date information about the firm in their decisions. Periodicity in performance measurement is problematic because the value implications of current activities may depend on uncertain future events and actions, which, moreover, may be affected by the current performance metric in the first place. Whether and how one can quantify the effects of firms' activities in finite timespans in a well-defined manner is therefore all but obvious. This paper studies the finite-period performance measurement problem in a classic stewardship context, where a business owner delegates management duties and seeks to incentivize the manager to commit effort and maximize value creation. The analysis gives insights into the joint incentive and information production roles of accounting, the optimal matching of revenues and expenses, the causal relationship between accruals and cash flows, and the benefits of separating ownership and control.

Prior research has focused on goal congruence in studying accrual accounting methods optimized for performance measurement purposes. Goal congruence means the alignment of the manager's incentives with the owner's objectives by means of a performance metric that, by design, the manager can only maximize by taking decisions that the owner desires. Existing results in this arena have been derived in settings where the value contributed by an individual manager's actions is well-defined and separable from that of other agents and events, and where agency conflicts between owner and manager are to some degree separable from the goal congruence problem (e.g., Reichelstein 1997; Dutta and Reichelstein 2005a; Baldenius, Dutta and Reichelstein 2007). The following analysis shows that changing these setup features has non-trivial implications.

To this end, two assumptions are made. First, the manager's term of employment is shorter than the remaining life of the firm, i.e., the owner must evaluate and reward the manager's performance before all long-term effects of the manager's actions have materialized in verifiable cash flows. Second, the firm's investment problem is not time-separable, in the sense that the incremental future benefit derived from resources spent today is a function of past and future events and resource expenditures. Hence, there is no unique, definitive way to determine how much of the outcomes observed in a given period should be attributed to a specific investment decision, and thus to the incumbent manager rather than to that manager's predecessors or successors. The objective is to design a performance metric that, given these conditions, incentivizes the manager to undertake value-maximizing actions each period, and to study its properties and implications.[1]

The managers in this model make investments in the form of both cash expenditures, which are verifiable, and personal effort, which is not directly observable to any third party, including the owner. These investments are made under uncertainty. The optimal expenditure and effort levels depend on environmental circumstances, whose future trajectory is stochastic and whose current state is observable only to the manager currently in charge. The history of investments and the evolution of the environment jointly determine the firm's cash revenue each period. A compensation scheme that induces a manager to invest optimally in this context must rely on accrual accounting in the following sense. First, the profit metric by which the manager's compensation is determined must adjust the current, observable revenue and expenditure cash flows because the revenue in a given period is only in part due to the manager's actions, and because the manager's current investment affects revenue in future periods. The accounting must further recognize that the firm's observable cash inflows and outflows, and their implications for future periods, are affected by actions and information visible only to the manager. The accounting method must therefore meet the incentive-compatibility conditions that, in equilibrium, the first-best investment strategy maximizes the accounting profit, and that the profit calculation includes correct conjectures, based on the observed cash flows, about the manager's effort level and the current environment. These conditions must hold sequentially over time, as the outcomes of the current period become inputs to the accounting process in the next period.

Under generic concavity assumptions, the unique first-best investing strategy is to respond to a better environment by investing more and generating higher cash revenue. Implementing this strategy as an incentive-compatible equilibrium requires a specific class of accounting methods that all must have the same, unique form on the set of possible first-best outcomes. This result is less trivial than it may appear at first glance because the existence of a unique first-best investment plan is not equivalent to identifying how much of the surplus value created by that plan can

[1] That management tenure and horizon have substantive effects on firm performance has also been documented empirically numerous times, e.g., by Dechow and Sloan (1991), Cheng (2004) and Ali and Zhang (2015).

be allocated to a specific time period and manager, as every unit of revenue the firm generates in that period is the joint product of all past investments working in concert. The interdependence of investments implies a multiplicity of possible ways to credit the manager with the firm's performance in any given period, even if the path of optimal investing decisions is unique. The difficulty in designing an accrual accounting method in this context is that conventional criteria of optimality, such as the implementation of investments with positive net present value, are not operational because the allocation of revenues to specific investments is arbitrary. Moreover, uncertainty and intertemporal dependence are not easily amenable to the conventional notion of matching investment costs to subsequent payoffs.[2] For example, deriving deterministic long-horizon depreciation schedules to attain incentive-compatibility, which prior research was able to accomplish for additively separable investments, is not possible here.[3]

An incentive-compatible accounting solution nonetheless exists because the problem does not require tracking the profits of individual investments through time. The reason is that the owner's objective is to solve a sequential decision problem with an incentive-compatibility constraint, not to measure the manager's performance in isolation. To induce the manager to act optimally, it suffices to localize the problem in each period by treating all past costs as sunk and assuming all subsequent decisions to be exogenously optimized by future incentive contracts. This approach requires careful calibration of the accounting method in its sensitivity to changes in investing behavior in a manner that, at any point on the continuum of possible first-best outcomes, aligns the value-maximizing investment decisions with the decisions that maximize the accounting profit by which the manager's performance is measured. Intuitively, the optimally designed performance measurement scheme holds the manager accountable for the entire amount of incremental value created under the manager's reign, notwithstanding that this value addition partly depends on both past and future actions that are outside the manager's control. The manager therefore has an incentive to invest optimally even though the performance metric does not separate out the manager's personal contribution to firm value.

---

[2] See, for example, Barker and Penman (2017) for a discussion of this problem.
[3] For example, Reichelstein (1997) and Dutta and Reichelstein (2005a) obtain goal congruence by measuring performance by residual income and depreciating investment expenditures over time by the so-called relative benefit rule.

As in standard accounting practice, the optimal profit metric in this performance measurement problem is a net amount of accrual-basis revenue, calculated by adjusting cash revenue receipts, and accrual-basis expenses, calculated by adjusting cash expenditures. To induce first-best investment, a unit increase in cash receipts must yield an increase in accrued revenue of larger magnitude, calibrated so that the manager internalizes not only the immediate cash payoff from the present investment but also any of its expected future profit effects. How large this rate of increase is depends on the reach of the manager's private information about the environment. If the information is not only pertinent to the current period but also reveals much about how investing today affects payoffs in the future, then the optimal rate of increase in accrued revenue, per unit of cash revenue, is higher the more the manager invests. Conversely, if the information is relevant mostly with respect to the current period and provides little updating about the future, then higher investment makes accrued revenue rise at a slower rate. The accounting scheme makes this calculation for every possible environment the manager might observe, given the equilibrium condition that the observable investment expenditure set by the manager is the optimal one for that environment.

The logic behind this design is that, the more the manager's information conveys about how investing today pays off in the future, the more these future effects dominate the investment decision, and hence less of the total benefit from marginal investment is reflected in today's incremental cash receipts. Then in order to reward the manager for investing more in a better environment, accrued revenue must become more responsive to cash flow. In the converse case when information speaks to the present but contains little news about future periods, the manager's adjusting of the investment to changes in the environment is largely reflected by changes in current-period cash payoff, so that the accrual-basis revenue is optimally more aligned with cash revenue. Charging an expense, calculated based on the investment cash expenditure, against the revenue makes this construction sustainable as an equilibrium outcome by forcing the manager to internalize the cost of generating the revenue. In analogy to the matching principle, the expense increases at a faster rate when the rate of revenue accrual is also rising.

The optimal sensitivity of the resulting net profit to changes in the manager's investment decisions is generally either strictly higher or lower than the corresponding sensitivity of firm value, so that the manager's incremental compensation does not coincide with the incremental value impact of the manager's actions. At an intuitive level, the reason lies in the manager's informational advantage over the owner, who can only conjecture the state of the environment based on the observed cash flows. Performance must be accounted for such that, at the first-best investment in any state, a change in the environment, which the manager cannot influence, has the same effect on compensation as a change in investment, which the manager controls. This construction counteracts the manager's incentive to manipulate the performance metric through inefficient investment choices.

A noteworthy implication of this design is that the optimal compensation scheme is not equivalent to having sold the firm to the manager. Incentive-compatibility requires the detaching of compensation from value accretion, which in turn necessitates the presence of both a decision-maker and a residual claimant, or owner, when economic actors have a horizon shorter than the life of the firm. Consolidating ownership and management responsibility in the same economic agent would be an inefficient organizational design because an owner-manager would invest suboptimally if anticipating that the firm will be sold to a new investor in the future. Divorcing the two roles allows for succession in ownership to remain independent of investing decisions. The model thus provides a plausible reason for the separation of ownership and control typically seen in large, long-lived enterprises.[4] Unlike many standard principal-agent models, the desirability of this separation does not require the assumption of risk-aversion or limited liability on the part of the agent. A directly related implication is that stock-based compensation, often considered as a means to align owners' and managers' interests, is detrimental because it amounts to a partial remerging of ownership and control and thus brings back the incentive problem that separation helps to avoid.

Formally, accounting implements a revelation mechanism in this agency problem. In equilibrium, the manager reveals the otherwise private information about effort and environment. The owner therefore remains apprised of the firm's status quo at all times and can sequentially negotiate efficient employment contracts as current managers retire and new ones are hired. In enabling this mechanism, accounting has information content beyond the revenue and expenditure cash flows used as inputs and is thus not merely an inconsequential transformation of cash flow

[4] The logic bears some similarity to Holmström's (1982) observation that providing efficient incentives in teams of agents is incompatible with a balanced budget constraint, a problem that can be resolved by introducing a principal who absorbs any budget surplus or shortfall that may arise when agents' incentive pay is calibrated to the optimum.

data. This information production role obtains even though the accounting process is fully transparent and relies only on known and publicly observable data. At the same time, incentivizing optimal investing decisions means that accounting also has a direct impact on real activities.

Managerial performance measurement determines the firm's compensation payments, which in turn affect the firm's total cash flow. The design of managerial performance measurement therefore influences how value creation is reflected in the firm's general-purpose financial reporting. The incentive-compatible compensation scheme requires that the manager is paid on the spot for the full value impact of the manager's investment decisions in the current period. Compensation payments therefore reward the manager not only for incremental current cash revenue but for all benefits of the manager's actions, including benefits only realized in future periods. As a result, higher investment leads to a net reduction in the firm's contemporaneous cash flow, even though the additional investment adds value in the long term. This directional misalignment of cash flow and value creation is not merely a reflection of the basic intuition that investment requires a present expenditure but generates benefits only in the future. While it is true that better investment opportunities imply higher expenditure even in the absence of the incentive problem examined here, the phenomenon is decidedly aggravated because the firm's investment cost includes managerial compensation that, to establish incentive-compatibility, pays the manager not only for the effort cost, i.e., for the underlying resource itself, but also for the surplus value credited to the manager's actions. The incentive problem therefore makes a quantitative difference: in the absence of incentive frictions, it is possible, but not necessary, that incremental investment expenditure outweighs the contemporaneous incremental cash revenue. With the incentive problem added, the incremental cash expenditure is, ceteris paribus, strictly higher and always outweighs the contemporaneous incremental cash revenue. If accrual accounting for financial reporting purposes is designed to undo this misalignment by deferring some of the investment cost, one should expect that, among businesses whose investments are difficult to evaluate and who therefore rely heavily on incentive pay, incremental expenditures lead to larger accrual changes than in businesses whose investing activities are more transparent to the outsider.

This paper has a close connection to prior work on accrual-based performance measurement. Reichelstein (1997, 2000), Dutta and Reichelstein (2005a) and Mohnen and Bareket (2007) demonstrate that revenue and cost allocation rules, when properly designed in a residual income framework, attain goal-congruence between principal and agent in various transaction settings.

Pfeiffer and Schneider (2007) show the robustness of these results to the introduction of adverse selection. Dutta and Reichelstein (2002), Dutta and Zhang (2002), and Baldenius, Dutta and Reichelstein (2007) likewise integrate accrual accounting for performance measurement with an explicit agency conflict by interacting the goal congruence objective with a moral hazard problem. These papers consider either single or time-separable investments, which entail deterministic matching of investment costs to future revenues as a solution to the incentive problem. Other models confine attention to linear contracts and accounting variables in reduced form but include leading indicator variables (Dikolli 2001; Smith 2002; Dutta and Reichelstein 2003) or stock price (Dutta and Reichelstein 2005b) in the compensation scheme, or allow for correlation in performance measurement error across time periods (Christensen, Feltham and Sabac 2005) or differences in managers' skills (Dutta 2008).

2. Model Setup

Consider the following discrete-time model of a firm that invests resources to generate cash inflows, hereafter referred to as (cash) revenues. Investments take the form of both verifiable cash expenditures and unobservable effort. Expenditures include purchases of, for example, capital assets, inventories and supplies. Effort should be thought of as work performed by employees in creative or managerial positions with complex job descriptions and substantial variety and discretion in their tasks and daily schedules, so that the impact these employees have on the firm's revenue cannot be observed directly.[5] The model considers the interaction of two parties: the firm's manager, who determines expenditures and commits effort, and the firm's owner, who hires and compensates the manager and receives the net profits of the business.

The mapping from investments to revenue has two important features. First, investments are not independent, in the sense that the marginal contribution to future revenue by an investment made today depends on the firm's history of prior investments. Second, revenues are not deterministic but depend jointly on investment and on exogenous, random events beyond the manager's control. In particular, revenue takes the form of a cash inflow m whose value in each pe-

[5] Examples include basic research and strategic decision-making. Some external vendors' services may also fall into this category, as indicated by the use of performance-based supply contracts in some industries.

riod $t = 1,2, ...$ depends on two state variables: a deterministic investment variable x_t and a stochastic environment variable θ_t. After its realization at the end of the period, the cash revenue $m(x_t, \theta_t)$ becomes observable and verifiable.

The environmental state variable $\theta_t \in \mathbb{R}$ summarizes all events external to the firm and follows a Markov process with commonly known distributional properties. The state variable θ_t can only be discovered by running the operations of the firm. In particular, the manager observes θ_t prior to determining the investment in period t, whereas the owner (or any other outsider) does not learn θ_t in any period, both present and future.[6] One should think of this situation as a simplified representation of the multitude of information that only someone immersed in the day-to-day management is privy to, including, for example, intra-firm personnel politics, the financial situation of customers and suppliers, direct feedback from employees and external business contacts via personal interaction, etc. Hence, the only way for the owner to observe θ_t directly would be to assume the manager's position and to run the firm personally. This setup implies that, from the manager's perspective, the current-period cash revenue m becomes deterministic once θ_t has been observed.[7]

The investment variable x_t is a function of the manager's current investment decision and the firm's history of past investments. In particular, the evolution of x_t follows the process

$$x_t = x_{t-1} + k_t + h_t \tag{1}$$

where x_{t-1} is the investment value at the end of the prior period, and k_t and h_t denote, respectively, the manager's expenditure and effort choices in period t. The transition model in (1) implies that investment in period t affects not only contemporaneous revenue but also the marginal return to investment in future periods, and thus future investment decisions.[8] Of the two components of x_t, the expenditure $k_t \in \mathbb{R}^+$ is verifiable and its cost is paid out of the firm's (and thus

[6] That the manager observes θ_t perfectly simplifies the notation but is not essential. One could alternatively provide the manager with an information signal that is imperfectly correlated with θ_t and restate the analysis as an optimization over expected outcomes conditional on the signal. The critical model feature is the information asymmetry between manager and owner.

[7] Without loss of insight, one can extend the analysis to a scenario in which cash revenue is additionally subject to a random shock ε_t that even the manager cannot foresee.

[8] The linear form of (1) simplifies the analysis but is not a necessary condition for the following results. Deriving clean analytical statements from a more general transition function $g(x_{t-1}, k_t, h_t)$ would, however, require additional regularity conditions without adding relevant insight.

the owner's) assets, whereas the effort $h_t \in \mathbb{R}^+$ is incurred privately by the manager and is unobservable to the owner. Let $c: k_t \rightarrow \mathbb{R}^+$ denote the cost of k_t, and let $q: h_t \rightarrow \mathbb{R}^+$ denote the monetary equivalent of the manager's effort cost. The cost functions c and q are convex, with $c'(0) = q'(0) = 0$. The revenue function $m: (x_t, \theta_t) \rightarrow \mathbb{R}^+$ is concave and monotonically increasing in x_t, with $m_\theta(0, \theta_t) = 0$ for all θ_t. All functions are continuously differentiable. Both current and future revenues have increasing differences in investment and the environment, i.e.,

$$m_{x\theta}(x_t, \theta_t) > 0$$

and

$$\frac{\partial}{\partial \theta_t} E(m_x(x_{t+t}, \theta_{t+1})|\theta_t) \geq 0$$

for any x_t and θ_t.[9] More favorable conditions thus increase the first-best amount of investment both in the current and, in expectation, in future periods.[10]

The owner hires the manager for a finite number of periods and offers some monetary compensation, which may be contingent on any verifiable information available, i.e., on all past and present cash revenues and expenditures. The life of the firm is assumed to be infinite, whereas all actors in the model have a finite horizon and therefore look to liquidate their assets, including any ownership share in the firm, at some finite, future point in time. In particular, the manager retires at the end of the employment contract period and hence must be rewarded at this time. Compensation must therefore be paid before all future effects of the manager's investment decisions have materialized in observable revenue.[11] The firm's total cash outflow thus consists of the investment expenditure c and of the manager's compensation, and the firm pays out the net

[9] Except when indicating time periods, subscripts denote partial derivatives throughout the text. All expectations of the form $E(m(x_{t+i}, \theta_{t+i})|\theta_t)$, with $i = 1,2, \ldots$, are taken over future states, conditional on the current state.

[10] The direction of the interaction between investment and environment in their impact on present and future cash revenue is not critical as long as monotonicity is maintained. It is, however, important that $m_{x\theta} \neq 0$ in the current period. The reason is that an efficient and incentive-compatible employment contract must amount to a mechanism that reveals θ_t, the manager's private information, as an equilibrium outcome. This inference must come from variations in current cash revenue because m does not become verifiable until realized and so only the effects of θ_t on present, but not on future, revenues can be used to measure the manager's performance. Requiring $m_{x\theta} \neq 0$ in the current period is not a strong assumption, however, as a scenario in which θ_t affects the future but is entirely orthogonal to present outcomes would be rather implausible.

[11] Letting compensation depend on outcomes in some finite number of periods post retirement would not change any insights from the analysis, as long as the manager's investment decisions have effects beyond the point at which compensation is finally settled.

cash flow (cash revenue less payments) to the owner at the end of each period. If payments exceed revenues, the owner makes a corresponding contribution of additional capital. Both owner and manager are risk-neutral, and the manager's reservation wage is set to zero.

3. Accounting and Contract Design

Given the risk-neutrality of the actors and the absence of other impediments, one should first investigate whether any potential agency problems can be forestalled if the owner either sells the firm to the manager or, equivalently, personally takes control of managing the business. As a baseline, consider a hypothetical scenario in which an owner-manager with an infinite horizon runs the firm. No incentive problem arises in this setting because the owner-manager receives all revenues while internalizing all investment costs. Intrinsic firm value, hereafter denoted by v, therefore equals the first-best optimum

$$v(x_{t-1}, \theta_t) = \max_{\{k_i, h_i\}_{i=t}^{\infty}} \sum_{i=t}^{\infty} \gamma^{i-t} E(m(x_i, \theta_i) - c(k_i) - q(h_i) | \theta_t) \tag{2}$$

at the beginning of period t, where $\gamma < 1$ is the firm's discount factor.

The object of interest is the optimal investing policy, i.e., the expenditure and effort levels k^* and h^* that, as functions of x_{t-1} and θ_t, maximize v in each period t and thereby solve (2). It is well known that, given the properties of the revenue and cost functions, the value function v in (2) is the unique solution to the Bellman equation

$$v(x_{t-1}, \theta_t) = \max_{k_t, h_t} \{m(x_t, \theta_t) - c(k_t) - q(h_t) + \gamma E(v(x_t, \theta_{t+1}) | \theta_t)\} \tag{3}$$

Further, given the concavity of the firm's net cash flow, v is concave in x_t, and the corresponding optimal investment choices $k^*(x_{t-1}, \theta_t)$ and $h^*(x_{t-1}, \theta_t)$ are unique in any period t (Lucas and Stokey 1989).[12] The following lemma summarizes these observations. All proofs can be found in the appendix.

Lemma 1. *There exists a unique first-best investment plan $(x_{t-1}, \theta_t) \to (k^*, h^*)$ that maximizes firm value at all t. The resulting value function $v(x_{t-1}, \theta_t)$ is concave.*

[12] Several generalizations of this model are possible, albeit not necessary for the results to come. For example, the cost functions $c(\cdot)$ and $q(\cdot)$ could each depend on $(x_{t-1}, k_t, h_t, \theta_t)$. Further, the problem need not be stationary, i.e., the revenue and cost functions could also change with t, and revenues and costs could contain period-specific error.

Lemma 1 implies that optimal expenditure and effort are characterized by the unique solutions to the (necessary and sufficient) first-order conditions

$$m_x(x_t, \theta_t) + \gamma E(v_x(x_t, \theta_{t+1})|\theta_t) - c'(k_t) = 0 \tag{4}$$

and

$$m_x(x_t, \theta_t) + \gamma E(v_x(x_t, \theta_{t+1})|\theta_t) - q'(h_t) = 0 \tag{5}$$

in any period t. Consider now a small increase in θ_t. In order to maintain (4) and (5), the manager adjusts expenditure and effort such that

$$\begin{aligned}&\big(m_{xx}(x_t, \theta_t) + \gamma E(v_{xx}(x_t, \theta_{t+1})|\theta_t)\big)\big(k_\theta^*(x_{t-1}, \theta_t) + h_\theta^*(x_{t-1}, \theta_t)\big) + m_{x\theta}(x_t, \theta_t)\\ &\quad + \gamma E_\theta(v_x(x_t, \theta_{t+1})|\theta_t) - c''(k_t)k_\theta^*(x_{t-1}, \theta_t) = 0\end{aligned}$$

and

$$\begin{aligned}&\big(m_{xx}(x_t, \theta_t) + \gamma E(v_{xx}(x_t, \theta_{t+1})|\theta_t)\big)\big(k_\theta^*(x_{t-1}, \theta_t) + h_\theta^*(x_{t-1}, \theta_t)\big) + m_{x\theta}(x_t, \theta_t)\\ &\quad + \gamma E_\theta(v_x(x_t, \theta_{t+1})|\theta_t) - q''(h_t)h_\theta^*(x_{t-1}, \theta_t) = 0\end{aligned}$$

where

$$E_\theta(v_x(x_t, \theta_{t+1})|\theta_t) \equiv \frac{\partial}{\partial \theta_t} E(v_x(x_t, \theta_{t+1})|\theta_t)$$

and all functions are evaluated at the first-best optimal values $k^*(x_{t-1}, \theta_t)$ and $h^*(x_{t-1}, \theta_t)$. Rearranging and solving for the optimal adjustments in investment yields

$$k_\theta^* = \frac{\big(m_{x\theta} + \gamma E_\theta(v_x)\big)q''}{c''q'' - \big(m_{xx} + \gamma E(v_{xx})\big)(c'' + q'')} \tag{6}$$

and

$$h_\theta^* = \frac{\big(m_{x\theta} + \gamma E_\theta(v_x)\big)c''}{c''q'' - \big(m_{xx} + \gamma E(v_{xx})\big)(c'' + q'')} \tag{7}$$

along the path of first-best investment choices.[13] Since v and m are concave and c and q are convex, (6) and (7) imply that k^* and h^* are increasing in θ_t for any initial x_{t-1}. An owner-manager with an infinite horizon would implement this first-best investment policy, which will serve as a reference point hereafter.

Consider now the more realistic scenario in which the owner-manager is looking to sell the firm at some finite time in the future (say, when reaching retirement age). In particular, suppose

[13] For better readability, the shorthand notation $m \equiv m(x^*(\theta_t), \theta_t)$, $v \equiv v(x^*(\theta_t), \theta_{t+1})$, $c \equiv c\big(k^*(\theta_t)\big)$, etc. will be used at times for functions evaluated at first-best values.

that the owner-manager expects to sell the firm at the end of period T to an outside buyer, who can observe the firm's history of revenues and expenditures up to T and the initial state x_0, but observes neither the owner-manager's effort h_t nor the state variable θ_t at any $t = 1, \ldots, T$.[14] Hence, the buyer offers some price equal to a conjectured firm value

$$\hat{v}(\{m_i, k_i\}_{i=1}^{T}) \equiv E(v(x_T, \theta_{T+1}) \big| \{m_i, k_i\}_{i=1}^{T})$$

at the end of period T. In addition to the selling price $\hat{v}$, the owner-manager also receives all of the firm's net cash flows up to the end of period T and therefore, in any period t, chooses an investment plan that solves

$$\max_{\{k_i,h_i\}_{i=t}^{T}} \sum_{i=t}^{T} \gamma^{i-t} E(m(x_i, \theta_i) - c(k_i) - q(h_i) | \theta_t) + \gamma^{T-t} E(\hat{v}(\{m_i, k_i\}_{i=1}^{T}) \big| \theta_t)$$

in anticipation of selling the firm for $\hat{v}$ after period T.

If the owner-manager were to apply the first-best investment policy identified above, the rational buyer would infer $\hat{v} = v$, i.e., the selling price of the firm would equal the first-best value function that solves (2). In critical difference to the valuation by a fully informed insider, however, this inferred continuation value does not depend directly on h_t and θ_t, which are unobservable to the outside buyer. Hence, if the buyer conjectured that the first-best investment policy has been implemented and offered a price of $\hat{v} = v$, a necessary condition to sustain this equilibrium outcome must be the absence of an incentive for the owner-manager to deviate from first-best investment at all $t \leq T$. Deviating from first-best investment without being detected is generally feasible because there exists a continuum of effort choices h_t and state variables θ_t that give rise to the same revenue amount and are thus indistinguishable to the buyer. In particular, suppose that, in a given period t, the buyer knew the beginning investment state x_{t-1}. Then an observed amount of revenue would be consistent with the first-best outcomes associated with the concurrently observed expenditure k_t as long as $m(x_t, \theta_t) = m\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)$, where $x^*(x_{t-1}, k_t)$ and $\theta^*(x_{t-1}, k_t)$ are the first-best investment level and environment associated with k_t and the beginning-of-period initial investment level x_{t-1}.[15]

For any given realization of θ_t, two deviation scenarios are possible. First, the owner-manager might overspend and set $k_t > k^*(x_{t-1}, \theta_t)$, while at the same time increasing the effort h_t

[14] Recall also that h_t and θ_t are unverifiable, so the owner cannot convey these values credibly to the buyer.
[15] That x^* and θ^* are unique follows from (6) and (7).

such that $m(x_t, \theta_t) = m\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)$ and thus making the buyer believe that the state is $\theta^*(x_{t-1}, k_t) \neq \theta_t$.[16] This maneuver is beneficial to the owner-manager if the resulting gain in the buyer's conjectured firm value $\hat{v}$ is large enough to justify the extra effort cost the owner-manager has to expend. Alternatively, the owner-manager might consider underspending and set $k_t < k^*(x_{t-1}, \theta_t)$, which permits a reduction in effort, again such that $m(x_t, \theta_t) = m\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)$. The buyer would then infer a state $\theta^*(x_{t-1}, k_t) < \theta_t$ and, accordingly, offer a lower purchase price $\hat{v}$ for the business. This deviation is optimal for the owner-manager if the loss in $\hat{v}$ is outweighed by the saving in effort cost. The following result shows that, except in a knife-edge case, either one of the two deviations would be profitable, so that an equilibrium with first-best investment is generally not sustainable.

Proposition 1. *If $E_\theta(v)m_x \neq E(v_x)m_\theta$ at the first-best investment for any θ_t, then there exists no equilibrium in which an owner-manager with a finite horizon makes first-best investments.*

Proposition 1 highlights that, without full observability of investments and economic conditions, an owner-manager cannot make a credible commitment to investing optimally. The reason is that, firstly, the owner-manager bears an investment cost whose benefits will, in part, only be realized after the end of the owner-manager's tenure at the firm, and, secondly, the transaction between the owner-manager and the buyer occurs *after* the owner-manager has received private information about the firm's economic condition. A similar information asymmetry problem arises if an owner-manager were to retire from managing duties at some time t and hire an external manager, but still retain ownership. Since the initial investment state x_{t-1} and the prior random state θ_{t-1} are the owner's private information, the negotiation of the initial compensation contract would again suffer from information asymmetry.

The upshot of this line of reasoning is that concentrating ownership and control (or management responsibility) in the hands of one party creates informational friction and thus a loss in firm value. One might therefore conjecture that separation of ownership and control could address the problem, and that a solution might take the form of a contractual arrangement by which the party who owns the firm at the end of the period promises a payment to a manager who

[16] Recall that $h_\theta^*, k_\theta^* > 0$, i.e., first-best expenditure and effort increase together.

makes the investments. This payment must be agreed upon *before* the manager obtains private information and have the property that the manager internalizes the owner's objective function, i.e., that maximizing the compensation payment also maximizes value creation. Specifically, suppose that the manager is hired to run the firm for one period, hereafter referred to as period t. (No substantive differences arise in a contract spanning several periods.) After observing θ_t, the manager determines the expenditure k_t and the effort h_t and, after the realization of the revenue m at the end of the period, receives compensation from the owner, which may possibly be contingent on m and k_t. The owner, removed from the daily operations of the firm, observes neither the manager's effort nor θ_t.

The objective is to find a compensation scheme that, in equilibrium, implements first-best investment. One should first observe that paying the manager a fixed salary would not accomplish this because the manager bears the effort cost q and would therefore set the effort level h_t to the minimum value, regardless of θ_t. Stock-based compensation would likewise fail because the firm's stock price reflects investors' conjectured firm value $\hat{v}$, which creates the same incentive problem as in Proposition 1.[17] A third candidate for a simple compensation scheme is paying the manager based on the firm's cash flow, i.e., setting compensation equal to

$$m(x_t, \theta_t) - c(k_t) + \bar{q}$$

where $\bar{q}$ is a fixed salary component set such that the manager receives an expected payoff, net of expected effort cost, equal to the reservation wage of zero and is therefore willing to sign the employment contract at the beginning of period t. One can readily tell that this cash flow-based scheme would not recover the first-best decision policy because the manager would optimize by setting expenditures and effort such that

$$m_x(x_t, \theta_t) - c'(k_t) = 0$$

and

$$m_x(x_t, \theta_t) - q'(h_t) = 0$$

which, compared to (4) and (5), are missing the future value elements. In other words, cash flow-based pay would make the manager neglect the long-term effects of investment.[18]

[17] Based on a similar logic, Dutta and Zhang (2002) demonstrate that the use of mark-to-market accounting in performance measurement fails to achieve incentive alignment. Stock price may, however, be a useful ingredient in a compensation scheme if part of the information it contains is both incentive-relevant and orthogonal to all other contractible variables in the model (Dutta and Reichelstein 2005b).

[18] This problem would persist even if the manager were hired for multiple periods, as long as the manager's tenure is shorter than the life of the firm.

Consider instead conditioning the manager's pay on a profit figure calculated on an accrual basis, i.e., on cash flow adjusted such that maximizing this accrual-basis profit solves the optimization problem in (3). Both cash revenue and expenditures are verifiable, and hence the accrual-basis income can be any mapping $\big(m(x_t, \theta_t), c(k_t)\big) \to \mathbb{R}$ or, equivalently, $(m(x_t, \theta_t), k_t) \to \mathbb{R}$, since the cost function c is invertible. Two equilibrium conditions must be met. First, optimizing the accrual income with respect to the value function v in (3) is only valid if the result is indeed incentive-compatible for all θ_t, because v assumes first-best investment to begin with. Second, compensation contracts and managers' actions need to be sequentially rational across periods, in the sense that the owner's belief about the investment state x_{t-1} and environment θ_{t-1} at the beginning of the period must be correct, given the compensation scheme and the equilibrium actions of the manager who ran the firm in the prior period $t-1$.

To establish the existence of such a compensation scheme, it will be useful to posit as an initial conjecture that accrual-basis income can be calculated such that a manager compensated based on this income makes first-best investments. The accrual income formula can then be optimized with respect to the first-best value function v if the manager's resulting decision incentives indeed turn out to yield the solution to (3). In equilibrium, the manager's chosen expenditure k_t will then reveal the state x_t. Specifically, consider accrual income of the form

$$p\big(k_t, m(x_t, \theta_t)\big) = r(k_t)m(x_t, \theta_t) - a(k_t) + \bar{q} - z\big(k_t, m(x_t, \theta_t)\big) \tag{8}$$

where r, a and z are unknown, continuously differentiable functions.[19] One can view r as an accrual adjustment to the firm's cash revenue m, and a as an accrual-basis expense charge. The constant term $\bar{q}$ is set such that expected compensation, as of the beginning of period t and conditional on θ_{t-1}, equals the manager's reservation wage of zero. The z-function will serve as a disciplinary device and will be designed to have a local maximum $z\big(k_t, m(x_t, \theta_t)\big) = 0$ on the level set of expenditure-revenue pairs that are consistent with first-best outcomes. The optimization problem in (3) is now changed to

$$\begin{aligned} &E(v(x_{t-1}, \theta_t)|\theta_{t-1}) \\ &\quad = \max_{p(\cdot)} E\Big(m(x_t, \theta_t) - c(k_t) - p\big(k_t, m(x_t, \theta_t)\big) + \gamma E(v(x_t, \theta_{t+1})|\theta_t)\Big|\theta_{t-1}\Big) \end{aligned} \tag{9}$$

subject to the incentive-compatibility constraint

[19] All elements of p technically also depend on the beginning investment level x_{t-1}, which is omitted for notational economy from hereon. Recall that, in equilibrium, the owner infers x_{t-1} from the results of the prior period.

$$(k_t, h_t) \in \arg\max_{k_t, h_t}\{p\big(k_t, m(x_t, \theta_t)\big) - q(h_t)\}$$

for all θ_t, the first-best optimality condition

$$(k_t, h_t) = \arg\max_{k_t, h_t}\{m(x_t, \theta_t) - c(k_t) - q(h_t) + \gamma E(v(x_t, \theta_{t+1})|\theta_t)\}$$

for all θ_t, and the participation constraint

$$E\big(p\big(k_t, m(x_t, \theta_t)\big) - q(h_t)\big|\theta_{t-1}\big) = 0$$

To solve (9), one can begin by conjecturing that the solution function p in (8) has a unique maximum in k_t and h_t for each θ_t. If this maximum were to coincide with first-best investment, the manager would choose effort and expenditure as solutions to the first-order conditions

$$p_m\big(k_t, m(x_t, \theta_t)\big)m_x(x_t, \theta_t) - q'(h_t) = r(k_t)m_x(x_t, \theta_t) - q'(h_t) = 0 \tag{10}$$

and

$$\begin{aligned} &p_k\big(k_t, m(x_t, \theta_t)\big) + p_m\big(k_t, m(x_t, \theta_t)\big)m_x(x_t, \theta_t) \\ &\quad = r'(k_t)m(x_t, \theta_t) + r(k_t)m_x(x_t, \theta_t) - a'(k_t) = 0 \end{aligned} \tag{11}$$

since, by construction, $z_k = z_m = 0$ at first-best investment. This incentive-compatible outcome obtains if the solutions to (10) and (11) coincide with the solutions to (4) and (5). By Lemma 1, the optimal investment plan is unique, and hence, in view of (6) and (7), the state transition function can be inverted to obtain, from a given expenditure k_t, the first-best effort $h^*(k_t)$ and the associated state $\theta^*(k_t)$.[20] Evaluating (10) at these values implies that the optimal revenue accrual coefficient r in (8) must take the form

$$r(k_t) = \frac{q'\big(h^*(k_t)\big)}{m_x\big(x^*(k_t), \theta^*(k_t)\big)} \tag{12}$$

where $x^*(k_t) = x_{t-1} + k_t + h^*(k_t)$. Similarly, evaluating (11) at first-best investment yields an accrued expense term a in (8) of the form

$$\begin{aligned} a(k_t) &= \int_0^{k_t} \Big(r(u)m_x\big(x^*(u), \theta^*(u)\big) + r'(u)m\big(x^*(u), \theta^*(u)\big)\Big)\, du \\ &= c(k_t) + \int_0^{k_t} r'(u)m\big(x^*(u), \theta^*(u)\big)\, du \end{aligned} \tag{13}$$

where the second equality follows from (4) and (5). The fixed compensation component

$$\bar{q} = E\Big(q\big(h^*(\theta_t)\big) - r\big(k^*(\theta_t)\big)m(x^*(\theta_t), \theta_t) + a\big(k^*(\theta_t)\big)\Big|\theta_{t-1}\Big) \tag{14}$$

---

[20] Recall that x_{t-1} is known in equilibrium before the contract is signed and can therefore be treated as a constant.

is set to make the manager's expected compensation cover the expected effort cost.

To ensure that the manager has no incentive to implement investments off the path of first-best choices, the z-function in (8) must be calibrated to impose a sufficiently high penalty whenever the realized amount of revenue is inconsistent with the expected first-best revenue given the manager's expenditure, i.e., when $m(x_t, \theta_t) \neq m\big(x^*(k_t), \theta^*(k_t)\big)$. A necessary condition to this end is that

$$\begin{aligned} &z\big(k_t, m(x_t, \theta_t)\big) \\ &\quad \geq r(k_t)\Big(m(x_t, \theta_t) - m\big(x^*(k_t), \theta^*(k_t)\big)\Big) + q(h_t) - q\big(h^*(k_t)\big) \end{aligned} \tag{15}$$

for all k_t and θ_t.[21] The following result establishes formally that (8), with its components defined as in the preceding discussion, indeed constitutes the only incentive-compatible accounting method to measure performance.

Proposition 2. *A compensation scheme is incentive-compatible if and only if performance is measured by (8), with r, a, $\bar{q}$ and z defined by (12) through (15).*

One should note that Proposition 2 does not describe a forcing contract that implements first-best investment solely by penalizing the manager when the realized expenditure and revenue numbers are inconsistent with a first-best outcome. In fact, the penalty function z is constant on the set of first-best outcomes and hence does not explain any of the variation in accounting profits realized in equilibrium.[22] The realized variation in p is entirely due to the revenue accrual r and the expense function a in (12) and (13), which identify the unique incentive-compatible solution to (9) on the set of possible equilibrium outcomes.[23] The proper specification of r and a is

[21] The optimal effort choice h_t is always uniquely determined by k_t and θ_t, given the convexity of the effort cost q.

[22] If the firm's cash revenue m is realized with error, the penalty z can only be made constant across first-best outcomes in expectation, rather than with certainty.

[23] The penalty function z is not unique off-equilibrium and can take any form that satisfies (15). In its arguably simplest form, the penalty function reduces the manager's compensation to zero at off-equilibrium outcomes, i.e.,

$$z\big(k_t, m(x_t, \theta_t)\big) = \big(r(k_t)m(x_t, \theta_t) - a(k_t)\big)\Big(1 - \mathbf{I}_{m^*}\big(m(x_t, \theta_t)\big)\Big)$$

so that

$$p\big(k_t, m(x_t, \theta_t)\big) = \big(r(k_t)m(x_t, \theta_t) - a(k_t)\big)\mathbf{I}_{m^*}\big(m(x_t, \theta_t)\big)$$

where the indicator function $\mathbf{I}_{m^*}$ equals unity whenever revenue and expenditure are consistent with a first-best outcome, i.e., when $m(x_t, \theta_t) = m\big(x^*(k_t), \theta^*(k_t)\big)$, and zero otherwise.

therefore critical in addressing the incentive problem identified in Proposition 1: in any given environment, the manager faces a continuum of expenditure and effort choices that generate revenue figures apparently consistent with first-best investment, of which all but one are inefficient. The accrual profit formula p in (8) aligns incentives in that, for each θ_t, the actual first-best investment from this continuum is the only investment choice that maximizes p, and thus the manager's compensation.

One can readily see from (12) to (15) that the primitive design element in all components of p is the accrual coefficient r. To make its role in creating incentive-compatibility more transparent, one can substitute the first-best optimality condition (5) into (12) to obtain

$$r(k_t) = 1 + \frac{\gamma E(v_x(x_t, \theta_{t+1})|\theta_t)}{m_x(x_t, \theta_t)} \tag{16}$$

where $x_t = x^*(k_t)$ and $\theta_t = \theta^*(k_t)$. On the surface, (16) is unsurprising: cash flow alone does not reflect the impact of current investment on future periods, and so r forces the manager to consider this missing part by including $E(v_x)$ in the accounting.

One might intuit that the resulting profit number should then measure the value added by the manager's investment decisions, but the accounting defined by (12) through (15) only aligns the optimality conditions for the manager's compensation with those for total firm value. To this end, r must scale cash flow by *marginal*, rather than total, present and future revenues, which leads to a performance metric that does not equate to incremental firm value. To understand the rationale behind calibrating accounting with respect to marginal payoffs, it is helpful to compare accrual profit and firm value in terms of their respective sensitivities to investment. Equating these sensitivities would be tantamount to selling the firm to the manager, which Proposition 1 implies would fail to attain first-best investment. The incentive-compatible profit metric in Proposition 2 does therefore not award the manager the exact amount of incremental firm value resulting from the manager's actions.

In particular, differentiating firm value totally in the original, frictionless case of an owner-manager with an infinite horizon shows that, in equilibrium, an increase in θ_t implies a change in firm value by

$$\frac{dv}{d\theta_t} = \frac{\partial v}{\partial \theta_t} = m_\theta(x_t, \theta_t) + \gamma E_\theta(v(x_t, \theta_{t+1})|\theta_t) \tag{17}$$

in view of the envelope theorem, where $x_t = x^*(\theta_t)$. By comparison, the same increase in θ_t changes the manager's equilibrium compensation, net of effort cost, by

$$\begin{aligned} \frac{d}{d\theta_t}\Big(p\big(k_t, m(x_t,\theta_t)\big) - q(h_t)\Big) &= \frac{\partial}{\partial\theta_t} p\big(k_t, m(x_t,\theta_t)\big) = r(k_t) m_\theta(x_t,\theta_t) \\ &= m_\theta(x_t,\theta_t) + \gamma E(v_x(x_t,\theta_{t+1})|\theta_t)\frac{m_\theta(x_t,\theta_t)}{m_x(x_t,\theta_t)} \end{aligned} \tag{18}$$

where $x_t = x^*(k_t)$, $k_t = k^*(\theta_t)$ and $h_t = h^*(\theta_t)$. The second equality in (18) obtains after substituting the first-order condition in (5) into (10) and replacing the revenue accrual coefficient r by (16). An incentive-compatible compensation scheme thus has a trajectory that, as a function of θ_t, is either steeper or flatter than the trajectory of firm value in a frictionless setting, because the effect of the manager's actions on future revenues is, in an optimally designed compensation scheme, not accounted for at its actual value. This observation illustrates why separation of ownership and control is requisite if the agency problem is to be resolved while still implementing first-best investment: the marginal benefit absorbed by the manager must be different from the marginal value created by the manager's actions. One might thus characterize the solution as renting, rather than selling, the firm to the manager, under a kind of profit-sharing scheme that gives the owner some residual claims.

One can see immediately that the marginal firm value in (17) coincides with the manager's marginal net compensation in (18) if and only if

$$E_\theta(v) m_x = E(v_x) m_\theta$$

which matches the knife-edge condition in Proposition 1 under which an owner-manager has no incentive to deviate locally from first-best investment even in the absence of an incentive scheme. Formally, the difference between value creation and managerial compensation thus arises because the ratio of marginal current revenue to marginal future value differs between increased investment and better environmental conditions. This observation connects directly to the rationale why the owner-manager in Proposition 1 would undertake suboptimal investment: since effort and environment are unobservable to the outsider, the manager could substitute one for the other to mimic first-best outcomes and thereby benefit at the owner's expense if compensation were naively determined according to (17).

Accrual accounting cancels this arbitrage opportunity. When $E_\theta(v) m_x > E(v_x) m_\theta$, environmental factors contribute relatively more to future value than to current revenue, in comparison to investment. In this case, the owner-manager in Proposition 1 sees an incentive to overinvest, which the accounting rule in Proposition 2 removes by undercompensating the manager for incremental value. In other words, the manager's marginal payoff from higher θ_t is dampened relative to the value added by the investment in that period, so that one would observe a low responsiveness of the manager's compensation to changes in the firm's market value. In the reverse case, investment (including the manager's unobservable effort) produces a greater marginal contribution to future value, relative to current revenue, than environmental factors do, which makes the owner-manager in Proposition 1 inclined to underinvest and reduce effort below the first-best level. The incentive scheme responds by making accrual-basis profit, and thus compensation, highly responsive to changes in value. The latter scenario applies to businesses facing a high degree of uncertainty about future outcomes, in the sense that the currently available information provides little insight into the firm's future environment.

Another tempting but incorrect reading of (16) is that, since r multiplies the cash revenue m and since $m_x, E(v_x) > 0$ implies $r > 1$, optimal accrual accounting for performance measurement amounts to an inflating of revenue above the amount of cash receipts. The incentive problem discussed here, however, makes no generalizable statement about the expected *levels* of cash flow and accrual profit because accrual profit only serves to calculate managerial compensation, whose expected value is exogenously given by the manager's outside wage option. What the observation that $r > 1$ instead implies is that, as a function of contemporaneous investment, accrual-basis revenue has a higher slope than cash revenue. In particular, r is linear in the ratio of marginal future value, $E(v_x)$, to marginal current cash revenue, m_x, so that investment with a high expected marginal impact on future periods implies high accrual adjustments per unit of current cash flow. In other words, investments with a long impact horizon lead to higher accruals than investments whose benefits are largely realized in the short run.

Including the investment cost in the analysis shows how this effect cascades down to net profit and net cash flow. By the envelope theorem, an increase in θ_t implies an increase in accrual profit by

$$\frac{d}{d\theta_t} p\big(k_t, m(x_t, \theta_t)\big) = r(k_t)\big(m_\theta(x_t, \theta_t) + m_x(x_t, \theta_t) h_\theta^*(x_t, \theta_t)\big) \tag{19}$$

By comparison, the same increase in θ_t changes net cash flow, excluding the manager's compensation, by

$$\begin{aligned}
&\frac{d}{d\theta_t}\big(m(x_t,\theta_t) - c(k_t)\big) \\
&\quad = m_\theta(x_t,\theta_t) + m_x(x_t,\theta_t)\big(k_\theta^*(x_t,\theta_t) + h_\theta^*(x_t,\theta_t)\big) - c'(k_t)k_\theta^*(x_t,\theta_t) \\
&\quad = m_\theta(x_t,\theta_t) + m_x(x_t,\theta_t)h_\theta^*(x_t,\theta_t) - \gamma E(v_x(x_t,\theta_{t+1})|\theta_t)k_\theta^*(x_t,\theta_t)
\end{aligned} \tag{20}$$

One can readily see that, in view of $r > 1$, the increase in p is strictly larger. Cash flow does not impound the future payoffs to the firm's current investment, which results in a mismatch of cash inflow and expenditure. In the extreme, cash flow may even decline if most of the benefits to the firm's incremental investment spending are realized in the future, i.e., if $E(v_x)$ in (20) is large. This directional misalignment of marginal costs and benefits does not arise in accrual accounting because the accrual coefficient r matches the two.

When interpreting r as a function of the investment expenditure k_t, one should bear in mind that the construction of r as part of an incentive-compatible profit metric anticipates first-best investment by the manager in equilibrium, and thus stating r formally as a function of investment expenditure k_t means evaluating its two components m_x and $E(v_x)$ at the optimal triple $\{k_t, h^*(k_t), \theta^*(k_t)\}$ associated with each k_t. In equilibrium, the manager predictably invests a higher amount k_t in response to a better environment θ_t. The magnitude of r thus effectively varies with θ_t as its primitive determinant on the set of possible first-best outcomes in any given period. Two opposing limit cases are shown in the following result.

Proposition 3. *The revenue accrual r becomes globally decreasing in the investment expenditure k_t as $\frac{E_\theta(v_x)}{m_{x\theta}} \to 0$ and globally increasing as $\frac{m_{x\theta}}{E_\theta(v_x)} \to 0$ on the set of first-best outcomes.*

The significance of Proposition 3 is that the optimal magnitude of accruals in performance measurement is, at its core, a function of the firm's information environment. The more the realization of θ_t reveals about investment returns in the future, relative to returns in the current period, the more the accrual adjustments per unit of cash flow increase in θ_t, even after the concurrent upward adjustment in first-best investment. The result follows from the observation that, given first-best investment, marginal revenue increases as the environment improves, or, in more technical terms, the total differential of both m_x and $E(v_x)$ is positive with respect to θ_t.[24]

In particular, if $E_\theta(v_x)$ is small relative to $m_{x\theta}$, the information the manager privately learns in the current period has little bearing on marginal revenues in future periods. When the manager then increases investment in response to higher θ_t, marginal cash revenue in the current period rises more strongly than expected marginal payoff in the future. As a result, keeping the manager incentivized to invest the first-best amount requires aligning accrual revenue more closely with the current-period cash inflow, and thus reducing r closer to unity. This scenario describes a business that faces high uncertainty about the future productivity of its current investments and possesses little information useful for forecasting the future, for example, firms relying heavily on high-risk, long-term research and development activity. The more a business of this type invests, the more accrual profit and cash flow are aligned. In the converse case, $m_{x\theta}$ is small relative to $E_\theta(v_x)$, which means that a higher θ_t in the current period makes relatively little difference to current marginal cash inflow but, by comparison, increases expected marginal payoff substantially in future periods. Here, making the manager internalize the future benefits of higher investment requires increasing r.[25] This scenario describes businesses whose performance can be forecasted well by current information. The more a firm of this type invests, the faster its accrual-basis revenue outpaces its cash receipts.

An implicit but important assumption underlying the analysis so far is that the manager optimizes decisions only with respect to the accrual profit in the current period and does not anticipate renewing the employment contract at the end of its term. This assumption is critical because the owner infers the (unobservable) end-of-period investment variable x_t and the environmental state variable θ_t as an equilibrium outcome and, based on both, offers a new contract to manager's successor in period $t+1$. If the new contract were to be offered to the incumbent manager and the incumbent manager had rational expectations during the original contract period, the anticipated renewal terms would factor into the manager's original investment decision. In all but

---

[24] This observation obtains from the first-order condition in (4). Maintaining (4) in response to an increase in θ_t requires an adjustment in investment by x_θ^* such that

$$m_{x\theta} + \gamma E_\theta(v_x) + (m_{xx} + \gamma E(v_{xx}) - c'')x_\theta^* = 0$$

That m_x and $E(v_x)$ both increase as a result now follows from $m_{x\theta}, E_\theta(v_x) > 0$, $m_{xx}, E(v_{xx}) < 0$ and $c'' > 0$.

[25] A further implication of a monotonically increasing revenue accrual is that all off-equilibrium expenditure-revenue pairs are suboptimal for the manager even in the absence of a penalty, so that the z-function in (8) can be set to zero everywhere.

the most fortuitous of circumstances, a distortion of incentives away from first-best results then becomes unavoidable, as the next result shows.

Corollary 2.1. *Rehiring the manager after the initial term of employment makes first-best investment unattainable as an incentive-compatible outcome if and only if*

$$\frac{d}{dk_t}E\left(p\left(k_{t+1}, m(x_{t+1}, \theta_{t+1})\middle|\hat{x}_t, \hat{\theta}_t\right) - q(h_{t+1})\middle|\theta_t\right) \neq 0$$

for any θ_t *and* $x_t = x^*(\theta_t)$.

A manager initially hired under a contract covering period t only but expecting to receive a renewal contract in period $t+1$ faces two considerations in the investment decision in period t that would not be present if employment ended after period t. First, changes in investment in period t alter the marginal return to investment in the future, which affect the payoff the manager's investment will generate under the anticipated renewal contract in period $t+1$. Second, changes in investment in period t influence the owner's beliefs about the manager's effort and about the environment, which affect the renewal contract terms the owner will offer. In particular, if the owner naively believed that the manager makes first-best investments, overinvestment in period t would, ceteris paribus, increase future revenue beyond the owner's expectation, while underinvestment would have the opposite effect. At the same time, over- or underinvestment would make the owner's inference about the current environment higher or lower than the actual θ_t and thus, depending on the relationship of θ_t to θ_{t+1}, make the owner offer a contract with an increased or decreased expected compensation amount. To avoid contamination of the manager's decision incentives in period t, these anticipated effects would have to neutralize each other exactly in all states, which, similar to Proposition 1, requires a rather implausible knife-edge scenario. In other words, incentive-compatibility is only attainable if the principal's ex-post beliefs about the agent's actions and information do not affect the agent's expected payoff, which is generally only possible if a new manager is hired at the end of the current manager's employment contract.[26]

[26] Dutta and Reichelstein (2003) similarly conclude that a series of employment contracts only yields optimal investment if the manager is replaced in each new contract. While their model setup differs substantially from what is presented here, the central cause is the same: after signing the original contract, the agent takes actions unobservable to

The accrual profit p determines the amount of compensation the manager receives, and p is calibrated to equal the manager's effort cost q in expectation. To see how the two differ across θ_t, recall that the optimal contract implements first-best investment in all states, and hence the manager increases effort by $q'(h_t)h_\theta^*(x_t,\theta_t)$ in response to a higher θ_t. The corresponding adjustment to the compensation payment is strictly larger, as one can verify by replacing the final term in (19) by (10) to obtain

$$\frac{d}{d\theta_t}p\big(k_t, m(x_t,\theta_t)\big) = r(k_t)m_\theta(x_t,\theta_t) + q'(h_t)h_\theta^*(x_t,\theta_t)$$

Incentive pay, although equal to effort cost in expectation, thus has a steeper reaction curve to θ_t, which implies that the variance of compensation exceeds the variance of the effort cost. One can read this observation as a statement about the effect of observability on the volatility of investment costs. If the cost functions c and q were the same, the amount and monetary value of expenditure and effort invested would be the same, but the need to induce optimal effort via incentive pay would create a higher sensitivity to economic conditions in the firm's cash outlay for effort than for the verifiable expenditure component. In practice, one would therefore expect a firm relying predominantly on 'invisible' investment, such as human capital, to have a higher variance in its cash outlay than, ceteris paribus, a firm investing primarily, say, in physical assets.

The volatility in incentive pay has implications for the firm's total net cash flow, i.e., the net amount of all cash receipts and payments, including the manager's pay. A useful point of reference is again a frictionless setting in which effort is contractible, so that the owner can pay the manager exactly according to the effort cost $q(h_t)$ incurred in each period while still obtaining first-best outcomes. The firm's total net cash flow in period t would then amount to

$$m(x_t,\theta_t) - c(k_t) - q(h_t)$$

with $k_t = k^*(\theta_t)$ and $h_t = h^*(\theta_t)$, and an increase θ_t would lead to an incremental cash flow of

$$m_\theta + m_x(k_\theta^* + h_\theta^*) - c'k_\theta^* - q'h_\theta^* = m_\theta - \gamma E(v_x)(k_\theta^* + h_\theta^*) \tag{21}$$

where the equality follows from (4) and (5). Similar to the ex-compensation cash flow in (20), (21) can be negative for high values of $E(v_x)$, i.e., in situations when the future component of the marginal payoff to present investment is large. Introducing the need for incentive pay means replacing q with p in this calculation, so that net cash flow becomes

---

the principal, and the resulting ex-post information asymmetry leaves incentive-compatibility intact only if the agent cannot exploit this informational advantage in a renewed interaction with the principal.

$$m(x_t, \theta_t) - c(k_t) - p\big(k_t, m(x_t, \theta_t)\big) = \big(1 - r(k_t)\big)m(x_t, \theta_t) - c(k_t) + a(k_t) + \bar{q}$$

An increase in θ_t now creates incremental cash flow of

$$\begin{aligned} & m_\theta + m_x(k_\theta^* + h_\theta^*) - c' k_\theta^* - p_m(m_\theta + m_x h_\theta^*) \\ & \quad = m_\theta + m_x h_\theta^* - \gamma E(v_x) k_\theta^* - r(m_\theta + m_x h_\theta^*) \\ & \quad = -m_\theta \frac{\gamma E(v_x)}{m_x} - \gamma E(v_x)(k_\theta^* + h_\theta^*) \end{aligned} \tag{22}$$

where the final equality obtains from (16). One can readily see that (22) is not only strictly less than (21) but also always negative, i.e., incentive pay creates an inverse relationship between incremental cash flow and investment even if the marginal future benefit to investment is small.[27] One should not, however, conclude from (22) that the correlation between investment and net cash flow is negative *across* time periods. Even though higher investment today lowers contemporaneous cash flow, the expected effect on the subsequent period is an increase in cash flow.

The analysis can readily be extended to studying the impact of the incentive problem on total firm value, by adding the future value component $E(v(x_t, \theta_{t+1})|\theta_t)$ to the net cash flow. An increase in θ_t thus changes firm value by

$$\begin{aligned} & \frac{d}{d\theta_t}\Big(m(x_t, \theta_t) - c(k_t) - p\big(k_t, m(x_t, \theta_t)\big) + \gamma E(v(x_t, \theta_{t+1})|\theta_t)\Big) \\ & \quad = -m_\theta(x_t, \theta_t) \frac{\gamma E(v_x(x_t, \theta_{t+1})|\theta_t)}{m_x(x_t, \theta_t)} + \gamma E_\theta(v(x_t, \theta_{t+1})|\theta_t) \end{aligned}$$

and so a more favorable environment implies a net increase in firm value if and only if

$$\frac{E_\theta(v)}{m_\theta} > \frac{E(v_x)}{m_x}$$

which is an expression already familiar from Propositions 1 and 2. Hence, if environmental factors have a stronger impact on future outcomes, relative to present outcomes, than investment does, the owner realizes an incremental net benefit, even after paying the manager. In the converse case, the manager is 'overcompensated' for better outcomes and paid more than the total increment in firm value. The intuition follows from the discussion around Proposition 2: the more incremental value depends on investment rather than on environmental factors, the more

[27] The difference between (21) and (22) naturally equals the manager's incremental pay, net of effort cost, of

$$-\left(1 + \frac{\gamma E(v_x)}{m_x}\right) m_\theta = -r m_\theta$$

the manager must be rewarded for better outcomes if the employment contract is to attain incentive-compatibility. Thus, even though the owner always extracts the surplus ex ante by adjusting the manager's fixed compensation component $\bar{q}$, managerial pay may seem 'excessive' ex post.

4. Conclusion

Economic actors tend to have limited horizons, so that measuring a firm's performance over finite time periods is a necessity in many agency and decision problems and therefore at the center of accrual-basis accounting. An inherent difficulty in performance measurement lies in the possibility that firms' investments can be interdependent across time, which makes an unambiguous attribution of outcomes to specific investment decisions impractical. In a stewardship context, this problem is acute because a firm owner hiring an outside manager for a specific time period must somehow isolate the effects of the manager's decisions if the compensation contract is to contain an incentive component. The model presented here develops a performance measurement rule that enables such an incentive-compatible compensation contract in an agency setting with hidden actions and information and non-time-separable investment payoffs.

The solution to the problem is an exercise in accrual accounting in that constructing an incentive-compatible performance metric requires targeted adjustments to the firm's cash flows, calibrated to counteract incentives to over- or underinvest. The firm's information environment determines the responsiveness of this performance metric to changes in cash expenditures and receipts. The more the information the manager observes at work reveals about the future, the higher the magnitude of the accrual adjustments; the more uncertainty remains at the end of the period, the more closely the performance metric aligns with cash flow. Although the assumption of risk-neutrality permits the implementation of first-best decisions, the payments to the manager do not match the value implications of the manager's actions, so that the optimal contract is not equivalent to the owner's having sold the firm to the manager. The delegation of decision-making duties to an outside manager is, in fact, more efficient than concentrating both roles in the same agent. The model therefore offers a context in which the optimal organizational design involves the separation of ownership and control.

References

Ali, A., & Zhang, W. (2015). CEO Tenure and Earnings Management. *Journal of Accounting and Economics, 59*(1), 60-79.

Baldenius, T., Dutta, S., & Reichelstein, S. (2007). Cost Allocation for Capital Budgeting Decisions. *The Accounting Review, 82*(4), 837-867.

Barker, R., & Penman, S. (2017). *Moving the Conceptual Framework Forward: Accounting for Uncertainty.* Working Paper, Columbia University.

Cheng, S. (2004). R&D Expenditures and CEO Compensation. *The Accounting Review, 79*(2), 305-328.

Christensen, P., Feltham, G., & Sabac, F. (2005). A Contracting Perspective on Earnings Quality. *Journal of Accounting and Economics, 39*(2), 265-294.

Dechow, P., & Sloan, R. (1991). Executive Incentives and the Horizon Problem: An Empirical Investigation. *Journal of Accounting and Economics, 14*(1), 51-89.

Dikolli, S. (2001). Agent Employment Horizons and Contracting Demand for Forward-Looking Performance Measures. *Journal of Accounting Research, 39*(3), 481-494.

Dutta, S. (2008). Managerial Expertise, Private Information, and Pay-Performance Sensitivity. *Management Science, 54*(3), 429-442.

Dutta, S., & Reichelstein, S. (2002). Controlling Investment Decisions: Depreciation- and Capital Charges. *Review of Accounting Studies, 7*(2), 253-281.

Dutta, S., & Reichelstein, S. (2003). Leading Indicator Variables, Performance Measurement, and Long-Term Versus Short-Term Contracts. *Journal of Accounting Research, 41*(5), 837-866.

Dutta, S., & Reichelstein, S. (2005). Accrual Accounting for Performance Evaluation. *Review of Accounting Studies, 10*(4), 527-552.

Dutta, S., & Reichelstein, S. (2005). Stock Price, Earnings, and Book Value in Managerial Performance Measures. *The Accounting Review, 80*(4), 1069-1100.

Dutta, S., & Zhang, X.-J. (2002). Revenue Recognition in a Multiperiod Agency Setting. *Journal of Accounting Research, 40*(1), 67-83.

Holmström, B. (1982). Moral Hazard in Teams. *Bell Journal of Economics, 13*(2), 324-340.

Mohnen, A., & Bareket, M. (2007). Performance Measurement for Investment Decisions under Capital Constraints. *Review of Accounting Studies, 12*(1), 1-22.

Pfeiffer, T., & Schneider, G. (2007). Residual Income-Based Compensation Plans for Controlling Investment Decisions Under Sequential Private Information. *Management Science, 53*(3), 495-507.

Reichelstein, S. (1997). Investment Decisions and Managerial Performance Evaluation. *Review of Accounting Studies, 2*(2), 157-180.

Reichelstein, S. (2000). Providing Managerial Incentives: Cash Flows versus Accrual Accounting. *Journal of Accounting Research, 38*(2), 243-269.

Sabac, F. (2007). Dynamic Agency with Renegotiation and Managerial Tenure. *Management Science, 53*(5), 849-864.

Smith, M. (2002). Gaming Nonfinancial Performance Measures. *Journal of Management Accounting Research, 14*(1), 119-133.

Stokey, N., & Lucas, R. (1989). *Recursive Methods in Economic Dynamics.* Cambridge: Harvard University Press.

Appendix

Proof of Lemma 1. The results follow from Stokey and Lucas (1989), chapter 9, theorems 9.6 and 9.8, and chapter 12.6, lemma 12.14.

Proof of Proposition 1. Suppose that the owner-manager has made first-best investment choices through period $t-1$ and that a potential outside buyer has correctly inferred x_{t-1} from the history of expenditures and revenues. Under a first-best investment policy, any observed pair of expenditure k_t and revenue m in period t then must be consistent in the sense that $m(x_t, \theta_t) = m\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)$. Then in order for a deviation from first-best expenditure to appear consistent with first-best outcomes, the owner-manager must adjust effort by some amount h_k such that, for the actual θ_t observed,

$$\begin{aligned}
&m_x(x_t, \theta_t)\big(1 + h_k(x_{t-1}, \theta_t)\big) \\
&\quad = m_x\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)\big(1 + h_k^*(x_{t-1}, \theta_t)\big) \\
&\quad + m_\theta\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)\theta_k^*(x_{t-1}, k_t) \\
&\quad = m_x\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)\left(1 + \frac{c''\big(k^*(x_{t-1}, k_t)\big)}{q''\big(h^*(x_{t-1}, k_t)\big)}\right) \\
&\quad + m_\theta\big(x^*(x_{t-1}, k_t), \theta^*(x_{t-1}, k_t)\big)\frac{1}{k_\theta^*(x_{t-1}, \theta_t)}
\end{aligned}$$

where the second equality follows from (6) and (7). Then if the owner-manager were to sell the firm in the next period and the buyer naively believed that first-best investment had occurred, the expenditure that maximizes the owner-manager's payoff must solve the necessary condition

$$m_x(1 + h_k) + E(v_x)(1 + h_k^*) + E_\theta(v)\theta_k^* = c' + q'h_k \tag{A1}$$

If the solution to (A1) were to coincide with the first-best investment policy, the owner-manager's choice of k_t and h_t must simultaneously also solve (4) and (5), which, after substitution into (A1), yields

$$E(v_x)(h_k^* - h_k) + E_\theta(v)\theta_k^* = 0 \Leftrightarrow E_\theta(v)m_x = E(v_x)m_\theta$$

Since the first-best continuation value v anticipates first-best investment in all future periods, this necessary condition must hold for all possible θ_t in all periods t in order for first-best investment to be incentive-compatible.

Proof of Proposition 2. After substitution of (12) and (13) into the first-order conditions (10) and (11), the compensation-maximizing choices of effort and expenditure must solve

$$r(k_t)m_x(x_t,\theta_t) - q'(h_t) = \frac{q'\big(h^*(k_t)\big)m_x(x_t,\theta_t)}{m_x\big(x^*(k_t),\theta^*(k_t)\big)} - q'(h_t) = 0$$

and

$$\begin{aligned} &r(k_t)\Big(m_x(x_t,\theta_t) - m_x\big(x^*(k_t),\theta^*(k_t)\big)\Big) \\ &\quad + r'(k_t)\Big(m(x_t,\theta_t) - m\big(x^*(k_t),\theta^*(k_t)\big)\Big) - z_k\big(k_t, m(x_t,\theta_t)\big) \\ &\quad - z_m\big(k_t, m(x_t,\theta_t)\big)m_k(x_t,\theta_t) = 0 \end{aligned} \tag{A2}$$

By design of the penalty function z, producing revenue $m(x_t,\theta_t) \neq m\big(x^*(k_t),\theta^*(k_t)\big)$ fails to maximize compensation, and so the manager creates expenditure and revenue consistent with first-best outcomes. In view of $z_k = z_m = 0$ at first-best outcomes, (A2) then reduces to

$$r(k_t)\Big(m_x(x_t,\theta_t) - m_x\big(x^*(k_t),\theta^*(k_t)\big)\Big) = 0 \tag{A3}$$

to which the only solution is investing $x = x^*$ because m is increasing and concave in x and because $m_{x\theta} > 0$. Given $x = x^*$, the only solution to (A3) is the first-best effort choice h^*, which implies that the expenditure k must also be first-best. To establish that any incentive-compatible profit metric must be equivalent to (8), observe that first-best effort and expenditure are unique for each θ_t, and thus designing p to solve (10) and (11) is both necessary and sufficient.

Proof of Proposition 3. By construction, r is evaluated at the first-best expenditure and effort levels $k^*(\theta_t)$ and $h^*(\theta_t)$ at any given θ_t. Likewise by construction, the first-order condition in (10) holds for all first-best investment choices, and thus an increase in θ_t implies that

$$r'm_x k_\theta^* + rm_{xx}(h_\theta^* + k_\theta^*) + rm_{x\theta} = q''h_\theta^*$$

Substituting (6) and (7) for k_θ^* and h_θ^* yields

$$r'm_x q'' = c''q''\left(1 - \frac{rm_{x\theta}}{m_{x\theta} + \gamma E_\theta(v_x)}\right) + r(c'' + q'')\frac{\gamma E(v_{xx})m_{x\theta} - \gamma E_\theta(v_x)m_{xx}}{m_{x\theta} + \gamma E_\theta(v_x)}$$

If $E_\theta(v_x) = 0$ and $m_{x\theta} > 0$, one obtains

$$r'm_x q'' = (1-r)c''q'' + r\gamma E(v_{xx})(c'' + q'') < 0$$

whereas the converse extreme case $m_{x\theta} = 0$ and $E_\theta(v_x) > 0$ implies

$$r'm_x q'' = c''q'' - rm_{xx}(c'' + q'') > 0$$

That $r' < 0$ in the former and $r' > 0$ in the latter case follows from $m_x > 0$ and $q'' > 0$.

Proof of Corollary 2.1. In equilibrium, the owner's inferences $\hat{x}_t$ and $\hat{\theta}_t$ about the firm's current investment and environment state variables must be correct for all possible outcomes, i.e., $\hat{x}_t = x_t$ and $\hat{\theta}_t = \theta_t$ for all x_t and θ_t. Based on these inferences, the owner offers a new contract that, in expectation and net of effort cost, yields compensation equal to the manager's reservation wage of zero, i.e.,

$$E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big) - q(h_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big) = 0$$

To make this offer strategy sustainable as an equilibrium outcome, the manager must face no incentive to deviate from first-best investment in period t and thereby to induce incorrect inferences $\hat{x}_t$ and $\hat{\theta}_t$, given the owner's beliefs. In particular, consider any state θ_t in period t and the associated first-best investment $k^*(\theta_t)$ and $h^*(\theta_t)$, and, without loss of generality, assume that the penalty function z in the accrual profit in period t is sufficiently steep to make deviations from outcomes consistent with first-best investment unattractive to the manager. The manager would therefore at most contemplate a deviation from k^* with a concurrent change in effort h_k that maintains first-best-consistent outcomes, as defined in the proof of Proposition 1. Given the owner's beliefs, the effect of this deviation on expected next-period compensation is

$$\begin{aligned}
&\frac{d}{dk_t} E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big) - q(h_{t+1})\big|\theta_t\big) \\
&\quad = E\big(r(k_{t+1}, \hat{x}_t) m_x(x_{t+1}, \theta_{t+1})\big)(1 + h_k) - \bar{q}_x\big(\hat{x}_t, \hat{\theta}_t\big)\frac{\partial \hat{x}_t}{\partial k_t} - \bar{q}_\theta\big(\hat{x}_t, \hat{\theta}_t\big)\frac{\partial \hat{\theta}_t}{\partial k_t} \qquad \text{(A4)} \\
&\quad = E\big(r(k_{t+1}, \hat{x}_t) m_x(x_{t+1}, \theta_{t+1})\big)\left(1 + h_k - \frac{\partial \hat{x}_t}{\partial k_t}\right) - \bar{q}_\theta\big(\hat{x}_t, \hat{\theta}_t\big)\frac{\partial \hat{\theta}_t}{\partial k_t}
\end{aligned}$$

by the envelope theorem, where $\bar{q}$ is the fixed compensation component, which the owner adjusts such that, given $\hat{x}_t$ and $\hat{\theta}_t$, the manager obtains the net reservation wage of zero. Then, given rational expectations by the owner in inferring x_t and θ_t, $\hat{x}_t = x_t$ and $\hat{\theta}_t = \theta_t$ in all states can only hold if (A4) is zero for all θ_t. Conversely, if (A4) equals zero at all first-best outcomes in period t, the original contract remains incentive-compatible and hence is sufficient to permit a first-best optimal renewal contract.