

Accounting for Owners' Capital

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Abstract. Financial accounting not merely conveys information but also controls the withdrawal of equity capital by the owners. In the presence of information asymmetry between owners and outside financiers, this control function implements three necessary and sufficient conditions under which firms make efficient, value-maximizing investments, namely, that (i) outside financiers are repaid in full; (ii) optimal investment maximizes owners' equity; and (iii) expected future dividends equal net firm value. The ability to pay dividends is thus not a byproduct of income measurement but its purpose. The solution rationalizes the commonly observed understatement of book value relative to market value as part of an incentive mechanism to induce optimal investment under uncertainty. Capital structure in this context neither is irrelevant nor has a single-point optimum but is constrained to a range of equally efficient choices. For any given capital structure, there exists a unique accounting system that maximizes distributable equity capital. The model moreover implies a sharp distinction between recognition and disclosure. While both convey the same information, only the former affects distributable equity capital, with all attendant incentive effects.

I. INTRODUCTION

The objectives of general-purpose financial accounting are a central matter of interest to both academic research and professional practice. The dominant paradigm views financial accounting as the conveying of information to existing and potential capital providers about an entity's past economic outcomes and hence, by implication, about its future prospects. Yet, the information perspective cannot by itself deliver an exact description of what constitutes effective, 'good' accounting since, without assuming frictions or costs that constrain the information communication process, one has little reason to prefer one method of accounting over another, so long as information content is preserved. A case in point, accounting-based representations of firm value admit an arbitrarily large number of accounting methods that all yield the same result. An alternative view argues for the utility of financial accounting in managerial performance measurement and in lending agreements, but the question remains why such private contracts should all have to rely on the same set of accounting standards. This paper seeks to point out by way of a simple model that concrete requirements for general-purpose financial accounting can in fact be derived from the basic premise that financial accounting does not merely provide decision-relevant information but primarily serves to measure the amount of equity capital available for distribution, and thereby balances the owners' right to access their funds against the need to incentivize efficient investment and to protect the financial interests of the entity's outside capital providers.

The critical assumption underlying all results to be presented here is that, for any given firm, some amount of value-relevant information is observable only to its owners and not to potential outside financiers, but that capital contributions from outsiders are needed because the owners' private wealth is insufficient to fund the firm optimally. Under these conditions, implementing value-maximizing investment requires a capital contribution and repayment mechanism that simultaneously (i) ensures the return of outsiders' capital; (ii) incentivizes owners to invest efficiently; and (iii) allows owners to draw dividends that eventually will exhaust firm value, net of liabilities. To see the difficulty of the problem, it suffices to consider the following simple example. Let intrinsic firm value be \$100, reflecting the expected net present value of future investments under optimal decision-making, and let the firm's liabilities, i.e., its repayment obligations to outsiders, be \$40. The value of the owners' equity one would then intuitively claim to be \$60, but this is only true if the \$60 can actually be paid out in dividends, either now or in the future,

and only the owners know this value. If one naively allowed the owners to determine at their discretion the equity capital available for distribution, they would face an incentive to overstate the amount, withdraw funds in excess of \$60, and leave the outside financiers to suffer losses. Conversely, if one deferred the payment of dividends indefinitely, owners would never see their money and their equity claims would be rendered worthless. A third simplistic approach might allow dividend payments only when all liabilities have been repaid, but owners would again take advantage of outsiders' inability to observe firm value and continue indefinitely to borrow and invest in the hope of a lucky success, even if firm value were to decline below the liability value later. By contrast, a balance sheet that continually tracks liabilities and owners' equity by a well-designed measurement rule can, as will be argued, address the problem effectively.

It has long been recognized that equity value is equal to the total amount of dividends a firm is expected to pay over its lifespan. Realizing, however, that dividend payout decisions are discretionary and therefore poorly suited for formulaic prediction, valuation theory has since shifted the unit of analysis to cash flows and earnings. Implicit in this substitution are two assumptions, namely, (i) that aggregate dividends inherently equal net intrinsic firm value; and (ii) that dividends follow, but do not cause, value creation and recognition. Neither assumption is valid if the role of accounting is to control capital flows in a world of incomplete information. That future dividends add up to firm value, net of liabilities, is true only if, in equilibrium, the income credited to owners' equity over time correctly reflects the value implications of the owners' investment decisions, which in turn requires accounting rules such that these investment decisions maximize the income, and thus the firm's dividends. Poorly designed accounting rules would instead either over-recognize earnings and lead to liabilities and equity in excess of firm value, and thus to insolvency, or under-recognize earnings and lead to firm value in excess of liabilities and equity ad infinitum, effectively turning part of firm value into unclaimed property. In sum, standard valuation theory considers earnings as a means for measuring value creation and views the ability to pay dividends as its consequence. The capital flow control perspective implies the reverse: earnings measurement rules arise in the first place from the need to calibrate dividends to equal net intrinsic firm value, while maintaining incentives for owners to invest efficiently. Only in the resulting equilibrium do earnings then acquire the property of measuring value creation.

A financial accounting system built around the need to control the payout of equity capital requires and rationalizes several properties seen in practice. First, the balance sheet must understate the value of owners' equity. Any attempt at recognizing distributable equity capital equal to the full net intrinsic value of the firm will create incentives for owners to overinvest and extract excess dividends that leave the firm insolvent, and the anticipation of this outcome prevents any equilibrium in which outside financiers are willing to fund the firm's investments to begin with. The incentive problem arises because full equity value recognition would require naively imputing future profits by taking on faith that any investment expenditure by the owners, no matter how large, is indeed profitable and made in optimal response to their private information. An incentive-compatible balance sheet instead records no more than the minimum net asset value that can be rationalized given the firm's observable history of investments and revenues and the decision incentives that this accounting approach creates for the owners. In particular, the higher the ratio of a firm's investment spending to its concurrent revenues, the more its book value must understate firm value, consistent with the convention in practice of delaying the recognition of value when expenditures produce only small immediate revenues but have large uncertain, long-horizon payoffs. As a result, firms with high revenue realizations but few growth opportunities can have higher book values than firms of significantly higher intrinsic value that stems largely from future, yet to be implemented opportunities. For any given capital structure, there exists a unique financial accounting system that, subject to these requirements, maximizes the amount of equity capital available for withdrawal by the owners in every period.

The above rationale behind the construction of the balance sheet sheds new light on the frequently studied market-to-book relationship. Rather than reflecting two independently created, differentially (in)complete measures of net firm value, market value and book value are causally and functionally connected, while serving distinct roles. The market value of a firm is the price at which current and prospective investors buy and sell their ownership rights and thus governs the relationship between different equity owners. The balance sheet (or book) value, on the other hand, is the amount of capital that these equity owners may transfer out of the firm into their private accounts and thereby remove from the reach of the non-owner claimants to the firm's resources. The balance sheet thus governs the relationship between equity owners and their firm's outside capital providers. Causal dependence between market and book value nonetheless arises,

because the market value equals expected future dividends, which are determined by the accounting rules that underlie the balance sheet. These accounting rules are written to incentivize efficient investment while securing the requisite funding from outside financiers, and thereby ensure that future dividends, and thus market value, equal net intrinsic firm value as an equilibrium outcome. That one can use the market value as an estimate of net intrinsic firm value is therefore an artifact of the financial accounting system and holds precisely because the balance sheet understates equity value and thereby incentivizes the efficiency in investment that supports the equilibrium market value in the first place.

Moreover, the capital flow control model puts several common ideas about capital structure in perspective. First, capital structure is flexible, in the sense that all feasible ratios of liabilities to equity are equally efficient and no single-point optimum exists, but feasibility is constrained by the book value of equity. Thus, even in the presence of informational friction, the capital structure irrelevance result of Miller-Modigliani need not give way to a unique solution. Rather, financing becomes bounded within a (potentially quite large) range of viable choices. In equilibrium, outside financiers are willing to fund the distribution of dividends to any amount, as long as the distributable equity position on the balance sheet remains positive. The minimum equity component of firm value therefore equals the amount of value not recognized as distributable equity capital. From this observation emerges a second conclusion, namely that the perennial question whether dividends are irrelevant or serve a discernible purpose needlessly assumes that dividends must either be the means to an end or else be superfluous. Dividends are in fact themselves the objective, rather than an inconsequential byproduct, of computing book value, and the construction of book value in turn answers the need for a mechanism to let owners access their equity capital, thereby giving equity ownership rights their value in the first place. That the distribution policy is arbitrary up to the amount of distributable equity capital on the balance sheet, and therefore cannot be uniquely optimized, does not diminish the pivotal role of dividends.

Another conception made untenable in this context is that of equity as a residual claim on the firm's resources. A hierarchy of claims, with owners being the last to receive their money, makes sense in the presence of a business-terminating event that forces a final settling-up among capital providers. Yet, whereas capital structure models routinely feature a terminal period, businesses in reality continue as a going concern indefinitely, and final settling-up occurs, with few exceptions, only in case of bankruptcy-induced liquidation. Meanwhile, companies can and do pay

dividends, issue equity, raise and repay debt, and engage in any other form of capital transaction in indiscriminate order and continually through time, and the parties to these transactions form a group with constantly changing membership. Existing owners may sell their rights to new investors, bank relationships are formed and terminated, supplier and customer credit arrangements come and go, etc. The two-sided balance sheet with its distinction between liabilities and equity creates a capital contribution and distribution mechanism that accommodates this ongoing renewal and replacement process. The precise terms of each party's contribution (debt capital, equity capital, hybrid versions) are of lesser importance. Critically, the mechanism must distinguish between the decision-making insiders (the owners) and everyone else if incentives for efficient investment and assurance of the repayment of all capital providers' funds are to be obtained.

In terms of accounting mechanics, valuation models (most prominently, the Ohlson model) have long recognized that dividends reduce current book value one-for-one, albeit by assumption rather than through an endogenous connection. With financial accounting assigned the task of controlling equity payouts in the face of informational friction, the inverse marginal one-to-one relation between book value and dividends becomes tautological, since book value is now *defined* as the upper bound on dividends. The effect of dividends on income measurement, and hence on *future* book value, is more complicated. In the Ohlson model, dividends are essentially income-neutral, save for a time-value-of-money adjustment to future income to recognize the cost of replacing equity with outsider capital. But with information asymmetry between owners and outsiders in play, the owners' decision incentives depend on the current and anticipated future balances of distributable equity capital and therefore change with every dividend payment. (Consider, for example, the owners' incentive to take additional risks as they withdraw their own equity capital toward zero and leave the outside financiers to bear an increasing share of the loss in case of adverse future outcomes.) Dividend payouts therefore require recalibration of future income measurement, such that the investment decisions that maximize owners' equity always remain the same as those maximizing total firm value. These dividend-induced adjustments lead, depending on the evolution of events, to higher or lower income in the future, with an expected value of zero. Hence, dividend policy has no impact on investment efficiency, and thus on firm value, only if the accounting rules for income recognition are made functionally dependent on the firm's net asset position, and thus on the dividend policy itself.

II. MODEL SETUP

Consider the following discrete-time model of a firm that makes periodic investments to generate future payoffs. Let $k_t \in \mathbb{R}^+$ denote the investment expenditure in period t . Investment is defined as any cash outlay that yields benefit in the form of future cash inflows and includes payments for any physical assets and intellectual, human and organizational resources that the firm acquires and deploys. (Modeling these resources explicitly is not necessary for the purpose of this discussion.) The cash inflow in period t , hereafter referred to as the firm's revenue, is a function

$$s_t = S(\mathbf{k}_t, \eta_t, \theta_t)$$

of the firm's investment history $\mathbf{k}_t = (k_1, \dots, k_t)$ and of random environmental factors $\eta_t, \theta_t \in \mathbb{R}^+$, which are independent of each other and outside the firm's control. (Throughout the text, capital letters will denote mappings and lower-case letters will denote realized values in a given period.) The firm's history of realized revenues through period t will be denoted by $\mathbf{s}_t = (s_1, \dots, s_t)$. Whereas the η_t are independent across periods, the θ_t have non-zero correlation in the time-series and follow a Markov process with

$$\Pr(\theta_{t+1} \leq x | \theta_t = a_1) > \Pr(\theta_{t+1} \leq x | \theta_t = a_2)$$

for all $x \in \mathbb{R}^+$ and $a_2 > a_1$, and

$$\lim_{t \rightarrow \infty} \Pr(\theta_t > \varepsilon) = 0$$

for any $\varepsilon > 0$. (The latter condition implies a finite firm value.) Having two environmental state variables, rather than just one, serves two purposes. First, if either only η_t or only θ_t were present, even somebody to whom the state is unobservable could trivially infer its value from the firm's expenditure and revenue flow. The model would in that case fail to represent the informational frictions that firms face in practice. Second, distinct short-term and long-term considerations enter the investment decision problem: an increase in current η_t implies, for lack of correlation with future η_t , an effect on investment profitability in the current period only, whereas variation in θ_t , by virtue of its correlation with future θ_t , induces investment decisions not fully explained by current-period revenue. (Alternative specifications, including correlation in the time series of η_t and between η_t and θ_t , are possible without changing the substance of the results. The critical model features are that (i) investment affects revenues in both current and future periods; and that (ii) optimal investment is determined by multiple, imperfectly correlated environmental factors with impact on future revenue.)

For ease of exposition, the analysis will assume the following properties of the revenue function S . Revenue is increasing, bounded and continuously differentiable in all its arguments, with

$$\lim_{\theta_t \rightarrow 0} S(\mathbf{k}_t, \eta_t, \theta_t) = 0$$

and

$$S(\mathbf{k}_t, \eta_t, 0) = S(\mathbf{k}_t, 0, \theta_t) = 0$$

for all \mathbf{k}_t , η_t and θ_t . Further, revenue is concave in investment and has increasing differences in investment and environment, i.e., in any period t ,

$$\frac{\partial^2 S}{\partial k_i \partial k_j} < 0$$

for all $i, j \leq t$, including $i = j$, and

$$\frac{\partial^2 S}{\partial k_t \partial \eta_t}, \frac{\partial^2 S}{\partial k_t \partial \theta_t} > 0$$

for all \mathbf{k}_t , η_t and θ_t in any given period t . For technical convenience, also let $\frac{\partial S}{\partial \eta_t} / \frac{\partial S}{\partial \theta_t}$ be non-increasing in k_t . (The latter condition is not strictly necessary for the points to be made in this paper but avoids unproductive technical discussion. Generally, any relationship that, for all sets of states with non-zero probability measure, maintains the single-crossing property in Lemma ___ below would suffice.)

All investment decisions are made by the firm's owners. Potential agency problems between owners and hired managers are not of interest here. The owners have limited capital of their own to invest, so that expenditures in excess of the firm's revenue proceeds must at least in part be financed by outside capital providers. These outsiders do not observe the firm's environmental states η_t and θ_t , either in or after period t . The owners, on the other hand, observe η_t and θ_t in each period, prior to making their investment decision. For reasons exogenous to the model, outsiders cannot join our firm as owners to resolve this information asymmetry. (Consider, for example, that outsiders such as suppliers, customers and bankers may have their own enterprises to look after, so that participating in our firm is not feasible for them. Also consider the inefficiency and confusion arising when an unnecessarily large number of individuals must make operating decisions jointly. Finally, observe that firms in practice almost invariably have substantial amounts of capital from outsiders.) The firm's realized expenditure and revenue cash flows, k_t

and s_t , are verifiable to both owners and outsiders. Outsiders collectively possess sufficient capital such that, if desired, any finite amount of investment by the firm could theoretically be funded in any given period. The market for capital is competitive, so that outsiders break even in expectation. The discount rate is normalized to zero and all parties are risk-neutral.

III. ANALYSIS AND DISCUSSION

The objective of the following analysis is to determine whether optimal investment is feasible despite the information asymmetry between owners and outsiders. To this end, it will be useful to discuss first the optimal investment policy and the attendant first-best firm value that would obtain if the owners could finance their enterprise entirely by themselves and no capital contribution from outsiders were ever necessary. Let first-best firm value in period t be denoted by

$$v_t = V(\mathbf{k}_{t-1}, \eta_t, \theta_t)$$

defined as the solution to the investment optimization problem

$$V(\mathbf{k}_{t-1}, \eta_t, \theta_t) = \max_{\{k_i\}_{i=t}^{\infty}} \sum_{i=t}^{\infty} E(S(\mathbf{k}_i, \eta_i, \theta_i) - k_i | \theta_t) \quad (1)$$

where expectations taken over future states η_i and θ_i . The right-hand side of (1) is the familiar representation of firm value as the total amount of expected future net cash flows. The corresponding Bellman equation is

$$V(\mathbf{k}_{t-1}, \eta_t, \theta_t) = \max_{k_t} (S(\mathbf{k}_t, \eta_t, \theta_t) + E(V(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) | \theta_t) - k_t) \quad (2)$$

It is well known that, given the properties of the revenue function, there exists a unique optimal investment plan

$$k_t^* = K^*(\mathbf{k}_{t-1}, \eta_t, \theta_t)$$

that solves (7), defined by

$$K^*: (\mathbf{k}_{t-1}, \eta_t, \theta_t) \rightarrow \arg \max_{k_t} (S(\mathbf{k}_t, \eta_t, \theta_t) + E(V(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) | \theta_t) - k_t) \quad (3)$$

and yielding a unique associated firm value v_t . For future reference, this first-best benchmark outcome is summarized in the following lemma. All proofs can be found in the Appendix.

Lemma 1. *There exists a unique first-best investment plan $K^*: (\mathbf{k}_{t-1}, \eta_t, \theta_t) \rightarrow k_t^*$ for all t . The associated firm value v is concave in k_i , for $i = 1, \dots, t - 1$.*

The firm's investment problem admits of the possibility that $k_t^* > s_t$, i.e., the optimal investment plan may involve, in at least some periods, more expenditure than can be covered by the firm's contemporaneous revenue inflow. This can occur for combinations of low η_t and high θ_t , when large expenditures are required at present while the related revenues arrive in later periods. Suppose now that the firm's owners are unable to cover these temporary cash flow deficits fully by their own means and therefore turn to outsiders for additional capital. Since outsiders are unable to observe η_t and θ_t , they cannot verify whether the amount the owners intend to invest indeed equals k_t^* and hence whether the firm's expected future net cash flows will be sufficient to pay back the outsiders' contributed capital.

The objective is to find a funding and repayment mechanism that, despite this informational friction, incentivizes the owners to undertake first-best investment in all periods. In particular, we will insist that this mechanism implement the following outcomes: (i) outsiders are willing to fund any amount of investment expenditure that the owners ask for in any given period, rationally anticipating that their funds will, in expectation, be repaid in full; (ii) the owners choose the optimal investment amount k_t^* in each period t ; and (iii) the combined expected future net payments to owners and outsiders equal total firm value, v_t , at all times.

The firm's owners optimize their investment decisions with respect to the value of their equity claims in the firm. This equity value must equal the total amount of payouts (dividends) the owners expect to receive in the future. A well-formed definition of equity value therefore specifies how much capital, at any given time t , is available to the owners for distribution. For this purpose, the firm creates a balance sheet on which it records a net asset (equity) value, hereafter denoted by b_t . We will also refer to b_t as the firm's (net) book value. Owners may pay out dividends to themselves, at their discretion, of up to the value of b_t , and every unit of dividends reduces b_t one-for-one. To establish some notation for later use, let dividends paid out in period t be denoted by w_t , and the history of dividends by $\mathbf{w}_t = (w_1, \dots, w_t)$.³ No further dividend distributions are allowed once b_t has been reduced to zero.⁴

Within these confines, one can now design b_t to serve as an arbiter in the firm's funding mechanism. In particular, the desired first-best funding and investing outcomes will obtain if our

³ Capital contributions by owners can readily be accommodated by allowing $w_t < 0$. All results remain the same.

⁴ Normalizing the lower bound to zero is arbitrary but convenient; generally, any value is permissible. For instance, the law may require a strictly positive amount of net book value to be retained in the firm at all times, or, conversely, outsiders may agree to let net assets go negative to a certain degree. Both scenarios appear in practice.

net asset formula enforces three necessary and sufficient conditions at all times, namely, (i) solvency, (ii) incentive-compatibility, and (iii) full property rights. Before examining each condition in detail, note first that net assets cannot depend directly on η_t and θ_t , as the latter are observable only to the owners. If left to their discretion, the owners could misreport η_t and θ_t opportunistically to overstate b_t and attempt to pay themselves dividends in excess of total firm value, in anticipation of which outsiders would refuse to fund the firm in the first place. We are therefore limited to making net assets a function

$$b_t = B(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t)$$

of the firm's histories of expenditures, \mathbf{k}_t , revenues, \mathbf{s}_t , and dividends, \mathbf{w}_t , which are verifiable to all parties. Moreover, that dividends, by definition, reduce net assets one-for-one directly implies the additional constraint

$$\frac{\partial B}{\partial w_i}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) = -1 \quad (4)$$

for all t and $i = 1, \dots, t$.

Consider first the solvency criterion. Solvency means that intrinsic firm value, v_t , is larger than the outsiders' claims against the firm. To make the idea concrete, let m_t denote the total amount of expected future net payments to outsiders, as of the end of period t . We will also refer to m_t as the firm's liabilities. Like net assets, liabilities can be defined as a function

$$m_t = M(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t)$$

of the firm's history of verifiable transactions. Since the model assumes complete capital markets, it will be of no relevance how much of m_t is scheduled to be paid in which future period to which specific outside financier. Indeed, if the funding mechanism works as intended, there will always be new outsiders willing to refinance m_t when needed. For this to be true, however, outsiders must believe that liabilities do not exceed firm value even if owners pay out the maximum amount of dividends, i.e., that $m_t \leq v_t$ at the end of every period t even when $b_t = 0$. This belief must be correct in equilibrium, given all observable past cash flows and the outsiders' conjectures about the owners' investing strategy.

Formally, outsiders' beliefs are given by an inference function that maps the firm's history of expenditures, \mathbf{k}_t , and of revenues, \mathbf{s}_t , into the environmental state θ_t and thus into the firm's current (and expected future) intrinsic value. In particular, if outsiders anticipated in equilibrium that owners follow the first-best investment plan in (3), this inference would, given the properties

of the revenue function $S(\cdot)$, map any given cash flow history into a unique θ_t (and, technically, also a unique η_t , but the latter has, by assumption, no relevance for future outcomes). Note that first-best investment is independent of dividends, so w_t plays no role here. For future reference, the following lemma gives a formal summary.

Lemma 2. *For any cash flow history (\mathbf{k}_t, s_t) , there exists a unique pair (η_t, θ_t) assigned by the inference function $\Gamma: (\mathbf{k}_t, s_t) \rightarrow \gamma_t$, where*

$$\gamma_t \equiv \{(\eta_t, \theta_t): (k_t = K^*(\mathbf{k}_{t-1}, \eta_t, \theta_t)) \wedge (s_t = S(\mathbf{k}_t, \eta_t, \theta_t))\}$$

In any period t , the net cash contribution from (or payment to) outsiders must equal the net of investment, revenue and dividends, i.e., $k_t - s_t + w_t$. Given the assumption of a competitive capital market in which liabilities earn zero return, the simplest attempt to account for the firm's liabilities from period to period is therefore

$$m_t = m_{t-1} + k_t - s_t + w_t \quad (5)$$

However, for any $m_{t-1} > 0$, there exist sufficiently unfavorable η_t and θ_t such that

$$v_t < m_{t-1} + k_t - s_t$$

which would violate the solvency condition $m_t \leq v_t$ even if dividends were zero, and hence (5) is infeasible. To remedy the problem, it is necessary to subject liabilities to gains and losses, large enough such that $m_t \leq v_t$ always holds. Outside financiers will accept if the expected gain or loss on every marginal unit of capital contributed in period t is zero in expectation, i.e., if

$$E(dm_{t+i} | \mathbf{k}_t, \gamma_t) = dk_t - ds_t + dw_t \quad (6)$$

subject to the constraint that $m_{t+i} \leq v_{t+i}$ for all $i = 1, 2, \dots$, and given on outsiders' inference γ_t , as defined in Lemma 2. There exist multiple solutions to (6) that ensure solvency. One special case is the standard debt-type structure

$$m_t = \min\{v_t, \bar{m}_{t-1} + k_t - s_t + w_t\} \quad (7)$$

where \bar{m}_{t-1} is the unique constant that solves (6).⁵ In (7), outsiders receive all firm value up to a threshold, mirroring the typical practice of declaring a firm bankrupt after a steep loss in asset value and handing over all ownership rights to the liability claimants.

⁵ The interest rate on the liabilities is thus $\bar{m}_{t-1}/m_{t-1} - 1 > 0$.

Implementing (6) requires inferring correctly, based on γ_t , whether the solvency condition $v_{t+i} \leq m_{t+i}$ indeed holds. The computation of γ_t in Lemma 2 in turn assumes first-best investment by the owners. Hence, a necessary equilibrium condition is that the net asset formula $B(\cdot)$ is creates incentive-compatibility. Incentive-compatibility means that first-best investment maximizes net asset value both at present and, in expectation, in all future periods, i.e.,

$$\arg \max_{k_t} E(B(\mathbf{k}_{t+i}, \mathbf{s}_{t+i}, \mathbf{w}_{t+i}) | \eta_t, \theta_t) = K^*(\mathbf{k}_{t-1}, \eta_t, \theta_t) \quad (8)$$

for all t , all $i = 0, 1, \dots$ and all η_t and θ_t , and under any future dividend payout strategy. Since equity value is equivalent to aggregate future dividends and since net assets determine dividend potential, (8) clearly implies that first-best investment maximizes equity value and hence is the owners' preferred choice.

An instructive but failing attempt at finding a $B(\cdot)$ that solves (8) is to equate the balance sheet value of equity directly with the intrinsic equity value of the firm, i.e.,

$$B(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) = E(V(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) | \gamma_t) - M(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) \quad (9)$$

where $V(\cdot)$ is given by (2). If implemented, (9) would incentivize overinvestment. To see why, note that (9) allows owners to pay out the value of all expected future surplus cash flows as dividends immediately. The expectation of these surplus cash flows is based not on the actual, unobservable environmental state θ_t but on its imputed value γ_t , which in turn is a function of the owners' investing decision k_t . Since γ_t assumes all investment decisions to be first-best responses to underlying θ_t and since all first-best k_t are increasing in θ_t , (9) assigns higher equity value the more the firm invests. By raising k_t to inefficiently high levels, the owners can thus effectively pay themselves a non-refundable advance on imputed future profits that will never materialize, leaving the outsiders with liability claims in excess of firm value and hence with a loss. The formal argument is given in Proposition 1 below.

Proposition 1. *Incentive-compatible accounting must understate net book value, b_t , relative to net intrinsic firm value, $v_t - m_t$.*

Proposition 1 rationalizes the well-known observation that companies' market values tend to exceed their book values.⁶ However, the result does not imply that merely understating net firm value on the balance sheet is by itself sufficient as a solution concept to the incentive-compatibility problem. As will become clear, the critical design flaw in (9) lies not in an overly generous crediting of distributable equity value per se but rather in the attempt to achieve perfect separation across firm types by assigning a different net asset value to each γ_t . With η_t and θ_t unobservable to the accounting system, any such attempt at full separation invariably creates opportunities for owners to raise the net asset value above $v_t - m_t$. An incentive-compatible net asset measurement rule $B(\cdot)$ must therefore involve pooling, i.e., groups of firms with different intrinsic values $v_t - m_t$ will need to be given the same balance sheet value b_t .

A graphical illustration may help to understand why and how these pools of firms with identical book values are, and indeed must be, constructed. First, draw an arbitrary firm's revenue curve, i.e., plot s_t as a function of k_t , and mark the point $(k_t, s_t) = (k_t^*, s_t^*)$ corresponding to that firm's (unique) first-best investment decision and the associated revenue, as shown in Figure 1A. To make the rationale concrete and transparent, suppose for the moment that the owners are interested in maximizing net assets at the end of the current period only. We will refer to these owners as *myopic*. (Fully rational owners will be introduced in due course.) Incentive-compatibility then obtains if the end-of-period net asset value b_t is higher at (k_t^*, s_t^*) than anywhere else on the revenue curve. A necessary condition to this end is that b_t remain constant in the direction of the firm's marginal revenue around (k_t^*, s_t^*) , shown in Figure 1A as the dotted tangent line. (Otherwise, the owners would prefer to increase or decrease their investment away from k_t^* .)

This requirement is equivalent to the first-order condition

$$\frac{dB}{dk_t}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) = \frac{\partial B}{\partial k_t}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) + \frac{\partial B}{\partial s_t}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) \cdot \frac{\partial S}{\partial k_t}(\mathbf{k}_t, \eta_t, \theta_t) = 0 \quad (10)$$

The level set of all investment-revenue pairs at which net asset value must be identical to our selected firm's optimal b_t can now be traced out by moving from (k_t^*, s_t^*) in Figure 1A to the 'neighboring' point (in the sense of an infinitesimal step) in the direction of our firm's marginal revenue line. There, one recalibrates the direction of travel to follow the marginal revenue line of

⁶ Recall, however, that the model assumes $b_t = 0$ as the threshold below which no further dividend distributions are permitted, whereas the effective threshold in practice may vary across firms. In particular, for strictly positive threshold values, book value can exceed market value without necessarily causing inefficient investment.

the firm for whom that neighboring point constitutes the first-best investment-revenue pair, as shown in Figure 1B, then moves in that direction to the next neighboring point, there reevaluates marginal revenue again, and so on, until one has traversed the entire support of k_t . Figure 1C illustrates the resulting iso-book-value line. One can, in fact, repeat this tracing exercise starting from *any* point in the k_t - s_t -diagram that constitutes the first-best outcome for some firm, and obtain a new iso-book-value line each time. By this heuristic, one can fill the k_t - s_t -diagram with a dense set of curves, each of which gives the level set of a particular net asset value. Then, by construction, the highest possible book value level set each firm can attain is the one tangent to its revenue curve at the point of first-best investment. Figure 1D shows an illustration. The mathematical equivalent to this visual approach is solving (7) for $B(\cdot)$, after replacing (η_t, θ_t) by γ_t . The formal solution is given in Lemma 3 below.

Lemma 3. *Given myopic owners, first-best investment k_t^* maximizes net asset value b_t if and only if $b_t = B(q_t)$, where $B(q_t)$ is monotonically increasing in q_t and $q_t = Q(\mathbf{k}_t, s_t)$ is the unique solution to the characteristic equation associated with (7), given $(\eta_t, \theta_t) = \gamma_t$.*

The q_t in Lemma 3 correspond to the book value level sets in Figure 1D. Each level set (also called a characteristic curve) is associated with a unique value q_t , and higher q_t correspond to higher level sets (and higher net asset values b_t). Note how the geometric logic behind the construction solves the original incentive problem of owners' overinvesting in a bid to overstate their net asset value. Any given level set q_t in Figure 1D includes firms with low k_t and s_t toward the left end of the curve, and firms with high k_t and s_t toward the right end. Firms with the same net asset value on their balance sheets can thus differ significantly in every other respect, including investment spending, reported revenue, expected future investment opportunities, intrinsic firm value (or market capitalization, for publicly traded entities), and any other financial metric based on these characteristics. The critical point, however, is that the firms at the right end of any given curve are those whose investments, although large and profitable, will produce most of their payoffs only in future periods. These firms' *current* revenues, s_t , although still higher than those of their peers to the left on their q_t -curve, therefore appear low *relative* to their investment expenditure k_t . This low revenue-to-investment ratio is vulnerable to imitation by firms that actually do not have good future prospects but do have high first-best marginal revenue and

could thus easily produce higher s_t with comparatively little additional k_t . But with the slope of the q_t -curves following first-best marginal revenues, these potential imitators will find that sticking to first-best investment will land them on a higher q_t -curve, and thus yield a higher net asset value, than overspending and copying the k_t and s_t of higher-valued firms with lower s_t -to- k_t ratios would (see Figure 1D again). An immediate implication of this design is that firms with few profitable investment opportunities but a particularly fast conversion of expenditures into revenues may end up with higher balance sheet equity than highly valuable firms whose revenue realizations happen with significant delay.

Assigning firms a net asset value b_t based on their level set q_t attains incentive-compatibility, but the associated mapping $q_t \rightarrow b_t$ is not unique. Lemma 3 specifies only that net asset value must be an increasing function of q_t , not what the rate of increase should be. In fact, even the trivial $B(q_t) = 0$ for all q_t in all periods is (weakly, at least) incentive-compatible. Yet, $B(q_t) = 0$ would clearly be inadequate because the firm's owners could never pay themselves any dividends. The equity capital would effectively be turned into unclaimed property and the owners' property rights be rendered worthless, similar to a savings account from which the depositor is never allowed to make a withdrawal. This expropriation is ruled out by the full property rights requirement, the third and final condition for the firm's funding mechanism to work as desired. Full property rights means that the value of the owners' equity equals the net of total intrinsic firm value, v_t , and the firm's liabilities, m_t . Formally, full property rights obtain if, in any period t , there exists a feasible dividend payout strategy such that

$$\sum_{i=t}^{\infty} E(w_i) = v_t - m_t \quad (11)$$

Consider now the class of functions $B(\cdot)$ that meet all three requirements of an effective funding mechanism, i.e., (i) solvency, $v_t \geq m_t$; (ii) incentive-compatibility, $b_t = B(q_t)$ and $B'(q_t) \geq 0$; and (iii) full property rights. Multiple solutions exist, and examining the lower and upper boundaries of the solution set is instructive. The most conservative approach to valuing the firm assumes that $v_t = 0$ at all t , and hence the lower bound on net asset value in any specific period t is clearly zero. The property rights requirement in (11), however, demands that all firm value in excess of liabilities must become available for distribution *eventually*, and hence $b_t = 0$ cannot be sustained indefinitely. If the firm generates positive cash flows for long enough, it will ultimately repay all extant liabilities and be left with negative m_t , i.e., surplus cash (or, in the

parlance of financial statement analysis, net financial assets). Then even under most restrictive approach, with $v_t = 0$ assumed throughout, there must exist, at the end of every period t , some finite future date $t + i$ at which

$$\Pr(b_{t+i} \geq \max\{0, -m_{t+i}\}) = 1 \quad (12)$$

and thus make any potential surplus available for payout. The dividend strategy $w_t = b_t$ then extracts the full equity value, as can be verified by noting that (12) implies the same payouts as

$$w_i = \max\{0, s_i - k_i - m_{i-1}\}$$

and hence

$$\begin{aligned} \sum_{i=t}^{\infty} E(w_i) &= \sum_{i=t}^{\infty} E(\max\{0, s_i - k_i - m_{i-1}\}) = \sum_{i=t}^{\infty} E(s_i - k_i - \min\{s_i - k_i - w_i, m_{i-1}\}) \\ &= v_t - m_t \end{aligned}$$

as required by (11). To see why (12) is necessary, observe that, if $b_t < \max\{0, -m_t\}$ at all t , there would exist some stock of equity capital that the owners can never access, violating (11). Now, while any arbitrarily restrictive approach is feasible as long as (12) holds, the more practically relevant question is how much equity value can *at most* be conceded for distribution, while still keeping the firm solvent and first-best efficient. To establish an upper bound on b_t , recall that all firms with the same q_t must have the same net asset value. Hence, solvency can only be guaranteed if, in every period t ,

$$B(q_t) \leq \min\{v_t - m_t | q_t\} \quad (13)$$

i.e., if net asset value is at most equal to the lowest net intrinsic firm value among all firms on the same q_t -curve. The maximum net asset value therefore obtains if (13) holds with equality. Proposition 2 below gives a formal summary.

Proposition 2. *Given myopic owners, the maximum net asset value that implements solvency, incentive-compatibility and full property rights is*

$$B(q_t) = \min\{v_t - m_t | q_t\}$$

for any $m_t = M(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t)$ such that $\min\{v_t - m_t | q_t\}$ is increasing in q_t .

The solution obtained so far (specifically, its incentive-compatibility mechanism) assumes that owners always optimize myopically with respect to the net asset value at the end of the current period. Realistically, of course, owners would understand that investment in period t not only impacts the end-of-period equity balance b_t but will also affect future investment opportunities and revenues, and thus future net asset values. The myopic solution $b_t = B(q_t)$ for all t would clearly be inadequate in this case because, except by fortuitous accident,

$$E \left(\frac{\partial B}{\partial q_{t+i}}(q_{t+i}) \cdot \frac{\partial Q}{\partial k_t}(\mathbf{k}_{t+i}, s_{t+i}) \middle| \theta_t \right) \neq 0$$

for $i = 1, 2, \dots$, and hence the effect of the current-period investment k_t on future periods would incentivize owners to deviate from the first-best decision strategy. To make the incentive-compatibility requirement consistent with fully rational owners, we will therefore insist from here on that investment in any period t maximize both the firm's concurrent net asset value and the expected net asset value in all future periods $t + i$ simultaneously.

The solution to this modified problem follows the same logic as in the myopic setting. Just as firms with different current-period investment-revenue pairs (k_t, s_t) need to be sorted into groups in order to attain incentive-compatibility with respect to the current net asset value b_t , so too firms with different sets (k_t, q_t, q_{t+1}) of current investment and current and next-period q -value will now need to be grouped in order to attain incentive-compatibility with respect to the next-period net asset value b_{t+1} . Considering periods t and $t + 1$ jointly, the first-order necessary condition for optimal investment $k_t = k_t^*$ then includes both (10) and

$$\frac{\partial B}{\partial k_t}(k_t, q_t, q_{t+1}) + \frac{\partial B}{\partial q_{t+1}}(k_t, q_t, q_{t+1}) \cdot E \left(\frac{\partial Q}{\partial k_t}(\mathbf{k}_{t+1}, s_{t+1}) \middle| k_t, q_{t+1}, \theta_t \right) = 0 \quad (14)$$

for every possible q_{t+1} .⁷ And just as making k_t^* the solution to (10) required construction of the variable q_t , so too making k_t^* the solution to (14) will require the construction of a new variable, hereafter denoted by

$$r_{t,t+1} = R(k_t, q_t, q_{t+1})$$

where the first subscript of $r_{t,t+1}$ indicates the investment period and the second subscript indicates the outcome period of interest. With incentive-compatibility in both periods t and $t + 1$

⁷ No derivative with respect to q_t appears because, at $k_t = k_t^*$,

$$\frac{\partial B}{\partial q_t}(k_t, q_t, q_{t+1}) \cdot \left(\frac{\partial Q}{\partial k_t}(\mathbf{k}_t, s_t) + \frac{\partial Q}{\partial s_t}(\mathbf{k}_t, s_t) \cdot \frac{\partial S}{\partial k_t}(\mathbf{k}_t, \eta_t, \theta_t) \right) = 0$$

by construction of q_t .

taken into account, the net asset value b_{t+1} then becomes a function of $r_{t,t+1}$ (as opposed to q_{t+1} , which, by Lemma 3, was sufficient to incentivize myopic owners). Extending this logic, one can readily see that $r_{t,t+1}$ must in turn be subjected to an incentive-compatibility constraint with respect to investment in period $t - 1$, requiring a variable $r_{t-1,t+1}$ that groups outcomes by $(k_{t-1}, q_{t-1}, r_{t,t+1})$, and that one can apply this logic iteratively to even earlier periods, successively solving conditions of the form

$$\begin{aligned} \frac{\partial B}{\partial k_i}(k_i, q_i, r_{i+1,t}) + \frac{\partial B}{\partial r_{i+1,t}}(k_i, q_i, r_{i+1,t}) \cdot E\left(\frac{\partial R}{\partial k_i}(k_{i+1}, q_{i+1}, r_{i+2,t}) \middle| q_{i+1}, \theta_i\right) \\ = 0 \end{aligned} \quad (15)$$

until one has reached the firm's founding period $i = 1$. With all periods considered, net asset value in any given period t must thus be a function of $r_{1,t}$, the variable that nests all incentive-compatibility conditions from period 1 through t . The analogue of Proposition 2 for fully rational owners is therefore as follows.

Proposition 3. *Given fully rational owners, the maximum net asset value that implements solvency, incentive-compatibility and full property rights is*

$$B(r_{1,t}) = \min\{v_t - m_t | r_{1,t}\}$$

for any $m_t = M(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t)$ such that $\min\{v_t - m_t | r_{1,t}\}$ is increasing in $r_{1,t}$, where $r_{1,t}$ is obtained by iteratively solving (15) after replacing θ_i by γ_i .

Since book value can at most equal the lowest firm value rationalized by $r_{1,t}$ and since every additional period t adds firms to the set, the degree to which book value understates intrinsic value, on average, increases in t . One should not, however, conclude immediately that older firms in practice therefore should have lower book-to-market ratios, because attrition due to business failure changes the population of firms as a function of age.

IV. CONCLUSION

The purpose and uses of financial accounting are long-standing topics of interest to researchers and practitioners, but the focus has predominantly been on information content and disclosure,

whereas relatively little attention has been given to the critical role that accounting plays in regulating the dividend distributions from owners' capital accounts. Owners' equity has value not because intrinsic firm value exceeds liabilities, but because the accretion of earnings permits the payout of dividends. Dividends are therefore not a byproduct of earnings measurement but its purpose. In the presence of information asymmetry between owners and outside financiers, the crediting of earnings to owners' equity must be calibrated to prevent excess withdrawals that would leave insufficient value in the firm to repay outsiders' claims, whilst on the other hand also ensuring that all surplus firm value does become available to the owners eventually, and all the while incentivizing owners to invest the firm's resources efficiently. Equity is therefore not a residual claim on the firm's resources but the value of expected future distributions permitted by accounting rules. The distinguishing feature of equity capital is that its owners possess privileged information and decision rights. An accounting system successfully optimized in this context must keep distributable equity capital on the balance sheet below the firm's net intrinsic value, consistent with the observation that market values in practice generally exceed book values. The optimal capital structure is not unique, but the equity component is bounded below by the amount of owners' capital not yet credited to the balance sheet for distribution. The capital-balance-regulation role of accounting also suggests a resolution to the well-known conundrum why it should matter whether value-relevant information is recognized in income or merely disclosed: only the recognition through income changes distributable equity capital, with specific implications for owners' decision incentives.

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APPENDIX

Proof of Lemma 1. The uniqueness of the investment strategy k_t^* and the concavity of the value function v follow from Stokey and Lucas (1989), chapter 9, theorems 9.6 and 9.8, and chapter 12, lemma 12.14.

Proof of Lemma 2. For any given investment history \mathbf{k}_{t-1} , first-best investment k_t^* is the unique solution to the necessary condition

$$\frac{\partial S}{\partial k_t}(\mathbf{k}_t^*, \eta_t, \theta_t) + E\left(\frac{\partial V}{\partial k_t}(\mathbf{k}_t^*, \eta_{t+1}, \theta_{t+1}) \middle| \theta_t\right) = 1$$

where $\mathbf{k}_t^* = (k_1, \dots, k_{t-1}, k_t^*)$. Consider the level set of states (η_t, θ_t) at which the same k_t^* solves (10), characterized by

$$\frac{\partial^2 S}{\partial k_t \partial \eta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) d\eta_t + \left(\frac{\partial^2 S}{\partial k_t \partial \theta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) + \frac{\partial E}{\partial \theta_t} \left(\frac{\partial V}{\partial k_t}(\mathbf{k}_t^*, \eta_{t+1}, \theta_{t+1}) \middle| \theta_t \right) \right) d\theta_t = 0$$

Then, in view of

$$\frac{\partial}{\partial k_t} \left(\frac{\partial S}{\partial \eta_t} / \frac{\partial S}{\partial \theta_t} \right) = \left(\frac{\partial S}{\partial \theta_t} \frac{\partial^2 S}{\partial k_t \partial \eta_t} - \frac{\partial S}{\partial \eta_t} \frac{\partial^2 S}{\partial k_t \partial \theta_t} \right) / \left(\frac{\partial S}{\partial \theta_t} \right)^2 \leq 0 \Leftrightarrow \frac{\partial S}{\partial \theta_t} / \frac{\partial S}{\partial \eta_t} \leq \frac{\partial^2 S}{\partial k_t \partial \theta_t} / \frac{\partial^2 S}{\partial k_t \partial \eta_t}$$

and

$$\frac{\partial E}{\partial \theta_t} \left(\frac{\partial V}{\partial k_t}(\mathbf{k}_t^*, \eta_{t+1}, \theta_{t+1}) \middle| \theta_t \right) > 0$$

the change in revenue on this level set is

$$\begin{aligned} & \frac{\partial S}{\partial \eta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) d\eta_t + \frac{\partial S}{\partial \theta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) d\theta_t \\ & < \left(\frac{\partial S}{\partial \eta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) - \frac{\partial S}{\partial \theta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) \right. \\ & \quad \left. \cdot \frac{\partial^2 S}{\partial k_t \partial \theta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) / \frac{\partial^2 S}{\partial k_t \partial \eta_t}(\mathbf{k}_t^*, \eta_t, \theta_t) \right) d\eta_t \leq 0 \end{aligned}$$

for all (η_t, θ_t) . Hence, there cannot exist multiple (η_t, θ_t) that generate the same (k_t^*, s_t) , and so the mapping $\Gamma: (\mathbf{k}_t, s_t) \rightarrow \gamma_t$, with

$$\gamma_t \equiv \{(\eta_t, \theta_t): (k_t = K^*(\mathbf{k}_{t-1}, \eta_t, \theta_t)) \wedge (s_t = S(\mathbf{k}_t, \eta_t, \theta_t))\}$$

assigns a unique γ_t to each first-best (\mathbf{k}_t, s_t) .

Proof of Proposition 1. Consider setting net asset value b_t equal to (9). Incentive-compatibility requires that b_t has a stationary point at the first-best investment level $k_t = K^*(\mathbf{k}_{t-1}, \eta_t, \theta_t)$. However, increasing k_t at this point yields

$$\begin{aligned}
& \frac{\partial B}{\partial k_t}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) \\
&= E \left(\frac{\partial V}{\partial k_t}(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) \Big| \gamma_t \right) + \frac{\partial E}{\partial \gamma_t}(V(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) | \gamma_t) \cdot \frac{\partial \Gamma}{\partial k_t}(\mathbf{k}_t, s_t) \\
&+ \frac{\partial M}{\partial s_t}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) \cdot \frac{\partial S}{\partial k_t}(\mathbf{k}_t, \eta_t, \theta_t) + \frac{\partial M}{\partial k_t}(\mathbf{k}_t, \mathbf{s}_t, \mathbf{w}_t) \\
&= E \left(\frac{\partial V}{\partial k_t}(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) \Big| \gamma_t \right) + \frac{\partial E}{\partial \gamma_t}(V(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) | \gamma_t) \cdot \frac{\partial \Gamma}{\partial k_t}(\mathbf{k}_t, s_t) \\
&+ \frac{\partial S}{\partial k_t}(\mathbf{k}_t, \eta_t, \theta_t) - 1 = \frac{\partial E}{\partial \gamma_t}(V(\mathbf{k}_t, \eta_{t+1}, \theta_{t+1}) | \gamma_t) \cdot \frac{\partial \Gamma}{\partial k_t}(\mathbf{k}_t, s_t) > 0
\end{aligned}$$

where the second equality follows from (6) and final equality follows from the first-order condition underlying (2). Owners can thus increase net asset value by overinvesting and so an accounting system with $b_t = v_t - m_t$ cannot attain incentive-compatibility.

Proof of Lemma 3. Incentive-compatibility obtains if the first-order condition in (10) holds after substituting θ_t by γ_t , yielding

$$\frac{\partial B}{\partial k_t}(\mathbf{k}_t, \mathbf{s}_t) + \frac{\partial B}{\partial s_t}(\mathbf{k}_t, \mathbf{s}_t) \cdot \frac{\partial S}{\partial k_t}(\mathbf{k}_t, \eta_t, \gamma_t) = \frac{\partial B}{\partial k_t}(\mathbf{k}_t, \mathbf{s}_t) + \frac{\partial B}{\partial s_t}(\mathbf{k}_t, \mathbf{s}_t) \cdot S'(\mathbf{k}_t, s_t) = 0$$

where

$$S': (\mathbf{k}_t, s_t) \rightarrow \frac{\partial S}{\partial k_t}((\mathbf{k}_t, \eta_t, \theta_t): (K^*(\mathbf{k}_{t-1}, \eta_t, \theta_t) = k_t) \wedge (S(\mathbf{k}_t, \eta_t, \theta_t) = s_t))$$

The characteristic equation is

$$ds_t = S'(\mathbf{k}_t, s_t) dk_t$$

which, by the Picard-Lindelöf theorem, has a unique solution, hereafter denoted by

$$q_t = Q(\mathbf{k}_t, s_t)$$

Now choose the solution function $b_t = B(q_t)$ to have $B' > 0$ everywhere. That first-best investment then maximizes b_t follows from

$$\frac{dB}{dk_t}(q_t) = B'(q_t) \cdot \left(\frac{\partial S}{\partial k_t}(\mathbf{k}_t, \eta_t, \theta_t) - \frac{\partial S}{\partial k_t}(\mathbf{k}_t, \gamma_t) \right)$$

which, in view of

$$\frac{\partial^2 S}{\partial k_t \partial \theta_t}(\mathbf{k}_t, \eta_t, \theta_t) > 0$$

is positive if and only if $k_t < k_t^*$. Conversely, if $B' < 0$ for some $k_t = \bar{k}$, then incentive-compatibility would fail for (η_t, θ_t) such that $k_t^* = \bar{k}$.

Proof of Proposition 2. That $B(q_t)$ must be increasing in q_t follows from Lemma 3. To establish that net asset value must be bounded above by $\min\{v_t - m_t | q_t\}$, consider any function

$$B(q_t) > \min\{v_t - m_t | q_t\}$$

Then for at least some q_t , there exist firms such that $b_t > v_t - m_t$, violating the solvency requirement.

Proof of Proposition 3. By Lemma 3, incentive-compatibility with respect to the effect of investment on contemporaneous book value requires a change of variables from (k_i, s_i) to (k_i, q_i) in all periods $i = 1, 2, \dots$. The inference function $\Gamma: (\mathbf{k}_t, s_t) \rightarrow \gamma_t$ established in Lemma 2 can thus equivalently be cast as $\Gamma: (\mathbf{k}_t, q_t) \rightarrow \gamma_t$, and the notation γ_t will therefore be used henceforth to denote inferences about (η_t, θ_t) given (\mathbf{k}_t, q_t) . Note that the inferred value of θ_t is decreasing in q_t for any given k_t . Further, note that any accounting system that maximizes distributable equity value must have $b_t = v_t - m_t$ for at least some firm type in every period t , since there would otherwise exist some constant $\varepsilon > 0$ that could be added to book value everywhere, without affecting owners' investment decisions. Incentive-compatibility thus requires

$$E\left(\frac{dB}{dk_t}(\mathbf{k}_{t+i-1}, \mathbf{q}_{t+i}) \middle| \theta_t\right) = 0$$

for all t and i . Otherwise, those firm types for whom $E(b_{t+i}) = E(v_{t+i} - m_{t+i})$ under first-best investment would increase book value to $E(b_{t+i}) > E(v_{t+i} - m_{t+i})$ for some i by over- or underinvesting in period t , which would lead to a violation of the solvency condition. Consider now the effect of investment k_{t-1} on net asset value in period t . The first-order necessary condition for k_{t-1} to maximize the expected value of b_t is

$$E\left(\frac{dB}{dk_{t-1}}(k_{t-1}, q_{t-1}, q_t) \middle| \theta_{t-1}\right) = 0$$

For each q_t , one has

$$\begin{aligned}
& \frac{dB}{dk_{t-1}}(k_{t-1}, q_{t-1}, q_t) \\
&= \frac{\partial B}{\partial k_{t-1}}(k_{t-1}, q_{t-1}, q_t) + \frac{\partial B}{\partial q_t}(k_{t-1}, q_{t-1}, q_t) \\
&\quad \cdot E \left(\frac{\partial Q}{\partial k_{t-1}}(\mathbf{k}_t, s_t) + \frac{\partial Q}{\partial s_t}(\mathbf{k}_t, s_t) \cdot \frac{\partial S}{\partial k_{t-1}}(\mathbf{k}_t, \eta_t, \theta_t) \Big| k_{t-1}, q_t, \theta_{t-1} \right) = 0
\end{aligned}$$

where the effect of k_{t-1} on q_{t-1} vanishes because, at $k_{t-1} = k_{t-1}^*$,

$$\frac{\partial B}{\partial q_{t-1}}(k_{t-1}, q_{t-1}, q_t) \cdot \left(\frac{\partial Q}{\partial k_{t-1}}(\mathbf{k}_{t-1}, s_{t-1}) + \frac{\partial Q}{\partial s_{t-1}}(\mathbf{k}_{t-1}, s_{t-1}) \cdot \frac{\partial S}{\partial k_{t-1}}(\mathbf{k}_{t-1}, \eta_{t-1}, \theta_{t-1}) \right) = 0$$

by construction of q_{t-1} . After replacing θ_{t-1} by γ_{t-1} , one obtains the characteristic equation

$$dq_t = E \left(\frac{\partial Q}{\partial k_{t-1}}(\mathbf{k}_t, s_t) + \frac{\partial Q}{\partial s_t}(\mathbf{k}_t, s_t) \cdot \frac{\partial S}{\partial k_{t-1}}(\mathbf{k}_t, \eta_t, \theta_t) \Big| k_{t-1}, q_t, \gamma_{t-1} \right) dk_{t-1}$$

which, by the Picard-Lindelöf theorem, has a unique solution, hereafter denoted by

$$r_{t-1,t} = R(k_{t-1}, q_{t-1}, q_t)$$

with $r_{t,t} \equiv q_t$. To establish that $k_{t-1} = k_{t-1}^*$ is the only solution that maximizes $E(b_t)$, note that

$$\frac{\partial E}{\partial \theta_{t-1}} \left(\frac{\partial Q}{\partial k_{t-1}}(\mathbf{k}_t, s_t) + \frac{\partial Q}{\partial s_t}(\mathbf{k}_t, s_t) \cdot \frac{\partial S}{\partial k_{t-1}}(\mathbf{k}_t, \eta_t, \theta_t) \Big| k_{t-1}, q_t, \theta_{t-1} \right) > 0$$

if q_t has increasing differences in k_{t-1} and θ_t . The latter condition indeed holds because

$$\begin{aligned}
& D_{k_{t-1}} D_{\theta_t} Q + D_{k_{t-1}} D_{k_t} Q \cdot \frac{\partial K^*}{\partial \theta_t} + D_{k_t} D_{\theta_t} Q \cdot \frac{\partial K^*}{\partial k_{t-1}} + D_{k_t} D_{k_t} Q \cdot \frac{\partial K^*}{\partial k_{t-1}} \cdot \frac{\partial K^*}{\partial \theta_t} > 0 \\
& \Leftrightarrow (D_{k_{t-1}} D_{\theta_t} Q) \cdot (D_{k_t} D_{k_t} Q) - (D_{k_t} D_{\theta_t} Q) \cdot (D_{k_{t-1}} D_{k_t} Q) < 0
\end{aligned}$$

where

$$D_x Q \equiv \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial s_t} \cdot \frac{\partial S}{\partial x}$$

and the equivalence relation obtains because

$$D_{k_t} D_{\theta_t} Q + D_{k_t} D_{k_t} Q \cdot \frac{\partial K^*}{\partial \theta_t} = 0$$

and

$$D_{k_{t-1}} D_{k_t} Q + D_{k_t} D_{k_t} Q \cdot \frac{\partial K^*}{\partial k_{t-1}} = 0$$

by construction of Q . The final inequality follows from

$$\frac{\partial^2 S}{\partial k_t \partial \theta_t} = \frac{\partial^2 S}{\partial k_{t-1} \partial \theta_t} \Rightarrow D_{k_{t-1}} D_{\theta_t} Q = D_{k_t} D_{\theta_t} Q$$

and from $D_{k_t}V = 0$, which implies

$$\frac{\partial K^*}{\partial \theta_t} = \frac{D_{k_t}D_{\theta_t}V}{D_{k_t}D_{k_t}V} > 0$$

and

$$\frac{\partial K^*}{\partial k_{t-1}} = \frac{D_{k_t}D_{k_{t-1}}V}{D_{k_t}D_{k_t}V} > -1$$

in view of

$$\frac{\partial^2 S}{\partial k_t \partial k_{t-1}} \geq \frac{\partial^2 S}{\partial k_t^2}$$

Now observe that the curves in the $k_{t-1}^*-s_{t-1}^*$ -space along which the underlying θ_{t-1} is constant are convex, in view of

$$\frac{\partial^2 S}{\partial k_{t-1} \partial \eta_{t-1}} + \frac{\partial^2 S}{\partial k_{t-1}^2} \cdot \frac{\partial K^*}{\partial \eta_{t-1}} > 0$$

and, at any given k_{t-1} , a higher curve corresponds to a lower θ_{t-1} . Then since revenue is concave in k_{t-1} , the revenue curve s_{t-1} can intersect the isoquant of any given θ_{t-1} at most twice, and so the first-order condition

$$\begin{aligned} \frac{\partial B}{\partial k_{t-1}} + \frac{\partial B}{\partial q_t} \cdot E(D_{k_{t-1}}Q | k_{t-1}, q_t, \theta_{t-1}) &= 0 \\ \Leftrightarrow E(D_{k_{t-1}}Q | k_{t-1}, q_t, \theta_{t-1}) - E(D_{k_{t-1}}Q | k_{t-1}, q_t, \gamma_{t-1}) &= 0 \end{aligned}$$

can have at most two solutions, of which only the higher value maximizes net assets, since the revenue curve crosses the θ_{t-1} -isoquant from above. That this maximizing value corresponds to $k_{t-1} = k_{t-1}^*$ is immediate since firm value is increasing along the isoquant and hence the lower investment point cannot be optimal. Iterating the solution with respect to periods prior to $t - 1$, one obtains

$$\begin{aligned} \frac{\partial B}{\partial k_{t-i}}(k_{t-i}, q_{t-i}, r_{t-i+1,t}) + \frac{\partial B}{\partial r_{t-i+1,t}}(k_{t-i}, q_{t-i}, r_{t-i+1,t}) \\ \cdot E(\mathcal{D}_{k_{t-i}}R(k_{t-i+1}, q_{t-i+1}, r_{t-i+2,t}) | k_{t-i}, r_{t-i+1,t}, \theta_{t-i}) &= 0 \end{aligned}$$

for all $i = 2, \dots, t - 1$, where

$$\begin{aligned}
& \mathcal{D}_{k_{t-i}} R(k_{t-i+1}, q_{t-i+1}, r_{t-i+2,t}) \\
& \equiv \frac{\partial R}{\partial q_{t-i+1}}(k_{t-i+1}, q_{t-i+1}, r_{t-i+2,t}) \cdot D_{k_{t-i}} Q(\mathbf{k}_{t-i+1}, s_{t-i+1}) \\
& + \frac{\partial R}{\partial r_{t-i+2,t}}(k_{t-i+1}, q_{t-i+1}, r_{t-i+2,t}) \\
& \cdot \left(\frac{\partial R}{\partial q_{t-i+2,t}}(k_{t-i+2}, q_{t-i+2}, r_{t-i+3,t}) \cdot D_{k_{t-i}} Q(\mathbf{k}_{t-i+2}, s_{t-i+2}) \right. \\
& + \frac{\partial R}{\partial r_{t-i+3,t}}(k_{t-i+2}, q_{t-i+2}, r_{t-i+3,t}) \\
& \cdot \left(\frac{\partial R}{\partial q_{t-i+3,t}}(k_{t-i+3}, q_{t-i+3}, r_{t-i+4,t}) \cdot D_{k_{t-i}} Q(\mathbf{k}_{t-i+3}, s_{t-i+3}) \right. \\
& \left. \left. + \frac{\partial R}{\partial r_{t-i+4,t}}(k_{t-i+3}, q_{t-i+3}, r_{t-i+4,t}) \cdot \left(\dots \cdot D_{k_{t-i}} Q(\mathbf{k}_t, s_t) \right) \right) \right)
\end{aligned}$$

The characteristic equations are

$$dr_{t-i+1,t} = E(\mathcal{D}_{k_{t-i}} R(k_{t-i+1}, q_{t-i+1}, r_{t-i+2,t}) | k_{t-i}, r_{t-i+1,t}, \gamma_{t-i}) dk_{t-i}$$

with solutions of the form $r_{t-i,t} = R(k_{t-i}, q_{t-i}, r_{t-i+1,t})$. That k_{t-i}^* uniquely maximizes $r_{t-i,t}$ now follows from the properties of $D_{k_{t-i}} Q$ given above.

FIGURE 1A

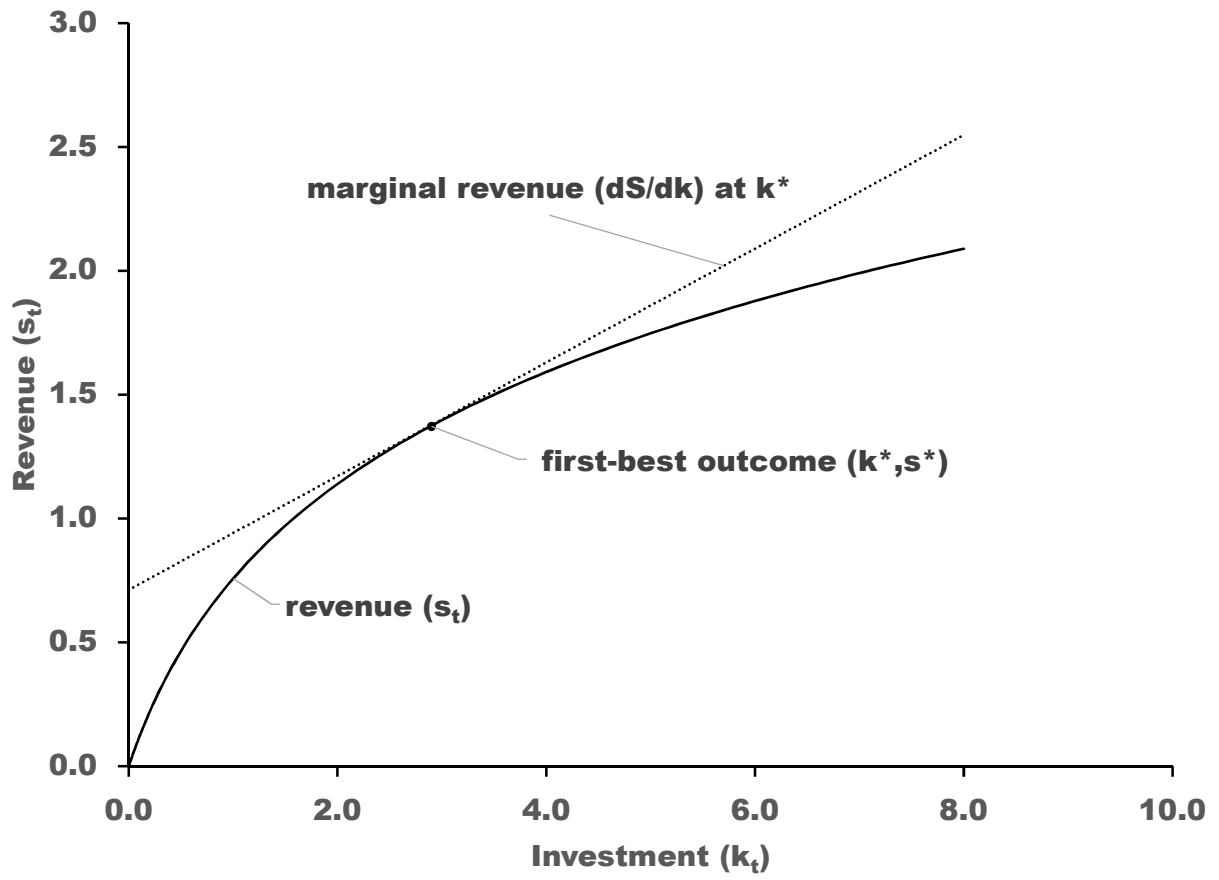


FIGURE 1B

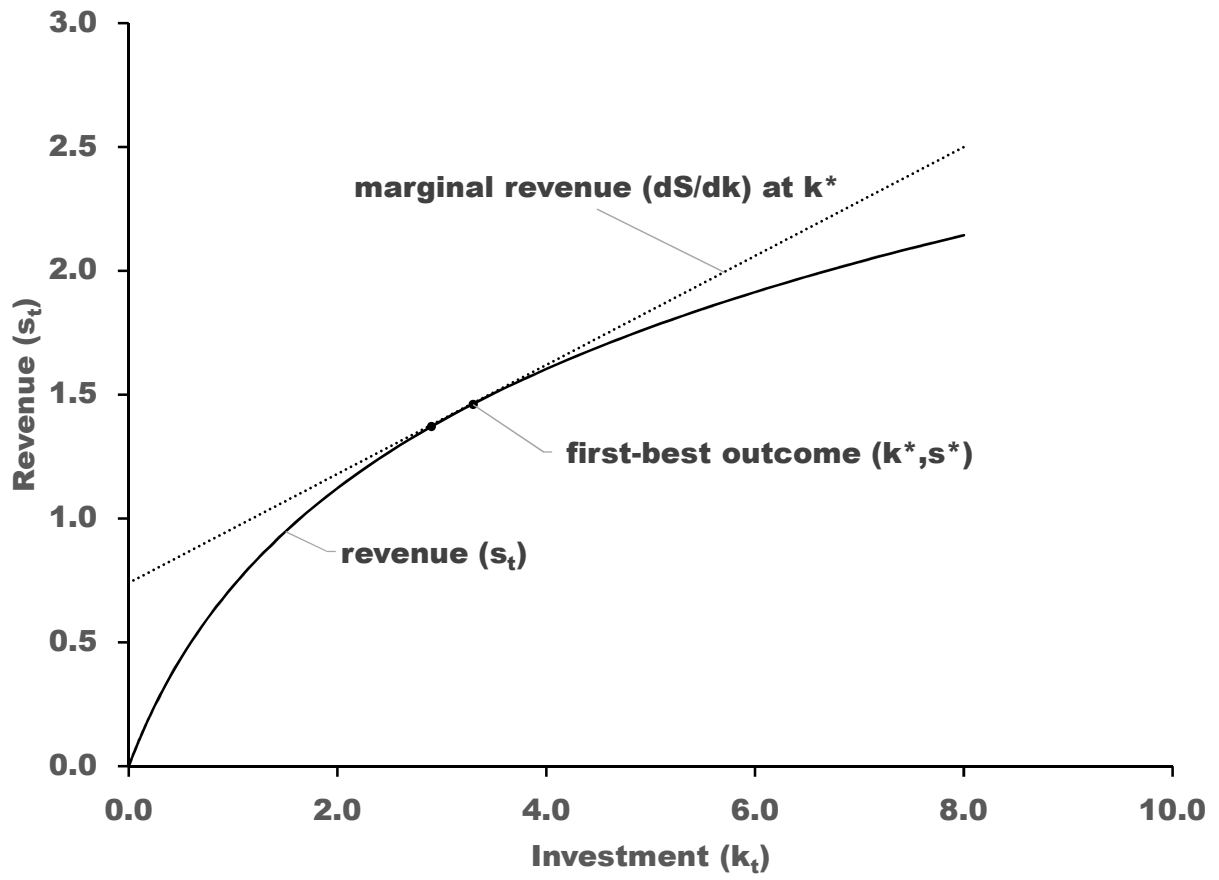


FIGURE 1C

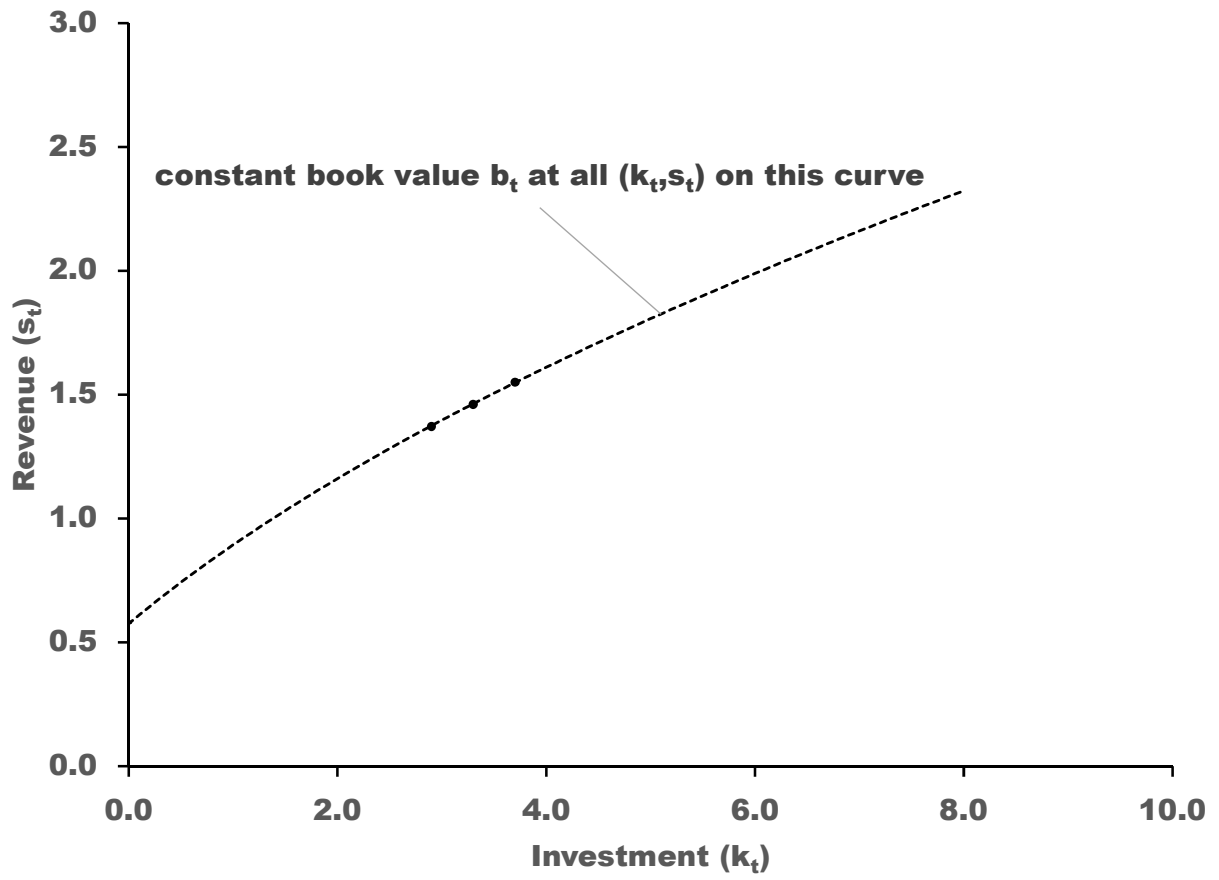


FIGURE 1D

