Managerial Optimism and Debt Covenants

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Abstract: This paper studies the effect of managerial optimism on the optimal tightness of debt covenants. One might expect that an entrepreneur who is optimistic about the future success of her project will choose a looser covenant to reduce the probability of a covenant violation and inefficient lender intervention (such as project liquidation). We show that the opposite is optimal: A more optimistic entrepreneur chooses a tighter debt covenant. In fact, the optimal covenant is so tight that it will lead to socially inefficient liquidations not only from the entrepreneur’s optimistic perspective but also from the lender’s unbiased perspective. In addition, we find that the optimal covenant threshold further tightens when the parties can sometimes observe the quality of the project prior to the lender’s intervention decision, but this effect only arises when the entrepreneur is optimistic.
1 Introduction

Empirical and survey evidence suggests that entrepreneurs and executives are overly optimistic about the chances of success of their own projects.\(^1\) One explanation for why entrepreneurs tend to be overly optimistic is developed in Van den Steen (2004) and Landier and Thesmar (2009). Individuals randomly under or overestimate the probability of success of their various opportunities and choose to pursue those opportunities that they believe have the greatest probability of success. As a consequence, individuals who forego other opportunities to start a new venture are typically those who, on average, overestimate the chances that their venture will be successful.\(^2\) Besides this "choice-driven" theory of optimism, there are several other explanations for managerial hubris that are grounded in the psychology literature; see for example, DeBondt and Thaler (1995), Malmendier and Tate (2005), and Gervais (2010).

In this paper, we study how entrepreneurial optimism affects the design of financial contracts, especially the optimal allocation of control rights in debt contracts. The allocation of control rights in debt contracts is typically governed by accounting-based covenants that transfer control rights to lenders if accounting signals fall below certain thresholds. A tighter covenant (i.e., a higher threshold) increases the likelihood that the lender gains control over the firm, permitting the lender to take actions against the entrepreneur’s will, such as liquidating the firm.

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\(^1\)See, e.g., Larwood and Whittaker (1977), Cooper et al. (1988), Arabsheibani et al. (2000), Malmendier and Tate (2005), Landier and Thesmar (2009), Ben-David et al. (2013).

\(^2\)See also De Meza and Southey (1996).
One might expect that entrepreneurs who are more optimistic about the chances of their success will choose looser covenants to remain in control and to prevent lenders from liquidating the firm. We show that the opposite is true. More optimistic entrepreneurs choose tighter covenants. In fact, the optimal covenant is so tight that it leads to socially excessive liquidations, not just from the optimistic entrepreneur’s point of view, but also from the lender’s point of view. We also show that the optimal covenant becomes even tighter when the two parties sometimes observe additional (noncontractible) information about the project prior to the lender’s intervention decision. This effect is present, however, only when the entrepreneur is optimistic about her project’s success probability.

Our model offers a novel explanation for the empirical evidence that covenants in debt contracts are set very tightly, and are often violated and subsequently waived (Chava and Roberts, 2005; Dichev and Skinner, 2002; Nini et al. 2012). Further, our model provides new empirical predictions relating the tightness of debt covenants to managerial optimism and the availability of information.

We study a model in which an entrepreneur raises capital from a lender to finance a new project. If the project is continued to completion, it will succeed if the state of the world is good and will fail if the state is bad. We assume the entrepreneur is optimistic in the sense that she believes the prior probability of the good state is higher than what the lender believes. At an interim date, a public accounting signal is released that is informative about the state. Based on the signal, both players use Bayes’ rule to update their prior beliefs. But since the entrepreneur’s prior is higher than the lender’s prior, her posterior is also higher. In an extension of the
model, we also assume that the players can directly observe the state at the interim date with some probability, but this information is not contractible. The party in control (either the entrepreneur or the lender) then chooses to continue the project or abandon it based on all the available information. Similar to Aghion and Bolton (1992) and Dessein (2005), we assume the entrepreneur enjoys private benefits of control when the project is continued. The debt contract includes a covenant that is contingent on the accounting signal. If the accounting signal falls below a certain threshold, the covenant is violated and the lender gains control over the liquidation decision. If the signal is above the threshold, the covenant is not violated and the entrepreneur remains in charge. After the signal is revealed, the two parties can renegotiate the contract if they wish to do so.

We first study the case where the parties can never observe the state so that the accounting signal is the only information relevant for the liquidation decision. When the two players have homogeneous priors, the optimal covenant implements the first-best liquidation decision given the available information. That is, the lender receives control and liquidates the project only if the accounting signal is sufficiently low such that liquidation is indeed the socially optimal action (which takes into account the entrepreneur’s control benefits). Although the entrepreneur always wishes to continue the project ex-post, she chooses the covenant threshold that implements first-best liquidation because the lender protects himself via a higher interest rate if he anticipates inefficient project continuations.

When the entrepreneur is optimistic relative to the lender, the two parties disagree on the first-best liquidation decision. Specifically, the entrepreneur believes that
the signal threshold above which project continuation is the socially optimal choice
is lower than what the lender believes. One might therefore expect that a more
optimistic entrepreneur chooses a looser covenant to retain control over the project
more often, and to prevent the lender from liquidating it. We show, however, that
this is not the case. Instead, the entrepreneur chooses a stricter covenant when she
is more optimistic.

The intuition for this result is as follows. When the entrepreneur chooses the
tightness of the covenant, she considers the following two effects. First, a lower
covenant threshold increases the probability that the entrepreneur retains control,
which permits her to continue the project. Continuation is desirable for the en-
trepreneur because she enjoys private benefits of control and because she receives
the residual output when the project succeeds, whereas liquidation leaves her empty
handed. Second, a lower covenant threshold diminishes the lender’s control and
hence protection. As a result, the lender demands a higher repayment amount, that
is, the face value of the debt increases. The higher face value, in turn, reduces the
entrepreneur’s residual in case of success. The optimal covenant threshold trades off
the benefits of more frequent project continuations with the costs of a higher face
value. The key here is that the higher face value affects the entrepreneur’s payoff
only when the project succeeds but not when it fails since in the latter case her
residual is zero anyway. A more optimistic entrepreneur believes that the project is
more likely to succeed and hence is more eager to keep the face value of the debt
low. To do so, the optimistic entrepreneur offers the lender greater protection via a
tighter covenant. As a result, the optimal covenant is stricter when the entrepreneur
is more optimistic.

We find that the optimal covenant is so tight that the lender sometimes receives control and liquidates the project although continuation is socially optimal even from the lender’s perspective. In these cases, the entrepreneur would like to renegotiate the contract to convince the lender to continue the project. But the entrepreneur is unable to do so because she cannot share her private benefits with the lender, similar to Aghion and Bolton (1992).

We then extend the model and assume the parties can sometimes observe the state before they make the liquidation decision. When the covenant is violated but the parties learn the state is in fact good, both players prefer project continuation and the lender simply waives the covenant. If the covenant is not violated but the parties learn the state is bad, they will engage in contract renegotiations to prevent inefficient continuation.

The fact that the parties can sometimes observe the state prior to the lender’s intervention decision does not affect the optimal covenant threshold when the two players have homogeneous priors. But it does affect the optimal covenant when the entrepreneur is optimistic about the state. Specifically, the prospect of learning the state induces an optimistic entrepreneur to choose an even tighter covenant. The tight covenant implies that the lender will receive control more frequently, which leads to excessive project terminations unless the parties observe that the state is good.

The reason why the entrepreneur chooses a stricter covenant when the state is more likely observable is the following. When the entrepreneur has control over
the continuation decision and the parties learn the state is bad, she will agree to liquidate the project but only in return for a lower face value that leaves her with a positive payoff in case of liquidation. In contrast, when the lender has control and the bad state is observed, he simply liquidates the project without compensating the entrepreneur. Having the control rights in case of the bad state is therefore valuable to the entrepreneur and costly to the lender. However, since the entrepreneur is optimistic, she places a smaller weight on the value of the control rights in the bad state than the lender. The entrepreneur therefore increases the covenant threshold to provide the lender with more control, which the lender rewards with a lower cost of capital. In short, the entrepreneur chooses to provide the lender with greater protection via a tighter covenant because the lender values this protection more than the entrepreneur.

Dessein (2005) and Gârleanu and Zwiebel (2009) provide an explanation for tight debt covenants that is based on asymmetric information at the contracting stage. In these papers, "good-type" entrepreneurs grant lenders greater control rights than "bad-type" entrepreneurs to signal their type. In our model, the allocation of control rights does not serve as a signalling tool because the two parties have symmetric information at the contracting stage.

Landier and Thesmar (2009) study a signalling model in which the entrepreneur is either rational or optimistic. In their model, there is no conflict of interest between a rational entrepreneur and the lender even when the former retains full control. A conflict of interest arises only when the entrepreneur is optimistic because the optimist believes the project will succeed for sure even when the interim signal reveals
the opposite. In contrast to our model, in Landier and Thesmar (2009) the optimal contract always implements the first-best decision (from the lender’s perspective) regardless of whether the entrepreneur is optimistic or rational.

2 Model

A risk-neutral penniless entrepreneur has a new investment idea that requires capital $I > 0$. To implement the new idea, the entrepreneur raises debt from a risk-neutral lender (e.g., a bank or venture capitalist). The outcome of the project (if not liquidated at an intermediate date) depends on the state of the world, $\omega \in \{0, 1\}$, which is initially not observable to any player. If the state is good, the project succeeds, and if the state is bad, the project fails. The entrepreneur may be more optimistic than the lender about the chances of future success. Specifically, the entrepreneur’s prior belief that the state is good is $\beta_E$ and the investor’s prior is $\beta_L$, with $\beta_E \geq \beta_L$. The players’ prior beliefs are common knowledge.

**Timing:** The game has five dates. At date 1, the entrepreneur and the lender agree on the terms of the debt contract and the lender provides capital $I$. At date 2, the accounting system generates a public signal that is informative about the state $\omega$. At the same time, the state of the world $\omega$ becomes observable to the two players with probability $p \geq 0$. At date 3, the parties can engage in mutually beneficial contract renegotiations. At date 4, the project is either liquidated or continued and the financial contract in place determines who has control over this decision. At date 5, payoffs are realized and shared between the two parties according to the contract
in place.

**Payoffs:** If the project is liquidated at date 4, cash flows are $L \in (0, I)$. If the project is continued, it succeeds and generates cash flows of $X$, with $X > I$, if the state is good, $\omega = 1$, and fails and generates zero cash flows, if the state is bad, $\omega = 0$. All cash flows are verifiable. Further, if the project is continued, the entrepreneur enjoys private benefits of control $B \in (0, L)$.

**Accounting signal:** At date 2, the accounting system generates a public signal $S \in [0, 1]$ that is informative of the state $\omega$. The state-dependent distribution function of the signal is $F_\omega(S)$ and the probability density function is $f_\omega(S) > 0$ for all values of $S$ and $\omega$. The ex-ante density is $f(S, \beta) = \beta f_1(S) + (1 - \beta) f_0(S)$. Conditional on $S$, the posterior probability of the good state is

$$\theta(S, \beta) = \frac{\beta f_1(S)}{\beta f_1(S) + (1 - \beta) f_0(S)}.$$  \hspace{1cm} (1)

The signal $S$ satisfies the monotone likelihood ratio property (MLRP), that is, $\frac{f_1(S)}{f_0(S)}$ is strictly increasing in the signal $S$ for all values of $S$. Thus, higher signals indicate that the state is more likely good. Further, we assume $\lim_{S \to 0} \frac{f_1(S)}{f_0(S)} = 0$ and $\lim_{S \to 1} \frac{f_1(S)}{f_0(S)} = \infty$, which implies that the signal becomes perfectly informative when it approaches 0 or 1.

At the same time, the state $\omega$ becomes observable (but not contractible) to the players with probability $p \geq 0$.

**Financial contract:** The debt contract specifies the amount $D$ the entrepreneur has to pay back the lender at date 5 ($D$ is the debt’s face value). The contract
also determines who has control over the liquidation decision at date 4 via a debt covenant. Specifically, if the accounting signal $S$ falls below a certain threshold, $S_C$, the covenant is violated and the lender receives control. If $S \geq S_C$, the covenant is not violated and the entrepreneur remains in charge. The lending market is competitive and the entrepreneur has all the bargaining power at date 1. The entrepreneur therefore chooses the $(D, S_C)$ combination that maximizes her expected utility subject to the constraint that the lender breaks even in equilibrium. We assume that $\beta_L X \geq I$ to ensure that the entrepreneur is able to obtain financing even when the contract does not provide the lender with any control and the entrepreneur always continues the project. Thus, different to Aghion and Tirole (1992), in our setting there is no need to grant the lender control rights in order to obtain financing.

**Renegotiation:** After the signal $S$ is released and the state is potentially observed, the parties can renegotiate the contract in place. At this time, the entrepreneur does not necessarily have all of the bargaining power because she can no longer shop around for the best offer. Specifically, we assume the entrepreneur can make a take-it-or-leave-it offer with probability $\gamma \in [0, 1]$, and the lender can make a take-it-or-leave-it offer with probability $(1 - \gamma)$. Thus, $\gamma$ represents the entrepreneur’s bargaining power at date 3. We assume that if two contracts lead to the same utility for the entrepreneur, she prefers the one with less renegotiation.
3 State is never observable

We start the analysis with the case in which the state $\omega$ is never observable, $p = 0$. The entrepreneur chooses the covenant threshold $S_C$ that maximizes her utility, subject to the lender’s participation constraint. We assume for now that the optimal covenant $S_C$ is such that the two parties will not want to renegotiate the contract after observing signal $S$. We show below (and more fully in the appendix) that this is indeed the case.

Consider first the lender’s optimal action when he has control over the liquidation decision. The lender will terminate the project if and only if

$$L > \theta(S, \beta_L) D.$$  (2)

When the lender liquidates the project he receives the whole liquidation value $L$ because the face value of the debt $D$ exceeds $L$ in equilibrium (which follows from $D > I$ and $I > L$). If, instead, the lender continues the project, he receives the face value $D$ when the project succeeds and nothing when it fails. Since the posterior $\theta(S, \beta_L)$ is increasing in $S$, continuation becomes more attractive for the lender when the signal $S$ is higher. We assume for now that the covenant $S_C$ is sufficiently small such that the lender will always want to liquidate the project when he gains control. That is, we assume that for all $S < S_C$ inequality (2) is satisfied. We show below that this is indeed the case when $S_C$ is chosen optimally.

When the entrepreneur receives control, she will always continue the project.
because continuation yields her benefits of control $B$ as well as the residual $X - D$ in case of success, whereas liquidation leaves her empty handed.

The lender’s and the entrepreneur’s ex ante utilities can now be written as:

$$U_L = \int_{S_C}^{1} \theta (S, \beta_L) D f (S, \beta_L) dS + \int_{0}^{S_C} L f (S, \beta_L) dS - I$$  \hspace{1cm} (3)

and

$$U_E = \int_{S_C}^{1} (\theta (S, \beta_E) (X - D) + B) f (S, \beta_E) dS,$$  \hspace{1cm} (4)

respectively.

The lender provides the required capital $I$ if she breaks even in equilibrium. The debt’s face value $D$ therefore solves $U_L = 0$. Since a higher covenant threshold $S_C$ provides the lender with greater protection, the lender is willing to reduce the face value $D$ if $S_C$ increases.

The entrepreneur chooses the covenant that maximizes her utility in (4) subject to $U_L = 0$, which leads to the result in the next proposition

**Proposition 1** When $p = 0$, the optimal covenant, denoted $S_C^0$, solves

$$\theta (S_C^0, \beta_L) X + B = L + (1 - \theta (S_C^0, \beta_L)) \left( \frac{\beta_E - \beta_L}{\beta_E (1 - \beta_L)} \right) B.$$  \hspace{1cm} (5)

All proofs are in the appendix. The equation in (5) yields several insights, starting with the following proposition.
**Proposition 2** When the two players have homogeneous priors, $\beta_E = \beta_L$, the entrepreneur chooses the covenant that implements the first-best liquidation decision.

To prove Proposition 2, substitute $\beta_E = \beta_L = \beta$ into (5) to obtain

$$\theta (S_C, \beta) X + B = L.$$ (6)

Let $S_{FB}$ (FB for first-best) denote the covenant threshold that solves (6). Note that the expected social value of continuation $\theta (S_C, \beta) X + B$ exceeds the liquidation value $L$ if and only if $S > S_{FB}$, implying that $S_C = S_{FB}$ implements the first-best decision. The entrepreneur chooses the covenant that maximizes social welfare because the lender always breaks even. Intuitively, if the entrepreneur deviates from $S_C = S_{FB}$ and chooses a looser covenant to allow her to continue the project more often, the lender responds by increasing the face value $D$ and the entrepreneur bears the cost of inefficient continuation.

When the entrepreneur is optimistic, $\beta_E > \beta_L$, the two players no longer agree on the first-best threshold. The first-best threshold from the entrepreneur’s perspective and the lender’s perspective are denoted by $S_{FB}^E$ and $S_{FB}^L$ and solve

$$\theta (S_{FB}^E, \beta_E) X + B = L \text{ and } \theta (S_{FB}^L, \beta_L) X + B = L,$$ (7)

respectively. We obtain $S_{FB}^E < S_{FB}^L$ because the entrepreneur’s posterior probability of success is higher than the lender’s for any signal $S$. Thus, there is a range of signals $S \in (S_{FB}^E, S_{FB}^L)$, for which the entrepreneur believes the socially optimal decision is
to continue the project, whereas the lender believes the socially optimal decision is to liquidate.

One might expect that the entrepreneur’s optimal contract involves a covenant threshold \( S_C \) that lies somewhere between \( S_{FB}^E \) and \( S_{FB}^L \) to take into account the preferences of both players. However, this is not the case. The optimal covenant actually exceeds both the lender’s and the entrepreneur’s first-best thresholds.

**Proposition 3** When the entrepreneur is optimistic relative to the lender, \( \beta_E > \beta_L \), the optimal covenant threshold lies above the lender’s first-best threshold, \( S_C^0 > S_{FB}^L \), and is increasing in \( \beta_E \). The optimal contract therefore leads to socially excessive liquidations even from the lender’s perspective.

The intuition for the result in Proposition 3 can be best explained by rewriting the entrepreneur’s ex ante utility as:

\[
U_E = (1 - \beta_E) * E [U_E|\omega = 0] + \beta_E * E [U_E|\omega = 1] \\
= (1 - \beta_E) B \int_{S_C}^{1} f_0 (S) dS + \beta_E (X + B - D) \int_{S_C}^{1} f_1 (S) dS,
\]

where \( E [U_E|\omega = 0] \) is the entrepreneur’s utility when the state is bad and \( E [U_E|\omega = 1] \) is her utility when the state is good.

Consider first the entrepreneur’s utility when the state is bad, \( E [U_E|\omega = 0] \). The entrepreneur enjoys private benefits of control \( B \) if the project is continued, which happens only if she retains control. Thus, if the entrepreneur’s sole goal was to maximize her utility in the bad state, she would choose the lowest possible covenant
$S_C$ that allows her to obtain the required capital $I$, which is $S_C = 0$ (since we assume that $\beta_L X > I$).

The entrepreneur’s utility when the state is good $E[U_E|\omega = 1]$ looks quite different. Again, the entrepreneur receives a positive utility only if she remains in control. However, since the project will be a success in the good state, the entrepreneur not only enjoys the private benefits $B$ but also receives the residual $X - D$. The covenant therefore affects the entrepreneur’s utility in the good state via two channels. On one hand, a lower covenant $S_C$ increases the probability that the project is continued and hence increases the probability that the entrepreneur receives $B + X - D$. On the other hand, a higher covenant threshold $S_C$ provides the lender with more protection, and renders him willing to reduce the face value $D$, which, in turn, increases the entrepreneur’s residual $X - D$. If the entrepreneur’s sole goal was to maximize her expected utility in the good state, she would choose the $S_C$ that solves $\theta (S_C, \beta_L) (X + B) = L$. Thus, the covenant threshold that maximizes the entrepreneur’s utility in the good state $E[U|\omega = 1]$ exceeds the covenant that maximizes the entrepreneur’s utility in the bad state $E[U|\omega = 0]$. When the entrepreneur is more optimistic ($\beta_E$ increases), she places a higher weight on $E[U|\omega = 1]$ and a lower weight on $E[U|\omega = 0]$, and hence chooses a stricter covenant. In short, when the entrepreneur is more optimistic, she is more concerned about the residual $X - D$ she receives in case of success. The optimistic entrepreneur is therefore more eager to reduce the face value $D$ and she can do so by offering the lender greater protection via a higher covenant $S_C$.

Proposition 3 shows that the optimal threshold $S_C$ exceeds the first-best level
even from the lender’s perspective $S_{FB}^L$. Thus, the covenant is tight and the lender receives control for a wide range of signals. This raises the question whether our initial assumption that the lender wishes to liquidate the project when he has control is indeed satisfied. We now show that this is the case, that is, inequality (2) is satisfied for all $S < S_{C}^0$. To do so, let $S_2$ denote the covenant that solves

$$
\theta (S_2, \beta_L) X = L.
$$

Since $\theta(S, \beta_L)$ is increasing in $S$, the lender prefers liquidation over continuation for any $S < S_2$ even when he received the total output $X$ in case of success. Rearranging (5) shows that the optimal covenant $S_{C}^0$ lies below $S_2$. As a result, the lender will liquidate the project whenever he gains control, as initially assumed.

We next address the question of contract renegotiations. When the signal $S$ lies in the range $[S_{FB}^L, S_{C}^0]$, the covenant is violated and the lender receives control and, as just discussed, prefers to liquidate the project. Nevertheless, for these $S$ values, both parties agree that the first-best decision is to continue the project. One might therefore expect that the two parties renegotiate the contract to implement the efficient decision and to share the surplus. This is, however, not the case because the entrepreneur’s benefits of control $B$ are not transferable. The most the entrepreneur can offer in an attempt to convince the lender to continue the project is a face value of $D = X$. But even for $D = X$ the lender prefers liquidation for all $S < S_{C}^0$ since, as just discussed, $S_{C}^0 < S_2$. For similar reasons the contract is also not renegotiated for any $S < S_{FB}^L$. Further, when the covenant is not violated, $S \geq S_{C}^0$, the entrepreneur
receives control and continues the project. In this case, both parties agree that continuing the project is the socially optimal decision and there is again no room for renegotiation.

In the appendix, we also consider cases where the covenant threshold $S_C$ takes extremely low or high values so that the parties will renegotiate the contract ex post. Covenants that induce ex post renegotiation could potentially be optimal for the entrepreneur because she is more optimistic about the good state than the lender. However, as we show in the appendix, these extreme covenants are never optimal when $p = 0$.

4 State is observable with probability $p$

We now study the case in which the state $\omega$ is observable at date 2 with probability $p$.

Consider first the lender’s optimal action when he has control over the continuation decision, which happens when $S < S_C$. If the lender learns that the state is good, he will simply waive the covenant (since $D > L$) allowing the entrepreneur to continue the project. In this case, there is no further contract renegotiation. If the lender observes the bad state, he will terminate the project. Finally, similar to the previous section, if the state is not observed, the lender prefers liquidation if $\theta(S, \beta_L)D < L$ and continuation otherwise. However, different from the previous section, the parties sometimes renegotiate the contract when the state is not observed even when the covenant is chosen optimally.
Suppose now the entrepreneur has control over the decision, which happens when $S \geq S_C$. If the entrepreneur observes the good state, she will continue the project. If the parties observe the bad state, they will renegotiate the contract to implement the efficient liquidation decision. Finally, if the state is not observed, the entrepreneur prefers continuation unless the parties renegotiate the contract.

We start the analysis by assuming that the two players have homogeneous beliefs, $\beta = \beta_L = \beta_E$. Similar to Proposition 2, we obtain the following result.

**Proposition 4** When the two parties have homogeneous priors, $\beta_E = \beta_L$, the entrepreneur chooses $S^*_C = S_{FB}$, and the parties implement the first-best liquidation decision for any $p$.

When the state is observable and the contract in place induces an inefficient liquidation decision, the parties always implement the efficient decision via renegotiation. When the state is not observable, the liquidation decision is determined solely by the accounting signal $S$. In this case, the entrepreneur has control and continues the project for all $S \geq S_{FB}$ and the lender has control and liquidates it for all $S < S_{FB}$. Again, given the available information, these decisions are efficient.

While the prospect of observing the state $\omega$ plays no role for the optimal covenant threshold when beliefs are homogeneous, it plays a key role when the entrepreneur is optimistic.

**Proposition 5** There is a threshold level $\hat{p}$, with $\hat{p} \in (0, 1)$, such that:

(i) For $p < \hat{p}$, the optimal covenant threshold is $S^*_C = S_C^\#$, where $S_C^\#$ is determined
by

$$\theta \left( S_C^\#, \beta_L \right) X + B = L + \left( 1 - \theta \left( S_C^\#, \beta_L \right) \right) \frac{(\beta_E - \beta_L) (B + (L - B) p \gamma)}{\beta_E (1 - \beta_L) (1 - p)}. \quad (10)$$

$S_C^*$ is increasing in $p$ for any $p \leq \hat{p}$. If the state is not observable, the project is continued if and only if $S \geq S_C^*$, leading to socially inefficient liquidation, even from the lender’s perspective, for all $S \in [S_{FB}^L, S_C^*)$;

(ii) For $p \geq \hat{p}$, the lender receives full control, $S_C^* = 1$. If the state is not observable, the project is continued if and only if $S \geq S_2$ leading to inefficient liquidation, even from the lender’s perspective, for all $S \in [S_{FB}^L, S_2]$.

Consider part (i) of Proposition 5. We show in the appendix that for $p < \hat{p}$ the optimal covenant threshold $S_C^* = S_C^\#$ lies below $S_2$. Recall from the previous section that $S_2$ is determined by $\theta(S_2, \beta_L) X = L$. Further, equation (10) shows that $S_C^\#$ exceeds the lender’s first-best threshold $S_{FB}^L$. From the previous section, we know that for $S_C \in (S_{FB}^L, S_2)$, the parties do not renegotiate the contract when the state is unobservable, and the lender prefers to liquidate whenever he has control and the entrepreneur prefers to continue whenever she has control. However, if the state is observable, the parties sometimes renegotiate the contract (or simply waive the covenant) to implement the efficient action as discussed earlier.

To explain why the optimal covenant increases in $p$ (as long as $p < \hat{p}$), we can write the first derivative of the entrepreneur’s utility with respect to the covenant $S_C$ as follows:
\[
\frac{dU_E}{dS_C} = (1 - p) \left[ -\frac{\beta_E}{\beta_L} \left( \theta (S_C, \beta_L) (X + B) - L \right) f (S_C, \beta_L) - (1 - \beta_E) B f_0 (S_C) \right] \\
+ p \left[ (\gamma L + (1 - \gamma)B) \left( \frac{\beta_E - \beta_L}{\beta_L} \right) f_0 (S_C) \right].
\]

The first term in square brackets in (11) captures \( \frac{dU_E(p=0)}{dS_C} \): the effect of a change in \( S_C \) on the entrepreneur’s utility assuming all players know that the state can never be observed \((p = 0)\). The second term in square brackets captures \( \frac{dU_E(p=1)}{dS_C} \): the effect of a change in \( S_C \) on the entrepreneur’s utility assuming all players know the state is always observable \((p = 1)\).

The optimal covenant threshold \( S_C^* \) increases with \( p \) for the following reasons. From the previous section we already know that the utility \( U_E(p = 0) \) is maximized when \( S_C = S_C^0 \). The optimal threshold \( S_C^0 \) balances the benefits and costs of giving the lender more control: on one hand, a tighter covenant \( S_C \) leads to a lower face value \( D \), on the other hand, it increases the probability that the lender liquidates the project although continuation is socially efficient. Further, the utility \( U_E(p = 1) \) is strictly increasing in \( S_C \). We explain why in the next paragraph. Since \( U_E(p = 0) \) is maximized for \( S_C^0 \) and \( U_E(p = 1) \) is strictly increasing in \( S_C \), the entrepreneur chooses a higher covenant threshold when \( p \) increases.

What remains to be established is why \( \frac{dU_E(p=1)}{dS_C} > 0 \). An increase in the covenant
increases the lender’s control but does not cause inefficient terminations because for
\( p = 1 \) the lender will always waive the covenant when he learns the state is good.
Thus, for \( p = 1 \), an increase in the covenant does not affect the payoffs of the two
players when the state is good. An increase in the covenant, however, does affect the
players’ ex post payoffs when the state is bad. To see this, suppose the entrepreneur
is in control and the parties observe the bad state. The entrepreneur can then exploit
her control rights and renegotiate with the lender to implement liquidation, which
yields her an expected payoff of \( \gamma L + (1 - \gamma) B \), where \( \gamma \) represents her bargaining
power. In contrast, when the lender has control and observes the bad state, he can
simply terminate the project without having to renegotiate with the entrepreneur.
Thus, giving the lender more control leaves him with a higher payoff in the bad state.
Since the lender just breaks even in expectation he will return this expected payoff
in the form of a lower face value \( D \). This trade is beneficial from the entrepreneur’s
perspective because she believes that the bad state is less likely to occur than what the
lender believes. In short, the entrepreneur wishes to offer the lender greater protection
in the bad state because the lender values this protection more than the entrepreneur.
This effect, which we label the "protection effect" is the reason why the entrepreneur
wishes to choose a tighter covenant when \( p \) increases.

Consider now part (ii) of Proposition 5, which shows that the entrepreneur gives
the lender unconditional control when \( p \geq \hat{p} \). The intuition for this result is as
follows. We know from part (i) of Proposition 5 that the covenant threshold \( S_C^\# \)
increases in \( p \). For a sufficiently high level of \( p \), say \( p = \bar{p} \), the threshold \( S_C^\# \) reaches
\( S_2 \). When \( S_C = S_2 \) a further increase in the covenant threshold no longer affects the
liquidation decision when the state is not observable. To see this, note that for all $S \in [S_2, S_C)$ the lender has control and prefers to liquidate the project for some $S$ values close to $S_2$ but both the entrepreneur and the lender agree that continuation is socially efficient. For $S > S_2$ renegotiation to implement continuation is now possible because $\theta(S, \beta_L)X > L$. Thus, for $S_C \geq S_2$ an increase in $S_C$ does not affect the liquidation decision but increases the entrepreneur’s utility via the protection effect. Specifically, we show in the appendix that for $S_C \geq S_2$, an increase in $S_C$ increases the entrepreneur’s ex ante utility by

$$
\frac{dU_E}{dS_C} = p(\gamma L + (1 - \gamma)B) \frac{\beta_E - \beta_L}{\beta_L} f_0 (S_C),
$$

which equals the second line of (11). An increase in $S_C$ grants the lender greater protection when the parties observe the bad state and the lender values this protection more than the optimistic entrepreneur. Thus, when the entrepreneur has to choose a covenant threshold from the range $[S_2, 1]$ she will choose $S_C = 1$ because granting the lender unconditional control best exploits the protection effect. Note that due to the protection effect the entrepreneur will also prefer $S_C = 1$ over some values of $S_C$ that are slightly smaller than $S_2$, even though those covenant thresholds implement a slightly more efficient liquidation decision when the state is not observed. As a result, there is a probability threshold, $\tilde{p}$, with $\tilde{p} < \bar{p}$, such that the entrepreneur chooses $S_C^* = 1$ if $p \geq \tilde{p}$, and $S_C^* = S_C^\#$ if $p < \tilde{p}$, where $S_C^\# < S_2$. 

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5 Empirical Implications

To be continued...

6 Appendix

Proof of Propositions 1 and 5.

Proposition 1 is a special case of Proposition 5 so we focus here on the proof of the latter.

There are several cases to consider. Case (i) is the case where the parties do not renegotiate the contract when the state is not observable. Cases (ii) and (iii) analyze scenarios where the covenant is so high that the parties sometimes engage in renegotiation to avoid inefficient liquidation. Case (iv) studies situations where the covenant is so low that the parties will sometimes engage in renegotiation to prevent continuation. To specify when the different cases apply, we use the following three thresholds, $S_1 < S_2 < S_3$, which are defined below. We first study these four cases and then specify the overall optimal covenant threshold.

Case (i): $S_C \in [S_1, S_2]$.

Thresholds $S_1$ and $S_2$ satisfy

$$L = \theta(S_1, \beta_L)D + \theta(S_1, \beta_E)(X - D) + B,$$

(13)

and

$$\theta(S_2, \beta_L)X = L,$$

(14)
respectively.

Suppose the state $\omega$ is not observable. A covenant threshold $S_C$ that lies in the range $[S_1, S_2]$ has the following two implications. First, $S_C > S_1$ implies that whenever the entrepreneur has control (which happens when $S > S_C$), she will continue the project, and there will be no contract renegotiations to implement liquidation. This follows because for $S > S_1$ the sum of the lender’s continuation payoff (from his perspective) and the entrepreneur’s continuation payoff (from her perspective) exceed the liquidation value; that is, the right-hand side of (13) exceeds the left-hand side. To prove the absence of renegotiation in detail assume that $S > S_C$ and that the lender has all of the bargaining power (which happens with probability $1 - \gamma$). He can then offer the entrepreneur to reduce the face value to $D_N = L - \theta (S, \beta_{E}) (X - D) + B$ in return for the control rights. This offer leaves the entrepreneur indifferent but can only be optimal for the lender if

$$L - \theta (S, \beta_{E}) (X - D) - B > \theta (S, \beta_{L}) D,$$

which cannot hold when $S > S_1$. Similar arguments apply when the entrepreneur has all of the bargaining power (which happens with probability $\gamma$).

Second, $S_C < S_2$ implies that whenever the lender has control (which occurs when $S < S_C$), he prefers to liquidate the project, and there is again no room for contract renegotiation. We already discussed this case in Section 3. When $S < S_C$, the entrepreneur may want to convince the lender to continue the project (or waive the covenant) but is unable to do so because she can offer him at most $D = X$, but
even for $D = X$ the lender prefers liquidation since $S_C < S_2$.

In short, $S_C \in [S_1, S_2]$ implies that the parties abstain from renegotiation when the state is not observed, and the lender (entrepreneur) always prefers liquidation (continuation) when he (she) is in control.

The lender’s ex ante utility is now given by

\[
U_L = \int_{S_C}^{1} \left( \theta(S, \beta_L) D + (1 - \theta(S, \beta_L)) p (1 - \gamma) (L - B) \right) f(S, \beta_L) dS 
+ \int_{0}^{S_C} \left( \theta(S, \beta_L) p D + (1 - p \theta(S, \beta_L)) L \right) f(S, \beta_L) dS - I = 0,
\]

which can be rewritten as:

\[
D \left( \int_{S_C}^{1} f_1(S) dS + p \int_{0}^{S_C} f_1(S) dS \right) \beta_L = I - L \beta_L (1 - p) \int_{0}^{1} f_1(S) dS \tag{16}
\]

\[-L(1 - \beta_L) \int_{0}^{S_C} f_0(S) dS - p(1 - \gamma) (L - B) (1 - \beta_L) \int_{S_C}^{1} f_0(S) dS.
\]

The entrepreneur’s ex ante utility is

\[
U_E = \int_{S_C}^{1} \theta(S, \beta_E) (X - D + B) f(S, \beta_E) dS + \int_{0}^{S_C} \theta(S, \beta_E) p (X - D + B) f(S, \beta_E) dS \tag{17}
+ \int_{S_C}^{1} \left( 1 - \theta(S, \beta_E) \right) \left( (1 - p) B + p \left( (1 - \gamma) B + \gamma L \right) \right) f(S, \beta_E) dS,
\]

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which can be rewritten as

\[ U_E = \beta_E (X - D + B) \left( \int_{S_C}^{1} f_1(S) dS + p \int_{0}^{S_C} f_1(S) dS \right) \]  

\[ + (1 - \beta_E) (B + p\gamma (L - B)) \int_{S_C}^{1} f_0(S) dS. \]

Equation (18) is instructive as it can be directly related to (8).

Substituting (16) into (18), to eliminate \( D \), yields

\[ U_E = \beta_E (X + B) \int_{S_C}^{1} f_1(S) dS + \beta_E ((1 - p) L + p(X + B)) \int_{0}^{S_C} f_1(S) dS \]

\[ + \frac{\beta_E}{\beta_L} L (1 - \beta_L) \int_{0}^{S_C} f_0(S) dS + p(L - B) \left( (1 - \gamma) \frac{\beta_E (1 - \beta_L)}{\beta_L} + \gamma (1 - \beta_E) \right) \int_{S_C}^{1} f_0(S) dS \]

\[ + (1 - \beta_E) B \int_{S_C}^{1} f_0(S) dS - \frac{\beta_E}{\beta_L} I \]

Taking the first-order condition of (19) gives:

\[ \frac{dU_E}{dS_C} = 0 = \beta_E (- (X + B) f_1(S_C) + ((1 - p) L + p(X + B)) f_1(S_C) \]

\[ + \frac{1}{\beta_L} L (1 - \beta_L) f_0(S_C) - p(L - B) \left( (1 - \gamma) \frac{1 - \beta_L}{\beta_L} + \gamma \frac{1 - \beta_E}{\beta_E} \right) f_0(S_C) \]

\[ - \frac{1 - \beta_E}{\beta_E} B f_0(S_C). \]
After some rearranging we obtain:

\[
\frac{dU_E}{dS_C} \frac{1}{f_0(S_C) \beta_E} = 0 = -(1-p) (X + B - L) \frac{f_1(S_C)}{f_0(S_C)} \\
+ \frac{1 - \beta_L}{\beta_L} L - p(L - B) \left( (1 - \gamma) \frac{1 - \beta_L}{\beta_L} + \gamma \frac{1 - \beta_E}{\beta_E} \right) - \frac{1 - \beta_E}{\beta_E} B,
\]

which leads to (10) in the main text.

Recall from the main text that the threshold that solves (21) is denoted by \( S^#_C \).

From (21) we obtain \( \frac{dU_E}{dS_C} > 0 \) when \( \frac{f_1(S_C)}{f_0(S_C)} < \frac{f_1(S^#_C)}{f_0(S^#_C)} \) and \( \frac{dU_E}{dS_C} < 0 \) when \( \frac{f_1(S_C)}{f_0(S_C)} > \frac{f_1(S^#_C)}{f_0(S^#_C)} \).

Due to the MLRP, we obtain \( \frac{dU_E}{dS_C} > 0 \) when \( S < S^#_C \) and \( \frac{dU_E}{dS_C} < 0 \) when \( S_C > S^#_C \),

which proves that \( S^#_C \) is an optimum.

To show that the \( S^#_C \) that solves (10) is increasing in \( p \), we rearrange (10) to obtain

\[
\Gamma \equiv \theta \left( S^#_C, \beta_L \right) X + B - L - \left( 1 - \theta \left( S^#_C, \beta_L \right) \right) \frac{(\beta_E - \beta_L) (B + (L - B) p \gamma)}{\beta_E (1 - \beta_L) (1-p)} = 0.
\]

Applying the implicit function theorem yields:

\[
\frac{dS^#_C}{dp} = - \frac{d\Gamma/dp}{d\Gamma/dS_C} > 0,
\]
where

\[
\frac{d\Gamma}{dp} = -\left(1 - \theta \left( S_C^\#, \beta_L \right) \right) \frac{\beta_E (\beta - \beta_L) (B + (L - B) \gamma)}{\beta_E (1 - \beta_L) (1 - p)^2} < 0,
\]

\[
\frac{d\Gamma}{dS_C} = \frac{d\theta \left( S_C^\#, \beta_L \right)}{dS_C} \left( X + \frac{\beta_E (\beta - \beta_L) (B + (L - B) \gamma)}{\beta_E (1 - \beta_L) (1 - p)} \right) > 0,
\]

and \( \frac{d\theta(S_C^\#; \beta_L)}{dS_C^\#} > 0 \) due to the MLRP.

Finally, since \( S_C^\# > S_{FB}^L \) and \( S_{FB}^L > S_1 \), we obtain \( S_C^\# > S_1 \). As shown further below, for the \( p \) values for which \( S_C^\# \) is the optimal solution (\( p < \hat{p} \)), we obtain \( S_C^\# < S_2 \).

**Case (ii):** \( S_C \in [S_2, S_3] \).

The threshold \( S_3 \) satisfies the equation \( \theta(S_3, \beta_L) D = L \). Suppose the state is not observable. In this case, a covenant that lies in the range \([S_2, S_3]\) has the following two implications. First, in the absence of renegotiation, the lender prefers to liquidate the project if he has control because \( S_C < S_3 \) implies that \( \theta(S, \beta_L) D < L \) for all \( S < S_C \).

Second, there is a range of signals, \( S \in [S_2, S_3] \), for which the lender is in control and prefers liquidation, but the two parties are willing to renegotiate the contract to implement the efficient continuation decision. Continuation is the socially optimal decision for \( S \in [S_2, S_3] \) even from the lender’s perspective because \( S_2 > S_{FB}^L \). The renegotiation process proceeds as follows. If the entrepreneur has all of the bargaining power, she will offer the lender to increase the face value of debt to \( D_N = \frac{L}{\theta(S, \beta_L)} \) in return for the control rights. This offer leaves the lender indifferent but increases
the entrepreneur’s utility from zero to $\theta (S, \beta_E) (X - D_N) + B$. If the lender has all of the bargaining power, he will demand an increase in the face value to $X$ in return for waiving the covenant. Since the entrepreneur’s utility is zero if the project is liquidated, she accepts the offer and at least gets her private benefits. The lender is then better off because his expected utility increases from $L$ to $\theta (S, \beta_L) X$, where $\theta (S, \beta_L) X > L$ follows from $S > S_2$. In sum, for $S_C \in [S_2, S_3]$, the lender prefers to liquidate if he is in control but the parties will renegotiate the contract to implement efficient continuation for all $S \in [S_2, S_C)$. As a result, as long as $S_C \in [S_2, S_3]$, an increase in the covenant threshold $S_C$ does not affect the continuation decision when the state is not observable because the parties always end up continuing the project for all $S > S_2$.

The lender’s break-even constraint is now:

$$I = \int_0^{S_2} \left( p \theta (S, \beta_L) D + (1 - p) \theta (S, \beta_L) L \right) f (S, \beta_L) dS$$

$$+ \int_{S_2}^{S_C} \left( p \theta (S, \beta_L) D + (1 - p) (1 - \gamma) \theta (S, \beta_L) X \right)$$

$$+(1 - p) \gamma L + p(1 - \theta (S, \beta_L)) L f (S, \beta_L) dS$$

$$+ \int_{S_C}^1 \left( \theta (S, \beta_L) D + p (1 - \theta (S, \beta_L)) (1 - \gamma) (L - B) \right) f (S, \beta_L) dS.$$

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The entrepreneur’s utility function is:

\[
U_E = \int_{0}^{s_2} p\theta (S, \beta_E) (X - D + B) f (S, \beta_E) dS \\
+ \int_{s_2}^{s_C} \left( p\theta (S, \beta_E) (X - D + B) + (1 - p) \left( B + \gamma \theta (S, \beta_E) \left( X - \frac{L}{\theta (S, \beta_L)} \right) \right) \right) f (S, \beta_E) dS \\
+ \int_{s_C}^{1} (\theta (S, \beta_E) (X - D) + B + p(1 - \theta (S, \beta_E)) \gamma (L - B)) f (S, \beta_E) dS.
\] (23)

Substituting the lender’s break-even constraint into the entrepreneur’s utility function to remove \( D \) yields:

\[
U_E = \int_{0}^{s_2} p\theta (S, \beta_E) (X + B) f (S, \beta_E) dS - \frac{\beta_E I}{\beta_L} + \frac{\beta_E}{\beta_L} \int_{0}^{s_2} (1 - p\theta (S, \beta_L)) Lf (S, \beta_L) dS \\
+ \frac{\beta_E}{\beta_L} \int_{s_2}^{s_C} ((1 - p)(1 - \gamma) \theta (S, \beta_L) X + (1 - p)\gamma L + p(1 - \theta (S, \beta_L))L) f (S, \beta_L) dS \\
+ \frac{\beta_E}{\beta_L} \int_{s_C}^{1} p(1 - \theta (S, \beta_L)) (1 - \gamma) (L - B) f (S, \beta_L) dS \\
+ \int_{s_2}^{s_C} \left( p\theta (S, \beta_E) (X + B) + (1 - p) \left( B + \gamma \theta (S, \beta_E) \left( X - \frac{L}{\theta (S, \beta_L)} \right) \right) \right) f (S, \beta_E) dS \\
+ \int_{s_C}^{1} (\theta (S, \beta_E) X + B + p(1 - \theta (S, \beta_E)) \gamma (L - B)) f (S, \beta_E) dS.
\] (24)
Taking the derivative of $U_E$ with respect to $S_C$ and simplifying yields

$$\frac{dU_E}{dS_C} = p\frac{(\beta_E - \beta_L)}{\beta_L} (\gamma L + (1 - \gamma)B) f_0(S_C).$$

(25)

Thus, for any $S_C$ in the range $[S_2, S_3]$, the entrepreneur prefers $S_C = S_3$.

**Case (iii):** $S_C \geq S_3$.

Suppose the parties do not observe the state. For $S_C \geq S_3$, the covenant threshold is so high that there exist signals, $S \in [S_3, S_C)$, for which the lender finds it optimal to continue the project even in the absence of renegotiation. Thus, for $S \in [S_3, S_C)$, the lender is in control and makes the socially efficient continuation decision. Similar to case (ii), as long as $S_C \geq S_3$, a change in $S_C$ does not affect the continuation decision since the project is always continued for all $S \geq S_3$ regardless of who is in control. Further, and again similar to case (ii), for all $S \in [S_2, S_3)$, the lender is in control and prefers liquidation but the two parties renegotiate the contract to implement the efficient continuation decision. Thus, for all $S_C \geq S_3$, the project is continued if and only if $S \geq S_2$ (assuming the state is not observable).
The lender’s break-even constraint is now:

\[
I = \int_0^{s_2} (p \theta (S, \beta_L) D + (1 - p \theta (S, \beta_L)) L) f(S, \beta_L)\,dS + \int_{s_2}^{s_3} (p \theta (S, \beta_L) D \, \theta) + (1 - p) (1 - \gamma) \theta (S, \beta_L) X + (1 - p) \gamma L + p(1 - \theta (S, \beta_L)L) f (S, \beta_L)\,dS \\
+ \int_{s_3}^{s_C} (\theta (S, \beta_L) D + p(1 - \theta (S, \beta_L)) L) f (S, \beta_L)\,dS \\
+ \int_{s_C}^1 (\theta (S, \beta_L) D + p(1 - \theta (S, \beta_L)) (1 - \gamma) (L - B)) f (S, \beta_L)\,dS.
\]

The entrepreneur’s utility function is:

\[
U_E = \int_0^{s_2} p \theta (S, \beta_E) (X - D + B) f(S, \beta_E)\,dS + \int_{s_2}^{s_3} p \theta (S, \beta_E) (X - D + B) + \left(1 - p\right) \left(B + \gamma \theta (S, \beta_E) \left(X - \frac{L}{\theta (S, \beta_E)}\right)\right) f(S, \beta_E)\,dS \\
+ \int_{s_3}^{s_C} \left(\theta (S, \beta_E) (X - D + B) + \left(1 - p\right)(1 - \theta (S, \beta_E)) B\right) f(S, \beta_E)\,dS \\
+ \int_{s_C}^1 \left(\theta (S, \beta_E) (X - D) + B + p(1 - \theta (S, \beta_E)) \gamma (L - B)\right) f(S, \beta_E)\,dS.
\]

Substituting the lender’s break-even constraint into the entrepreneur’s utility
function to remove $D$ yields:

\[
U_E = \int_0^{S_2} p\theta(S, \beta_E) (X + B) f(S, \beta_E) \, dS - \frac{\beta_E}{\beta_L} \int_0^{S_2} \left(1 - p\theta(S, \beta_L)\right) L f(S, \beta_L) \, dS \\
+ \frac{\beta_E}{\beta_L} \int_0^{S_3} \left(1 - p\right) \left(1 - \gamma\right) \theta(S, \beta_L) X + \left(1 - p\right) \gamma L + p(1 - \theta(S, \beta_L)) L f(S, \beta_L) \, dS \\
+ \frac{\beta_E}{\beta_L} \int_0^{S_C} \left(p(1 - \theta(S, \beta_L)) L f(S, \beta_L) \, dS + \frac{\beta_E}{\beta_L} \int_0^{S_C} \left(1 - p(1 - \theta(S, \beta_L)) \left(1 - \gamma\right) (L - B) f(S, \beta_L) \, dS \\
+ \int_0^{S_2} \left(p\theta(S, \beta_E) (X + B) + \left(1 - p\right) \left(B + \gamma\theta(S, \beta_E) \left(X - \frac{L}{\theta(S, \beta_L)}\right)\right) f(S, \beta_E) \, dS \\
+ \int_0^{S_2} \left(1 - \theta(S, \beta_E) (X + B) + \left(1 - p\right) \left(1 - \theta(S, \beta_E) B\right) f(S, \beta_E) \, dS \\
+ \int_0^{S_C} \left(1 - \theta(S, \beta_E) X + B + p(1 - \theta(S, \beta_E)) \gamma (L - B)\right) f(S, \beta_E) \, dS. \tag{28}
\]

Taking the derivative of $U_E$ with respect to $S_C$ and simplifying yields

\[
\frac{dU_E}{dS_C} = p\frac{\beta_E - \beta_L}{\beta_L} \left(\gamma L + (1 - \gamma) B\right) f_0(S_C). \tag{29}
\]

Note, the effect of a change in $S_C$ on $S_3$ on the entrepreneur’s utility cancels out.

Since $\frac{dU_E}{dS_C} > 0$, for any $S_C$ in the range $[S_3, 1]$, the entrepreneur prefers $S_C = 1$.

Case (iv): $S_C \in [0, S_1]$.

Suppose the state is not observable. When the covenant lies below $S_1$ the two parties may engage in renegotiation to implement liquidation. Specifically, for $S \in [S_C, S_1]$,
the entrepreneur has control and prefers to continue the project, but from the lender’s perspective the socially optimally decision is liquidation since \( S_1 < S_{FB}^L \). Liquidation is not necessarily the socially optimal decision from the entrepreneur’s perspective since \( S_1 > S_{FB}^E \). Nevertheless, the parties renegotiate the contract to implement liquidation for \( S \in [S_C, S_1] \) because for \( S < S_1 \) the liquidation value \( L \) exceeds the sum of the expected continuation payoff for the lender from the lender’s perspective and the expected continuation payoff for the entrepreneur from the entrepreneur’s perspective, that is,

\[
L > \theta(S, \beta_L)D + \theta(S, \beta_E)(X - D) + B.
\]

Specifically, when the entrepreneur has all the bargaining power, she offers the lender a face value of \( D_N = \theta(S, \beta_L)D \) and grants him control rights. Since \( D_N = \theta(S, \beta_L)D < L \) for \( S < S_1 \) (which follows from \( S_1 < S_2 \)), the lender will use the control rights to liquidate the project and receive \( D_N \). The entrepreneur is then better off because her expected utility increases from \( \theta(S, \beta_E)(X - D) + B \) to \( L - \theta(S, \beta_L)D \), which is positive if

\[
L - \theta(S, \beta_L)D > \theta(S, \beta_E)(X - D) + B.
\]

This inequality is indeed satisfied for \( S < S_1 \). The renegotiation process works in a similar fashion when the lender has all of the bargaining power.
The lender’s break-even constraint is now given by:

\[ I = \int_{S_1}^{1} (\theta (S, \beta_L) D + p (1 - \theta (S, \beta_L)) (1 - \gamma) (L - B)) f (S, \beta_L) dS \]

\[ + \int_{0}^{S_1} (p\theta (S, \beta_L) D + (1 - p\theta (S, \beta_L)) L) f (S, \beta_L) dS + \int_{S_C}^{s_1} p\theta (S, \beta_L) D f (S, \beta_L) dS \]

\[ + \int_{S_C}^{1} \gamma (1 - p) \theta (S, \beta_L) D f (S, \beta_L) dS \]

\[ + \int_{S_C}^{s_1} (1 - \gamma) ((1 - p) (L - \theta (S, \beta_L) (X - D) - B) + p(1 - \theta (S, \beta_L) (L - B)) f (S, \beta_L) dS. \]

The Entrepreneur’s ex ante utility is:

\[ U_E = \int_{S_1}^{1} (\theta (S, \beta_E) (X - D) + B + p (1 - \theta (S, \beta_E)) \gamma (L - B)) f (S, \beta_E) dS \]

\[ + \int_{0}^{s_1} p\theta (S, \beta_E) (X - D + B) f (S, \beta_E) dS \]

\[ + \int_{S_C}^{s_1} \gamma ((1 - p) (L - \theta (S, \beta_L) D) + p(1 - \theta (S, \beta_E)) L) f (S, \beta_E) dS \]

\[ + \int_{S_C}^{s_1} (1 - \gamma) ((1 - p) (\theta (S, \beta_E) (X - D) + B) + p(1 - \theta (S, \beta_E)) B) f (S, \beta_E) dS. \]

We substitute the lender’s break-even constraint into the entrepreneur’s utility function to eliminate most of \( D \). We do not completely substitute for \( D \) because it
would not cancel out neatly. This gives

$$U_E = \int_{S_1}^{1} \left( \theta (S, \beta_E) X + B + p (1 - \theta (S, \beta_E)) \gamma (L - B) \right) f (S, \beta_E) dS - \frac{\beta_E}{\beta_L} I \tag{32}$$

$$+ \frac{\beta_E}{\beta_L} \int_{S_1}^{1} p (1 - \theta (S, \beta_L)) (1 - \gamma) (L - B) f (S, \beta_L) dS$$

$$+ \frac{\beta_E}{\beta_L} \int_{S_1}^{S_C} (1 - p \theta (S, \beta_L)) L f (S, \beta_L) dS + \frac{\beta_E}{\beta_L} \int_{S_C}^{S_1} \gamma (1 - p) \theta (S, \beta_L) D f (S, \beta_L) dS$$

$$+ \frac{\beta_E}{\beta_L} \int_{S_1}^{S_C} (1 - \gamma) ((1 - p) (L - \theta (S, \beta_E) (X - D) - B) + p (1 - \theta (S, \beta_L) (L - B)) f (S, \beta_L) dS$$

$$+ \int_{0}^{S_1} p \theta (S, \beta_E) (X + B) f (S, \beta_E) dS$$

$$+ \int_{S_C}^{S_1} \gamma ((1 - p) (L - \theta (S, \beta_L) D) + p (1 - \theta (S, \beta_E)) L) f (S, \beta_E) dS$$

$$+ \int_{S_C}^{S_1} (1 - \gamma) ((1 - p) (\theta (S, \beta_E) (X - D) + B) + p (1 - \theta (S, \beta_E)) B) f (S, \beta_E) dS.$$
any other $S_C < S_1$, i.e., $U_E(S_C = S_1) \geq U_E(S_C \leq S_1)$, which can be shortened to

$$\frac{\beta_E}{\beta_L} \int_{S_C}^{S_1} (1 - p\theta(S, \beta_L)) L f(S, \beta_L) dS \geq \frac{\beta_E}{\beta_L} \int_{S_C}^{S_1} \gamma (1 - p) \theta(S, \beta_L) D f(S, \beta_L) dS$$

(33)

$$+ \frac{\beta_E}{\beta_L} \int_{S_C}^{S_1} (1 - \gamma) ((1 - p) (L - \theta(S, \beta_E)(X - D) - B) + p(1 - \theta(S, \beta_L)(L - B)) f(S, \beta_L) dS$$

$$+ \int_{S_C}^{S_1} \gamma (1 - p) (L - \theta(S, \beta_L) D) + p(1 - \theta(S, \beta_E)) L) f(S, \beta_E) dS$$

$$+ \int_{S_C}^{S_1} (1 - \gamma) ((1 - p) (\theta(S, \beta_E)(X - D) + B) + p(1 - \theta(S, \beta_E)) B) f(S, \beta_E) dS.$$  

Further simplifications yield

$$0 \leq \int_{S_C}^{S_1} \gamma (L - (1 - p) \theta(S, \beta_L) D) f_0(S) \left( \frac{\beta_E}{\beta_L} - 1 \right)$$

(34)

$$+ (1 - \gamma) ((1 - p) (\theta(S, \beta_E)(X - D) + B) + pB) \left( \frac{\beta_E}{\beta_L} - 1 \right) f_0(S) dS.$$  

This proves that for any $S_C$ in the range $[S_1, S_2]$, the entrepreneur chooses $S_C = S_2$. We know from case (i) that $S_C = S_2$ is inferior to $S_C^\#$.  

**Optimal solution:** Suppose first that $p = 0$. From the analysis in cases (ii) and (iii) we know that $\frac{dU_E}{dSC} = 0$ for all $S_C \in [S_2, 1]$ when $p = 0$. From the analysis in case (i) we know that $S_C = S_1$ dominates any other $S_C < S_1$. Further, for $p = 0$, the optimality condition in (10) simplifies to the one in (5). Rearranging condition (5)
to

$$
\theta (S_C^0, \beta_L) X = L - \frac{(\beta_E - \beta_E \beta_L) - (1 - \theta (S_C^0, \beta_L)) (\beta_E - \beta_L)}{\beta_E (1 - \beta_L)} B
$$

(35)

shows that $S_C^0 < S_2$. We already have established in case (i) that $S_C^0 > S_1$ for any $p$, implying that $S_C^0 > S_1$. Thus, for $p = 0$, the optimal solution is $S_C = S_C^0$.

Suppose now that $p > 0$. We know from cases (ii) and (iii) that for any $S_C \in [S_2, 1]$ it holds that $\frac{dU_E}{dS_C} > 0$ and the entrepreneur prefers $S_C = 1$. We next show that there exists a threshold $\hat{p} \in (0, 1)$ such that for $p > \hat{p}$ the corner solution $S_C = 1$ is optimal. Since, as just established, for $p = 0$ the optimal covenant is $S_C^0$, the threshold $\hat{p}$ is greater than zero. To prove that $\hat{p} < 1$, we show first that there exists a threshold $\tilde{p}$ such that $S_C^0(\tilde{p}) = S_2$. Substituting $\theta (S_2, \beta_L) X = L$ and $S_C^0 = S_2$ into (10) and solving for $p$ yields

$$
\tilde{p} = \frac{(\beta_E - \beta_E \beta_L) - (\beta_E - \beta_L) (1 - \theta (S_2, \beta_L))}{B (\beta_E - \beta_E \beta_L) - \gamma (B - L) (\beta_E - \beta_L) (1 - \theta (S_2, \beta_L))} B,
$$

which shows that $\tilde{p}$ exists with $\tilde{p} \in (0, 1)$. When $p \geq \tilde{p}$, then given the range $S_C \in [S_1, S_2]$, the entrepreneur prefers $S_C = S_2$. However, because $\tilde{p} > 0$, we know from cases (ii) and (iii) that $S_C = 1$ is strictly better for the entrepreneur than $S_C = S_2$, and hence $S_C = 1$ must be the overall optimal solution for $p = \tilde{p}$. This establishes that $\hat{p} < \tilde{p}$. Thus, for all $p < \hat{p}$, the optimal covenant is $S_C^* = S_C^0$ and for all $p \geq \hat{p}$, the optimal covenant is $S_C^* = 1$. 

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References


