Asymmetric Reporting

Christopher S. Armstrong
carms@wharton.upenn.edu

Daniel J. Taylor
dtayl@wharton.upenn.edu

Robert E. Verrecchia
verrecchia@wharton.upenn.edu

The Wharton School
University of Pennsylvania

This Draft: April 4, 2014

We thank Paul Fischer and Mirko Heinle for very useful discussions and the The Wharton School for financial support.
Asymmetric Reporting

Abstract:

We generalize the CAPM to a setting where a regulator requires firms to report earnings before their shares are publicly traded but does not specify the reporting system that maps economic income into reported earnings. We show that under fairly mild conditions, a risk-averse entrepreneur (as representative of the initial owners of the firm) will endogenously choose a reporting system where reported earnings reflect economic income to a greater extent in bad states than in good states. The intuition for this result is that by virtue of being risk averse, the entrepreneur’s utility is concave; as such, he derives greater marginal benefit from reducing uncertainty in bad states. Thus, the entrepreneur will be more willing to undertake costly verification in bad states, and this differential level of verification results in reported earnings that reflect economic income to a greater extent in bad states. Additionally, we show that the choice of reporting system endogenously affects firms’ systematic risk. Firms where reported earnings reflect economic income to a greater extent in bad states have higher (lower) covariance with the market portfolio in good (bad) states. Strikingly, these results obtain even in the most basic of circumstances, where earnings serve only a valuation role and there are no agency problems or contracting considerations.

Keywords: endogenous reporting; the CAPM; mandatory disclosure; asymmetric reporting; asymmetric timeliness; expected returns;

JEL Classification: G11, G12, G14, G31
1 Introduction

A large literature examines the relation between properties of the financial reporting system and various capital market outcomes. Many of the studies in this literature either explicitly or implicitly take the properties of the reporting system as given. In this paper, we take a different approach. Rather than focus solely on the capital market consequences of financial reporting, we attempt to understand the fundamental economic forces that determine the properties of the reporting system and, in turn, the effect of these forces on capital markets.

One of the most studied—and controversial—properties of financial reporting is the asymmetry in the extent to which reported earnings reflect contemporaneous economic income (e.g., Ball, Kothari, and Nikolaev, 2013). A wide variety of accounting rules require the recognition of expected losses but only allow the recognition of realized gains. For example, when the value of property, plant, and equipment (PP&E) declines, earnings reflect the decline in value, but when the value of PP&E increases, earnings do not reflect the increase in value. Existing economic theory suggests that the asymmetry in reported earnings is exclusively the result of economic forces related to the stewardship role of earnings: the role of earnings in mitigating agency problems and facilitating efficient contracting (see Armstrong, Guay, and Weber, 2011, and Kothari, Ramanna, and Skinner, 2011, for reviews). Many studies argue that if it were not for the stewardship role of accounting, reported earnings would symmetrically reflect economic income (e.g., Holthausen and Watts, 2001; Guay and Verrecchia, 2006; Ball et al., 2008).

In this paper, we examine whether there are additional economic forces beyond agency.

1 Many studies refer to the observed asymmetry in the extent to which reported earnings reflect contemporaneous economic income as “conservatism” or “conditional conservatism.” We eschew the use of such terms, because different studies have different opinions on what “conservatism” means, how to model it, and how it manifests. For example, Kwan et al. (2001), Guay and Verrecchia (2006), and Gigler et al., (2012) all claim to model “conservatism’” but do so very differently. Instead, we use the phrase “asymmetry in the extent to which reported earnings reflect contemporaneous economic income” or “asymmetric reporting” because it is a precise description of the theoretical constructs that we model. “Conservatism” as typically construed (e.g., Watts, 2003) can be thought of as one type of asymmetric reporting system.
and contracting that could give rise to the asymmetry in reported earnings, and, in turn, whether this asymmetry is valuable even in the absence of agency conflicts. We develop a parsimonious model to study the endogenous choice of reporting system and whether that system entails asymmetric treatment of economic income. We extend the classic Capital Asset Pricing Model (CAPM) to a setting where a regulator requires each firm to report earnings prior to selling shares to investors. This setting corresponds to the requirement that firms must file financial statements with the SEC prior to being publicly traded. Although the regulator requires each firm to report earnings, the regulator does not specify the reporting system that maps economic income into reported earnings. Instead, a firm’s initial owners (i.e., the entrepreneur) choose the reporting system conditional on their expectation of economic income. We refer to the firm as being in a “good state” if expected income is high and in a “bad state” if expected income is low. The entrepreneur’s choice of reporting system is represented by the extent to which reported earnings reflect contemporaneous economic income (i.e., reporting precision), and we assume that more precise reporting requires greater verification, which comes at an increasing cost to the entrepreneur.

In modeling the entrepreneur’s choice of reporting precision, we deliberately abstract away from agency conflicts and contracting considerations. This ensures that our results are solely attributable to the valuation role of earnings rather than a stewardship role. In our model, the sole purpose of reported earnings is to facilitate valuation by investors. Nevertheless, and contrary to conventional wisdom, we show that under very mild conditions, reported earnings reflect contemporaneous economic income to a greater extent in the bad state: a property that we refer to as “asymmetric reporting in favor of bad news.” This property comports with the notion of asymmetric timeliness in the empirical literature—that reported earnings reflect contemporaneous bad news to a greater extent than good news.

We establish a necessary and sufficient condition for the optimal reporting system to be
asymmetric in favor of bad news. We characterize the condition in terms of four exogenous parameters: (i) the entrepreneur’s risk aversion; (ii) the firm’s systematic risk; (iii) the volatility of economic income; and (iv) the minimum level of reporting precision required by the regulator. We solve for the optimal reporting precisions in both states when this condition is met, and show that the magnitude of the asymmetry is decreasing in the expected economic income in the bad state and increasing in expected economic income in the good state. The intuition for these results is that because the entrepreneur is risk-averse, he derives greater benefit from a reduction in uncertainty in the bad state. As a consequence, the entrepreneur will be more willing to undertake costly verification in the bad state, and this differential level of verification results in reported earnings reflecting economic income to a greater extent in the bad state. We also show that mandated (i.e., exogenous) changes in accounting standards that increase the minimum level of reporting precision in both states reduce the asymmetry in reported earnings, which in turn reduces welfare. The intuition for this result is that the verification costs associated with reporting more precisely accrue in both states, whereas the benefits accrue primarily in the bad state.

Finally, we show that when reported earnings are asymmetric in favor of bad news, the firm’s systematic risk is also asymmetric: investors’ assessment of the firm’s covariance with the market portfolio is lower in bad states than in good states. The intuition for this result is that reporting is more precise in the bad state, and, in the CAPM, more precise reporting reduces investors’ assessments of the firm’s systematic risk (e.g., Lambert et al. 2007). Consequently, the firm’s systematic risk is lower (higher) in the bad (good) state. To the best of our knowledge this prediction is unique to our model. Prior literature on the role of asymmetric reporting in reducing agency conflicts suggests asymmetric reporting affects the mean of the firm’s cash flow; as such, it has exclusively an indirect effect on the firm’s cost of capital. In our analysis, asymmetric reporting has an additional direct effect that operates through investors’ assessment of the firm’s covariance with the market portfolio.
Empirical researchers can use this insight to distinguish among the various explanations for why reported earnings asymmetrically reflect economic income. For example, if the asymmetry is solely attributable to the stewardship role of earnings, then we would not expect any asymmetry in systematic risk.

Two key features of our model distinguish it from prior research. First, our model generalizes the reporting space to allow the entrepreneur’s choice of reporting precision to vary with the expectation of economic income. Prior work that focuses on state contingent reporting examines how expected economic income affects voluntary disclosure when managers have private information (e.g., Bagnoli and Watts, 2007). In contrast, we focus on how expected economic income affects the information conveyed by mandatory disclosure (e.g., earnings) in the absence of private information. In particular, we characterize the conditions under which reported earnings reflect economic income to a greater extent in the bad state. In this regard, our model links expected income with reporting precision, and in turn the firm’s systematic risk.

Second, we focus exclusively on the valuation role of earnings. Our focus on the valuation role of earnings is not intended to suggest that stewardship considerations play no role in the asymmetry in reported earnings, but rather that such considerations are not necessary to generate asymmetry. Perhaps the most striking aspect of our analysis is the simplicity of the model that is needed to generate our results. For example, our chief requirements are that the entrepreneur is risk averse and verification is costly. These requirements seem descriptive of a broad range of circumstances, suggesting that a reporting system that entails asymmetric treatment of bad news is optimal in a wide variety of settings, even in the absence of agency and contracting considerations. Our analysis suggests that the valuation role of earnings is an important driving force behind the asymmetry in reported earnings that has heretofore been overlooked.

By focusing on the valuation role of earnings, we contribute to the relatively small litera-
ture on the endogenous choice of reporting precision in the absence of agency and contracting motivations (e.g., Titman and Trueman, 1986; Stocken and Verrecchia, 2004; see Stocken, 2012 for a review). In Titman and Trueman (1986), the entrepreneur observes economic income with noise, and chooses a more (less) precise reporting system when his private information suggests that such income is high (low). In Stocken and Verrecchia (2004) a manager has private information that is not conveyed by the reporting system and chooses a less (more) precise reporting system when he has more (less) precise private information. Our model is very different from these studies in that it represents a generalization of the CAPM where the entrepreneur commits to a reporting system in advance of observing economic income. In our model, the entrepreneur does not have private information, and thus there is no information asymmetry between the entrepreneur and investors. This parsimonious setup makes transparent both the condition under which asymmetric reporting is optimal, as well as the effect of asymmetric reporting on asset prices. It also illustrates the generality of our results, insofar as information asymmetry, agency conflicts, and contracting considerations are likely to reinforce the benefit of asymmetric reporting.

Our paper makes two main contributions to the literature. First, we contribute to the literature on the endogenous design of financial reporting systems. While much of the empirical literature takes the properties of observed reporting systems as given and examines their effects on capital markets and contracts, it is important to understand the forces that shape the properties of reporting systems. Empirical analyses along these lines are inherently difficult because historical data are conditional on observed markets, institutions, and political processes (both past and present). Accordingly, the limited research that exists on optimal reporting systems is theoretical in nature and focuses almost exclusively on agency conflicts (e.g., Chen et al., 2007; Goex and Wagenhofer, 2009; Caskey and Laux, 2013).\(^2\)

\(^2\) Gigler et al. (2009), Gox and Wagenhofer (2009), and Caskey and Hughes (2012) focus on principal-agent problems as it relates to debt contracting, and Chen et al. (2007), Bertomeu et al. (2013), and Caskey and Laux (2013), and Gao (2013) focus on principal-agent problems as it relates to compensation
Our analysis illustrates that although agency conflicts and contracting considerations are undoubtedly important economic forces that shape financial reporting, they need not be the only forces. While one thought experiment is to consider what the reporting system would look like if it were shaped exclusively by the *stewardship* role of reporting (e.g., Kothari, Ramanna, and Skinner, 2011), another is to consider what the reporting system would look like if it were shaped exclusively by the *valuation* role of reporting (as in this paper). To the extent the resulting systems are similar, the (normative) predictions of the valuation and stewardship perspectives are not in conflict.

Second, we contribute to the literature on the relation between financial reporting and the cost of capital. Existing theoretical work in this literature takes reporting precision as given and examines whether and how it affects a firm’s cost of capital (e.g., Hughes et al., 2007; Lambert, et al., 2007). One important limitation of this work is that it ignores the choice of reporting precision. For example, these analyses suggest that absent some (unspecified) cost, firms will gravitate to the corner solution of reporting with the highest precision possible. We extend this literature by modeling a risk-averse entrepreneur’s endogenous choice of reporting precision, derive the conditions under which asymmetric reporting emerges as the optimum, and show that it is priced in the sense that it affects firms’ cost of capital. In particular, we show that when reported earnings more precisely reflect economic income (e.g., in bad states), firms have lower systematic risk and therefore lower cost of capital. In other words, we show that an important consequence of asymmetric reporting precision is that a firm’s Beta—and therefore its risk-premium—differs across states. Thus, our model can be viewed as a Conditional Capital Asset Pricing Model (or Conditional CAPM) in which firms’ Betas are endogenously influenced both by properties of the reporting system as well as the state. In this regard, our paper answers Cochrane’s (2013) call to endogenize investors’ assessments and earnings manipulation. While some studies examine limited liability explanations, these models are also premised on the existence of agency conflicts (e.g., Kwon et al., 2001).
of systematic risk rather than simply treat systematic risk as an exogenous parameter.

The remainder of the paper is as follows. Section 2 presents and solves the model and derives the necessary and sufficient conditions under which asymmetric reporting in favor of bad news is optimal. Section 3 discusses the comparative statics. Section 4 summarizes and concludes.

2 Analysis

2.1 Setting

In this section we characterize the reporting environment in our model. In particular, we extend the CAPM to a setting where a regulator requires firms to publicly report earnings before trade occurs, and the entrepreneur, as representative of the initial owners of the firm, commits to a reporting system that maps economic income into reported earnings. The entrepreneur’s choice of reporting system is represented by his choice of reporting precision, which determines the extent to which reported earnings reflect economic income. We assume that, although the entrepreneur does not observe realized economic income at the time he chooses reporting precision, he can condition his choice of reporting precision on expected economic income.

Figure 1 summarizes the timeline of our model. In period $t = 0$, in anticipation of having to report earnings, the entrepreneur commits to a reporting system that maps economic income into reported earnings. In period $t = 1$, investors observe reported earnings and subsequently trade shares of $J$ firms and a risk-free asset. In period $t = 2$, economic income is realized and paid to investors in the form of a liquidating dividend (the firm’s terminal cash flow).

To facilitate the discussion, henceforth we use a tilde, i.e., $\tilde{\cdot}$, to denote a random variable.
Formally, the reported earnings of firm $j$ are characterized as

$$
\tilde{r}_j = \tilde{V}_j + \tilde{\delta}_j,
$$

where $\tilde{r}_j$ is reported earnings, $\tilde{V}_j$ is economic income, and $\tilde{\delta}_j$ is noise in reported earnings.

We assume that $\tilde{V}_j$ is correlated across firms and follows a multivariate normal distribution with mean $E[\tilde{V}_j]$, variance $Var[\tilde{V}_j]$, and precision $\Pi_{\delta_j}$, where precision is the reciprocal of variance, and we represent the economic income of the market by $\tilde{V}_M$, where $\tilde{V}_M = \sum_{k=1}^{J} \tilde{V}_k$.

We assume that $\tilde{\delta}_j$ follows a multivariate normal distribution with mean 0, variance $Var[\tilde{\delta}_j]$, and precision $\Pi_{\delta_j}$, and that $\tilde{\delta}_j$ is independent of $\tilde{V}_j$.

A standard interpretation of $\Pi_{\delta_j}$ is that it represents the extent to which reported earnings reflect economic income: larger values of $\Pi_{\delta_j}$ indicate that the firm’s reported earnings, $\tilde{r}$, are a “less noisy”—or, equivalently, a “more precise”—measure of its economic income, $\tilde{V}$. As is standard in the literature, the entrepreneur’s choice of the reporting system is operationalized by the choice of reporting precision, $\Pi_{\delta_j}$. We assume that the regulator sets a non-zero minimum level of $\Pi_{\delta_j}$ (referred to as $\Pi_\delta$). The role of this assumption is fairly benign: it ensures that reported earnings contain at least some information, and rules out the corner solution that earnings are pure noise (i.e., $\Pi_{\delta_j} = 0$).

We allow for state-contingent reporting by assuming that although the entrepreneur does not know the firm’s realized economic income at the time he commits to a reporting system (i.e., $\Pi_{\delta_j}$), he can condition the choice of the reporting system on expected earnings, $E[\tilde{V}]$.

We refer to the firm as being in a good state if $E[\tilde{V}] = \mu_G$ and a bad state if $E[\tilde{V}] = \mu_B$, where $\mu_G > \mu_B$. If the entrepreneur’s chosen reporting precision is the same in both the good and bad states, then reported earnings reflect economic income to the same extent in both states, and reporting is said to be symmetric. If the entrepreneur’s chosen reporting precision is greater in the good state, then reported earnings reflect economic income to a
greater extent in the good state, and reporting is said to be *asymmetric in favor of good news*. Lastly, if the entrepreneur’s chosen reporting precision is greater in the bad state, then reported earnings reflect economic income to a greater extent in the bad state, and reporting is said to be *asymmetric in favor of bad news*. Note that our notion of “state” is firm-specific: firm $j$ could be in a good state while firm $k$ is in a bad state. We focus the discussion and analysis on a two state world (i.e. good state or bad state) for parsimony and to develop the intuition for our results. In subsequent analysis we discuss how our results extend to a continuum of states.

Finally, we make two key assumptions regarding the entrepreneur and his objective function. First, for the entrepreneur’s choice of reporting system to be meaningful, reporting must be costly. Accordingly, we assume that more precise reporting (i.e., increasing in the extent to which reported earnings reflect economic income) entails an explicit cost to the entrepreneur of $\frac{1}{2} \Pi^2_{\delta_j}$. One can think of the explicit cost of more precise reporting as coming from the costs of verification, with more precise reporting naturally requiring a greater degree of verification. Because the regulator requires a minimum reporting precision of $\Pi_\varphi$, the minimal cost to the entrepreneur of complying with the regulatory lower bound is $\frac{1}{2} \Pi^2_\varphi$.

Second, we assume the entrepreneur is risk-averse (i.e., has concave utility). Specifically, we assume that the entrepreneur has a negative exponential utility function represented by $U(x)$, where

$$U(x) = -\exp[-\rho \cdot x],$$

and $\rho > 0$ represents the entrepreneur’s constant, absolute risk aversion. The entrepreneur chooses $\Pi_{\delta_j}$ at time $t = 0$ to maximize his expected utility, subject to the fact that $\Pi_{\delta_j}$ entails a cost of $\frac{1}{2} \Pi^2_{\delta_j}$.

Let $P_j(\hat{r}_j)$ represent the price of firm $j$ as a function of reported earnings $\hat{r}_j$. We represent
the entrepreneur’s (of firm $j$) objective function as:

$$\max_{\Pi_{\delta_j}} \left\{ E[U(P_j(\tilde{r}_j))] - \frac{1}{2} \Pi_{\delta_j}^2 \right\}. \quad (1)$$

Note that in eqn. (1), the price of the firm, $P_j(\tilde{r}_j)$, is uncertain (random) because the entrepreneur does not know in advance the realization of reported earnings when he commits to a reporting system. In other words, the entrepreneur chooses the extent to which reported earnings reflect economic income through his choice of $\Pi_{\delta_j}$, but he has no influence over the realization of $\tilde{r}$; as a consequence, he has no influence over the realization of $P_j(\tilde{r}_j)$.

### 2.2 CAPM Pricing in the Absence of Reporting

Our next step is to discuss the derivation of asset prices and risk premia in the absence of reporting. These results serve as a useful benchmark with which to compare our subsequent results regarding exogenous reporting. In the absence of reporting, the entrepreneur and the reporting system play no role in the model, and $N$ identically informed investors simply trade shares of $J$ firms and a risk-free asset in period $t = 1$. In the absence of reporting, the setting corresponds identically to that of the CAPM, and prices are set accordingly. For $N$ identically informed investors each with constant absolute risk tolerance $\tau$ prices are given by

$$P_j = \frac{E[\tilde{V}_j] - \frac{1}{N\tau} Cov[\tilde{V}_j, \tilde{V}_M]}{1 + R_f}, \quad (2)$$

where: $P_j$ is the price of firm $j$ in period $t = 1$, $N\tau$ represents the economy’s total risk tolerance; $Cov[\tilde{V}_j, \tilde{V}_M]$ represents the covariance of the firm’s terminal cash flow with that of the market portfolio; and $R_f$ is the risk-free rate. The derivation of eqn. (2) is provided in Appendix A and mirrors that in Lambert et al. (2007). Following Lambert et al. (2007), throughout the paper we characterize prices in terms of investors’ risk tolerance rather than risk aversion. Risk tolerance is simply the reciprocal of risk aversion, and so which
claims in the paper can be found in Appendix B. As is standard in the literature, eqn. (2) makes clear that the firm is priced based on its expected terminal cash flow less a discount for systematic risk (i.e., \( \frac{1}{N^T} \text{Cov} \left[ \tilde{V}_j \cdot V_M \right] \)).

### 2.3 CAPM Pricing in the Presence of Endogenous Reporting

We now derive asset prices and risk premia in the context of the (endogenous) reporting environment described above. The entrepreneur’s objective is to maximize his expected utility at time \( t = 1 \) (when investors convene to trade shares) as a function of the price of firm \( j \) conditional on reported earnings, where the entrepreneur determines the extent to which reported earnings reflect economic income through his choice of \( \Pi_{\delta_j} \). Let \( P_j(r_j) \) represent the price of firm \( j \) conditional on the realization of reported earnings (i.e., conditional on \( \tilde{r}_j = r_j \)). Using standard results from Bayesian statistics in conjunction with eqn. (2), conditional on reported earnings \( r_j \), prices are given by

\[
P_j(r_j) = \frac{E[\tilde{V}_j] + \frac{\Pi_{\delta_j}}{\Pi_{\delta_j} + \Pi_j}(r_j - E[\tilde{V}_j]) - \frac{1}{N^T \Pi_{\delta_j} + \Pi_j} \text{Cov} \left[ \tilde{V}_j \cdot V_M \right]}{1 + R_f}.
\]

Note that prices in the presence of reporting (eqn. (3)) are similar to those in the absence of reporting (eqn. (2)), with two important differences. As before, the firm is priced based on its expected economic income less a discount for systematic risk, except that investors’ assessments of the firm’s expected economic income and systematic risk are now conditional on reported earnings. This implies that investors’ assessments are conditional on the reporting system that generated reported earnings.

There are two extra terms in the numerator. The first, \( E[\tilde{V}_j] + \frac{\Pi_{\delta_j}}{\Pi_{\delta_j} + \Pi_j}(r_j - E[\tilde{V}_j]) \), represents investors assessment of the firm’s economic income conditional on reported earn-
nings. The second, $\frac{1}{N^T \Pi_{\delta_j} + \Pi_{\delta_j}} \text{Cov} \left[ \bar{V}_j \cdot \bar{V}_M \right]$, represents investors’ assessment of the firm’s systematic risk conditional on reported earnings. The additional term $\frac{\Pi_{\delta_j}}{\Pi_{\delta_j} + \Pi_{\delta_j}}$ captures how an entrepreneur’s choice of reporting precision, $\Pi_{\delta_j}$, affects investors’ assessment of the firm’s systematic risk: more precise reporting (i.e., higher $\Pi_{\delta_j}$) implies a lower assessment of systematic risk (see also, Lambert et al., 2007). In this regard, investors’ assessments of systematic risk conditional on observing reported earnings represents an amalgamation of information risk, embodied by $\frac{\Pi_{\delta_j}}{\Pi_{\delta_j} + \Pi_{\delta_j}}$, and intrinsic systematic risk embodied by $\frac{1}{N^T \text{Cov} \left[ \bar{V}_j \cdot \bar{V}_M \right]}$.

It is important to emphasize the distinction between the intrinsic systematic risk of the firm and investors’ assessment of the systematic risk conditional on observing reported earnings. In our model, intrinsic systematic risk is exogenous and represents the primitive correlation between terminal cash flows, whereas investors’ assessment of systematic risk is endogenous and depends on the entrepreneur’s (endogenous) choice of reporting precision, $\Pi_{\delta_j}$. Thus, by endogenizing the choice of reporting precision, we effectively endogenize the firm’s systematic risk (as it manifests in prices). To the best of our knowledge this is the first model to endogenize both reporting precision and the firm’s systematic risk in the context of the CAPM. We solve for the entrepreneur’s optimal choice of reporting precision in the next section.

### 2.4 Optimal precision choice

As is standard in any optimization problem, in choosing an optimal level of reporting precision, $\Pi_{\delta_j}$, the entrepreneur trades off the marginal benefit of an increase in reporting precision against the marginal cost. In our model, there are two costs and one benefit.

The first cost involves the explicit cost to the entrepreneur of reporting earnings with precision $\Pi_{\delta_j}$: we interpret this cost as the cost of effort. This cost operates through $-\frac{1}{2} \Pi_{\delta_j}^2$ and is straightforward: the marginal effect of an increase in $\Pi_{\delta_j}$ entails a marginal cost of
\[ \Pi_{\delta_j} \] to the entrepreneur.

The second cost is more subtle and operates through \[ E[U(P_j(\tilde{r}_j))] \]. More precise reporting creates an additional risk for the entrepreneur, and because the entrepreneur is risk averse, an increase in risk decreases his expected utility. To explain this effect, consider the extreme case where the entrepreneur reports nothing to investors (this violates the lower bound of \( \Pi_{\varphi} \), but we abstract from this for a moment). In this circumstance, investors assess the firm’s economic income to be \[ E[\tilde{V}_j] \]. Alternatively, suppose that the entrepreneur reports \( r_j \). In this circumstance, investors assess the firm’s economic income to be \[ E[\tilde{V}_j] + \frac{\Pi_{\delta_j}}{\Pi_{\varphi} + \Pi_{\delta_j}} (r_j - E[\tilde{V}_j]) \], where the latter could be higher or lower than \( E[\tilde{V}_j] \) depending upon whether the realization of reported earnings, \( r_j \), is greater or less than \( E[\tilde{V}_j] \). Thus, from the entrepreneur’s perspective, reporting earnings is tantamount to participating in a lottery (\( r_j \) is unknown at the time the entrepreneur chooses reporting precision), and because the entrepreneur is risk averse, he abhors lotteries: the entrepreneur prefers the “sure thing” of \( E[\tilde{V}_j] \). The marginal effect of an increase in the “lottery” as a consequence of an increase in \( \Pi_{\delta_j} \) is not as straightforward to characterize mathematically as the prior cost because it involves the computation of \( E[U(P_j(\tilde{r}_j))] \), and we defer that computation to the Appendix; nonetheless, it represents a cost that is separate and apart from the cost of effort (i.e., separate from \( -\frac{1}{2} \Pi_{\delta_j}^2 \)).

The benefit is that as the entrepreneur commits to report more precisely, he reduces investors’ assessment of the firm’s systematic risk: in effect, an increase in reporting precision reduces \[ \frac{1}{N \tau} \frac{\Pi_{\delta_j}}{\Pi_{\varphi} + \Pi_{\delta_j}} Cov[\tilde{V}_j \cdot \tilde{V}_M] \]. For example, the marginal effect of an increase in \( \Pi_{\delta_j} \) on \[ \frac{1}{N \tau} \frac{\Pi_{\delta_j}}{\Pi_{\varphi} + \Pi_{\delta_j}} Cov[\tilde{V}_j \cdot \tilde{V}_M] \] yields a marginal benefit (a reduction in systematic risk) of \[ \frac{1}{N \tau} \frac{\Pi_{\delta_j}}{(\Pi_{\varphi} + \Pi_{\delta_j})^2} Cov[\tilde{V}_j \cdot \tilde{V}_M] \]. A standard interpretation of this benefit is that it represents a reduction in “information risk”; as such, it provides a favorable boost to the price of firm \( j \).

To summarize, in choosing an optimal level of reporting precision the entrepreneur trades off the marginal benefit of reducing investors’ assessments of the firm’s systematic risk against
the marginal costs of exerting effort and participating in a lottery. Henceforth we refer to the marginal benefit of reducing investors’ assessments of the firm’s systematic risk net of the marginal cost of participating in a lottery (both of which operates through \( E [U (P_j (\tilde{r}_j))] \)) as the *net marginal benefit* of an increase in \( \Pi_{\delta_j} \). When the net marginal benefit is negative, it does not justify incurring the cost of effort, and the entrepreneur will choose the lowest level of reporting precision allowed by the regulator, \( \Pi_\varnothing \). When the net marginal benefit is positive, the entrepreneur will choose a \( \Pi_{\delta_j} \) such that the net marginal benefit that operates through \( E [U (P_j (\tilde{r}_j))] \) equals the marginal cost of effort that operates through \( -\frac{1}{2} \Pi_{\delta_j}^2 \) (subject to the regulatory lower bound \( \Pi_\varnothing \)).

Let \( \Pi^*_\delta_j \) represent the entrepreneur’s optimal choice of precision. In the Appendix we prove the following result.

**Lemma.** *One can characterize the entrepreneur’s optimal choice of precision, \( \Pi^*_\delta_j \), as*

\[
\Pi^*_\delta_j = \max \left\{ \frac{d}{d \Pi_{\delta_j}} \left[ E [U (P_j (\tilde{r}_j))] \right] \left| \Pi_{\delta_j}^*, \Pi_\varnothing \right. \right\}.^4
\]

This characterization presumes that the optimization problem is well behaved insofar as it yields a well-defined solution: we discuss this issue in the Appendix in conjunction with proving this result.

### 2.5 Reporting precision in good versus bad states

At time \( t = 0 \) before the entrepreneur of firm \( j \) chooses reporting precision, he comes to know the state of the firm (i.e., \( E \left[ \hat{V}_j \right] = \mu_G \) or \( E \left[ \hat{V}_j \right] = \mu_B \)). Here there are two possible assumptions one could make about investors. The first assumption is that at time \( t = 0 \) investors are also aware of the state: in effect, *the state of firm \( j \) is common knowledge.*

---

[^4]: In words, the expression \( \frac{d}{d \Pi_{\delta_j}} |_{\Pi_{\delta_j}^*} E [U (P_j (\tilde{r}_j))] |_{\Pi_{\delta_j}^*, \Pi_\varnothing} \) means: the derivative of \( E [U (P_j (\tilde{r}_j))] \) with respect to \( \Pi_{\delta_j} \), as evaluated at \( \Pi_{\delta_j} = \Pi^*_\delta_j \).
This removes any information asymmetry between the entrepreneur and investors. The second assumption is that at time $t = 0$ investors do not know the state, but because investors have rational expectations, they can make an inference about the state based on the entrepreneur’s actions or choices when he provides his report. As it relates to the analysis below, both assumptions yield qualitatively similar results. Thus, we start out assuming that at time $t = 0$ investors also know whether $E[\tilde{V}_j]$ equals $\mu_G$ or $\mu_B$, and then later discuss any change in our results as a consequence of assuming that investors do not know the state of the firm. In the Appendix we prove the following result.

**Proposition.** The entrepreneur’s (unique) choice of reporting precision in the bad state is never less than the choice in the good state, and typically higher when the firm’s intrinsic systematic risk is greater than one-half the entrepreneur’s risk aversion times the variance of the firm’s economic income:

$$\frac{1}{N} Cov \left[ \tilde{V}_j, \tilde{V}_M \right] > \frac{1}{2} \rho Var \left[ \tilde{V}_j \right].$$

(4)

Note our deliberate use of the word “typically” in the statement of the Proposition, as well as other results that we report below. When eqn. (4) holds, the entrepreneur’s preferred choice of reporting precision is higher in the bad state, and reported earnings reflect economic income to a greater extent in the bad state (as compared to the good state). These preferred choices, however, are subject to the requirement that they exceed the regulatory lower bound of $\Pi_\varphi$. When the choice of reporting precision in the bad state does not exceed $\Pi_\varphi$, then the entrepreneur is forced to choose $\Pi_\varphi$ in both the good and bad states (because the bad state choice is always higher than the good state choice). Thus, “typically” relates to the requirement that the choice of $\Pi_{\delta_j}$ in the bad state exceeds the regulatory lower bound of $\Pi_\varphi$.

The intuition underlying the Proposition is that when the inequality in eqn. (4) does
not hold, the net marginal benefit of an increase in $\Pi_{\delta_j}$ on $E[U(P_j(\bar{r}_j))]$ is non-positive. Consequently, and irrespective of whether firm $j$ is in the good or bad state, the entrepreneur chooses the lowest level of precision allowed, $\Pi_{\delta}$. Because the level of precision is identical in both states, the choice of reporting precision in the bad state is no lower than the choice in the good state.

Alternatively, when the inequality in eqn. (4) holds, the net marginal benefit of an increase in $\Pi_{\delta_j}$ on $E[U(P_j(\bar{r}_j))]$ is positive. In the good state, however, the net marginal benefit of an increase in $\Pi_{\delta_j}$ is lower than in the bad state. The reason why it is lower is that by virtue of being risk averse, the entrepreneur’s utility is concave; concavity implies that the marginal benefit of an increase in price in the good state is less than a commensurate increase in price in the bad state. An alternative way to state this intuition is that concavity implies that the entrepreneur becomes increasingly more sated as outcomes become increasingly better: satiation implies that the entrepreneur derives less marginal utility from an increase in price (and hence less marginal utility from a reduction in uncertainty). Because the net marginal benefit of an increase in $\Pi_{\delta_j}$ is lower in the good state than in the bad state, the optimal choice of reporting precision is higher in the bad state than in the good state (when eqn. (4) holds, and assuming that the former exceeds the regulatory lower bound $\Pi_{\delta}$).

It is important to emphasize the role risk aversion plays in ensuring that the entrepreneur’s choice of reporting precision is asymmetric in favor of bad news. Risk aversion implies the marginal benefit of an increase in price (reduction in uncertainty) is greater in the bad state. If the entrepreneur were risk neutral, the marginal benefit of an increase in price would be the same in both states and consequently reporting would be symmetric: the entrepreneur’s choice of reporting precision would be the same in both states, and reporting precision would be higher than if the entrepreneur were risk averse. This emphasizes the two key assumptions of our model: a risk averse entrepreneur and costly reporting.
2.6 Additional considerations

To explain why one achieves qualitatively similar results if investors do not know the state of firm \( j \) at time \( t = 0 \), suppose this is the case. When the inequality in eqn. (4) holds, investors with rational expectations will conjecture the firm’s state based on the entrepreneur’s choice of reporting precision, and in equilibrium this conjecture will hold. Thus, effectively rational investors will always know (or can infer) the state.

When the inequality in eqn. (4) does not hold, let us assume that the \textit{ex ante} probability of being in the good (bad) state is \( q \) \((1 - q)\). Note that this uncertainty is idiosyncratic to firm \( j \) and therefore does not manifest in investors' assessments of the firm’s systematic risk. Here, investors will conjecture that \( E \left[ \tilde{V}_j \right] = q \mu_G + (1 - q) \mu_B \) and this conjecture will also hold because when the inequality in eqn. (4) does not hold, the entrepreneur chooses the lowest level of reporting precision allowed (i.e., \( \Psi_\phi \)) in both states. In short, when eqn. (4) does not hold, the price of the firm when investors are uninformed about the state is different than when investors are informed. In particular, in the former case, \( E \left[ \tilde{V}_j \right] \) in eqn. (3) is a weighted average of both \( \mu_G \) and \( \mu_B \). Nevertheless, because the Proposition does not depend on \( E \left[ \tilde{V}_j \right] \), the statement of the Proposition continues to hold.

3 Comparative Statics

In this section we discuss three comparative statics. The first comparative static speaks to the asymmetry in the extent to which reported earnings reflect economic income in the two states—specifically how the magnitude of the asymmetry is affected by expected economic income. The second comparative static speaks to how properties of the financial reporting system endogenously determine firms’ systematic risk—specifically how the asymmetry in reported earnings endogenously affects investors’ assessment of the firm’s covariance with
the market portfolio, i.e., “Beta.”. The third comparative static speaks to the role of the regulator—specifically the welfare implications of altering the minimum level of reporting precision required by the regulator.

### 3.1 Magnitude of Reporting Asymmetry

In the Appendix, we prove the following as a corollary to the Proposition.

**Corollary 1.** Assuming eqn. (4) holds, the difference in the optimal reporting precision in the bad state and the good state for firm \( j \) typically increases as either: 1) \( \mu_B \) decreases; 2) \( \mu_G \) increases; or 3) \( \mu_G \) increases and \( \mu_B \) decreases simultaneously.

The intuition that underlies Corollary 1 is that a decrease (increase) in the firm’s expected economic income in the bad (good) state increases (decreases) the net marginal benefit from an increase in \( \Pi_{\delta_j} \) in the bad (good) state. Consequently, the entrepreneur chooses a higher (lower) level of \( \Pi_{\delta_j} \) in the bad (good) state. This, in turn, increases the magnitude of the asymmetry in the extent to which reported earnings reflect economic income in the two states.

An implication of Corollary 1 is that it is not necessary to bifurcate the analysis into just two states: good and bad. Corollary 1 implies that one could posit a continuum of states ranging from very bad to very good as a consequence of \( E\left[\bar{V}_j\right] \), and, provided that the inequality in eqn. (4) holds, the entrepreneur’s choice of reporting precision will decrease as firm \( j \)’s expected economic income, \( E\left[\bar{V}_j\right] \), increases, *ceteris paribus*. We formally prove this intuition in the following corollary.\(^5\)

**Corollary 2.** Assuming eqn. (4) holds, the entrepreneur’s choice of reporting precision

\(^5\) Note that the proof to Corollary 2 is a generalization of the proof to Corollary 1, just as the statement of Corollary 2 is a generalization of the statement to Corollary 1. As such, the proof to Corollary 2 renders the proof to Corollary 1 redundant.
typically decreases as firm j’s expected economic income increases (ceteris paribus).

3.2 Asymmetric reporting and systematic risk

In the context of the results reported above, there are two points worth emphasizing, especially as it relates to future empirical work. First, regarding the endogenous determinants of systematic risk, recall from eqn. (3) that investors’ assessment of the firm’s systematic risk conditional on reported earnings takes the form, $\frac{1}{N_t} \Pi_{\theta_j} \left[ \frac{\Pi_{\theta_j}}{\Pi_{\theta_j} + \Pi_{\theta}} Cov \left( \tilde{V}_j \cdot \tilde{V}_\theta \right) \right]$. In this regard, investors’ assessments of the firm’s covariance with the market portfolio $\frac{\Pi_{\theta_j}}{\Pi_{\theta_j} + \Pi_{\theta}} Cov \left( \tilde{V}_j \cdot \tilde{V}_M \right)$, represents an amalgamation of (endogenous) information risk, embodied by $\frac{\Pi_{\theta_j}}{\Pi_{\theta_j} + \Pi_{\theta}}$, and (exogenous) intrinsic systematic risk embodied by $Cov \left( \tilde{V}_j \cdot \tilde{V}_M \right)$. As a consequence, more precise reporting (i.e., higher $\Pi_{\theta_j}$) reduces investors’ assessments of the firm’s covariance with the market portfolio. Thus, when reported earnings reflect economic income to a greater extent in the bad state (i.e., eqn. (4) holds), investors’ assessment of the firm’s systematic risk is lower in the bad state (as compared to the good state). Although this result follows directly from the Proposition, we nonetheless codify it as follows.

Observation 1. Assuming eqn. (4) holds, conditional on observing earnings, investors’ assessments of firm j’s systematic risk is typically lower in the bad state and higher in the good state.

Second, an important implication of our approach to endogenizing both reporting precision and systematic risk is that it links expected economic income and priced risk. Because the firm’s expected economic income determines reporting precision, and, in turn, reporting precision determines investors’ assessment of the firm’s systematic risk, our analysis suggests that higher levels of expected economic income will be associated with higher levels of systematic risk. Although this observation follows directly from Observation 1 in conjunction
with Corollary 2, we nonetheless codify it as follows.

**Observation 2.** *Assuming eqn. (4) holds, conditional on observing earnings, investors’ assessments of firm j’s systematic risk typically increases as the firm’s expected economic increases.*

Importantly, prior literature on the role of asymmetric reporting in reducing agency costs and facilitating efficient contracting suggests asymmetric reporting will manifest in the mean of the firm’s cash flow. In contrast, in our model (premised exclusively on the valuation role of earnings) asymmetric reporting manifests in the firm’s covariance with the market portfolio. To the best of our knowledge this prediction is unique to our model. Thus, empirical researchers can use the above “observations” to distinguish among the various explanations for why reported earnings asymmetrically reflect economic income. If the asymmetry is solely attributable to the stewardship role of earnings, then we would not expect Observations 1 and 2 to manifest in the data.

### 3.3 Welfare Analysis

As a final comparative static, we discuss welfare implications of an exogenous change in the minimum level of reporting precision required by the regulator, $\Pi_\otimes$. One can think of an exogenous change in the minimum level of reporting precision required by the regulator as a the adoption of new accounting standards that alter the extent to which reported earnings reflect economic income *without regard to the state*. For example, in the absence of manipulation and illiquidity, the application of fair value accounting (to the entire balance sheet), should result in reported earnings that symmetrically reflect economic income. Of course, any welfare analysis regarding regulatory action has to carefully distinguish between the welfare of the entrepreneur, as a representation of the current ownership of firm $j$, and the welfare of investors.
First, consider the circumstance where the minimum level of reporting precision required by the regulator *increases*. From the perspective of the entrepreneur, an increase in $\Pi_\sigma$ is welfare debilitating. When the inequality in eqn. (4) *does not hold*, the entrepreneur will choose the lowest level of reporting precision allowed by the regulator, $\Pi_\sigma$. In this circumstance, increasing $\Pi_\sigma$ is bad for the entrepreneur because it requires that he incur greater costs than he would otherwise prefer (i.e., recall that the cost to the entrepreneur of complying with the regulatory lower bound is $\frac{1}{2}\Pi_\sigma^2$). When the inequality in eqn. (4) *holds*, there exists a unique optimal reporting precision where the net marginal benefits equal the cost of effort. To the extent that the new higher level of $\Pi_\sigma$ exceeds this optimal level, however, the entrepreneur is forced to choose $\Pi_\sigma$. In this circumstance, increasing $\Pi_\sigma$ is also bad for the entrepreneur: the lowest level of reporting precision allowed by the regulator exceeds the optimal level of reporting precision, such that the costs associated with $\Pi_\sigma$ exceed the net marginal benefits. In short, an increase in the minimum level of reporting precision required by the regulator cannot make the entrepreneur better off, and may make him worse off.

Now consider the welfare of investors. In the context of the CAPM, the term “investors” is really a reference to potential investors: individuals who have no current ownership stake in the firm, but nonetheless compete against other individuals (who also have no ownership stake) to purchase the firm’s shares. In other words, in the CAPM (and in our model) prices are set based on competition among individuals who have no ownership stakes. The net payoff to each investor in firm $j$ is simply the (realized) liquidating dividend less the (realized) share price, $V_j - P_j$. Thus, investors would prefer to pay as little as possible to purchase shares. In our model, more precise reporting increases price by reducing systematic risk. Although this works in favor of the entrepreneur (as a representation of the current ownership of the firm), it works against potential investors who are competing for shares of
the firm.\footnote{To elaborate on this point, if investors could hold the price of firm $j$ fixed, they would prefer more precise reporting because this serves to reduce systematic risk and investors are risk averse. The problem is that investors cannot collude to hold prices fixed when reporting precision increases: competition among them bids up the price of firm $j$ when reporting precision increases. From a utility (or welfare) perspective, the \textit{net} effect of an increase in price and a reduction in systematic risk leaves investors worse off.} Thus, to the extent an increase in $\Pi_\sigma$ forces the entrepreneur to increase reporting precision, investors are made worse off. In short, as was the case for the entrepreneur, an increase in the minimum level of reporting precision required by the regulator cannot make investors better off, and may make them worse off. Taken together, this implies that an increase in the minimum level of reporting precision can not make anyone better off, and potentially makes everyone in the economy worse off.

Next, consider the circumstance where the minimum level of reporting precision required by the regulator \textit{decreases}. The same logic and intuition applies, but in the opposite direction. A decrease in $\Pi_\sigma$ relaxes the regulatory constraint on the entrepreneur. If the optimal level of reporting precision was previously above $\Pi_\sigma$, and thus the regulatory constrain \textit{was not binding}, then decreasing $\Pi_\sigma$ and relaxing the constraint has no effect. If the optimal level of reporting precision was previously below $\Pi_\sigma$, and thus the regulatory constraint \textit{was binding}, then decreasing $\Pi_\sigma$ and relaxing the constraint makes everyone better off. Thus, a decrease in the minimum level of reporting precision can not make anyone worse off, and potentially makes everyone in the economy better off. We codify the discussion above as a corollary (where the discussion serves as the proof).

\textbf{Corollary 3.} \textit{An increase in the minimum level of reporting precision required by the regulator, $\Pi_\sigma$, never increases welfare and potentially makes everyone in the economy worse off. A decrease in the minimum level of reporting precision required by the regulator never decreases welfare and potentially makes everyone in the economy better off.}

In Corollary 3, it is important to emphasize the role played by the assumption that more precise reporting entails an explicit cost. In the absence of an explicit cost, the entrepreneur
would move would move to the “corner solution” of the highest level of reporting precision achievable when the inequality in eqn. (4) holds. Thus, the only reason why an increase in the minimum level of reporting precision required by the regulator is welfare debilitating for the entrepreneur is because increased reporting precision entails an explicit cost. In this regard, the cost of capital is not synonymous with welfare. While an increase in the minimum level of reporting precision required by the regulator may reduce systematic risk (and hence reduce the cost of capital), because reporting is costly, this reduction may not be welfare enhancing.

4 Conclusion

One of the most studied—and controversial—properties of financial reporting is the asymmetry in the extent to which reported earnings reflect contemporaneous economic income. Existing economic theory suggests that the asymmetry in reported earnings is exclusively the result of the stewardship role of earnings: the role of earnings in mitigating agency problems and facilitating efficient contracting. In this paper, we examine whether there are additional economic forces beyond the stewardship role of earnings that could give rise to the asymmetry in reported earnings, and, in turn, whether this asymmetry is valuable even in the absence of agency conflicts.

We develop a parsimonious model to study the endogenous choice of reporting system and whether that system entails asymmetric treatment of economic income. We extend the classic Capital Asset Pricing Model (CAPM) to a setting where a regulator requires that firms report earnings prior to selling shares to investors but does not specify the reporting system that maps economic income into reported earnings. Instead, a risk-averse entrepreneur (as representative of the firms’ initial owners) endogenously chooses the extent to which reported earnings reflect economic income as a function of expect economic income (i.e., the “state”).
In modeling the reporting system we deliberately abstract away from agency conflicts and contracting considerations. This ensures that our results are solely attributable to the valuation role of earnings. Nevertheless, and despite the absence of agency problems and contracts, we show that under very mild conditions an entrepreneur will endogenously choose a reporting system that entails asymmetric treatment of economic income: a reporting system where reported earnings reflect economic income to a greater extent in the bad state (as compared to the good state).

In addition, we show that the choice of reporting system endogenously affects investors’ assessments of firms’ systematic risk. Specifically, we show that when the optimal reporting system entails asymmetric treatment of economic income, the firm’s systematic risk is also asymmetric. Firms where reported earnings reflect economic income to a greater extent in bad states have lower (higher) covariance with the market portfolio in bad (good) states.

Our model and the accompanying results provide a number of novel insights for empirical research. First, we demonstrate that asymmetry in reported earnings can exist under even in the absence of agency and contracting considerations. Thus, our analysis illustrates that although agency conflicts and contracting considerations are undoubtedly important economic forces that shape financial reporting, they need not be the only forces. The valuation role of earnings alone, can result in an earnings number that asymmetrically reflects economic income. As a consequence, our model predicts asymmetry in reported earnings even in firms without debt (i.e. no debt contracts) and where agency problems are minimal.

Second, prior literature on the role of asymmetric reporting in reducing agency conflicts suggests asymmetric reporting affects the mean of the firm’s cash flow; as such, it has exclusively an indirect effect on the firm’s cost of capital. In our analysis, asymmetric reporting has an additional direct effect that operates through investors’ assessment of the firm’s covariance with the market portfolio. To the best of our knowledge, ours is the first model to predict that asymmetric reporting has an endogenous effect on systematic risk and
firms’ cost of capital. Empirical researchers that examine the capital market consequences of financial reporting can test whether asymmetric reporting manifests in firms’ betas in the spirit of a Conditional CAPM, and potentially use this prediction to distinguish among the various explanations for the asymmetry in reported earnings. For example, if the asymmetry is solely attributable to the stewardship role of earnings, then we would not observe asymmetry in systematic risk.

Another important implication of our model stems from the intuition that underlies our results. The key tension in our model relates to risk-averse entrepreneurs balancing the explicit cost of providing an earnings number that more precisely reflects economic income against the benefit of a reduction in systematic risk. Explicitly articulating this tradeoff highlights that a regulatory increase in the minimum level of required reporting precision may in fact, be welfare debilitating. While an increase in the minimum level of reporting precision may reduce systematic risk and the cost of capital, because reporting is costly, this reduction may not be welfare enhancing either for the firm’s current or future owners. In this regard, our results emphasize that the cost of capital is not synonymous with welfare.
References


Appendix A

Derivation of eqn. (??).

Consider an economy with \( J \) firms, indexed by the subscript \( j = 1, 2, ..., J \), and a risk-free bond. We assume that the risk-free rate of return is \( R_f \); that is, an investment of $1 in the risk-free bond yields a return of $1 + R_f$. Let \( \tilde{V}_j \) and \( P_j \) represent the uncertain cash flows of firm \( j \) and the market equilibrium price of firm \( j \), respectively. Along with the \( J \) firms, we introduce a perfectly competitive market for firm shares comprised of \( N \) investors, indexed by the subscript \( i = 1, 2, ..., N \), where \( N \) is large. Let \( U(c) \) represent investor \( i \)'s utility preference for an amount of cash \( c \). Each investor has a negative exponential utility function: that is, \( U(c) \) is defined by \( U(c) = -\exp \left( -\frac{1}{\tau} c \right) \), where \( \tau > 0 \) describes each investor’s (constant) tolerance for risk.

Now consider the market price for firm \( j \) that prevails in a perfectly competitive market in which \( N \) investors compete to hold shares in each firm, as well as a risk-free bond. Let \( \tilde{D}_i = \{D_{i1}, D_{i2}, ..., D_{ij}, ..., D_{iJ}\} \) represent the \( 1 \times J \) vector of investor \( i \)'s demand for ownership in \( J \) firms, where \( D_{ij} \) represents investor \( i \)'s demand for firm \( j \) expressed as percentage of the total firm; let \( \tilde{D}_i^* = \{D_{i1}^*, D_{i2}^*, ..., D_{ij}^*, ..., D_{iJ}^*\} \) represent her vector of endowed ownership in firms, where \( D_{ij}^* \) represents her endowment in firm \( j \) expressed as a percentage of the total firm; and let \( \tilde{P} = \{P_1, P_2, ..., P_j, ..., P_J\} \) represent the vector of firm prices, where once again \( P_j \) represents the price of firm \( j \). Let \( B_i \) and \( B_i^* \) represent investor \( i \)'s demand for a risk-free bond and her endowment in bonds, respectively. Each investor solves

\[
\max_{D_i, B_i} E \left[ -\exp \left( -\frac{1}{\tau} \{\tilde{D}_i \{\tilde{V}_1, \tilde{V}_2, ..., \tilde{V}_j\}' + (1 + R_f) B_i\} \right) \right] \tag{A1}
\]

subject to the budget constraint

\[
\tilde{D}_i \tilde{P}' + B_i = \tilde{D}_i^* \tilde{P}' + B_i^*.
\]
Taking the expectation of eqn. (A1) and substituting in the relation $B_i = \bar{D}_i \bar{P}' + B_i^* - \bar{D}_i \bar{P}'$ yields the following expression

$$
\max_{D_i, B_i} \left\{ - \exp\left[ - \frac{1}{\tau} (\bar{D}_i \{ E \left[ \tilde{V}_1 \right] - (1 + R_f) P_1, E \left[ \tilde{V}_2 \right] - (1 + R_f) P_2, \ldots, E \left[ \tilde{V}_J \right] - (1 + R_f) P_J \} + (1 + R_f) \bar{D}_i P' + (1 + R_f) B_i^* + \frac{1}{2 \tau^2} D_i \Delta D_i^\prime \right] \right\},
$$

(A2)

where $\Lambda$ is an $J \times J$ covariance matrix whose $s, t$-th term is $\text{Cov} \left[ \tilde{V}_s, \tilde{V}_t \right]$.

The first-order condition that maximizes eqn. (A2) with respect to $D_{ij}$ reduces to

$$
0 = E \left[ \tilde{V}_j \right] - (1 + R_f) P_j - \frac{1}{\tau} \sum_{k=1}^J D_{jk} \text{Cov} \left[ \tilde{V}_j, \tilde{V}_k \right].
$$

(A3)

Because collectively investors have claims to the cash flows of the entire firm, for each $k$ it must be the case that $\sum_{i=1}^N D_{ik} = 1$. Thus, summing over both sides of eqn. (A3) with respect to $i$ yields

$$
0 = N \left( E \left[ \tilde{V}_j \right] - (1 + R_f) P_j \right) - \frac{1}{\tau} \sum_{i=1}^N \sum_{k=1}^J D_{ik} \text{Cov} \left[ \tilde{V}_j, \tilde{V}_k \right],
$$

or

$$
0 = N \left( E \left[ \tilde{V}_j \right] - (1 + R_f) P_j \right) - \frac{1}{\tau} \sum_{k=1}^J \text{Cov} \left[ \tilde{V}_j, \tilde{V}_k \right].
$$

This, in turn, implies that the price for firm $j$ is given by

$$
P_j = \frac{E \left[ \tilde{V}_j \right] - \frac{1}{N \tau} \text{Cov} \left[ \tilde{V}_j, \sum_{k=1}^J \tilde{V}_k \right]}{1 + R_f}.
$$

Q.E.D.
Appendix B

Proof of the Lemma. The report $\tilde{r}_j$ has a normal distribution with mean $E\left[\tilde{V}_j\right]$ and variance $\frac{1}{\pi_{v_j}} + \frac{1}{\pi_{s_j}} = \frac{\pi_{v_j} + \pi_{s_j}}{\pi_{v_j} \pi_{s_j}}$. Thus, when

$$P_j(r_j) = \frac{E\left[\tilde{V}_j\right] + \frac{\pi_{s_j}}{\pi_{v_j} + \pi_{s_j}} \left( r_j - E\left[\tilde{V}_j\right] \right) - \frac{\pi_{v_j}}{\pi_{v_j} + \pi_{s_j}} \frac{1}{N \tau} Cov \left[\tilde{V}_j \cdot \tilde{V}_M\right]}{1 + R_f},$$

using standard results relating to moment generating functions, $E\left[U \left( P_j (\tilde{r}_j) \right)\right]$ reduces to

$$E\left[U \left( P_j (\tilde{r}_j) \right)\right] = -\sqrt{\frac{\pi_{v_j} \pi_{s_j}}{2\pi \left( \pi_{v_j} + \pi_{s_j} \right)}} \times \int_{-\infty}^{\infty} \exp \left[ -\frac{\rho}{1 + R_f} \left( E\left[\tilde{V}_j\right] + \frac{\pi_{s_j}}{\pi_{v_j} + \pi_{s_j}} \left( r_j - E\left[\tilde{V}_j\right] \right) - \frac{\pi_{v_j}}{\pi_{v_j} + \pi_{s_j}} \frac{1}{N \tau} Cov \left[\tilde{V}_j \cdot \tilde{V}_M\right] \right) \right] \times \exp \left[ -\frac{1}{2} \left( \frac{\pi_{v_j} \pi_{s_j}}{\pi_{v_j} + \pi_{s_j}} \right) (r_j - E\left[\tilde{V}_j\right]) \right] dr_j = -\exp\left[ -\frac{\rho}{1 + R_f} \left( E\left[\tilde{V}_j\right] - \frac{1}{2} \frac{\rho}{1 + R_f \pi_{v_j} \left( \pi_{v_j} + \pi_{s_j} \right)} \pi_{s_j} - \frac{\pi_{v_j}}{\pi_{v_j} + \pi_{s_j}} \frac{1}{N \tau} Cov \left[\tilde{V}_j \cdot \tilde{V}_M\right] \right) \right].$$

This, in turn, implies that

$$\frac{d}{d\pi_{s_j}} E\left[U \left( P_j (\tilde{r}_j) \right)\right] = \rho \frac{1}{1 + R_f} \left( \frac{\pi_{v_j}}{\left( \pi_{v_j} + \pi_{s_j} \right)^2} \left( \frac{1}{N \tau} Cov \left[\tilde{V}_j \cdot \tilde{V}_M\right] - \frac{\rho \pi_{v_j}^{-1}}{2 \left( 1 + R_f \right)} \right) \right) \times \exp\left[ -\frac{\rho}{1 + R_f} \left( E\left[\tilde{V}_j\right] - \frac{1}{2} \frac{\rho}{1 + R_f \pi_{v_j} \left( \pi_{v_j} + \pi_{s_j} \right)} \pi_{s_j} - \frac{\pi_{v_j}}{\pi_{v_j} + \pi_{s_j}} \frac{1}{N \tau} Cov \left[\tilde{V}_j \cdot \tilde{V}_M\right] \right) \right].$$

Recall that $\pi_{v_j}^{-1} = Var \left[\tilde{V}_j\right]$. Thus, if

$$\frac{1}{N \tau} Cov \left[\tilde{V}_j \cdot \tilde{V}_M\right] - \frac{\rho Var \left[\tilde{V}_j\right]}{2 \left( 1 + R_f \right)} \leq 0,$$

(5)
then the entrepreneur chooses as his optimal level of reporting precision \( \Pi_{\delta_j}^* = \Pi_{\sigma} \) because the net marginal benefit that operates through \( E[U(P_j(\tilde{r}_j))] \) is non-positive. Alternatively, suppose

\[
\frac{1}{N\tau} Cov \left[ \tilde{V}_j \cdot \tilde{V}_M \right] - \frac{\rho \text{Var} \left[ \tilde{V}_j \right]}{2 (1 + R_f)} > 0. \tag{6}
\]

Recall that the marginal cost that operates through \(-\frac{1}{2}\Pi_{\delta_j}^2 \) is \(-\Pi_{\delta_j} \). Thus, when eqn. (6) holds the entrepreneur chooses as his optimal \( \Pi_{\delta_j}^* \)

\[
\Pi_{\delta_j}^* = \frac{d}{d\Pi_{\delta_j}} E[U(P_j(\tilde{r}_j))] \bigg|_{\Pi_{\delta_j}^*}. \tag{7}
\]

Eqn. (7) does not provide a closed-form expression for \( \Pi_{\delta_j}^* \), but the solution is nonetheless unique because when eqn. (6) holds, the optimization problem in eqn. (1) is concave. Of course, it may be the fact that the \( \Pi_{\delta_j}^* \) computed in eqn. (7) is less-than-or-equal-to \( \Pi_{\sigma} \), in which case the entrepreneur chooses \( \Pi_{\delta_j}^* = \Pi_{\sigma} \). Thus, one can characterize the choice of \( \Pi_{\delta_j}^* \) as

\[
\Pi_{\delta_j}^* = \max \left\{ \frac{d}{d\Pi_{\delta_j}} E[U(P_j(\tilde{r}_j))] \bigg|_{\Pi_{\delta_j}^*}, \Pi_{\sigma} \right\}. 
\]

Q.E.D.

**Proof of the Proposition.** To prove this result, one is only required to show that if the \( \Pi_{\delta_j}^* \) computed in eqn. (7) is greater than \( \Pi_{\sigma} \), then \( \Pi_{\delta_j}^* \) is higher in the bad-state than the good-state. Let \( \Pi_{\delta_j}^{G*} \) solve \( \Pi_{\delta_j}^{G*} = \frac{d}{d\Pi_{\delta_j}} E[U(P_j(\tilde{r}_j, \mu_G))] \bigg|_{\Pi_{\delta_j}^{G*}} \) in the good-state (i.e., \( E[\tilde{V}_j] = \mu_G \)), and \( \Pi_{\delta_j}^{B*} \) solve \( \Pi_{\delta_j}^{B*} = \frac{d}{d\Pi_{\delta_j}} E[U(P_j(\tilde{r}_j, \mu_B))] \bigg|_{\Pi_{\delta_j}^{B*}} \) in the bad-state (i.e., \( E[\tilde{V}_j] = \mu_B \)). Our proof is by contradiction. In other words, we assume \( \Pi_{\delta_j}^{B*} < \Pi_{\delta_j}^{G*} \) and show this to be false when, in addition, \( \mu_B < \mu_G \). When eqn. (6) holds, \( E[U(P_j(\tilde{r}_j))] \) is concave in \( \Pi_{\delta_j} \). Thus,
when \( \Pi_{\delta_j}^{B*} < \Pi_{\delta_j}^{G*} \) and \( \mu_B < \mu_G \)

\[
\Pi_{\delta_j}^{G*} = \frac{d}{d \Pi_{\delta_j}} E \left[ U \left( P_j (\tilde{r}_j, \mu_G) \right) \right] |_{\Pi_{\delta_j}^{G*}} \leq \frac{d}{d \Pi_{\delta_j}} E \left[ U \left( P_j (\tilde{r}_j, \mu_G) \right) \right] |_{\Pi_{\delta_j}^{B*}}
\]

\[
= \frac{\rho}{1 + R_f} \left( \frac{\Pi_{v_j}}{\left( \Pi_{v_j} + \Pi_{\delta_j}^{B*} \right)^2} \left( \frac{1}{N \tau} \text{Cov} \left[ \tilde{V}_j \cdot \tilde{V}_M \right] - \frac{\Pi_{v_j}^{-1}}{2 (1 + R_f)} \right) \right)
\times \exp \left[ - \frac{\rho}{1 + R_f} \left( \mu_B - \frac{1}{2} \frac{\rho}{1 + R_f} \frac{\Pi_{\delta_j}^{B*}}{\Pi_{v_j} + \Pi_{\delta_j}^{B*}} - \frac{\Pi_{v_j}^{-1}}{2 (1 + R_f)} \right) \right]
\]

\[
< \frac{\rho}{1 + R_f} \left( \frac{\Pi_{v_j}}{\left( \Pi_{v_j} + \Pi_{\delta_j}^{B*} \right)^2} \left( \frac{1}{N \tau} \text{Cov} \left[ \tilde{V}_j \cdot \tilde{V}_M \right] - \frac{\Pi_{v_j}^{-1}}{2 (1 + R_f)} \right) \right)
\times \exp \left[ - \frac{\rho}{1 + R_f} \left( \mu_B - \frac{1}{2} \frac{\rho}{1 + R_f} \frac{\Pi_{\delta_j}^{B*}}{\Pi_{v_j} + \Pi_{\delta_j}^{B*}} - \frac{\Pi_{v_j}^{-1}}{2 (1 + R_f)} \right) \right]
\]

\[
= \frac{d}{d \Pi_{\delta_j}} E \left[ U \left( P_j (\tilde{r}_j, \mu_B) \right) \right] |_{\Pi_{\delta_j}^{B*}} = \Pi_{\delta_j}^{B*},
\]

where the first inequality follows from \( \Pi_{\delta_j}^{B*} < \Pi_{\delta_j}^{G*} \) and concavity of \( E \left[ U \left( P_j (\tilde{r}_j) \right) \right] \) and the second inequality follows from \( \mu_B < \mu_G \); collectively these inequalities contradict the claim that \( \Pi_{\delta_j}^{B*} < \Pi_{\delta_j}^{G*} \). Q.E.D.

**Proof of the Corollary 2.** Define \( G \left( \Pi_{\delta_j}; \mu_G \right) = \Pi_{\delta_j} - \frac{d}{d \Pi_{\delta_j}} E \left[ U \left( P_j (\tilde{r}_j, \mu_G) \right) \right] \) and \( B \left( \Pi_{\delta_j}; \mu_B \right) = \Pi_{\delta_j} - \frac{d}{d \Pi_{\delta_j}} E \left[ U \left( P_j (\tilde{r}_j, \mu_B) \right) \right] \). Assuming eqn. (4), let \( \Pi_{\delta_j}^{G*} \) solve \( G \left( \Pi_{\delta_j}^{G*}; \mu_G \right) = 0 \) and let \( \Pi_{\delta_j}^{B*} \) solve \( B \left( \Pi_{\delta_j}^{B*}; \mu_B \right) = 0 \). It is a straightforward exercise to show that both

\[
\frac{\partial \Pi_{\delta_j}^{G*}}{\partial \mu_G} = -\frac{\partial G \left( \Pi_{\delta_j}^{G*}, \mu_G \right)}{\partial \mu_G} < 0 \quad \text{and} \quad \frac{\partial \Pi_{\delta_j}^{B*}}{\partial \mu_B} = -\frac{\partial B \left( \Pi_{\delta_j}^{B*}, \mu_B \right)}{\partial \mu_B} < 0.
\]

Thus, an increase in \( \mu_G \) implies a decrease in \( \Pi_{\delta_j}^{G*} \), whereas a decrease in \( \mu_B \) implies an increase in \( \Pi_{\delta_j}^{B*} \). Q.E.D.