Cost Allocation For Capital Budgeting

(PRELIMINARY AND INCOMPLETE)

Tim Baldenius∗
Sunil Dutta†
Stefan Reichelstein‡

October 2005

∗Graduate School of Business, Columbia University, tb171@columbia.edu
†Haas School of Business, University of California at Berkeley, dutta@haas.berkeley.edu
‡Graduate School of Business, Stanford University, reichelstein@stanford.edu
1 Introduction

Capital budgeting decisions are frequently complex because they affect multiple entities such as departments or divisions within a firm. Decision externalities may arise because the divisional projects are mutually exclusive due to limited investment budgets. Alternatively, firms may be in a position to acquire common assets which can be of use to multiple divisions at the same time. The coordination problem then is to determine whether the aggregated individual benefits justify the common investment expenditure.\footnote{A recent example faced by a California semiconductor manufacturer was that several product lines (profit centers) were using a common fabrication facility. The product line managers regularly encouraged the firm’s central office to upgrade the manufacturing equipment. The firm’s central office was skeptical to undertake these upgrades partly because the firm did not have a charging mechanism which properly allocated the cost of additional investments among the different users.}

The management literature provides only scant evidence regarding firms’ actual capital budgeting practices; see for example Kaplan and Atkinson (1998) and Taggart (1988). One approach seems to be that firms set hurdle rates which must be met by individual projects in order to receive funding. In this context, Poterba and Summers (1995) provide evidence that these hurdle rates frequently exceed a firm’s actual cost of capital by a substantial margin. To understand this discrepancy, it would seem essential to have either a theoretical framework or anecdotal evidence suggesting how divisional capital budgets translate into subsequent hurdle rates.

Our largely normative perspective in this paper is to examine a class of capital budgeting mechanisms wherein the divisions initially report private information and a central planner commits to a decision rule contingent on the reports received. To provide proper incentives, the central planner can impose cost charges which depend on the initial divisional reports. These cost charges may be comprised of both depreciation and capital charges. We confine attention to mechanisms that satisfy the usual accounting convention that the sum of all depreciation charges across time periods and across divisions be equal to the initial investment expenditure. Our main objective is to characterize the incentive provisions that can be implemented by this
We first consider a scenario in which the firm must pick one out of \( n \) mutually exclusive divisional projects. Under the so-called \textit{Competitive Hurdle Rate Mechanism} (CHR), the hurdle rate is unaffected by the report of the winning division whose project is funded. In the special case where the divisions are ex-ante identical, the competitive hurdle rate reduces to the second highest internal rate of return. The CHR mechanism also specifies a depreciation schedule which achieves intertemporal matching so that the value of a particular project is reflected in a time consistent manner in the divisional performance measure.\(^2\) As a consequence, the CHR mechanism is strongly incentive compatible in the sense that divisional managers have a dominant strategy incentive to report their information truthfully regardless of their intertemporal preferences, i.e., regardless of the weights (such as discount factors or bonus coefficients) they attach to different time periods. In fact, the CHR mechanism is shown to be the only mechanism that is \textit{satisfactory}, i.e., strongly incentive compatible and resulting in the funding of the highest positive NPV project.

To understand the uniqueness of the CHR mechanism, which is effectively a multi-period version of the second price auction mechanism, it is instructive to consider a setting in which divisional managers are equally patient and discount future payoffs at the firm's cost of capital. To obtain dominant strategy incentives, the mechanism must then amount to a Groves scheme (Groves 1973). Since we postulate that a division will not be charged unless its project is funded, the class of feasible Groves mechanisms reduces to the so-called \textit{Pivot} mechanism: divisions are charged if and only if their report is pivotal in that it changes the resource allocation decision.

The public choice literature has demonstrated the impossibility of finding dominant strategy mechanisms that attain efficient outcomes and achieve balanced transfers among the participants.\(^3\) In fact, a central feature of the Pivot mechanism (or

\(^2\)Invoking earlier results on goal congruent performance measures (e.g., Rogerson 1997, Reichelstein 1997), we find that such intertemporal matching can be achieved by the so-called relative benefit depreciation rule in conjunction with the residual income performance measure.

\(^3\)In an accounting context, this has been noted by Pfaff (1994).
equivalently, the second price auction for an indivisible private good) is that it runs a surplus. In our multiperiod framework, we find that the CHR mechanism is nominally balanced in the sense that the sum of the depreciation charges is equal to the initial acquisition cost of the winning division. In real terms, however, the CHR mechanism runs a surplus because the competitive hurdle rate will always exceed the firm’s cost of capital.

Beginning with the work of Antle and Eppen (1985), the capital budgeting literature has emphasized that, with a single agent, agency costs result in capital rationing. In essence, the principal is better off foregoing marginally profitable projects in order to economize on the agent’s informational rents. One way to implement such capital rationing is to raise the hurdle rate. With multiple competing divisions, we find that the CHR mechanism can be adapted to a second-best mechanism. Like in most adverse selection models, optimal second-best mechanisms are obtained by imputing the agents’ virtual rather than their true profitability parameters. As intuition would suggest, more competition, as measured by the number of agents vying for funds, always lowers the principal’s agency costs. On the other hand, the implications of the agency problem for the competitive hurdle rate are ambiguous. For instance, if the severity of all divisional moral hazard problems goes up uniformly by the same factor (e.g., due to a deterioration of the firm’s management control system), the competitive hurdle rate may not change at all.\textsuperscript{4}

When the capital budgeting problem concerns the acquisition of a shared assets to which all divisions have access, our findings are in several respects “dual” to the ones obtained in connection with exclusive assets. Invoking similar criteria as before, we identify the so-called \textit{Pay-the-Minimum-Necessary} (PMN) mechanism as the unique satisfactory capital budgeting mechanism. In present value terms, this mechanism charges every division the critical value required so that the investment in the joint asset just breaks even. To implement this rule in a time consistent fashion, the

\textsuperscript{4}This finding is related to the result that under certain conditions the second price auction is an optimal mechanism; see, for example, Myerson (1981). In particular, optimality obtains if the bidder with the highest intrinsic value for the object also has the highest virtual value.
divisions are assigned shares of the joint asset in proportion to their critical valuations. Furthermore, the capital charge rate under the PMN mechanism is given by the internal rate of return corresponding to the critical valuations.

In contrast to our findings for exclusive assets, the PMN mechanism is not a multiperiod version of the Pivot mechanism. In particular, the capital charge rate under the PMN mechanism is below the firm’s cost of capital. As a consequence, the PMN mechanism runs a deficit in real terms: while the sum of all depreciation charges across time periods and divisions is equal to the acquisition cost of the joint asset, the present value of all depreciation and capital charges is less than the initial acquisition cost. Intuitively, the need for such subsidization arises because with shared assets the divisions no longer compete but instead exert a positive externality upon one another.

In the presence of hidden action problems, the PMN mechanism can also be adapted to a second-best contracting mechanism. Now, however, the capital charge rate will increase unambiguously with higher divisional agency costs, essentially because the investment decision is driven by the sum of the virtual valuations (rather than their maximum, as in the case of exclusive assets). We conclude that for shared assets the resulting agency-adjusted capital charge rate can either be above or below the firm’s cost of capital. The need for incentive compatible reporting tends to push the capital charge rate below the firm’s cost of capital, while agency costs tend to push in the opposite direction.

In terms of prior research on managerial incentives for investment decisions, our analysis builds directly on the earlier work by Rogerson (1997) and Reichelstein (1997). In their one-agent settings without moral hazard, the capital charge rate must be set equal to the firm’s cost of capital in order to obtain goal congruent performance measures. In contrast, Dutta and Reichelstein (2002) and Christensen, Feltham and Wu (2002) demonstrate that the capital charge rate should be adjusted to reflect agency costs resulting either from informational rents or managerial risk aversion.\(^5\) In a setting with multiple divisions and symmetric information, Wei (2004)

\(^5\)Baldenius (2003) and Dutta (2003) link the choice of the capital charge rate to the possibility of
shows that suitable fixed cost allocations can alleviate divisional underinvestment problems. Bareket (2001) and Mohnen (2004) consider cost allocation and revenue recognition rules in a single-agent setting where one manager has to pick one from among mutually exclusive projects. Finally, Bernardo, Cai and Luo (2004) consider a single-period model where two managers seek funding for projects and each can take an action that affects the project outcome of the respective other manager.

The remainder of the paper is organized as follows. Section 2 examines satisfactory capital budgeting mechanisms for both exclusive and shared assets. Hidden action problems and second-best contracting mechanisms are the subject of Section 3. We conclude in Section 4.

2 Satisfactory Capital Budgeting Mechanisms

We examine mechanisms for coordinating investment decisions in a firm with \( n \) divisions and a central office. At some initial date, the firm can acquire a capital asset with a useful life of \( T \) years. The firm’s central office faces an incentive and coordination problem because the division managers have private information that is necessary in evaluating the profitability of alternative decisions. To study the most common forms of interdivisional coordination problems, we consider two scenarios: (i) \textit{exclusive} and (ii) \textit{shared} assets.

In the case of exclusive assets, each division is assumed to have one initial investment opportunity. Each divisional project, if undertaken, generates future cash flows only for that particular division. Due to exogenous (and unmodeled) constraints, the firm can finance at most one of the \( n \) divisional projects. In this setting, the projects are “private goods” and the divisions compete for scarce investment capital. In contrast, in the alternative setting of shared assets, the firm has access to a common investment project, which, if undertaken, generates future revenues for \textit{all} \( n \) divisions. In that sense, the common investment project is a “public good” which

empire benefits that the managers may derive from new investments. In Baldenius and Ziv (2003) the capital charge rate is adjusted to reflect tax consequences of investment decisions.
benefits all divisions.

Without loss of generality, each division’s status quo operating cash flow (in the absence of any new investment) is normalized to zero. When division $i$ has access to the payoffs of the new investment (exclusive or shared), its periodic operating cash flows take the form:

$$c_{it} = x_{it} \cdot \theta_i.$$  

(1)

Here, $\theta_i \in \Theta_i = [\theta_i, \bar{\theta}_i]$ with $\bar{\theta}_i \geq 0$ denotes the profitability parameter of division $i$. The vector of profitability parameters of all $n$ divisions will be denoted by $\theta = (\theta_1, ..., \theta_n)$. Adopting standard notation, we let $\theta_{-i} \equiv \theta \setminus \{\theta_i\}$ describe the profitability profile of all divisions other than $i$. While the divisional profitability parameter $\theta_i$ is assumed to be known only to manager $i$, the intertemporal distribution of future cash flows, as represented by the vector $X_i = (x_{1i}, ..., x_{iT}) \in \mathbb{R}_+^T$, is commonly known.\(^7\)

The firm’s cost of capital is given by $r \geq 0$ with $\gamma = 1/(1 + r)$ representing the discount factor. If division $i$ has access to the asset, the present value of its cash flows is

$$PV_i(\theta_i) = \sum_{t=1}^{T} \gamma^t \cdot x_{it} \cdot \theta_i = \Gamma \cdot X_i \cdot \theta_i,$$

where $\Gamma \equiv (\gamma, ..., \gamma^T)$.

Initially, we do not specify an agency problem with moral hazard and instead focus on the choice of goal congruent performance measures for divisional managers.\(^8\)

Following earlier literature in this area, a performance measure is said to be goal congruent if it induces managers to make decisions that maximize the present value

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\(^6\)If the effective useful life of a certain project is $\tau < T$ periods, we simply set $x_{it} = 0$ for all $\tau \leq t \leq T$.

\(^7\)Thus the firm’s central management is assumed to know not only the useful life of the asset but also the intertemporal pattern of cash flows, e.g., they may on average be uniform across time periods. Similar assumptions are made in Rogerson (1997), Reichelstein (1997), Bareket (2001), Baldenius and Ziv (2003), Wei (2004) and Dutta and Reichelstein (2005).

\(^8\)We will introduce a formal agency model in Section 3.
of firm-wide cash flows. In our search for goal congruent performance indicators, we confine attention to accounting-based metrics of the form

$$\pi_{it} = Inc_{it} - \hat{r} \cdot A_{i,t-1},$$

(2)

where $A_{it}$ denotes book value of division $i$’s asset at the end of period $t$, and $\hat{r}$ is a capital charge rate applied to the beginning book value. We note that the class of performance measures in (2) encompasses the most common accounting performance metrics such as income, residual income, and operating cash flow. The net asset value at the end of period $t$ is given by

$$A_{it} = A_{i,t-1} - d_{it} \cdot A_{i0},$$

where $d_{it}$ denotes the period-$t$ depreciation percentage for division $i$ in period $t$ and $A_{i0}$ represents the initial asset value assigned to division $i$. Given comprehensive income measurement, income in period $t$ is calculated as:

$$Inc_{it} = c_{it} + A_{it} - A_{i,t-1} = c_{it} - d_{it} \cdot A_{i0}.$$ 

To create managerial incentives, the central office has two principal instruments: the capital charge rate $\hat{r}$ and the depreciation rules $\{d_{it}\}_{t=1}^T$. In our search for alternative capital budgeting mechanisms, we impose throughout the nominal “tidiness” condition that the sum of all depreciation charges across agents and across time periods is equal to the amount initially invested.

Following earlier work on goal congruent performance measures, we allow for the possibility that a division manager may attach different weights to future outcomes than the principal who is interested in the present value of future cash flows. Let $u_i = (u_{1i}, \ldots, u_{ti})$ denote non-negative weights that manager $i$ attaches to the sequence of performance measures $\pi_i = (\pi_{1i}, \ldots, \pi_{Ti})$. At the beginning of period 1, manager $i$’s objective function can thus be written as $\sum_{t=1}^T u_{it} \cdot E[\pi_{it}]$. One can think of the weights $u_i$ as reflecting a manager’s discount factor as well as the bonus coefficients.
attached to the periodic performance measures. We require performance measures to have a “robustness” property such that the desired incentives hold even if the intertemporal weights $u_i$ can vary freely in some open neighborhood in $\mathcal{V}_i \subset \mathbb{R}_+^T$. For instance, $\mathcal{V}_i$ could be a neighborhood around $(u \cdot \gamma, u \cdot \gamma^2, ..., u \cdot \gamma^T)$ for some constant bonus coefficient $u$.

To specify the goal congruence requirement formally, let $\pi_{it}(\tilde{\theta}_i, \tilde{\theta}_{-i} | \theta_i)$ denote manager $i$’s period $t$ performance measure when his true type is $\theta_i$, but he reports $\tilde{\theta}_i$ and the other $n-1$ managers report $\tilde{\theta}_{-i}$. We say that a performance measure satisfies strong incentive compatibility if:

$$\sum_{t=1}^{T} u_{it} \cdot \pi_{it}(\theta_i, \theta_{-i} | \theta_i) \geq \sum_{t=1}^{T} u_{it} \cdot \pi_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i), \quad \text{for all } i, \theta_i, \tilde{\theta}_i, \tilde{\theta}_{-i}, u_i \in \mathcal{V}_i. \quad (3)$$

Our requirement of strong incentive compatibility amounts to dominant-strategy incentives for truthful reporting in a setting where the designer is also uncertain about managers’ intertemporal preferences. This notion of strong incentive compatibility therefore combines aspects of the classic public choice literature on dominant-strategy implementation (e.g., Groves 1973, Green and Laffont 1979) with the more recent literature on goal congruence for multiperiod decision problems (e.g., Rogerson 1997, Reichelstein 1997).

2.1 Exclusive Assets

This section examines capital budgeting mechanisms for a setting in which the divisions compete for scarce investment capital. In particular, we suppose that the firm can fund at most one of the divisional projects because of capital constraints. The interpretation is that, while the capital cost of financing a single project is $r$, this

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\[9\]In order to assess the robustness of a particular mechanism one would like $\mathcal{V}_i$ to be as large as possible, e.g., the entire $\mathbb{R}_+^T$. On the other hand, any necessity result pointing to the uniqueness of a particular mechanism becomes more powerful if derived with reference to a smaller set $\mathcal{V}_i$. For now, it is useful to view $u_{it}$ as a summary statistic for the manager’s discount factor and the bonus coefficients in his compensation function. In Section 3, the coefficients $u_{it}$ will emerge endogenously from the underlying agency problem.
cost would become prohibitively large if all available projects were to be financed. Division $i$’s investment opportunity requires an initial cash outlay of $b_i$. The net present value (NPV) of division $i$’s investment project is then given by

$$NPV_i(\theta_i) \equiv PV_i(\theta_i) - b_i.$$  

Division $i$’s internal rate of return is denoted by $r^o_i(\theta_i)$, that is,

$$\sum_{t=1}^{T} (1 + r^o_i(\theta_i))^{-t} \cdot x_{it} \cdot \theta_i - b_i = 0.$$  

The internal rate of return $r^o_i(\theta_i)$ exists and is unique because $\theta_i > 0$ and $x_{it} \geq 0$. The first-best investment decision rule calls for selecting the highest NPV project provided that NPV is positive; i.e., provided the corresponding internal rate of return exceeds the firm’s cost of capital $r$.

We use the indicator variable $I_i \in \{0, 1\}$ to denote whether division $i$’s investment project is undertaken. Given our assumption that the firm can fund at most one project, a feasible investment policy must satisfy $\sum_{i=1}^{n} I_i \leq 1$. In response to managers’ reports about their projects’ profitability parameters $\theta_i$, a capital budgeting mechanism specifies:

- An investment decision rule, $I_i : \Theta \to \{0, 1\}$ such that $\sum_{i=1}^{n} I_i(\theta) \leq 1$. Division $i$’s beginning balance equals $A_{i0} = b_i \cdot I_i(\theta)$;
- A capital charge rate, $\hat{r} : \Theta \to (-1, \infty)$;
- A depreciation schedule, $d_i : \Theta \to \mathbb{R}^T$, satisfying $\sum_{t=1}^{T} d_{it}(\theta) = 1$ if $I_i(\theta) = 1$, and $d_{it}(\theta) \equiv 0$ if $I_i(\theta) = 0$.

Note that the class of mechanisms we consider imposes a “no-play-no-pay” condition: a division is charged only if its project receives funding. A capital budgeting mechanism for exclusive assets is said to be satisfactory if it (i) selects the highest positive

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10Alternatively, suppose the divisions are directly competing to carry out a project that needs to be performed only once within the firm.

11Because manager $i$ will be burdened with investment costs (depreciation and capital charges) only if $I_i = 1$ (because $A_{i0} = 0$ whenever $I_i = 0$), it is without loss of generality to set a uniform, firmwide capital charge rate $\hat{r}(\cdot)$ and to set all depreciation percentages equal to zero for $I_i = 0$. 

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NPV project, and (ii) is strongly incentive compatible for all $n$ managers. It will be notationally convenient to denote the highest positive NPV project by

$$NPV^1(\theta) \equiv \max_i \{NPV_i(\theta), 0\}.$$ 

Efficient project selection requires that $I_i^*(\theta) = 1$ only if $NPV^1(\theta) = NPV_i(\theta)$. We also define

$$NPV^1(\theta_{-i}) = \max_{j \neq i} \{NPV_j(\theta), 0\}.$$ 

For a given $\theta_{-i}$, let $\theta_i^*(\theta_{-i})$ denote the lowest value of division $i$’s profitability parameter for which its project is at least as profitable as any of the other $n-1$ projects. That is,

$$NPV_i(\theta_i^*(\theta_{-i})) = NPV^1(\theta_{-i}).$$

Put differently, at $\theta_i = \theta_i^*(\theta_{-i})$, division $i$’s NPV would just tie with the highest NPV of the remaining divisions, provided that value is positive.\(^{13}\) Note that this definition of $\theta_i^*(\theta_{-i})$ implies that $I_i^*(\theta) = 1$ only if $\theta_i \geq \theta_i^*(\theta_{-i})$.\(^{14}\)

For the class of capital budgeting mechanisms we consider, the performance measure in (2) takes the form:

$$\pi_{it} = (x_{it} \cdot \theta_i - z_{it} \cdot b_i) \cdot I_i,$$

where

$$z_{it} = d_{it} + \hat{r} \cdot \left(1 - \sum_{r=1}^{t-1} d_{ir}\right)$$

denotes the sum of depreciation and interest charges in period $t$. Following Rogerson (1997), we refer to $\{z_{it}\}_{t=1}^T$ as an intertemporal cost allocation scheme. Earlier studies on goal congruence have observed that for any given $\hat{r}$ there is a one-to-one mapping

\(^{12}\)Note that $NPV^1(\theta_{-i}) \neq NPV^1(\theta)$ if, and only if, $NPV_i(\theta_i) = NPV^1(\theta)$.

\(^{13}\)While $\theta_i^*(\theta_{-i})$ depends on all distributional cash flow parameters ($X_1, ..., X_n$) and on the cash outlay amounts ($b_1, ..., b_n$), this dependence is ignored in the notation to avoid clutter.

\(^{14}\)To rule out uninteresting corner solutions and to ensure the critical profitability type $\theta_i^*(\theta_{-i})$ is always well defined for all $\theta_{-i}$, we assume throughout that, for all $i$, $NPV_i(\bar{\theta}_i) = L$ and $NPV_i(\bar{\theta}_i) = H$ for some $H > L$.
between depreciation and intertemporal cost allocation schemes. In particular, there exists a unique intertemporal cost allocation such that

$$z_{it} \cdot b_i = \frac{x_{it}}{\sum_{\tau=1}^{T}(1+\hat{r})^{-\tau} \cdot x_{i\tau}} \cdot b_i.$$  \hspace{1cm} (6)

The importance of this so-called relative benefit cost allocation rule is that the resulting residual income measure in each period is proportional to division $i$’s NPV when evaluated at the discount rate $\hat{r}$. Thus, for a capital budgeting problem with a single agent, the principal can achieve goal congruence by setting $\hat{r} = r$. The unique depreciation schedule that gives rise to the cost allocation charges in (6) is referred to as the relative benefit depreciation rule.\(^{15}\) For future reference, it is useful to observe that if the discount rate $\hat{r}$ is set equal to $r^\iota(\theta_i)$, then the relative benefit cost allocations in (6) satisfy the equation

$$\sum_{\tau=1}^{T}(1+r^\iota(\theta_i))^{-\tau} \cdot x_{i\tau} \cdot b_i = x_{it} \cdot \theta_i,$$ \hspace{1cm} (7)

since, by definition, the NPV evaluated at the internal rate of return is zero and, by construction of the relative benefit rule, $\pi_{it}$ in (4) must then be zero in all periods.

The following capital budgeting mechanism will be called the Competitive Hurdle Rate (CHR) mechanism:

(i) $I_i(\theta) = I^*_i(\theta);$  
(ii) $\hat{r} = r^*(\theta) \equiv r^\iota(\theta^\iota_i(\theta_{-i}))$ if $I^*_i(\theta) = 1;$  
(iii) $\{d_{it}(r^*(\theta))\}_{t=1}^{T}$ is the relative benefit depreciation schedule based on the competitive hurdle rate $r^*(\theta)$.

The competitive hurdle rate is the internal rate of return of the winning division evaluated at the critical profitability type, $\theta^\iota_i(\theta_{-i})$. Division $i$’s report does not affect

\(^{15}\)It is well known that this rule amounts to the annuity depreciation method in case the $x_{it}$’s are constant across time periods. On the other hand, if cash flows were to decline geometrically at a rate of $\alpha$ over an infinite horizon, relative benefit depreciation would amount to a declining balance method with decline factor $1-\alpha$.  

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the competitive hurdle rate provided the order of the NPVs remains unchanged. The 
CHR mechanism simplifies considerably when all divisions are ex-ante identical with 
regard to $X_i$ and $b_i$. The rank order of the NPVs then is identical to the rank order of 
the internal rates of return and therefore the competitive hurdle rate for the winning 
division, say division $i$, simply equals the second-highest internal rate of return: $r^*_i = 
\max_{j\neq i}\{r^*_j(\theta_j), r\}$. In this context, it is also readily seen that the CHR mechanism can 
be viewed as a delegation mechanism: divisions report their internal rates of return 
and decide on their own whether to proceed with their divisional projects with the 
capital charge rate set at the second-highest reported internal rate of return.

The relative benefit depreciation schedule in (iii) ensures a proper intertemporal 
matching of the initial investment expenditure with future cash returns. In particular, 
it follows directly from (7) that:

$$
z_{it} \cdot b_i = \frac{x_{it}}{\sum_{\tau=1}^{T} (1 + r^\theta(\theta_{\tau}))^{-\tau} \cdot x_{i\tau}} \cdot b_i = x_{it} \cdot \theta^*_i(\theta_{-i}). \tag{8}$$

Thus the performance measure under the competitive hurdle rate mechanism in (4) 
reduces to

$$
\pi_{it} = x_{it} \cdot [\theta_i - \theta^*_i(\theta_{-i})] \cdot I_i, \tag{9}
$$

making it a dominant strategy for each manager to report truthfully. This incentive 
holds not only in aggregate over the entire planning horizon but also on a period-
by-period basis, as equation (9) shows. Specifically, we find that for the winning 
division:\footnote{We denote $NPV_i(\theta_i | \tilde{r}) \equiv \sum_{t=1}^{T}(1 + \tilde{r})^{-t} \cdot x_{it} \cdot \theta_i - b_i$ as the divisional NPV measured with some generic discount rate $\tilde{r}$, and continue to write $NPV_i(\theta_i) \equiv NPV_i(\theta_i | r)$.}

$$
\pi_{it}(\theta) = \frac{x_{it}}{\Gamma^*(\theta) \cdot X_i} \cdot NPV_i(\theta_i | r^*(\theta)),
$$

where $\pi_{it}(\theta) \equiv \pi_{it}(\theta_i, \theta_{-i} | \theta_i)$, $NPV_i(\theta_i | r^*(\theta)) \equiv \Gamma^*(\theta) \cdot X_i \cdot \theta_i - b_i$, and $\Gamma^*(\theta) \equiv ((1 + r^*(\theta))^{-1}, \ldots, (1 + r^*(\theta))^{-T})$. We conclude that the competitive hurdle rate mechanism is a satisfactory mechanism. The following result below shows that this mechanism is in fact the only satisfactory capital budgeting mechanism that meets the 
requirements of strong incentive compatibility.
Proposition 1 For exclusive assets, the competitive hurdle rate (CHR) mechanism is the unique satisfactory capital budgeting mechanism.\footnote{All proofs are in Appendix A.}

To understand why the CHR mechanism is the only satisfactory mechanism, it is useful first to consider settings in which each manager discounts future at the firm’s discount rate, i.e, $u_{it} = \gamma^t$ for each $i$ and $t$. In this special case, intertemporal matching is of no importance and it suffices to ensure dominant strategy incentive compatibility over the entire planning horizon (rather than on a period-by-period basis). We note that every (dominant-strategy) incentive compatible mechanism must be a Groves scheme. (e.g., Green and Laffont 1979). A Groves scheme achieves dominant-strategy reporting incentives by ensuring that each manager’s payoff is the same as the social surplus up to a constant. Since manager $i$ already internalizes his own surplus ($\text{NPV}_i$), his transfer must be equal to an “externality payment,” i.e., the maximized surplus of the remaining $n - 1$ divisions. That is, the present value of intertemporal cost charges must equal

$$
\sum_{t=1}^{T} \gamma^t \cdot z_{it}(\theta) = - \sum_{j \neq i} \text{NPV}_j(\theta_j) \cdot I^*_j(\theta) + h_i(\theta_{-i}) \tag{10}
$$

where $h_i(\cdot)$ is an arbitrary function of $\theta_{-i}$. The total surplus of the remaining $n - 1$ divisions, given by the first term on the right hand side of (10), is equal to zero when $I_i(\theta_i) = 1$, and equal to $\text{NPV}^1(\theta_{-i})$ when $I^*_i(\theta) = 0$. Equation (10) can therefore be written as

$$
\sum_{t=1}^{T} \gamma^t \cdot z_{it}(\theta) = - \text{NPV}^1(\theta_{-i}) \cdot [1 - I^*_i(\theta)] + h_i(\theta_{-i}).
$$

The no-play-no-pay condition immediately implies that $h_i(\theta_{-i}) = \text{NPV}^1(\theta_{-i})$ and therefore:

$$
\sum_{t=1}^{T} \gamma^t \cdot \pi_{it} = [\text{NPV}_i(\theta_i) - \text{NPV}^1(\theta_{-i})] \cdot I^*_i(\theta_i, \tilde{\theta}_{-i}) \tag{11}
$$

In conclusion, when all divisions discount future payoffs at the principal’s cost of capital, $r$, a satisfactory capital budgeting mechanism for exclusive assets must take
the form of a second price auction in which the winning division is charged, in present value terms, the second highest NPV.

Since our notion of strong incentive compatibility requires truthful reporting to be a dominant strategy for each manager for an entire neighborhood of \( u_i \), however, the managerial performance measure must reflect value creation only over the entire planning horizon but also on a period-by-period basis. As argued above, the relative benefit cost allocation scheme based on the competitive hurdle rate achieves this stronger requirement because:

\[
\pi_{it} = x_{it} \cdot [\theta_i - \theta_i^* (\theta_{-i})] \\
= \frac{x_{it}}{\Gamma^*(\theta) \cdot X_i} \cdot [NPV_i(\theta_i|r^*(\theta)) - NPV_i(\theta_i^*(\theta_{-i})|r^*(\theta))] \\
= \frac{x_{it}}{\Gamma^*(\theta) \cdot X_i} \cdot NPV_i(\theta_i|r^*(\theta)).
\]

The remaining question is whether there exist other combinations of capital charge rates and depreciation schedules that can yield the same periodic cost charges as those in (8). It turns out that the non-linear mapping between \( z_i \equiv \{z_{it}\}_{t=1}^T \) and \( (\hat{r}, d_i) \), for

\[
d_i = \{d_{it}\}_{t=1}^T,
\]

as defined by equations (5) is one-to-one and therefore the CHR is the unique satisfactory mechanism. We state this technical result as a separate lemma because it will be used later.\(^{18}\)

**Lemma 1** Let \( D = \{d_1, \ldots, d_T \mid \sum_{t=1}^T d_t = 1\} \) denote the set of all depreciation schedules. The mapping \( f_t : D \times (-1, \infty) \rightarrow \mathbb{R}_+^T \), with

\[
f_t(d, \hat{r}) \equiv d_t + \hat{r} \cdot \left(1 - \sum_{\tau=1}^{t-1} d_\tau\right) = z_t
\]

is one-to-one and onto, and \( \sum_{t=1}^T (1 + \hat{r})^{-t} \cdot f_t(d, \hat{r}) = 1 \) for all \( d \) and \( \hat{r} \).

We note that the CHR mechanism is a form of the so-called Pivot mechanism, which in turn belongs to the class of Groves mechanisms in (10). A well-known result

\(^{18}\)This technical result generalizes Corollary 3 in Rogerson (1997).
in the public choice literature is that the Pivot mechanism runs a surplus in the sense that the sum of all monetary transfers to the agents is negative. The CHR mechanism has precisely this feature in the sense that the present value of the cost charges to the winning division, when measured at the principal’s cost of capital, exceeds the cost of investment:

\[
\sum_{t=1}^{T} (1 + r)^{-t} \cdot z_{it}(\theta) \cdot b_i > \sum_{t=1}^{T} (1 + r^*(\theta))^{-t} \cdot z_{it}(\theta) \cdot b_i = b_i.
\]

Given the usual tidiness conditions that \(\sum_{t=1}^{T} d_{it} = 1\) and \(A_{i0} = b_i \cdot I_i\), the remaining instrument for implementing a mechanism that runs a surplus in real terms is the capital charge rate \(\hat{r}\). We state this finding separately so as to contrast it with some of our later findings.\(^{19}\)

**Corollary 1** The hurdle rate under the CHR mechanism, \(r^*(\theta)\), exceeds the firm’s cost of capital, \(r\).

Alternatively, if one were to impose the requirement that the capital charge rate be equal to the firm’s cost of capital, i.e., \(\hat{r}(\theta) \equiv r\), the initial capitalized asset amount for the winning division would have to be inflated to \(A_{i0} = PV_i(\theta^*(\theta_{-i})) \geq b_i\).

We note, however, that this solution would violate our requirement that accounting measurements must satisfy the clean surplus relation.

### 2.2 Shared Assets

We now examine the design of capital budgeting mechanisms for a setting in which the asset acquired is shared among the \(n\) divisions in the sense that all divisions can have simultaneous access to the asset and derive future cash benefits from it. Applicable examples include cost reducing investments in a manufacturing process.

\(^{19}\)Corollary 1 seems at odds with Bareket (2001) and Mohnen (2004) who also consider the problem of picking one out of mutually exclusive projects, yet in their models the hurdle rate can be kept at \(r\). This different prediction arises because we consider multiple managers competing for funds, whereas in their models there is a single manager who can have only one of many projects approved.
that is used by all divisions or "lumpy" investments in capacity which alleviate any subsequent capacity constraints.

The "public" investment project requires an initial cash outlay of $b$ and generates (incremental) operating cash flow in the amount of $x_{it} \cdot \theta_i$ for divisions $1 \leq i \leq n$ and periods $1 \leq t \leq T$. As before, the profitability parameters $\theta_i$ are divisional private information. The corporate NPV of the project equals

$$NPV(\theta) = \sum_{i=1}^{n} PV_i(\theta_i) - b$$

and the corresponding internal rate of return $r^\theta(\theta)$ is implicitly defined by

$$\sum_{i=1}^{n} \sum_{t=1}^{T} (1 + r^\theta(\theta))^{-t} \cdot x_{it} \cdot \theta_i = b.$$

The first-best investment rule $I^*(\theta)$ calls for the investment to be made whenever $NPV(\theta) \geq 0$ or, equivalently, whenever $r^\theta(\theta) \geq r$.

For computing the divisional performance measure in (2), the initial investment cost $b$ is first allocated across divisions by assigning each division a share of the common cost, $\lambda_i \cdot b$. This amount is capitalized on the divisional balance sheet and subsequently depreciated over the next $T$ periods according to a depreciation schedule $d_i = (d_{i1}, ..., d_{iT})$. As a result, the book value of division $i$’s asset at the end of period $t$ is given by:

$$A_{it} = A_{i,t-1} - d_{it} \cdot \lambda_i \cdot b = \left(1 - \sum_{r=1}^{t} d_{ir}\right) \cdot \lambda_i \cdot b.$$

With shared assets, a capital budgeting mechanism consists of:

- An investment rule $I: \Theta \rightarrow \{0, 1\}$;
- Asset shares $\lambda_i : \Theta \rightarrow [0, 1]$ satisfying the requirement $\sum_{i=1}^{n} \lambda_i(\theta) = 1$;
- A capital charge rate $\hat{r} : \Theta \rightarrow (-1, \infty)$;
- Depreciation schedules $d_i : \Theta \rightarrow \mathbb{R}_+^T$ satisfying the conditions that $\sum_{t=1}^{T} d_{it}(\theta) = 1$ if $I(\theta) = 1$, and $d_{it}(\theta) \equiv 0$ if $I(\theta) = 0$. 

In direct analogy to our earlier terminology, a capital budgeting mechanism is said to be *satisfactory* if it is (i) strongly incentive compatible and (ii) the project is undertaken, if and only if \( \text{NPV}(\theta) \geq 0 \). Since the divisions cannot be charged when the joint asset is not acquired, we are imposing again a no-play-no-pay condition.

Let \( \theta_i^*(\theta_{-i}) \) denote the *critical* profitability parameter of division \( i \), that is, the value of \( \theta_i \) at which the project breaks even, given the valuations \( \theta_{-i} \) of all other divisions. Thus, \( \theta_i^*(\theta_{-i}) \) is defined implicitly by

\[
PV_i(\theta_i^*(\theta_{-i})) + \sum_{j \neq i} PV_j(\theta_j) = b. \tag{13}
\]

**Assumption 1** Each division is essential in that \( \sum_{j \neq i} PV_j(\theta_j) + PV_i(\theta_i) < b \) for all \( i \) and \( \theta_{-i} \).

Assumption 1 ensures that \( \theta_i^*(\theta_{-i}) \geq \theta_i \) for all \( i \) and \( \theta_{-i} \). The corresponding restriction on the range of divisional profitability parameters \( \Theta_i \) is easier to satisfy in a setting with just a “small” number of participating divisions. As demonstrated at the end of this subsection, our main results can be extended to environments beyond those conforming to Assumption 1.

The following capital budgeting mechanism will be called the *Pay-the-Minimum-Necessary* (PMN) mechanism:

(i) \( I(\theta) = I^*(\theta) \);

(ii) \( \hat{r}(\theta) = r^*(\theta) \equiv r^o(\theta_i^*(\theta_{-i}), ..., \theta_n^*(\theta_{-n})) \);

(iii) \( \lambda_i(\theta) = \frac{\Gamma^*(\theta) \cdot X_i \cdot \theta_i^*(\theta_{-i})}{\sum_{j=1}^n \Gamma^*(\theta) \cdot X_j \cdot \theta_j^*(\theta_{-j})} \);

(iv) Depreciation is calculated according to the relative benefit rule based on the capital charge rate \( r^*(\theta) \);

where, as before, \( \Gamma^*(\theta) \equiv ((1 + r^*(\theta))^{-1}, ..., (1 + r^*(\theta))^{-T}) \). The PMN mechanism sets the capital charge rate equal to the project’s internal rate of return at the critical
profitability levels $\theta_i^*(\theta_{-i})$. When all $X_i$ are the same, the asset sharing rule in (iii) allocates the initial investment cost simply in proportion to the divisions’ critical profitability levels, that is, $\lambda_i(\theta) = \frac{\theta_i^*(\theta_{-i})}{\sum_{j \neq i} \theta_j^*(\theta_{-i})}$. More generally, these shares are determined in proportion to the divisional present values generated by the common asset when evaluated at the critical profitability parameters. If the aggregate present values of all divisions other than $i$ approaches $b$ (note that it cannot exceed $b$ by Assumption 1), then division $i$’s assigned ownership share $\lambda_i(\theta)$ will tend to be small; whereas it will tend be large when the aggregate valuation of the other divisions is small. These characterizations follow directly upon observing that the denominator of $\lambda_i(\theta)$ in (iii) is just equal to $b$, due to the definition of the internal rate of return.

The relative benefit depreciation rule in (iv) ensures that the cost charge to division $i$ in period $t$ is given by $z_{it}(\theta) \cdot \lambda_i \cdot b = x_{it} \cdot \theta_i^*(\theta_{-i}) = x_{it} \cdot \gamma^t \cdot \sum_{t=1}^{T} z_{it}(\theta) \cdot \gamma^t = - \left( \sum_{j \neq i} PV_j(\theta_j) - b \right) \cdot I^*(\theta_i, \theta_{-i}) + h_i(\theta_{-i}).$

and hence $\pi_{it}(\theta) = x_{it} \cdot [\theta_i - \theta_i^*(\theta_{-i})]$. Therefore, the PMN mechanism provides strong incentives for each division to report its private information truthfully.

**Proposition 2** Suppose Assumption 1 holds. For shared assets, the Pay-the-Minimum-Necessary (PMN) mechanism is the unique satisfactory capital budgeting mechanism.

To provide some intuition for the uniqueness of the PMN mechanism, suppose first that divisional managers receive a constant share of residual income in each period as compensation and they discount future payoffs at the firm’s cost of capital $r$. Every dominant-strategy mechanism must then again be a Groves mechanism which in the present setting takes the form:

$$\lambda_i(\theta) \cdot b \cdot \sum_{t=1}^{T} z_{it}(\theta) \cdot \gamma^t = - \left( \sum_{j \neq i} PV_j(\theta_j) - b \right) \cdot I^*(\theta_i, \theta_{-i}) + h_i(\theta_{-i}).$$
The no-play-no-pay condition requires that the remainder term \( h_i(\cdot) \) be equal to zero and therefore

\[
\lambda_i(\theta) \cdot b \cdot \sum_{t=1}^{T} z_{it}(\theta) \cdot \gamma^t = PV_i(\theta^*_i(\theta_{-i})) \cdot I^*(\theta_i, \theta_{-i}).
\]  

(15)

Expressed in this form, the label PMN mechanism becomes most transparent: in terms of discounted future cost charges, each division is charged the minimum amount required to make the project acceptable.\(^{20}\) Strong incentive compatibility requires that the aggregate cost charge in (15) be annuitized so that

\[
z_{it}(\theta) \cdot \lambda_i(\theta) \cdot b = x_{it} \cdot \theta^*_i(\theta_{-i}) \cdot I^*(\theta_i, \theta_{-i}).
\]  

(16)

Dividing by \( \lambda_i(\theta) \), Lemma 1 implies that the above intertemporal cost allocation can only be generated by the relative benefit benefit depreciation rule and the capital charge rate \( r^*(\theta) \). From there, it follows directly that the asset shares \( \lambda_i(\theta) \) must be the ones specified under the PMN mechanism.

We note that Propositions 1 and 2 differ markedly in their prescriptions regarding the capital charge rates. While the competitive hurdle rate for exclusive assets is higher than the owner’s cost of capital (Corollary 1), the reverse conclusion is obtained for shared assets. Specifically, we have:

\[
r = r^o(\theta_{-i}, \theta_i^*(\theta_{-i})) \\
\geq r^o(\theta_1^*(\theta_{-1}), \ldots, \theta_n^*(\theta_{-n})) \\
= r^*(\theta),
\]

where the inequality follows from the facts that (i) the internal rate of return \( r^o(\theta_1, \ldots, \theta_n) \) is monotone increasing in each of its arguments and (ii) \( \theta_i \geq \theta_i^*(\theta_{-i}) \) for any positive corporate NPV project. To summarize:

**Corollary 2** Under the PMN mechanism, the capital charge rate, \( r^*(\theta) \), is less than the firm’s cost of capital, \( r \).

\(^{20}\)This property addresses a common concern with regard to cost allocations, e.g., Hodak (1997).
Investment in the shared assets therefore is “subsidized” by the central office in real terms by way of a reduced capital charge rate. An alternative way to implement this mechanism would be for the central office to set \( \hat{r} = r \) while lowering the initial capital allocation for each division to

\[
\hat{\lambda}_i(\theta_{-i}) = \frac{b - \sum_{j \neq i} PV_j(\theta_j | r)}{b}.
\]

However, because \( \sum_{i=1}^n \hat{\lambda}_i(\theta_{-i}) = \frac{1}{b} [n \cdot b - (n-1) \cdot \sum_{i=1}^n PV_i(\theta_i)] < 1 \) whenever \( I(\theta) = 1 \), this solution would violate the fundamental tidiness requirement that the sum of all depreciation charges across time periods and participating divisions be equal to the initial investment outlay.

Since the PMN mechanism is satisfactory in the dominant strategy sense, it must be a time-consistent Groves mechanism. A key feature of Groves mechanisms for a binary public decision is that, contingent on the project being undertaken, the cost charge to any division must be independent of its own report. The capital charge rate in (ii) and the asset sharing rules in (iii) may suggest that the PMN mechanism violates this property. However, while both \( \hat{r}(\theta) \) and \( \lambda_i(\theta) \) vary non-trivially with manager \( i \)'s report \( \theta_i \), equation (14) shows that the resulting sum of depreciation and capital charges, \( z_{it} \), is indeed independent of \( \theta_i \).

Unlike the CHR mechanism identified above, the PMN mechanism is not a multi-period version of the familiar Pivot mechanism. In the public choice literature, it is usually assumed that the cost of the public investment is first divided in some arbitrary, say equal, fashion among all agents (e.g., Kreps 1990, Mas-Colell et al. 1995). One of the key features of the Pivot mechanism is that an agent receives a non-zero transfer payment only if he is “pivotal” in the sense that his report alters the social decision. The PMN mechanism also does have the feature that divisions are being charged only when they are pivotal (in fact, our Assumption 1 implies that all agents are pivotal). Yet, the Pivot mechanism does not satisfy our no-play-no-pay condition since an agents can be charged even if the project is not undertaken. As observed above, Pivot mechanisms always generate a “budget surplus” since the agents collec-
tively pay more than than they receive in monetary transfers. The PMN mechanism does not satisfy that property since the total charges in real terms (i.e., evaluated at the owner’s cost of capital) are less than the initial cost of investment.

It is instructive to contrast the findings for exclusive and shared assets in terms of how the capital charge rate varies with changes in the environment. Recall that the capital charge rate in the competitive hurdle rate mechanism is (weakly) increasing in the number of competing divisions. The opposite is true for shared assets. Specifically, suppose \( \theta^o \equiv (\theta_1^o, \ldots, \theta_n^o) \) is the profitability profile of the existing \( n \) divisions and let \( r^*(\theta^o) \) be the capital charge rate for the corresponding PMN mechanism. It is easy to show that if an additional division with profitability parameter \( \theta_{n+1} \) participates in the asset acquisition decision, then \( r^*(\theta^o, \theta_{n+1}) < r^*(\theta^o) \), regardless of type \( \theta_{n+1} \) of the new entrant. Intuitively, Groves mechanisms entail transfer payments equal to the expected externality an agent’s valuation imposes on all other parties. This externality is negative for exclusive assets due to more competition for a scarce resource, but the externality is positive for shared assets because of an increase in the aggregate willingness-to-pay.

Finally, we demonstrate that the PMN mechanism can be extended to settings in which Assumption 1 does not hold. The immediate issue then is that the critical profitability cutoffs \( \theta^*_{i}(\theta_{-i}) \) may be less than \( \theta^i \). To adapt the PMN mechanism to such parameter configurations, define \( \hat{\theta}_i(\theta_{-i}) = \max\{\theta_i, \theta^*_{i}(\theta_{-i})\} \). Instead of Assumption 1, it then suffices to have the following weaker condition:

**Assumption 2** For all \( \theta \), there exists some division \( k \) such that \( \hat{\theta}_k(\theta_{-k}) > 0 \).

Thus, for all type profiles \( \theta \), there must be at least one essential division. Assumption 2 is likely to be satisfied in settings where there are either one or a few larger divisions (in terms of the magnitude of their \( PV_i \)’s) whose valuation is critical for the project’s overall profitability. Given Assumption 2, the PMN mechanism can be modified by replacing \( \theta^*_{i}(\theta_{-i}) \) with \( \hat{\theta}_i(\theta_{-i}) \). As a consequence, non-essential agents whose critical cutoff \( \hat{\theta}_i(\theta_{-i}) \) equals the boundary value \( \theta_i \) will be assigned an ownership share
\( \lambda_i(\theta) = 0 \). While the modified PMN mechanism is again satisfactory, it is no longer the unique solution among the class of capital budgeting mechanisms we consider. In particular, it is not necessary that non-essential divisions be charged zero. However, uniqueness of the modified PMN mechanism would emerge if one were to impose an additional participation condition akin to Moulin’s (1986) “no-free-ride” condition, requiring that no division be worse off by participating in the mechanism.

3 Hidden Actions and Incentive Contracting

3.1 Second-Best Mechanisms

Our analysis has so far taken as given that divisional managers seek to maximize divisional income and that the objective of the central office is to maximize the firmwide NPV by choice of goal congruent performance measures. It is natural to ask whether the insights about goal congruent mechanisms remain viable once the model is extended to include an explicit agency problem, and the desired incentives are derived from a unified optimization program. Specifically, suppose divisional operating cash flow in period \( t \) is now given by

\[
    c_{it} = a_{it} + x_{it} \cdot \theta_i \cdot I_i,
\]

where \( a_{it} \in [0, \bar{a}_{it}] \) denotes productive effort chosen privately by manager \( i \) in period \( t \) and \( I_i \in \{0, 1\} \) indicates generically (for shared and exclusive assets) whether division \( i \) has access to the asset. Manager \( i \) observes \( \theta_i \) before contracting. The central office and all other managers share the same beliefs about \( \theta_i \) given by the cumulative distribution \( F_i(\theta_i) \) with strictly positive density over the entire support \( \Theta_i \).\(^{21}\) We assume the usual monotone inverse hazard rate condition; i.e., \( H_i(\theta_i) = [1 - F_i(\theta_i)] / f_i(\theta_i) \) is decreasing in \( \theta_i \) for all \( i \). While the central office can observe the divisional operating cash flows in each period, it is unable to disentangle the investment- from the

\(^{21}\) Alternatively, the managers may learn their \( \theta_i \)-parameters after entering into the contract, but they cannot be prevented from quitting the job if their participation constraints, spelled out below, are not satisfied.
effort-related components.

Manager $i$’s date 0 utility payoff is given by

$$U_i = \sum_{t=1}^{T} \gamma^t \cdot [s_{it} - v_{it}(a_{it})],$$

where $s_{it}$ denotes his compensation in period $t$ and $v_{it}(\cdot)$ is his disutility from exerting effort $a_{it}$ in period $t$. The function $v_{it}(\cdot)$ is increasing and convex with $v'_{it}(0) = 0$, for all $i$ and $t$. Given this structure, it is only the present value of compensation payments that matters to each manager, provided all parties can commit to a $T$-period contract.

In our setting, a message-contingent mechanism specifies an investment decision rule $I_i(\tilde{\theta}) \in \{0, 1\}$, as well as “target cash flows” $c_i(\tilde{\theta}) \equiv (c_{i1}(\tilde{\theta}), \ldots, c_{iT}(\tilde{\theta}))$ to be delivered by each division and managerial compensation payments $s_i(\tilde{\theta}) \equiv (s_{i1}(\tilde{\theta}), \ldots, s_{iT}(\tilde{\theta}))$, contingent on the managers’ reports $\tilde{\theta}$. For any such mechanism, let $U_i(\tilde{\theta}_i, \tilde{\theta}_{-i} | \theta_i)$ denote manager $i$’s utility contingent on his own true profitability parameter $\theta_i$, reports $\tilde{\theta}_{-i}$ submitted by the other managers, and his own report $\tilde{\theta}_i$. Assuming truthful reporting on the part of the other managers, this yields

$$U_i(\tilde{\theta}_i, \tilde{\theta}_{-i} | \theta_i) \equiv \sum_{t=1}^{T} \gamma^t \cdot [s_{it}(\tilde{\theta}) - v_{it}(a_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i))],$$

where

$$a_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i) = \min \left\{a_{it} \left| a_{it} + x_{it} \cdot \theta_i \cdot I_i(\tilde{\theta}_i, \theta_{-i}) \geq c_{it}(\tilde{\theta}_i, \theta_{-i}) \right. \right\}$$

is the minimum effort that manager $i$ has to exert so as to achieve the required periodic cash flow target, $c_{it}(\tilde{\theta}_i, \theta_{-i})$ in each period.

By the Revelation Principle we may focus on direct revelation mechanisms which induce the managers to reveal their information truthfully. The central office’s optimization problem can then be stated as follows:
\[
\mathcal{P} : \max_{(c_i(\theta), s_i(\theta), I_i(\theta))_{i=1}^n} E_\theta \left\{ \sum_{i=1}^n \sum_{t=1}^T \gamma^t \cdot [c_{it}(\theta) - s_{it}(\theta)] - B(\theta) \right\}
\]

subject to:

(i.e) for exclusive assets: \( \sum_{i=1}^n I_i(\theta) \leq 1 \) and \( B(\theta) = \sum_{i=1}^n b_i \cdot I_i(\theta) \),

(is) for shared assets: \( I_i(\theta) = I_j(\theta) = I(\theta) \), for all \( i, j \), and \( B(\theta) = b \cdot I(\theta) \),

(ii) \( E_{\theta_i}[U_i(\theta, \theta_{-i} | \theta_i)] \geq E_{\theta_i}[U_i(\tilde{\theta}_i, \theta_{-i} | \theta_i)] \), for all \( \theta_i, \tilde{\theta}_i \) and \( i \),

(iii) \( E_{\theta_i}[U_i(\theta, \theta_{-i} | \theta_i)] \geq 0 \), for all \( \theta_i \) and \( i \).

Constraints (i.e) and (is) ensure feasibility of the investment rule for exclusive and shared assets, respectively, and specify the resulting initial investment amounts. The incentive compatibility constraints (ii) require that truthful reporting constitute a Bayesian-Nash equilibrium. The participation constraints (iii) are required to hold on an interim basis, i.e., each manager must break even in expectation over the other managers’ possible types. We denote the solution to this program by \( (c^*_i(\theta), s^*_i(\theta), I^*_i(\theta))_{i=1}^n \) and refer to it as the second-best solution.

The managers will earn informational rents on account of their private information. Each manager can underreport \( \tilde{\theta}_i < \theta_i \) and at the same time reduce his effort whenever \( I_i(\tilde{\theta}_i, \theta_{-i}) = 1 \). The basic tradeoff for the central office is that manager \( i \)'s information rents will be increasing both in the induced effort levels, \( (a_{it}, ..., a_{iT}) \), as well as in the set of states in which division \( i \) has access to the asset. Applying standard arguments from the adverse selection literature based on “local” incentive constraints (e.g., Laffont and Tirole, 1993), it can be shown that manager \( i \)'s interim informational rent for any \( \theta_i \) (i.e., in expectation over other managers’ types) equals

\[
E_{\theta_i}[U_i(\theta, \theta_{-i} | \theta_i)] = E_{\theta_i} \left[ \int_{\tilde{\theta}_i}^{\theta_i} \sum_{t=1}^T \gamma^t \cdot v'_{it}(a_{it}(q_i, \theta_{-i} | q_i)) \cdot x_{it} \cdot I_i(q_i, \theta_{-i}) \ dq_i \right].
\] (17)
The expression in (17) illustrates the above tradeoff: manager \( i \)'s informational rent can be reduced either by creating lower powered effort incentives (there will be no rent if \( a_{it} = 0 \)) or by curtailing the set of states \( \theta \) in which \( I_i(\theta) = 1 \).

To simplify the exposition and sharpen our predictions regarding the optimal investment decision rule, we focus on a setting in which each manager’s marginal cost of effort is sufficiently small so that the central office always finds it worthwhile to induce maximum effort in each period, i.e., \( a_{it}(\theta_i, \theta_{-i} \mid \theta_i) \equiv \bar{a}_{it} \). Inducing the maximum level of effort will indeed be optimal provided \( v'_{it}(\bar{a}_{it}) \) is sufficiently small relative to the other parameters of the model. For brevity, we denote \( v'_{it} \equiv v'_{it}(\bar{a}_{it}) \).

As shown in the proof of Lemma 2, the following condition ensures that the central office will indeed seek to induce the maximum level of effort in each period:

\[
1 - v'_{it} - v''_{it}(\bar{a}_{it}) \cdot x_{it} \cdot H_i(\theta_i) \geq 0 \tag{18}
\]

for each \( i \) and \( t \). Condition (18) says that even if division \( i \) is allowed to invest for all \( \theta_i \geq \bar{\theta}_i \), the marginal return from effort, which has been normalized to one, is sufficiently large for the central office to prefer high effort despite the corresponding increase in expected informational rents.

We use the expression in (17), evaluated at \( a_{it} = \bar{a}_{it} \), to solve for each manager’s compensation payments \( s_{it}(\theta) \) in \( P \). The central office’s optimization problem then simplifies to the following program (see Appendix B for a detailed derivation):

\[
P': \max_{(i_s, \theta)_{i=1}^n} \mathbb{E}_\theta \left\{ \sum_{i=1}^n \left[ \sum_{t=1}^T \gamma^t \cdot [\bar{a}_{it} - v_{it}(\bar{a}_{it})] + [PV_i(\theta_i) - \kappa_i \cdot H_i(\theta_i)] \cdot I_i(\theta) \right] - B(\theta) \right\},
\]

subject to \((i_s), (i_v))\),

where \( \kappa_i \equiv \sum_{t=1}^T \gamma^t \cdot v'_{it} \cdot x_{it} \). The reduced objective function in \( P' \) reflects that the expected value of manager \( i \)'s interim informational rents (i.e., \( \mathbb{E}_\theta[U(\theta_i, \theta_{-i} \mid \theta_i)] \)) is equal to the expected value of \( \kappa_i \cdot H_i(\theta_i) \cdot I_i(\theta) \). To characterize the optimal investment decision rule, it will be useful to define

\[
\phi_i(\theta_i) \equiv PV_i(\theta_i) - \kappa_i \cdot H_i(\theta_i), \tag{19}
\]
as the present value of division \( i \)'s \textit{virtual} cash flows (i.e., the present value of cash flows net of the manager’s informational rents), \( VNPV_i(\theta_i) = \phi_i(\theta_i) - b_i \) as the corresponding \textit{virtual divisional NPV} for exclusive assets, and \( VNPV(\theta) = \sum_{i=1}^{n} \phi_i(\theta_i) - b \) as the \textit{virtual corporate NPV} for shared assets. Given (18), the optimal investment rule for exclusive assets then is given by \( I_i(\theta) = 1 \) if and only if

\[
VNPV_i(\theta_i) \geq \max_j \{VNPV_j(\theta_j), 0\},
\]

while for shared assets it is \( I(\theta) = 1 \) if and only if

\[
VNPV(\theta) \geq 0.
\]

In the presence of asymmetric information and managerial moral hazard problems, the relevant criterion for the optimal investment decision rule must be based on the present value of \textit{virtual}, or agency-adjusted, cash flows. As a consequence, for the exclusive asset case, the central office will choose to fund division \( i \)'s project only if its virtual divisional NPV exceeds both zero and the virtual NPVs of the other projects. Similarly, for the shared asset scenario, the common investment project will be undertaken if and only if the virtual corporate NPV is positive.

In the following two subsections, we ask whether the mechanisms identified as satisfactory in the goal congruence framework of Section 2 can be adapted to generate optimal incentives in the presence of agency problems. We say that a capital budgeting mechanism is \textit{optimal} if and only if there exists linear compensation schemes

\[
\{ s_{it}(\pi_{it} | \tilde{\theta}) = \alpha_{it} + \beta_{it} \cdot \pi_{it} \}
\]

for \( t = 1, ..., T \) and \( i = 1, ..., n \), that achieve the same payoffs for the firm as the second-best scheme identified in Lemma 2.

3.2 Optimal Mechanisms: Exclusive Assets

The second-best investment rule calls for funding a project if and only if its \textit{virtual} NPV exceeds both zero and the virtual NPVs of all other projects. That is, \( I_i(\theta) = 1 \),
if and only if $\text{VNPV}_i(\theta_i) = \text{VNPV}^1(\theta)$, where $\text{VNPV}^1(\theta) \equiv \max_{1 \leq j \leq n}\{\text{VNPV}_j(\theta_j), 0\}$. Accordingly, we now denote the *agency-adjusted* profitability cutoff value for division $i$ by $\theta^{**}_i(\theta_{-i})$ which is implicitly defined by\(^{22}\)

$$\text{VNPV}_i(\theta^{**}_i(\theta_{-i})) = \text{VNPV}^1(\theta_{-i}).$$

The corresponding agency-adjusted competitive hurdle rate of division $i$, $r^{**}_i(\theta)$, is defined to be the internal rate of return of its agency-adjusted critical project; i.e.:

$$r^{**}_i(\theta) \equiv r^o_i(\theta^{**}_i(\theta_{-i})). \quad (23)$$

Suppose now that each manager is offered a linear compensation scheme of the form in (22) with bonus coefficients $\beta_{it} = v'_{it}$. If the performance measure is based on the competitive hurdle rate mechanism (i.e., the capital charge rate is equal to the agency-adjusted competitive hurdle rate and the asset valuation is based on the relative benefit depreciation rule), then

$$\pi_{it}(\bar{\theta}_i, \theta_{-i} | \theta_i) = v'_{it} \cdot I_i(\bar{\theta}_i, \theta_{-i}) \cdot x_{it} \cdot [\theta_i - \theta^{**}_i(\theta_{-i})]. \quad (24)$$

It is therefore a dominant strategy for each manager to report his information truthfully because a project makes a positive contribution to his performance measure if and only if $\theta_i > \theta^{**}_i(\theta_{-i})$. Our next result shows that this mechanism is indeed optimal. Furthermore, even though the optimization program in $P$ is stated in terms of Bayesian-Nash incentive compatibility and interim participation constraints, the central office obtains dominant strategy incentives and ex-post satisfaction of the participation constraints for “free.”\(^{23}\)

\(^{22}\)To ensure that well-defined profitability cutoffs exist, we again assume that, for all $i$, $\text{VNPV}_i(\bar{\theta}_i) = L$ and $\text{VNPV}_i(\bar{\theta}_i) = H$ for some $H > L$.

\(^{23}\)Unlike Proposition 1, Proposition 3 only speaks to the sufficiency, and not necessity, of the CHR mechanism. Given all parties can commit to the contract, there is some indeterminacy as to how the divisional cost charges are applied to the periods. However, following similar lines as in Dutta and Reichelstein (2002), necessity of the CHR mechanism can be established by invoking a robustness notion when the severity of the divisional agency problem itself, i.e., the vector $(v'_{i1}, ..., v'_{iT})$, is subject to uncertainty.
Proposition 3  The competitive hurdle rate (CHR) mechanism based on the agency-adjusted capital charge rate \( \hat{r}(\theta) = r^{**}_i(\theta) \) is an optimal mechanism.

It can be easily verified from Equation (24) that the present value of the intertemporal cost charges for the winning division is equal to the second highest positive virtual NPV. The CHR mechanism can therefore again be interpreted as a multiperiod version of the second-price auction mechanism. If managers are ex-ante identical with regard to \( F_i(\theta_i) = F(\theta), \kappa_i = \kappa, b_i = b \) and \( X_i = X \), then it will suffice to ask each manager to report his internal rate of return \( r_i \). With \( r^c > r \) denoting the internal rate of return of a project whose virtual NPV is zero, it is then optimal to set the capital charge rate for the winning division equal to \( r_i^{**}(r_1, \ldots, r_n) = \max_{j \neq i} \{ r_j, r^c \} \).

In the general case of ex-ante different managers, the divisional internal rates of returns need to be properly calibrated through the \( r_i^{o}(\theta_i^{**}(\theta_{-i})) \) function so as to account for cross-sectional differences in the agency problems (i.e., \( \kappa_i \) and \( H_i(\cdot) \)) and in the specifics of the investment projects (i.e., \( X_i \) and \( b_i \)).

The case of ex ante identical managers also helps illustrate the impact of an exogenous change in the number of divisions competing for the scarce capital. The firm will benefit from the increased competition among divisions in two ways. First, the informational rents of the winning manager will be reduced. To see this, note that for ex ante identical managers, the winning manager’s informational rent is given by \( U_i(\theta_i, \theta_{-i} | \theta_i) = \kappa_i \cdot [\theta_i - \theta_i^{**}(\theta_{-i})] = \kappa_i \cdot [\theta_i - \max_{j \neq i} \{ \theta_j, r^c \}] \), where \( \theta^c \) denotes the agency adjusted break-even profitability in a single agent setting (i.e., \( r^c \equiv r^o(\theta^c) \)). Clearly, the winning manager’s informational rents are decreasing in \( n \), in expectation. Second, the underinvestment problem will be mitigated. To see this, note that the ex ante probability of underinvestment is equal to \( [F(\theta^c) - F(\theta^o)]^n \), where \( \theta^o \) denotes the break-even profitability (i.e., \( NPV(\theta^o) = 0 \)).

A natural question is how the agency-adjusted hurdle rate under the CHR mechanism, \( r^{**}(\theta) \), compares with the hurdle rate identified in Proposition 1 in the absence of an explicit agency problem. The well-known rationing result from the single period capital budgeting literature may suggest that the hurdle rate should always increase
as divisional agency problems become more severe. We note that this prediction generalizes to our multi-divisional setting only as far as the winning manager is concerned. To illustrate, let us consider a setting with two divisions. Suppose division 1 has the highest positive virtual NPV project. We use $\kappa = (\kappa_1, \kappa_2)$ as measures of the relative severities of the two agency problems and let $\phi_i(\theta_i | \kappa_i)$ denote the present value of virtual cash flows as a function of $\kappa_i$. Division 1’s cutoff profitability type for given $\kappa$ is then given by

$$\phi_1(\theta_1^*(\theta_2 | \kappa) | \kappa_1) - b_1 \equiv \phi_2(\theta_2 | \kappa_2) - b_2,$$

(25)

and the resulting hurdle rate equals $\hat{r}(\theta | \kappa) = r_1^*(\theta_1^*(\theta_2 | \kappa))$. We note from (19) that $\phi_i(\cdot)$ is increasing in $\theta_i$ and decreasing in $\kappa_i$. Consider now an exogenous increase in $\kappa_2$, i.e., manager 2’s agency problem becomes more severe. As a result, the right-hand side of (25) decreases. To restore the identity, manager 1’s cutoff type $\theta_1^*(\theta_2 | \kappa)$ and therefore the competitive hurdle rate $\hat{r}(\cdot)$ must go down. The reverse will hold for an increase in $\kappa_1$. This differential impact on the winner’s hurdle rate can best be understood using the following arguments: As $\kappa_2$ goes up, the opportunity cost of investing in division 1 declines, resulting in a lower hurdle rate; as $\kappa_1$ goes up, the net profitability of his own project (adjusted for informational rents) goes down, resulting in a higher hurdle rate.

We now generalize this insight to the case of ex ante heterogenous divisions. For comparative statics purposes, we express the competitive hurdle rate $r_i^*(\theta | \kappa)$ as parameterized by $\kappa = (\kappa_1, ..., \kappa_n)$.

**Corollary 3** The agency-adjusted competitive hurdle rate $r_i^*(\theta | \kappa)$ is increasing in $\kappa_i$, but decreasing in $\kappa_j$ for all $j \neq i$.

Put differently, the CHR mechanism allows the principal to “handicap” a division that is subject to a more severe agency problem. Returning again to the case of a firm with two divisions, suppose $\theta_1$ and $\theta_2$ are identically distributed and the two projects are ex-ante identical with regard to their cash outlays and intertemporal patterns.
of cash flows, \((b_i, X_i)\), so that \(NPV_1(\theta) = NPV_2(\theta)\) for all \(\theta\). If \(\kappa_1 > \kappa_2\), the moral hazard problem is more severe for manager 1. Our analysis then shows that manager 1 will face a higher hurdle rate than manager 2.

An interesting special case arises when the two divisions are identical with respect to the managerial agency problems and ex ante information asymmetries, i.e., \(\kappa_i = \kappa\) and \(F_i(\theta) = F(\theta)\), \(i = 1, 2\), but the two projects differ in their initial cash outlays and cash flows over time. The parameter \(\kappa\) may capture the quality of the firm’s management control system. The identity in (25) then becomes:

\[
NPV_1(\theta_1^*(\theta_2 | \kappa)) - \kappa \cdot H(\theta_1^*(\theta_2 | \kappa)) \equiv NPV_2(\theta_2) - \kappa \cdot H(\theta_2).
\]

Now suppose \(\kappa\) increases, indicating that the management control system becomes uniformly less stringent. The left-hand side of this cutoff condition will decrease by \(H(\theta_1^*(\theta_2 | \kappa))\) whereas the right-hand side decreases by \(H(\theta_2)\). If division 1’s project is ex ante more profitable in that \(NPV_1(\theta) > NPV_2(\theta)\) for all \(\theta\), then \(\theta_1^*(\theta_2 | \kappa) < \theta_2\) for any \(\kappa\) and \(\theta_2\). Since \(H(\theta)\) is a decreasing function, \(\theta_1^*(\theta_2 | \kappa)\) and the corresponding hurdle rate must increase to restore the identity. Generalizing this insight, we find:

**Corollary 4** Suppose \(\kappa_i = \kappa\) and \(F_i(\theta) \equiv F(\theta)\) for all \(i\). The agency-adjusted competitive hurdle rate \(r_i^*(\theta | \kappa)\) for the investing division is increasing in \(\kappa\) if \(NPV_i(\theta) \geq NPV^1(\theta_{-i})\) for all \(\theta\), and decreasing in \(\kappa\) if \(NPV_i(\theta) < NPV^1(\theta_{-i})\).

If manager \(i\)’s project is ex ante more profitable than those of the other divisions, then his profitability cutoff will be relatively low, resulting in sizable information rents for favorable realizations of \(\theta_i\). As \(\kappa\) uniformly increases, the central office will weigh rent extraction more heavily and therefore raise the hurdle rate. The reverse holds if the ex ante profitability of manager \(i\)’s project is relatively low.

\(^{24}\)Division \(i\)’s project will be more profitable ex ante than that of division \(j\) provided \(b_1 < b_2\) and/or the vector \(X_1\) is more “front-loaded” than \(X_2\).
3.3 Optimal Mechanisms: Shared Assets

We now return to the shared asset setting and examine whether the PMN mechanism identified in connection with the goal congruence framework can be used to generate optimal incentives in the presence of managerial agency problems. We recall from Lemma 2 that the second-best investment decision calls for the joint asset to be acquired if and only if the firmwide virtual NPV is positive. That is, \( I(\theta) = 1 \), if and only if

\[
VNPV(\theta) \equiv \sum_{i=1}^{n} \phi_i(\theta_i) - b \geq 0
\]

In analogy with our definition of \( \theta^*_i(\theta_{-i}) \), we now define the *agency-adjusted* divisional profitability cutoffs \( \theta^{**}_i(\theta_{-i}) \) by

\[
\phi_i(\theta^{**}_i(\theta_{-i})) \equiv b - \sum_{j \neq i} \phi_j(\theta_j).
\]

(26)

In the agency setting, the capital charge rate in the PMN mechanism is therefore given by

\[
\hat{r}(\theta) = r^{**}(\theta) \equiv r^o(\theta^{**}_1(\theta_{-1}),...,\theta^{**}_n(\theta_{-n})),
\]

(27)

and the asset sharing rule takes the form:

\[
\lambda_i(\theta) = \frac{\Gamma^{**}(\theta) \cdot X_i \cdot \theta^{**}_i(\theta_{-i})}{\sum_{i=1}^{n} \Gamma^{**}(\theta) \cdot X_j \cdot \theta^{**}_j(\theta_{-j})},
\]

(28)

with \( \Gamma^{**}(\theta) \equiv ((1 + r^{**}(\theta))^{-1},..., (1 + r^{**}(\theta))^{-T}) \)

When divisional managers are compensated on the basis of linear schemes of the form in (22) with bonus coefficients \( \beta_{it} = v'_{it} \), they have incentives to provide the optimal amount of efforts in each period. If the performance measure is based on the PMN mechanism (i.e., the capital charge rate is equal to \( r^{**}(\theta) \), the asset sharing rule is as given in (28), and the depreciation is based on the relative benefit rule according to the hurdle rate rate \( r^{**}(\theta) \)), the performance measure becomes:

\[
\pi_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i) = v'_{it} \cdot I_i(\tilde{\theta}_i, \theta_{-i}) \cdot x_{it} \cdot [\theta_i - \theta^{**}_i(\theta_{-i})]
\]

(29)
As a consequence, the joint project makes a positive contribution to manager $i$’s performance measure if and only if $\theta_i > \theta_\star^\star(\theta_{-i})$, i.e., $\text{VNPV}(\theta) \geq 0$. Truthful reporting therefore again is a dominant-strategy equilibrium and we obtain the following:\footnote{Assumption 1 was introduced in Section 2.2 to ensure that the profitability cutoffs are in the interior of the managers’ type supports. Since $\theta_\star^\star(\theta_{-i})$ is higher for any positive $\kappa_i$ than for $\kappa_i = 0$ (i.e., absent a moral hazard problem), Assumption 1 ensures interior cutoffs also with agency problems.}

**Proposition 4** Given Assumption 1, the pay-the-minimum-necessary (PMN) mechanism based on the agency-adjusted capital charge rate of $r^\star(\theta)$ in (27) and the agency-adjusted asset sharing rule in (28) is optimal.

In contrast to exclusive assets, the cutoff condition in (26) implies an unambiguous comparative statics result on how the relative severity of the agency problem affects the capital charge rate. To this end, we again express the hurdle rate $r^\star(\theta | \kappa)$ as a function of $\kappa = (\kappa_1, ..., \kappa_n)$.

**Corollary 5** The agency-adjusted hurdle rate $r^\star(\theta | \kappa)$ under the PMN mechanism is increasing in $\kappa_i$ for all $i$.

As the agency problem associated with a given division becomes more severe, its contribution $\phi_i(\cdot)$ to the total virtual NPV declines. This in turn has a negative externality on the other divisions as expressed by a higher capital charge rate. It is straightforward to construct examples for which the resulting capital charge rate can exceed the firm’s cost of capital $r$, which contrasts with Corollary 2.

## 4 Conclusion

Interdependencies between divisions are ubiquitous in the capital budgeting process yet have received little attention in the literature. This paper has developed a unified framework that allows for positive externalities among divisions (shared assets) or negative externalities (exclusive assets). Our analysis has illustrated commonalities
among these two scenarios as well as distinct differences, in particular with regard to hurdle rates. We found that capital charge rates without managerial moral hazard tend to exceed the cost of capital $r$ for exclusive assets, while the reverse holds for shared assets. As incentive problems become more severe, the hurdle rate will go up unambiguously for shared assets, but not necessarily so for exclusive assets. Our analysis therefore generates a rich set of empirically testable hypothesis regarding the cross-sectional variation in hurdle rates. This is an important area for future empirical research given the centrality of hurdle rates for the resource allocation process.

An omitted factor in our analysis is risk. Investments returns in practice are risky and managers in general are risk averse. Prior studies have addressed this issue within single-agent frameworks (Christensen et al. 2002, Dutta and Reichelstein 2002). While it would be desirable to extend these studies to settings with many privately informed agents, the Gibbard-Satterthwaite’s impossibility theorem (e.g., Mas Colell et al. 1995) implies that under fairly general conditions there does not exist any dominant-strategy incentive compatible capital budgeting mechanism that achieves $\text{NPV}$ maximization for risk averse agents. The search for optimal mechanisms would therefore have to be confined to Bayesian ones.

Another simplifying assumption underlying our analysis was that collusion among divisional managers can effectively be prevented by the central office. It is well known that Groves schemes are susceptible to agents agreeing on side-contracts which make the principal worse off (Green and Laffont 1979). It would be desirable in future research to address optimal collusion-proof capital budgeting mechanisms for a variety of alternative asset usage scenarios.
Appendix A: Proofs

Proof of Lemma 1

For any given $z \equiv (z_1, \ldots, z_T) \in \mathbb{R}_+^T$, the mapping

$$f_t(d, \hat{r}) \equiv d_t + \hat{r} \cdot \left(1 - \sum_{\tau=1}^{t-1} d_{\tau}\right) = z_t$$

(30)

defines a set of $T$ non-linear equations in $(d, \hat{r})$. To prove the result, we will show that the above system of equations has a unique solution in $D \times (-1, \infty)$ for each $z \in \mathbb{R}_+^T$.

Solving the first $T - 1$ equations in (30) recursively for $(d_1, \ldots, d_{T-1})$, we get

$$d_t = \hat{r} \cdot \sum_{\tau=1}^{t-1} (1 + \hat{r})^{t-\tau-1} \cdot z_\tau - \hat{r} \cdot (1 + \hat{r})^{t-1}, \text{ for } t = 1, \ldots, T - 1.$$

Substituting this solution into the last component of equation (30) gives

$$z_T = (1 + \hat{r}) \cdot \left[(1 + \hat{r})^{T-1} - (1 + \hat{r})^{T-2} \cdot z_1 - (1 + \hat{r})^{T-3} \cdot z_2 - \ldots - (1 + \hat{r}) \cdot z_{T-2} - z_{T-1}\right].$$

Multiplying both sides by $\hat{\gamma} \equiv (1 + \hat{r})^{-1}$ (recall that by assumption $\hat{\gamma}$ is non-zero) and simplifying, we get:

$$\sum_{t=1}^{T} \hat{\gamma}^t \cdot z_t = 1.$$

Thus, there is a unique solution to the system of $T$ equations. Moreover, the capital charge rate $\hat{r}$ must equal the discount rate at which the present value of the cost allocation charges $z_t$ is equal to one.

Proof of Proposition 1.

We first prove that the CHR mechanism is a satisfactory mechanism. Under the competitive hurdle rate mechanism, the asset valuation rule is based on the relative benefit depreciation schedule and the capital charge rate is equal to the division’s
competitive hurdle rate $r_t^i(\theta_t^i(\theta_{-i}))$. This implies that period $t$ performance measure is given by:

$$
\pi_{it}(\tilde{\theta}_i, \theta_{-i} \mid \theta_i) = \left( x_{it} \cdot \theta_i - \frac{x_{it}}{\sum_{\tau=1}^{T} [1 + r_t^i(\theta_t^i(\theta_{-i}))]^{-\tau} \cdot x_{i\tau}} \cdot b_i \right) \cdot I^*_i(\tilde{\theta}_i, \theta_{-i})
$$

$$
= x_{it} \cdot [\theta_i - \theta_t^i(\theta_{-i})] \cdot I^*_i(\tilde{\theta}_i, \theta_{-i}),
$$

(31)

From (31), we note that in each period, division $i$’s payoff depends on its own report $\tilde{\theta}_i$ only through the impact on the decision rule $I^*_i(\cdot)$. Furthermore, (31) shows that each manager has strong incentives to induce $I^*_i = 1$ when $\theta_i \geq \theta_t^i(\theta_{-i})$, and $I^*_i = 0$ when $\theta_i < \theta_t^i(\theta_{-i})$. Consequently, the CHR mechanism satisfies our requirement of strong incentive compatibility, i.e.,

$$
\pi_{it}(\theta_i, \theta_{-i} \mid \theta_i) \geq \pi_{it}(\tilde{\theta}_i, \theta_{-i} \mid \theta_i),
$$

for each $i$, $t$, $\theta_i$, $\tilde{\theta}_i$, and $\theta_{-i}$.

We now prove the necessity part, i.e., the CHR mechanism is the unique satisfactory mechanism in the class described in (4). We first show that any strongly incentive compatible mechanism must have the property that for all $\tilde{\theta}_i > \theta_t^i(\theta_{-i})$:

$$
z_{it}(\tilde{\theta}_i, \theta_{-i}) = \frac{x_{it}}{b_i} \cdot \theta_t^i(\theta_{-i}).
$$

(32)

If $u_i$ can vary in some open neighborhood, strong incentive compatibility implies directly that for any $\theta_i, \tilde{\theta}_i > \theta_t^i(\theta_{-i})$:

$$
z_{it}(\tilde{\theta}_i, \theta_{-i}) = z_{it}(\theta_i, \theta_{-i}).
$$

Suppose now that for some period $t$,

$$
z_{it}(\theta_t^i(\theta_{-i}), \theta_{-i}) = \frac{x_{it}}{b_i} \cdot \theta_t^i(\theta_{-i}) + \Delta_i(\theta_{-i}).
$$

Suppose division $i$’s profitability parameter is $\tilde{\theta}_i = \theta_t^i(\theta_{-i}) + \epsilon_i$. Incentive compatibility requires that:

$$
\sum_{t=1}^{T} u_{it} \cdot [\epsilon_i \cdot x_{it} - \Delta_i(\theta_{-i}) \cdot b_i] \geq 0.
$$

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Conversely, for \( \tilde{\theta}_i = \theta_i^*(\theta_{-i}) - \epsilon_i \) incentive compatibility requires:

\[
\sum_{t=1}^{T} u_{it} \cdot \left[ -\epsilon_i \cdot x_{it} - \Delta_i(\theta_{-i}) \cdot b_i \right] \leq 0.
\]

Since both of the above inequalities have to hold for any \( \epsilon_i > 0 \) and for all \( u_i \) in some open neighborhood, Equation (32) must hold.

Finally, by Lemma 1 there exists a unique depreciation schedule and a unique capital charge rate, \( \hat{r}(\theta) = r^*(\theta) \), given by \( \sum_{t=1}^{T} (1 + r^*(\theta))^{-t} \cdot z_{kt} = 1 \), that implements the intertemporal cost allocation scheme:

\[
z_i(\theta) = \left( \frac{x_{i1}}{b_i} \cdot \theta_i^*(\theta_{-i}), \ldots, \frac{x_{iT}}{b_i} \cdot \theta_i^*(\theta_{-i}) \right).
\]

As argued in the text, one solution is provided by the capital charge rate \( \hat{r}(\theta) = r^*(\theta) \) combined with the relative benefit depreciation rule and this must therefore be the unique solution.

\[\blacksquare\]

**Proof of Proposition 2.**

We again begin with the sufficiency part by showing that the Pay-the-minimum-Necessary (PMN) mechanism satisfies

\[
\pi_{it}(\theta_i, \theta_{-i} | \theta_i) \geq \pi_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i), \quad \text{for all } \theta, \tilde{\theta}_i, i, t.
\]

Under the PMN mechanism, for \( I(\theta) = 1 \) we have:

\[
\pi_{it}(\theta_i, \theta_{-i} | \theta_i) = x_{it} \cdot \theta_i - z_{it}(\theta_i) \cdot \lambda_i(\theta_i) \cdot b
\]

\[
= x_{it} \cdot \left[ \theta_i - \frac{b}{\Gamma^*(\theta) \cdot X_i \cdot \lambda_i(\theta)} \right]
\]

\[
= x_{it} \cdot \left[ \theta_i - \frac{b}{\Gamma^*(\theta) \cdot X_i \cdot \sum_{j=1}^{n} \Gamma^*(\theta) \cdot X_j \cdot \theta_j^*(\theta_{-j})} \right]
\]

\[
= x_{it} \cdot \left[ \theta_i - \theta_i^*(\theta_{-i}) \right]
\]

\[\geq 0.\]
The inequality holds because \( \theta_i \geq \theta^*_i(\theta_{-i}) \) for all \( i \), if and only if \( I(\theta) = 1 \). Moreover, \( \pi_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i) \) is independent of \( \tilde{\theta}_i \) for any \( \tilde{\theta}_i \) resulting in \( I(\theta) = 1 \), and similar arguments apply to the case where \( I(\theta) = 0 \). This proves the sufficiency part of Proposition 2.

Regarding the necessity part of the proof, we write manager \( i \)'s

\[
\pi_{it}(\tilde{\theta} | \theta_i) = [x_{it} \cdot \theta_i - z_{it}(\theta) \cdot \lambda_i(\theta) \cdot b] \cdot I(\theta)
\]

Using arguments parallel to the ones in the proof of Proposition 1, we can show that \( z_{it}(\theta_i, \theta_{-i}) = x_{it} \cdot \theta^*_i(\theta_{-i})/(\lambda_i(\theta) \cdot b) \) for any \( \theta_i \geq \theta^*_i(\theta_{-i}) \). By Lemma 1 there exist a unique capital charge rate \( \tilde{r}(\theta) \) and a unique depreciation schedule, \( d_i(\theta) \), implementing the intertemporal cost allocation

\[
z_i(\theta) = \left( x_{i1} \cdot \frac{\theta^*_i(\theta_{-i})}{\lambda_i(\theta) \cdot b}, ..., x_{iT} \cdot \frac{\theta^*_i(\theta_{-i})}{\lambda_i(\theta) \cdot b} \right),
\]

for any \( i \).

It remains to demonstrate the uniqueness of the asset sharing rule, \( (\lambda_i(\theta), ..., \lambda_n(\theta)) \). Relative benefit depreciation ensures that

\[
x_{it} \cdot \theta^*_i(\theta_{-i}) = z_{it}(\theta) \cdot \lambda_i(\theta) \cdot b = \frac{x_{it}}{\Gamma^*(\theta) \cdot X_i} \cdot \lambda_i(\theta) \cdot b.
\]

\[\iff \lambda_i(\theta) = \frac{\Gamma^*(\theta) \cdot X_i \cdot \theta^*_i(\theta_{-i})}{\sum_{j=1}^{n} \Gamma^*(\theta) \cdot X_j \cdot \theta^*_j(\theta_{-j})} \]

This completes the proof of Proposition 2.

Proof of Proposition 3

For a given profile of types other than division \( i, \theta_{-i} \), let \( U_{it} (\tilde{\theta}_i, \theta_{-i} | \theta_i) \) denote type \( \theta_i \) manager’s utility payoff in period \( t \) when he reports \( \tilde{\theta}_i \). Therefore, the manager’s
total utility payoffs are given by \( U_i(\bar{\theta}_i, \theta_{-i} | \theta_i) \equiv \sum_{t=1}^{T} \gamma^t \cdot U_{it}(\bar{\theta}_i, \theta_{-i} | \theta_i) \). Under the competitive hurdle rate mechanism,

\[
U_{it}(\bar{\theta}_i, \theta_{-i} | \theta_i) = \alpha(\bar{\theta}_i, \theta_{-i}) + \beta(\bar{\theta}_i, \theta_{-i}) \cdot [a_{it} + I_{i}^*(\bar{\theta}_i, \theta_{-i}) \cdot (x_{it} \cdot \theta_i - z_{it}(\bar{\theta}_i, \theta_{-i}))] - v_{it}(a_{it}),
\]

where \( I_{i}^*(\cdot, \cdot) \) denotes the optimal investment decision rule and \( z_{it}(\cdot, \cdot) \) denotes the sum of depreciation and interest charges, as defined in (5).

For each division, the central office chooses the compensation parameters \( \{\alpha_{it}, \beta_{it}\}_{t=1}^{T} \) such that \( \beta_{it}(\bar{\theta}) = v_{it}' \) and \( \alpha_{it}(\bar{\theta}) = v_{it}(\bar{a}_{it}) - v_{it}' \cdot \bar{a}_{it} \) for any report profile \( \bar{\theta} \). This choice of bonus coefficients ensures that \( a_{it} = \bar{a}_{it} \) for each \( i \) and \( t \). Furthermore, the relative benefit depreciation schedule corresponding to the agency-adjusted competitive hurdle rate \( r_{i}^{**}(\theta_{-i}) \) implies that \( z_{it}(\bar{\theta}, \theta_{-i}) = x_{it} \cdot \theta_i^{**}(\theta_{-i}) \). As a consequence, manager \( i \)'s total utility payoff becomes:

\[
U_i(\bar{\theta}_i, \theta_{-i} | \theta_i) = \kappa_i \cdot I_{i}^*(\bar{\theta}_i, \theta_{-i}) \cdot [\theta_i - \theta_i^{**}(\theta_{-i})]
\]

(33)

where \( \kappa_i \equiv \sum_{t=1}^{T} \gamma^t \cdot v_{it}' \cdot x_{it} \).

It is clear from (33) that manager \( i \)'s participation constraint holds for each \( \theta_{-i} \), and each manager has a strong dominant strategy incentive to report his information truthfully. Furthermore, a comparison with (37) reveals that manager \( i \)'s interim utility payoffs \( E_{\theta_{-i}}[U_i(\theta_i, \theta_{-i} | \theta_i)] \) from (33) coincide with those of the optimal revelation mechanism.

\[\blacksquare\]

**Proof of Corollary 3**

As parameterized by \( \kappa \), division \( i \)'s cutoff profitability parameter \( \theta_i^{**}(\theta_{-i} | \kappa) \) is defined by the equation:

\[
VNPV_i(\theta_i^{**}(\theta_{-i} | \kappa)) = \max_{j \neq i} \{VNPV_j(\theta_j | \kappa_j), 0\}.
\]

Since the virtual NPV is given by

\[
VNPV_i(\theta_i | \kappa_i) = NPV_i(\theta_i) - \kappa_i \cdot H_i(\theta_i),
\]

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it follows that the virtual NPV is uniformly decreasing in $\kappa_i$. This immediately implies that $\theta^{**}_i(\theta_{-i} | \kappa)$ is uniformly increasing in $\kappa_i$ and uniformly decreasing in $\kappa_j$ for all $j \neq i$. The result follows since the agency-adjusted competitive hurdle rate $r^{**}_i(\theta | \kappa)$ is equal to $r^o_i(\theta^{**}_i(\theta_{-i} | \kappa))$, and $r^o_i(\cdot)$ is an increasing function.

**Proof of Corollary 4**

Given uniform agency problems (i.e., $\kappa_j = \kappa$ for all $j$), take division $i$ to be the one with the highest virtual NPV and division $j$ to be the one with the second-highest virtual NPV. The condition describing division $i$’s cutoff profitability parameter then can be rewritten as follows:

$$NPV_i(\theta^*_i(\theta_{-i} | \kappa)) - \kappa \cdot H(\theta^*_i(\theta_{-i} | \kappa)) - [NPV_j(\theta_j) - \kappa \cdot H(\theta_j)] \equiv 0. \quad (34)$$

By the Implicit Function Theorem,

$$\frac{d\theta^*_i(\theta_{-i} | \kappa)}{d\kappa} = \frac{\kappa \cdot [H(\theta^*_i(\theta_{-i} | \kappa)) - H(\theta_j)]}{\Gamma \cdot X_i - \kappa \cdot H'(\theta^*_i(\theta_{-i} | \kappa))}.$$ 

Since the denominator of this ratio is positive and the inverse hazard $H(\cdot)$ is a decreasing function, we have

$$\text{sign} \left( \frac{d\theta^*_i(\theta_{-i} | \kappa)}{d\kappa} \right) = \text{sign}(\theta_j - \theta^*_i(\theta_{-i} | \kappa)). \quad (35)$$

Consider first the case where $NPV_i(\theta) \geq NPV^1(\theta_{-i}) = NPV_j(\theta_j)$ and suppose that, contrary to our claim, $\theta^*_i(\theta_{-i} | \kappa)$ (and hence $r^*_i(\theta | \kappa)$) were decreasing in $\kappa$. That is, $\theta_j < \theta^*_i(\theta_{-i} | \kappa)$ by (35). Using this last inequality for the left-hand side of (34) gives

$$NPV_i(\theta^*_i(\theta_{-i} | \kappa)) - \kappa \cdot H(\theta^*_i(\theta_{-i} | \kappa)) - [NPV_j(\theta_j) - \kappa \cdot H(\theta_j)]$$

$$> NPV_i(\theta_j) - NPV_j(\theta_j)$$

$$\geq 0,$$

contradicting (34). These arguments can be reversed to complete the proof. ■
Proof of Proposition 4

Under the PMN mechanism, manager $i$’s utility payoffs in period $t$ are given by:

$$U_{it}(\tilde{\theta}_i, \theta_{-i} | \theta_i) = \alpha(\tilde{\theta}_i, \theta_{-i}) + \beta(\tilde{\theta}_i, \theta_{-i}) \cdot [a_{it} + I^*(\tilde{\theta}_i, \theta_{-i}) \cdot (x_{it} \cdot \theta_i - z_{it}(\tilde{\theta}_i, \theta_{-i}))] - v_{it}(a_{it}).$$

As in the proof of Proposition 3, high managerial efforts can be induced by setting $\beta_{it}(\tilde{\theta}) = v_{it}'$ for each report $\tilde{\theta}$. Combined with the PMN asset allocation rule $\{\lambda_i(\tilde{\theta}_i, \theta_i)\}_{i=1}^N$, relative benefit depreciation corresponding to the hurdle rate $r^{**}(\tilde{\theta}_i, \theta_{-i})$ ensures that $z_{it}(\tilde{\theta}, \theta_{-i}) = x_{it} \cdot \theta^{**}_i(\theta_{-i})$. When the fixed payments are chosen such that $\alpha_{it}(\tilde{\theta}) = v_{it}(\bar{a_{it}}) - v_{it}' \cdot \bar{a_{it}}$, manager $i$’s total utility payoffs become:

$$U_i(\tilde{\theta}_i, \theta_{-i} | \theta_i) = \kappa_i \cdot I^*(\tilde{\theta}_i, \theta_i) \cdot [\theta_i - \theta^{**}_i(\theta_{-i})]. \quad (36)$$

Clearly, the above PMN mechanism ensures that manager $i$’s participation constraint holds for each $\theta_{-i}$, and each manager has a strong dominant strategy incentive to report his information truthfully. Furthermore, each manager’s interim utility payoffs $E_{\theta_{-i}}[U_i(\theta_i, \theta_{-i} | \theta_i)]$ from (36) coincide with those of the optimal revelation mechanism as given in (37).

Proof of Corollary 5

Since $\phi_i(\theta_i) = V_i(\theta_i) - \kappa_i \cdot H_i(\theta_i)$, equation (26) implies that the cutoff-profitability parameter $\theta^{**}_i(\theta_{-i} | \kappa)$ is uniformly increasing in $\kappa_j$ for all $1 \leq j \leq n$. The result follows because:

$$r^{**}(\theta | \kappa) = r^o(\theta^{**}_1(\theta_{-1} | \kappa), \cdots, \theta^{**}_n(\theta_{-n} | \kappa)),$$

and $r^o(\cdot)$ increases in each of its arguments.
Appendix B: Derivation of Relaxed Program $\mathcal{P}'$

The participation constraint $E_{\theta_{-i}}[U_i(\theta_i, \theta_{-i} | \theta_i)] \geq 0$ in the principal’s original program $\mathcal{P}'$ will hold with equality for the lowest type $\theta_i$. This boundary condition combined with the fact that $a_{it}(\theta_i, \theta_{-i} | \theta_i) = c_{it}(\theta_i, \theta_{-i}) - x_{it} \cdot \theta_i \cdot I_i(\theta_i, \theta_{-i})$ and the “local” incentive compatibility condition implies that manager $i$ will earn the following informational rents:

$$E_{\theta_{-i}}[U_i(\theta_i, \theta_{-i} | \theta_i)] = E_{\theta_{-i}} \left[ \int_{\theta_i}^{\theta_t} \sum_{t=1}^{T} \gamma^t \cdot v_{it}'(a_{it}(q_i, \theta_{-i} | q_i)) \cdot x_{it} \cdot I_i(q_i, \theta_{-i}) \, dq_i \right].$$

(37)

Integrating by parts yields that manager $i$’s expected informational rents are given by:

$$E_{\theta}[U_i(\theta_i, \theta_{-i} | \theta_i)] = E_{\theta_{-i}} \left[ \int_{\theta_i}^{\theta_t} \sum_{t=1}^{T} \gamma^t \cdot v_{it}'(a_{it}(\theta)) \cdot x_{it} \cdot I_i(\theta) \cdot H_i(\theta_i) \right] f_i(\theta_i) \, d\theta_i,$$

(38)

where

$$a_{it}(\theta) \equiv a_{it}(\theta_i, \theta_{-i} | \theta_i).$$

Since $U_i(\theta_i, \theta_{-i} | \theta_i) \equiv \sum_{t=1}^{T} \gamma^t \cdot [s_{it}(\theta_i, \theta_{-i} | \theta_i) - v_{it}(a_{it}(\theta))],$ substituting (38) into the objective function in $\mathcal{P}$ yields:

$$\max_{(a_{it}(\theta), I_i(\theta))_{i=1}^{n}} E_{\theta} \left[ \sum_{i=1}^{n} \Pi_i(\theta) - B(\theta) \right]$$

subject to$(i_e), (i_a)$,

where

$$\Pi_i(\theta) \equiv \sum_{t=1}^{T} \gamma^t \cdot \{ a_{it}(\theta) - v_{it}(a_{it}(\theta)) + x_{it} \cdot [\theta_i - v_{it}'(a_{it}(\theta)) \cdot H_i(\theta_i)] \cdot I_i(\theta) \}.$$  

For any given investment rule $I_i(\theta)$, the central office will choose $a_{it}$ to maximize:

$$a_{it} - v_{it}(a_{it}) - v_{it}'(a_{it}) \cdot x_{it} \cdot H_i(\theta_i) \cdot I_i(\theta)$$

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Condition (18) implies that 1 \(- v_{it}'(\bar{a}_{it}) - v_{it}''(\bar{a}_{it}) \cdot x_{it} \cdot H_{i}(t) \cdot I(\theta) > 0 \) for all \( \theta \) and each \( I_{i}(\theta) \in \{0, 1\} \). It is therefore optimal to induce the highest level of effort \( \bar{a}_{it} \) for all \( \theta \). Consequently, the optimization program in \( P \) simplifies to the program in \( \hat{P}' \), and the optimal investment rules for the exclusive and shared asset settings are as given by (20) and (21), respectively.

To complete the proof, we need to show that the resulting scheme is globally incentive compatible. As shown in Mirrlees (1971), a mechanism is incentive compatible provided it is locally incentive compatible, and \( \frac{\partial U_i(\bar{\theta}_i, \theta_{-i} \mid \theta_i)}{\partial \theta_i} \) is weakly increasing in \( \bar{\theta}_i \). For the above mechanism:

\[
\frac{\partial U_i(\bar{\theta}_i, \theta_{-i} \mid \theta_i)}{\partial \theta_i} = \sum_{t=1}^{T} \gamma^t \cdot v_{it}'(a_{it}(\bar{\theta}_i, \theta_{-i} \mid \theta_i)) \cdot x_{it} \cdot I_{i}(\bar{\theta}_i, \theta_{-i}),
\]

which is increasing in \( \bar{\theta}_i \) since \( v_{it}'(a_{it}(\cdot, \theta_{-i} \mid \theta_i)) \) is increasing in \( \bar{\theta}_i \) and the optimal \( I_{i}(\cdot, \theta_{-i}) \) is an upper-tail investment policy in both settings. \( \blacksquare \)
References


