Managerial Expertise, Private Information and Pay-Performance Sensitivity

Sunil Dutta*
Haas School of Business
University of California, Berkeley

September 2005

*I would like to thank the conference participants at the University of Bern and workshop participants at INSEAD for their useful comments on this paper.
Managerial Expertise, Private Information and Pay-Performance Sensitivity

Abstract

This paper characterizes optimal pay-performance sensitivities of compensation contracts for managers who have private information about their skills, and those skills affect their outside employment opportunities. The model presumes that the rate at which a manager’s opportunity wage increases in his expertise depends on the nature of that expertise, i.e., whether it is general or firm-specific. The analysis demonstrates that when managerial expertise is largely firm-specific (general), the optimal pay-performance sensitivity is lower (higher) than its optimal value in a benchmark setting of symmetric information. Furthermore, when managerial skills are largely firm-specific (general), the optimal pay-performance sensitivity decreases (increases) as managerial skills become a more important determinant of firm performance. Taken together, the results predict that pay-performance sensitivities will be higher for managers of firms in which managerial expertise is (i) a relatively more important determinant of performance, and (ii) more mobile.
1 Introduction

A large body of research, theoretical as well as empirical, has investigated magnitudes and determinants of pay-performance sensitivities of executive compensation contracts. Theoretical research has primarily relied on moral hazard models in which optimal pay-performance sensitivities reflect the usual tradeoff between risk and effort incentives. Empirical research testing for predictions of standard moral hazard agency theory has, however, had only limited success.\(^1\) This lack of empirical validity may not be entirely surprising, since unobservable efforts are not likely to be the only driving force underlying executive compensation contracts. Top executives also possess information and expertise which are equally, if not more, important drivers of performance.\(^2\) The speed of technological change in recent years has further increased the importance of managerial expertise in modern business enterprises.\(^3\) The purpose of this paper is to characterize optimal compensation contracts when managers have private information about their skills.

The paper considers an agency relationship between a firm’s owner and its manager. The manager has expertise which is valuable to the firm. In particular, the firm’s expected output is increasing in the level of managerial expertise known privately to the manager. The model distinguishes between general and firm-specific expertise. While the returns of firm-specific expertise can only be realized inside the firm, general expertise has value inside as well as outside the firm.\(^4\) It is therefore natural to expect that the firm-specificity of managerial expertise will be one of the key determinants of the manager’s alternative employment opportunities. The model presumes that if the manager’s expertise is entirely firm-specific, his reservation wage does not vary with his ability. In contrast, if managerial expertise is general, the manager’s opportunity wage, in self employment or alternative employment, is increasing in his skills.

When the manager has private information about his expertise, an optimal compensation

\(^1\)For instance, a negative association between risk and pay-performance sensitivity is one of the central predictions of this theory. As reviewed in Prendergast (2002), however, the empirical evidence has been quite mixed.

\(^2\)In reviewing the extant literature on executive compensation contracts, Murphy (1999) writes: “CEOs have superior skills or information. Unobservable actions cannot be the driving force underlying executive contracts…”

\(^3\)For a review of the changing nature of business enterprises, see Zingales (2000) and Rajan and Zingales (2000).

\(^4\)For a further discussion of the distinction between firm-specific and general expertise, see Becker (1964).
contract must not only solve the usual incentive problem of motivating the manager to work hard, but also ensure that the manager does not benefit from misrepresenting his private information. To understand how asymmetric information about managerial expertise, and its nature, affect the choice of optimal pay-performance sensitivity, suppose that the manager does not contribute any productive effort. If the manager is risk-averse and managerial expertise is entirely firm-specific, efficient risk-sharing requires that the pay-performance sensitivity of the optimal compensation contract must be equal to zero—that is, the manager must receive a fixed wage contract. However, when managerial expertise is general or transferable, an optimal contract must include a performance-based component. The reason is that when the value of the manager’s outside employment opportunities is increasing in his productivity, he has a natural incentive to exaggerate his expertise in an attempt to secure higher wages from the owner. To prevent the manager from overstating his ability, the manager is required to take some of his compensation in the form of performance-based pay. Put differently, the manager is asked to “back-up” his claims about his productivity by “buying” shares in the firm’s output. Therefore, relative to a first-best setting in which the manager’s opportunity wage is known to both parties, the optimal compensation contract in the asymmetric information setting is “higher-powered”.

My analysis shows that the optimal pay-performance sensitivity must increase with managerial expertise in order to screen managers of different abilities; that is, more productive managers must receive higher powered incentives. This implies a “reverse” causality between performance and pay-performance sensitivity. Conventional wisdom based on standard moral hazard models suggest a positive association between performance and incentives because higher-powered incentives induce managers to work more diligently which in turn generates better performance. In contrast my analysis shows that performance and pay-performance sensitivity are positively correlated because more expert managers, who are inherently more productive, must be optimally provided with higher-powered incentives in order to separate them from less productive managers.

When the firm’s output depends on both managerial expertise and managerial efforts, the manager faces countervailing incentives. On the one hand, the manager would like to exaggerate the value of his outside options in an attempt to convince the owner that a more generous compensation package is in order. As noted above, this forces the owner to use higher-powered incentive contracts. On the other hand, the manager now also has an
incentive to contribute as little effort as possible. Consequently, the manager will prefer to lower the owner’s performance expectations by claiming to be less expert than he really is. Such an understatement of ability allows the manager to shirk and then ascribe the resulting poor performance to lower productivity. By understating the value of the performance-based component of his compensation, the manager essentially attempts to extract a higher salary from the owner. Notice that this incentive to under-report applies only to the extent that the manager’s compensation is tied to firm performance. As the pay-performance sensitivity increases, the manager’s incentives to understate his ability becomes stronger, and therefore the manager extracts larger amount of rents from his information. To economize on the manager’s informational rents, the owner compromises on provision of effort incentives by lowering the pay-performance sensitivity of the manager’s compensation contract. The optimal pay-performance sensitivity of the manager’s compensation contract depends on which of these two countervailing incentives dominate.

The firm-specificity of managerial expertise turns out to be one of the key determinants of the manager’s reporting incentives. If managerial skills are largely firm-specific, the manager does not have a sufficiently strong reason to exaggerate his expertise, since the value of his outside options does not increase with his ability at a sufficiently high rate. Therefore, the manager’s dominant incentive is to understate his expertise in an attempt to lower the owner’s performance expectations. To mitigate this under-reporting incentive, the owner finds it optimal to lower the pay-performance sensitivity of the manager’s compensation contract. Thus, when managerial expertise is sufficiently firm-specific, the optimal contract is of lower power than what would be optimal in a symmetric information setting. Moreover, my analysis shows that the optimal pay-performance sensitivity decreases monotonically as managerial expertise becomes a more significant determinant of firm performance.

In contrast when managerial skills are sufficiently general or mobile, the value of the manager’s outside options increase with his expertise at a sufficiently high rate. As a consequence, now the manager’s dominant incentive is to exaggerate the level of his expertise (and hence his reservation wage) in an attempt to convince the owner to offer him a more generous compensation package. In such cases, optimal screening requires that the pay-performance sensitivity is higher than its optimal value in the benchmark symmetric information model. Furthermore, the analysis shows that the optimal pay-performance sensitivity increases monotonically as managerial expertise becomes a more important determinant of
firm output. My analysis also shows that, regardless of whether managerial expertise is general or firm-specific, the optimal pay-performance sensitivity is always increasing in managerial ability.

Whether managerial expertise has the overall effect of increasing or decreasing the incentive intensity of optimal compensation contracts depends on the nature of managerial expertise; that is, whether it is general or firm-specific. Taken together, my results predict that firms, in which managerial expertise is a relatively more important factor of production and more general or mobile, are more likely to offer higher-powered compensation contracts. It is commonly argued that specialized managerial expertise plays a more prominent role in knowledge-intensive new economy firms. It is also evident that managerial expertise tends to be more mobile in these firms. Empirical and anecdotal evidence suggest that performance-based pays (through stock options and other performance measures) play a more prominent role in knowledge-intensive new-economy firms than in traditional firms. These observations are consistent with my prediction that pay-performance sensitivities will be higher for managers of firms in which managerial expertise is (i) a more important determinant of performance, and (ii) more mobile.

The question of pay-performance sensitivity has received considerable attention in the executive compensation literature. In a seminal paper, Jensen and Murphy (1990) empirically investigates the extent to which CEO compensation is tied to firm performance. They find a statistically significant, but economically small, relationship between CEO pay and firm performance. This evidence has raised concerns about whether the relation between pay and performance is strong enough. Another well-documented empirical regularity in the executive compensation literature is that pay-performance sensitivities tend to vary quite widely across firms and industries. My paper generates some potential explanations for these empirical findings. First, it shows that when managers have asymmetric information about their skills and those skills are largely firm-specific, managers will optimally receive weaker incentives than those predicted by standard moral hazard agency models. Second, my paper

---

5See, for instance, Anderson et al. (2000); Core and Guay (2001); Ittner et al. (2003); and Murphy (2003).

6Jensen and Murphy (1990) find that the average CEO receives only $3.25 for every $1000 increase in firm value. Hall and Liebman (1998) examine more recent data on executive compensation, and find that the average pay-performance sensitivity is somewhat higher than that documented in Jensen and Murphy (1990).

7See Murphy (1999) and the references therein.
shows that optimal pay-performance sensitivities will vary systematically with managerial expertise. In addition, my analysis generates predictions about how pay-performance sensitivities relate to firm and industry characteristics, the extent of private information, and the nature of managers’ outside opportunities. These results can help explain some of the cross-sectional heterogeneity observed in executive compensation contracts.\footnote{See Milbourn (2003) for an alternative explanation of the cross-sectional heterogeneity in pay-performance sensitivities.}

The remainder of the paper is organized as follows. Section 2 considers a model of managerial expertise without an effort incentive problem. Section 3 introduces a moral hazard problem into the model, solves for the optimal contract, and derives some comparative statics results. Section 4 concludes the paper.

## 2 Basic Model

I model a one-period principal-agent relationship between a risk-neutral owner (principal) and a risk-averse manager (agent) of a firm. The firm’s gross output depends on the level of managerial expertise $\theta$. In this section, I consider a setting in which the manager does not contribute any productive effort.\footnote{Section 3 extends this analysis to a setting in which the output depends on managerial expertise as well as managerial efforts.} As a function of $\theta$, the output is given by:

$$x = \gamma \cdot \theta + \varepsilon$$

where $\gamma > 0$ is a known constant which reflects the marginal product of managerial expertise inside the firm. The noise term $\varepsilon$ is the realization of a normally distributed random variable with mean zero and variance $\sigma^2$.

The manager has private pre-contract information about his expertise $\theta$. The owner’s prior beliefs regarding $\theta$ are represented by a distribution $F(\theta)$ with positive density $f(\theta)$ on the interval $[\bar{\theta}, \bar{\theta}]$. I impose the regularity conditions that $F(\theta)$ and $f(\theta)$ are increasing functions of $\theta$, and the inverse hazard rate, $\frac{1 - F(\theta)}{f(\theta)}$, is a decreasing function of $\theta$. These monotonicity conditions are commonly imposed in the private information agency literature.\footnote{These conditions are satisfied by many commonly-used distributions such as Uniform, Normal, Exponential, and Gamma. See Bagnoli and Bergstrom (2004) for other examples of distributions that have these properties.}
As a function of his end-of-period compensation $s$, the manager’s risk preferences are given by a negative exponential utility function of the form:

$$U(s) = -exp(-\rho \cdot s),$$

where $\rho > 0$ is the manager’s coefficient of absolute risk aversion.

Consistent with much of the prior work in the agency literature, I presume that the owner designs a compensation contract that the manager can accept or reject. The manager will accept the contract if and only if his expected utility from the contract exceeds his reservation utility. In the asymmetric information agency literature, it is typically assumed that the agent’s reservation utility is independent of his private information.\textsuperscript{11} While this may be a reasonable assumption in some situations, it is not likely to hold in general. I model a more general form for the value of the manager’s outside options. In particular, the manager’s reservation wage, in self employment or alternative employment, is assumed to be (weakly) increasing in his ability. The rate at which the reservation wage increases with managerial expertise depends on the nature of that expertise; that is, whether managerial skills are firm-specific or general. In the polar case when managerial expertise is entirely firm-specific, the manager’s reservation utility is independent of ability. As managerial expertise becomes less firm-specific and more general, the value of the manager’s outside options increases in his expertise at an increasingly higher rate.\textsuperscript{12}

To capture these features in the model, I assume that the manager’s (risk-free) reservation wage is given by:

$$w(\theta) = w_0 + \lambda \cdot \gamma \cdot \theta,$$

\textsuperscript{11}For asymmetric information models of procurement and regulation, see Baron and Myerson (1982), Laffont and Tirole (1984), and Laffont and Tirole (1993). Papers that have used asymmetric information agency models to characterize optimal intrafirm resource allocation decisions include Antle and Eppen (1985), Balduin (2003), Dutta and Reichelstein (2002), Harris, Kriebel and Raviv (1982), and Harris and Raviv (1996).

\textsuperscript{12}This assumption of type-dependent reservation utility is one of the major distinctions between my model and the earlier work in the asymmetric information agency literature. With a few exceptions, the earlier work in the asymmetric information agency literature assumes that the reservation utility does not vary with the agent’s private information. For exceptions, see Lewis and Sappington (1989a) and (1989b) and Maggi and Rodriguez-Clarke (1995). These papers consider regulation settings in which a regulated firm’s reservation price depends on its private cost information. Dutta (2003) considers a capital investment setting in which the manager’s reservation utility is a function of his private information.
where \( w_0 \) and \( \lambda \) are known constants satisfying \( w_0 \geq 0 \) and \( \lambda \geq 0 \). Therefore, the manager’s reservation utility is equal to \(-exp[-\rho \cdot w(\theta)]\).

Notice that the the parameter \( \lambda \) measures the rate at which the manager’s reservation wage \( w(\cdot) \) increases with his expertise \( \theta \). When \( \lambda \) is relatively small, managerial expertise is relatively firm-specific, and the manager’s reservation wage is largely independent of \( \theta \). In the polar case of \( \lambda = 0 \), the manager’s opportunity wage is independent of his ability. In this case, the model reduces to a “standard” private information agency setting in which the reservation utility does not depend on the agent’s private information.

As managerial expertise becomes more general, the parameter \( \lambda \) increases and the manager’s opportunity wage becomes a steeper function of his ability. One can think of \( \lambda \cdot \gamma \) as a measure of the marginal productivity of managerial expertise outside the firm because the manager’s opportunity wage \( w(\cdot) \) increases in \( \theta \) at this rate. Recall that \( \gamma \) represents the marginal productivity of managerial expertise inside the firm. I assume that the manager is at least as productive inside the firm as outside the firm; that is, \( \lambda \leq 1 \).

For notational convenience, the “base” level of the manager’s outside opportunity wage, \( w_0 \), is normalized to zero.

The risk-neutral owner’s objective is to design a compensation contract so as to maximize her expected profits net of compensation payments to the manager. Formally, the owner designs a compensation schedule \( s(\hat{\theta}, x) \) which specifies the manager’s compensation as a function of: (i) the manager’s report \( \hat{\theta} \) about his expertise \( \theta \), and (ii) the realized firm performance \( x \). By the revelation principle, it is without loss of generality to consider only those compensation schedules that induce the manager to reveal his private information truthfully; that is \( \hat{\theta} = \theta \) for all \( \theta \in [\underline{\theta}, \bar{\theta}] \). We can thus think of the owner as offering a menu of compensation contracts \( s(\theta, \cdot) \) indexed by the manager’s truthful announcement of his private information \( \theta \).

For tractability, I restrict the focus of my analysis to linear compensation contacts of the form:

\[
s(\hat{\theta}, x) = \alpha(\hat{\theta}) + \beta(\hat{\theta}) \cdot x. \tag{3}
\]

Here, \( \alpha(\hat{\theta}) \) denotes the manager’s payment contingent on his report \( \hat{\theta} \), but independent of the realized value of \( x \). Consequently, \( \alpha(\cdot) \) is referred to as salary or fixed wage. The slope

\[\text{This is reasonable, since there is likely to be synergy between the incumbent manager and firm technology.}\]
coefficient $\beta(\cdot)$ measures the pay-performance sensitivity of the compensation contract. The restriction to linear contracts allows me to focus sharply on pay-performance sensitivity.\footnote{If the manager were risk-neutral, the owner’s contracting problem could be easily solved without the linearity restriction. With a risk-neutral agent, it can be shown that the linear contract of the form in (3), which is exogenously imposed in my model of a risk-averse manager, is indeed optimal for a risk-neutral manager. This implies that the restriction to linear contracts entails an arbitrarily small loss as the manager’s degree of risk aversion becomes arbitrarily small. Dutta and Reichelstein (2002) also model a privately informed risk-averse agent and impose the linearity restriction on compensation contracts. They, however, do not consider the case of type-dependent reservation utility.}

Let $EU(\hat{\theta}, \theta)$ denote the manager’s expected utility when he self-selects the contract $s(\hat{\theta}, x) = \alpha(\hat{\theta}) + \beta(\hat{\theta}) \cdot x$ by reporting $\hat{\theta}$, but his true type is $\theta$. That is,

$$EU(\hat{\theta}, \theta) = \int_{-\infty}^{\infty} -exp \left[ -\rho \cdot \left( \alpha(\hat{\theta}) + \beta(\hat{\theta}) \cdot (\gamma \cdot \theta + \varepsilon) \right) \right] g(\varepsilon) d\varepsilon,$$

Since $g(\varepsilon)$ is a normal density function, the manager’s expected utility takes the following certainty equivalent form:

$$EU(\hat{\theta}, \theta) = -exp[-\rho \cdot CE(\hat{\theta}, \theta)]$$

where $CE(\hat{\theta}, \theta)$ denotes the certainty equivalent of the manager’s expected utility, and is given by the following mean-variance expression:

$$CE(\hat{\theta}, \theta) = \alpha(\hat{\theta}) + \beta(\hat{\theta}) \cdot \gamma \cdot \theta - \frac{1}{2} \cdot \rho \cdot (\beta(\hat{\theta}))^2 \cdot \sigma^2$$

Given (4) and (5), the manager’s preferences over linear compensation contracts can be conveniently represented by $CE(\hat{\theta}, \theta)$. Let $CE(\theta)$ denote the manager’s certainty equivalent when he reports his private information truthfully; i.e., $CE(\theta) \equiv CE(\theta, \theta)$.

The risk-neutral owner chooses the parameters of the linear compensation plan $s(\theta, x)$ to maximize the expected value of her net profit. The owner’s optimization problem is as follows:

$$\max_{\{\alpha(\theta), \beta(\theta)\}} \int_{\theta}^{\hat{\theta}} \left[ \gamma \cdot \theta - \alpha(\theta) - \beta(\theta) \cdot \gamma \cdot \theta \right] f(\theta) d\theta$$

subject to:
The objective function of the above optimization program represents the expected value of the owner’s payoffs net of compensation payments to the manager. The participation constraints in (i) ensure that the manager is always guaranteed at least his reservation wage \( w(\theta) \). The incentive compatibility constraint in (ii) ensure that when the true value of managerial expertise is \( \theta \), the manager (weakly) prefers to truthfully reveal this fact rather than claim to possess some other level of expertise, \( \hat{\theta} \).

Since the manager is risk-averse and there is no hidden effort problem, a simple fixed wage contract would be optimal if the value of \( \theta \) were known to the both parties. In such a symmetric information setting, the manager would optimally receive a fixed compensation of \( \lambda \cdot \gamma \cdot \theta \) for each \( \theta \). In the setting under consideration, however, the value of \( \theta \) is the manager’s private information. The following result shows that a flat wage contract cannot be optimal as long as managerial skills are general (i.e., \( \lambda > 0 \)):

**Proposition 1** The optimal pay-performance sensitivity is given by:

\[
\beta^*(\theta) = \min \left\{ \lambda, \frac{\gamma}{\rho \cdot \sigma^2} \cdot \frac{F(\theta)}{f(\theta)} \right\}.
\]

**Proof:** All proofs are in the appendix.

Proposition 1 shows that a performance-based contract is optimal even in the absence of a managerial moral hazard problem. The reason is that when the manager’s reservation wage increases in his expertise and the manager has private information about it, he will have an incentive to overstate the value of his outside options in an attempt to extract higher compensation from the owner. To prevent the manager from such overstatement, the owner finds it optimal to rely on a performance-contingent contract in which more expert managers receive more generous compensation through higher performance-based pays rather than through higher fixed wages.

To gain a better understanding, it is instructive to consider the incentive consequences of a fixed wage contract. To ensure the manager’s participation with a fixed contract, the
manager’s fixed wage $\alpha(\theta)$ must be chosen so that it is at least equal to his opportunity wage of $\lambda \cdot \gamma \cdot \theta$. If the owner were to simply rely on the manager’s report in setting the manager’s fixed wage, however, the manager would have in incentive to exaggerate the value of $\theta$. In fact, the manager would always claim to be of the highest type; that is, he would always report $\hat{\theta} = \bar{\theta}$. To induce truthful reporting and ensure participation from all managerial types through a fixed wage contract, it must therefore be the case that:

$$\alpha(\theta) = \lambda \cdot \gamma \cdot \bar{\theta}$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Under such a contract, however, the manager would earn rents—he would receive more than his opportunity wage for all values of $\theta$ except when $\theta = \bar{\theta}$.

As an alternative to the above fixed wage contract, consider the following performance-based contract:

$$s(x) = \frac{1}{2} \cdot \rho \cdot \lambda^2 \cdot \sigma^2 + \lambda \cdot x$$  \hspace{1cm} (7)

Observe that the contract in (7) entirely eliminates the manager’s incentives for overstating the value of $\theta$. To see this, notice that the manager’s compensation in (7) is independent of his report, and therefore he has no reason to misreport his private information. Since the pay-performance sensitivity is set at $\lambda$, the expected value of the manager’s bonus earnings is exactly equal to his opportunity wage of $\lambda \cdot \gamma \cdot \theta$. Furthermore, the fixed wage of $\frac{1}{2} \cdot \rho \cdot \lambda^2 \cdot \sigma^2$ ensures that the risk-averse manager is adequately compensated for bearing the output risk. Consequently, the compensation contract in (7) ensures that each type of manager earns exactly his reservation utility. Note, however, that this contract is inefficient from a risk-sharing perspective because it requires the owner to pay a risk premium in the amount of $\frac{1}{2} \cdot \rho \cdot \lambda^2 \cdot \sigma^2$. At the optimum, the owner balances her objectives of appropriating the manager’s informational rents and minimizing the cost of imposing risk on the manager.

Since the manager’s informational rents can be entirely eliminated by setting $\beta(\theta) = \lambda$, it will never be optimal to set the pay-performance sensitivity above $\lambda$. Figure 1 depicts the optimal pay-performance sensitivity as a function of managerial expertise. For all $\theta$ in an upper-tailed interval $[\theta^*, \bar{\theta}]$, it is indeed optimal to set the pay-performance sensitivity equal to $\lambda$. In this region, the manager’s participation constraint binds; i.e., the manager earns no informational rents.\footnote{As shown in the appendix, $\theta^* < \bar{\theta}$ if and only if $\gamma \cdot \frac{F(\bar{\theta})}{f(\bar{\theta})} > \lambda \cdot \rho \cdot \sigma^2$.} When $\theta$ is below $\theta^*$, the pay-performance sensitivity increases...
monotonically in $\theta$ from a value of zero at $\theta = \theta_0$ to a value of $\lambda$ at $\theta = \theta^*$. In this region, the owner finds it optimal to provide the manager with some informational rents in order to reduce the manager’s risk exposure and the attendant risk premium.

![Figure 1](image.png)

Note that the pay-performance sensitivity is increasing in managerial expertise $\theta$, i.e., more competent managers are required to take more of their compensation in the form of bonus payments rather than fixed salaries.\(^\text{16}\) To understand why $\beta^*(\theta)$ must be increasing in $\theta$, observe that the owner has to provide appropriate incentives to low type managers to prevent them from overstating their expertise. To provide such incentives, the owner must either reward lower type managers with higher rents for reporting low $\theta$, or must somehow punish them for announcing high $\theta$. The optimal incentive scheme relies on both rewards and punishments to induce truthful reporting. To punish exaggeration of expertise by lower type managers, higher type managers are required to take a bigger fraction of their overall compensation in the form of performance-based variable pay (rather than fixed pay). This mitigates the incentive for lower types to exaggerate their expertise because any potential gain from such misreporting is offset by a corresponding reduction in their bonus earnings.

The result that $\beta^*(\cdot)$ is increasing in $\theta$ implies a “reverse” causality between performance and pay-performance sensitivity. Under the conventional wisdom based on standard\(^\text{16}\) Since $\frac{\kappa(\theta)}{f(\theta)}$ is increasing in $\theta$, it can be easily verified that $\beta^*(\theta)$ is increasing.
moral hazard models, performance and pay-performance sensitivity are positively correlated because higher pay-performance sensitivity induces the manager to exert higher effort which in turn leads to higher performance. In contrast, Proposition 1 shows that performance and pay-performance sensitivity are positively correlated because the optimal screening of managers requires that more expert managers, who happen to be inherently more productive, receive higher-powered incentives.

Finally, note that, for all values of $\theta$, the pay-performance sensitivity is decreasing in the degree to which the managerial expertise is firm-specific. That is, $\beta^*(\theta)$ is uniformly decreasing in $\lambda^{-1}$. This suggests a higher level of pay-performance sensitivity in industries in which managerial human capital is relatively more mobile.

3 Model of Managerial Effort and Expertise

This section introduces a managerial moral hazard incentive problem into the model. The output now depends on managerial expertise as well as managerial efforts. To model this, I presume that the firm’s production function takes the following form:

$$x = a + \gamma \cdot \theta + \varepsilon$$

where $a \geq 0$ denotes the manager’s choice of productive effort. The interpretation is that the manager can increase the firm’s expected output by taking a personally costly and unobservable action $a$. The parameter $\gamma \geq 0$ measures the marginal product of managerial expertise relative to the marginal product of managerial efforts (which has been normalized to one). That is, $\gamma$ reflects the relative importance of managerial expertise in determining the expected firm performance.

As a function of his compensation $s$ and effort choice $a$, the manager’s utility is given by:

$$U(s, a) = -exp \left[ -\rho \cdot (s - \frac{k}{2} \cdot a^2) \right],$$

where $k > 0$ is a known constant, and $\frac{k}{2} \cdot a^2$ denotes the manager’s disutility of efforts measured in monetary terms.\textsuperscript{17} The manager’s (risk- and work-free) reservation wage is

\textsuperscript{17}The assumption of a quadratic cost function is made for expositional convenience. All of the results hold as long as the manager’s disutility is an increasing and convex function of his effort.
again assumed to be given by (2). As before, the analysis is restricted to linear compensation contracts of the form in (3).

When the manager’s true expertise is $\theta$, but he self-selects the contract $s(\hat{\theta}, x) = \alpha(\hat{\theta}) + \beta(\hat{\theta}) \cdot x$ by reporting $\hat{\theta}$, it can be shown that the manager’s certainty equivalent takes the following mean-variance form:

$$CE(\hat{\theta}, \theta) = \alpha(\hat{\theta}) + \beta(\hat{\theta}) \cdot [a + \gamma \cdot \theta] - \frac{1}{2} \cdot [k \cdot a^2 + \rho \cdot (\beta(\hat{\theta}))^2 \cdot \sigma^2]$$  \hspace{1cm} (8)

As before, let $CE(\theta) \equiv CE(\theta, \theta)$ denote the manager’s certainty equivalent when he reports his private information truthfully. Given the mean-variance representation of the manager’s expected utility in (8), the manager’s effort incentive compatibility constraint can be replaced with the corresponding first-order-condition:

$$a(\theta) = \frac{\beta(\theta)}{k}$$

Therefore, the owner’s optimization problem becomes:

$$\max_{\{a(\theta), \alpha(\theta), \beta(\theta)\}} \int_{\theta}^{\hat{\theta}} \left[ \gamma \cdot \theta + a(\theta) - \alpha(\theta) - \beta(\theta) \cdot (\gamma \cdot \theta + a(\theta)) \right] f(\theta) \, d\theta$$ \hspace{1cm} (9)

subject to:

(i) $CE(\theta) \geq \lambda \cdot \gamma \cdot \theta$ for all $\theta$,
(ii) $CE(\theta) \geq CE(\hat{\theta}, \theta)$ for each $\hat{\theta}$ and $\theta$,
(iii) $a(\theta) = \frac{\beta(\theta)}{k}$ for all $\theta$.

As a benchmark, let us first consider a symmetric information setting in which the value of $\theta$ is known to the both parties. The truth-telling constraints in (ii) drop out, and the contracting problem reduces to the familiar linear contracting model of pure moral hazard.\footnote{See Holmstrom and Milgrom (1991). Linear contracting models have also been used quite frequently in the accounting literature. See, for instance, Bushman and Indjejikian (1993) and Feltham and Xie (1992). Lambert (2001) provides a comprehensive review of the linear contracting framework and its applications in accounting.}

In choosing the optimal pay-performance sensitivity, the owner trades off the cost of exposing the manager to risk against the benefit of providing effort incentives. Using the well-known
arguments, it can be shown that the optimal pay-performance sensitivity in this benchmark setting is given by:

\[
\beta^0 = \frac{1}{1 + k \cdot \rho \cdot \sigma^2}
\]  

(10)

In the setting under consideration, however, the value of \( \theta \) is the manager’s private information. Therefore, the optimal solution to the owner’s contracting problem has to take into account the incentive compatibility constraints in (ii), and the type-contingent participation constraints in (i).

### 3.1 Firm-Specific Expertise

First, suppose that managerial expertise is entirely firm-specific; that is, \( \lambda = 0 \). Since the reservation utility no longer depends on the manager’s private information, the owner’s optimization program reduces to a standard adverse selection problem. As shown in the appendix, the incentive compatibility condition in combination with the participation constraints implies that the manager’s certainty equivalent must take the form:

\[
CE(\theta) = \int_\theta^0 \gamma \cdot \beta(u) \, du
\]  

(11)

The above equation shows that if the manager is provided with non-trivial effort incentives (i.e., if \( \beta > 0 \)), the manager will earn informational rents since \( CE(\theta) \) will exceed his reservation wage of zero.

Notice that the expected output level is increasing in \( \theta \). Therefore, for any given pay-performance sensitivity \( \beta \), the manager’s expected bonus (i.e., \( \beta \cdot \gamma \cdot \theta \)) is also increasing in \( \theta \). If the the value of \( \theta \) were known to the owner, a more expert manager would receive a correspondingly smaller salary. That is, in the symmetric information setting, the manager’s salary would be a decreasing function of his expertise. Since the manager has private information about \( \theta \), however, such a compensation scheme would induce the manager to understate his expertise in an attempt to receive higher salary. To ensure that the manager does not benefit from any misreporting of his expertise, the manager must be provided with informational rents. Furthermore, to deter a higher type manager from mimicking as lower type, the higher type manager must be provided with a greater amount
of rents than the lower type. That is, the manager’s informational rents must be increasing in \( \theta \).

**Proposition 2**  
If managerial expertise is entirely firm-specific (that is, \( \lambda = 0 \)), the optimal pay-performance sensitivity is given by:

\[
\beta^*(\theta) = \max\left\{0, \beta^0 \cdot \left(1 - k \cdot \gamma \cdot \frac{1 - F(\theta)}{f(\theta)}\right)\right\}.
\]

(12)

Proposition 2 shows that the optimal pay-performance sensitivity is lower than its optimal value in the benchmark setting of symmetric information; that is, \( \beta^*(\theta) < \beta^0 \) for all \( \theta < \bar{\theta} \).\(^{19}\)

Recall that the manager has an incentive to understate the value of \( \theta \). To encourage him to reveal his private information truthfully, the manager must be provided with informational rents. Equation (11) shows that the amount of informational rents needed to restore truthful reporting is increasing in the pay-performance sensitivity of the manager’s compensation contract. A fixed wage contract in which \( \beta(\theta) = 0 \) for all \( \theta \) would eliminate any need for paying informational rents, since the manager would then have no reason to misrepresent his private information. Such a fixed wage contract, however, would fail to generate any effort incentives. The optimal pay-performance sensitivity \( \beta^*(\theta) \) in Proposition 2 balances the owner’s conflicting objectives of providing efficient effort incentives, which requires that the pay-performance sensitivity is equal to \( \beta^0 \), and minimizing the manager’s informational rents, which requires that the manager’s pay is independent of performance.

For certain parameters, it can even be optimal to provide no effort incentives to lower type managers. To see that this can indeed happen, suppose \( \gamma = 2, k = 1 \), and \( \theta \) is uniformly distributed over the unit interval. Note that the inverse hazard rate \( \frac{1 - F(\theta)}{f(\theta)} \) becomes equal to \( 1 - \theta \), and hence the expression for the optimal pay-performance sensitivity simplifies to:

\[
\beta^*(\theta) = \max\{0, \beta^0 \cdot (2 \cdot \theta - 1)\}.
\]

The optimal pay-performance sensitivity schedule for this case is depicted in Figure 2.

---

\(^{19}\)Since the inverse hazard rate function at \( \theta = \bar{\theta} \) is zero, i.e., \( \frac{1 - F(\theta)}{f(\theta)} = 0 \), equation (12) shows that the highest type manager (i.e., \( \theta = \bar{\theta} \)) will receive efficient effort incentives (i.e., \( \beta^*(\theta) = \beta^0 \)). This is the well-known “no distortion at the top” result.
Thus, the manager is provided with a fixed wage contract (i.e., $\beta^*(\theta) = 0$) for all $\theta < 0.5$. To understand why this is optimal, notice that when lower type managers do not receive any performance-based pay, higher type managers do not have any incentive to mimic as lower types. This reduces the amount of rents commanded by higher types for revealing their private information. The expected value of benefits that the owner forgo by not providing any effort incentives for low values of $\theta$ is more than offset by her savings from reduced information rents for high values of $\theta$.

It is also worth noting that the optimal pay-performance sensitivity is increasing in $\theta$, since the inverse hazard rate $\frac{1 - F(\cdot)}{f(\cdot)}$ is decreasing. In this model of moral hazard and asymmetric information, therefore, performance and pay-performance sensitivity are positively correlated for two different reasons: (i) more productive managers receive higher-powered incentives, and (ii) higher-powered incentives induce higher efforts which lead to better performance.

### 3.2 General Expertise

Having characterized the optimal solution when managerial expertise is completely firm-specific, I now consider the general case of $\lambda > 0$. The preceding analysis shows that when managerial expertise is entirely firm-specific and there is an effort incentive problem,
the manager has an incentive to understate his ability in an attempt to lower the owner’s performance expectations. In contrast, the analysis in Section 2 has shown that when the manager’s outside opportunity wage is increasing in $\theta$ and there is no moral hazard problem, the manager has an incentive to exaggerate his expertise in an attempt to convince the owner that the reservation wage is high. When there is a managerial moral hazard problem and the value of the manager’s outside options increase in his ability, the manager faces countervailing incentives.\footnote{Lewis and Sappington (1989a) examine countervailing incentives in a regulation setting in which a regulated firm privately observes a state variable that impacts the firm’s marginal and fixed costs in opposite directions. They show that when countervailing incentives arise, pooling generally characterizes the equilibrium contract. Maggi and Rodriguez-Clare (1995) extend this work and provide sufficient conditions under which the optimal contract is separating. See also Lewis and Sappington (1989b).} The manager would like to claim that his outside opportunity wage is high, but his productivity inside the firm is low. Since the manager’s productivity inside the firm and employment opportunities outside the firm are positively correlated, however, the manager cannot credibly make these two claims simultaneously.

The following result characterizes the optimal pay-performance sensitivity:

**Proposition 3**  
(i) When managerial expertise is sufficiently firm-specific; specifically, when $\lambda \leq \beta^0$, the optimal pay-performance sensitivity is given by:

\[
\beta^*(\theta) = \max \left\{ \lambda, \beta^0 \cdot \left( 1 - k \cdot \gamma \cdot \frac{1 - F(\theta)}{f(\theta)} \right) \right\}
\]

(ii) When managerial expertise is sufficiently general; specifically, when $\lambda \geq \beta^0$, the optimal pay-performance sensitivity is given by:

\[
\beta(\theta) = \min \left\{ \lambda, \beta^0 \cdot \left( 1 + k \cdot \gamma \cdot \frac{F(\theta)}{f(\theta)} \right) \right\}
\]

Proposition 3 reveals several features of the optimal contract that warrant some discussion. Consider first the case when managerial expertise is sufficiently firm-specific, i.e., $\lambda < \beta^0$. Note that the optimal pay-performance sensitivity is strictly lower than its optimal value in the benchmark setting of symmetric information. That is, $\beta^*(\theta) < \beta^0$ for all
Moreover, the optimal pay-performance sensitivity is bounded from below at \( \lambda \) for all values of \( \theta \in [\bar{\theta}, \tilde{\theta}] \).

To understand these features of the optimal contract, it is useful to consider the incentive properties of the following contract:

\[
s(\theta, x) = \frac{1}{2} \cdot k \cdot a^* + \frac{1}{2} \cdot \rho \cdot \lambda^2 \cdot \sigma^2 + \lambda \cdot (x - a^*),
\]

where \( a^* \equiv \frac{\lambda}{k} \) denotes the induced effort choice. Observe that this contract satisfies the manager’s incentive compatibility constraints; the manager has no reason to misrepresent his private information because his compensation does not depend on his report. The pay-performance sensitivity of the contract in (13) is equal to \( \lambda \), which ensures that the manager’s expected bonus is exactly equal to his opportunity wage of \( \lambda \cdot \gamma \cdot \theta \) for all values of \( \theta \). Moreover, the fixed wage, given by the first two terms on the right-hand side of (13), is chosen so as to adequately compensate the manager for his cost of providing effort and bearing risk. Therefore, contract (13) ensures that \( CE(\theta) = \lambda \cdot \gamma \cdot \theta \) for each value of \( \theta \); that is, the manager does not earn any rents. Since the only reason for distorting the manager’s effort incentives is to reduce his informational rents, and since contract (13) can eliminate the manager’s rents entirely, \( \lambda \) is the lower bound on the optimal pay-performance sensitivity.

While the contract in (13) could eliminate the manager’s informational rents, it would generally be inefficient from the perspective of providing managerial effort incentives. In particular, since \( \lambda < \beta^0 \), the pay-performance sensitivity of (13) is lower than its optimal value in the benchmark setting of symmetric information. If the pay-performance sensitivity is not exactly equal to \( \lambda \), however, the manager would have incentives to misrepresent his private information. When \( \lambda \) is sufficiently low, the manager’s outside options are relatively insensitive to his productivity inside the firm. Therefore, the manager’s incentive to understate his expertise (to lower the owner’s performance expectations) dominates his incentive to overstate it (to convince the owner that his opportunity wage is high). The intensity of the manager’s incentives to understate his productivity, and hence the amount of informational rents needed to restore truthful reporting, is increasing in the pay-performance sensitivity of the contract. The owner thus faces a tradeoff between providing efficient effort incentives (which requires that \( \beta^* = \beta^0 \)) and appropriating the manager’s informational rents (which is optimal only if \( \theta = \tilde{\theta} \).

---

21 The efficient level of effort incentives (i.e., \( \beta^* = \beta^0 \)) is optimal only if \( \theta = \tilde{\theta} \).
requires that $\beta^* = \lambda$). At the optimum, the owner generally compromises on both fronts; that is, the optimal effort incentives are generally set above $\lambda$ and below $\beta^0$.

The optimal pay-performance sensitivity as a function of $\theta$ is depicted in Figure 3. The optimal pay-performance sensitivity is equal to $\lambda$ for a lower-tailed interval of types, $[\bar{\theta}, \theta_1]$. In this interval, the manager receives earns no informational rents. For $\theta > \theta_1$, the manager earns informational rents and the optimal pay-performance sensitivity increases monotonically from a value of $\lambda$ at $\theta = \theta_1$ to $\beta^0$ at $\theta = \bar{\theta}$.\footnote{As shown in the appendix, the interval $[\bar{\theta}, \theta_1]$ is non-degenerate (i.e., $\bar{\theta} < \theta_1$) if and only if $\beta^0 \cdot [1 - k \cdot \gamma \cdot H(\bar{\theta})] < \lambda$, where $H(\cdot) \equiv [1 - F(\theta)]/f(\cdot)$. When this condition does not hold, the participation constraint binds only for the lowest type.}

![Figure 3](image)

Figure 3

Now consider the case when $\lambda$ is above $\beta^0$. Proposition 3 shows that the optimal pay-performance sensitivity is strictly higher than $\beta^0$ for all $\theta > \bar{\theta}$. Furthermore, the optimal incentive intensity is always below $\lambda$. To understand the intuition for these results, note that when $\lambda$ is relatively large, the manager’s opportunity wage $w(\theta)$ increases in $\theta$ at a sufficiently high rate. Consequently, the dominant incentive for the manager is to overstate the true value of $\theta$ in an attempt to convince the owner that his reservation wage is high. To prevent such misrepresentation of private information, the owner chooses to pay higher compensation to more expert managers through higher pay-performance sensitivities (rather...}
than through higher salaries). As a result, the optimal pay-performance sensitivity is higher than its optimal value in the benchmark setting of symmetric information (i.e., $\beta^*(\theta) > \beta^0$).

Since the manager’s informational rents can be entirely eliminated by contract (13) in which the pay-performance sensitivity is equal to $\lambda$, and since $\lambda > \beta^0$, it would never be optimal to set the pay-performance sensitivity above $\lambda$. As depicted in Figure 4, the optimal pay-performance sensitivity is equal to this upper bound for types in an upper-tailed interval $[\theta_2, \bar{\theta}]$. The participation constraint binds for all types in this interval. In contrast, all manager types below this region earn informational rents. The optimal pay-performance sensitivity increases monotonically from a value of $\beta^0$ at $\theta = \theta_0$ to $\lambda$ at $\theta = \theta_2$.

$$\beta^*(\theta)$$

![Figure 4](image)

It is interesting to note that the manager’s incentives to understate and overstate are in perfect balance when $\lambda = \beta^0$. In this knife-edge case, there is no conflict between the owner’s objectives of appropriating the manager’s informational rents and providing effort incentives, and hence contract in (13) is optimal for all values of $\theta$. That is, the manager receives the same effort incentives as he would in the benchmark setting of symmetric information, since $\beta^*(\theta) = \lambda = \beta^0$ for all $\theta$.

---

23 The interval $[\theta_2, \bar{\theta}]$ has a positive measure (i.e., $\theta_2 < \bar{\theta}$) if and only if $\beta^0 \cdot [1 + k \gamma \cdot h(\bar{\theta})] > \lambda$, where $h(\cdot) = F(\cdot)/f(\cdot)$. Otherwise, the participation constraint binds for the highest type, and all other types earn informational rents.
To derive cross-sectional predictions on optimal pay-performance sensitivities, let us consider how the optimal contract varies with; (i) $\gamma$, which represents the relative weight of the manager’s ability in the firm’s production function, and (ii) $\lambda$, which measure the inverse of firm-specificity of managerial expertise.

**Proposition 4**  
(i) Suppose managerial expertise is sufficiently firm-specific (i.e., $\lambda$ is less than $\beta^0$). The optimal pay-performance sensitivity decreases as managerial expertise becomes a relatively more important determinant of firm performance. That is, $\beta^*(\theta)$ is uniformly decreasing in $\gamma$.

(ii) Suppose managerial expertise is sufficiently general (i.e., $\lambda$ is greater than $\beta^0$). The optimal pay-performance sensitivity increases as managerial expertise becomes a relatively more important determinant of firm performance. That is, $\beta^*(\theta)$ is uniformly increasing in $\gamma$.

(iii) The optimal pay-performance sensitivity is decreasing in the degree to which managerial expertise is firm-specific. That is, $\beta^*(\theta)$ is uniformly decreasing in $\lambda^{-1}$ for all $\gamma$.

If managerial expertise is largely firm-specific, the optimal pay performance sensitivity declines as managerial expertise becomes a more important determinant of the output. In contrast, when managerial expertise is relatively general, the optimal pay-performance sensitivity increases as managerial expertise becomes a more crucial determinant of performance. The last part of Proposition 4 shows that the optimal pay-performance sensitivity decreases monotonically as managerial human capital becomes more firm-specific (and, hence, less mobile). Taken together, these results predict that the new-economy knowledge-intensive firms—in which managerial expertise is a more crucial factor of production and is more mobile—are likely to set higher pay-performance sensitivities in their managerial compensation contracts. This prediction appears to be consistent with the common observation and empirical evidence that performance-based compensation is more prominent in knowledge-intensive new economy firms than in traditional firms.\(^{24}\)

\(^{24}\)Anderson et al. (2000); Core and Guay (2001); Murphy (2003); and Ittner et al. (2003) provide empirical evidence on the differences between new economy and traditional firms in their reliance on performance-based plans.
4 Conclusion

Managerial expertise or human capital plays a significant role in running modern business organizations. However, managerial expertise has received little attention in the executive compensation literature. This paper introduces managerial expertise in an agency model and examines its influence on the choice of optimal compensation contracts. In particular, the paper investigates how managerial expertise affects the optimal pay-performance sensitivity.

The analysis shows that a key determinant of the optimal pay-performance sensitivity is the firm-specificity of managerial expertise. When managerial skills are largely firm-specific, the optimal pay-performance sensitivity is lower than its optimal value in a benchmark setting of symmetric information. Furthermore, the optimal pay-performance sensitivity decreases monotonically as managerial expertise becomes a more significant factor of production. In contrast, if managerial expertise is relatively general, the optimal pay-performance sensitivity is higher than its optimal value in the benchmark setting, and the optimal pay-performance sensitivity increases monotonically as managerial expertise becomes a more important determinant of performance. These results predict that pay-performance sensitivities will be higher for managers in new economy, high tech firms-in which managerial expertise is more critical and also more mobile than for managers in traditional firms.

In my analysis, the manager is exogenously endowed with a given level and form (i.e., general or firm-specific) of expertise. Future research could extend this analysis to endogenize managers’ expertise acquisition decisions. A multiperiod extension of my model can be used to examine the choice of optimal incentive plans that not only motivate managers to use their skills appropriately, but also provide them with incentives to acquire appropriate skills.
Appendix

Proof of Proposition 1
As a function of the manager’s true type $\theta$ and report $\hat{\theta}$, let $CE_e(\hat{\theta}, \theta) \equiv CE(\hat{\theta}, \theta) - \lambda \cdot \gamma \cdot \theta$ denote the manager’s certainty equivalent in excess of his reservation wage. As before, define $CE_e(\theta) \equiv CE_e(\theta, \theta)$. The participation and incentive compatibility constraints in the owner’s optimization program in (6) can then be written as follows:

$$CE_e(\theta) \geq 0 \text{ for all } \theta.$$

(14)

$$CE_e(\theta) \geq CE_e(\hat{\theta}, \theta) \text{ for all } \theta \text{ and } \hat{\theta}.$$

(15)

Using standard arguments, it can be shown that the necessary and sufficient conditions for the above incentive compatibility constraints to hold are:

(i) $CE'_e(\theta) = \gamma \cdot (\beta(\theta) - \lambda)$

(ii) $\beta(\theta)$ is increasing

for almost all $\theta \in [\underline{\theta}, \bar{\theta}]$.\(^{25}\) Conditions (i) and (ii) above imply that the manager’s excess certainty equivalent is convex in his expertise; that is., $CE''_e \geq 0$ almost everywhere. This in turn implies that the participation constraint in (14) will bind at most along a single (possibly degenerate) interval of $[\underline{\theta}, \bar{\theta}]$.

Suppose the participation constraint binds for some interval $[\theta_1, \theta_2]$. This implies that $CE'_e(\theta) = 0$ for $\theta \in [\theta_1, \theta_2]$. Condition (i) above then yields that $\beta(\theta) = \lambda$ for all $\theta \in [\theta_1, \theta_2]$ and:

$$CE_e(\theta) = \begin{cases} 
-\int_{\theta_1}^{\theta} \gamma \cdot (\beta(t) - \lambda) \, dt & \text{if } \theta < \theta_1 \\
0 & \text{if } \theta \in [\theta_1, \theta_2] \\
\int_{\theta_2}^{\theta} \gamma \cdot (\beta(t) - \lambda) \, dt & \text{if } \theta > \theta_2 
\end{cases}$$

(16)

\(^{25}\)See, for instance, Baron and Myerson (1982).
By definition, \( CE_e(\theta) = \alpha(\theta) + \beta(\theta) \cdot \gamma \cdot \theta - \frac{1}{2} \cdot \rho \cdot (\beta(\theta))^2 \cdot \sigma^2 - \lambda \cdot \gamma \cdot \theta \). Substituting this in (16), one can solve for the manager’s expected compensation \( \alpha(\theta) + \beta(\theta) \cdot \gamma \cdot \theta \). Substituting the resulting expression into the owner’s objective function and integrating by parts show that the owner’s optimization problem in (6) simplifies to choosing \( \{\beta(\theta), \theta_1, \theta_2\} \) so as to maximize:

\[
\int_{\theta_1}^{\theta_2} \left[ \gamma \cdot \theta \cdot (1 - \lambda) - \frac{1}{2} \cdot \rho \cdot (\beta(\theta))^2 \cdot \sigma^2 + \gamma \cdot (\beta(\theta) - \lambda) \cdot h(\theta) \right] f(\theta) \, d\theta \\
+ \int_{\theta_1}^{\theta_2} \left[ \gamma \cdot \theta (1 - \lambda) - \frac{1}{2} \cdot \rho \cdot (\beta(\theta))^2 \cdot \sigma^2 - \gamma \cdot (\beta(\theta) - \lambda) \cdot H(\theta) \right] f(\theta) \, d\theta \tag{17}
\]

where,

\[
h(\theta) \equiv \frac{F(\theta)}{f(\theta)}, \tag{18}
\]

and

\[
H(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)}. \tag{19}
\]

The above optimization problem must be solved subject to the constraints that \( \beta(\theta) \) is increasing and \( \beta(\theta) = \lambda \) for \( \theta \in [\theta_1, \theta_2] \).

The objective function can be optimized pointwise. Note that the third integral in expression (17) is decreasing in \( \beta(\cdot) \) pointwise. Therefore, the expected profit maximization requires that \( \beta(\theta) \) must be set equal to \( \lambda \), which is the lowest value consistent with the constraints that \( \beta(\theta) \) is increasing and \( \beta(\theta) = \lambda \) for \( \theta \in [\theta_1, \theta_2] \). This implies that \( \theta_2 = \overline{\theta} \), and the participation constraint will bind along a possibly degenerate upper-tailed interval \( [\theta_1, \overline{\theta}] \). As a consequence, the owner’s problem simplifies to choosing \( \{\beta(\theta), \theta_1\} \) in order to maximize:

\[
\int_{\theta_1}^{\theta_1} \left[ \gamma \cdot \theta \cdot (1 - \lambda) - \frac{1}{2} \cdot \rho \cdot (\beta(\theta))^2 \cdot \sigma^2 + \gamma \cdot (\beta(\theta) - \lambda) \cdot h(\theta) \right] f(\theta) \, d\theta \\
+ \int_{\theta_1}^{\overline{\theta}} \left[ \gamma \cdot \theta (1 - \lambda) - \frac{1}{2} \cdot \rho \cdot (\beta(\theta))^2 \cdot \sigma^2 - \gamma \cdot (\beta(\theta) - \lambda) \cdot H(\theta) \right] f(\theta) \, d\theta \tag{20}
\]

subject to the constraints that \( \beta(\theta) \) is increasing and \( \beta(\theta) = \lambda \) for \( \theta \in [\theta_1, \overline{\theta}] \).
For a given $\theta_1$, the first-order condition with respect to $\beta(\theta)$ combined with the constraint that $\beta(\theta) = \lambda$ for all $\theta \geq \theta_1$ yield the following choice for the optimal pay-performance sensitivity:

$$
\beta^*(\theta) = \begin{cases} 
\hat{\beta}(\theta) \equiv \frac{\gamma \cdot h(\theta)}{\rho \cdot \sigma^2} & \text{if } \theta \in [\theta, \theta_1) \\
\lambda & \text{if } \theta \in (\theta_1, \bar{\theta}] 
\end{cases} 
$$

where the monotonicity constraint requires that $\hat{\beta}(\theta) \leq \lambda$ for $\theta < \theta_1$. To characterize the optimal choice of $\theta_1$, let $\pi(\theta_1)$ denote the maximal value of the objective function in (20) for a given value of $\theta_1$. Applying Leibnitz’s rule and simplifying yield:

$$
\pi'(\theta_1) = \frac{1}{2} \cdot f(\theta_1) \cdot \rho \cdot \sigma^2 \cdot \gamma \cdot (\hat{\beta}(\theta_1) - \lambda)^2. 
$$

Claim: If the optimal $\theta_1$ is interior, then $\beta^*(\theta_1) = \hat{\beta}(\theta_1) = \lambda$.\footnote{That is, the optimal pay-performance schedule cannot have a discontinuity at $\theta = \theta_1$.}

Proof: The monotonicity condition rules out $\hat{\beta}(\theta_1) > \lambda$. If $\hat{\beta}(\theta_1) < \lambda$, equation (22) implies that $\pi'(\theta_1) > 0$. Since $\theta_1$ is interior by assumption, this implies that the owner can increase his expected profit $\pi(\cdot)$ by choosing a slightly higher value of $\theta_1$ without violating the monotonicity constraint. This contradicts the fact that $\theta_1$ is optimal, thereby proving the claim that $\hat{\beta}(\theta_1) = \lambda$ for interior $\theta_1$.

Since $h(\cdot) \equiv \frac{f(\theta)}{f(\theta)}$ is an increasing function of $\theta$, $\hat{\beta}(\theta)$ increases monotonically from a value of zero at $\theta = \underline{\theta}$ to a value of $\frac{\gamma \cdot h(\theta)}{\rho \cdot \sigma^2}$ at $\theta = \bar{\theta}$. Therefore:

- (i) If $\hat{\beta}(\theta) > \lambda$, the optimal $\theta_1$ is interior and given by the unique solution to the equation $\hat{\beta}(\theta_1) = \lambda$.
- (ii) If $\hat{\beta}(\theta) \leq \lambda$, the optimal $\theta_1$ is equal to the upper bound $\bar{\theta}$.

Combining these results yield:

$$
\beta^*(\theta) = \min \left\{ \lambda, \frac{\gamma \cdot h(\theta)}{\rho \cdot \sigma^2} \right\} 
$$
Proof of Proposition 2:

Using standard arguments, it can be shown that the incentive compatibility constraint in program (9) is equivalent to the conditions:

(i) $CE'(\theta) = \gamma \cdot \beta(\theta)$, and

(ii) $\beta(\theta)$ is increasing

for almost all $\theta \in [\underline{\theta}, \overline{\theta}]$. When $\lambda = 0$, the participation constraint in program (9) simplifies to the condition that $CE(\theta) \geq 0$ for all $\theta$. Conditions (i) and (ii) above imply that the participation constraint will bind for the lowest type. Consequently, condition (i) yields

$$CE(\theta) = \int_{\underline{\theta}}^{\theta} \gamma \cdot \beta(t) \, dt.$$ Integrating by parts gives

$$\int_{\underline{\theta}}^{\theta} CE(\theta) \cdot f(\theta) \, d\theta = \int_{\underline{\theta}}^{\theta} [\gamma \cdot \beta(\theta) \cdot H(\theta)] \, d\theta.$$ (23)

where $H(\cdot)$ is the inverse hazard rate as defined in (19).

Substituting $CE(\theta) = \alpha(\theta) + \beta(\theta) \cdot (a + \gamma \cdot \theta) - \frac{1}{2} \cdot [k \cdot a^2 + \rho \cdot (\beta(\theta))^2 \cdot \sigma^2]$, equation (23) can be written as follows:

$$\int_{\underline{\theta}}^{\theta} \left[ a(\theta) + \beta(\theta) \cdot (a + \gamma \cdot \theta) \right] f(\theta) \, d\theta = \int_{\underline{\theta}}^{\theta} \left[ \frac{1}{2} \cdot [k \cdot a^2 + \rho \cdot \sigma^2 \cdot (\beta(\theta))^2] + \gamma \cdot \beta(\theta) \cdot H(\theta) \right] \, d\theta$$

After substituting the above equation and the effort incentive compatibility condition $a = \frac{\beta(\theta)}{k}$ into the objective function of program (9), the owner’s optimization problem simplifies to choosing $\beta(\theta)$ so as to maximize:

$$\int_{\underline{\theta}}^{\theta} \left[ \gamma \cdot \theta + \beta(\theta) \cdot \left( \frac{1}{k} - \gamma \cdot H(\theta) \right) - \frac{1}{2} \cdot (\beta(\theta))^2 \cdot \left( \frac{1}{k} + \rho \cdot \sigma^2 \right) \right] \, d\theta$$ (24)

subject to the constraints that $\beta(\theta) \geq 0$ for all $\theta$, and $\beta(\cdot)$ is increasing.

I first solve the above maximization problem without the monotonicity constraint that $\beta(\cdot)$ is increasing, and verify later that the solution to the relaxed problem satisfies this
constraint. Pointwise maximization of the objective function in (24) yields that \( \beta(\theta) = 0 \) is optimal if \( 1 - k \cdot \gamma \cdot H(\theta) < 0 \). Since the inverse hazard rate \( H(\cdot) \) is decreasing, \( 1 - k \cdot \gamma \cdot H(\theta) > 0 \) for all \( \theta \in [\underline{\theta}, \bar{\theta}] \) provided that:

\[
1 - k \cdot \gamma \cdot H(\theta) > 0. \tag{25}
\]

Consequently, when (25) holds, the optimal value of \( \beta(\theta) \) is interior (i.e., strictly positive) for all \( \theta \in [\underline{\theta}, \bar{\theta}] \), and given by the following first-order condition:

\[
\beta(\theta) = \beta^0 \cdot [1 - k \cdot \gamma \cdot H(\theta)] \tag{26}
\]

where \( \beta^0 \equiv (1 + k \cdot \rho \cdot \sigma^2)^{-1} \) denotes the benchmark pay-performance sensitivity.

When (25) does not hold, the optimal pay-performance schedule is given by:

\[
\beta(\theta) = \begin{cases} 
0 & \text{if } \theta \in [\underline{\theta}, \theta^*] \\
\beta^0 \cdot [1 - k \cdot \gamma \cdot H(\theta)] & \text{if } \theta \in [\theta^*, \bar{\theta}]
\end{cases}
\]

where \( \theta^* \in (\underline{\theta}, \bar{\theta}) \) uniquely solves the following equation:

\[
1 - k \cdot \gamma \cdot H(\theta^*) = 0
\]

Combining the above two cases yields

\[
\beta(\theta) = \max \{0, \beta^0 \cdot (1 - k \cdot \gamma \cdot H(\theta))\} \tag{27}
\]

Note that \( \beta(\cdot) \) in (27) is increasing because the inverse hazard rate \( H(\cdot) \) is decreasing. This implies that the pay-performance schedule in (27) satisfies the monotonicity constraint and is therefore optimal.

**Proof of Proposition 3:** As in the proof of Proposition 1, it is convenient to work with the *excess* certainty equivalent expression:

\[
CE_e(\hat{\theta}, \theta) \equiv CE(\hat{\theta}, \theta) - \lambda \cdot \gamma \cdot \theta,
\]

As before, let \( CE_e(\theta) \equiv CE_e(\theta, \theta) \). It can be verified that the necessary and sufficient conditions for the incentive compatibility constraint in program (9) to hold are:
(i) \( CE'_e(\theta) = \gamma \cdot [\beta(\theta) - \lambda] \), and

(ii) \( \beta(\theta) \) is increasing

for almost all \( \theta \in [\underline{\theta}, \bar{\theta}] \). Conditions (i) and (ii) above imply that the manager’s excess certainty equivalent is convex in \( \theta \); that is, \( CE''_e(\theta) \geq 0 \) almost everywhere. This in turn implies that the participation constraint will bind at most along a single (possibly degenerate) interval of \([\underline{\theta}, \bar{\theta}]\). Suppose the participation constraint binds along the interval \([\theta_1, \theta_2]\). This implies that \( CE'_e(\theta) = 0 \) for \( \theta \in [\theta_1, \theta_2] \). Condition (i) then implies equation (16).

Using the definition of \( CE_e(\theta) \), I first solve equation (16) for the expected compensation of type \( \theta \) manager; i.e., \( \alpha(\theta) + \beta(\theta) \cdot [a + \gamma \cdot \theta] \). I then substitute the resulting expression into the owner’s objective function and integrate by parts. A subsequent substitution of the manager’s effort incentive compatibility condition (i.e., \( a(\theta) = \beta(\theta)/k \)) and simplification reveal than the owner’s optimization problem simplifies to choosing \( \{\beta(\theta), \theta_1, \theta_2\} \) so as to maximize:

\[
\int_{\underline{\theta}}^{\theta_1} \left[ \gamma \cdot \theta \cdot (1 - \lambda) + \frac{\beta(\theta)}{k} - \frac{1}{2} \cdot (\beta(\theta))^2 \cdot \left( \frac{1}{k} + \rho \cdot \sigma^2 \right) + \gamma \cdot (\beta(\theta) - \lambda) \cdot h(\theta) \right] f(\theta) \, d\theta \\
+ \int_{\theta_1}^{\theta_2} \left[ \gamma \cdot \theta \cdot (1 - \lambda) + \frac{\lambda}{k} - \frac{1}{2} \cdot \rho \lambda^2 \cdot \left( \frac{1}{k} + \rho \cdot \sigma^2 \right) \right] f(\theta) \, d\theta \\
+ \int_{\theta_2}^{\bar{\theta}} \left[ \gamma \cdot \theta \cdot (1 - \lambda) + \frac{\beta(\theta)}{k} - \frac{1}{2} \cdot (\beta(\theta))^2 \cdot \left( \frac{1}{k} + \rho \cdot \sigma^2 \right) - \gamma \cdot (\beta(\theta) - \lambda) \cdot H(\theta) \right] f(\theta) \, d\theta
\]

subject to the constraints that \( \beta(\theta) \) is increasing and \( \beta(\theta) = \lambda \) for all \( \theta \in [\theta_1, \theta_2] \). The functions \( h(\cdot) \) and \( H(\cdot) \) are as defined in equations (18) and (19), respectively.

The pointwise unconstrained maximization over \( \theta < \theta_1 \) and \( \theta > \theta_2 \) combined with the constraint \( \beta(\theta) = \lambda \) for \( \theta \in [\theta_1, \theta_2] \) yields the following pay-performance schedule:

\[
\beta^*(\theta) = \begin{cases} 
\beta^0 \cdot [1 + k \cdot \gamma \cdot h(\theta)] > \beta^0 & \text{if } \theta < \theta_1 \\
\lambda & \text{if } \theta \in [\theta_1, \theta_2] \\
\max \{0, \beta^0 \cdot [1 - k \cdot \gamma \cdot H(\theta)]\} < \beta^0 & \text{if } \theta > \theta_2
\end{cases}
\]  

(28)

Notice that if both \( \theta_1 \) and \( \theta_2 \) are interior, the pay-performance schedule in (28) cannot satisfy the monotonicity constraint that \( \beta(\cdot) \) is increasing in \( \theta \).
To characterize the optimal values of \( \theta_1 \) and \( \theta_2 \), let us consider the following three cases:

**Case I:** \( \lambda < \beta^0 \)

When \( \lambda < \beta^0 \), it follows from equation (28) that the monotonicity constraint can be met only if \( \theta_1 = \overline{\theta} \). For a given \( \theta_2 \), the owner will optimally choose:

\[
\beta^*(\theta) = \begin{cases} 
\lambda & \text{if } \theta \in [\overline{\theta}, \theta_2) \\
\hat{\beta}(\theta) \equiv \beta^0 \cdot [1 - k \cdot \gamma \cdot H(\theta)] & \text{if } \theta \in (\theta_2, \overline{\theta}] 
\end{cases} \tag{29}
\]

where the monotonicity constraint also requires that \( \hat{\beta}(\theta) \geq \lambda \) for all \( \theta > \theta_2 \).

To characterize the optimal value of \( \theta_2 \), let \( \pi(\theta_2) \) denote the maximal value of the owner’s objective function as a function of \( \theta_2 \). Applying Leibnitz’s rule and simplifying yield:

\[
\pi'(\theta_2) = -\frac{1}{2} \cdot \beta^0 \cdot [\hat{\beta}(\theta_2) - \lambda]^2. \tag{30}
\]

If the optimal value of \( \theta_2 \) is interior (i.e., strictly greater than \( \overline{\theta} \)), then I claim that \( \beta^*(\theta_2) = \hat{\beta}(\theta_2) = \lambda \). To prove this claim, note that the monotonicity constraint rules out \( \hat{\beta}(\theta_2) < \lambda \). If \( \hat{\beta}(\theta_2) > \lambda \), then equation (30) implies that \( \pi'(\cdot) < 0 \). This implies that the owner can increase his expected profit by choosing a slightly lower value of \( \theta_2 \), which contradicts the assumption that \( \theta_2 \) is optimal. This proves the claim.

Since \( H(\cdot) \) is a decreasing function of \( \theta \), \( \hat{\beta}(\theta) \) increases monotonically from a value of \( \hat{\beta}(\overline{\theta}) \) at \( \theta = \overline{\theta} \) to a value of \( \beta^0 \) at \( \theta = \overline{\theta} \). This implies that:

(i) If \( \hat{\beta}(\overline{\theta}) < \lambda \), the optimal value of \( \theta_2 \) is interior and given by the unique solution to the equation \( \hat{\beta}(\theta_2) = \lambda \).

(ii) If \( \hat{\beta}(\overline{\theta}) \geq \lambda \), the optimal value of \( \theta_2 \) is equal to the lower bound \( \overline{\theta} \).

Combining these two cases yields:

\[
\beta^*(\theta) = \max \{ \lambda, \beta^0 \cdot [1 - k \cdot \gamma \cdot H(\theta)] \}
\]

**Case II:** \( \lambda > \beta^0 \)

Since \( \lambda > \beta^0 \), equation (28) reveals that the monotonicity constraint can be met only if \( \theta_2 = \overline{\theta} \). For a given \( \theta_1 \), the optimal pay-performance schedule is then given by:
\[
\beta^*(\theta) = \begin{cases} 
\tilde{\beta}(\theta) \equiv \beta^0 \cdot [1 + \gamma \cdot h(\theta)] & \text{if } \theta \in (\theta_1, \bar{\theta}) \\
\lambda & \text{if } \theta \in [\tilde{\theta}, \theta_1) 
\end{cases}
\] (31)

Using the similar arguments as in Case I, it can be shown that:

(i) If \(\tilde{\beta}(\bar{\theta}) > \lambda\), then \(\theta_1\) is interior and solves the equation \(\tilde{\beta}(\theta_1) = \lambda\).

(ii) If \(\tilde{\beta}(\bar{\theta}) \leq \lambda\), then the optimal value of \(\theta_1\) is equal to the upper bound \(\bar{\theta}\).

Combining these two cases yields

\[
\beta^*(\theta) = \min \{\lambda, \beta^0 \cdot [1 + k \cdot \gamma \cdot h(\theta)]\}
\]

Case III: \(\lambda = \beta^0\)

Equation (28) shows that the monotonicity constraint will be satisfied if and only if \(\theta_1 = \tilde{\theta}\) and \(\theta_2 = \bar{\theta}\). As a result, the participation constraint binds for all types, and \(\beta^*(\theta) = \lambda\) for all \(\theta \in [\tilde{\theta}, \bar{\theta}]\).

Proof of Proposition 4: Proposition 4 follows by differentiating the expressions for optimal pay-performance sensitivities in Proposition 3 with respect to \(\gamma\) and \(\lambda\).
References


