The Role of Disaggregated Accounting Data in Detecting and Suppressing Earnings Management*

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Abstract

We study the role of disaggregated accounting data in the context of earnings management. Our analysis is rooted in the notion that different earnings components tend to be (noisily) proportional to each other by their fundamental economic nature. We argue that these fundamental proportions between the components of earnings are likely to be distorted by reporting manipulations. The distortion in proportions that involve an income item and an expense item is due to the managerial wish to bias incomes and expenses in opposite directions. In other cases, diversity in the managerial ability to manipulate different earnings components generates a reporting bias that distorts their original proportions. Users of financial statements can thus utilize the disaggregated earnings data in order to identify deviations of the reported earnings components from their expected fundamental proportions and thereby detect biases in reporting. Being aware of the informational role of disaggregated accounting data in indicating earnings management, firms may adopt a more cautious approach in managing their earnings reports. Our analysis, therefore, suggests that disaggregated accounting information, via its role in detecting earnings management, might serve as a powerful mechanism for suppressing misreporting incentives.

Keywords: Information asymmetry; Accounting; Financial reporting; Earnings management; Disaggregated accounting data; Financial ratios.

JEL classification: D82; G14; M41.
1. Introduction

Accounting earnings are typically the aggregation of many components, some of which have to be disclosed within the income statement, while others may be disclosed in notes to the financial statements. Though the informational content of such disaggregated accounting data is the focus of numerous studies in accounting (e.g., Lipe 1986; Rayburn 1986; Wilson 1987; Barth, Beaver and Wolfson 1990; Barth 1991; Ohlson and Penman 1992; Amir 1996; Sloan 1996; Nissim and Penman 2001; Barth, Clinch and Shibano 2003), it has been largely ignored by the extensive literature on earnings management. Extant studies consider biases in reporting earnings mostly at the aggregated level of the reported earnings (e.g., Dye 1988; Stein 1989; Fischer and Verrecchia 2000; Kirschenheiter and Melumad 2002; Fischer and Stocken 2004; Ewert and Wagenhofer 2005; Guttman, Kadan and Kandel 2006). By analyzing reporting manipulations at the disaggregated level of the earnings, this study explores the role that disaggregated accounting data play in the context of earnings management.

Our analysis is rooted in the notion that earnings components tend to be proportional to each other by their fundamental economic nature. Examples include the ratios that usually exist between revenues and certain expense items, such as the cost of goods sold, marketing expenses and administration expenses. Influenced as they are by changes in the business environment, such ratios are noisy in most situations. Their degree of noisiness depends on the nature of the earnings components involved, as well as on industry and firm-specific characteristics. We argue that reporting manipulations are likely to distort the fundamental proportions between the earnings components. The distortion in proportions that involve an incomes item and an expenses item is due to the managerial wish to bias incomes and expenses in opposite directions. In other cases, diversity in the managerial ability to manipulate different earnings components generates a reporting bias that distorts their original proportions.
The leeway of managers in reporting earnings within accounting conventions indeed seems to vary in its degree across different components of earnings. Due to the inherent differences in their nature, some of the earnings components are more easily managed than others. It is generally accepted, for example, that accrual-based items (such as bad debts and loss reserves, depreciations and amortizations, asset impairments) are more easily managed than cash-based items. High diversity in the managerial discretion in reporting different earnings components is also typical of segment reporting, where the consolidated earnings of a firm are decomposed into earnings from its operations in various business or geographical segments.

We demonstrate the distorting impact of reporting manipulations on the original proportions between the different components of earnings and explore its implications for the ability of users of financial statements to detect earnings management and for the propensity of managers to engage in earnings management. To do so, we model a single-period reporting game where a manager of a publicly traded firm has the ability and incentives to manage the mandatory periodical earnings reported to capital market investors. In our model, the firm’s manager, whose compensation is linked to the firm’s stock price, chooses a reporting strategy based on her rational expectations about the market pricing rule. The investors, in turn, invoke their rational expectations regarding the manager’s reporting strategy when pricing the firm in an effort to detect earnings manipulations. Following to Fischer and Verrecchia (2000), we incorporate an exogenous noise in the model, which does not allow investors to perfectly resolve the reporting bias in pricing the firm. Our departure from the traditional setting of earnings management is made in considering a disaggregated earnings report rather than an aggregate report.

An analysis of the equilibrium in our reporting game highlights the power of disaggregated accounting data in detecting and suppressing reporting manipulations. In
equilibrium, recognizing the manager’s incentives to bias incomes and expenses in opposite directions and knowing that she faces different degrees of discretion in manipulating different components of the earnings report, investors rationally infer that earnings management is likely to distort the original proportions of the earnings components. Furthermore, they can utilize the disaggregated earnings report in order to compute the actual deviation of the reported earnings components from their fundamental economic proportions. Such deviation may either stem from an economic noise that underlies the stochastic relationship between the earnings components, or result from a reporting bias. It thus provides investors with a noisy indicator of earnings management, which enables them to more accurately evaluate the firm based on the earnings report. The firm’s manager, who is aware of investors’ ability to utilize the reported disaggregated earnings data in order to imperfectly detect the bias in reporting, becomes more reluctant to manipulate the firm’s earnings report in the first place. Our analysis, therefore, demonstrates that disaggregated accounting information, via its role in indicating reporting manipulations, might serve as an effective mechanism that suppresses managerial misreporting incentives. Utilizing a comparative statics analysis, we further show that disaggregated earnings information is more effective in detecting and suppressing earnings management when it consists of items that are fundamentally more tightly proportional to each other. The analysis also points to the merits of disaggregating earnings into pure-incomes and pure-expenses components or alternatively decomposing earnings into items that are very diverse in the extent to which their reporting can be managed.

The paper proceeds as follows. The next section models the reporting game. The equilibrium in this game is derived and analyzed in Section 3, demonstrating the role of disaggregated accounting data in detecting and suppressing reporting manipulations. In an attempt to provide accounting standard setters with useful guidance in selecting among alternative disaggregating accounting procedures, Section 4 considers the sensitivity
of the equilibrium outcomes to the main modeling parameters. In Section 5, we discuss the empirical implications of our study. The final section summarizes and offers concluding remarks. Proofs appear in the appendix.

2. **Model**

In this section, we present a rational expectations model, which describes a single-period reporting game between a manager of a publicly traded firm and investors in the capital market. The manager of the firm, whose compensation is linked to the price at which the firm is traded in the market, exercises discretion over the costly bias in the earnings reported to the investors. In designing the model, we built on the earnings management setup of Fischer and Verrecchia (2000). We deviate from their model by considering a disaggregated earnings report rather than an aggregate earnings report. This allows us to flush out the important role of accounting disaggregation in the context of earnings management. The remainder of this section details the parameters and assumptions underlying the model, which are all assumed to be common knowledge unless otherwise indicated.

We consider a firm that is traded in a capital market for one period. The earnings that the firm yields during the given period constitute its equity value.\(^1\) We model the firm’s uncertain earnings as a random variable \(\tilde{\pi}\). To capture in the model the essence of earnings as an accounting aggregator, we assume that the firm’s earnings \(\tilde{\pi}\) is the sum of two components \(\tilde{\pi}_1\) and \(\tilde{\pi}_2\). That is, \(\tilde{\pi} = \tilde{\pi}_1 + \tilde{\pi}_2\). The earnings components \(\tilde{\pi}_1\) and \(\tilde{\pi}_2\) are assumed to be normally distributed random variables with means \(\mu_1\) and \(\mu_2\), respectively, and variances \(\sigma_1^2\) and \(\sigma_2^2\), respectively.

\(^1\) The assumption that the equity value of the firm equals its earnings is a simplifying assumption that fits the single-period nature of the model. However, the analysis can be generalized to the case where the earnings measure is a noisy estimator of the firm’s value without qualitatively affecting the results.
and \( \sigma^2 \), respectively, where the covariance between them is \( \sigma_{12} \). \(^2\) It follows, therefore, that the aggregate earnings variable \( \tilde{\pi} \) is normally distributed with mean \( \mu = \mu_1 + \mu_2 \) and variance \( \sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\sigma_{12} \).

The manager privately observes the realization \( \pi \) of the firm’s earnings \( \tilde{\pi} \), as well as the realizations \( \pi_1 \) and \( \pi_2 \) of the two earnings components \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \), respectively. Based on her private information, she issues an earnings report \( r = r_1 + r_2 \), which includes a report \( r \) of the aggregate earnings \( \pi \) and two additional reports \( r_1 \) and \( r_2 \) of the two earnings components \( \pi_1 \) and \( \pi_2 \), respectively. We assume that the manager exercises discretion in reporting, which allows her to bias the reported earnings. We further assume, however, that the reporting bias is costly for the manager. \(^3\) Biases and manipulations in reporting earnings can be associated with a variety of costs. When earnings management involves the carrying out of inefficient real transactions, it is associated with the cost of distorting value. In other cases, it might be associated with litigation costs, reputation erosion costs, costs that emerge from conflicts with auditors and audit committees, and the costs of the anticipated mean-reversion in future reports.

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\(^2\) The normal distribution of the two variables \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) implies that they can have both negative and positive realizations. This allows us to capture earnings components that mix both incomes and expenses, and thus might be positive in some cases and negative in other cases. Nevertheless, we can also capture in the model an income-based component by assuming that the corresponding variable \( \tilde{\pi}_i \sim N(\mu_i, \sigma_i^2) \) has a positive mean \( \mu_i \) that exceeds by a large amount the standard deviation \( \sigma_i^2 \), so that the probability of a negative realization of \( \tilde{\pi}_i \) is very close to zero. Similarly, we can capture in the model an expense-based component by assuming that the corresponding variable \( \tilde{\pi}_i \sim N(\mu_i, \sigma_i^2) \) has a negative mean \( \mu_i \) that its absolute value exceeds by a large amount the standard deviation \( \sigma_i^2 \), so that the probability of a positive realization of \( \tilde{\pi}_i \) is very close to zero.

\(^3\) This assumption is typical of earnings management models, distinguishing them from cheap talk models where misreporting is costless (e.g., Crawford and Sobel 1982) and precluding “babbling” equilibria in which no information is conveyed.
As the managerial leeway in reporting might vary in its degree across the different components of the reported earnings, we allow the manager to bias both earnings components, but at potentially different costs. In common with the literature (e.g., Fischer and Verrecchia 2000; Dye and Sridhar 2004; Guttman, Kadan and Kandel 2006), we assume a quadratic cost function. Unlike prior models, however, our cost function is quadratic not only in the overall reporting bias, but also in the reporting bias in each of the earnings components. That is, when the manager observes earnings of \( \pi = \pi_1 + \pi_2 \) and reports \( r = r_1 + r_2 \), she bears a cost of 
\[
 c_1 (r_1 - \pi_1)^2 + c_2 (r_2 - \pi_2)^2 + c_3 (r - \pi)^2 ,
\]
where \( c_1, c_2, c_3 > 0 \). The terms \( c_1 (r_1 - \pi_1)^2 \) and \( c_2 (r_2 - \pi_2)^2 \) in the cost function capture costs that are related to the reporting bias in a certain component of the earnings, such as the costs of distorting value due to inefficient real transactions that are carried out just for reporting purposes, or the costs that arise due to conflicts with auditors and audit committees. The term \( c_3 (r - \pi)^2 \) in the cost function captures costs that are related to the overall reporting bias, but are not sensitive to the bias in particular earnings components, such as litigation costs or reputation erosion costs. The ratio \( \frac{c_1}{c_2} \) measures the extent to which the second earnings component is manageable relative to the first earnings component. The higher the ratio \( \frac{c_1}{c_2} \), the more manageable is the second component relative to the first component.\(^4\)

The manager’s compensation is linked to the firm’s stock price, and thus the manager makes her reporting decision in conjunction with her expectations about the impact of the

\(^4\) The extreme case where \( c_1 \) converges to infinity (and so is the ratio \( \frac{c_1}{c_2} \)) describes situations where only the second component of the earnings report is manageable. The opposite extreme case where \( c_2 \) converges to infinity (and so the ratio \( \frac{c_1}{c_2} \) converges to zero) describes situations where only the first component of the earnings report is manageable. In both these extreme cases, the reporting of one of the earnings components must be truthful, and this component can thus be viewed as a public signal.
earnings report on the market price of the firm. For any given earnings report \( r = r_1 + r_2 \), we define \( P(r_1, r_2) \) as the market price of the firm. After observing the earnings components \( \pi_1 \) and \( \pi_2 \), the manager designs her optimal earnings report \( r = r_1 + r_2 \) in order to influence the rationally anticipated market price of the firm \( P(r_1, r_2) \), but subject to the cost \( c_1(r_1 - \pi_1)^2 + c_2(r_2 - \pi_2)^2 + c_3(r - \pi)^2 \) associated with the reporting bias. We denote by \( x \) the marginal benefit to the manager from shifting upward the market price of the firm by one additional unit. Hence, the utility of the manager takes the form

\[
xP(r_1, r_2) - c_1(r_1 - \pi_1)^2 - c_2(r_2 - \pi_2)^2 - c_3(r - \pi)^2,
\]

given that she reports \( r = r_1 + r_2 \) after observing \( \pi = \pi_1 + \pi_2 \). Similarly to Fischer and Verrecchia (2000) and subsequent papers, it is assumed that \( x \) is privately known only to the manager. The investors, on the other hand, do not observe \( x \) and they consider it as the realization of some normally distributed random event \( \bar{x} \) with mean \( \mu_x \) and variance \( \sigma_x^2 \). That is, the investors face some degree of uncertainty about the manager’s reporting objective.\(^5\) This uncertainty on the part of investors precludes fully revealing equilibria, where the market is capable of perfectly backing out the reporting bias.

Investors are assumed to be rational and risk-neutral. Accordingly, they set the firm’s market price equal to the firm’s expected value conditional on all the available information. In particular, they use their expectations about the manager’s reporting strategy in an effort to detect the reporting bias and thereby most effectively utilize the information conveyed in the

\(^5\) Following Fischer and Verrecchia (2000), uncertainty on the part of investors about the reporting objective of managers is widely assumed in the disclosure literature (e.g., Fischer and Stocken 2004; Dye and Sridhar 2004; Ewert and Wagenhofer 2005; Einhorn 2007). Such uncertainty is modeled by assuming imperfect information of investors with respect to either the benefit to the manager from shifting the firm’s market price or the costs that she bears when biasing the report.
earnings report in pricing the firm. Given that \( \pi_1 \) and \( \pi_2 \) are the realizations of the earnings components \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \), respectively, and \( x \) is the realization of the random variable \( \tilde{x} \), we define \( B_i(\pi_1, \pi_2, x) \) as the reporting bias in the \( i \)'th \( (i = 1, 2) \) earnings component. Although investors do not observe \( \pi_1 \), \( \pi_2 \) and \( x \), they rationally anticipate that the manager reports
\[
r_1 = \pi_1 + B_1(\pi_1, \pi_2, x) \quad \text{and} \quad r_2 = \pi_2 + B_2(\pi_1, \pi_2, x)
\]
when she observes \( \tilde{\pi}_1 = \pi_1 \), \( \tilde{\pi}_2 = \pi_2 \) and \( \tilde{x} = x \). Therefore, the incremental information that investors elicit from the manager’s disaggregated earnings report \( r = r_1 + r_2 \), beyond the already available public information, is
\[
\tilde{\pi}_1 + B_1(\tilde{\pi}_1, \tilde{\pi}_2, \tilde{x}) = r_1 \quad \text{and} \quad \tilde{\pi}_2 + B_2(\tilde{\pi}_1, \tilde{\pi}_2, \tilde{x}) = r_2.
\]

Figure 1 provides a timeline depicting the sequence of events in the model. At the beginning, investors establish their prior beliefs about the firm’s uncertain earnings \( \tilde{\pi} = \tilde{\pi}_1 + \tilde{\pi}_2 \) and about the random event \( \tilde{x} \). Then, the manager privately observes the realizations \( \pi_1 \) and \( \pi_2 \) of the earning components \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \), respectively, and the realization \( x \) of the random event \( \tilde{x} \). Based on her private information, the manager issues the firm’s disaggregated earnings report \( r = r_1 + r_2 \), where \( r_1 = \pi_1 + B_1(\pi_1, \pi_2, x) \) and \( r_2 = \pi_2 + B_2(\pi_1, \pi_2, x) \). Subsequently, using the earnings report and all other available public information, investors set the firm’s market price \( P(r_1, r_2) \). At the end of the period, the firm’s earnings \( \tilde{\pi} = \tilde{\pi}_1 + \tilde{\pi}_2 \) are realized and become common knowledge.

The model is, therefore, a single-period game with two interrelated decisions made by two players – the reporting decision of the firm’s manager and the pricing decision of the investors. Equilibrium in the model consists of three functions: \( B_1, B_2 : \mathbb{R}^3 \rightarrow \mathbb{R} \) that represent the manager’s strategy in biasing the two components of the earnings report, and \( P : \mathbb{R}^2 \rightarrow \mathbb{R} \)
that represents the pricing rule applied by the investors. In equilibrium, the manager chooses the reporting strategy \((B_1, B_2)\) based on her rational expectations about the market pricing rule \(P\), which, in turn, is based on the investors’ rational expectations regarding the manager’s reporting strategy \((B_1, B_2)\). Equilibrium is formally defined as a vector of three functions \((B_1, B_2 : \mathbb{R}^3 \to \mathbb{R}, \, P : \mathbb{R}^2 \to \mathbb{R})\) that satisfies two conditions. The first equilibrium condition pertains to the manager’s reporting strategy \((B_1, B_2)\) in biasing the disaggregated earnings report, requiring for any \(\pi_1, \pi_2, x \in \mathbb{R}\) that

\[
(B_1(\pi_1, \pi_2, x), B_2(\pi_1, \pi_2, x)) \in \arg \max_{(b_1, b_2)} xP(\pi_1 + b_1, \pi_2 + b_2) - c_1b_1^2 - c_2b_2^2 - c_3(b_1 + b_2)^2.
\]

The second equilibrium condition describes the market pricing rule \(P\), imposing

\[
P(r_1, r_2) = E[\tilde{\pi} \mid \tilde{\pi}_1 + B_1(\tilde{\pi}_1, \tilde{\pi}_2, \tilde{x}) = r_1, \, \tilde{\pi}_2 + B_2(\tilde{\pi}_1, \tilde{\pi}_2, \tilde{x}) = r_2]
\]

for any disaggregated earnings report \(r_1, r_2 \in \mathbb{R}\).

We restrict the analysis to equilibria with a linear pricing rule. That is, we impose the pricing rule \(P(r_1, r_2)\) to be a linear function of the two reported earnings components \(r_1\) and \(r_2\). Linear equilibria are commonly assumed in the earnings management literature.\(^6\) When combined with a quadratic biasing cost function and a normal distribution of the firm’s equity value and earnings, a linear pricing rule enables a tractable analysis and yields equilibrium outcomes that can be easily characterized and intuitively explained. As linearity restrictions are commonly made in empirical research, the assumption of a linear pricing rule also allows us to link our analytical results to empirical findings and to draw new predictions that are empirically testable.

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\(^6\) An exception is the analysis of Guttman, Kadan and Kandel (2006), which explains kinks and discontinuities in the distribution of the reported earnings by focusing on non-linear equilibria.
3. **Equilibrium Analysis**

We start the analysis by investigating the fundamental proportion between the two earnings components $\pi_1$ and $\pi_2$. For this purpose, we define the covariance ratio

$$\lambda = \frac{\text{cov}(\pi_1, \pi_2)}{\text{cov}(\pi_1, \pi_2)} = \frac{\sigma_1^2 + \sigma_{12}^2}{\sigma_2^2 + \sigma_{12}^2}.$$  

7 The ratio $\lambda$ can be either positive or negative, but it must satisfy $\lambda \neq -1$, because the variance $\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\sigma_{12}$ of the earnings variable $\pi$ is positive. The following observation indicates that the normally distributed random variable $\pi_2 - \lambda \pi_1$, denoted $\tilde{\varepsilon}$, is independent of the firm’s earnings and value $\pi$.

**Observation 1.** Let $\lambda = (\sigma_2^2 + \sigma_{12})/(\sigma_1^2 + \sigma_{12})$. Then, $\tilde{\varepsilon} = \pi_2 - \lambda \pi_1$ is a normally distributed variable with mean $\mu_\varepsilon = \mu_2 - \lambda \mu_1$ and variance $\sigma_\varepsilon^2 = \sigma_1^2 + \lambda^2 \sigma_2^2 - 2\lambda \sigma_{12}$, where $\text{cov}(\pi, \tilde{\varepsilon}) = 0$.

Since the random variable $\pi_2 - \lambda \pi_1$ is value-independent, the measure $\lambda$ can be viewed as the fundamental proportion between the earnings components $\pi_1$ and $\pi_2$. Using this interpretation, the mean $\mu_\varepsilon$ of the random variable $\pi_2 - \lambda \pi_1$ captures the expected deviation of the earnings components from their fundamental proportion $\lambda$, whereas its variance $\sigma_\varepsilon^2$ captures the extent to which the proportion $\lambda$ is noisy. For example, in the special case where $\pi_1$ represents the revenues and $\pi_2$ represents the expenses, $\lambda$ measures the fundamental proportion between the variable costs and the revenues, $\mu_\varepsilon$ measures the average level of the fixed costs, and $\sigma_\varepsilon^2$ measures the extent of noise that underlies this stochastic relationship between the revenues and the expenses.

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7 Since $\text{var}(\pi) = \sigma_1^2 + \sigma_2^2 + 2\sigma_{12}$ is strictly positive, either the covariance $\text{cov}(\pi_1, \pi) = \sigma_1^2 + \sigma_{12}$ or the covariance $\text{cov}(\pi, \pi_2) = \sigma_2^2 + \sigma_{12}$ must differ from zero. So, without loss of generality, it is assumed that $\text{cov}(\pi, \pi_1) = \sigma_1^2 + \sigma_{12} \neq 0$, ensuring that $\lambda$ is always well-defined.
While the random variables \( \tilde{\pi}_1, \tilde{\pi}_2 \sim N(\mu_1, \mu_2, \sigma_{\pi_1}^2, \sigma_{\pi_2}^2, \sigma_{\pi_{12}}^2) \) are the primitives of the model, it is more convenient to characterize the equilibrium properties in terms of the common distribution of the random variables \( \tilde{\pi}_1 = \tilde{\pi}_1 + \tilde{\pi}_2, \ \tilde{\epsilon} = \tilde{\pi}_2 - \lambda \tilde{\pi}_1 \sim N(\mu, \mu_\pi, \sigma_\pi^2, \sigma_\epsilon^2, 0) \), where 

\[
\lambda = (\sigma_\pi^2 + \sigma_{\pi_{12}}^2)/(\sigma_\pi^2 + \sigma_{\pi_{12}}^2), \ \mu = \mu_1 + \mu_2, \ \mu_\pi = \mu_2 - \lambda \mu_1, \ \sigma_\pi^2 = \sigma_\pi^2 + \sigma_{\pi_{12}}^2 + 2\sigma_{\pi_{12}}^2 \text{ and }
\]

\[
\sigma_\epsilon^2 = \sigma_\pi^2 + \lambda^2 \sigma_\pi^2 - 2\lambda \sigma_{\pi_{12}}. \text{ As } \lambda, \ \mu, \ \mu_\pi, \ \sigma_\pi^2 \text{ and } \sigma_\epsilon^2 \text{ unequivocally determine } \mu_1, \ \mu_2, \ \sigma_\pi^2, \ \sigma_{\pi_{12}}^2, \ \text{the variables } \tilde{\pi}_1 = \tilde{\pi}_1 + \tilde{\pi}_2 \text{ and } \tilde{\epsilon} = \tilde{\pi}_2 - \lambda \tilde{\pi}_1 \text{ can substitute for the variables } \pi_1 \text{ and } \pi_2 \text{ in constituting the primitives of the model.}^8 \text{ Accordingly, when applying the equilibrium analysis, we refer to } \lambda, \ \mu, \ \mu_\pi, \ \sigma_\pi^2 \text{ and } \sigma_\epsilon^2 \text{ as primitive parameters, so changes in each of them are made while keeping the others intact.}

Though our results hold for any set of parameters \( \lambda, \ \mu, \ \mu_\pi, \ \sigma_\pi^2 \text{ and } \sigma_\epsilon^2 \), we especially interested in the more descriptive situations where the expected deviation \( \mu_\pi \) of the earnings components from their fundamental proportion is lower than their expected aggregate value \( \mu \).

In such situations, the sign of \( \lambda \) equals the sign of \( \mu_1, \mu_2 \).^9 It follows thus that a negative proportion \( \lambda \) between the two variables \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) exists when they are likely to have opposite signs. This is typical of situations where one of these variables represents incomes and the other represents expenses (and thus equals the additive inverse of the expenses). A positive \( \lambda \), on the other hand, captures all other situations where the two earnings components are likely to share the same sign, implying that both of them represent incomes or that both of them represent expenses.

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^8 Explicitly, \( \mu_1 = (\mu - \mu_\pi)/(1 + \lambda), \ \mu_2 = (\lambda \mu + \mu_\pi)/(1 + \lambda), \ \sigma_{\pi_1}^2 = (\sigma_\pi^2 + \sigma_\epsilon^2)/(1 + \lambda)^2, \ \sigma_\pi^2 = (\lambda \sigma_\pi^2 + \sigma_\epsilon^2)/(1 + \lambda)^2 \text{ and } \sigma_{\pi_{12}}^2 = (\lambda \sigma_\pi^2 - \sigma_\epsilon^2)/(1 + \lambda)^2. \)

^9 Assuming without loss of generality that \( \left|\lambda\right| > 1 \), it follows from the inequality \( \mu_\pi < \mu \) that \( \lambda \) and \( \mu_1, \mu_2 = (\mu - \mu_\pi)/(\lambda \mu + \mu_\pi)/(1 + \lambda)^2 \) share the same sign.
Since the manager wishes to bias incomes and expenses in opposite directions, the driving force behind our results for negative values of $\lambda$ is the diversity in the manager’s incentives with respect to the direction of the bias in the two earnings components. When $\lambda$ is positive, the manager seeks to bias the two earnings components in the same direction, so the diversity in the manager’s ability to bias the two earnings components becomes the crucial force that drives the results. Therefore, we distinguish between three cases where the proportion $\lambda$ is positive: (i) $\lambda = c_1 / c_2$, (ii) $0 < \lambda < c_1 / c_2$, and (iii) $\lambda > c_1 / c_2$. In the case of $\lambda = c_1 / c_2$, the fundamental proportion $\lambda$ between the two earnings components coincides exactly with the ratio $c_1 / c_2$ of their marginal biasing costs. That is, the marginal cost of biasing the second earnings component $\pi_2$ by one additional unit equals the marginal cost of proportionally biasing the first earnings component $\pi_1$ by the addition of $\lambda$ units. We interpret this as implying that the two earnings components $\pi_1$ and $\pi_2$ are equally manageable.

The case $0 < \lambda < c_1 / c_2$ captures situations where the first earnings component is less manageable than the second earnings component. The opposite occurs in the case of $\lambda > c_1 / c_2$, where the first earnings component is more manageable than the second earnings component. Throughout the analysis, we focus on situations where $\lambda \neq c_1 / c_2$, using the case of $\lambda = c_1 / c_2$ as a benchmark case.

3.1. The Benchmark case of $\lambda = c_1 / c_2$

We first consider the edge case of $\lambda = c_1 / c_2$, where the fundamental proportion $\lambda$ between the two earnings components is positive and coincides exactly with the ratio $c_1 / c_2$ of

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10 It should be noted that the case $\lambda > c_1 / c_2$ is symmetric to the case $0 < \lambda < c_1 / c_2$ subject to an exchange in the indexes of the two earnings components.
their marginal biasing costs. The positive sign of $\lambda$ implies that the two earnings components are likely to share the same sign, and thus the manager wishes to bias both of them in the same direction. The fact that $\lambda$ coincides exactly with $c_1 / c_2$ implies that the manager also faces the same degree of leeway in manipulating the two earnings components. The case of $\lambda = c_1 / c_2$ is therefore the one and only case where the reporting bias preserves the fundamental proportion between the two earnings components. It thus provides a natural point of reference, serving in the analysis as a benchmark case. Proposition 1 establishes the existence and uniqueness of equilibrium in the benchmark case of $\lambda = c_1 / c_2$ and characterizes its form.

**Proposition 1.** In the benchmark case of $\lambda = c_1 / c_2$, there exists a unique linear equilibrium $(B_1^0, B_2^0 : \mathbb{R}^2 \rightarrow \mathbb{R}, P^0 : \mathbb{R}^2 \rightarrow \mathbb{R})$. The equilibrium satisfies $B_1^0(\pi_1, \pi_2, x) = xc_2\beta^0 / c$, $B_2^0(\pi_1, \pi_2, x) = xc_\beta^0 / c$ and $P^0(r_1, r_2) = \alpha^0 + \beta^0(r_1 + r_2)$ for any $\pi_1, \pi_2, x, r_1, r_2 \in \mathbb{R}$, where $c = 2(c_1c_2 + c_1c_3 + c_2c_3)$, and $\alpha^0, \beta^0$ are scalars such that $0 < \beta^0 < 1$.

It follows from Proposition 1 that the benchmark case of $\lambda = c_1 / c_2$ yields equilibrium outcomes that are consistent with traditional models of earnings management, which assume an aggregate earnings report. In particular, the equilibrium outcomes in the benchmark case coincide exactly with those of Fischer and Verrecchia (2000). The benchmark case of $\lambda = c_1 / c_2$ yields a unique linear equilibrium with an overall reporting bias of $B_1^0(\pi_1, \pi_2, x) + B_2^0(\pi_1, \pi_2, x) = xc(c_1 + c_2)\beta^0 / c$, where $c = 2(c_1c_2 + c_1c_3 + c_2c_3)$. The sign of the reporting bias is the same as the sign of $x$. Its absolute value is increasing in the importance that the manager attaches to the market price of the firm, as captured by the absolute value of $x$. The absolute value of the reporting bias is also increasing in the weight $\beta^0$ assigned by the pricing function $P^0(r_1, r_2) = \alpha^0 + \beta^0(r_1 + r_2)$ to the reported earnings $r = r_1 + r_2$, but it is decreasing in the marginal biasing costs $c_1, c_2$ and $c_3$. 

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Since investors do not observe the realization $x$ of the random event $\tilde{x}$, they cannot precisely detect the reporting bias $B_1^0(\pi_1, \pi_2, x) + B_2^0(\pi_1, \pi_2, x) = x(c_1 + c_2)\beta^0 / c$, and thus cannot decipher the manager’s private information from her report. Being unable to undo the reporting bias, the market uses the earnings report $r = r_1 + r_2$ only as a noisy signal of the underlying earnings realization $\tilde{x}$. Consequently, the weight $\beta^0$ that the benchmark equilibrium pricing function $P^0(r_1, r_2) = \alpha^0 + \beta^0(r_1 + r_2)$ assigns to the aggregate reported earnings $r = r_1 + r_2$ is lower than 1.

More importantly, in pricing the firm, the market places the same weight $\beta^0$ on the two observable components $r_1$ and $r_2$ of the reported earnings. That is, when $\lambda = c_1 / c_2$, investors only use the aggregate earnings datum $r$ in pricing the firm and find its decomposition into the components $r_1$ and $r_2$ redundant. Disaggregated data cannot assist investors in identifying the reporting bias when $\lambda = c_1 / c_2$, because the equilibrium reporting bias is proportionally embedded in the two components of the earnings report as reflected by $B_2^0(\pi_1, \pi_2, x) / B_1^0(\pi_1, \pi_2, x) = c_1 / c_2 = \lambda$. Investors rationally infer that the reporting bias preserves the fundamental proportion $\lambda$ between the two earnings components. They thus attribute any deviation of the reported earnings components from their expected proportion to economic noise, so such deviation is not informative to them in detecting the reporting bias.

The benchmark equilibrium presented in Proposition 1 constitutes an important reference point when evaluating the power of disaggregated earnings data in mitigating earnings management. This is because the benchmark equilibrium appears to be the unique linear equilibrium that the model yields for any value of $\lambda$ under an aggregate reporting regime, where the manager provides only the aggregate report $r$ without detailing its components $r_1$ and $r_2$. We formally state this result in the following corollary.
Corollary to Proposition 1. The benchmark equilibrium \((B_1^0, B_2^0 : \mathbb{R}^3 \to \mathbb{R}, P^0 : \mathbb{R}^2 \to \mathbb{R})\) presented in Proposition 1 is the unique linear equilibrium for any value of \(\lambda\) under an aggregate reporting regime, where the earnings report includes only the aggregate datum \(r\) (and all other modeling assumptions are kept intact).

3.2. The Case of \(\lambda \neq c_1 / c_2\)

Having analyzed the benchmark case of \(\lambda = c_1 / c_2\), we now turn to analyzing the more frequent situations where the fundamental proportion \(\lambda\) between the earnings components deviates from the ratio \(c_1 / c_2\) of their marginal biasing costs. Since the costs ratio \(c_1 / c_2\) is positive by definition, \(\lambda \neq c_1 / c_2\) implies that either the manager has different incentives regarding the direction of the bias in the two earnings components (when \(\lambda\) is negative) or she has different abilities in biasing the two earnings components (when \(\lambda\) is positive). It appears that in both situations, the manager’s optimal reporting bias distorts the fundamental proportion between the two earnings components, which makes the disaggregated earnings data useful in detecting and suppressing earnings management. While we cannot rule out the existence of multiple linear equilibria when \(\lambda \neq c_1 / c_2\), we establish in Proposition 2 the existence of a linear equilibrium and characterize the form of any such equilibrium.

Proposition 2. In the case of \(\lambda \neq c_1 / c_2\), there exists at least one linear equilibrium \((B_1, B_2 : \mathbb{R}^3 \to \mathbb{R}, P : \mathbb{R}^2 \to \mathbb{R})\). Any such linear equilibrium satisfies

\[
B_1(\pi_1, \pi_2, x) = x(c_2 \beta + c_3(\beta - \gamma)) / c, \quad B_2(\pi_1, \pi_2, x) = x(c_1 \gamma - c_3(\beta - \gamma)) / c, \quad \text{and}
\]

\[
P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2 \quad \text{for any } \pi_1, \pi_2, x, r_1, r_2 \in \mathbb{R}, \text{ where } c = 2(c_1 c_2 + c_1 c_3 + c_2 c_3) \text{ and } \alpha, \beta, \gamma \text{ are scalars. The pricing coefficients } \beta \text{ and } \gamma \text{ satisfy } \beta \neq \gamma \text{ and}
\]

\[
0 < c_2 \beta + c_1 \gamma < (c_1 + c_2) \beta^0.
\]
According to Proposition 2, when $\lambda \neq c_1 / c_2$, the equilibrium pricing function

$$P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2$$

assigns different weights to the two components of the reported earnings $r_1$ and $r_2$.\(^{11}\) Hence, unlike the benchmark case of $\lambda = c_1 / c_2$, where investors do not use the reported disaggregated earnings data and the pricing function

$$P^0(r_1, r_2) = \alpha^0 + \beta^0 (r_1 + r_2)$$

assigns an identical weight of $\beta^0$ to both earnings components, disaggregated earnings data become informative to investors when $\lambda$ deviates from $c_1 / c_2$, enabling them to more accurately evaluate the firm. Proposition 2 further suggests that the overall reporting bias $B_1(\pi_1, \pi_2, x) + B_2(\pi_1, \pi_2, x) = x(c_2 \beta + c_1 \gamma) / c$ in situations where $\lambda \neq c_1 / c_2$ is of lower magnitude relative to the benchmark bias

$$B_1^0(\pi_1, \pi_2, x) + B_2^0(\pi_1, \pi_2, x) = x(c_1 + c_2) \beta^0 / c .$$

The explanation behind the results of Proposition 2 is the disproportional partition of the equilibrium reporting bias across the two earnings components, which distorts their fundamental proportion $\lambda$ (that is, $B_1(\pi_1, \pi_2, x) / B_2(\pi_1, \pi_2, x) \neq \lambda$). This enables investors to utilize the reported disaggregated earnings data in reducing to some extent the exogenous noise $\tilde{x}$ embedded in the reporting bias, though they are still incapable of perfectly identifying and backing out the reporting bias $B_1(\pi_1, \pi_2, x) + B_2(\pi_1, \pi_2, x) = x(c_2 \beta + c_1 \gamma) / c$. Based on the disaggregated earnings report, they can compute the extent $r_2 - \lambda r_1$ to which the reported earnings components $r_1$ and $r_2$ deviate from the fundamental proportion $\lambda$. The observable measure $r_2 - \lambda r_1$ is the sum of two unobservable values: $\pi_2 - \lambda \pi_1$ and $r_2 - \pi_2 - \lambda (r_1 - \pi_1)$. The former is the realization of the value-independent variable $\bar{\epsilon} = \bar{\pi}_2 - \lambda \bar{\pi}_1$, whereas the

\(^{11}\) When $0 < \lambda < c_1 / c_2$, the weight $\beta$ of $r_1$ in the pricing function is higher than the weight $\gamma$ of $r_2$. The opposite occurs when $\lambda > c_1 / c_2$. In both cases, the market places more weight on the less manageable component of the earnings report.
latter is the realization of \( B_1(\bar{x}_1, \bar{x}_2, \bar{x}) - \lambda B_2(\bar{x}_1, \bar{x}_2, \bar{x}) = \tilde{x}(c_1\gamma - \lambda c_2\beta - (1 + \lambda)c_1(\beta - \gamma))/c \).

Knowing this, investors recognize that any deviation of \( r_2 - \lambda r_1 \) from \( \mu_e \) could be due to either the economic noise embedded in the stochastic relationship between the earnings components or the noise \( \tilde{x} \) embedded in the reporting bias \( r - \pi \). By allowing them to diminish some of the noise \( \tilde{x} \) that underlies the reporting bias, the observable measure \( r_2 - \lambda r_1 \) serves investors as a noisy indicator of earnings management. Hence, in pricing the firm, investors utilize the estimator \( r_2 - \lambda r_1 \) in addition to the aggregate earnings measure \( r = r_1 + r_2 \) (which is the one and only relevant signal in the benchmark case). Accordingly, the following corollary to Proposition 2 offers an alternative representation of the firm’s market price

\[
P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2 \text{ as a function of } r_1 + r_2 \text{ and } r_2 - \lambda r_1.
\]

**Corollary to Proposition 2.** *In the case of \( \lambda \neq c_1 / c_2 \), the equilibrium pricing function

\[
P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2 \text{ presented in Proposition 2 can be rewritten as}
\]

\[
P(r_1, r_2) = \alpha + \delta (r_1 + r_2) + \tau(r_2 - \lambda r_1) \text{ for any } r_1, r_2 \in \mathbb{R}, \text{ where } \delta = (\beta + \lambda \gamma)/(1 + \lambda) \text{ and}
\]

\[
\tau = (\gamma - \beta)/(1 + \lambda). \text{ The pricing coefficient } \delta \text{ satisfies } 0 < \beta^0 < \delta < 1. \text{ The pricing coefficient } \tau \text{ is negative for } \lambda < c_1 / c_2 \text{ and positive for } \lambda > c_1 / c_2.
\]

Since investors are more capable of detecting the reporting bias in situations where \( \lambda \neq c_1 / c_2 \) relative to the benchmark case of \( \lambda = c_1 / c_2 \), the aggregate earnings measure \( r = r_1 + r_2 \) becomes more informative to them, and thus the weight \( \delta \) that they place on it is higher than the benchmark weight \( \beta^0 \) (though it is still less than 1). To elaborate on the weight \( \tau \) that investors place on the indicator \( r_2 - \lambda r_1 \) of earnings management, it is convenient to distinguish between negative and positive values of \( \lambda \).

When \( \lambda \) is negative, so that one of the earnings components represents incomes and
the other represents expenses, the wish of the manager to bias incomes and expenses in opposite directions generates a reporting bias that distorts the fundamental proportion between the two components. In this case, the indicator \( r_2 - \lambda r_1 \) increases (decreases) when \( x \) is positive (negative) and the manager biases earnings upward (downward). Therefore, the pricing function assigns a positive weight \( \delta \) to the aggregate reported earnings \( r = r_1 + r_2 \) and a negative weight \( \tau \) to the indicator \( r_2 - \lambda r_1 \) of earnings management. That is, for any fixed level of aggregate reported earnings \( r = r_1 + r_2 \), the firm’s market price is negatively related to the estimator \( r_2 - \lambda r_1 \) that noisily indicates the extent of the reporting bias.

When \( \lambda \) is positive, the manager wishes to bias the two earnings components in the same direction, but she can more easily bias one of the components relative to the other. In the case \( 0 < \lambda < c_1 / c_2 \), the first earnings component is less manageable than the second, so investors can infer that the reporting bias works to increase (decrease) the value of \( r_2 - \lambda r_1 \) when \( x \) is positive (negative) and the manager biases earnings upward (downward). As a result, the pricing function assigns a positive weight \( \delta \) to the aggregate reported earnings \( r = r_1 + r_2 \) and a negative weight \( \tau \) to the indicator \( r_2 - \lambda r_1 \) of earnings management. The opposite occurs in the case of \( \lambda > c_1 / c_2 \). Here, the first earnings component is more manageable than the second, so investors infer that the reporting bias works to decrease (increase) the value of \( r_2 - \lambda r_1 \) when \( x \) is positive (negative) and the manager biases earnings upward (downward). In this case, therefore, the pricing function still assigns a positive weight \( \delta \) to the aggregate reported earnings \( r = r_1 + r_2 \), but now it places a positive weight \( \tau \) on the indicator \( r_2 - \lambda r_1 \) of earnings management.

In all situations where \( \lambda \neq c_1 / c_2 \), the manager, who recognizes the power of disaggregated accounting data in enabling the market to imperfectly detect the reporting
bias, becomes more reluctant to manipulate the earnings report, and thus her optimal choice of
the reporting bias is lower compared to the benchmark case. This is reflected in Proposition 2
by the inequality \(0 < c_2 \beta + c_1 \gamma < (c_1 + c_2) \beta^0\), which implies that the equilibrium overall
reporting bias \(B_1(\pi_1, \pi_2, x) + B_2(\pi_1, \pi_2, x) = x(c_2 \beta + c_1 \gamma) / c\) is lower in magnitude than the
benchmark reporting bias \(B_1^0(\pi_1, \pi_2, x) + B_2^0(\pi_1, \pi_2, x) = x(c_1 + c_2) \beta^0 / c\). We thus conclude
that disaggregated accounting information, via its role in imperfectly detecting the reporting
bias, works to suppress managerial misreporting incentives.

We emphasize, however, that despite their restricting impact on managerial ability to
engage in earnings management, disaggregated accounting data are not necessarily undesirable
from the viewpoint of managers. It has been well established in the literature that managers
can be worse off with the option to bias their earnings reports (see Stein 1989). Managers
might end up taking costly actions to bias their reporting even when they know that they are
unable to fool the market. Managers are trapped into such inefficient behavior because they
take the market’s conjectures as fixed, knowing that investors will suspect their report in any
case. Disaggregated accounting data might mitigate this problem, as they enable investors to
better assess the reporting bias and thereby allow managers to decrease their compelled
engagement in inefficient actions of earnings management.

Additional interesting insights arise when we consider the reporting bias in each of the
earnings components separately. According to Proposition 2, the reporting bias in the first
earnings component is \(B_1(\pi_1, \pi_2, x) = x(c_2 \beta + c_1 (\beta - \gamma)) / c\), whereas the reporting bias in the
second earnings component is \(B_2(\pi_1, \pi_2, x) = x(c_1 \gamma - c_2 (\beta - \gamma)) / c\). The term \(xc_2 \beta / c\) in
\(B_1(\pi_1, \pi_2, x)\) implies that the manager is more inclined to bias the first earnings component
when its coefficient \(\beta\) in the pricing function \(P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2\) is higher. Similarly,
the term \( xc_1 \gamma / c \) in \( B_2(\pi_1, \pi_2, x) \) implies that the manager is more inclined to bias the second earnings component when its coefficient \( \gamma \) in the pricing function is higher. The terms 

\[ + c_3(\beta - \gamma) / c \quad \text{and} \quad - c_3(\beta - \gamma) / c \]

in \( B_1(\pi_1, \pi_2, x) \) and \( B_2(\pi_1, \pi_2, x) \), respectively, reflect the propensity of the manager to shift the bias from one of the earnings components to the other (relative to the optimal biases under an aggregate earnings report). The manager increases the bias in one of the earnings components by an amount of \( c_3|\beta - \gamma| / c \), while decreasing the bias in the other component by the same amount. This shift of the reporting bias from one component to the other is beneficial to the manager, because it fogs the deviation of the reported earnings components from their fundamental proportion (as captured by the indicator \( r_2 - \lambda r_1 \)), and thereby decreases the market’s estimate of the reporting bias, without actually changing the overall reporting bias.

4. **Comparative Statics Analysis**

The analysis provided in the previous section highlights the importance of disaggregated accounting data in mitigating reporting manipulations, shedding light on the merits of expanded disaggregated disclosure in cases where the accounting system allows a relatively high degree of discretion in reporting. In this section, we attempt to provide accounting policy-makers with useful guidance in selecting among alternative disaggregating accounting procedures. We do so by utilizing a comparative statics analysis to investigate how the exact structure of disaggregated accounting information affects its effectiveness in detecting and suppressing earnings management.\(^\text{12}\)

\(^{12}\) When multiple equilibria exist, all the comparative statics results presented in this section remain the same within each equilibrium.
The comparative statics analysis focuses on two equilibrium outcomes. The first is the variance of the firm’s true earnings $\bar{\pi}$ conditional on the disaggregated earnings report, which is used to compare the effectiveness of alternative disaggregating procedures in detecting earnings management. The second is the absolute value of the overall reporting bias, which is used to compare the effectiveness of alternative disaggregating procedures in suppressing earnings management. Specifically, a lower level of the conditional variance of $\bar{\pi}$ implies that the disaggregating procedure is more effective in detecting earnings management. Similarly, a lower absolute value of the overall reporting bias implies that the disaggregating procedure is more effective in suppressing earnings management. We analyze the sensitivity of these two equilibrium outcomes to the modeling parameters $\lambda$ and $\sigma^2$ that depict important properties of the disaggregated data.

We first analyze the sensitivity of the equilibrium outcomes to the fundamental proportion $\lambda$ between the earnings components. We are able to provide conclusive comparative statics results only when $\lambda$ is positive. Focusing on positive values of $\lambda$, where the two earnings components are likely to share the same sign, we especially interested in analyzing how the equilibrium relates to the extent to which the proportion $\lambda$ differs from the ratio $c_1 / c_2$ of the biasing costs. It follows from Propositions 1 and 2 that disaggregated accounting information is effective in mitigating earnings management only when $\lambda \neq c_1 / c_2$ and the managerial leeway in manipulating reporting is different for the various earnings components. Proposition 3 complements this argument by showing that the effectiveness of disaggregated accounting information in this role monotonically increases when the departure of $\lambda$ from $c_1 / c_2$ grows larger and the ability to manage the different earnings components becomes more diverse.
Proposition 3. Assume $\lambda \geq 0$. Both the variance of $\bar{\pi}$ conditional on the disaggregated earnings report and the absolute value of the overall reporting bias initially increase in $\lambda$, reaching a maximum at $\lambda = c_1 / c_2$, and then decrease in $\lambda$.

The more $\lambda$ departs from $c_1 / c_2$, the greater the diversity in the ability to manage the two earnings components, and the larger the disproportion in their reporting bias. As a result, the deviation $r_2 - \lambda r_1$ of the reported earnings components from their fundamental proportion becomes more indicative of the reporting bias. Consequently, the market’s ability to detect the reporting bias improves, enhancing the informational quality of the earnings report. The manager, being aware that the market is better able to discern earnings management, becomes more cautious in manipulating the earnings report, so the magnitude of the overall reporting bias decreases. Proposition 3 therefore points to the advantages of disaggregating accounting procedures that decompose earnings into components that are very different in the extent to which their reporting can be manipulated, such as pure cash-based items and pure accrual-based items.

While Proposition 3 pertains only to positive values of $\lambda$, particular attention should be also paid to situations where $\lambda$ is negative, so that one of the earnings components represents incomes and the other represents expenses. Since the costs ratio $c_1 / c_2$ is positive by definition, any negative $\lambda$ must satisfies $\lambda \neq c_1 / c_2$. It follows thus from Proposition 2 that disaggregated earnings report that decomposes earnings into incomes and expenses is always effective in mitigating earnings management, regardless of the exact degree of leeway in managing the different earnings components. When $\lambda$ is negative, even if the manager exercises the same degree of discretion in managing the two earnings components, her wish to bias the incomes and expenses in opposite directions generates a reporting bias that distorts the fundamental proportion between the two components. Under such circumstances, the
observable measure \( r_2 - \lambda r_1 \) is always indicative about the reporting bias, regardless of the biasing costs \( c_1 \) and \( c_2 \), improving the informational quality of the disaggregated earnings report in pricing the firm and reducing the managerial misreporting incentives. This sheds light on the merits of disaggregating accounting procedures that decompose earnings into pure-income and pure-expense components. However, the dependence of the equilibrium outcomes on \( \lambda \) takes a rather complicated shape when \( \lambda \) is negative, which does not enable us to provide clear-cut comparative statics results for negative values of \( \lambda \). This is due to the existence of two forces that are at work when \( \lambda \) is negative. The first is the diversity in the manager’s incentives regarding to the direction of the bias in the two components and the second is the diversity in the manager’s abilities to bias the two components. This is unlike the case of a positive \( \lambda \), where the manager wishes to bias the two earnings components in the same direction, so the only force at work is the diversity in her ability to bias the two components.

We now turn to analyzing the sensitivity of the equilibrium outcomes to the extent to which the fundamental proportion between the earnings components \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) is noisy, as captured by \( \sigma^2_\varepsilon \). Proposition 4 demonstrates that, when \( \sigma^2_\varepsilon \) decreases and the fundamental proportion between \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) becomes less noisy, disaggregated accounting data become more effective in detecting and suppressing earnings management.

**Proposition 4.** Assume \( \lambda \neq c_1 / c_2 \). Both the variance of \( \tilde{\pi} \) conditional on the disaggregated earnings report and the absolute value of the overall reporting bias increase in \( \sigma^2_\varepsilon \).

When \( \sigma^2_\varepsilon \) decreases and the fundamental proportion between the earnings components \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) becomes less noisy, the deviation \( r_2 - \lambda r_1 \) of the reported earnings components from their fundamental proportion provides a more accurate indicator of the reporting bias.
Consequently, the ability of investors to detect the reporting bias improves as $\sigma^2_{\varepsilon}$ decreases, and the earnings report becomes more informative to them in evaluating the firm. This reduces the manager’s incentives to manipulate the earnings report, so the reporting bias decreases as $\sigma^2_{\varepsilon}$ decreases. It thus follows from Proposition 4 that the effectiveness of disaggregated accounting information in detecting and suppressing earnings management is monotonically increasing in the strength of the fundamental proportion between its components. This suggests that accounting standards setters should seek to decompose earnings into components that fundamentally exhibit the most stable proportions.

5. **Empirical Implications**

Assessing the usefulness of accounting earnings to investors has been the focus of numerous empirical studies. Our study contributes to this line of research by providing a rational explanation for the effect of earnings management on the association between stock prices and accounting earnings. In particular, our analysis points out to a potential misspecification in empirical models commonly used in the literature. More specifically, it follows from Proposition 2 that including the aggregate earnings measure $r = r_1 + r_2$ as a single explanatory variable of share prices in a regression model may be insufficient. The corollary to Proposition 2 points to the deviation $r_2 - \lambda r_1$ of the earnings components from their expected economic proportion as an additional explanatory variable that captures the extent of the reporting bias. The equilibrium price function $P(r_1, r_2) = \alpha + \delta (r_1 + r_2) + \tau (r_2 - \lambda r_1)$ therefore suggests a methodological refinement that might improve the specification of the relation between share prices and earnings. Proposition 4 further predicts that the explanatory power of the variable $r_2 - \lambda r_1$ is especially large when the regression model is applied to firms that exhibit less noisy proportions between their earnings components.
In addition to the empirical predictions regarding the way investors utilize disaggregated earnings data in detecting earnings management and in pricing firms, our study provides testable empirical predictions with respect to the managerial tendency to engage in earnings management. Proposition 4 predicts a lower level of earnings management for firms that are characterized by less noisy proportions between their earnings components. A lower level of earnings management is also expected in firms that provide more detailed disclosures on the components of earnings. Similarly, our study predicts a reduction in earnings management following the issuance of an accounting standard that mandates additional disclosure on the components of earnings (for example, mandating segment disclosures of sales and operating income).

Empirical examination of our predictions requires the careful design of proxies for the economic relationships between different earnings components. Financial ratios seem natural candidates for this role. Particular attention should be given, however, to the fact that financial ratios are measured using reported earnings data rather than the underlying (unobservable) true earnings data. This problem could be mitigated by averaging the financial ratios of firms over several reporting periods, provided that deviations of the reported earnings items from the corresponding true earnings items mean-revert over time. The theory developed in our study points to financial ratios that involve only income statement components, such as net profit margin, operating profit margin, gross profit margin, and effective tax rate. Nevertheless, financial ratios based on balance sheet line items, such as asset turnover, might be employed in empirical analysis as well. This is so because of the linkage between the income statement and the balance sheet. In particular, any item that appears in the income statement is associated with a change in balance sheet items. A similar argument can be applied to financial ratios that involve items from the cash flow statement, as these ratios capture relationships between cash-based earnings components and accrual-based earnings components.
While most of our analytical results have not been directly tested, there is empirical evidence consistent with our predictions. For example, in their recent study, Hirst, Koonce and Venkataraman (2007) find that disaggregation in management earnings forecasts enhances their credibility. Consistent with our predicted pricing rule, Lev and Thiagarajan (1993) argue that the deviation of the effective tax rate (measured as income tax expense divided by pre-tax income) from the statutory tax rate is a sign of lower earnings quality. Also, Ohlson and Penman (1992) find that different components of earnings have different valuation coefficients, but these coefficients converge in magnitude as earnings are accumulated over a sufficiently long period of time. They attribute this result to the diversity in the measurement errors embedded in different earnings components, which diminishes as the time window becomes longer. Our study provides a slightly different interpretation of their result, suggesting that diversity in the ability to manage the different earnings components might be the reason for their different valuation coefficients. Over a long period of time, if earnings management is mean-reverting, the difference in valuation coefficients shrinks. In the context of our model, diversity in the ability to manage the different earnings components might also be the reason for the empirical evidence documenting that investors place a higher value on the cash components of earnings than on accrual-based earnings components (see Sloan 1996).

6. **Concluding Remarks**

This paper highlights accounting disaggregation as a powerful tool in detecting and suppressing earnings management. It suggests that expanded disaggregated accounting disclosure is especially advantageous in cases where accounting standards allow management to select among several reporting alternatives. In cases where the reporting discretion is substantial, restrictions on the use of materiality considerations might also be recommended in order to avoid the withholding of information that can assist in mitigating earnings
management. The analysis also offers guidance to accounting policy-makers in selecting among alternative accounting disaggregating procedures. In particular, it demonstrates that disaggregated earnings information is most effective in mitigating earnings management when its components are most tightly proportional to each other due to their fundamental economic nature. The analysis further points to the merits of decomposing the earnings report into pure-income and pure-expense components, or alternatively – decomposing it into components that are very different in the extent to which their reporting can be manipulated.

While our study sheds light on considerations that could be important to accounting standards setters in selecting the proper accounting disaggregation procedure, these considerations obviously should be regarded in the context of other considerations that are not analyzed in this study. They should be viewed, for example, in light of the differences in the proprietary costs implied by different disaggregating accounting procedures. They should be also viewed in light of the differences in the likelihood of different kinds of disaggregated data to be voluntarily disclosed by managers even in the absence of mandatory requirements (see Einhorn 2005).

We also emphasize that the conclusions drawn from our analysis only pertain to disaggregated information that details additive line items of an aggregate accounting measure, such as incomes and expenses that are accumulated into the net earnings measure. These conclusions do not necessarily hold, however, with respect to other types of detailed accounting data. Dye and Sridhar (2003), for example, consider accounting information that details the substitutable (non-additive) objective and subjective measures on which a single summary accounting datum is based, such as the historical cost and the fair value measures that underlie the familiar Lower Cost or Market accounting measurement. They demonstrate circumstances where this kind of detailed accounting data could enhance managerial
misreporting incentives, rather than suppress them, because more weight is attached to subjective measures.

Our results are consistent with extant empirical evidence. In addition, the results offer hitherto unidentified empirical predictions with regard to the way that investors utilize disaggregated earnings data and ratio analysis in detecting earnings management when pricing firms. They also provide interesting testable predictions with regard to the managerial tendency to engage in earnings management. In a companion paper, we provide empirical evidence that support our predictions.
APPENDIX

Using symmetry considerations, we assume throughout the appendix that \(-1 < \lambda \leq c_1 / c_2\).

This assumption does not detract from the generality of the proofs, because

\[
\lambda = \frac{\sigma_1^2 + \sigma_{12}}{(1 + \lambda)^2 c_1} > c_1 / c_2 \quad \text{implies} \quad 1 / \lambda = \frac{\sigma_1^2 + \sigma_{12}}{(1 + \lambda)^2 c_1} < c_2 / c_1,
\]

whereas \(\lambda < -1\) implies \(1 / \lambda > -1\). So, the proofs apply to the case of \(\lambda > c_1 / c_2\) or \(\lambda < -1\) after exchanging the indexes of the two earnings components. To further streamline the presentation of the proofs, we use additional notation throughout the appendix. Specifically, we define the following three functions for any \(v, w > 0\):

\[
F(v, w) = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} c^{-1} \left( c_1 + c_2 - (c_1 - \lambda c_2) \frac{\sigma^2 v w}{(\sigma_x v)^2 + \sigma_v^2} \right),
\]

\[
G(v, w) = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} c^{-1} \left( c_1 - \lambda c_2 - (c_1 + \lambda^2 c_2) \frac{\sigma^2 v w}{(\sigma_x v)^2 + \sigma_v^2} \right),
\]

\[
H(v, w) = (c_1 + \lambda^2 c_2 + (1 + \lambda)^2 c_3) v - (c_1 - \lambda c_2) w \frac{(1 + \lambda)^2}{\sigma^2 + (\sigma_x v)^2} \frac{\sigma_v^2}{\sigma_v^2 + (\sigma_x w)^2}.
\]

**Proof of Observation 1.** Since \(\tilde{\pi}_1\) and \(\tilde{\pi}_2\) are normally distributed, so is their linear combination \(\tilde{\varepsilon} = \tilde{\pi}_2 - \lambda \tilde{\pi}_1\). The mean of \(\tilde{\varepsilon} = \tilde{\pi}_2 - \lambda \tilde{\pi}_1\) equals \(\mu_\varepsilon = \mu_2 - \lambda \mu_1\). The variance of \(\tilde{\varepsilon} = \tilde{\pi}_2 - \lambda \tilde{\pi}_1\) equals \(\sigma_\varepsilon^2 = \sigma_1^2 + \lambda^2 \sigma_1^2 - 2 \lambda \sigma_{12}\). Lastly, 

\[
\text{cov}(\tilde{\pi}, \tilde{\varepsilon}) = \text{cov}(\tilde{\pi}, \tilde{\pi}_2) - \lambda \text{cov}(\tilde{\pi}, \tilde{\pi}_1).
\]

Substituting \(\lambda = \text{cov}(\tilde{\pi}, \tilde{\pi}_2) / \text{cov}(\tilde{\pi}, \tilde{\pi}_1)\), we get 

\[
\text{cov}(\tilde{\pi}, \tilde{\varepsilon}) = 0.
\]
As a basis for the proofs of Propositions 1-4, we state and prove Lemmata 1-6.

**Lemma 1.** Any linear equilibrium in the model is associated with a pair \( v \) and \( w \) that solves the equations \( v = F(v, w) \) and \( w = G(v, w) \). Any such pair defines an equilibrium of the following form: \( P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2 = \alpha + \delta(r_1 + r_2) + \tau(r_2 - (\varphi + \lambda r_1)) \),

\[
B_1(\pi_1, \pi_2, x) = x(c_2 \beta + c_3 (\beta - \gamma))/c \quad \text{and} \quad B_2(\pi_1, \pi_2, x) = x(c_1 \gamma - c_3 (\beta - \gamma))/c ,
\]

where

\[
\alpha = \mu - \delta(\mu + \frac{\sigma_v^2}{\sigma_x^2 + \sigma_e^2} v \mu), \quad \delta = \frac{\sigma_v^2}{\sigma_x^2 + \sigma_e^2}, \quad \tau = -\delta \cdot \frac{\sigma^2_{v w}}{\sigma_x^2 + \sigma_e^2}, \quad \beta = \delta - \lambda \tau , \quad \gamma = \delta + \tau .
\]

**Proof of Lemma 1.** We look for equilibrium with a linear pricing function. Thus, we assume there exist scalars \( \alpha, \beta \) and \( \gamma \), such that \( P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2 \). The manager’s optimal reporting strategy satisfies

\[
(\pi_1, \pi_2, x), B_1(\pi_1, \pi_2, x)) = \arg\max_{(b_1, b_2)} x(\alpha + \beta(\pi_1 + b_1) + \gamma(\pi_2 + b_2)) - c_1 b_1^2 - c_2 b_2^2 - c_3 (b_1 + b_2)^2 .
\]

Using the notation \( U(b_1, b_2) = x(\alpha + \beta(\pi_1 + b_1) + \gamma(\pi_2 + b_2)) - c_1 b_1^2 - c_2 b_2^2 - c_3 (b_1 + b_2)^2 \), the first-order conditions are

\[
\frac{\partial U}{\partial b_1} = x \beta - 2c_1 b_1 - 2c_3 (b_1 + b_2) = 0 \quad \text{and} \quad \frac{\partial U}{\partial b_2} = x \gamma - 2c_2 b_2 - 2c_3 (b_1 + b_2) = 0 .
\]

The function \( U(b_1, b_2) \) gets its maximal value at

\[
b_1 = x(c_2 \beta + c_3 (\beta - \gamma))/c \quad \text{and} \quad b_2 = x(c_1 \gamma - c_3 (\beta - \gamma))/c ,
\]

where the first-order conditions hold, and also

\[
\frac{\partial^2 U}{\partial b_1 \partial b_2}(b_1, b_2) = -2c_1 - 2c_3 < 0 , \quad \frac{\partial^2 U}{\partial b_2 \partial b_2}(b_1, b_2) = -2c_2 - 2c_3 < 0 , \quad \text{and} \quad \frac{\partial^2 U}{\partial b_1 \partial b_2}(b_1, b_2) = -2c_1 - 2c_3 - 2c_3 < 0 .
\]

Hence, the optimal reporting biases in the two earnings components are

\[
B_1(\pi_1, \pi_2, x) = x(c_2 \beta + c_3 (\beta - \gamma))/c \quad \text{and} \quad B_2(\pi_1, \pi_2, x) = x(c_1 \gamma - c_3 (\beta - \gamma))/c .
\]
$B_2(\pi_1, \pi_2, x) = x(c_1' - c_3(\beta - \gamma))/c$, so the overall reporting bias is $x(c_2\beta + c_1\gamma)/c$.

Since $\lambda \neq -1$, the signals $r_1 + r_2$ and $r_2 - \lambda r_1$ are equivalent to the signals $r_1$ and $r_2$. To streamline the presentation, we henceforth use the notation $v = (c_2\beta + c_1\gamma)/c$ and

$w = (c_1\gamma - \lambda c_2\beta - (1 + \lambda)c_3(\beta - \gamma))/c$. Using this notation, the reported aggregate earnings measure $r_1 + r_2$ is the realization of the random variable $\tilde{\pi} + \tilde{\pi}v$, whereas $r_2 - \lambda r_1$ is the realization of the random variable $\tilde{\pi}_2 - \lambda \tilde{\pi}_1 + \tilde{\pi}w$, which equals $\tilde{\pi} + \tilde{\pi}w$. Recall now that both $\tilde{\pi} \sim N(\mu_\pi, \sigma_\pi^2)$ and $\tilde{\pi} \sim N(\mu_\pi, \sigma_\pi^2)$ are independent of the true earnings variable

$\tilde{\pi} \sim N(\mu_\pi, \sigma_\pi^2)$. Hence, using the properties of the normal distribution, we get that

$E[\tilde{\pi} \mid \tilde{\pi} + \tilde{\pi}v = r_1 + r_2, \tilde{\pi} + \tilde{\pi}w = r_2 - \lambda r_1]$ equals

$$
\mu + \frac{\sigma^2}{\sigma^2 + \mu^2 \text{VAR}[\tilde{\pi} \mid \tilde{\pi} + \tilde{\pi}w = r_2 - (\phi + \lambda r_1)]} (r_1 + r_2 - \mu - v E[\tilde{\pi} \mid \tilde{\pi} + \tilde{\pi}w = r_2 - \lambda r_1])
$$

or

$$
\mu + \frac{\sigma^2}{\sigma^2 + (\sigma_v^2)v^2} (r_1 + r_2 - \mu - v E[\tilde{\pi} \mid \tilde{\pi} + \tilde{\pi}w = r_2 - \lambda r_1])
$$

Again using the properties of the normal distribution, we also get that $E[\tilde{\pi} \mid \tilde{\pi} + \tilde{\pi}w = r_2 - \lambda r_1]$ equals

$$
\mu_x + \frac{\sigma^2}{\sigma_x^2 + \mu_x^2} ((r_2 - \lambda r_1 - \mu_x)/w - \mu_x) \quad \text{or} \quad \frac{\sigma^2}{\sigma_x^2 + \mu_x^2} \mu_x + \frac{\sigma^2}{\sigma_x^2 + \mu_x^2} (r_2 - \lambda r_1 - \mu_x).
$$

Consequently, $P(r_1, r_2) = \alpha + \beta r_1 + \gamma r_2 = \alpha + \delta(r_1 + r_2) + \tau(r_2 - \lambda r_1),$

$B_1(\pi_1, \pi_2, x) = x(c_2\beta + c_3(\beta - \gamma))/c$ and $B_2(\pi_1, \pi_2, x) = x(c_1\gamma - c_3(\beta - \gamma))/c$, where

$$
\alpha = \mu - \delta(\mu + \frac{\sigma^2}{\sigma_x^2 + \mu_x^2} v \mu_x) - \tau \mu_x,
\beta = \delta - \lambda \tau, \quad \gamma = \delta + \tau.
$$

Note that $v = (c_2\beta + c_1\gamma)/c$ implies

\[31\]
\[ v = (c_2(\delta - \lambda \tau) + c_1(\delta + \tau)) / c = ((c_1 + c_2)\delta + (c_1 - \lambda c_2)\tau) / c, \] and thus \( v \) satisfies the equation \( v = F(v, w) \). Similarly, \( w = (c_1\gamma - \lambda c_2\beta - (1 + \lambda)c_3(\beta - \gamma)) / c \) implies

\[ \omega = (c_1(\delta + \tau) - \lambda c_2(\delta - \lambda \tau) + (1 + \lambda)^2 c_3 \tau) / c = ((c_1 - \lambda c_2)\delta + (c_1 + \lambda^2 c_2 + (1 + \lambda)^2 c_3)\tau) / c, \] and thus \( w \) satisfies the equation \( w = G(v, w) \).

**Lemma 2.** If \( v \) and \( w \) satisfy the equations \( v = F(v, w) \) and \( w = G(v, w) \), then they also satisfy the equation \( H(v, w) = 0 \).

**Proof of Lemma 2.** Multiplying the equation \( v = F(v, w) \) by \( c_1 + \lambda^2 c_2 + (1 + \lambda)^2 c_3 \) and then subtracting the equation \( w = G(v, w) \) multiplied by \( c_1 - \lambda c_2 \), we get after rearranging the equation \( H(v, w) = 0 \).

**Lemma 3.** If \( v \) and \( w \) satisfy the equations \( v = F(v, w) \) and \( w = G(v, w) \), then they also satisfy \( v, w \geq 0 \) and \( (c_1 - \lambda c_2)v \geq (c_1 + c_2)w \).

**Proof of Lemma 3.** We first show that \( vw \geq 0 \). That is, we show that either both \( v \) and \( w \) are non-negative or both of them are non-positive. Suppose by contradiction that \( v \geq 0 \) and \( w < 0 \). In the case, the left side of the equation \( w = G(v, w) \) is negative, while \( \lambda \leq c_1 / c_2 \) implies that the right side of the equation is non-negative – a contradiction. Suppose now by contradiction that \( v < 0 \) and \( w \geq 0 \). In this case, the left side of the equation \( v = F(v, w) \) is negative, while \( \lambda \leq c_1 / c_2 \) implies that the right side of the equation is non-negative – a contradiction.

We next show that \( (c_1 - \lambda c_2)v \geq (c_1 + c_2)w \). Since \( \delta = \frac{\sigma^2_v}{\sigma^2_v + (\sigma \times \sigma)^2} + \frac{\sigma^2_w}{\sigma^2_w + (\sigma \times \sigma)^2} \) is positive, the fact that \( vw \geq 0 \) implies that \( \tau = -\delta \cdot \frac{\sigma^2_v w}{(\sigma \times \sigma)^2 + \sigma^2_w} \) is non-positive. When substituting
\[ v = (c_2 \beta + c_1 \gamma) / c = c_1 (\delta - \lambda \tau) + c_1 (\delta + \tau) / c \] and
\[ w = (c_1 \gamma - \lambda c_2 \beta - (1 + \lambda) c_1 (\beta - \gamma)) / c = (c_1 (\delta + \tau) - \lambda c_2 (\delta - \lambda \tau) + (1 + \lambda)^2 c_1 \tau) / c, \]
rearranging, we get that \((c_1 - \lambda c_2) v - (c_1 + c_2) w = -(1 + \lambda)^2 \tau / 2.\) Based on \(\tau \leq 0,\) it follows that \(-(1 + \lambda)^2 \tau / 2\) is non-negative, implying that \((c_1 - \lambda c_2) v \geq (c_1 + c_2) w.\)

Lastly, we preclude the possibility that both \(v\) and \(w\) are negative. Suppose by contradiction that \(v < 0\) and \(w < 0.\) In this case, it follows from the equation \(v = F(v, w)\) that
\[ c_1 + c_2 - (c_1 - \lambda c_2) \frac{\sigma^2_{wv}}{(\sigma_x)^2 + \sigma^2_\varepsilon} \leq 0. \] It follows from \(v, w < 0\) and \((c_1 - \lambda c_2) v \geq (c_1 + c_2) w\) that
\[ c_1 + c_2 - (c_1 - \lambda c_2) \frac{\sigma^2_{wv}}{(\sigma_x)^2 + \sigma^2_\varepsilon} \geq c_1 + c_2 - (c_1 + c_2) \frac{\sigma^2_{w^2}}{(\sigma_x)^2 + \sigma^2_\varepsilon} = (c_1 + c_2) \frac{\sigma^2_\varepsilon}{(\sigma_x)^2 + \sigma^2_\varepsilon} > 0. \] Thus, the right side of the equation \(v = F(v, w)\) is positive, implying that \(v > 0\) – a contradiction.

**Lemma 4.** Any pair \(v\) and \(w\) that solves equations \(v = F(v, w)\) and \(w = G(v, w)\) satisfies the inequalities
\[ F(v, w) \leq \frac{c^{-1}(c_1 + c_2)}{\sigma^2 + (\sigma_x)^2 + (\sigma_x v)^2} \] and
\[ G(v, w) \leq \frac{c^{-1}(c_1 - \lambda c_2)}{\sigma^2 + (\sigma_x)^2 + (\sigma_x v)^2}. \]

**Proof of Lemma 4.** By Lemma 3, \(v, w \geq 0\) and \((c_1 - \lambda c_2) v \geq (c_1 + c_2) w.\) Thus,
\[ c_1 + c_2 - (c_1 - \lambda c_2) \frac{\sigma^2_{wv}}{(\sigma_x)^2 + \sigma^2_\varepsilon} \leq (c_1 + c_2)(1 - \frac{\sigma^2_{w^2}}{(\sigma_x)^2 + \sigma^2_\varepsilon}) = (c_1 + c_2) \frac{\sigma^2_\varepsilon}{(\sigma_x)^2 + \sigma^2_\varepsilon}. \] As
\[ \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \] is positive, it follows that
\[ \frac{\sigma^2_\varepsilon}{\sigma^2 + (\sigma_x)^2} \]
\[
F(v,w) \leq \frac{\sigma^2}{\sigma^2 + (\sigma_v v)^2} c^{-1}(c_1 + c_2) \frac{\sigma_e^2}{(\sigma_e^2 + (\sigma_y w)^2 + \sigma^2_e) + (\sigma_v v)^2} c^{-1}(c_1 + c_2).
\]

Again using Lemma 3, \( v, w \geq 0 \) and \( (c_1 - \lambda c_2) v \geq (c_1 + c_2) w \) imply
\[
c_1 - \lambda c_2 - (c_1 + \lambda c_2 + (1 + \lambda)^2 c_3) \frac{\sigma_{wv}^2}{(\sigma_v w)^2 + \sigma_e^2} \leq c_1 - \lambda c_2 - \frac{(c_1 + \lambda c_2 + (1 + \lambda)^2 c_3)(c_1 + c_2)}{c_1 - \lambda c_2} \frac{\sigma_{wv}^2}{(\sigma_v w)^2 + \sigma_e^2}.
\]

Since
\[
\frac{(c_1 + \lambda c_2 + (1 + \lambda)^2 c_3)(c_1 + c_2)}{c_1 - \lambda c_2} \geq c_1 - \lambda c_2,
\]
we get
\[
c_1 - \lambda c_2 - (c_1 + \lambda c_2 + (1 + \lambda)^2 c_3) \frac{\sigma_{wv}^2}{(\sigma_v w)^2 + \sigma_e^2} \leq (c_1 - \lambda c_2)(1 - \frac{\sigma_{wv}^2}{(\sigma_v w)^2 + \sigma_e^2}) = (c_1 - \lambda c_2) \frac{\sigma_e^2}{(\sigma_v w)^2 + \sigma_e^2}.
\]

As
\[
\frac{\sigma^2}{\sigma^2 + (\sigma_v v)^2} \frac{\sigma_e^2}{(\sigma_v w)^2 + \sigma_e^2}
\]
is positive, it follows that
\[
G(v,w) \leq \frac{\sigma^2}{\sigma^2 + (\sigma_v v)^2} c^{-1}(c_1 - \lambda c_2) \frac{\sigma_e^2}{(\sigma_v w)^2 + \sigma_e^2} \frac{\sigma^2}{\sigma^2 + (\sigma_v w)^2 + (\sigma_v v)^2} c^{-1}(c_1 - \lambda c_2).
\]

**Lemma 5.** There exists at least one pair of \( v \) and \( w \) that solves the equations \( v = F(v,w) \) and \( w = G(v,w) \).

**Proof of Lemma 5.** Using Lemma 3, we focus on non-negative values of \( v \) and \( w \). Based on Lemma 2, any pair \( v \) and \( w \) that solves equations \( v = F(v,w) \) and \( w = G(v,w) \) must also satisfy the equation \( H(v,w) = 0 \). The function \( H(v,w) \) is increasing in \( v \). For any non-negative \( w \), \( H(0,w) \) is negative and \( \lim_{v \to +\infty} H(v,w) = +\infty \). So, for any non-negative \( w \), there exists a unique positive value of \( v \), denoted \( h(w) \), such that \( H(h(w),w) = 0 \).

Substituting \( v = h(w) \) in the equation \( w = G(v,w) \), we get the equation

\[
K(w) = w - G(h(w),w) = 0.
\]

Since \( K(0) \) is negative and \( \lim_{w \to +\infty} K(w) = +\infty \) by Lemma 4, there exists at least one positive value of \( w \) that satisfies the equation \( K(w) = 0 \).
Lemma 6. When $\lambda = c_1 / c_2$, the equations $v = F(v, w)$ and $w = G(v, w)$ are reduced to

$$v = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \cdot \frac{c_1 + c_2}{c}$$

and $w = 0$.

Proof of Lemma 6. Based on Lemma 3, $v$ and $w$ must be non-negative. Hence, when substituting $\lambda = c_1 / c_2$ in the equation $w = G(v, w)$, the left side is positive, whereas the right side is negative. This implies that $w = 0$. Substituting $w = 0$ in the equation $v = F(v, w)$, we get the equation

$$v = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \cdot \frac{c_1 + c_2}{c}.$$

Proof of Proposition 1 and its corollary. By Lemma 6, when $\lambda = c_1 / c_2$, $v = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \cdot \frac{c_1 + c_2}{c}$ and $w = 0$. By Lemma 3, $v \geq 0$. For non-negative values of $v$, the left of the equation $v = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \cdot \frac{c_1 + c_2}{c}$ is increasing in $v$ from zero to infinity, whereas the right side is positive and decreasing in $v$. Hence, there is a unique (positive) solution $v$ to the equation, which implies a unique equilibrium. Denoting this solution by $v^0$, the equilibrium pricing coefficients are $\alpha^0 = \mu - \beta^0 (\mu + v^0 \mu_x)$ and $\beta^0 = \frac{\sigma^2}{\sigma^2 + (\sigma_x v^0)^2}$, where $0 < \beta^0 < 1$. The same equilibrium emerges under an aggregate reporting regime for any value of $\lambda$, because an aggregate reporting regime implies that the indicator $\tilde{\varepsilon} + \tilde{x}w$ is not available to investors when pricing the firm, which is equivalent to assuming $w = 0$ because $\tilde{\varepsilon}$ is an independent random variable.

Proof of Proposition 2 and its corollary. The existence of equilibrium and its form follow from Lemmata 1 and 5. To show that $v < v^0$, we use the equation $v^0 = \frac{\sigma^2}{\sigma^2 + (\sigma_x v^0)^2} \cdot \frac{c_1 + c_2}{c}$ implied by Lemma 6, replacing $v$ in the left side of the equation $v = F(v, w)$ by...
\[
\frac{\sigma^2}{\sigma^2 + (\sigma_x v^0)^2} \cdot \frac{c_1 + c_2}{c} \quad \text{and replacing } v \text{ in the right side of the equation by } v^0. \quad \text{We get that the}
\]
left side of the equation exceeds \[
\frac{\sigma^2}{\sigma^2 + (\sigma_x w)^2} \cdot \frac{c^{-1}(c_1 + c_2) \epsilon}{\epsilon + (\sigma_x v^0)^2}
\]
right side of the equation is lower than \[
\frac{\sigma^2}{\sigma^2 + (\sigma_x w)^2} \cdot \frac{c^{-1}(c_1 + c_2) \epsilon}{\epsilon + (\sigma_x v^0)^2}
\]
lower than the left side. Since \( F(v, w) \) is decreasing in \( v \), it follows that \( v \) must be lower than \( v^0 \). Since \( v = (c_2 \beta + c_1 \gamma) / c \) and \( v^0 = \beta^0 (c_1 + c_2) / c \), the inequality \( 0 < v < v^0 \) is equivalent to \( 0 < c_2 \beta + c_1 \gamma < (c_2 + c_1) \beta^0 \). Also, \( \beta^0 = \frac{\sigma^2}{\sigma^2 + (\sigma_x v^0)^2} \) and \( \delta = \frac{\sigma^2}{\sigma^2 + (\sigma_x v^0)^2} \frac{\sigma_x^2}{\sigma_x^2 + (\sigma_x w)^2} \), together with the inequality \( v < v^0 \), imply \( 0 < \beta^0 < \delta < 1 \). Since \( v \) and \( w \) are positive, we get that \( \tau = -\delta \frac{\sigma_x^2 w^v}{(\sigma_x w)^2 + \sigma_x^2} \) is negative. It thus follows from \( \beta = \delta - \lambda \tau, \gamma = \delta + \tau \) that
\[
0 < \gamma < \beta^0 < \beta < 1 \quad \text{when } 0 < \lambda < c_1 / c_2.
\]

**Proof of Proposition 3.** We first show that \( w \) decreases in \( \lambda \) for \( 0 \leq \lambda \leq c_1 / c_2 \). It follows from Lemma 3 that \( \omega \geq 0 \). It follows from the proofs of Propositions 1 and 2 that \( w \) is zero in the benchmark case of \( \lambda = c_1 / c_2 \) and strictly positive when \( \lambda < c_1 / c_2 \). Hence, to show that \( w \) monotonically decreases in \( \lambda \) for any \( \lambda \leq c_1 / c_2 \), we need to show that there are no two different values of \( \lambda \) with the same value of \( w \). Suppose by contradiction that there exist two values \( \lambda_L \) and \( \lambda_H \) of \( \lambda \), such that \( \lambda_L < \lambda_H \), for which \( w \) gets the same value \( w_{LH} \). Denote by \( v_L \) and \( v_H \) the values of \( v \) for \( \lambda = \lambda_L \) and \( \lambda = \lambda_H \), respectively. Utilizing the equation \( w = G(v, w; \lambda) \), we get that \( w_{LH} = G(v_L, w_{LH}; \lambda_L) = G(v_H, w_{LH}; \lambda_H) \). The function \( G(v, w; \lambda) \)
is decreasing in both $v$ and $\lambda$ (when $\lambda$ is positive), so $\lambda_L < \lambda_H$ implies $v_L > v_H$. Since the function $F(v,w;\lambda)$ is decreasing in $v$ and increasing in $\lambda$, and given that $\lambda_L < \lambda_H$ and $v_L > v_H$, we get $F(v_L,w_{LH};\lambda_L) < F(v_H,w_{LH};\lambda_H)$. Utilizing now the equation $v = F(v,w)$, this implies that $v_L < v_H$ – a contradiction. This completes the proof that $w$ decreases in $\lambda$ for $\lambda \leq c_1 / c_2$.

We now show that $v$ increases in $\lambda$ for $\lambda \leq c_1 / c_2$. It follows from Proposition 2 that $v = (c_2 \beta + c_1 \gamma) / c$ in the benchmark case of $\lambda = c_1 / c_2$ is strictly higher compared to the case where $\lambda < c_1 / c_2$. Hence, to show that $v$ monotonically increases in $\lambda$ for any $\lambda \leq c_1 / c_2$, we need to show that there are no two different values of $\lambda$ with the same value of $v$. Suppose by contradiction that there exist two values $\lambda_L$ and $\lambda_H$ of $\lambda$, such that $\lambda_L < \lambda_H$, for which $v$ gets the same value $v_{LH}$. Denote also by $w_L$ and $w_H$ the values of $w$ for $\lambda = \lambda_L$ and $\lambda = \lambda_H$, respectively. Since $w$ decreases in $\lambda$, $w_L > w_H$. Observe that

$$F(v,w) = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \frac{\sigma_x^2}{\sigma_x^2 + (\sigma_x w)^2} c^{-1} \left( c_1 + c_2 - (c_1 - \lambda c_2) \frac{\sigma_x^2 w}{(\sigma_x w)^2 + \sigma_x^2} \right)$$

can be rewritten as

$$F(v,w) = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} \frac{\sigma_x^2}{\sigma_x^2 + (\sigma_x w)^2} c^{-1} (c_1 + c_2) \left( \frac{\sigma_x^2}{(\sigma_x w)^2 + \sigma_x^2} + \frac{(\sigma_x w)^2}{(\sigma_x w)^2 + \sigma_x^2} \right) \frac{c_1 - \lambda c_2}{c_1 + c_2} \frac{\sigma_x^2 w}{c_1 + c_2 \left( (\sigma_x w)^2 + \sigma_x^2 \right)}.$$  

After rearranging, we get

$$F(v,w) = \frac{\sigma^2}{\sigma^2 + (\sigma_x v)^2} c^{-1} (c_1 + c_2) \left( \frac{\sigma_x^2}{\sigma_x^2 + (\sigma_x w)^2} \right) \frac{\sigma_x^2 w}{(c_1 + c_2) \left( (\sigma_x w)^2 + \sigma_x^2 \right)}.$$  

For any $\lambda, v = (c_2 \beta + c_1 \gamma) / c = c_2 (\delta - \lambda \tau) + c_1 (\delta + \tau) / c$ and

$$w = (c_1 \gamma - \lambda c_2 \beta - (1 + \lambda) c_3 (\beta - \gamma)) / c = (c_1 (\delta + \tau) - \lambda c_2 (\delta - \lambda \tau) + (1 + \lambda)^2 c_3 \tau) / c$$ imply

$$w - \frac{c_1 - \lambda c_2 \gamma}{c_1 + c_2} \frac{1}{2(c_1 + c_2)}.$$  

Hence, $\frac{\sigma_x^2 w}{\sigma_x^2} \left( w - \frac{c_1 - \lambda c_2 \gamma}{c_1 + c_2} \right) = \frac{\sigma_x^2 w (1 + \lambda)^2 \tau}{2 \sigma_x^2 (c_1 + c_2)}$, which
equals \[-\frac{\sigma^2 \sigma^4 (1 + \lambda)^2 w^2 v}{2 \sigma^2 (c_1 + c_2) (\sigma^2 (\sigma^2 (\sigma^2 w^2 + \sigma^2) + (\sigma^2 v)^2)^2) \sigma^2}.\] This leads to

\[F(v,w) = \frac{\sigma^2}{\sigma^2 (c_1 + c_2) (\sigma^2 (\sigma^2 w^2 + \sigma^2) + (\sigma^2 v)^2)^2} \cdot \frac{\sigma^2 \sigma^4 (1 + \lambda)^2 w^2 v}{2 \sigma^2 (c_1 + c_2) (\sigma^2 (\sigma^2 w^2 + \sigma^2) + (\sigma^2 v)^2)^2).\] Now,

based on \(w_L > w_H\), it follows that \(v_{_LH} = F(v_{_LH},w_L) < F(v_{_LH},w_H) = v_{_LH}\) — a contradiction.

This completes the proof that \(v\) increases in \(\lambda\) for \(\lambda \leq c_1 / c_2\), implying that the absolute value of the overall reporting bias \(|v|\) increases in \(\lambda\).

Based on the properties of the normal distribution, the variance of \(\tilde{v}\) conditional on the disaggregated earnings report equals

\[\text{VAR}[\tilde{\epsilon} + \tilde{\epsilon}x + w\tilde{\epsilon} + \tilde{\epsilon}] = \sigma^2 \sigma^2 \sigma^2 \sigma^2 \sigma^2 w^2 + \sigma^2 \sigma^2 \sigma^2 \sigma^2 \sigma^2 v^2,\] which is decreasing in \(w\) and increasing in \(v\). Since \(w\) decreases in \(\lambda\) and \(v\) increases in \(\lambda\), it follows that

\[\text{VAR}[\tilde{\epsilon} + \tilde{\epsilon}x + w\tilde{\epsilon} + \tilde{\epsilon}]\] increases in \(\lambda\).

**Proof of Proposition 4.** We first show that \(w\) is monotonic in \(\sigma^2\). For this, we need to show that there are no two different values of \(\sigma^2\) with the same value of \(w\). Suppose by contradiction that there exist two values \(\sigma^2\) and \(\sigma^2\) of \(\sigma^2\), such that \(\sigma^2 < \sigma^2\), for which \(w\) gets the same value \(w_{_LH}\). Denote by \(v_L\) and \(v_H\) the values of \(v\) for \(\sigma^2 = \sigma^2\) and \(\sigma^2 = \sigma^2\), respectively. Based on Lemma 2, \(H(v_L,w_{_LH} ; \sigma^2) = 0\) and \(H(v_H,w_{_LH} ; \sigma^2) = 0\).

The function \(H(v,w ; \sigma^2)\) is increasing in both \(v\) and \(\sigma^2\), so \(\sigma^2 < \sigma^2\) implies \(v_L > v_H\).

Observe also that \(H(v_L,w_{_LH} ; \sigma^2) = H(v_H,w_{_LH} ; \sigma^2) = 0\) and \(v_L > v_H\) imply that

\[\delta^2 = \frac{\sigma^2}{\sigma^2 (\sigma^2 v_L)^2} > \frac{\sigma^2}{\sigma^2 (\sigma^2 v_H)^2} = \delta^2.\] Since

\[w_{_LH} = G(v_L,w_{_LH} ; \sigma^2) = G(v_H,w_{_LH} ; \sigma^2),\] the inequality \(\delta^2 > \delta^2\) implies
\[
\frac{\sigma_x^2 w_{LH} v_L}{\sigma_x^2 w_{LH}^2 + \sigma_{dl}^2} > \frac{\sigma_x^2 w_{LH} v_H}{\sigma_x^2 w_{LH}^2 + \sigma_{dl}^2}. \]
So,
\[
v_L = \frac{F(v_L, w_{LH} ; \sigma_{dl})}{G(v_L, w_{LH} ; \sigma_{dl})} < \frac{F(v_H, w_{LH} ; \sigma_{dl})}{G(v_H, w_{LH} ; \sigma_{dl})} = \frac{v_H}{w_{LH}},
\]
implies \( v_L < v_H \) — a contradiction. This completes the proof that \( w \) is monotonic in \( \sigma_x \).

Since \( w = 0 \) for \( \sigma_x = 0 \), \( w \) is monotonically increasing in \( \sigma_x \).

We now show that \( v \) is monotonically increasing in \( \sigma_x \). Suppose by contradiction that there exist two values \( \sigma_{xL} \) and \( \sigma_{xH} \) of \( \sigma_x \), such that \( \sigma_{xL} < \sigma_{xH} \), for which \( v \) gets the values \( v_L \) and \( v_H \), respectively, and \( v_L > v_H \). Denote also by \( w_L \) and \( w_H \) the values of \( w \) for \( \sigma_x = \sigma_{dl} \) and \( \sigma_x = \sigma_{dl} \), respectively. Since \( w \) increases in \( \sigma_x \), \( w_L < w_H \). Based on Lemma 2,
\[
H(v_L, w_L ; \sigma_{dl}) = H(v_H, w_L ; \sigma_{dl}) = 0.
\]
This implies that
\[
\delta_L = \frac{\sigma^2}{\sigma^2 + (\sigma_x v_L)^2} > \frac{\sigma^2}{\sigma^2 + (\sigma_x v_H)^2} = \delta_H.
\]
Therefore,
\[
w_L = \frac{F(v_L, w_L ; \sigma_{dl})}{G(v_L, w_L ; \sigma_{dl})} < \frac{F(v_H, w_L ; \sigma_{dl})}{G(v_H, w_L ; \sigma_{dl})} = \frac{v_H}{w_L}.
\]
However, it follows from \( v_L > v_H \) and
\[
w_L < w_H \] that \( \frac{v_L}{w_L} > \frac{v_H}{w_H} \) — a contradiction. Consequently, we get that \( v \) increases in \( \sigma_x \), implying that the absolute value of the overall reporting bias \( |v| \) increases in \( \sigma_x \).

We next show that \( \delta \) is monotonic in \( \sigma_x \). For this, we need to show that there are no two different values of \( \sigma_x \) with the same value of \( \delta \). Suppose by contradiction that there exist two values \( \sigma_{xL} \) and \( \sigma_{xH} \) of \( \sigma_x \), such that \( \sigma_{xL} < \sigma_{xH} \), for which \( \delta \) gets the same value \( \delta_{LH} \).

Denote by \( v_L \) and \( v_H \) the values of \( v \) for \( \sigma_x = \sigma_{dl} \) and \( \sigma_x = \sigma_{dl} \), respectively. Denote also by \( w_L \) and \( w_H \) the values of \( w \) for \( \sigma_x = \sigma_{dl} \) and \( \sigma_x = \sigma_{dl} \), respectively. Since \( w \) and \( v \)
increase in $\sigma^I$, $w_L < w_H$ and $v_L < v_H$. Since $\delta$ gets the same value $\delta_{HL}$ for $\sigma^I = \sigma_{dl}$ and $\sigma^I = \sigma_{dl}$, the inequality $v_L = F(v_L, w_L; \sigma_{dl}) < F(v_H, w_H; \sigma_{dl}) = v_H$ implies

$$\frac{\sigma^2 w_L v_L}{\sigma^2 w_L^2 + \sigma_{dl}^2} > \frac{\sigma^2 w_H v_H}{\sigma^2 w_H^2 + \sigma_{dl}^2}.$$ So, $v_L = \frac{F(v_L, w_L; \sigma_{dl})}{G(v_L, w_L; \sigma_{dl})} < \frac{F(v_H, w_H; \sigma_{dl})}{G(v_H, w_H; \sigma_{dl})} = \frac{v_H}{w_H}$. However,

$$\delta_L = \frac{\sigma^2}{\sigma^2 + (\sigma_L v_L)^2} = \frac{\sigma^2}{\sigma^2 + (\sigma_L w_L)^2} = \delta_L$$ implies

$$(\sigma^2 v_L^2) \left( \frac{\sigma^2}{\sigma_{dl}^2 + (\sigma_L w_L)^2} \right) = (\sigma^2 v_H^2) \left( \frac{\sigma^2}{\sigma_{dl}^2 + (\sigma_L w_H)^2} \right) = \sigma^2 \left( \frac{\sigma^2_{dl}}{\sigma^2_{dl} + (\sigma_L w_H)^2} \right).$$ Hence, it follows from $v_L < v_H$ that

$$w_L / v_L > w_H / v_H \sigma_{dl},$$ or $v_L \sigma_{dl} / w_L > v_H \sigma_{dl} / w_H$. Since $\sigma^I < \sigma_{HL}$, we get

$$v_L / w_L > v_H / w_H$$ — a contradiction. This completes the proof that $\delta$ is monotonic in $\sigma^I$.

Since $\delta = \sigma^2 / (\sigma^2 + \sigma^2_{dl} v^2)$ for $\sigma^I = 0$ and $v$ increases in $\sigma^I$, $\delta$ is decreasing in $\sigma^I$. So,

$$VAR[\tilde{\pi} + \tilde{v}x, \tilde{w}x + \tilde{\epsilon}] = \sigma^2 \sigma^2_{\sigma} \sigma^2_{\sigma} v^2 / (\sigma^2_{\sigma} w^2 + \sigma^2_{\sigma} + \sigma^2_{\sigma} \sigma^2_{\sigma} v^2) = \sigma^2 (1 - \delta)$$ increases in $\sigma^I$. 

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REFERENCES


Investors establish their prior beliefs about the firm’s earnings $\tilde{\pi} = \pi_1 + \pi_2$ and about the random event $\tilde{x}$.

The firm’s manager privately observes the realizations $\pi_1$ and $\pi_2$ of the earnings components $\tilde{\pi}_1$ and $\tilde{\pi}_1$, and the realization $x$ of the random event $\tilde{x}$.

The manager issues the firm’s disaggregated earnings report $r = r_1 + r_2$, where

\[ r_1 = \pi_1 + B_1(\pi_1, \pi_2, x) \]

and

\[ r_2 = \pi_2 + B_2(\pi_1, \pi_2, x) \]

Investors set the firm’s market price $P(r_1, r_2)$.

The firm’s earnings $\tilde{\pi} = \pi_1 + \pi_2$ are realized and become commonly known.

**Figure 1.** The figure provides a timeline depicting the sequence of events in the model.