On the rationale behind the market premium (discount) for meeting or beating (missing) analysts’ earnings forecasts*

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Abstract: Research on security analysts provides strong empirical evidence of a market valuation premium (discount) for firms whose earnings reports meet or beat (miss) prior analysts’ forecasts, after controlling for the earnings news. The present study suggests a theoretical foundation to this seemingly anomalous pricing pattern. The study is based on the argument that the observed pricing effect of analysts’ earnings forecasts might be the rational consequence of the practice of earnings management, rather than the cause of earnings management activities as conventionally perceived in the literature. This argument is demonstrated by showing that the market premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts might be simply an adjustment that the market applies to the reported (managed) earnings in order to extract the underlying true (unmanaged) earnings measure.

Keywords: Information asymmetry; Accounting; Financial reporting; Reporting bias; Earnings management; Security analysts; Earnings forecast; Earnings expectations.

JEL classification: D82; G14; M41; M43.

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1. **Introduction**

Research on security analysts provides strong empirical evidence of a market valuation premium (discount) for firms whose earnings reports meet or beat (miss) prior analysts’ forecasts, after controlling for the earnings news (e.g., Bartov, Givoly and Hayn, 2002; Kasznik and McNichols, 2002). This well known pricing pattern is asymmetric. The market premium associated with beating an analyst’s earnings forecast by a certain amount appears to be lower than the market discount associated with missing the forecast by the same amount. The stock price tends to drop dramatically when the earnings target set by the analysts is missed by even a tiny amount. The empirically documented pricing effect of analysts’ earnings forecasts seems anomalous, as it implies that earnings forecasts of analysts are not subsumed by the subsequent earnings announcement. They continue to play an important valuation role even after the earnings have already been realized and reported to the market. Described differently, the change in a stock price over a certain period is not only a function of the earnings surprise for this period, as measured relative to the expectations held by the market participants at the beginning of the period. It also depends on the manner in which the market expectations have changed over the period, as reflected in the earnings forecasts provided to the market by analysts during the period. That is, similar firms with the very same market expectations at the beginning of a certain year (quarter) may be priced differently by the market, even following the announcement of identical annual (quarterly) earnings reports, merely because the analysts’ earnings forecasts during the year (quarter) were different. Although the rationale behind the market valuation premium (discount) for meeting or beating (missing) analysts’ earnings forecasts has not been explored in the literature yet, it nevertheless appears to be justified on economic grounds as it is empirically shown to be a leading indicator of future performance.
The objective of this study is to provide a theoretical foundation for the market pricing premium (discount) empirically observed following earnings announcements that meet or beat (miss) prior analysts’ forecasts. While this seemingly anomalous pricing pattern is commonly viewed as being responsible for applying excessive pressure on managers to manipulate their earnings reports in order to meet the target set by the analysts (e.g., Degeorge, Patel and Zeckhauser, 1999; Burgstahler and Eames, 2006), the present study attempts to counter-intuitively demonstrate that it might be the other way around. That is, the study is based on the argument that the documented pricing effect of analysts’ earnings forecasts might be the rational consequence of the practice of earnings management, rather than the cause of earnings management activities as conventionally perceived in the literature. This argument is demonstrated by showing that the market pricing premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts could be simply an adjustment that the capital market investors apply to the managed earnings report in order to extract the underlying true earnings measure. In its focus on the observed pricing effect of analysts’ earnings forecasts, the study is most closely related to the recent work of Trueman and Versano (2012), who indicate the industry expertise of security analysts as another explanation for the observed market pricing premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts.

In an effort to explore the linkage between the pricing implications of analysts’ earnings forecasts and the practice of earnings management, a security analyst is introduced into the conventional setting of earnings management. The resulting model follows prior models that capture interactive decisions of analysts and managers (e.g., Dutta and Trueman, 2002; Mittendorf and Zhang, 2005; Beyer, 2008; Langberg and Sivaramakrishnan, 2008; Langberg and Sivaramakrishnan, 2010) or interactive decisions of speculators and managers (e.g., Fischer and Stocken, 2004). Unlike prior models, however, the model at the basis of this study is designed to
explain the market pricing premium (discount) empirically observed following earnings announcements that meet or beat (miss) prior analysts’ forecasts. The model depicts a two-stage reporting game with three players: an equity analyst who covers the securities of a publicly traded firm, the privately informed manager of this firm, and the capital market investors. In the first stage of the reporting game, the analyst collects private information about the firm and based on it strategically issues to the capital market investors an earnings forecast that is an attempt to most accurately predict the forthcoming earnings report of the firm. Subsequently, the firm’s manager, whose compensation is linked to the market price of the firm, privately observes the firm’s earnings realization and exercises discretion over the costly bias of a mandatory earnings report published to the investors. An investigation of the equilibrium in the reporting game highlights two alternative situations that might trigger a market pricing rule that incorporates a premium (discount) for meeting or beating (missing) the analyst’s forecast.

The analysis starts by showing that uncertainty of the capital market investors with respect to the managerial reporting objective is sufficient to rationally explain the observed continuing pricing effect of analysts’ earnings forecasts even after the earnings have already been realized and publicly announced. To demonstrate this argument, an exogenous noise in the fashion of Fischer and Verrecchia (2000) is incorporated into the manager’s utility function, which does not allow the investors to perfectly identify and back out the manager’s reporting bias. The investors thus consider the earnings report only as a noisy signal of the underlying true earnings measure. Therefore, the analyst’s prior earnings forecast is not subsumed by the manager’s earnings announcement. It rather serves the investors as an additional noisy signal of the true earnings measure that enables them to imperfectly detect the bias hidden in the earnings report and adjust for it when pricing the firm. Accordingly, the market price of the firm after the earnings announcement incorporates both the manager’s earnings report and the analyst’s prior earnings forecast.
Being motivated by the wish to minimize the forecast error, the analyst accounts for earnings management in designing his earnings forecast. In circumstances where the analyst’s private information regarding the manager’s forthcoming reporting bias is sufficiently accurate relative to the publicly known information, the resulting earnings forecast serves the market as a very noisy estimator of the true (unmanaged) earnings measure, because the investors are incapable of perfectly backing out the component in the forecast that captures the expected bias in the earnings report. This component of the forecast works thus as an additional noise that adds to the already noisy information that the analyst’s forecast is based on. It should be noted, however, that even though the direct information that the analyst’s earnings forecast conveys about the true earnings realization is very noisy, it also conveys valuable indirect information about the manager’s reporting bias that can serve the market to interpret the earnings report and better utilize it in pricing the firm. The direct information that the analyst’s forecast provides about the true earnings realization positively affects the market price of the firm following the earnings announcement, but the indirect information that the forecast offers with regard to the manager’s reporting bias has a countervailing negative effect on the price. When the indirect information embedded in the analyst’s forecast dominates its direct information, the net pricing effect of the analyst’s earnings forecast is negative. The resulting post-earnings announcement price is positively related to the earnings report and negatively related the analyst’s prior forecast. This provides a theoretical ground for the observed market pricing premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts.

While the uncertainty of the investors about the reporting objective of managers can rationalize the empirically documented market premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts, the exact same pricing pattern can also be rationally explained in circumstances where the managerial reporting incentives are commonly known. To
demonstrate this argument, the equilibrium in the model is again analyzed under a different modeling of the manager’s utility function. The utility function of the manager, which is now taken as commonly known, is altered by assuming that the manager bears an additional cost when failing to meet (either beating or missing) the analyst’s earnings forecast, besides the cost associated with biasing the earnings report relative to the true earnings realization. The marginal cost associated with missing the analyst’s earnings forecast is allowed to exceed the marginal cost of beating the forecast. This cost structure may be attributed to the wish of managers to maintain good relationships with analysts and not embarrassing them by reporting earnings that deviate from their prior forecasts (especially when the earnings report fall short of the analyst’s earnings forecast). A similar cost structure can also arise when the manager’s compensation scheme is based on the benchmark set by the analyst’s earnings forecast (e.g., Matsunaga and Park, 2001). Under such circumstances, the manager might not get a bonus when the earnings report misses the target set by the analyst’s forecast, but reporting high earnings that exceed the target is also costly (albeit probably less costly) to the manager because it reduces her flexibility to manage future earnings reports upward (e.g., Healy, 1985).

The costs that the manager bears when deviating from the true earnings realization and when deviating from the prior earnings forecast of the analyst induce her to use a weighted average of the true earnings realization and the analyst’s earnings forecast as a basis for the earnings report. Being aware of the incentives of the manager to meet the analyst’s benchmark, the investors rationally infer that the analyst’s earnings forecast is embedded in the manager’s earnings report, and thus rationally neutralize it from the earnings report in order to elicit the underlying true earnings measure. The resulting market price of the firm after the earnings announcement is therefore again positively related to the earnings report and negatively related the analyst’s prior forecast. Here, however, due to the asymmetric structure of the cost associated with positive and negative
deviations of the earnings report from the analyst’s forecast, the equilibrium pricing rule better reconciles with the empirical findings, as it reflects a premium for beating the analyst’s forecast that is lower than the discount associated with missing the forecast. This suggests that it is not the observed pricing effect of analysts’ earnings forecasts that induces managers to manipulate their earnings reports in order to meet the target set by the analysts as conventionally surmised. It is rather the managerial wish to meet the analysts’ benchmark that leads to the observed pricing effect of analysts’ forecasts.

The paper proceeds as follows. The next section presents and analyzes a basic reporting model, which yields equilibrium where the analyst’s earnings forecast is totally subsumed by the subsequent earnings announcement. The base model, which serves as a starting point to the analysis, is then extended in Sections 3 and 4 in order to explore circumstances where the analyst’s earnings forecast continues to affect the market price of the firm even after the earnings have already been realized and announced. Section 3 analyzes the equilibrium outcomes that emerge when introducing into the base model uncertainty of the capital market investors with respect to the manager’s reporting objective. In Section 4, the equilibrium outcomes that the model yields are again analyzed under a different modeling of the manager’s utility function, which is now taken as commonly known but extended to include an additional cost that the manager bears when failing to meet the analyst’s earnings forecast. The extensions analyzed in Sections 3 and 4 provide two alternative rational explanations for the empirically documented market premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts. The unifying theme underlying these two explanations is the view of the observed pricing effect of analysts’ earnings forecasts as an adjustment that the investors apply to the reported (managed) earnings in order to extract the underlying true (unmanaged) earnings measure. The final section summarizes and offers concluding remarks. Proofs appear in the appendix.
2. The Base Model

This section describes the basic framework underlying the analysis (henceforth, the base model), whose parameters and assumptions are all assumed to be common knowledge unless otherwise indicated. The base model deviates from the conventional setting of earnings management by the introduction of a financial analyst as an additional player. It considers a firm that is traded in a capital market for one period and depicts a reporting game with three players: a privately informed analyst who follows the securities of the firm, the privately informed manager of the firm, and the capital market investors. The uncertain equity value of the firm is represented by a normally distributed random variable $\tilde{v}$ with mean $\mu$ and variance $\sigma^2$. A correlated random variable, denoted $\tilde{e}$, represents an accounting earnings measure. The earnings measure $\tilde{e}$ serves as a noisy estimator of the firm’s equity value $\tilde{v}$ that takes the form $\tilde{e} = \tilde{v} + \tilde{e}_1$, where $\tilde{e}_1$ is an independent normally distributed noise term with mean zero and variance $\sigma_1^2$. Though the realization of the earnings measure $\tilde{e}$ is an important piece of information in evaluating the equity value of the firm, it is unobservable to the investors. The investors, however, do observe two imperfect public reports about the earnings realization, which are sequentially provided to them by the analyst who covers the firm’s equity and by the firm’s manager.

Prior to the realization of the earnings measure, the analyst produces an earnings forecast that is published to the capital market investors. The information that the analyst collects as a basis for the earnings forecast is modeled as a noisy signal of the firm’s earnings represented by the random variable $\tilde{s} = \tilde{e} + \tilde{e}_2$, where $\tilde{e}_2$ is an independent normally distributed noise term with mean zero and variance $\sigma_2^2$. Due to the importance of forecast accuracy for analysts’ reputation and compensation (e.g., Hong and Kubick, 2003; Mikhail, Walther and Willis, 1999; Stickel, 1992), and in common with prior literature, it is assumed that the analyst’s objective is to minimize his forecast
error. As he wishes to most accurately predict the firm’s forthcoming (potentially biased) earnings report, the analyst’s strategic earnings forecast, as published to the investors, might differ from his privately known true expectations $E[\tilde{e}|\tilde{s} = s]$ of the earnings realization. This strategic behavior is consistent with the findings of Givoly, Hayn and Yoder (2012), who empirically document that analysts account for earnings management in their earnings forecasts. The analyst’s reporting bias is not associated with any direct cost but is nevertheless indirectly limited by his goal of avoiding forecast errors. The function $F: \mathbb{R} \rightarrow \mathbb{R}$ represents the analyst’s reporting strategy, where $F(s)$ is the analyst’s earnings forecast, given that $s \in \mathbb{R}$ is the realization of his private signal $\tilde{s}$. The difference $F(s) - E[\tilde{e}|\tilde{s} = s]$ is thus the analyst’s reporting bias.

After the analyst’s earnings forecast is published, the manager privately observes the realization $e$ of the earnings measure $\tilde{e}$ and issues an earnings report to the capital market investors. The manager exercises discretion in reporting, which allows her to bias the earnings report, but at a cost. Biases in earnings reports can be associated with a variety of costs, such as litigation costs, reputation erosion costs, costs that emerge from conflicts with auditors and audit committees, and the costs of reducing the reporting flexibility in future reports. As the manager’s compensation is linked to the firm’s stock price, the manager makes the reporting decision in accordance with her expectations about the impact of the earnings report on the market price of the firm, taking into account the cost associated with biasing the report. In common with the earnings management literature (e.g., Stein, 1989; Fischer and Verrecchia, 2000; Dye and Sridhar, 2004; Guttman, Kadan and Kandel, 2006; Einhorn and Ziv, 2012; Amir, Einhorn and Kama, 2012), it is assumed that the biasing cost takes the standard quadratic form. Specifically, when the manager observes an earnings realization of $e$ and reports $r$, she bears a cost of $c(r - e)^2$, where $c > 0$. The manager’s reporting strategy is represented by the function $R: \mathbb{R}^2 \rightarrow \mathbb{R}$. Given that $f$ is the
earnings forecast reported by the analyst and $e$ is the realization of the earnings measure $\tilde{e}$, $R(f,e)$ is the manager’s earnings report and $R(f,e) - e$ is thus the manager’s reporting bias.

The investors are assumed to be risk neutral. Accordingly, they set the firm’s equity price in the capital market to be equal to the firm’s expected value conditional on all the information available to them at the pricing date. Initially, the investors set the equity price of the firm to be $\mu$. Then, following the arrival of information to the market, they update the price of the firm twice – first after the analyst issues his forecasting report, and once again after the manager issues her earnings report. The analysis focuses on the pricing procedure applied by the investors after the earnings announcement, which is represented by the function $P: \mathbb{R}^2 \rightarrow \mathbb{R}$. Specifically, for any content $f \in \mathbb{R}$ of the analyst’s forecast and any content $r \in \mathbb{R}$ of the manager’s report, $P(f,r)$ is the market equity price of the firm after the arrival of the manager’s earnings report.

Figure 1 provides a timeline depicting the sequence of events in the model. At the beginning, all players establish their prior beliefs about the random variables $\tilde{v}$, $\tilde{e}$, and $\tilde{s}$. Afterward, the analyst privately observes the realization $s$ of the signal $\tilde{s}$. Based on this information, the analyst reports to the market the earnings forecast $f = F(s)$ in an attempt to minimize the expected forecast error, as captured by $E[(f - R(f,\tilde{e}))^2|\tilde{e} = s]$. Subsequently, the earnings measure $\tilde{e}$ is realized and the manager privately observes its realization $e$. Based on the privately observed earnings realization $e$ and the publicly observed earnings forecast $f$ published by the analyst, the manager issues an earnings report $r = R(f,e)$. The manager’s report is designed to maximize her utility function $xP(f,r) - c(r - e)^2$, where $x \in \mathbb{R}$ denotes a commonly known benefit to the manager from shifting the market price of the firm upward by one additional unit.

Then, using the analyst’s report $f$ and the manager’s report $r$, the risk-neutral investors update the
The firm’s market price $P(f,r)$ to be $E[\tilde{v} \mid F(\tilde{s}) = f, R(f, \tilde{e}) = r]$. At the end, the earnings realization becomes commonly known and the manager bears the cost $c(r-e)^2$ of biasing the earnings report.

**[FIGURE 1]**

We look for Bayesian equilibrium in the base model. Equilibrium is therefore formally defined by three functions: the analyst’s reporting strategy $F : \mathbb{R} \rightarrow \mathbb{R}$, the manager’s reporting strategy $R : \mathbb{R}^2 \rightarrow \mathbb{R}$, and the investors’ pricing rule $P : \mathbb{R} \rightarrow \mathbb{R}$, which satisfy the following three conditions for any $s,f,e,r \in \mathbb{R}$:

(i) $F(s) \in \arg\min_{f,e,r} E[(f - R(f, \tilde{e}))^2 \mid \tilde{s} = s]$;

(ii) $R(f,e) \in \arg\max_{f,e,r} xP(f,r) - c(r-e)^2$;

(iii) $P(f,r) = E[\tilde{v} \mid F(\tilde{s}) = f, R(f, \tilde{e}) = r]$.

The first equilibrium condition pertains to the analyst’s reporting strategy $F$. This condition captures the wish of the analyst to issue an earnings forecast that most accurately predicts the forthcoming earnings report of the manager. It is thus based on the analyst’s rational expectations about the manager’s reporting strategy $R$. The second equilibrium condition pertains to the manager’s reporting strategy $R$, describing her incentives to bias the earnings report in order to shift the expected market price of the firm, but subject to the costs associated with the reporting bias. It is therefore based on the manager’s rational expectations about the pricing rule $P$ applied by the investors. The third equilibrium condition describes the market pricing rule $P$ of the risk-
neutral investors, who set the price of the firm after the earnings announcement to be equal to its expected value based on all the publicly available information at this date. The investors invoke their rational expectations regarding the reporting strategies $F$ and $R$ of the analyst and the manager, respectively, in an effort to detect opportunistic biases in reporting and thereby most effectively utilize the earnings forecast, as well as the earnings report, in their pricing rule.

We restrict the analysis to equilibria with a linear pricing rule $P: \mathbb{R}^2 \to \mathbb{R}$. Accordingly, the post-earnings announcement price $P(f, r)$ of the firm is assumed to be a linear function of the analyst’s earnings forecast $f$ and the manager’s earnings report $r$. Linear equilibria are commonly assumed in the earnings management literature. When combined with a quadratic biasing cost function and a normal distribution of the firm’s equity value and earnings measure, a linear pricing rule enables a tractable analysis and yields equilibrium outcomes that can be analytically characterized and intuitively explained.

Not surprisingly, the base model yields standard equilibrium outcomes, consistent with those established by earlier literature (e.g., Stein, 1989; Fischer and Verrecchia, 2000). It yields, in particular, an equilibrium market price that relies solely on the manager’s earnings report, and does not incorporate the prior analyst’s forecast. The base model thus provides a natural point of

\[\text{\footnotesize\textsuperscript{2}}\text{ An exception is the analysis of Guttman, Kadan and Kandel (2006), which explains kinks and discontinuities in the distribution of the reported earnings by focusing on non-linear equilibria.}\]

\[\text{\footnotesize\textsuperscript{3}}\text{ Einhorn and Ziv (2012) show that requiring the pricing rule to be linear in a conventional earnings management setting is equivalent to requiring the out-of-equilibrium beliefs to be reasonable in the sense of the D1 criterion of Cho and Kreps (1987).}\]
reference to the analysis and serves to highlight the triggers that cause the investors to utilize the analyst’s forecast in pricing the firm after the earnings announcement. The following observation indicates the existence and uniqueness of a linear equilibrium in the base model and characterizes its form.

**Observation 1.** There exists a unique linear equilibrium \(( F : \mathbb{R} \rightarrow \mathbb{R}, \ R : \mathbb{R}^2 \rightarrow \mathbb{R}, \ P : \mathbb{R}^2 \rightarrow \mathbb{R} )\) in the base model. The equilibrium takes the following form for any \( s, e, f, r \in \mathbb{R} : \)

\[
F(s) = E[\hat{e}\big|\tilde{s} = s] + x\beta / 2c = (1 - \lambda)\mu + \lambda s + x\beta / 2c, \ R(f, e) = e + x\beta / 2c \text{ and } P(f, r) = \alpha + \beta r,
\]

where \( \lambda = \frac{\sigma^2 + \sigma_i^2}{\sigma^2 + \sigma_i^2 + \sigma^2_\lambda}, \ \alpha = (1 - \beta)\mu - \beta^2 x / 2c \text{ and } \beta = \frac{\sigma^2}{\sigma^2 + \sigma_i^2}. \)

It follows from Observation 1 that, in equilibrium, the manager’s earnings report is

\[
R(f, e) = e + x\beta / 2c,\]

deviating from the earnings realization \( e \) by a reporting bias of \( x\beta / 2c \). The sign of the manager’s reporting bias \( x\beta / 2c \) is the same as the sign of \( x \). Its absolute value is increasing in the importance that the manager attaches to the market price of the firm, as captured by the absolute value of \( x \), increasing in the weight \( \beta \) assigned by the pricing function

\[
P(f, r) = \alpha + \beta r \]

to the earnings report \( r \), and decreasing in the marginal cost \( c \) associated with the bias. Observation 1 further indicates that the analyst’s equilibrium earnings forecast is

\[
F(s) = (1 - \lambda)\mu + \lambda s + x\beta / 2c.\]

The analyst relies on the realization \( s \) of the private signal \( \tilde{s} \), which implies that the true earnings realization is expected to be \( E[\hat{e}\big|\tilde{s} = s] = (1 - \lambda)\mu + \lambda s \). In addition, in an attempt to minimize his forecast error, the analyst anticipates the forthcoming reporting bias \( x\beta / 2c \) of the manager and incorporates it in his forecast. Consequently, the analyst’s reporting bias is identical to the manager’s reporting bias.
By Observation 1, the equilibrium market price of the firm following the earnings announcement is \( P(f, r) = \alpha + \beta r \), relying solely on the earnings report \( r \). The investors can precisely detect the reporting bias \( x\beta/2c \) hidden in the earnings report, and thus can perfectly decipher the manager’s private information from her report. To adjust for the manager’s reporting bias, the investors subtract their estimate \( x\beta/2c \) of the bias from the earnings report and rationally infer that the adjusted report \( r - x\beta/2c \) captures the realization \( e \) of the earnings measure \( \bar{e} \).

Being capable of perfectly unraveling the reporting bias and adjusting for it, the weight \( \beta \) that the equilibrium pricing function \( P(f, r) = (1 - \beta)\mu + \beta(r - x\beta/2c) \) assigns to the adjusted earnings report \( r - x\beta/2c \) is \( \beta = \frac{\sigma^2}{\sigma^2 + \sigma_i^2} \), averaging it with the market’s prior expectation \( \mu \) of the firm’s equity value. It should be noted that, even though the manager knows that she is unable to fool the market, she nevertheless ends up taking costly actions to bias her reporting. As has been well established in the literature (Stein, 1989), the manager is trapped into such inefficient behavior because she takes the market’s conjectures as fixed, knowing that the investors will suspect her report in any case.

Although the investors can also infer the reporting bias \( x\beta/2c \) hidden in the analyst’s earnings forecast, and thus can perfectly deduce that the realization \( s \) of the analyst’s private signal \( \tilde{s} \) is \( \lambda^{-1}(f - (1 - \lambda)\mu - x\beta/2c) \), the analyst’s earnings forecast nevertheless becomes irrelevant to them after the earnings announcement. This is because the analyst’s private signal \( \tilde{s} \) is a noisy estimator of the true earnings measure \( \bar{e} \), so its content is fully subsumed by the true earnings realization as inferred by the investors after observing the manager’s earnings report. In Sections 3 and 4, two alternative extensions of the base model are analyzed where the analyst’s earnings forecast is not subsumed by the earnings announcement. The resulting pricing rule under each of
these two extensions incorporates both the analyst’s earnings forecast and the manager’s earnings report in a way that reconciles with extant empirical findings.

3. **Extension A – Introducing Uncertain Reporting Objective**

   In this section, in the spirit of Fischer and Verrecchia (2000), uncertainty of the capital market investors about the manager’s reporting objective is incorporated into the base model by assuming that $x$ is unobservable to the investors.\(^4\) Hence, in the extended model considered in this section, the investors consider $x$ as the realization of some independent normally distributed random event $\tilde{x}$ with mean $\mu_x$ and variance $\sigma_x^2$.\(^5\) That is, the investors face some degree of uncertainty about the manager’s reporting objective, which does not allow them to perfectly detect her reporting bias and thus precludes fully revealing equilibria. The analyst, who seeks to minimize the forecast error, collects private information not only about the firm, but also with respect to the manager’s reporting objective. Information about the firm serves the analyst in forecasting the true future realization of the earnings measure, while information about the manager’s reporting incentives assists him in predicting the manager’s forthcoming reporting bias. The assumption is

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\(^4\) Investors usually do not possess timely and perfect information about managers’ precise compensation schemes, their time and risk preferences, beliefs and future plans. They are thus likely to face some degree of uncertainty about the managers’ reporting goals. Following Fischer and Verrecchia (2000), uncertainty on the part of investors about the reporting objective of managers is widely assumed in the disclosure literature (e.g., Fischer and Stocken, 2004; Dye and Sridhar, 2004; Ewert and Wagenhofer, 2005; Einhorn, 2007; Amir, Einhorn and Kama, 2012).

\(^5\) Since $\tilde{x}$ is normally distributed, it might be either positive or negative, implying that the manager might have the incentive to either inflate or deflate the market price of the firm. While managerial incentives to inflate the stock price are prevalent, there are points in time at which managers have the incentive to drive the stock price of their firm downward. Incentives to deflate prices might exist, for example, prior to stock option awards, management buyout offers, repurchasing shares, covering employee stock options, and in anticipating contract renegotiations with labor unions. Nevertheless, such scenarios can be approximately excluded from the model by assuming that the mean $\mu_x$ of $\tilde{x}$ is a positive number that exceeds its variance $\sigma_x^2$ by a large amount, so that the probability that the manager wishes to deflate the price is very close to zero.
thus that the analyst possesses private information about the managerial reporting incentives that is not available to the investors. For simplicity and in order to sharpen the analysis and its insights, it is assumed that $x$ is observable to the analyst. The results of the analysis, however, qualitatively prevail under an alternative assumption that the analyst possesses imperfect information about $x$, as long as this information is sufficiently precise relative to the publicly known information. All other features of the extended model are as described in Section 2. The special case of $\sigma^2 = 0$, where the manager’s reporting objective is commonly known, coincides with the base model.

Equilibrium in the extended model, referred to as extension A, is represented by three functions $F_A: \mathbb{R}^2 \to \mathbb{R}$, $R_A: \mathbb{R}^3 \to \mathbb{R}$ and $P_A: \mathbb{R}^2 \to \mathbb{R}$. The function $F_A$ represents the analyst’s reporting strategy, where $F_A(s, x)$ is the analyst’s earnings forecast, given that $s \in \mathbb{R}$ is the realization of the signal $\tilde{s}$ and $x \in \mathbb{R}$ is the realization of $\tilde{x}$. The function $R_A$ represents the manager’s reporting strategy, where $R_A(f, e, x)$ is the manager’s earnings report, given that $f$ is the analyst’s earnings forecast, $e$ is the realization of the earnings measure $\tilde{e}$ and $x \in \mathbb{R}$ is the realization of the variable $\tilde{x}$. The function $P_A$ represents the pricing procedure applied by the investors, where $P_A(f, r)$ is the market price of the firm after the earnings announcement, given that $f \in \mathbb{R}$ is the analyst’s earnings forecast and $r \in \mathbb{R}$ is the manager’s earnings report. In equilibrium, the functions $F_A$, $R_A$ and $P_A$ satisfy the following three conditions for any $s, x, f, e, r \in \mathbb{R}$:

(i) $F_A(s, x) \in \arg\min_{f \in \mathbb{R}} E[(f - R_A(f, e, x))^2] | s = \tilde{s}]$;

(ii) $R_A(f, e, x) \in \arg\max_{e \in \mathbb{R}} xP_A(f, r) - c(r - e)^2$;

(iii) $P_A(f, r) = E[\tilde{\nu} \mid F_A(\tilde{s}, \tilde{x}) = f, R_A(f, \tilde{e}, \tilde{x}) = r]$. 

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Proposition 2 indicates the existence and uniqueness of a linear equilibrium in extension A and characterizes its form.

**Proposition 2.** There exists a unique linear equilibrium \((F_A : \mathbb{R}^2 \rightarrow \mathbb{R}, R_A : \mathbb{R}^3 \rightarrow \mathbb{R})\) in extension A, which takes the following form for any \(s, x, f, e, r \in \mathbb{R}\):

\[
F_A(s, x) = E[\hat{e} | \tilde{s} = s] + x \beta_A / 2c = (1 - \lambda)\mu + \lambda s + x \beta_A / 2c, \quad R_A(f, e, x) = e + x \beta_A / 2c \quad \text{and}
\]

\[
P_A(f, r) = \alpha_A + \beta_A r - \gamma_A f = \alpha_A + (\beta_A - \gamma_A) r + \gamma_A (r - f), \quad \text{where} \quad \lambda = \frac{\sigma^2 + \sigma_i^2}{\sigma^2 + \sigma_i^2 + \sigma_j^2},
\]

\[
\alpha_A = (1 - \beta_A + \gamma_A)\mu - (\beta_A - \gamma_A)\beta_A\mu / 2c, \quad \beta_A = \frac{\sigma^2}{\sigma^2 + \sigma_i^2} \quad \text{and}
\]

\[
\gamma_A = \frac{\sigma^2}{\sigma^2 + \sigma_i^2} \frac{(\sigma^2 + \sigma_i^2 + \sigma_j^2)\sigma^4 / 4c^2}{(\sigma^2 + \sigma_i^2 + \sigma_j^2)\sigma^2, \sigma^4 / 4c^2 + (\sigma^2 + \sigma_i^2)^3}.
\]

It follows from Proposition 2 that the manager’s earnings report in equilibrium equals

\[
R_A(f, e, x) = e + x \beta_A / 2c, \quad \text{deviating from the true earnings measure} \ e \ \text{by a reporting bias of} \ x \beta_A / 2c. \ \text{Like the equilibrium outcomes of the base model, the sign of the manager’s reporting bias} \ x \beta_A / 2c \ \text{is the same as the sign of} \ x. \ \text{Its absolute value is increasing in the importance that the manager attaches to the market price of the firm, as captured by the absolute value of} \ x, \ \text{increasing in the weight} \ \beta_A \ \text{assigned by the pricing function} \ P_A(f, r) = \alpha_A + \beta_A r - \gamma_A f \ \text{to the earnings report} \ r, \ \text{but decreasing in the marginal cost} \ c \ \text{associated with the bias. The analyst relies on the content} \ s \ \text{of the private signal} \ \tilde{s}, \ \text{which implies that the earnings realization is expected to be}
\]

\[
E[\hat{e} | \tilde{s} = s] = (1 - \lambda_A)\mu + \lambda_A s. \ \text{So, the analyst’s equilibrium earnings forecast}
\]

\[
F_A(s, x) = (1 - \lambda_A)\mu + \lambda_A s + x \beta_A / 2c, \ \text{as presented in Proposition 2, is biased by the amount of}
\]

\[
x \beta_A / 2c. \ \text{Again, as with the base model, the analyst accounts for the forthcoming earnings management in an attempt to minimize the forecast error, and thus includes in his forecast a bias of}
that captures the expected managed component in the manager’s earnings report.

Proposition 2 further suggests that the equilibrium market price of the firm, following the manager’s earnings announcement, takes the form $P_A(f, r) = \alpha_A + \beta_A r - \gamma_A f$. The following corollary summarizes the properties of the equilibrium pricing coefficients.

**Corollary to Proposition 2.** The pricing coefficients satisfy $\beta_A > \gamma_A \geq 0$. The pricing coefficient $\gamma_A$ equals zero if and only if $\sigma_x^2 = 0$, and it is positive and increasing in $\sigma_x^2$ for any $\sigma_x^2 > 0$. The pricing coefficient $\gamma_A$ is also decreasing in $c$, converging to zero when $c$ converges to infinity.

The pricing coefficient $\gamma_A$ is positive as long as $\sigma_x^2 > 0$. Hence, unlike the equilibrium outcomes of the base model (captured by the special case of $\sigma_x^2 = 0$), the analyst’s prior earnings forecast $f$ is still utilized by the investors in pricing the firm even though the earnings have already been realized and reported to the market. Since the investors do not observe the realization $x$ of the random event $\tilde{x}$, they cannot precisely detect the reporting bias $x\beta_A / 2c$ hidden in the earnings report, and thus cannot perfectly adjust for the bias when pricing the firm. Being unable to unravel the manager’s reporting bias, the market uses the earnings report $r$ only as a noisy signal of the underlying true earnings measure $e$. Therefore, the analyst’s prior forecast $f$ is not subsumed by the manager’s earnings report. It rather serves the investors as an additional noisy signal of the earnings measure $e$. Accordingly, the market price of the firm after the earnings announcement incorporates both the manager’s earnings report $r$ and the analyst’s earnings forecast $f$.

The analyst’s forecast $F_A(s, x) = (1 - \lambda)\mu + \lambda s + x\beta_A / 2c$ serves the market as a very noisy estimator of the true earnings measure. The investors are incapable of perfectly backing out the component $x\beta_A / 2c$ in the forecast that captures the forthcoming bias in the earnings report. This
component of the forecast thus works as an additional noise that adds to the noise \( \tilde{\epsilon}_2 \) already embedded in the analyst’s private signal \( \tilde{s} \). However, even though the direct information that the analyst’s earnings forecast conveys about the true earnings realization is very noisy, it does convey valuable indirect information about the manager’s reporting bias that can serve the market to better interpret the earnings report. The direct information embedded in the analyst’s forecast about the true earnings realization positively affects the market price of the firm after the earnings announcement, but the indirect information that the forecast provides regarding the manager’s reporting bias has a countervailing negative effect on the price. As its indirect role in imperfectly detecting the manager’s reporting bias dominates its direct role in estimating the true earnings realization, the net pricing effect of the analyst’s earnings forecast is negative.\(^6\) The market price of the firm after the earnings announcement is therefore positively related to the manager’s earnings report and negatively related to the analyst’s earnings forecast. This result is consistent with Lundholm (1988) and Einhorn (2005), who demonstrate situations where the presence of two correlated signals evokes an inverse price-signal relation.

In order to reconcile the equilibrium pricing rule with extant empirical findings, it is useful to represent it as \( P_A(f, r) = \mu + (\beta_A - \gamma_A)(r - \mu - \mu_A \beta_A / 2c) + \gamma_A (r - f) \). Hence, the price change \( P_A(f, r) - \mu \) over the entire period, which begins before the arrival to the market of the analyst’s earnings forecast and ends after the manager’s earnings announcement, equals

\[(\beta_A - \gamma_A)(r - \mu - \mu_A \beta_A / 2c) + \gamma_A (r - f) \],

where both pricing coefficients \( \beta_A - \gamma_A \) and \( \gamma_A \) are

\(^6\) This result can be generalized to the case where the analyst possesses imperfect information about \( x \), as long as this information is sufficiently precise relative to the publicly known information. When the incremental private knowledge of the analyst about \( x \) is too low, the net pricing effect of the analyst’s earnings forecast is positive.
positive provided that $\sigma^2_x > 0$. It thus appears that the price change over the entire period is not only a function of the earnings surprise for the period, $r - \mu - \mu_x \beta_x/2c$, as measured relative to the expectations held by the market participants at the beginning of the period. It also depends on the manner in which the market expectations have changed over the period, as reflected by $r - f$.

Consistent with extant empirical evidence, the equilibrium price change in the entire period explains a market premium (discount) of $\gamma_d$ for any unit of deviation upward (downward) of the earnings report relative to the analyst’s prior earnings forecast, after controlling for the earnings surprise, $r - \mu - \mu_x \beta_x/2c$, for the period. It should be noted, however, that the term $\gamma_d(r - f)$ in the pricing rule does not represent any market reward (penalty) for meeting or beating (missing) analysts’ earnings forecasts as commonly claimed in the literature. It is simply an adjustment that the market applies to the managed earnings report in order to extract the underlying true earnings number. The rationale behind the empirical findings of a positive association between $r - f$ and the future performance of the firm also becomes apparent from the analysis, because $r - f$ is the realization of $(1 - \lambda)(\tilde{v} - \mu) + (1 - \lambda)\tilde{e}_1 - \lambda \tilde{e}_2$, which is a noisy estimator of the firm’s value $\tilde{v}$.

When $\sigma^2_x$ increases and the uncertainty of the investors with respect to the manager’s reporting objective becomes more severe, the investors rely more heavily on the analyst’s earnings

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7 Empirical studies documenting the market premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts (e.g., Bartov, Givoly and Hayn, 2002; Kasznik and McNichols, 2002) show that quarterly/annual stock returns are positively related to the end-of-quarter/year earnings surprise (measured relative to the latest pre-earnings announcement forecast of analysts) after controlling for the overall earnings surprise for the entire quarter/year (measured relative to an earlier earnings forecast provided by analysts at the beginning of the quarter/year). Hence, their estimate of the earnings expectation held by the market at the beginning of the quarter/year, as captured by $\mu + \mu_x \beta_x/2c$ in the model, is based on an earlier earnings forecast of analysts.
forecast in their effort to adjust for the manager’s reporting bias, and thus the pricing coefficient \( \gamma_A \) increases. Uncertainty on the part of investors about the managerial reporting objective can however explain a market premium (discount) for meeting or beating (missing) the analyst’s earnings forecast only in an environment where earnings management can exist. Therefore, when the biasing cost \( c \) increases, so that the managerial capability of managing earnings decreases, the pricing coefficient \( \gamma_A \) decreases as well. In circumstances where earnings management is impossible and the manager thus truthfully reports the earnings realization, the analyst’s prior earnings forecast becomes irrelevant to the investors after the earnings announcement. Therefore, in the edge case where \( c \) converges to infinity, the pricing coefficient \( \gamma_A \) converges to zero and the pricing rule is reduced to \( P_A(f,r) = \mu + \beta_A(r - \mu) \), as in the base model.

4. **Extension B - Introducing Additional Reporting Costs**

The analysis given in the previous section highlights the uncertainty of the investors about the managerial reporting objective as a rational explanation for the empirically documented market premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts. The equilibrium pricing rule that emerges from a model with uncertain reporting objective exhibits, however, a market premium for beating the analyst’s forecast that is symmetric to the market discount associated with missing the analyst’s forecast. To explain the empirical evidence about an asymmetric market reaction to the two scenarios, the equilibrium in the base model is again analyzed in this section under an extended modeling of the manager’s utility function, which is now taken as commonly known. Specifically, it is assumed that the manager might bear an additional cost when failing to meet (either beating or missing) the analyst’s earnings forecast, besides the cost
associated with biasing the earnings report relative to the true earnings realization. This additional cost is assumed to be quadratic in the deviation $r - f$ of the manager’s earnings report from the analyst’s prior earnings forecast, albeit possibly asymmetrically for negative and positive deviations. Specifically, the cost that the manager incurs is represented by the function $K(r - f)$, which equals $k_1(r - f)^2$ if $r < f$ and $k_2(r - f)^2$ otherwise, where $k_1$ and $k_2$ are non-negative scalars such that $k_2 \geq k_1 \geq 0$. Such a cost structure can stem from the manager’s need to maintain good relationships with the analyst, which makes her disinclined to embarrass him by announcing an earnings report that deviates from his prior earnings forecast, especially when the earnings report falls short of the forecast. A similar cost structure can also arise when the manager’s compensation scheme is based on the benchmark set by the analyst’s earnings forecast (e.g., Matsunaga and Park, 2001). In this case, the manager might not get a bonus when the earnings report misses the target set by the analyst’s forecast, but reporting high earnings that exceed the target is also costly (albeit probably less costly) to the manager because it reduces her flexibility to manage future earnings reports upward (e.g., Healy, 1985). All other features of the base model remain intact in the extended model considered in this section. The special case of $k_1 = k_2 = 0$ coincides with the base model.

Equilibrium in the extended model, referred to as extension B, is represented by three functions $F_B : \mathbb{R} \to \mathbb{R}$, $R_B : \mathbb{R}^2 \to \mathbb{R}$ and $P_B : \mathbb{R}^2 \to \mathbb{R}$. The function $F_B$ represents the analyst’s reporting strategy, where $F_B(s)$ is the analyst’s earnings forecast, given that $s \in \mathbb{R}$ is the

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8 The market uncertainty about the managerial reporting objective and the additional reporting cost are added into the base model separately in order to isolate the effect of each of these two features and demonstrate that each of them is solely sufficient to support a pricing rule that incorporates a premium (discount) for meeting or beating (missing) the analyst’s earnings forecast.
realization of his private signal $\tilde{s}$. The function $R_b$ represents the manager’s reporting strategy, where $R_b(f,e)$ is the manager’s earnings report, given that $f$ is the analyst’s earnings forecast and $e$ is the earnings realization. The function $P_b$ represents the market pricing rule, where $P_b(f,r)$ is the firm’s market price after the earnings announcement, given that $f \in \mathcal{R}$ is the analyst’s earnings forecast and $r \in \mathcal{R}$ is the manager’s earnings report. In equilibrium, the functions $F_b$, $R_b$ and $P_b$ satisfy the following three conditions for any $s,f,e,r \in \mathcal{R}$:

(i) $F_b(s) \in \arg\min_{f \in \mathcal{R}} E[(f - R_b(f,\tilde{e}))^2 | \tilde{s} = s]$;

(ii) $R_b(f,e) \in \arg\max_{f \in \mathcal{R}} \mathcal{E}[P_b(f,r) - c(r-e)^2 - K(r-f)]$;

(iii) $P_b(f,r) = E[\tilde{v} | F_b(\tilde{s}) = f, R_b(f,\tilde{e}) = r]$.

The equilibrium in extension B is first analyzed for the special case of $k_1 = k_2 = k \geq 0$, where the cost function $K(r-f)$ is symmetric around zero and the manager thus incurs the same marginal cost when missing or beating the analyst’s earnings forecast. Proposition 3 indicates the existence and uniqueness of a linear equilibrium in the case of $k_1 = k_2 = k \geq 0$ and characterizes its form.

**Proposition 3.** When $k_1 = k_2 = k \geq 0$, there exists a unique linear equilibrium $(F_b : \mathcal{R} \to \mathcal{R}, R_b : \mathcal{R}^2 \to \mathcal{R}, P_b : \mathcal{R} \to \mathcal{R})$ in the extension B, which takes the following form for any $s, f, e, r \in \mathcal{R}$:

$s, f, e, r \in \mathcal{R}: F_b(s) = E[\tilde{e} | \tilde{s} = s] + x\beta_b / 2c = (1 - \lambda)\mu + \lambda s + x\beta_b / 2c$,

$$R_b(f,e) = \frac{ce + kf}{c + k} + x\beta_b / 2(c + k) = \frac{c(e + x\beta_b / 2c) + kf}{c + k},$$

$$P_b(f,r) = \alpha_b + \beta_b r - \gamma_b f = \alpha_b + (\beta_b - \gamma_b) r + \gamma_b (r - f),$$

where $\lambda = \frac{\sigma^2 + \sigma_1^2}{\sigma^2 + \sigma_1^2 + \sigma_2^2}$,

$$\alpha_b = (1 - \beta_b + \gamma_b)\mu - (\beta_b - \gamma_b)\beta_b x / 2c , \quad \beta_b = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k}{c} \quad \text{and} \quad \gamma_b = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{k}{c}.$$
Proposition 3 suggests that, in the case of $k_1 = k_2 = k \geq 0$, the manager’s earnings report is

$$R_b(f, e) = \frac{ce + kf}{c + k} + x\beta_b / 2(c + k).$$

The costs $c(r - e)^2$ and $k(r - f)^2$ that the manager bears when deviating from the earnings forecast $f$ of the analyst and when deviating from the true earnings realization $e$, respectively, induce her to use the weighted average $\frac{ce + kf}{c + k}$ as a basis for the earnings report. The weight $\frac{c}{c + k}$ that the manager places in her earnings report on the true earnings realization $e$ is increasing in the marginal cost $c$ of deviating from the earnings realization and decreasing in the marginal cost $k$ of deviating from the analyst’s forecast. Similarly, the weight $\frac{k}{c + k}$ that the manager assigns in her earnings report to the analyst’s forecast $f$ is increasing in the marginal cost $k$ of deviating from the analyst’s forecast and decreasing in the marginal cost $c$ of deviating from the true earnings realization. Trying to influence the firm’s market price, the manager further shifts the earnings report by an additional bias of $x\beta_b / 2(c + k)$. The sign of this additional bias is the same as the sign of $x$. Its absolute value is increasing in the absolute value of $x$, increasing in the weight $\beta_b$ assigned by the pricing function $P_b(f, r) = \alpha_b + \beta_b r - \gamma_b f$ to the earnings report, and decreasing in the marginal reporting costs $c$ and $k$. The total bias embedded in the earnings report equals $ce + kf + x\beta_b / 2(c + k) - e$ or $k(f - e) / c + x\beta_b / 2(c + k)$, and is thus decreasing in the true earnings realization $e$ and increasing in the analyst’s earnings forecast $f$.

This is unlike the base model and extension A, where the manager’s equilibrium reporting bias is independent of the earnings realization and the analyst’s earnings forecast. In designing his forecast, the analyst relies on the realization $s$ of his private signal $\tilde{s}$, which implies that the earnings realization is expected to be $E[\tilde{e} | \tilde{s} = s] = (1 - \lambda)\mu + \lambda s$. Seeking to minimize the forecast error
\[
\frac{ce + kf}{c + k} + x\beta_B / 2(c + k) - f, \text{ which equals } \frac{c(e + x\beta_B / 2c - f)}{c + k}, \text{ the analyst shifts his forecast by the amount of } x\beta_B / 2c. \text{ It follows thus from Proposition 3 that the equilibrium earnings forecast of the analyst equals } F_B(s) = (1 - \lambda)\mu + \lambda s + x\beta_B / 2c. \text{ Proposition 3 further indicates that the equilibrium market price of the firm, following the earnings announcement, takes the form } P_B(f, r) = \alpha_B + \beta_B r - \gamma_B f. \text{ The properties of the equilibrium pricing coefficients are described by the following corollary.}

**Corollary to Proposition 3.** The pricing coefficients satisfy \( \beta_B > \gamma_B \geq 0 \). The pricing coefficient \( \gamma_B \) equals zero if and only if \( k = 0 \), and it is positive and increasing in \( k \) for any \( k > 0 \). The pricing coefficient \( \gamma_B \) is also decreasing in \( c \), converging to zero when \( c \) converges to infinity.

The pricing coefficient \( \gamma_B \) is positive as long as \( k > 0 \). This implies that, unlike the equilibrium outcomes of the base model (captured by the special case of \( k = 0 \) ), the analyst’s earnings forecast \( f \) is utilized by the investors in pricing the firm even though the earnings have already been realized and reported to the market. Intuitively, being aware of the incentives of the manager to meet the analyst’s benchmark, the investors rationally infer that the analyst’s forecast is embedded in the manager’s earnings report, and thus neutralize it from the earnings report in order to decipher the true earnings measure \( e \). Invoking their rational expectations about the manager’s reporting strategy, the investors adjust the earnings report \( r = \frac{ce + kf}{c + k} + x\beta_B / 2(c + k) \) as follows:

\[
\frac{c + k}{c} r - \frac{k}{c} f - x\beta_B / 2c. \text{ The investors rationally infer that the adjusted earnings report } \frac{c + k}{c} r - \frac{k}{c} f - \beta_B x / 2c \text{ captures the true earnings realization } e. \text{ They thus set the price of the firm
after the earnings announcement to be \( P_b(f,r) = E[v^2 \mid \epsilon] = \frac{c+k}{c}r - \frac{k}{c}f - x\beta_b / 2c \), which equals

\[
\mu + \frac{\sigma^2}{\sigma^2 + \sigma^2_\epsilon} \left( \frac{c+k}{c}r - \frac{k}{c}f - x\beta_b / 2c - \mu \right)
\]

and can be equivalently represented as \( \alpha_b + \beta_b r - \gamma_b f \).

In order to reconcile the equilibrium pricing rule with extant empirical findings, it is useful to recast it as \( P_b(f,r) = \mu + (\beta_b - \gamma_b)(r - \mu - \mu_c \beta_b / 2c) + \gamma_b (r - f) \). Hence, the price change \( P_b(f,r) - \mu \) over the entire period, which begins before the arrival to the market of the analyst’s earnings forecast and ends after the manager’s earnings announcement, equals

\[
(\beta_b - \gamma_b)(r - \mu - \mu_c \beta_b / 2c) + \gamma_b (r - f),
\]

where both pricing coefficients \( \beta_b - \gamma_b \) and \( \gamma_b \) are positive provided that \( k > 0 \). Hence, the price change over the entire period is again not only a function of the earnings surprise for the period, \( r - \mu - \mu_c \beta_b / 2c \), but also depends on the path by which the market expectations have changed over the period, as reflected in \( r - f \). Consistent with extant empirical evidence, the equilibrium price change over the entire period explains a market premium (discount) of \( \gamma_b \) for any unit of deviation upward (downward) of the earnings report relative to the analyst’s prior earnings forecast, after controlling for the earnings surprise, \( r - \mu - \mu_c \beta_b / 2c \), for the period. Similarly to extension A, the term \( \gamma_b (r - f) \) in the pricing rule does not represent any market reward (penalty) for meeting or beating (missing) analysts’ earnings forecasts, as commonly claimed in the literature. It is just an adjustment that the market applies to the managed earnings report in order to extract the underlying true earnings measure. Proposition 3 thus suggests that it is not the observed pricing effect of analysts’ earnings forecasts that induces managers to manipulate their earnings reports in order to meet the target set by the analysts, as conventionally surmised. It is rather the managerial wish to meet the analysts’ benchmark that leads to the observed pricing effect of analysts’ forecasts. The equilibrium outcomes presented in
Proposition 3 also rationalize the empirically documented positive association between $r - f$ and the future performance of the firm, because $r - f$ is the realization of

$$\frac{c}{c + k}((1 - \lambda)(\bar{v} - \mu) + (1 - \lambda)e_i - \lambda e_2)$$

which is a noisy estimator of the firm’s value $\bar{v}$.

The magnitude of the marginal premium (discount) $\gamma$ , which is associated with any unit of deviation upward (downward) of the earnings report relative to the analyst’s earnings forecast, is increasing in the marginal cost $k$ that the manager bears when failing to meet the analyst’s forecast. As the cost $k$ increases, the manager’s earnings report places more weight of the analyst’s forecast, and thus the required market adjustment of the earnings report is more significant. The cost $k(r - f)^2$ can, however, explain a market premium (discount) for meeting or beating (missing) the analyst’s forecast only in an environment where earnings management can emerge. Consequently, when the biasing cost $c$ increases, so that the managerial capability to manage earnings decreases, the pricing coefficient $\gamma$ decreases as well. In situations where earnings management is impossible because the biasing cost $c$ converges to infinity, the pricing coefficient $\gamma$ converges to zero and the pricing rule is reduced to $P_b(f, r) = \mu + \beta_b(r - \mu)$, as in the base model.

Having analyzed the case $k_1 = k_2 = k \geq 0$ where the cost function $K(r - f)$ is symmetric around zero, the case $k_1 > k_2 \geq 0$ of an asymmetric cost function is next analyzed. It appears that when the cost function $K(r - f)$ is asymmetric around zero, equilibrium with a linear pricing rule does not exist. However, there exists equilibrium with a monotonically increasing (non-linear) pricing function, which is presented in Proposition 4 (even though its uniqueness is not established).

**Proposition 4.** When $k_1 > k_2 \geq 0$, there exists an equilibrium $(F_b : \mathbb{R} \to \mathbb{R}, R_b : \mathbb{R}^2 \to \mathbb{R})$, $P_b : \mathbb{R}^2 \to \mathbb{R}$ in extension $B$, which is characterized by a scalar $\varphi \in \mathbb{R}$ and a monotonically
increasing function $G: \mathbb{R} \rightarrow \mathbb{R}$, such that for any $s, e, f, r \in \mathbb{R}$: $F_s(s) = (1 - \lambda) \mu + \lambda s + \varphi$.

$$R_b(f, e) = \begin{cases} \frac{c(e + \beta_b(1)x/2c) + k_if}{c + k_1} & \text{if } e \leq f - \beta_b(1)x/2c \\ \frac{c(G(e) + \beta_b(2)x/2c) + k_2f}{c + k_2} & \text{if } e > f - \beta_b(1)x/2c \end{cases}$$

\[
\lambda = \frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 + \sigma_1^2 + \sigma_2^2}, \quad \alpha_b(i) = (1 - \beta_b(i) + \gamma_b(i))\mu - (\beta_b(i) - \gamma_b(i))\beta_b(i)x/2c, \quad \beta_b(i) = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_1^2} \frac{c + k_1}{c}
\]

and $\gamma_b(i) = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_1^2} \frac{k_i}{c}$ for any $i = 1, 2$. The scalar $\varphi$ satisfies $\beta_b(2)x/2c < \varphi < \beta_b(1)x/2c$. The function $G(e)$ is increasing and convex in $e$ and satisfies $G(f - \beta_b(1)x/2) = f - \beta_b(2)x/2$ for any $f \in \mathbb{R}$ and $\lim_{e \to \pm \infty}(G(e) - e) = 0$. For any $f \in \mathbb{R}$, $R_b(f, e) > \frac{c(e + x\beta_b(2)/2c) + k_2f}{c + k_2}$.

$$\lim_{e \to \pm \infty} R_b(f, e) - \frac{c(e + x\beta_b(2)/2c) + k_2f}{c + k_2} = 0, \quad P_b(f, r) < \alpha_b(2) + \beta_b(2)r - \gamma_b(2)f \quad \text{and}$$

$$\lim_{r \to \pm \infty} \alpha_b(2) + \beta_b(2)r - \gamma_b(2)f - P_b(f, r) = 0.$$

The shape of the manager’s reporting function in the case $k_i > k_2 \geq 0$ is graphically illustrated in Figure 2. The horizontal axis describes all the possible realizations, $e$, of the earnings measure $\tilde{e}$. The solid graph is the earnings report $R_b(f_1, e)$ as a function of the true earnings realization $e$ and for a fixed content $f_1$ of the analyst’s earnings forecast, while the dotted graph is the earnings report $R_b(f_2, e)$ as a function of the true earnings realization $e$ for another fixed content $f_2$ of the analyst’s earnings forecast, such that $f_1 < f_2$. As graphically illustrated, the manager’s
reporting function \( R_a(f, e) \) is increasing in both the true earnings realization \( e \) and the analyst’s earnings forecast \( f \). For any earnings forecast \( f \), the manager’s reporting function is continuous in the true earnings realization \( e \), though it consists of two different functions. The manager applies the linear reporting function \( \frac{c(e + x\beta_a (1) / 2c) + kf}{c + k_1} \) up to the point of meeting the analyst’s earnings forecast, but from this point forward she applies another reporting function,

\[
\frac{c(G(e) + x\beta_a (2) / 2c) + k_2f}{c + k_2},
\]

which is convex in \( e \) and approaches the linear asymptote \( \frac{c(e + x\beta_a (2) / 2c) + k_2f}{c + k_2} \) as \( e \) increases.

The broken shape of the manager’s reporting function is the result of the asymmetry of the cost function \( K(r - f) \) around zero. As the cost of missing the analyst’s forecast exceeds the cost of beating the forecast, it is more important to the manager to reduce the deviation of the reported earnings from the analyst’s forecast when the true earnings realization is too low to allow her to meet the forecast. Therefore, the reporting function \( \frac{c(e + x\beta_a (1) / 2c) + kf}{c + k_1} \) applied to low earnings realizations is less sensitive to the earnings realization and more sensitive to the analyst’s forecast than the asymptotic reporting function \( \frac{c(e + x\beta_a (2) / 2c) + k_2f}{c + k_2} \) applied to high earnings realizations. These two linear functions are disconnected, because the function

\[
\frac{c(e + x\beta_a (1) / 2c) + kf}{c + k_1}
\]

results in an earnings report that meets the analyst’s forecast at the point \( e = f - x\beta_a (1) / 2c \), whereas the function \( \frac{c(e + x\beta_a (2) / 2c) + k_2f}{c + k_2} \) results in an earnings report that meets the forecast at the higher point \( e = f - x\beta_a (2) / 2c \). The convex function
\[
\frac{c(G(e) + x_\beta_e(2)/2c) + k_2f}{c + k_2}
\]
that the manager applies when the earnings realization is above 

\[e = f - x_\beta_e(1)/2c\]
serves to connect the two linear functions. As a result, for intermediate earnings realizations that belong to the region \((f - x_\beta_e(1)/2c, f - x_\beta_e(2)/2c)\), the manager’s earnings report is slightly above the analyst’s forecast and is almost insensitive to the earnings realization.

[FIGURE 2]

Proposition 4 further indicates that the equilibrium earnings forecast of the analyst equals 

\[F_\beta(s) = (1 - \lambda)\mu + \lambda s + \phi, \text{ where } x_\beta_e(2)/2c < \phi < x_\beta_e(1)/2c.\]

In designing his forecast, the analyst relies on the realization \(s\) of his private signal \(\tilde{s}\), which implies that the earnings realization is expected to be 

\[E[\tilde{e} | \tilde{s} = s] = (1 - \lambda)\mu + \lambda s.\]

Here, unlike the case of a symmetric cost function, the analyst cannot perfectly predict the bias embedded in the manager’s report because it depends on the unknown earnings realization. Therefore, in an effort to minimize the forecast error, the analyst shifts his forecast by the amount of \(\phi\), which is the average shift required based on his private information \(s\). The shift \(\phi\) in the analyst’s forecast thus lies between the shift \(x_\beta_e(1)/2c\) required for low earnings realizations and the lower shift \(x_\beta_e(2)/2c\) required for high earnings realizations.

This implies that the mean \(E[\tilde{e} | \tilde{s} = s]\) of the earnings measure \(\tilde{e}\) conditional on the analyst’s private information belongs to the region \((f - x_\beta_e(1)/2c, f - x_\beta_e(2)/2c)\) of earnings realizations, where the manager’s earnings report is slightly above the analyst’s forecast and is very insensitive to the earnings realization. That is, the mass of earnings realizations that belong to the middle of the true earnings distribution yields an accumulation of similar earnings reports that beat the analyst’s forecast by a tiny amount. This property of the distribution of the reported earnings reconciles with empirical findings (e.g., Degeorge, Patel and Zeckhauser, 1999; Burgstahler and Eames, 2006).
The shape of the equilibrium pricing rule in the case \( k_1 > k_2 \geq 0 \) is graphically illustrated in Figure 3. The horizontal axis describes all the possible contents of the manager’s earnings report, \( r \). The solid graph is the market price \( P_b(f_1, r) \) of the firm as a function of the content \( r \) of the manager’s earnings report and for a fixed content \( f_1 \) of the analyst’s earnings forecast, while the dotted graph is the price \( P_b(f_2, r) \) as a function of the content \( r \) of the manager’s earnings report for another fixed content \( f_2 \) of the analyst’s earnings forecast, such that \( f_1 < f_2 \). As graphically illustrated, the equilibrium pricing rule \( P_b(f, r) \) is increasing in both the earnings report \( r \) and the analyst’s earnings forecast \( f \). Since the manager’s reporting function is monotonically increasing in the true earnings realization, the investors can perfectly adjust for the manager’s reporting bias and accurately extract the true earnings realization from the manager’s report. As the managerial reporting bias depends upon whether the earnings report is below the forecast or above it, so too does the market adjustment of the biased earnings. This results in a broken pricing function \( P_b(f, r) \). In pricing the firm, the investors use the increasing linear pricing function

\[
\alpha_b(1) + (\beta_b(1) - \gamma_b(1))r + \gamma_b(1)(r - f)
\]

when the earnings report does not exceed the analyst’s forecast (i.e., \( r \leq f \)). However, for earnings reports that exceed the analyst’s forecast (i.e., \( r > f \)), they apply a different pricing function, which is concave in the earnings report \( r \) and approaches the linear asymptote \( \alpha_b(2) + (\beta_b(2) - \gamma_b(2))r + \gamma_b(2)(r - f) \) as \( r \) increases. The following corollary describes the properties of the pricing coefficients.

**Corollary to Proposition 4.** When \( k_1 > k_2 \geq 0 \), the pricing coefficients satisfy \( \alpha_b(1) < \alpha_b(2) \), \( \beta_b(1) > \beta_b(2) > 0 \), \( \gamma_b(1) > \gamma_b(2) \geq 0 \), and \( \beta_b(1) - \gamma_b(1) = \beta_b(2) - \gamma_b(2) = \frac{\sigma^2}{\sigma_i^2 + \sigma_f^2} > 0 \). The differences \( \alpha_b(2) - \alpha_b(1) \), \( \beta_b(1) - \beta_b(2) \) and \( \gamma_b(1) - \gamma_b(2) \) are decreasing in \( k_1 - k_2 \), converging to
zero when \( k_1 - k_2 \) converges to zero. The pricing coefficient \( \gamma_\beta(1) \) is positive and increasing in \( k_1 \). The pricing coefficient \( \gamma_\beta(2) \) equals zero if and only if \( k_2 = 0 \), and it is positive and increasing in \( k_2 \) for any \( k_2 > 0 \). The pricing coefficients \( \gamma_\beta(1) \) and \( \gamma_\beta(2) \) are decreasing in \( c \), converging to zero when \( c \) converges to infinity.

The inequality \( \beta_\beta(1) > \beta_\beta(2) \) implies that the pricing function applied by the market to earnings reports below the point of meeting the analyst’s forecast is more sensitive to the earnings report relative to the asymptotical pricing function applied to earnings reports that exceed this point by a sufficient amount. The pricing function is however very steep for earnings reports that are slightly above the forecast. Consistent with empirical evidence, this implies that the stock price drops dramatically when the earnings report misses the analyst’s forecast by even a tiny amount relative to the case of beating the forecast by a tiny amount. The equilibrium pricing rule explains a premium that approaches \( \gamma_\beta(2) \) for any unit of deviation upward of the earnings report relative to the analyst’s forecast, after controlling for the earning news. It however explains a higher discount of \( \gamma_\beta(1) \) for any unit of deviation downward of the earnings report relative to the analyst’s forecast, after controlling for the earning news. The inequality \( \gamma_\beta(1) > \gamma_\beta(2) \geq 0 \) suggests that the premium associated with beating an analyst’s earnings forecast by a certain (albeit not too low) amount is lower than the market discount associated with missing the forecast by the same amount. This property of the pricing rule is illustrated in Figure 3 by the distance between the solid graph and the dotted graph, which is relatively wide when the earnings report misses the analyst’s forecast, but becomes much narrower when the earnings report beats the forecast by a sufficient amount.

[FIGURE 3]

In reconciling the equilibrium outcomes of Proposition 4 with empirical findings, it should be noted that empirical studies documenting the market premium (discount) associated with
meeting or beating (missing) analysts’ forecasts assume a piecewise linear pricing rule that switches from one linear function to another linear function at the point where the earnings report exactly meets the analyst’s forecast. The empirical estimate of the pricing rule \( P_b(f, r) \) deriving from these studies is thus likely to be the linear function \( \alpha_b(1) + (\beta_b(1) - \gamma_b(1))r + \gamma_b(1)(r - f) \) when \( r \leq f \) and the linear function \( \alpha_b(2) + (\beta_b(2) - \gamma_b(2))r + \gamma_b(2)(r - f) \) when \( r > f \). This may explain the empirical findings of a discontinuity in the price reaction to earnings news around the point of meeting the analysts’ threshold, as well as the asymmetric price reaction to scenarios of missing the analysts’ threshold by a certain amount relative to scenarios of beating it by the same amount.

The magnitude of the discount \( \gamma_b(1) \) associated with any unit of downward deviation of the earnings report relative to the analyst’s earnings forecast is increasing in the cost \( k_1 \). Similarly, the magnitude of the asymptotical premium \( \gamma_b(2) \) associated with any unit of upward deviation of the earnings report relative to the analyst’s forecast is increasing in the cost \( k_2 \). The difference between \( \gamma_b(1) \) and \( \gamma_b(2) \) is thus decreasing in the difference between \( k_1 \) and \( k_2 \). As \( k_1 - k_2 \) converges to zero, both \( \gamma_b(1) \) and \( \gamma_b(2) \) approach \( \gamma_b \) and the pricing rule of Proposition 4 coincides with the pricing rule of Proposition 3. The managerial cost \( K(r - f) \) can of course explain a market premium (discount) for meeting or beating (missing) the analyst’s earnings forecast only in an environment where earnings management can emerge. Consequently, both pricing coefficients \( \gamma_b(1) \) and \( \gamma_b(2) \) are decreasing in the biasing cost \( c \). In situations where earnings management is impossible because the biasing cost \( c \) converges to infinity, both pricing coefficient \( \gamma_b(1) \) and \( \gamma_b(2) \) converge to zero and the pricing rule coincides with that of the base model.

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5. **Concluding Remarks**

   This study explores the rationale behind the market pricing premium (discount) empirically observed following earnings announcements that meet or beat (miss) prior analysts’ forecasts. It provides two alternative explanations for this seemingly anomalous pricing pattern. The unifying idea underlying the two explanations is the view of the observed pricing effect of analysts’ earnings forecasts as the rational consequence of the practice of earnings management. This idea is established by demonstrating that the market premium (discount) associated with meeting or beating (missing) analysts’ forecasts might be simply an adjustment that the market applies to the managed earnings report in order to extract the underlying true earnings measure. Hence, while the empirically documented pricing effect of analysts’ earnings forecasts is often perceived as being the reason that managers feel excessively pressured to engage in earnings management activities, this study counter-intuitively demonstrates that it might be the other way around.

   It is shown that, in environments with earnings managements, two factors can trigger a pricing rule that incorporates a market premium (discount) for meeting or beating (missing) analysts’ earnings forecasts. The first factor is uncertainty on the part of the capital market investors with respect to the managerial reporting incentives. The second trigger is the costs that managers bear when their earnings reports deviate from the analysts’ threshold. Each of these two factors is solely sufficient to support a pricing rule that reconciles with extant empirical findings, but both of them can do so only in circumstances where earnings management can emerge. This yields the empirical prediction that the market premium (discount) associated with meeting or beating (missing) analysts’ earnings forecasts is expected to be much more salient in situations where earnings management activities are identified or more likely to take place (for example, following the issuance of accounting standards that expand the managerial reporting discretion).
Appendix – Proofs

Proof of Observation 1. Looking for equilibrium with a linear pricing function, it is assumed there exist scalars $\alpha$, $\beta$ and $\gamma$, such that $P(f, r) = \alpha + \beta r - \gamma f$. It follows thus that

$$R(f, e) = \arg \max_{r \in \mathbb{R}} x(\alpha + \beta r - \gamma f) - c(r - e)^2.$$ The first-order condition is $x\beta - 2c(r - e) = 0$. The manager’s optimal report is $R(f, e) = e + x\beta / 2c$, where the first-order condition holds, and so does the second-order condition: $-2c < 0$. The analyst’s report thus satisfies

$$F(s) \in \arg \min_{f \in \mathbb{R}} E[(f - (\tilde{e} + x\beta / 2c))^2 | s = s].$$ Hence,

$$F(s) = E[\tilde{e} | s = s] + x\beta / 2c = (1 - \lambda)\mu + \lambda s + x\beta / 2c,$$ where $\lambda = \frac{\sigma^2 + \sigma_1^2}{\sigma^2 + \sigma_1^2 + \sigma_2^2}$.

The investors rationally infer that both the analyst’s earnings forecast $f$ and the manager’s earnings report $r$ are biased by the amount of $x\beta / 2c$. They therefore know that

$$\lambda^{-1}(f - (1 - \lambda)\mu - x\beta / 2c)$$ is the realization $s$ of the analyst’s private signal $\tilde{s}$ whereas $r - x\beta / 2c$ is the realization $e$ of the earnings measure $\tilde{e}$. Since $\tilde{s}$ is a noisy signal of the earnings measure $\tilde{e}$, the earnings forecast is subsumed by the earnings news and is not informative to the investors in pricing the firm after the earnings announcement. So, the market price of the firm after the earnings announcement is $P(f, r) = E\left(\tilde{e} | r - x\beta / 2c\right) = \mu + \frac{\sigma^2}{\sigma^2 + \sigma_1^2} (r - x\beta / 2c - \mu)$, implying

$$\alpha = (1 - \beta)\mu - x\beta^2 / 2c, \quad \beta = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \text{ and } \gamma = 0. \square$$

Proof of Proposition 2. Looking for equilibrium with a linear pricing function, it is assumed there exist scalars $\alpha_A$, $\beta_A$ and $\gamma_A$, such that $P_A(f, r) = \alpha_A + \beta_A r - \gamma_A f$. It follows thus that

$$R_A(f, e, x) = \arg \max_{r \in \mathbb{R}} x(\alpha_A + \beta_A r - \gamma_A f) - c(r - e)^2.$$ The first-order condition is $x\beta_A - 2c(r - e) = 0$. 

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The manager’s optimal report is \( R_A(f,e,x) = e + x\beta_A / 2c \), where the first-order condition holds, and so does the second-order condition: \(-2c < 0\). It follows now that the analyst’s report satisfies \( F_A(s,x) \in \arg\min_{f, \mathbb{E}} E[(f-(\overline{e} + x\beta_A / 2c))^2 | \overline{y} = s] \). Hence, \( F_A(s,x) = E[\overline{e}|\overline{y} = s] + x\beta_A / 2c \) or

\[
F_A(s,x) = (1 - \lambda_A) \mu + \lambda_A s + x\beta_A / 2c , \text{ where } \lambda_A = -\frac{\sigma_x^2 + \sigma_s^2}{\sigma_x^2 + \sigma_s^2 + \sigma_x^2} .
\]

The manager’s earnings report \( e + x\beta_A / 2c \) is the realization of the random variable \( \overline{\epsilon} + \overline{x}\beta_A / 2c \). The reporting bias is unobservable to investors, because they are uncertain about the realization of \( \overline{x} \), and thus consider the earnings report as a noisy signal of the true earnings measure. Since the publicly observable analyst’s earnings forecast \( f \) is the realization of \((1 - \lambda_A) \mu + \lambda_A \overline{s} + \overline{x}\beta_A / 2c \), it can serve investors as an additional noisy signal about the true earnings realization. The investors can infer that \( \lambda_A^{-1} (f - (1 - \lambda_A) \mu) \) is the realization of the random variable \( \overline{s} + \lambda_A^{-1} \overline{x}\beta_A / 2c \), which equals \( \overline{\epsilon} + \overline{s} + \lambda_A^{-1} \overline{x}\beta_A / 2c \). They accordingly update the price \( P_A(f,r) \) to be

\[
E[\overline{\epsilon} + \overline{x}\beta_A / 2c = r, \overline{\epsilon} + \overline{s} + \lambda_A^{-1} \overline{x}\beta_A / 2c = \lambda_A^{-1} (f - (1 - \lambda_A) \mu)] .
\]

This implies that

\[
\alpha_A = (1 - \beta_A + \gamma_A) \mu - \beta_A (\beta_A - \gamma_A) \mu / 2c , \text{ with } \beta_A = \frac{\sigma_x^2 (\text{var}(\tilde{b}) - \text{cov}(\tilde{a},\tilde{b}))}{\text{var}(\tilde{a}) \text{var}(\tilde{b}) - \text{cov}(\tilde{a},\tilde{b})^2} \text{ and }
\]

\[
\gamma_A = -\frac{\lambda_A^{-1} \sigma_x^2 (\text{var}(\tilde{a}) - \text{cov}(\tilde{a},\tilde{b}))}{\text{var}(\tilde{a}) \text{var}(\tilde{b}) - \text{cov}(\tilde{a},\tilde{b})^2} , \text{ where } \tilde{a} = \overline{\epsilon} + \overline{x}\beta_A / 2c \text{ and } \tilde{b} = \overline{\epsilon} + \overline{s} + \lambda_A^{-1} \overline{x}\beta_A / 2c . \text{ Since }
\]

\[
\text{var}(\tilde{a}) = \sigma_x^2 + \sigma_s^2 + \lambda_A^{-2} (\beta_A / 2c)^2 , \text{ var}(\tilde{b}) = \sigma_x^2 + \sigma_s^2 + \lambda_A^{-2} (\beta_A / 2c)^2 \text{ and }
\]

\[
\text{cov}(\tilde{a},\tilde{b}) = \sigma_x^2 + \lambda_A^{-1} \sigma_x^2 (\beta_A / 2c)^2 , \text{ we get after rearranging }
\]

\[
\beta_A = \sigma_x^2 \left( \frac{\sigma_x^2 + \lambda_A^{-1} (\lambda_A^{-1} - 1) \sigma_x^2 (\beta_A / 2c)^2}{\sigma_x^2 (\beta_A / 2c)^2 + (\sigma_x^2 + \sigma_s^2)(\sigma_x^2 + (\lambda_A^{-1} - 1) \sigma_x^2 (\beta_A / 2c)^2)} \right) \text{ and }
\]

\[
\gamma_A = -\lambda_A^{-1} \sigma_x^2 \left( \frac{(1 - \lambda_A^{-1}) \sigma_x^2 (\beta_A / 2c)^2}{\sigma_x^2 (\beta_A / 2c)^2 + (\sigma_x^2 + \sigma_s^2)(\sigma_x^2 + (\lambda_A^{-1} - 1) \sigma_x^2 (\beta_A / 2c)^2)} \right) . \text{ After substituting }
\]

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\[ \lambda_A = \frac{\sigma^2 + \sigma_1^2}{\sigma^2 + \sigma_1^2 + \sigma_2^2} \] and rearranging, we get \( \beta_A = -\frac{\sigma^2}{\sigma^2 + \sigma_1^2} \) and

\[ \gamma_A = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{(\sigma^2 + \sigma_1^2 + \sigma_2^2) \sigma_2^4 / 4c^2}{(\sigma^2 + \sigma_1^2 + \sigma_2^2) \sigma_2^4 / 4c^2 + (\sigma^2 + \sigma_1^2) \sigma_2^4 / 4c^2} \] . The pricing rule \( P_A(f, r) = \alpha_A + \beta_A r - \gamma_A f \) can now be rewritten as \( P_A(f, r) = \alpha_A + (\beta_A - \gamma_A) r + \gamma_A (r - f) \), where

\[ \beta_A - \gamma_A = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{(\sigma^2 + \sigma_1^2)^4}{(\sigma^2 + \sigma_1^2 + \sigma_2^2) \sigma_2^4 / 4c^2 + (\sigma^2 + \sigma_1^2) \sigma_2^4 / 4c^2} > 0 \] and \( \gamma_A \geq 0 \). The pricing coefficient

\[ \gamma_A = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{(\sigma^2 + \sigma_1^2 + \sigma_2^2) \sigma_2^4 / 4c^2}{(\sigma^2 + \sigma_1^2 + \sigma_2^2) \sigma_2^4 / 4c^2 + (\sigma^2 + \sigma_1^2) \sigma_2^4 / 4c^2} \] equals zero if and only if \( \sigma_2^2 = 0 \), and it is positive and increasing in \( \sigma_2^2 \) for any \( \sigma_2^2 > 0 \). \( \square \)

**Proof of Proposition 3.** Looking for equilibrium with a linear pricing function, it is assumed there exist scalars \( \alpha_B, \beta_B \) and \( \gamma_B \), such that \( P_B(f, r) = \alpha_B + \beta_B r - \gamma_B f \) . It follows thus that

\[ R_B(f, e) = \arg \max_{r \in \mathbb{R}} x(\alpha + \beta_B r - \gamma_B f) - c(r - e)^2 - k(r - f)^2 . \] The first-order condition is

\[ x\beta_B - 2c(r - e) - 2k(r - f) = 0 . \] The manager’s optimal report is \( R_B(f, e) = \frac{ce + kf}{c + k} + x\beta_B / 2(c + k) \), where the first-order condition holds, and so does the second-order condition: \( -2c - 2k < 0 \). The analyst’s earnings forecast thus satisfies \( F_B(s) \in \arg \min_{f \in \mathbb{R}} E\left[ f - \left( \frac{ce + kf}{c + k} + x\beta_B / 2(c + k) \right) \right]^2 \mid \tilde{s} = s \] or

\[ F_B(s) \in \arg \min_{f \in \mathbb{R}} E\left[ \left( c(f - e - x\beta_B / 2c) \right) \right]^2 \mid \tilde{s} = s \]. Hence,

\[ F_B(s) = E\left[ \tilde{c} \mid \tilde{s} = s \right] + x\beta_B / 2c = (1 - \lambda_B) \mu + \lambda_B s + x\beta_B / 2c \] , where \( \lambda_B = \frac{\sigma^2 + \sigma_1^2}{\sigma^2 + \sigma_1^2 + \sigma_2^2} \).
The investors rationally infer that \( \lambda^\dagger(f - (1 - \lambda_g)\mu - x\beta_0 / 2c) \) is the realization \( s \) of the analyst’s private signal \( \tilde{s} \), whereas \( \frac{c + k}{c}r - \frac{k}{c}f - x\beta_0 / 2c \) is the realization \( e \) of the earnings measure \( \tilde{e} \).

Since \( \tilde{s} \) is a noisy signal of the earnings measure \( \tilde{e} \), the market price of the firm after the earnings announcement is

\[
P_b(f, r) = E \left( \frac{c + k}{c} r - \frac{k}{c} f - x\beta_0 / 2c \right) = \mu + \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \left( \frac{c + k}{c} r - \frac{k}{c} f - x\beta_0 / 2c - \mu \right).
\]

This implies

\[
\alpha_0 = (1 - \beta_0 + \gamma_0)\mu - \beta_0(\beta_0 - \gamma_0)x / 2c, \quad \beta_0 = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k}{c} \quad \text{and} \quad \gamma_0 = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{k}{c}.
\]

The pricing rule \( P_b(f, r) = \alpha_0 + \beta_0 r - \gamma_0 f \) can be now rewritten as \( P_b(f, r) = \alpha_0 + (\beta_0 - \gamma_0)r + \gamma_0(r - f) \), where \( \beta_0 - \gamma_0 = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} > 0 \) and \( \gamma_0 \geq 0 \). The pricing coefficient \( \gamma_0 = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{k}{c} \) equals zero if and only if \( k = 0 \), and it is positive and increasing in \( k \) for any \( k > 0 \).

**Proof of Proposition 4.** The proof shows that there exists a unique pair of scalar \( \varphi \in \mathcal{R} \) and function \( G : \mathcal{R} \to \mathcal{R} \) that constitutes an equilibrium that takes the form described in the proposition. The proof does not, however, preclude the existence of other equilibria that take a different form.

When \( r \leq f \), the earnings report \( r \) maximizes \( x(\alpha_0(l) + \beta_0(l)r - \gamma_0(l)f) - c(r - e)^2 - k_i(r - f)^2 \). The first-order condition is \( x\beta_0(l) - 2c(r - e) - 2k_i(r - f) = 0 \). Therefore, \( r = \frac{c(e + x\beta_0(l)/2c) + k_i f}{c + k_i} \), where the first-order condition holds, and so does the second-order condition: \( -2c - 2k_i < 0 \). Note
that \( \frac{c(e + x\beta_0(1)/2c) + k_i f}{c + k_i} \leq f \) if and only if \( e \leq f - x\beta_0(1)/2c \). Therefore, the earnings report equals \( \frac{c(e + x\beta_0(1)/2c) + k_i f}{c + k_i} \) for any \( e \leq f - x\beta_0(1)/2c \).

When \( r > f \), the earnings report \( r \) maximizes

\[
x \left( \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \mu + \frac{\sigma^2}{\sigma^2 + \sigma_1^2} G'(c) \left( \frac{c + k_f}{c} r - \frac{k_f}{c} f - \beta_0 (2) x/2c \right) \right) - c(r - e)^2 - k_2 (r - f)^2.
\]

The first-order condition is

\[
x \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k_f}{c} - \frac{1}{G'(e)} 2c(r - e) - 2k_2 (r - f) = 0.
\]

This yields the following differential equation:

\[
G'(e) = \frac{x \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k_f}{c} G'(e) + cG(e) + \beta_0 (2) x/2c + k_2 f}{2c(r - e) + 2k_2 (r - f)}.
\]

we get

\[
G'(e) = \frac{x \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k_f}{c}}{(2c + 2k_2) cG(e) + cG(e) + x\beta_0 (2) x/2c + k_2 f - 2ce - 2k_2 f}.
\]

Define \( g(e) = G(e) - e \) and note that \( g'(e) = G'(e) - 1 \). We can thus recast the differential equation

\[
G'(e) = \frac{x\beta_0 (2)}{2c(G(e) - e) + x\beta_0 (2)} \quad \text{into} \quad \frac{dg}{de} = \frac{x\beta_0 (2)}{2cg(e) + x\beta_0 (2)} - 1,
\]

where

\[
g(f - x\beta_0 (1)/2c) = x(\beta_0 (1) - \beta_0 (2))/2c, \quad \text{in order to ensure indifference between reporting}
\]

\[
\frac{c(e + x\beta_0 (1)/2c) + k_i f}{c + k_i} \quad \text{or reporting} \quad \frac{cG(e) + \beta_0 (2) x/2c + k_2 f}{c + k_2} \quad \text{at the point} \quad e = f - x\beta_0 (1)/2c.
\]

One solution to the differential equation \( \frac{dg}{de} = \frac{x\beta_0 (2)}{2cg(e) + x\beta_0 (2)} - 1 \) is the constant function \( g(e) = 0 \), but this solution does not satisfy the condition \( g(f - x\beta_0 (1)/2c) = x(\beta_0 (1) - \beta_0 (2))/2c > 0 \). To find other solutions, we rewrite the differential equation \( \frac{dg}{de} = \frac{x\beta_0 (2)}{2cg(e) + x\beta_0 (2)} - 1 \) as
\[
\int \left( \frac{x \beta_2}{2 \gamma + x \beta_2} - 1 \right) \, dg = \int \, de 	ext{ or } \int \left( \frac{2 \gamma + x \beta_2}{x \beta_2} \right) \left( 1 - \frac{2 \gamma + x \beta_2}{x \beta_2} \right) \, dg = \int \, de \text{ or } \\
\int \left( 1 + \frac{2 \gamma}{x \beta_2} \right) \left( \frac{2 \gamma}{x \beta_2} \right) \, dg = \int \, de . \]
The function \( g(e) \) is thus defined for any \( e \in \mathbb{R} \) by the implicit equation
\[
-g(e) - \frac{x \beta_2}{2 \gamma} \ln \left( \frac{2 \gamma}{x \beta_2} g(e) \right) = e + \omega , \text{ where } \omega \text{ is the scalar satisfying} \\
g(f - x \beta_2/2c) = x(\beta_2) - (\beta_2) / 2c \text{ or} \\
-x(\beta_2) - x(\beta_2) / 2c \ln \left( \frac{\beta_2 - \beta_2}{\beta_2} \right) = f - x \beta_2 / 2c + \omega . \text{ The left side of the equation} \\
-g(e) - \frac{x \beta_2}{2 \gamma} \ln \left( \frac{2 \gamma}{x \beta_2} g(e) \right) = e + \omega \text{ is defined only for positive values of } g(e) \text{ and is decreasing in } g(e) . \text{ Hence, for any } e \geq f - x \beta_2 / 2c , \text{ there exists a unique solution } g(e) , \text{ which is positive and decreasing in } e , \text{ where} \\
g(f - x \beta_2(1) / 2c) = x(\beta_2(1) - \beta_2(2)) / 2c > 0 \text{ and} \\
\lim_{e \to \infty} g(e) = 0 . \text{ Hence, } R_2(f, e) > \frac{c(e + x \beta_2 / 2c) + k_2 f}{c + k_2} \text{ for any } r > f \text{ and} \\
\lim_{e \to \infty} R_2(f, e) - \frac{c(e + x \beta_2 / 2c) + k_2 f}{c + k_2} = 0 .
\]

In setting the price \( P_2(f, r) \), the investors can perfectly extract the true earnings realization \( e \) from the reported earnings \( r \), because the manager’s equilibrium earnings report \( r = R_2(f, e) \) is monotonically increasing in \( e \). When \( r \leq f \), the investors know that the earnings report is
\[
r = \frac{c(e + x \beta_2(1) / 2c) + k_2 f}{c + k_2} , \text{ and thus deduce that} \\
\frac{c + k_1}{c} r - \frac{k_1 f}{c} - x \beta_2(1) / 2c \text{ is the true earnings realization. They accordingly set the price to be} 
\]

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\[
E\left[\tilde{e}e\right] = \frac{c + k_1}{c} r - \frac{k_1 f}{c} - x\beta_g(1)/2c \quad \text{when}
\]
\[
r \leq f \quad \text{So,} \quad \alpha_g(1) = (1 - \beta_g(1) + \gamma_g(1))\mu - (\beta_g(1) - \gamma_g(1))\beta_g(1)x/2c \quad , \beta_g(1) = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k_1}{c} \quad \text{and}
\]
\[
\gamma_g(1) = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{k_1}{c} \quad \text{When } r > f \quad , \text{the investors know that the earnings report is}
\]
\[
r = \frac{c(G(e) + \beta_g(2)x/2c) + k_2 f}{c + k_2} \quad , \quad \text{and thus deduce that} \quad G^{-1}\left(\frac{c + k_2}{c} r - \frac{k_2}{c} f - x\beta_g(2)/2c\right) \quad \text{is the true earnings realization. They accordingly set the price to be}
\]
\[
E\left[\tilde{e}e\right] = G^{-1}\left(\frac{c + k_2}{c} r - \frac{k_2}{c} f - x\beta_g(2)/2c\right) \quad \text{when} \quad r > f \quad . \quad \text{As } g(e) \text{ is positive and decreasing in } e \quad , \text{approaching to zero when } e \text{ converges to infinity, we get that} \quad P_g(f,r) < \alpha_g(2) + \beta_g(2)r - \gamma_g(2)f \quad \text{for any } r > f \quad \text{and}
\]
\[
\lim_{r \to \infty} P_g(f,r) = \alpha_g(2) + \beta_g(2)r - \gamma_g(2)f \quad , \text{where}
\]
\[
\alpha_g(2) = (1 - \beta_g(2) + \gamma_g(2))\mu - (\beta_g(2) - \gamma_g(2))\beta_g(2)x/2c \quad , \beta_g(2) = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{c + k_2}{c} \quad \text{and}
\]
\[
\gamma_g(2) = \frac{\sigma^2}{\sigma^2 + \sigma_1^2} \cdot \frac{k_2}{c} \quad .
\]

The analyst’s report satisfies \( F_g(s) \in \arg \min_{f \in \mathbb{R}} E[(f - R(f, \tilde{e}))^2|\tilde{s} = s] \). Hence,
\[
F_g(s) = E[\tilde{e}|\tilde{s} = s] + \varphi, \quad \text{where} \quad \varphi = q(\varphi)\beta_g(1)/2c + (1 - q(\varphi))\beta_g(2)/2c + \int_{E[\tilde{e}|\tilde{s} = s] + \varphi - x\beta_g(1)/2c}^{\infty} g(e)h(e)de,
\]
\[
q(\varphi) = H(E[\tilde{e}|\tilde{s} = s] + \varphi - x\beta_g(1)/2c), \quad h \quad \text{and} \quad H \quad \text{are, respectively, the probability density function and the cumulative distribution function for the random variable } \tilde{e}|\tilde{s} = s \quad . \text{The derivative of}
\]
\[
q(\varphi)\beta_g(1)/2c + (1 - q(\varphi))\beta_g(2)/2c \quad \text{with respect to} \quad \varphi \quad \text{is}
\]
The derivative of \( \int_{E[\varphi]=1}^{\varphi=\beta/2c} g(e)h(e)de \) with respect to \( \varphi \) is

\[
-\beta \frac{x(\beta(1) - \beta(2))}{2c}.
\]

Hence, the right side of the equation

\[
\varphi = q(\varphi) + \frac{(1 - q(\varphi))\beta(2)}{2c} + \int_{E[\varphi]=1}^{\varphi=\beta/2c} g(e)h(e)de
\]

is independent of \( \varphi \), implying that the equation yields a unique solution \( \varphi \), which satisfies \( \beta(2)x/2c < \varphi < \beta(1)x/2c \).
References


Figures

All players establish their prior beliefs

The analyst observes a private noisy signal about the earnings realization and issues the earnings forecast

The manager privately observes the earnings realization and issues the earnings report

The investors set the firm’s market price

The earnings realization becomes publicly known and the manager bears the biasing cost

Figure 1. The figure provides a timeline depicting the sequence of events in the model.
Figure 2. The figure illustrates the equilibrium earnings report in extension B when $k_1 > k_2 \geq 0$. The horizontal axis describes all the possible realizations, $e$, of the earnings measure $\tilde{e}$. The solid line is the earnings report $R_{\tilde{e}}(f_1, e)$ as a function of the true earnings realization $e$ and for a fixed content $f_1$ of the analyst’s earnings forecast, while the dotted line is the earnings report $R_{\tilde{e}}(f_2, e)$ as a function of the true earnings realization $e$ for another fixed content $f_2$ of the analyst’s earnings forecast, such that $f_1 < f_2$. 
Figure 3. The figure illustrates the equilibrium pricing rule of extension B when $k_1 > k_2 \geq 0$. The horizontal axis describes all the possible contents of the manager’s earnings report, $r$. The solid line is the market price $P_B(f_1, r)$ of the firm as a function of the content $r$ of the manager’s earnings report and for a fixed content $f_1$ of the analyst’s earnings forecast, while the dotted line is the market price $P_B(f_2, r)$ of the firm as a function of the content $r$ of the manager’s earnings report for another fixed content $f_2$ of the analyst’s earnings forecast, such that $f_1 < f_2$. 