Necessary Qualities of Accounting Information to Serve a Confirmatory Role

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We study the following two mathematically equivalent questions: 1) what are the necessary qualities of accounting information to provide a confirmatory role for voluntary disclosures, and 2) when is it worthwhile to induce communication of an agent's private post-decision information in an optimal contract? We find the communication of such private information to be strictly valuable only if the incremental informativeness of the private information about the agent's action is higher for favorable realizations of the public signal than for unfavorable realizations. We also find that, when meeting the revelation constraints required to use communication has adverse incentive effects, communication is strictly valuable if and only if the public information is incrementally more informative about the agent's private information than it is about the agent's chosen action.

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1. Introduction

One of the longest-lived and ongoing areas of investigation in accounting research is studying whether mandatory financial reporting is an important source of information to investors. Starting from Ball and Brown (1968), numerous studies have documented that accounting earnings are informative to the capital market in that stock prices, on average, contemporaneously adjust to the public release of accounting earnings. However, several other studies have noted that the size of these price reactions are small, as evidenced by the low explanatory power of the earnings-return regressions (e.g., Lev [1989] and Ball and Shivakumar [2008]). From this they conclude either that financial reporting is not fulfilling its objective (Lev [1989]) or that the economic role of financial reporting lies outside that of providing timely information to investors (Ball and Shivakumar [2008] and Ball, Jayaraman and Shivakumar [2009]).

Gigler and Hemmer (1998) propose an alternative role for financial reporting that is consistent with these empirical findings. In their model audited financial reports serve a disciplining or "confirmatory role" in providing credibility to management's more informative and more timely voluntary disclosures. They hypothesize that the low explanatory power in earnings-returns regressions are due to the fact that accounting information is usually backward looking while stock prices are forward looking and most if not all of the information contained in the financial reports is preempted by managements' voluntary disclosures. Nevertheless, the accounting information is of utmost importance, for without the ability to check and potentially punish management for making misleading statements, the voluntary disclosures they make to communicate to investors would lack credibility and therefore be ignored. Indeed it is only when mandatory reports are not fulfilling their role of lending credibility to voluntary disclosures that we would see strong contemporaneous price reactions to the less timely and less informative accounting information.

Consistent with this theory, Ball, Jayaraman and Shivakumar (2009) empirically document a complementary relationship between the amount by which investors react to management voluntary disclosures and the level of verification to which the mandatory financial reports are subjected (as measured by audit fees). From this they conclude that the usefulness of financial reporting depends on its contribution to the total information environment and therefore cannot be evaluated in isolation, as is commonly done in the aforementioned earnings-returns studies. While alternative views on the economic role of financial reporting, such as the confirmatory role, are being more widely accepted and studied in recent literature (e.g., Ball (2001), Ball and Shivakumar (2008) and Ball, Jayaraman and Shivakumar (2009)), the issue of "what is the actual economic role of financial reporting" is far from decided, so far from it that Ball (2008) identifies this very question as the biggest unanswered question in accounting.

In this paper we broaden insight into the confirmatory role of financial reporting by expanding on the analysis provided in Gigler and Hemmer (1998).[1] In their model, following Dye (1983), it is assumed that the information provided in mandatory disclosures is necessarily a less informative statistic about the value of a firm than is the private information available to managers. As a result, equilibrium voluntary disclosures are more informative than mandatory disclosures and therefore there is no market response to the mandatory disclosure. The only role for mandatory disclosure is to confirm that the voluntary disclosure is provided truthfully. However, in reality, low as the explanatory power of earnings is, the market still reacts to earnings announcements. It therefore appears that mandatory disclosures must serve (at least) a dual role; providing information incremental to the voluntary disclosures in addition to potentially confirming the information contained in voluntary disclosures. This suggests that the sufficient statistic ordering between public and private information assumed in Gigler and Hemmer (1998) is not entirely accurate. More importantly, since it is sufficient but not necessary condition for accounting information to serve as a confirmation for voluntary disclosure, their model does little to help us understand what confirmatory information would look like in comparison to a more direct primary source of information. The main objective of this paper is to derive necessary conditions for earnings to serve a confirmatory role, thereby providing directly observable qualities which can help distinguish whether the information provided in financial reports is better suited to being used as a primary or confirmatory source of information.

[1] A similar model is used in Gigler and Hemmer (2001) and (2002) and our analysis is equally relevant to those papers. We reference the 1998 paper alone in the body of the text simply to avoid clutter.

We find that for voluntary disclosures to be made credible by using accounting information in a confirmatory role, it must be that the incremental informativeness of the voluntary disclosure is higher when the accounting information is favorable than when it is unfavorable. Hence the requirement for accounting information to be useful in a confirmatory role stands in stark contrast to the condition for accounting information to be useful as primary source of information -- which is whenever it is incrementally informative for any possible realization.

Within the framework of our model, the question of when financial reporting can serve a confirmatory role is analytically equivalent to the question of when communication of agent private information is valuable from a contracting perspective. Dye (1983) provided a partial answer to the second question. He shows that a sufficient condition for communication to be strictly valuable in a principal-agent model with private post-decision information is that the private information is a sufficient statistic for the pair of public and private information with respect to the agent's choice of action. We provide a more complete answer, albeit in a simpler setting, by deriving a necessary condition for communication to be valuable in a binary version of the model of Dye (1983) (as used by Gigler and Hemmer (1998)). The methodology that allows us to characterize the necessary condition is adapted from Rajan and Reichelstein (2009). Our derivation centers on modelling the principal-agent problem as a constrained Lagrange optimization problem in which the first order conditions serve as necessary and sufficient conditions for an interior optimal solution.

Holmstrom (1979) provides a necessary and sufficient condition for public information to have value. He finds that as long as any public information has marginal information content about the agent's action it should be included in the contract -- no matter how noisy the additional information might be. The basic idea underlying this result is that if the additional information is very noisy, the contract need only reference the information by a small amount. And since even risk-averse agents are "risk neutral in the small", in the limit, the risk premium associated with using the noisy information is second order relative to the incentive benefits provided by the incremental informativeness of the information. The result depends crucially on the verifiability of the public signal.

When information is only privately known by the agent there is the additional concern that the agent might misrepresent the private information (and, possibly, shirk as well). Therefore, in addition to the potential risk premium associated with using this noisy information, there is a potential cost to the principal of inducing the agent to report the information truthfully. It turns out that when the private information is a sufficient statistic for the private and public information with respect to the agent's action choice, that this additional "truth-telling constraint" can be met without imposing additional risk on the agent. However, when this sufficiency requirement is not met this is not generally true, that is meeting the truth-telling constraint requires imposing additional risk on the agent. As a result, whether the private information is valuable for contracting or not comes down to a tradeoff between the additional information it contains about the agent's action and the amount of risk (or noise) inherent in the information.

We provide a mathematical notion which embodies this tradeoff, namely, the risk-adjusted information content. It characterizes how useful the information would be for the principal to infer the agent's action (or, in equilibrium, deter the agent from taking undesired action since the agent always carry out the desired action) adjusting for the risk that information imposes on the agent (i.e., how noisy is the information when the agent carries out the desired action). We show that contracting on this private information is valuable only if the risk-adjusted incremental information content of the favorable realization of the private signal is sufficiently high.

The remainder of the paper is organized as follows. In the next section we present the model and derive the necessary and sufficient conditions for communication to be efficiency-enhancing. Section 3 concludes. All of the major proofs are contained in the appendix.

2. The Model

The model is a typical principal-agent setting with agent private post-decision information (see, for example, Dye (1983) and Gigler and Hemmer (1998)). A principal (i.e., shareholders) hires an agent (i.e., a manager) to carry out a task which has value implications for the firm. Shareholders are not able to observe the manager's actions and are therefore unable to direct the manager's choice of action. Upon choosing his action the manager privately learns something about the value added by his action. He can choose whether or not to report this private information to the shareholders, i.e., disclosure of the private information to the market is "voluntary". Since the shareholders can always rationally ignore the manager's voluntary disclosure if they feel it is not credible, such voluntary disclosures will never reduce the efficiency of the relationship between the managers and shareholders. Whether they provide a strict efficiency improvement is not obvious and is the primary focus of this paper.

In addition to the manager's voluntary disclosures, shareholders can observe an objective but noisy measure of the value added by the manager's action, which we will call "accounting income". Measurement and disclosure of accounting income is mandatory and occurs after the manager has acted and privately learned something about the value of his action. Notice that if voluntary disclosures are either not made or are not credible, accounting income is the sole information available to shareholders and therefore the "primary" source of information to capital markets. If on the other hand the voluntary disclosure is viewed as credible, the capital market will price the firm based on both the accounting information and the voluntary disclosure. How much the price reflects one source of information visa vis the other depends on how value relevant the accounting information is compared to the manager's private information.

Gigler and Hemmer (1998) showed how the credibility of the voluntary disclosure rests on the ability of the accounting income to provide corroborating confirming information to what is reported in the voluntary disclosure. They assumed that the value relevance of the manager's private information subsumed that of the accounting income and, as a result, the voluntary disclosure was always efficiency enhancing. Here we relax the assumption on the comparative value relevance of the two pieces of information and study when the accounting information can and cannot support credible voluntary disclosure.

We derive necessary and sufficient conditions for voluntary disclosure to be strictly valuable when the action, the income, and the private information are all represented as binary variables. Hence, the manager chooses an action $a \in \{a_l, a_h\}$ that is unobservable to the principal. To make the problem of providing the manager with incentives interesting we assume that the manager and shareholders have preferences for different actions. This is done by assuming that action a_h adds more (expected) value to the firm than action a_l, yet the manager has a personal preference for action a_l. Specifically, we assume that the shareholders are risk-neutral with representative utility function $U_P = V(a) - s$, where $V(a)$ represents the action dependent value of the firm, with $V(a_h) > V(a_l)$ and s denotes the payment made to compensate the agent for his action. The agent is strictly risk-averse with the assumed utility function $U_A = U(s) - e(a)$, where $e(a_h) = e > 0$ and we normalize $e(a_l) = 0$.

Let $x \in \{x_L, x_H\}$ represent the measure of accounting income and use the notation $p_h = p(x_H \mid a_h) > p_l = p(x_H \mid a_l)$ to capture the fact that accounting income is a noisy measure of the value added by the manager's action. In the benchmark case, where only accounting information is available to the shareholders, we have the standard model of Grossman and Hart (1983). To facilitate solving the problem in utility space rather than payment space we denote $U_H = U_A(s(x_H))$ and $U_L = U_A(s(x_L))$. Therefore, the optimal payment scheme $s(x_H)$ is chosen such that

$$\{U_L, U_H\} \in \underset{\{\hat{U}_L, \hat{U}_H\}}{\operatorname{argmin}} \; p_h * U^{-1}(\hat{U}_H) + (1 - p_h) * U^{-1}(\hat{U}_L)$$

subject to $$p_h * \hat{U}_H + (1 - p_h) * \hat{U}_L \geq \underline{U} + e \qquad \text{IR}$$

$$p_h * \hat{U}_H + (1 - p_h) * \hat{U}_L - e \geq p_l * \hat{U}_H + (1 - p_l) * \hat{U}_L \qquad \text{IC}$$

where $\underline{U}$ denotes the reservation utility for the agent and U^{-1} is the inverse utility function of the agent. The two constraints are respectively for individual rationality and incentive compatibility. It is a standard result that the two constraints bind and that the resulting optimal contract, denoted $\{U_L^*, U_H^*\}$, is given by:

$$U_H^* = \underline{U} + e * \frac{1 - p_l}{p_h - p_l} \text{ and } U_L^* = \underline{U} - e * \frac{p_l}{p_h - p_l}.$$

Now we introduce the manager's private information, denoted by $y \in \{y_L, y_H\}$. Again to capture the idea that the manager's private information is also a noisy representation of the value added by his actions, let $q_h = p(y_H \mid a_h) > q_l = p(y_H \mid a_l)$. We utilize the revelation principle as proposed by Myerson (1979) and compensate the agent based on the accounting income x as well as his report of y, denoted $\hat{y}$. Since y (and therefore $\hat{y}$) is also binary, the optimal contract (in terms of manager utility) consists of four numbers, $U(s(x,\hat{y}))$, which we denote $U_{H\hat{H}}$, $U_{H\hat{L}}$, $U_{L\hat{H}}$ and $U_{L\hat{L}}$. The revelation principle states that optimal contract solves the following problem:

$$\{U_{HH}, U_{HL}, U_{LH}, U_{LL}\} \in$$

$$\underset{\{U_{H\hat{H}}, U_{H\hat{L}}, U_{L\hat{H}}, U_{L\hat{L}}\}}{\operatorname{argmin}} \; p(x_H, y_H \mid a_h) * U^{-1}(U_{H\hat{H}}) + p(x_H, y_L \mid a_h) * U^{-1}(U_{H\hat{L}})$$
$$+ p(x_L, y_H \mid a_h) * U^{-1}(U_{L\hat{H}}) + p(x_L, y_L \mid a_h) * U^{-1}(U_{L\hat{L}})$$

subject to

$$p(x_H, y_H \mid a_h) * U_{H\hat{H}} + p(x_H, y_L \mid a_h) * U_{H\hat{L}} + p(x_L, y_H \mid a_h) * U_{L\hat{H}} + p(x_L, y_L \mid a_h) * U_{L\hat{L}} - e$$
$$\geq \underline{U} \qquad \text{IR}$$

$$\geq p(x_H, y_H \mid a_l) * U_{H\hat{j}} + p(x_H, y_L \mid a_l) * U_{H\hat{k}} + p(x_L, y_H \mid a_l) * U_{L\hat{j}} + p(x_L, y_L \mid a_l) * U_{L\hat{k}}$$
$$\forall \hat{j}, \hat{k} = H, L \qquad \text{IC}_{\hat{j},\hat{k}}$$

$$p(x_H, y_H \mid a_h) * U_{H\hat{H}} + p(x_L, y_H \mid a_h) * U_{L\hat{H}} \geq p(x_H, y_H \mid a_h) * U_{H\hat{L}} + p(x_L, y_H \mid a_h) * U_{L\hat{L}}$$
$$TT_{y_H, a_h}$$

$$p(x_H, y_L \mid a_h) * U_{H\hat{L}} + p(x_L, y_L \mid a_h) * U_{L\hat{L}} \geq p(x_H, y_L \mid a_h) * U_{H\hat{H}} + p(x_L, y_L \mid a_h) * U_{L\hat{H}}.$$
$$TT_{y_L, a_h}$$

The constraints are the usual individual rationality constraint, the action incentive compatibility constraints for all possible reporting strategies and two revelation, or truth-telling, constraints -- which induce the manager to truthfully disclose his private information to shareholders. It is straightforward to show that the individual rationality and one of the action incentive compatibility constraints bind. For the two revelation constraints, rearranging terms gives

$$p(x_H, y_H \mid a_h) * [U_{H\hat{H}} - U_{H\hat{L}}] \geq p(x_L, y_H \mid a_h) * [U_{L\hat{L}} - U_{L\hat{H}}] \qquad TT_{y_H, a_h}$$

$$p(x_H, y_L \mid a_h) * [U_{H\hat{H}} - U_{H\hat{L}}] \leq p(x_L, y_L \mid a_h) * [U_{L\hat{L}} - U_{L\hat{H}}]. \qquad TT_{y_L, a_h}$$

Now if we denote $U_{H\hat{H}} - U_{H\hat{L}} \equiv \delta_H$ and $U_{L\hat{L}} - U_{L\hat{H}} \equiv \delta_L$, the revelation constraints can be rewritten as

$$\delta_H \geq \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)} \delta_L \text{ and} \qquad TT_{y_H, a_h}$$

$$\delta_H \leq \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} \delta_L. \qquad TT_{y_L, a_h}$$

Note that if the optimal values for δ_H and δ_L are both zero communication of agent private information has no value, since the optimal contract when $\hat{y}$ is used is the same as the optimal contract when it is not. In the language of voluntary disclosure, if the optimal values for δ_H and δ_L are both zero, then both parties would find it optimal to suppress voluntary disclosure -- accounting income would have only an informational role and no confirmatory role. If at least one of them is non-zero voluntary disclosure is strictly beneficial, with accounting income serving a confirmatory role.

It is straightforward to show that the individual rationality constraint binds. However, because we have four action incentive compatibility constraints (corresponding to always telling the truth, always lying, always claiming to have observed y_H and always claiming to have observed y_L) it is not immediately clear which one binds.

We now introduce the following assumptions:

Assumption 1: $E(x \mid y_H, a_i) \geq E(x \mid y_L, a_i) \quad i = h, l$ and

Assumption 2: $E(x \mid y_j, a_h) \geq E(x \mid y_j, a_l) \quad j = H, L$.

Assumptions 1 and 2 together can be made without loss of generality as they are essentially labeling conventions. Assumption 2 states that x_H is "good news" in that it is more likely that the manager has taken the desired level of effort. for given levels of effort. Assumption 1 is identical to the assumption in Gigler and Hemmer (2004) that matches between the public and private signals are more likely than mismatches regardless of the effort taken.

With these assumptions, we can narrow down the set of potentially binding action incentive compatibility constraints:

Lemma 1. *Given assumptions 1 and 2 as well as the two revelation constraints, the binding action incentive constraints are either*

$$p(x_H, y_H \mid a_h) * U_{H\hat{H}} + p(x_H, y_L \mid a_h) * U_{H\hat{L}} + p(x_L, y_H \mid a_h) * U_{L\hat{H}} + p(x_L, y_L \mid a_h) * U_{L\hat{L}} - e$$
$$\geq p(x_H, y_H \mid a_l) * U_{H\hat{H}} + p(x_H, y_L \mid a_l) * U_{H\hat{L}} + p(x_L, y_H \mid a_l) * U_{L\hat{H}} + p(x_L, y_L \mid a_l) * U_{L\hat{L}} \quad \mathrm{IC}_{H,L}$$

or

$$p(x_H, y_H \mid a_h) * U_{H\hat{H}} + p(x_H, y_L \mid a_h) * U_{H\hat{L}} + p(x_L, y_H \mid a_h) * U_{L\hat{H}} + p(x_L, y_L \mid a_h) * U_{L\hat{L}} - e$$
$$\geq p(x_H, y_H \mid a_l) * U_{H\hat{L}} + p(x_H, y_L \mid a_l) * U_{H\hat{L}} + p(x_L, y_H \mid a_l) * U_{L\hat{L}} + p(x_L, y_L \mid a_l) * U_{L\hat{L}} \quad \mathrm{IC}_{L,L}$$

Proof. See appendix.

The lemma is a result of the fact that, to induce truth-telling, agents are in effect rewarded for correctly "forecasting" the earnings by paying someone who reports y_L more when x_L is realized than when x_H is realized. This results in separation because agents who have seen y_L have lower posterior expectations on the earnings than those who have seen y_H. However, providing a reward for correctly forecasting low earnings may have adverse incentive effects -- because the agent might find it beneficial to *always* report y_L, shirk and save the disutility, and at the same time increase the likelihood of x_L. This is why $\mathrm{IC}_{L,L}$ might bind. On the other hand, when a reward for correctly forecasting earnings can be made without any adverse incentive effects, as is the case when y is a sufficient statistic for x and y with respect to a, we need only worry about the agent shirking when he tells the truth. This is the case where $\mathrm{IC}_{H,L}$ binds.

We can now solve for U_{HH}, U_{HL}, U_{LH} and U_{LL} as functions of δ_H and δ_L using the binding individual rationality constraint and separately for each of the binding $\mathrm{IC}_{H,L}$ or $\mathrm{IC}_{L,L}$ constraints. The results are presented in the following Lemma.

Lemma 2. *When* $\mathrm{IC}_{H,L}$ *binds, we have*

$$U_{HH} = U_H + [1 + \frac{p(x_H, y_H \mid a_l)(1-p_h) - p(x_H, y_H \mid a_h)(1-p_l)}{p_h - p_l}]\delta_H$$

$$+ \frac{p(x_L, y_L \mid a_l)(1-p_h) - p(x_L, y_L \mid a_h)(1-p_l)}{p_h - p_l}\delta_L$$

$$U_{HL} = U_H + \frac{p(x_H, y_H \mid a_l)(1-p_h) - p(x_H, y_H \mid a_h)(1-p_l)}{p_h - p_l}\delta_H$$

$$+ \frac{p(x_L, y_L \mid a_l)(1-p_h) - p(x_L, y_L \mid a_h)(1-p_l)}{p_h - p_l}\delta_L$$

$$U_{LH} = U_L + \frac{p(x_H, y_H \mid a_h)p_l - p(x_H, y_H \mid a_l)p_h}{p_h - p_l}\delta_H + \frac{p(x_L, y_L \mid a_h)p_l - p(x_L, y_L \mid a_l)p_h}{p_h - p_l}\delta_L$$

$$U_{LL} = U_L + \frac{p(x_H, y_H \mid a_h)p_l - p(x_H, y_H \mid a_l)p_h}{p_h - p_l}\delta_H + [1 + \frac{p(x_L, y_L \mid a_h)p_l - p(x_L, y_L \mid a_l)p_h}{p_h - p_l}]\delta_L.$$

When $\mathrm{IC}_{L,L}$ *binds, we have*

$$U_{HH} = U_H + [1 - \frac{p(x_H, y_H \mid a_h)(1-p_l)}{p_h - p_l}]\delta_H + \frac{p(x_L, y_H \mid a_h)(1-p_l)}{p_h - p_l}\delta_L$$

$$U_{HL} = U_H - \frac{p(x_H, y_H \mid a_h)(1-p_l)}{p_h - p_l}\delta_H + \frac{(1-p_l(1-p_h) - p(x_L, y_L \mid a_h)(1-p_l)}{p_h - p_l}\delta_L$$

$$U_{LH} = U_L + \frac{p(x_H, y_H \mid a_h)p_l}{p_h - p_l}\delta_H + \frac{p(x_L, y_L \mid a_h)p_l - p_h(1-p_l)}{p_h - p_l}\delta_L$$

$$U_{LL} = U_L + \frac{p(x_H, y_H \mid a_h)p_l}{p_h - p_l}\delta_H + [1 + \frac{p(x_L, y_L \mid a_h)p_l - p_h(1-p_l)}{p_h - p_l}]\delta_L.$$

We divide our analysis and the derivation of our main results into two cases to better illustrate the methodology we employ. The first case is the simplest and is a relatively straightforward application of a methodology used in Rajan and Reichelstein (2009). The second case is where we generalize their methodology and derive our main results.

2.1 When *x* and *y* are statistically independent conditional on *a*

When x and y are statistically independent conditional on a, $p(x,y\,|\,a)=p(x\,|\,a)*p(y\,|\,a)$ for all x,y,a, so we have $\frac{p(x_L,y_H\,|\,a_h)}{p(x_H,y_H\,|\,a_h)}=\frac{p(x_L\,|\,a_h)*p(y_H\,|\,a_h)}{p(x_H\,|\,a_h)*p(y_H\,|\,a_h)}=\frac{p(x_L\,|\,a_h)*p(y_L\,|\,a_h)}{p(x_H\,|\,a_h)*p(y_L\,|\,a_h)}=\frac{p(x_L,y_L\,|\,a_h)}{p(x_H,y_L\,|\,a_h)}$. (Note that this is the case where Assumption 1 holds only weakly.) Therefore the two revelation constraints reduce to one: $\delta_H=\frac{1-p_h}{p_h}\delta_L$. Also in this case, as is shown in the proof of the next lemma, $\mathrm{IC}_{L,L}$ binds. The intuition for $\mathrm{IC}_{L,L}$ binding comes from the fact hen x and y are statistically independent conditional on a, the earnings uncertainty facing the manager is independent of the private information y they observed. Therefore, any reward offered to separate managers who correctly forecast x_L. when they see and report y_L is equally as attractive to managers who have seen y_H. Consequently, meeting the truth-telling constraint for managers who observe y_L will necessarily have adverse incentive consequences.

Knowing that both the individual rationality and $\mathrm{IC}_{L,L}$ constraints are binding allows us to substitute the appropriate results of Lemma 2 into the objective function, giving an unconstrained programming problem of only one variable, δ_L. Evaluating the first order necessary and sufficient condition at $\delta_L=0$ gives the following Lemma.

Lemma 3. *In the binary case when the manager's private information y and the accounting income x are independent conditional on effort, communication of the manager's private information is strictly valuable.*

Proof. See Appendix.

Lemma 3 can be viewed as a sufficient condition for communication to be valuable, much like Dye's (1983) sufficient statistic condition. While interesting in its own right, as a sufficient condition, it does little in helping us determine what accounting information must look like in order to serve a confirmatory role. Next we generalize the statistical relationship between x and y to characterize necessary conditions for accounting to serve a confirmatory role.

2.2 When *x* and *y* are statistically dependent conditional on *a*

When x and y are not statistically independent conditional on a, it is not generally the case that $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} = \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}$ so we are unable to reduce the set of two revelation constraints to a single one, because the manager's expectations about the realization of x depends on the y he sees. Nevertheless, we are able to extend the method of Rajan and Reichelstein (2009) to this case in order to obtain our main result. We rewrite the optimization problem as

$$\{\delta_H, \delta_L\} \in \underset{\{\delta_{\hat{H}}, \delta_{\hat{L}}\}}{\operatorname{argmin}}\ p(x_H, y_H \mid a_h) * U^{-1}(U_{HH}) + p(x_H, y_L \mid a_h) * U^{-1}(U_{HL})$$
$$+ p(x_L, y_H \mid a_h) * U^{-1}(U_{LH}) + p(x_L, y_L \mid a_h) * U^{-1}(U_{LL})$$

subject to

$$\delta_{\hat{H}} \geq \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)} \delta_{\hat{L}} \qquad TT_{y_H, a_h}$$

$$\delta_{\hat{H}} \leq \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} \delta_{\hat{L}} \qquad TT_{y_L, a_h}$$

Note that in this problem U_{HH}, U_{HL}, U_{LH} and U_{LL} are given by Lemma 2 and, as can be seen in Lemma 2, these payments are functions of both δ_H and δ_L, although the exact format depends on which action incentive constraint binds. We write the Lagrangian for the optimization problem as

$$L = p(x_H, y_H \mid a_h) * U^{-1}(U_{HH}) + p(x_H, y_L \mid a_h) * U^{-1}(U_{HL}) + p(x_L, y_H \mid a_h) * U^{-1}(U_{LH})$$
$$+ p(x_L, y_L \mid a_h) * U^{-1}(U_{LL}) + \lambda_1 * [\frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)} \delta_L - \delta_H] + \lambda_2 * [\delta_H - \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} \delta_L]$$

In order to extend the method of Rajan and Reichelstein (2009) to the more general case we first derive the following lemma.[2]

Lemma 4. *For any given strictly concave utility function U, there exists a finite number K such that* $\delta_i \in [-K, K]$ *for* $i = H, L$ *and the objective in the above minimization problem is convex with respect to* $\delta_{\hat{H}}$ *and* $\delta_{\hat{L}}$.[3]

Proof. See Appendix.

Lemma 4 establishes this as a convex programming problem on a compact set and, therefore, that the first order conditions (with non-negative Lagrangian multiples) are necessary and sufficient for a global solution.

When $\mathrm{IC}_{H,L}$ binds, the first order conditions for δ_H and δ_L respectively are

$$\frac{p(x_H, y_H \mid a_h)}{U'_{HH}} * [1 + \frac{p(x_H, y_H \mid a_l)(1-p_h) - p(x_H, y_H \mid a_h)(1-p_l)}{p_h - p_l}]$$
$$+ \frac{p(x_H, y_L \mid a_h)}{U'_{HL}} * \frac{p(x_H, y_H \mid a_l)(1-p_h) - p(x_H, y_H \mid a_h)(1-p_l)}{p_h - p_l}$$
$$+ \frac{p(x_L, y_H \mid a_h)}{U'_{LH}} * \frac{p(x_H, y_H \mid a_h) p_l - p(x_H, y_H \mid a_l) p_h}{p_h - p_l}$$
$$+ \frac{p(x_L, y_L \mid a_h)}{U'_{LL}} * \frac{p(x_H, y_H \mid a_h) p_l - p(x_H, y_H \mid a_l) p_h}{p_h - p_l} + \lambda_2 - \lambda_1 = 0 \qquad (1)$$

$$\frac{p(x_H, y_H \mid a_h)}{U'_{HH}} * \frac{p(x_L, y_L \mid a_l)(1-p_h) - p(x_L, y_L \mid a_h)(1-p_l)}{p_h - p_l}$$
$$+ \frac{p(x_H, y_L \mid a_h)}{U'_{HL}} * \frac{p(x_L, y_L \mid a_l)(1-p_h) - p(x_L, y_L \mid a_h)(1-p_l)}{p_h - p_l}$$
$$+ \frac{p(x_L, y_H \mid a_h)}{U'_{LH}} * \frac{p(x_L, y_L \mid a_h) p_l - p(x_L, y_L \mid a_l) p_h}{p_h - p_l}$$

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[2] Note that this lemma is proved for the optimization problem where $\mathrm{IC}_{H,L}$ binds, the case where $\mathrm{IC}_{L,L}$ binds can be proved similarly.

[3] We have shown that for any given strictly concave utility function and any given probability structure, δ_H and δ_L are bounded. If any of the likelihood ratios of the joint probability distribution of *x* and *y* conditional on *a* goes to zero or infinity in a way that still satisfies assumptions 1 and 2 then this Lemma would not hold. It is clear that in those cases *y* should be included in the contract. But then it can be verified that the inequalities in Propositions 7 and 8 hold. Therefore, those propositions are still the general characterizations of the necessary conditions for communication to be valuable.

$$+\frac{p(x_L, y_L \mid a_h)}{U'_{LL}} * [1 + \frac{p(x_L, y_L \mid a_h) p_l - p(x_L, y_L \mid a_l) p_h}{p_h - p_l}]$$

$$+\lambda_1 \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)} - \lambda_2 \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} = 0 \qquad (2)$$

Whereas when $\mathrm{IC}_{L,L}$ binds, the first order conditions for δ_H and δ_L respectively are

$$\frac{p(x_H, y_H \mid a_h)}{U'_{HH}} * [1 - \frac{p(x_H, y_H \mid a_h)(1 - p_l)}{p_h - p_l}]$$

$$-\frac{p(x_H, y_L \mid a_h)}{U'_{HL}} * \frac{p(x_H, y_H \mid a_h)(1 - p_l)}{p_h - p_l}$$

$$+\frac{p(x_L, y_H \mid a_h)}{U'_{LH}} * \frac{p(x_H, y_H \mid a_h) p_l}{p_h - p_l}$$

$$+\frac{p(x_L, y_L \mid a_h)}{U'_{LL}} * \frac{p(x_H, y_H \mid a_h) p_l}{p_h - p_l} + \lambda_2 - \lambda_1 = 0 \qquad (3)$$

$$\frac{p(x_H, y_H \mid a_h)}{U'_{HH}} * \frac{p(x_L, y_H \mid a_h)(1 - p_l)}{p_h - p_l}$$

$$+\frac{p(x_H, y_L \mid a_h)}{U'_{HL}} * \frac{p(x_L, y_H \mid a_h)(1 - p_l)}{p_h - p_l}$$

$$+\frac{p(x_L, y_H \mid a_h)}{U'_{LH}} * \frac{p(x_L, y_L \mid a_h) p_l - p_h(1 - p_l)}{p_h - p_l}$$

$$+\frac{p(x_L, y_L \mid a_h)}{U'_{LL}} * \frac{p(x_L, y_L \mid a_h) p_l - p_h(1 - p_l)}{p_h - p_l}$$

$$+\lambda_1 \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)} - \lambda_2 \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} = 0 \qquad (4)$$

Proposition 5. *A necessary and sufficient condition for communication of the agent's private information to have no value when* $\mathrm{IC}_{H,L}$ *binds (when* $\mathrm{IC}_{L,L}$*) is that equations (1) and (2) ((3) and (4)), when evaluated at* $\delta_H = \delta_L = 0$, *would result in* $\lambda_1 \geq 0$ *and* $\lambda_2 \geq 0$.

Proof. (Sufficiency) Suppose we have the first order condition satisfied at $\delta_H = \delta_L = 0$ with non-negative Lagrange multipliers λ_1 and λ_2. Because of Lemma 4 we know that the first order conditions are sufficient for an interior optimal solution therefore the optimal solution for the program is that $\delta_H = \delta_L = 0$ and thus the optimal contract for the problem is that the optimal contract is independent of the private information y and thus communication is of no value.

Proof. (Necessity) Suppose communication is of no value, then it must be that the optimal contract is independent of the private information y, which results in $\delta_H = \delta_L = 0$ being the optimal solution for the program. But from Lemma 4 we know that for $\delta_H = \delta_L = 0$ to be the optimal interior solution, the first order conditions have to be satisfied with the Lagrange multipliers being non-negative.

The proof is based entirely on the fact that the first order conditions are necessary and sufficient conditions for obtaining an optimal interior solution and that the Lagrangian multipliers must be non-negative. (A negative Lagrangian multiplier would result in violating the constraint.)

We now evaluate the first order conditions $\delta_H = \delta_L = 0$ for $\mathrm{IC}_{H,L}$ binding and $\mathrm{IC}_{L,L}$ binding, respectively. Since in this case, $U_{HH} = U_{HL} = U_H$ and $U_{LL} = U_{LH} = U_L$, equations (1) and (2) constitute two linear equations with two unknowns. And when x and y are not statistically independent conditional on a, $\frac{p(y_H \mid x_H, a_h)}{p(y_L \mid x_H, a_h)} \neq \frac{p(y_H \mid x_L, a_h)}{p(y_L \mid x_L, a_h)}$, which results in $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} \neq \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}$. Therefore, equations (1) and (2) are linearly independent, implying a unique solution for λ_1 and λ_2. The solution is given in the following lemma with the proof being omitted.

Lemma 6. *Solving the first order condition and evaluating at* $\delta_H = \delta_L = 0$ *when* $\mathrm{IC}_{H,L}$ *binds gives*

$$\lambda_1 * [\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} - \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}] =$$

$$-[\frac{1}{U'_H}-\frac{1}{U'_L}]\{p(x_L,y_L\,|\,a_h)+\frac{1-p_h}{p_h-p_l}[p(x_L,y_L\,|\,a_h)p_l-p(x_L,y_L\,|\,a_l)p_h]$$

$$+\frac{1-p_h}{p_h-p_l}\frac{p(x_L,y_L\,|\,a_h)}{p(x_H,y_L\,|\,a_h)}[p(x_H,y_H\,|\,a_h)p_l-p(x_H,y_H\,|\,a_l)p_h]\} \qquad (5)$$

and

$$\lambda_2*[\frac{p(x_L,y_L\,|\,a_h)}{p(x_H,y_L\,|\,a_h)}-\frac{p(x_L,y_H\,|\,a_h)}{p(x_H,y_H\,|\,a_h)}]=$$

$$-[\frac{1}{U'_H}-\frac{1}{U'_L}]\{p(x_L,y_H\,|\,a_h)+\frac{p_h}{p_h-p_l}[p(x_L,y_L\,|\,a_l)(1-p_l)-p(x_L,y_L\,|\,a_h)(1-p_l)]$$

$$+\frac{p_h}{p_h-p_l}\frac{p(x_L,y_H\,|\,a_h)}{p(x_H,y_H\,|\,a_h)}[p(x_H,y_H\,|\,a_l)(1-p_h)-p(x_H,y_H\,|\,a_h)(1-p_l)]\} \qquad (6)$$

Solving the first order condition and evaluating at $\delta_H=\delta_L=0$ *when* $\mathrm{IC}_{L,L}$ *binds gives*

$$\lambda_1*[\frac{p(x_L,y_L\,|\,a_h)}{p(x_H,y_L\,|\,a_h)}-\frac{p(x_L,y_H\,|\,a_h)}{p(x_H,y_H\,|\,a_h)}]=$$

$$[\frac{1}{U'_H}-\frac{1}{U'_L}]\{-\frac{p_l(1-p_h)}{p_h-p_l}p(x_H,y_H\,|\,a_h)\frac{p(x_L,y_L\,|\,a_h)}{p(x_H,y_L\,|\,a_h)}$$

$$+\frac{p_h(1-p_l)}{p_h-p_l}p(x_L,y_H\,|\,a_h)\} \qquad (7)$$

and

$$\lambda_2*[\frac{p(x_L,y_L\,|\,a_h)}{p(x_H,y_L\,|\,a_h)}-\frac{p(x_L,y_H\,|\,a_h)}{p(x_H,y_H\,|\,a_h)}]=[\frac{1}{U'_H}-\frac{1}{U'_L}]p(x_L,y_H\,|\,a_h)\,. \qquad (8)$$

Given Assumption 1, the coefficients on λ_1 and λ_2 on the left hand sides of equations (5) to (8) are all positive. We can now formally state the necessary and sufficient conditions for communication to be strictly valuable when $\mathrm{IC}_{H,L}$ and $\mathrm{IC}_{L,L}$ respectively binds.

Proposition 7. *When* $\mathrm{IC}_{H,L}$ *binds communication of the agent's private information y is strictly valuable if and only if either*

$$\frac{p_h-p_l}{p_h}-\frac{p(x_H,y_L\,|\,a_h)-p(x_H,y_L\,|\,a_l)}{p(x_H,y_L\,|\,a_h)}>$$

$$\frac{(1-p_h)-(1-p_l)}{(1-p_h)} - \frac{p(x_L, y_L \mid a_h) - p(x_L, y_L \mid a_l)}{p(x_L, y_L \mid a_h)} \tag{9}$$

or

$$\frac{p(x_H, y_H \mid a_h) - p(x_H, y_H \mid a_l)}{p(x_H, y_H \mid a_h)} - \frac{p_h - p_l}{p_h} >$$

$$\frac{p(x_L, y_H \mid a_h) - p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} - \frac{(1-p_h)-(1-p_l)}{(1-p_h)}. \tag{10}$$

Proof. See Appendix.

Proposition 8. *When* $\mathrm{IC}_{L,L}$ *binds communication of the agent's private information y is strictly valuable if and only if*

$$\frac{p_h}{(1-p_h)} * \frac{(1-p_l)}{p_l} < \frac{p(x_H \mid y_H, a_h)}{p(x_H \mid y_L, a_h)} * \frac{p(x_L \mid y_L, a_h)}{p(x_L \mid y_H, a_h)} \tag{11}$$

Proof. See Appendix.

The intuition underlying Proposition 7 is fairly straightforward. Recall that in order to meet the two truth-telling constraints (when they are linearly independent), both δ_H and δ_L are necessarily greater than or equal to zero. This means that the wages must be set such that $U(x_H, \hat{y}_H) \geq U(x_H, \hat{y}_L)$ and $U(x_L, \hat{y}_L) \geq U(x_L, \hat{y}_H)$. In the first case, when x_H is realized, a manager who sees and reports y_H is paid more than one who reports y_L. The use of the report of y_H therefore requires paying the agent more for generating a favorable private signal – moving the contract payments in the direction of providing better incentives. However, when x_L is realized, the truth-telling constraints require paying a manager who sees and reports y_L more than one who reports y_H. Here using the report requires moving the contract payments *against* the incentive benefit of using y. Truth-telling in effect requires giving up any information benefit that can be had by using y when x turns out to be low. Proposition 8 states that any improvement in the informativeness of signals gained by adding y must arise when x is high, because any incentive benefit gained when x is low must be given up.

Contrasting Proposition 8 to Holmstrom's (1979) result is useful. The terms in expressions (9) and (10) are the discrete analogs to Holmstrom's informativeness expression, $\frac{f_a}{f}$. He shows that when y is public, including it in the contract is strictly valuable if and only if $\frac{f_a(x,y\,|\,a)}{f(x,y\,|\,a)} \neq \frac{f_a(x\,|\,a)}{f(x\,|\,a)}$ for *any* (x,y) combination. Proposition 8 states that requiring truthful revelation of y places an additional intuitive restriction on the characteristics of information to be useful for contracting.

Now let's look at Proposition 8. When $\mathrm{IC}_{L,L}$ binds inducing the agent to tell the truth requires more reward for reporting y_L. This in turn generates an adverse incentive effect from using the reports – putting more pressure on the public signal x to provide incentives. The public signal now must play the dual role of providing incentives both for the desired action and for truth-telling. Proposition 8 states that, if the public signal is not better for inferring whether the manager told the truth than for inferring whether he chose the desired action, it is not worthwhile using it to provide the additional action incentives needed overcome the adverse incentives created by the truth-telling constraints. Note that the left-hand side of expression (11) is the product of the "action" likelihood ratios for the two realizations of the public signals. This measures the public signals power in inferring which action generated the signal. The right-hand side is the product of the "private information" likelihood ratios. It is an analogous measure for the power of x in inferring y (given the desired action a_h is chosen).

Next we introduce one more assumption on the joint distribution of signals in order to characterize when each action incentive constraint binds in terms of exogenous variables and in orde4r to unify the two cases of Propositions 7 and 8 into one set of necessary and sufficient conditions.

Assumption 3: $E(y\,|\,x_k,a_h) \geq E(y\,|\,x_k,a_l) \quad k = H,L$.

Assumption 3 states that high values of y are good news, regardless of the realization of x. In particular it rules out cases were low values of y might be interpreted as good from an action incentive perspective when they are coupled with low values of x.

Specifically, it ensures that $\frac{p(x_L, y_L \mid a_l)}{p(x_L, y_L \mid a_h)} \geq \frac{p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)}$.

Lemma 9. *When x and y are not statistically independent conditional on a, Assumptions 1,2 and 3 imply that*

$$\mathrm{IC}_{H,L} \textit{ binds if and only if } p(x_H \mid y_H, a_l) \geq p(x_H \mid y_L, a_h) \tag{12}$$

and

$$\mathrm{IC}_{L,L} \textit{ binds if and only if } p(x_H \mid y_H, a_l) \leq p(x_H \mid y_L, a_h). \tag{13}$$

Proof. See Appendix.

The inequality in (12) can be interpreted as y being more important to the manager than a in forecasting x, in which case his reporting strategy would not depend on the action he chose. Therefore, if truth-telling is met in equilibrium (i.e., for a_h) it will also be satisfied off-equilibrium (i.e., for a_l). This is why $\mathrm{IC}_{H,L}$ binds. On the other hand, if a is more important to the manager than y in forecasting x (as reflected by the inequality in (13)), the reporting strategy of the manager will depend on the action he chose.

Lemma 9 would allow us to restate Propositions 7 and 8 in terms of exogenous variables, however, we can actually go one step further (because of Assumption 3) and combine these Propositions into one Proposition which characterizes necessary and sufficient conditions for communication to have strict value for all of our cases.

Proposition 10. *When x and y are not statistically independent conditional on a, Assumptions 1,2 and 3 imply that communication of the agent's private information y is strictly valuable if and only if*

$$\frac{p_h}{(1-p_h)} * \frac{(1-p_l)}{p_l} < \frac{p(x_H \mid y_H, a_h)}{p(x_H \mid y_L, a_h)} * \frac{p(x_L \mid y_L, a_h)}{p(x_L \mid y_H, a_h)} \tag{11}$$

and either

$$\frac{p_h - p_l}{p_h} - \frac{p(x_H, y_L \mid a_h) - p(x_H, y_L \mid a_l)}{p(x_H, y_L \mid a_h)} > \frac{(1-p_h)-(1-p_l)}{(1-p_h)} - \frac{p(x_L, y_L \mid a_h) - p(x_L, y_L \mid a_l)}{p(x_L, y_L \mid a_h)} \tag{9}$$

or

$$\frac{p(x_H, y_H \mid a_h) - p(x_H, y_H \mid a_l)}{p(x_H, y_H \mid a_h)} - \frac{p_h - p_l}{p_h} >$$

$$\frac{p(x_L, y_H \mid a_h) - p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} - \frac{(1-p_h)-(1-p_l)}{(1-p_h)}. \qquad (10)$$

Proof. See Appendix.

Proposition 10 illustrates that the two cases we have analyzed are not qualitatively different. Rather in both cases the public information must be *both* better at inferring the private information than at inferring action *and* the additional informativeness of the private signal about action must arise when the public signal is good news. Which incentive constraint binds merely reflects which of these two requirements is “binding”. In the case where action is more important for the manager in forecasting earnings the public information must be more useful in inferring the private information (which ensures that the incremental informativeness of the private information occurs when earnings are high). When the manager’s private information is more useful to him in forecasting action, if the private information is more informative when earnings are high, then the earnings are necessarily more powerful in inferring the private information than in inferring the manager’s action.

3. Conclusion

We derive a necessary and sufficient condition for communication of private information to be strictly valuable and interpret the condition as necessary and sufficient for mandatory disclosure to serve a confirmatory role for voluntary disclosures. Earlier work in this area has assumed that manager's private information is strictly more informative than the information contained in mandatory disclosures about the value enhancing efforts of management. This results in an equilibrium where managers always reveal every piece of their private information and consequently markets rationally ignore mandatory disclosures in setting prices. However, neither of these features of the equilibrium are particularly realistic. By generalizing the possible statistical relationship between private and public information, our model characterizes the conditions under which management would not disclose their private information and under which the capital market does use mandatory disclosures to value firms even when management does make voluntary disclosures. More importantly, by characterizing necessary conditions for information to play a confirmatory role, we can determine whether or not a particular information system is useful in providing confirmatory information.

Our results showed that whether or not communication of the private information is optimal depends on 1) the relative strength of the incremental informativeness of the private information over the support of the public information and 2) whether the correlation between private and public information is strong enough.

Relative to Dye (1983) and Gigler and Hemmer (1998), which requires the private information to be a sufficient statistic of the information pair, our results provide a more relaxed and in our opinion, more realistic justification for 1) the prevalence of the reliance of private reports in real world contracts and 2) the empirical fact that earnings release have information content but the information content is very small.

Appendix

Proof of Lemma 1: The only difference in the action incentive compatibility constraints is in their right-hand sides:

$$p(x_H, y_H \mid a_l)*U_{H\hat{H}} + p(x_H, y_L \mid a_l)*U_{H\hat{L}} + p(x_L, y_H \mid a_l)*U_{L\hat{H}} + p(x_L, y_L \mid a_l)*U_{L\hat{L}} \qquad \text{IC}_{H,L}$$

$$p(x_H, y_H \mid a_l)*U_{H\hat{L}} + p(x_H, y_L \mid a_l)*U_{H\hat{L}} + p(x_L, y_H \mid a_l)*U_{L\hat{L}} + p(x_L, y_L \mid a_l)*U_{L\hat{L}} \qquad \text{IC}_{L,L}$$

$$p(x_H, y_H \mid a_l)*U_{H\hat{H}} + p(x_H, y_L \mid a_l)*U_{H\hat{H}} + p(x_L, y_H \mid a_l)*U_{L\hat{H}} + p(x_L, y_L \mid a_l)*U_{L\hat{H}} \qquad \text{IC}_{H,H}$$

$$p(x_H, y_H \mid a_l)*U_{H\hat{L}} + p(x_H, y_L \mid a_l)*U_{H\hat{H}} + p(x_L, y_H \mid a_l)*U_{L\hat{L}} + p(x_L, y_L \mid a_l)*U_{L\hat{H}} \qquad \text{IC}_{L,H}$$

To prove Lemma 1 we show that the right-hand sides of $\text{IC}_{H,L}$ and $\text{IC}_{L,L}$ are at least as big as the right-hand sides of $\text{IC}_{H,H}$ and $\text{IC}_{L,H}$. From the truth-telling constraints we have $\delta_H \geq \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}\delta_L$ and $\delta_H \leq \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}\delta_L$, implying $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}\delta_L \geq \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}\delta_L$. Assumption 1 therefore implies $\delta_L \geq 0$ (except possibly when $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} = \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}$), which from the truth-telling constraints in turn implies $\delta_H \geq 0$. In this case the right-hand side of $\text{IC}_{L,L}$ is greater than the right-hand side of $\text{IC}_{L,H}$ and the right-hand side of $\text{IC}_{H,L}$ is greater than the right-hand side of $\text{IC}_{H,H}$ if and only if $p(x_H, y_L \mid a_l)\delta_H \leq p(x_L, y_L \mid a_l)\delta_L$, which is implied by $\delta_H \leq \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}\delta_L$ and the fact that $\frac{p(x_L, y_L \mid a_l)}{p(x_H, y_L \mid a_l)} \geq \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}$ from Assumption 2.

The case where $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} = \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}$ is covered in the proof of Lemma 3, where it is shown that $\text{IC}_{H,L}$ binds. Q.E.D.

Proof of Lemma 3: When *x* and *y* are independent conditional on *a*, $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} = \frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}$ and so the two truth-telling constraints reduce to one, $p(x_H, y_H \mid a_h) * \delta_H = p(x_L, y_H \mid a_h) * \delta_L$ (which we can further reduce through factorization to $p_h * \delta_H = (1 - p_h) * \delta_L$ when *x* and *y* are conditionally independent. If we assume that δ_H and δ_L are greater than or equal to zero, this in conjunction with Assumption 2 implies that $\mathrm{IC}_{L,L}$ binds. This is because the right-hand side of $\mathrm{IC}_{L,L}$ is greater than the right-hand side of $\mathrm{IC}_{H,L}$ if and only if $p(x_H, y_H \mid a_l) * \delta_H \leq p(x_L, y_H \mid a_l) * \delta_L$. While Assumption 2 implies that $p(x_H, y_H \mid a_h) \geq p(x_H, y_H \mid a_l)$ and that $p(x_L, y_H \mid a_l)$ $\geq p(x_L, y_H \mid a_h)$, so that $\mathrm{IC}_{L,L}$ binds if δ_H and δ_L are greater than or equal to zero.

The contract payments in the expected cost minimization problem, $U_{H\hat{H}}$, $U_{H\hat{L}}$, $U_{L\hat{H}}$ and $U_{L\hat{L}}$, are as characterized in Lemma 2 for the case where $\mathrm{IC}_{L,L}$ binds with δ_H replaced by $\frac{(1 - p_h)}{p_h}\delta_L$. Hence, the cost minimization problem is reduced to choosing a value for the single variable, δ_L. The first-order condition with respect to δ_L evaluated at $\delta_L = 0$ is $(1 - p_h)q_h * [\frac{1}{U'(U^{-1}(U_H))} - \frac{1}{U'(U^{-1}(U_L))}] > 0$. Therefore, the optimal δ_L would be less than zero, contradicting the assumption that δ_H and δ_L are greater than or equal to zero and the result that $\mathrm{IC}_{L,L}$.

If we start by assuming that δ_H and δ_L are less than or equal to zero, the right-hand side of $\mathrm{IC}_{H,L}$ is greater than the right-hand side of $\mathrm{IC}_{L,L}$ and so $\mathrm{IC}_{H,L}$. Now the first-order condition with respect to δ_L evaluated at $\delta_L = 0$ is $p_h(q_h - q_l) * [\frac{1}{U'(U^{-1}(U_H))} - \frac{1}{U'(U^{-1}(U_L))}] > 0$. Therefore, the optimal δ_L would be strictly less than zero and so communication is strictly valuable. Q.E.D.

Proof of Lemma 4: We prove the case when $\mathrm{IC}_{H,L}$ binds as the proof would be essentially the same for the programming problem when $\mathrm{IC}_{L,L}$ binds. First we prove that the optimal solution always has δ_H and δ_L bounded by some constant K (which possibly depends on the utility function of the agent), meaning that we can limit our optimization problem to a compact set. Suppose to the contrary that either δ_H or δ_L is unbounded. Then because of the two constraints, the only possibility for either δ_H or δ_L to be unbounded is for *both* to go to $+\infty$ or $-\infty$. Without loss of generality, consider the case where both go to $+\infty$. Since $U_{HH} = U_{HL} + \delta_H$ and $U_{LL} = U_{LH} + \delta_L$, either U_{HH} or U_{LL} would approach $+\infty$, which would obviously not minimize expected cost, unless either U_{HL} or U_{LH} approaches $-\infty$ at the same speed or faster than δ_H or δ_L approaches $+\infty$. Then either U_{HH} or U_{LL} or both would be a finite value, but in this case the individual rationality constraint would be violated as the expected utility would approach $-\infty$. If either U_{HL} or U_{LH} approaches $-\infty$ at a faster speed than δ_H or δ_L approaches $+\infty$, then we would have similar violation of the individual rationality constraint. If either U_{HL} or U_{LH} approaches $-\infty$ at a slower speed than δ_H or δ_L approaches $+\infty$ then either U_{HH} or U_{LL} approaches $+\infty$. However because of the convexity of the inverse utility function, this would result in a larger expected payment than the original non-communication contract with $U_{HH} = U_{HL} = U_H$ and $U_{LH} = U_{LL} = U_L$ (by applying Jensen's inequality). Since the objective function as well as the constraints are continuous with respect to δ_H and δ_L, we can bound the range of δ_H and δ_L and thus obtain the bound K. Detailed expressions of K with respect to the utility function are available from the authors upon request.

Next we prove that the objective function is convex with respect to the variables δ_H and δ_L. Denote the objective function as $F(\delta_H, \delta_L)$. It is sufficient to show that $\frac{\partial^2 F}{\partial \delta_H^2} \geq 0$, $\frac{\partial^2 F}{\partial \delta_L^2} \geq 0$ and $\frac{\partial^2 F}{\partial \delta_L^2}\frac{\partial^2 F}{\partial \delta_L^2} - (\frac{\partial^2 F}{\partial \delta_H \partial \delta_L}) \geq 0$. To prove the first two inequalities, just note that the second order derivatives are each a sum of four terms, where each term is a square (the coefficient of each U with respect to δ_H or δ_L) times $-\frac{U''}{U'^2}$ and thus non-negative. To prove the last inequality, again notice that it consists of four terms, with each term being some positive coefficient times $(a^2b^2 - (ab)^2)$ for some a and b, and therefore is equal to zero. Thus the objective function is convex with respect to the variables. The details of the algebraic expressions are omitted here for simplicity and are available from the authors upon request. Q.E.D.

Proof of Proposition 7: When $\mathrm{IC}_{H,L}$ binds equation (5) of Lemma 6 gives that λ_1 evaluated at $\delta_H = \delta_L = 0$ is less than zero if and only if

$$p(x_L, y_L \mid a_h) + \frac{(1-p_h)}{p_h - p_l}[p(x_L, y_L \mid a_h)p_l - p(x_L, y_L \mid a_l)p_h]$$

$$+\frac{(1-p_h)}{p_h - p_l}\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}[p(x_H, y_H \mid a_h)p_l - p(x_H, y_H \mid a_l)p_h] > 0.$$

$$\Leftrightarrow \frac{p(y_L \mid x_L, a_h) - p(y_L \mid x_L, a_l)}{p(y_L \mid x_L, a_h)}\frac{(1-p_l)}{(1-p_h)} > \frac{p(y_L \mid x_H, a_h) - p(y_L \mid x_H, a_l)}{p(y_L \mid x_H, a_h)}\frac{p_l}{p_h}.$$

$$\Leftrightarrow \frac{p_h - p_l}{p_h} - \frac{p(x_H, y_L \mid a_h) - p(x_H, y_L \mid a_l)}{p(x_H, y_L \mid a_h)} >$$

$$\frac{(1-p_h)-(1-p_l)}{(1-p_h)} - \frac{p(x_L, y_L \mid a_h) - p(x_L, y_L \mid a_l)}{p(x_L, y_L \mid a_h)}.$$

Similarly, equation (6) of Lemma 6 gives that λ_2 evaluated at $\delta_H = \delta_L = 0$ is less than zero if and only if

$$p(x_L, y_L \mid a_h) + \frac{p_h}{p_h - p_l}[p(x_L, y_L \mid a_l)(1-p_h) - p(x_L, y_L \mid a_h)(1-p_l)]$$

$$+\frac{p_h}{p_h - p_l}\frac{p(x_L, y_H \mid a_h)}{p(x_H, y_H \mid a_h)}[p(x_H, y_H \mid a_l)(1-p_h) - p(x_H, y_H \mid a_h)(1-p_l)] < 0.$$

$$\Leftrightarrow (p_h - p_l) + p_h(1-p_l)\frac{p(y_H \mid x_L, a_h) - p(y_H \mid x_L, a_l)}{p(y_H \mid x_L, a_h)}$$

$$< \frac{p(y_H \mid x_H, a_h)p_h(1-p_l) - p(y_H \mid x_H, a_l)p_l(1-p_h)}{p(y_H \mid x_H, a_h)}.$$

$$\Leftrightarrow \frac{p(x_H, y_H \mid a_h) - p(x_H, y_H \mid a_l)}{p(x_H, y_H \mid a_h)} - \frac{p_h - p_l}{p_h} >$$

$$\frac{p(x_L, y_H \mid a_h) - p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} - \frac{(1-p_h)-(1-p_l)}{(1-p_h)}. \qquad \text{Q.E.D.}$$

Proof of Proposition 8: When $\mathrm{IC}_{L,L}$ binds equation (7) of Lemma 6 gives that λ_1 evaluated at $\delta_H = \delta_L = 0$ is less than zero if and only if

$$\frac{p_h(1-p_l)}{p_h - p_l} p(x_L, y_H \mid a_h) - \frac{p_l(1-p_h)}{p_h - p_l} p(x_H, y_H \mid a_h) \frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} < 0$$

$$\Leftrightarrow \frac{p_h}{(1-p_h)} * \frac{(1-p_l)}{p_l} < \frac{p(x_H \mid y_H, a_h)}{p(x_H \mid y_L, a_h)} * \frac{p(x_L \mid y_L, a_h)}{p(x_L \mid y_H, a_h)}.$$

Equation (8) of Lemma 6 gives that λ_2 evaluated at $\delta_H = \delta_L = 0$ is less than zero if and only if $p(x_L, y_H \mid a_h) < 0$, which is never satisfied. Hence, communication is strictly valuable if and only if $\Leftrightarrow \frac{p_h}{(1-p_h)} * \frac{(1-p_l)}{p_l} < \frac{p(x_H \mid y_H, a_h)}{p(x_H \mid y_L, a_h)} * \frac{p(x_L \mid y_L, a_h)}{p(x_L \mid y_H, a_h)}$. Q.E.D.

Proof of Lemma 9: First we show that TT_{y_L, a_h} binds if Assumption 3 holds. Assumption 3 implies that $p(y_H \mid x_L, a_h) \geq p(y_H \mid x_L, a_l)$, which in turn implies that $\frac{p(y_L \mid x_L, a_l)}{p(y_L \mid x_L, a_h)} \geq \frac{p(y_H \mid x_L, a_l)}{p(y_H \mid x_L, a_h)}$. We next show that this implies we show that TT_{y_L, a_h} binds. Suppose that TT_{y_L, a_h} is slack. Consider lowering $U_{L\hat{L}}$ by ε and increasing $U_{L\hat{H}}$ by $\varepsilon \frac{p(x_L, y_L \mid a_h)}{p(x_L, y_H \mid a_h)}$. This variation decreases δ_L but because TT_{y_L, a_h} is not binding, ε can be made sufficiently small so that the constraint is still satisfied. This variation satisfies TT_{y_H, a_h}, i.e., $\delta_H * p(x_H, y_H \mid a_h) \geq \delta_L * p(x_L, y_H \mid a_h)$. The proposed variation continues to satisfy this constraint because it lowers the right-hand side without affecting the left-hand side. It remains to check whether the two possibly binding IC constraints are satisfied.

For $\mathrm{IC}_{H,L}$, the change in the left-hand side is $\varepsilon * p(x_L, y_H \mid a_h) * \dfrac{p(x_L, y_L \mid a_h)}{p(x_L, y_H \mid a_h)}$ $-\varepsilon * p(x_L, y_L \mid a_h) = 0$, while the change in the right-hand side is $-\varepsilon * p(x_L, y_L \mid a_l)$ $\varepsilon * p(x_L, y_H \mid a_l) * \dfrac{p(x_L, y_L \mid a_h)}{p(x_L, y_H \mid a_h)} = -\varepsilon * [p(x_L, y_L \mid a_l) - p(x_L, y_H \mid a_l)\dfrac{p(x_L, y_L \mid a_h)}{p(x_L, y_H \mid a_h)}] \leq 0$, where the negative sign is implied by the fact that $\dfrac{p(x_L, y_L \mid a_l)}{p(x_L, y_L \mid a_h)} \geq \dfrac{p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)}$ $\Leftrightarrow \dfrac{p(y_L \mid x_L, a_l)}{p(y_L \mid x_L, a_h)} \geq \dfrac{p(y_H \mid x_L, a_l)}{p(y_H \mid x_L, a_h)}$, and so $\mathrm{IC}_{H,L}$ is satisfied. The change in the left-hand side of $\mathrm{IC}_{L,L}$ is also zero and the change in the right-hand side is $-\varepsilon * p(x_L, y_L \mid a_l)$ $-\varepsilon * p(x_L, y_H \mid a_l) < 0$, so it too is satisfied. Finally note that the proposed variation reduces the expected payment to the manager while satisfying all of the constraints, contradicting the hypothesis that TT_{y_L, a_h} is not binding.

Since TT_{y_L, a_h} binds, $\delta_H * p(x_H, y_L \mid a_h) = \delta_L * p(x_L, y_L \mid a_h)$. Recall from the proofs of Lemmas 1 and 3 that $\mathrm{IC}_{H,L}$ binds if and only if $p(x_H, y_H \mid a_l) * \delta_H \geq p(x_L, y_H \mid a_l) * \delta_L$. Which, when TT_{y_L, a_h} binds, is equivalent to $\dfrac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} \geq \dfrac{p(x_L, y_H \mid a_l)}{p(x_H, y_H \mid a_l)}$. And this inequality is satisfied if and only if $\dfrac{p(x_L \mid y_L, a_h)}{p(x_H \mid y_L, a_h)} \geq \dfrac{p(x_L \mid y_H, a_l)}{p(x_H \mid y_H, a_l)}$, which in turn is satisfied if and only if $p(x_H \mid y_H, a_l) \geq p(x_H \mid y_L, a_h)$. This completes the proof for the case where $\mathrm{IC}_{H,L}$ binds. The case where $\mathrm{IC}_{L,L}$ binds is proved in the same fashion and so we omit it here. Q.E.D.

Proof of Proposition 10: We first show that when Assumption 3 is satisfied the necessary and sufficient conditions in Proposition 7 can be reduced to the single condition (9). To show this we show that in this case $\lambda_2 < 0 \Rightarrow \lambda_1 < 0$. Therefore, *either* λ_1 or λ_2 is less than zero if and only if $\lambda_1 < 0$. It is fairly straightforward to show that

$$\frac{p(x_H, y_H \mid a_h) - p(x_H, y_H \mid a_l)}{p(x_H, y_H \mid a_h)} - \frac{p_h - p_l}{p_h} >$$

$$\frac{p(x_L, y_H \mid a_h) - p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} - \frac{(1-p_h)-(1-p_l)}{(1-p_h)} \Leftrightarrow$$

$$(1-p_h)p_l \frac{p(y_H \mid x_H, a_h) - p(y_H \mid x_H, a_l)}{p(y_H \mid x_H, a_h)} > p_h(1-p_l)\frac{p(y_H \mid x_L, a_h) - p(y_H \mid x_L, a_l)}{p(y_H \mid x_L, a_h)}.$$

Through some more involved algebra this expression can be shown to be equivalent to

$$\frac{p(x_H, y_L \mid a_h)}{p(x_H, y_H \mid a_h)}[\frac{p_h - p_l}{p_h} - \frac{p(x_H, y_L \mid a_h) - p(x_H, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}] >$$

$$\frac{(1-p_h)-(1-p_l)}{(1-p_h)} - \frac{p(x_L, y_L \mid a_h) - p(x_L, y_L \mid a_h)}{p(x_L, y_L \mid a_h)}.$$

$$\Rightarrow [\frac{p_h - p_l}{p_h} - \frac{p(x_H, y_L \mid a_h) - p(x_H, y_L \mid a_h)}{p(x_H, y_L \mid a_h)}] >$$

$$\frac{(1-p_h)-(1-p_l)}{(1-p_h)} - \frac{p(x_L, y_L \mid a_h) - p(x_L, y_L \mid a_h)}{p(x_L, y_L \mid a_h)}.$$

$$\Rightarrow \lambda_1 < 0.$$

Next we show that when $\mathrm{IC}_{H,L}$ binds, satisfaction of condition (9) implies satisfaction of condition (11). Then we show that when $\mathrm{IC}_{L,L}$ binds, satisfaction of (11) implies satisfaction of (9). Together this implies that (9) and (11) constitute necessary and sufficient conditions for both cases.

We rewrite (11) in the form

$$\frac{p(x_H, y_H \mid a_h)}{p(x_H, y_L \mid a_h)} \frac{p(x_L, y_L \mid a_h)}{p(x_L, y_H \mid a_h)} > \frac{p_h}{(1-p_h)} \frac{(1-p_l)}{p_l} \tag{14}$$

and (9) as

$$\frac{p(y_H \mid x_H, a_h) - p(y_H \mid x_H, a_l)}{p(y_L \mid x_H, a_h)} \frac{p(y_L \mid x_L, a_h)}{p(y_H \mid x_L, a_h) - p(y_H \mid x_L, a_l)} > \frac{p_h}{(1-p_h)} \frac{(1-p_l)}{p_l}. \tag{15}$$

We show that (14) implies (9) when $\mathrm{IC}_{L,L}$ binds by showing that the left-hand side of (15) is greater than the left-hand side of (14) when $\mathrm{IC}_{L,L}$ binds.

$$\frac{p(y_H \mid x_H, a_h) - p(y_H \mid x_H, a_l)}{p(y_L \mid x_H, a_h)} \frac{p(y_L \mid x_L, a_h)}{p(y_H \mid x_L, a_h) - p(y_H \mid x_L, a_l)} >$$

$$\frac{p(x_H, y_H \mid a_h)}{p(x_H, y_L \mid a_h)} \frac{p(x_L, y_L \mid a_h)}{p(x_L, y_H \mid a_h)}$$

$$\Leftrightarrow \frac{p(x_H, y_H \mid a_h) - \frac{p_h}{p_l} p(x_H, y_H \mid a_l)}{p(x_L, y_H \mid a_h) - \frac{(1-p_h)}{(1-p_l)} p(x_L, y_H \mid a_l)} > \frac{p(x_H, y_H \mid a_h)}{p(x_L, y_H \mid a_h)}$$

$$\Leftrightarrow \frac{(1-p_h)}{(1-p_l)} \frac{p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} > \frac{p_h}{p_l} \frac{p(x_H, y_H \mid a_l)}{p(x_H, y_H \mid a_h)}$$

$\Leftrightarrow \frac{p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} \frac{p(x_H, y_H \mid a_h)}{p(x_H, y_H \mid a_l)} > \frac{(1-p_h)}{(1-p_l)} \frac{p_h}{p_l}$, which is satisfied if (14) holds if $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} < \frac{p(x_L, y_H \mid a_l)}{p(x_H, y_H \mid a_l)}$. This inequality is in turn satisfied if and only if condition (13) of Lemma 9 is satisfies, i.e., if and only if $\text{IC}_{L,L}$ binds.

To show that (9) implies (11) when $\text{IC}_{H,L}$ binds, we note that not-(11) implies not-(9) if and only if the left-hand side of (14) is greater than the left-hand side of (15).

$\Leftrightarrow \frac{(1-p_h)}{(1-p_l)} \frac{p_h}{p_l} > \frac{p(x_L, y_H \mid a_l)}{p(x_L, y_H \mid a_h)} \frac{p(x_H, y_H \mid a_h)}{p(x_H, y_H \mid a_l)}$. And this inequality is satisfied if (14) is not satisfied and $\frac{p(x_L, y_L \mid a_h)}{p(x_H, y_L \mid a_h)} > \frac{p(x_L, y_H \mid a_l)}{p(x_H, y_H \mid a_l)}$, i.e., if and only if $\text{IC}_{H,L}$ binds. Q.E.D.

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