Manipulability vs. Information Asymmetries about
the Manipulability of Accounting Numbers

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1. Introduction

Many of the Financial Accounting Standards Board’s (FASB’s) recent standards and ongoing projects emphasize fair value measurement in an attempt to increase the relevance of financial statements. This can be seen as part of the FASB’s broader emphasis on decision usefulness. Arguably, these same standards and projects have reduced or have the potential to reduce the reliability of accounting, as is to be expected in the familiar tradeoff between relevance and reliability.

Ijiri (1975) and Christensen and Demski (2003) emphasize the stewardship and information content roles of accounting. Both recognize the hidden (out-of-equilibrium) role of accounting in ensuring managerial actions and reports are held in check. Ijiri (1975) studies the axiomatic structure of historical cost accounting and compares historical cost accounting to other valuation methods along three dimensions: accountability, performance measurement, and economic decisions in general.

In Paton and Littleton (1940), a focus on measuring effort and accomplishment leads to their advocating “verifiable, objective evidence” and historical cost-based valuation. Paton and Littleton write (p. 7) “[i]n general, the only definite facts available to represent exchange transactions objectively and to express them homogenously are the price-aggregates involved in the exchanges; hence such data constitute the basic subject matter of accounting.”

Ijiri and Jaedicke (1966) view reliability as having two components: objectivity and bias. Objectivity is defined as the variance in observed measurements produced by independent (and truthful) measurers/accountants. (The FASB’s definition of verifiability corresponds to Ijiri and Jaedicke’s definition of objectivity.) Ijiri (1975)
goes on to develop the idea of “hardness,” which asks how far apart measurers can push the measures if they have incentives to take extreme positions. That is, hardness encompasses both objectivity and a lack of manipulability.

This paper studies a principal-agent (both are risk neutral) model of moral hazard with two performance measures: one perfectly hard (not manipulable) and one soft (partially manipulable). Our basic premise is that soft performance measures are not only subject to manipulation but also that there is often an information asymmetry between managers and investors about the degree of manipulability. Such an information asymmetry arises because of previous accounting choices (e.g., the creation of a hidden/secret reserve) or a better understanding of: the inherent objectivity of a specific firm’s estimates, opportunities for structuring transactions to achieve accounting objectives, the toughness of the particular audit engagement partner, etc.

We derive conditions under which it is optimal to exclude the soft performance measures from the incentive contract. In our model, it is the information asymmetry about manipulability, not manipulability per se, that drives the soft performance measure out of the contract.

To some extent, our result should be expected by accountants who recognize the importance of hardness as a central consideration in accounting. Nevertheless, we think we have something new to say. First, even though our results are consistent with what accountants think reasonable, existing models are not. Reconciling the two is itself an interesting puzzle. Second, we provide a different lens through which to view hardness and, in particular, stress the role of an information asymmetry about the extent of possible manipulation. Third, different economic forces come into play than in the standard
In addition to reports with high likelihood ratios (the only factor in the standard risk-neutral model), the following are also desirable: large probability outcomes and outcomes for which the agent’s marginal productivity is large.

In their final chapter, Paton and Littleton note that historical-cost-based valuation does not preclude the inclusion of market values in a second column. More recently, the International Accounting Standards Board (IASB) is considering (now in a joint project with the FASB) a multi-column income statement, where the columns are before re-measurement, re-measurement, and after re-measurement. If one thinks of the re-measurements column as less reliable than the before re-measurements column, the IASB format is a reliability-based disaggregation. (For a related approach that emphasizes the reliability of accounting estimates rather than re-measurements, see Glover, Ijiri, Levine, and Liang (2004).)

Part of the motivation for the FASB’s recent Exposure Draft on Fair Value Measurements is that “many constituents have raised concerns about the ability to develop reliable estimates of fair value in certain circumstances, in particular, in the absence of quoted prices” (FASB, 2004). The proposed standard is intended to partly remedy the situation by providing more unified guidance on fair values and developing a reliability-oriented hierarchy for fair values. The reliability-oriented hierarchy plays a role in both measurement and disclosure. The hierarchy emphasizes subjectivity in managerial inputs (estimates/forecasts) that have to be relied on when market prices or other market inputs are not available and labels such estimates Level 3 Estimates (the least reliable).

Our model is consistent with these reliability-based disaggregations in that we
derive conditions under which softer performance measures are excluded from compensation contracts. It seems natural to separate accounting numbers well suited for contracting from those that are not. The softer numbers may still be useful for valuation and/or interact in a subtler way with the hard numbers than captured in our single-period model.

We also study the case in which one of the accounting numbers is subjective (in the sense if Ijiri and Jaedicke) but not manipulable. The conditions change somewhat relative the setting with manipulation, and a second more complicated case emerges. We present an example that is somewhat counterintuitive in that the required level of subjectivity needed for it to be optimal to omit the subjective performance measure from the contract is larger when the performance measure can be manipulated than when it cannot. The principal takes the potential manipulation into account in designing the contract, which enables her to reduce the size of the incentive payment.

A common criticism of the principal-agent literature is that the derived optimal contracts are typically quite complex relative to contracts observed in practice. For example, in the standard model of moral hazard (Holmstrom, 1979), all informative variables are included in the optimal contract no matter how little additional information content they have. Also, these theoretically optimal contracts often suffer from the problem that they perform extremely poorly if the environment is slightly different than assumed. That is, the theoretical contracts are not robust to even slight misspecifications of the parameters. The section of our paper on subjectivity without manipulation can be viewed as a robustness result. When probabilities are subject to misspecification, it is optimal to exclude low probability events from contracts and focus on high probability
events.

An important debate in accounting is whether probabilities should be used in triggering recognition (as is traditional in accounting) or instead solely in measurement. The notion of a critical event in revenue recognition can be viewed as a focus on large probability events or large changes in probabilities. Both large probabilities and large changes in probabilities (the agent’s marginal productivity) emerge as important forces in our model.

The remainder of this paper is organized into four sections. In section 2, we present the basic model and its solution as a benchmark. In Section 3, we study the manipulability problem. In Section 4, we study subjectivity in performance measurement in the absence of manipulation. In Section 5, we apply the idea to a revenue recognition problem. Section 6 concludes the paper.

2. The Basic Model

The model is simple. A risk-neutral principal would like to motivate a risk-neutral agent to supply a hidden and personally costly action \( a = a_h \) rather than \( a_l < a_h \). (For a development of the risk-neutral agency model, see Innes (1990).) The implicit assumption is that the parameters are such that the high action generates a benefit to the principal sufficiently large that she always prefers to motivate high effort instead of low effort.

The agent’s action gives rise to transactions \( t^1 \) and \( t^2 \), which can each take on realizations of \( t_l \) or \( t_h \). \( t^1 \) and \( t^2 \) are then converted into accounting reports \( r^1 \) and \( r^2 \) through a combination of self reporting and verification. \( r^1 \) is a perfectly hard performance measure in the sense that it cannot be manipulated. For simplicity, when \( t^1 \)
is \( t_j \), it will be reported as \( r_j \), \( j \in \{L, H\} \). \( r^2 \) is softer: with probability \( \theta \), the manager will be able to report \( r^2 \) as she chooses \( (r_L, r_H) \). Given the assumptions we will soon make, it is optimal for the agent to over-report rather than under-report. We will take this behavior as given.

If the agent chooses \( a_H \), the probabilities over the set of possible \((t^1, t^2)\) transactions \( \{(t_L, t_L), (t_L, t_H), (t_H, t_L), (t_H, t_H)\} \) are \( \{p_{LL}, p_{LH}, p_{HL}, p_{HH}\} \). If the agent chooses \( a_L \), the corresponding probabilities are \( \{q_{LL}, q_{LH}, q_{HL}, q_{HH}\} \).

The contract the principal designs to motivate the agent to choose \( a_H \) can be based only on the final reports \( r^1 \) and \( r^2 \). Importantly, we have deliberately ruled out a report on \( \theta \) in an attempt to gain some understanding about the conditions under which soft reports will be excluded from contracts. The contract is simply a report-contingent payment \( s(r^1, r^2) \).

The agent (who knows \( \theta \)) has an induced probability distribution over \((r^1, r^2)\) realizations as follows:

<table>
<thead>
<tr>
<th>( a_H )</th>
<th>( (r_L, r_L) )</th>
<th>( (r_L, r_H) )</th>
<th>( (r_H, r_L) )</th>
<th>( (r_H, r_H) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1-\theta)p_{LL} )</td>
<td>( p_{LH} + \theta p_{LL} )</td>
<td>( (1-\theta)p_{HL} )</td>
<td>( p_{HH} + \theta p_{HL} )</td>
<td></td>
</tr>
<tr>
<td>( a_L )</td>
<td>( (1-\theta)q_{LL} )</td>
<td>( q_{LH} + \theta q_{LL} )</td>
<td>( (1-\theta)q_{HL} )</td>
<td>( q_{HH} + \theta q_{HL} )</td>
</tr>
</tbody>
</table>

The ordering of the payments is determined by the standard likelihood ratios. For tractability, we assume the likelihood ratios are ordered as follows:

**Likelihood Ratios:** \( p_{LL}/q_{LL} < p_{LH}/q_{LH} < 1 < p_{HL}/q_{HL} < p_{HH}/q_{HH} \).

The agent’s payoff is the payment he receives less the cost of effort, also denoted by \( a_H \) or \( a_L \). The principal’s objective is to minimize the expected payment to the agent.
E[s(r^1,r^2)] subject to the constraints that choosing a_h be incentive compatible for the agent and that the payments s(r^1,r^2) be nonnegative. The usual individual rationality constraint is omitted. Alternatively (but with the same result), the individual rationality constraint is dominated by the other constraints if we assume a_L and the agent’s reservation utility are both zero.

As a benchmark, we formulate and solve the principal’s program assuming the problem is a standard moral hazard problem in which both performance measures are objectively determined (no subjectivity or manipulation, captured by \( \theta = 0 \)):

\[
\begin{align*}
\text{Min} & \quad \sum_{j,k=L,H} p_{jk} s(r_j,r_k) \\
\text{such that:} & \\
\sum_{j,k=L,H} p_{jk} s(r_j,r_k) - a_H & \geq \sum_{j,k=L,H} q_{jk} s(r_j,r_k) - a_L \quad \text{(Incentive Compatibility)} \\
s(r_j,r_k) & \geq 0, \forall j,k \quad \text{(Nonnegativity)}
\end{align*}
\]

By solving the incentive compatibility constraint as an equality, it is straightforward to show the following well-known result.

**Observation.** If both performance measures are objective (\( \theta = 0 \)), the solution is to make a single bonus payment of \((a_H - a_L)/(p_{HH} - q_{HH})\) for the report \((r_H,r_H)\) with the highest associated likelihood ratio \(p_{HH}/q_{HH}\). All other payments are zero.

3. Manipulation

We now specify the program for the case that the second performance measure is subject to manipulation and the extent of the opportunity for manipulation is known only to the agent. The principal knows only that \( \theta \) is uniformly distributed over the interval \([0,\theta_{\max}]\).
Min \sum_{j=L,H} \left( \frac{\theta_{\text{max}}}{2} p_{jL} s(r_j, r_L) + (p_{jH} + \frac{\theta_{\text{max}}}{2} p_{jL}) s(r_j, r_H) \right)

subject to

\sum_{j=L,H} \left( (1 - \theta)p_{jL} s(r_j, r_L) + (p_{jH} + \theta p_{jL}) s(r_j, r_H) - a_H \right) \geq 0

\sum_{j=L,H} \left( (1 - \theta)q_{jL} s(r_j, r_L) + (q_{jH} + \theta q_{jL}) s(r_j, r_H) - a_L \right), \forall \theta \in [0, \theta_{\text{max}}]

(Incentive Compatibility)

s(r_j, r_k) \geq 0, \forall j, k

(Nonnegativity)

We also assume motivating high effort is always feasible, for any \theta. As another benchmark, first consider the case that manipulation is possible but the probability of successful manipulation is common knowledge.

**Proposition 1.** With the possibility of manipulation (\theta > 0) but common knowledge about that possibility, the optimal contract always uses both performance measures.

**Proof.** The principal’s problem is a linear program. Let \lambda be the multiplier on the incentive compatibility constraint and the choice variable in the dual program. Consider the following proposed solution to the primal and the dual.

\[
s(r_L, r_L) = s(r_L, r_H) = s(r_H, r_L) = 0, s(r_H, r_H) = \frac{(a_H - a_L)}{p_{HH} + \theta p_{HL} - q_{HH} - \theta q_{HL}},
\]

and \[
\lambda = \frac{p_{HH} + \theta p_{HL}}{p_{HH} + \theta p_{HL} - q_{HH} - \theta q_{HL}}.
\]

Under the proposed solution, the objective functions of the dual and the primal are equal. By the duality theorem of linear programming, the above is then indeed a solution as long as it is feasible in both the primal and the dual, which it is. This completes the proof.
We now return to the setting of primary interest in which there is an information asymmetry about the probability of successful manipulation.

**Proposition 2.** With both the possibility of manipulation and an information asymmetry about the probability of successful manipulation ($\theta$ is both greater than 0 and not common knowledge), the optimal contract relies on the hard performance measure, $r^1$, alone if and only if the following condition holds:

$$\frac{\varepsilon}{2} > \frac{(p_{HH}q_{HL} - p_{HL}q_{HH})}{p_{HL}(p_{HL} + p_{HH} - q_{HL} - q_{HH})}.$$

**Proof.** Again, let $\lambda$ be the multiplier on the incentive compatibility constraint and the choice variable in the dual program. Consider two potential solutions to the primal and the dual.

**Solution 1:** \(s(r_L, r_L) = s(r_L, r_H) = s(r_H, r_L) = 0, s(r_H, r_H) = \frac{(a_H - a_L)}{p_{HH} - q_{HH}}, \text{ and } \lambda = \frac{p_{HH} + \frac{\varepsilon}{2}p_{HL}}{p_{HH} - q_{HH}}.\)

**Solution 2:** \(s(r_L, r_L) = s(r_L, r_H) = s(r_H, r_L) = s(r_H, r_H) = \frac{(a_H - a_L)}{p_{HH} + p_{HL} - q_{HH} - q_{HL}}, \text{ and } \lambda = \frac{p_{HH} + p_{HL}}{p_{HH} + p_{HL} - q_{HH} - q_{HL}}.\)

Under each proposed solution, the objective functions of the dual and the primal are equal. By the duality theorem, the above are solutions as long as they are feasible in both the primal and the dual. The condition given in the statement of the proposition determines which of these solutions is feasible, which completes the proof.

Consider the following examples.
Example 1

$$\theta_{max} = 0$$

$$\{p_{LL}, p_{LH}, p_{HL}, p_{HH}\} = \{0.09, 0.5, 0.4, 0.01\}$$

$$\{q_{LL}, q_{LH}, q_{HL}, q_{HH}\} = \{0.2, 0.6, 0.2, 0\}$$

$$a_H = 1, a_L = 0.$$

Optimal Contract: $s(r_L, r_L) = s(r_L, r_H) = s(r_H, r_L) = 0, s(r_H, r_H) = 100$

$E(s) = 1$

Example 1 is the usual model. In this case, we know the optimal solution makes a single bonus payment for the report with the highest likelihood ratio. Since $q_{HH} = 0$, the first-best solution is obtained: $E(s) = 1$.

Example 2

$$\theta_{max} = 0.10$$

$$\{p_{LL}, p_{LH}, p_{HL}, p_{HH}\} = \{0.09, 0.5, 0.4, 0.01\}$$

$$\{q_{LL}, q_{LH}, q_{HL}, q_{HH}\} = \{0.2, 0.6, 0.21, 0\}$$

$$a_H = 1, a_L = 0.$$

Optimal Contract: $s(r_L, r_L) = s(r_L, r_H) = 0; s(r_H, r_L) = s(r_H, r_H) = 5$

$E(s) = 2.05$

In Example 2, if $\theta = 0.1$, the required incentive payment is $(a_H - a_L)(p_{HH} - q_{HH}) = 1/0.01 = 100$, which is not costly when $\theta = 0$ but is costly for all other $\theta$s. In particular, the expected payment would be $[(0.01) + (0.05)(0.4)](100) = 3$ if both the hard and the soft performance measure are used to determine payment. Things become worse if $\theta_{max}$ is larger. By instead relying on the hard performance measure alone, the optimal payment
is \((a_{HH}-a_{HL})/ [(p_{HH} + p_{HL}) - (q_{HH} - q_{HL})] = 1/(0.41- 0.21) = 5\), and the expected payment is 
\((0.41)(5) = 2.05\).

4. Subjectivity without Manipulation

Suppose instead there is subjectivity in measuring the second performance measure, but no manipulation. The revised program follows.

\[
\begin{align*}
\text{Min} & \quad \sum_{j,k=L,H;m \neq k} [(1-\theta_{\max})p_{jk} + \theta_{\max}p_{jm}]s(r_j, r_k) \\
& \quad \sum_{j,k=L,H;m \neq k} [(1-\theta/2)p_{jk} + \theta/2p_{jm}]s(r_j, r_k) - a_{H} \geq \\
& \quad \sum_{j,k=L,H;m \neq k} [(1-\theta/2)q_{jk} + \theta/2q_{jm}]s(r_j, r_k) - a_{L}, \forall \theta \in [0,\theta_{\max}] \\
& \quad s(r_j, r_k) \geq 0, \forall j, k
\end{align*}
\]

(Incentive Compatibility)

(Nonnegativity)

Somewhat surprisingly, removing manipulation complicates things in that a second case is created.

**Proposition 3.** With the possibility of subjectivity but not manipulability, the optimal contract relies on the hard performance measure, \(r^1\), alone if and only if one of the following cases holds:

\[
\begin{align*}
p_{HH} - q_{HH} & \leq p_{HL} - q_{HL} \quad \text{and} \quad \frac{\varepsilon}{2} > \frac{(p_{HH}q_{HL} - p_{HL}q_{HH})}{(p_{HL} - p_{HH})(p_{HH} + p_{HL} - q_{HL} - q_{HH})} \\
p_{HH} - q_{HH} & > p_{HL} - q_{HL} \quad \text{and} \quad \frac{\varepsilon}{2} > \frac{(p_{HH}q_{HL} - p_{HL}q_{HH})}{p_{HH}(p_{HH} - q_{HH} + 3q_{HL}) - p_{HL}(p_{HL} + 3q_{HH} - q_{HL})}.
\end{align*}
\]

**Proof.** The proof is analogous to the proof of Proposition 2.
Consider the following example.

### Example 3

\[ \theta_{\text{max}} = 1/3 \]
\[ \{p_{LL}, p_{LH}, p_{HL}, p_{HH}\} = \{0.2, 0.2, 0.2, 0.4\} \]
\[ \{q_{LL}, q_{LH}, q_{HL}, q_{HH}\} = \{0.5, 0.4, 0.1, 0\} \]
\[ a_H = 1, a_L = 0. \]

**Information Asymmetry about Manipulability:**

Optimal Contract: \[ s(r_L, r_L) = s(r_L, r_H) = s(r_H, r_L) = 0, s(r_H, r_H) = 2.5 \]

\[ \mathbb{E}(s) = 1.0833 \]

**Information Asymmetry about Subjectivity Only:**

Optimal Contract: \[ s(r_L, r_L) = s(r_L, r_H) = 0, s(r_H, r_L) = s(r_H, r_H) = 2 \]

\[ \mathbb{E}(s) = 1.2 \]

In Example 3, when manipulability is allowed, the extra flexibility is taken into account by the principal in designing the contract. As a result, the principal is better off under manipulation than subjectivity alone. With subjectivity alone, if the principal wants to use both performance measures, the optimal \( s(r_H, r_H) \) bonus payment is 3.3333. This is derived from the incentive compatibility constraint for the largest \( \theta \):

\[ s(r_H, r_H) = \frac{1}{[(2/3)(0.4)+(1/3)(0.2)−(2/3)(0)−(1/3)(0.1)]} = 3.3333. \]

With the addition of manipulability, the agent can ensure he receives the bonus payment with at least probability \( p_{HH} \). The required \( s(r_H, r_H) \) is only \( 1/(0.4−0) = 2.5 \). This smaller bonus is paid out more often: \( \mathbb{E}(s) = [(0.4)+(0.1667)(0.2)](2.5) = 1.0833 \).

However, the smaller probability under subjectivity alone is not enough to overcome the larger bonus: \( [(0.8333)(0.4)+(0.1667)(0.2)](3.3333) = 1.2222 \). In fact, it is better to leave the subjective performance measure out of the contract. Again, perhaps
surprisingly, the required $\theta$ for the second performance measure to be omitted from the contract is larger under manipulability ($\theta_{\text{max}}>0.8$) than under subjectivity only ($\theta_{\text{max}}>0.3077$).

5. Revenue Recognition

The following variation is also presented in Glover (2004). The agent is responsible for a sale to a customer, which can have one of three possible outcomes: no transaction ($x_1$), product ordered by customer ($x_2$), product delivered to customer ($x_3$), and product paid for by customer ($x_4$). Building on the work of Antle and Demski (1989) and Liang (2000), the principal designs and publicly commits to both (i) an accounting system to recognize revenue when a critical event ($x_1, x_2, x_3, \text{or } x_4$) occurs and (ii) payments to the agent conditional on whether revenue has been recognized or not, $s(\text{revenue})$ and $s(\text{no revenue})$.

The principal's beliefs over the transactions $x_1, x_2, x_3,$ and $x_4$ are $p_1, p_2, p_3,$ and $p_4$ if the agent supplies $a_H$ and $q_1, q_2, q_3,$ and $q_4$ if the agent supplies $a_L$. The principal designs the contract to be robust to any corresponding beliefs $p'$ and $q'$ the agent might have, where $p'$ and $q'$ are any probability vectors with a Euclidian distance of less than or equal to $\varepsilon$ from $p$ and $q$, respectively.

Make the standard monotone likelihood ratio property assumption $p_1/q_1 < p_2/q_2 < p_3/q_3 < p_4/q_4$ and the non-standard assumption this is also true for any $p'$ and $q'$ within $\varepsilon$ of $p$ and $q$. To keep things simple, suppose also that the first two ratios are less than one and the second two ratios are greater than one (for both $p/q$ and $p'/q'$).

If $\varepsilon = 0$, we are in the standard model, and the likelihood ratios determine everything. The optimal accounting system is to recognize revenue only when $x_4$ is realized, and the optimal contract is to set $s(\text{no revenue}) = 0$ and $s(\text{revenue}) = \frac{a_H - a_L}{p_4 - q_4}$.
Note the usual notion of accomplishment (the earned test) associated with a critical event is absent.

Assume instead the principal believes \( p' \) and \( q' \) are uniformly distributed over the slice of the Euclidean ball of distance \( \varepsilon \) in the probability simplex centered on \( p \) and \( q \) (the intersection of the ball and the simplex).

If \( \varepsilon > \frac{\sqrt{6}}{4}(\frac{p_3 q_3 - p_3 q_4}{p_3}) \), the optimal critical event is \( x_3 \) with \( s(\text{revenue}) = \frac{a_H - a_L}{(p_3 + p_4) - (q_3 + q_4) - 4\varepsilon/\sqrt{6}} \); otherwise, the optimal critical event is \( x_4 \) with \( s(\text{revenue}) = \frac{a_H - a_L}{p_4 - q_4 - 4\varepsilon/\sqrt{6}} \). In any case, \( s(\text{no revenue}) = 0 \).

The size of the probabilities matters. A large \( p_3 \) favors treating \( x_3 \) as the critical event. This seems to bring us closer to the accountant's notion of a critical event.

6. Concluding Remarks

This paper emphasizes the distinction between subjectivity, softness (manipulability), and information asymmetries regarding these two properties. The information asymmetry regarding subjectivity or softness can drive the subjective performance measure out of the optimal contract in our model. This is not true of subjectivity or softness itself. Also, an information asymmetry about subjectivity alone can drive the performance measure out of the contract sooner than an information asymmetry about softness.

The FASB continues to incorporate additional fair value measurements within the financial statements. Recent projects include their exposure draft on employee stock options and their revenue recognition project. At the same time, the PCAOB and others have raised questions about the “auditability” of some of the fair value measurements.
The FASB prefers the term “verifiability” (Ijiri and Jaedicke’s objectivity) and the broader concept of reliability (verifiability, representation faithfulness, and neutrality). However, the FASB also recognizes the importance of using the most verifiable inputs possible and the benefit of reliability-based disaggregations. Within this context, it seems natural to re-examine these concepts using the modern tools of information economics.
References


