Accounting for Cash Flow Hedges: Mark-to-Market vs. Historical Cost†

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1. Introduction.

Following several widely publicized losses in the 1990’s attributed to investments in derivatives (Orange County, Metallgesellschaft, Barings Corporation, etc.), the Financial Accounting Standards Board [1998] adopted SFAS 133 which requires firms to recognize all derivatives as assets or liabilities and measure them at fair market value. Changes in fair market value are reported as gains or losses in periodic earnings or in "other comprehensive income.” In the case of “fair value hedges”, where derivatives are used to hedge the value of a recognized asset or liability or of an unrecognized firm commitment, the change in the market value of the derivative is offset against the change in market value of the underlying asset or liability and both are reported in the earnings statement. In the case of "cash flow hedges”, where derivatives are used to hedge the variability in cash flows from a forecasted transaction, the change in the market value of derivatives is reported in "other comprehensive income” with no offset until the forecasted transaction is realized. If the forecasted transaction cannot be described with "sufficient specificity” or its occurrence cannot be established as likely, the gains or losses on the derivative transaction are reported immediately in the earnings statement, rather than in other comprehensive income. We refer to the reporting of such gains or losses on derivatives as "mark-to-market” accounting. In contrast to mark-to-market accounting, historical cost accounting would report derivative positions and gains or losses on derivatives only on the dates these trades settle.

In this paper we examine mark-to-market accounting for cash flow hedges, and contrast it to "historical cost” accounting. For purposes of this comparison, we do not distinguish between the reporting of gains or losses in "earnings” and reporting in "other comprehensive income” because it would seem that from an informational perspective the two forms of reporting are equivalent. In the case of cash flow hedges, it should be apparent that neither accounting system provides an accurate picture of the firm’s true financial position. We seek to identify the nature of the errors induced by each accounting system and the economic consequences of such errors.
The following examples are suggestive of the errors in reporting under each of the two accounting systems and the nature of the dilemma faced by accountants in choosing between them. In all of these examples, \( p_1, p_2, p_3 \) are the prices of the derivative security at dates 1, 2 and 3, respectively. We assume that \( p_2 > p_1 \) is realized, and that \( p_2 \) is an unbiased estimate of \( p_3 \). The firm adopts its derivative position at date 1, both the derivative and the real transaction settle at date 3 at price \( p_3 \), and date 2 is an interim date. The number of derivatives sold is \( z > 0 \) and the real transaction underlying the derivative is \( q \geq 0 \). As shown later, if \( z \neq q \), the difference represents a speculative position undertaken by the firm.

**Example 1**

Suppose the firm’s speculative position is zero, so that \( z = q > 0 \). This means the firm has shed all price risk and its true income is \( p_1q > 0 \). However, at the interim date 2, mark-to-market accounting will report a loss of \( (p_1 - p_2)z \). Clearly, this loss is fictitious and irrelevant. At date 3, when all transactions are settled, the income of the firm will be \( (p_1 - p_3)z + pq \). Since \( z = q \), \(-p_3z\) is perfectly offset by \(+pq\) and the firm’s income is simply \( p_1q \). Thus, mark-to-market accounting makes no sense when the firm never speculates and simply hedges the price uncertainty associated with its real transaction.

**Example 2**

Now, suppose the firm’s derivative position is purely speculative. As before, \( z > 0 \), but now \( q = 0 \). The firm has taken a short position in the derivatives market, but there is no underlying real transaction so the hedge component of the derivative is absent. In this case, at date 3, when the derivative trade is settled, the firm’s reported income will be \( (p_1 - p_3)z < 0 \) if \( p_3 > p_1 \). Since \( p_2 \) is the best estimate of \( p_3 \), reporting a loss of \( (p_1 - p_2)z \) at the interim date provides an unbiased picture of the firm’s true income. Thus, mark-to-market accounting makes eminent sense when the firm is using derivatives purely as a vehicle for speculation. Historical cost accounting would withhold important information because (as shown later), in the absence of an accounting report,
outsiders would rationally conjecture that the manager must have taken a long position in the derivatives market and would therefore assess positive profits for the firm.

**Example 3**

Now, suppose the firm’s derivatives position contains both a hedge component and a speculative component. As before, its aggregate (and observable) derivatives trade in the market is \( z > 0 \), but \( q > z > 0 \). To fix ideas, suppose the firm expects to sell 100 units of some commodity at date 3 \( (q = 100) \), and is observed to sell 60 units of the derivative at date 1 \( (z = 60) \). As argued in Kanodia et al [2000], this situation should be thought of as one where the firm has sold its entire real transaction of 100 units in the futures market at date 1, but has also taken a speculative long position of 40 units because it expects future prices to rise. In general, letting \( s \) be the firm’s speculative position, the firm’s derivative position should be thought of as \( z = q + s \). In the current example, \( s = -40 \) units. At date 3, when all transactions are settled, the firm’s income will be \( (p_1 - p_3)s + p_1q \) and, at date 2, the best estimate of the firm’s income is \( (p_1 - p_2)s + p_1q > 0 \). Yet mark-to-market accounting would report a loss of \( (p_1 - p_2)z \) at date 2. The firm’s manager, expecting prices to rise, acted in the best interests of the shareholders, ex post it turned out that she was on the right side of the market, yet mark-to-market accounting reports a gloomy assessment at the interim date! Historical cost accounting would have avoided this error.

**Example 4**

As a final example, suppose the manager has taken a speculative position on the wrong side of the market and her aggregate derivatives trade also has a hedge component. In this case, \( s > 0 \), \( q > 0 \), and \( z = q + s > 0 \). As in example 3, the profit of the firm at date 3 will be \( (p_1 - p_3)s + p_1q \), which may be positive or negative. Under mark-to-market accounting, a loss of \( (p_1 - p_2)z \) is reported at date 2. An unbiased estimate of the true loss imbedded in the date 3 income is \( (p_1 - p_2)s \). Since \( z > s \), mark-to-market accounting overstates the loss and presents too gloomy a picture of the firm’s financial condition. On the other hand, historical cost accounting
would lead to an assessment of speculative gains rather than losses. Here, it is unclear which of the two is the lesser evil, mark-to-market or historical cost accounting.

The above examples may seem overly simplistic. Outsiders know that the accounting system reports with error and will seek to correct these errors and assess the firm’s date 3 income as best as they can in the light of information available to them. However, we show that the errors in outsiders’ assessments have the same general direction as the the errors in reporting.

The fundamental reason for the inevitability of reporting errors is that the firm’s manager is likely to possess two kinds of private information. Because the real transaction underlying the derivative will occur in the future and is therefore uncertain, the firm’s manager is likely to possess private information regarding its distribution. Additionally, because the price at which the real transaction will occur is uncertain (hence the motivation to hedge), the manager is likely to be better informed about this future price too. As shown in Kanodia et al [2000], any derivative trade undertaken by the manager consists partly of a hedge component that reflects her beliefs about the magnitude of the real transaction, and partly of a speculative component that reflects her beliefs of the future price at which the real transaction will occur. Since both these beliefs are unobservable, the accounting for a derivative position cannot disentangle the hedge component from the speculative component. Hence, when the market value of the derivative fluctuates during interim periods, mark-to-market accounting reports the loss or gain on the entire derivative transaction. When there is an interim loss on the derivative, mark-to-market accounting overstates that loss, because the loss on the hedge component will actually reverse and only the loss on the speculative component is persistent and relevant. Similarly, mark-to-market accounting overstates any interim gains from derivatives.

On the other hand, historical cost accounting causes potential errors by inducing outsiders to hold too rosy a picture of the firm’s financial position. Under historical cost, just because accounting reports are silent about the firm’s derivatives position, outsiders do not naively assume that the
firm’s derivatives position is zero. Outsiders form rational beliefs about the firm’s speculative and hedge trades in the light of observed fluctuations in the interim price of the derivative and in the light of other information. Statistically, it is expected that traders with private information about future prices will make profits from speculative trades. Hence, regardless of whether the interim price of the derivative security moves up or down, outsiders rationally believe that the manager must have correctly anticipated this movement and earned speculative profits, and the larger the price change the larger is the speculative profit assessed by outsiders. Thus, when the manager is mistakenly on the wrong side of the market, historical cost accounting induces an overly rosy assessment of the firm’s financial position and the later reporting of a large loss comes as an unpleasant surprise.

What are the economic consequences of outsiders mistaken beliefs? We model these economic consequences in terms of the "solvent” or "insolvent” of the firm. A firm is solvent when, at date 2, the (unknown) best assessment of its date 3 income, \((p_1 - p_2)s + p_1q\), is positive, and insolvent when this quantity is negative. Our examples suggest that mark-to-market accounting will correctly identify a truly insolvent firm, but could cause a truly solvent firm to be assessed as insolvent. On the other hand, historical cost accounting will correctly identify a solvent firm but may cause an insolvent firm to be perceived as solvent. The consequences of the latter error are painfully obvious from the Orange County and Barings Corporation examples. Perhaps the possibility of this kind of error dominated the thinking of FASB when they mandated mark-to-market accounting. The cost associated with a solvent firm being perceived as insolvent is more subtle. Firms that are perceived as insolvent could be "abandoned" through a flight of capital, a denial of credit, a reluctance by suppliers to provide the inputs (raw materials and labor) needed for production, or a reluctance to purchase the goods that the firm seeks to offer. Any such form of abandoning will damage the potential real transaction that is supposed to occur in the future and which was the basis of the derivatives trade. Thus a solvent firm that is perceived to be insolvent by a sufficient number of outsiders would actually become insolvent, thus confirming
outsiders perceptions. To capture this idea, we model the reporting of derivatives as occurring in
the context of a coordination game, where individuals decide, after receiving the accounting report
at date 2, whether to abandon the firm or stay with the firm (see Morris and Shin [2001] for a
survey of the coordination /global games literature).

Relation to extant literature.

The remainder of the paper is organized as follows.

2. The Model.

There are three dates. At date 1, the firm is endowed with a potential real transaction $q$
that will occur at date 3. This real transaction could be interpreted as the potential sale of some
commodity that the firm is equipped to make. There is uncertainty in the magnitude of the real
transaction, and outsiders view $\tilde{q}$ as a random variable drawn from a Normal distribution with
mean $\bar{q}$ and variance $\sigma_q^2$. The date 3 price $\tilde{p}_3$ at which the real transaction will occur is also
uncertain. There is a derivatives market in which the firm’s manager can choose to hedge the
date 3 price uncertainty and additionally choose any speculative position she wishes. Derivative
positions are taken at date 1 and settled at date 3 at price $p_3$. The price in the derivatives market
at date 1 is $p_1 > 0$. Date 2 is an interim date at which the derivatives price changes randomly from
$p_1$ to $\tilde{p}_2$. Let $z$ denote the number of derivatives sold by the manager at date 1 (positive amounts
denote sales, negative amounts denote purchases). For simplicity we do not allow dynamic hedging,
i.e. the manager cannot revise her derivatives position at date 2.

The relationship between $p_1, p_2$ and $p_3$ is as follows:

\[ p_1 > 0 \text{ is exogenously given,} \]

\[ \tilde{p}_2 = p_1 + \tilde{\theta} + \tilde{\epsilon}, \quad (1) \]
The random variables $\tilde{\theta}$ and $\tilde{\epsilon}$ are drawn from Normal distributions with zero means and variances $\sigma_\theta^2$ and $\sigma_\epsilon^2$, respectively. The random variables $\tilde{q}, \tilde{\theta}, \tilde{\epsilon}$ are independent of each other. The assumption $p_3 = p_2$ is not essential to the analysis and is made here to avoid notational clutter (all that is needed is $E(p_3|p_2) = p_2$).

There is a continuum of “outsiders” contained in the unit interval $[0, 1]$. At date 2, each outsider decides whether to “abandon” the firm or stay with the firm. We do not operationalize exactly what is meant by abandoning the firm. As discussed in the introduction, such abandoning could be interpreted as a flight of capital, the refusal to supply essential inputs that are needed to produce the real transaction that is sold at date 3, the reluctance to buy the goods that are to be sold at date 3, etc. If a fraction $\lambda$ of these outsiders abandon the firm, the magnitude of the real transaction to occur at date 3 is reduced by the quantity $b\lambda$, where $b > 0$ is a known parameter. All economic agents are identically risk averse with preferences stated in terms of means and variances. If $\tilde{w}$ is the terminal income of the firm at date 3, an individual outsider $i$ abandons the firm if $E_i(\tilde{w}) - \frac{1}{2}\rho\text{variance}(\tilde{w}) < 0$, and stays with the firm otherwise. The expectation and variance in this expression is conditioned by the information available to individual $i$, which will differ with the accounting system in place and will be specified later. The parameter $\rho > 0$ is common knowledge and represents the degree of risk aversion.

The information available to economic agents is as follows. At date 1, the firm’s manager privately obtains information superior to that of outsiders regarding the magnitude of the real transaction to occur at date 3 as well as superior information regarding the future spot price $p_3$ (equivalently, $p_2$). Specifically, the manager learns the exact value of $q$ and the value of the random variable $\tilde{\theta}$ that affects the prices $p_2$ and $p_3$. The manager takes this information into account in determining the derivatives position of the firm. Outsiders obtain their information
at date 2, before they make their abandon/stay decisions. The realized interim price $p_2$ and the accounting report are public information, so these are contained in the information set of each outsider. Additionally, each outsider privately observes an idiosyncratic noisy signal of the real transaction $q$ with which the firm was endowed. Specifically, outsider $i$ observes,

$$x_i = q + \tilde{\eta}_i$$

(2)

where the idiosyncratic noise term $\tilde{\eta}_i$ is independently and identically distributed across outsiders and is drawn from a Normal distribution with zero mean and variance $\sigma_{\eta}^2$.

3. The Firm’s Derivatives decision.

We derive here a benchmark decision rule that determines the firm’s derivatives position. In a first best world where there are no outsiders who could abandon the firm, the firm’s date 3 income is,

$$\tilde{w}_f = (p_1 - \tilde{p}_2) z + \tilde{p}_2 q$$

At date 1, conditional on its knowledge of $q$ and $\theta$, the firm assesses:

$$E(\tilde{w}_f | q, \theta) = [p_1 - E(\tilde{p}_2 | \theta)] z + E(\tilde{p}_2 | \theta) q$$

$$Var(\tilde{w}_f | q, \theta) = z^2 var(\tilde{p}_2 | \theta) + q^2 var(\tilde{p}_2 | \theta) + 2zq cov[(p_1 - \tilde{p}_2), \tilde{p}_2 | \theta]$$

The firm chooses its derivative position $z$ to maximize $E(\tilde{w}_f | q, \theta) - \frac{1}{2} \rho Var(\tilde{w}_f | q, \theta)$. Noting that $cov[(p_1 - \tilde{p}_2), \tilde{p}_2 | \theta] = -var(\tilde{p}_2 | \theta)$, the firm’s decision problem is equivalently,
Max \[ z \left[ p_1 - E(\tilde{p}_2|\theta) \right] z - \frac{1}{2} \rho z^2 \text{var}(\tilde{p}_2|\theta) + \rho z q \text{var}(\tilde{p}_2|\theta) \]

The first order condition to this maximization problem yields the firm’s derivatives policy:

\[ z = q + \frac{p_1 - E(\tilde{p}_2|\theta)}{\rho \text{var}(\tilde{p}_2|\theta)} \]

The first term of the above expression, which depends only on the anticipated real transaction, is the hedge component of the firm’s derivatives position. The second term, which depends on the firm’s beliefs regarding future spot prices, is the speculative component. Henceforth we use \( s \equiv \frac{p_1 - E(\tilde{p}_2|\theta)}{\rho \text{var}(\tilde{p}_2|\theta)} \) to denote the firm’s speculative position, so that \( z = q + s \). Since \( E(\tilde{p}_2|\theta) = p_1 + \theta \), and \( \text{var}(\tilde{p}_2|\theta) = \sigma^2 \) the firm’s speculation policy is,

\[ s = -\frac{\theta}{\rho \sigma^2} \quad (3) \]

Notice that if the firm believes that future prices will be higher than the current price, it sells less than its anticipated production in the derivatives market, in effect taking a long position. In this case, \( s < 0 \) and \( z < q \). Conversely, if the firm believes that future prices will be lower than the current price, it sells more than its hedging needs, the difference being a speculative short position in the derivatives market. In this latter case, \( s > 0 \) and \( z > q \).

The derivatives policy characterized above will generally not be optimal when there are outsiders who make inferences about the firm’s financial condition and decide whether to abandon the firm or not. In principle, the optimal derivatives policy will vary with the accounting regime (mark-to-market or historical cost) that is imposed on the firm. At the least, the firm ought to anticipate potential damages to its real transaction from future abandonment decisions and factor
this anticipation into its hedging and speculative positions. Unfortunately, the characterization of such optimal derivatives policies turns out to be a formidable task, so we take the benchmark derivatives policy characterized above as exogenously given, impose it on both accounting regimes, and proceed to examine the performance of mark-to-market accounting and historical cost accounting.

4. Equilibrium in the Historical Cost Regime

Under historical cost, derivative transactions are not recorded at all until the date of settlement. Such transactions are considered executory in nature and do not result in any cash flows until the settlement date. In fact, in the simple abstraction we model, there are no cash flows at all until date 3, so there is no accounting report provided at date 2. Thus, in a historical cost regime, the information available to outsiders at date 2 consists solely of the publicly observed price $p_2$, described in (1), and the idiosyncratic signal $x_i$ described in (2).

Given the observation of $p_2$ at date 2 and knowing that $p_3 = p_2$, outsiders perceive the terminal income of the firm as:

$$\tilde{w} = (p_1 - p_2) \tilde{z} + p_2(\tilde{q} - b\lambda)$$

In the above expression, the firm’s derivative position, being unobservable, is replaced by a random variable $\tilde{z}$ and the quantity $b\lambda$ is the damage to the firm’s future transaction arising from a fraction $\lambda$ of outsiders abandoning the firm. Outsiders’ beliefs regarding $\tilde{z}$ and $\lambda$ will be derived below. Knowing that the firm’s derivatives policy consists of a hedge component which equals the firm’s observation of $\tilde{q}$ and a speculative component $\tilde{s} = \frac{-\theta}{\rho \sigma_z^2}$, outsiders’ perception of $\tilde{w}$ can be equivalently expressed as:

$$\tilde{w} = (p_1 - p_2)(\tilde{q} + \tilde{s}) + p_2(\tilde{q} - b\lambda) = (p_1 - p_2) \tilde{s} + p_1 \tilde{q} - p_2 b\lambda$$
Since $p_2$ contains information on $\theta$ and therefore on the firm’s speculative position $s$, and $x_i$ contains information on $q$, outsider $i$’s expectation of $\bar{w}$ is,

$$E(\bar{w}|p_2, x_i) = (p_1 - p_2) E(\bar{s}|p_2) + p_1 E(\bar{q}|x_i) - p_2 b E(\lambda|p_2, x_i)$$

Each of the expectations in the above expression is calculated below. Given the firm’s speculative policy $\tilde{s} = -\frac{\theta}{\rho \sigma_\epsilon^2}$, $(\tilde{s}, \tilde{p}_2)$ is joint Normally distributed with prior means $E(\tilde{s}) = 0$ and $E(\tilde{p}_2) = p_1$. Therefore,

$$E(\tilde{s}|p_2) = -\frac{1}{\rho \sigma_\epsilon^2} E(\theta|p_2) = -\frac{1}{\rho \sigma_\epsilon^2} \beta_p (p_2 - p_1) \quad (4)$$

where, $\beta_p = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\epsilon} > 0$, which implies that the expected speculative profits assessed by outsiders is,

$$(p_1 - p_2) E(\tilde{s}|p_2) = \frac{1}{\rho \sigma_\epsilon^2} \beta_p (p_1 - p_2)^2 \quad (5)$$

We have shown, in (5), that regardless of whether prices in the derivatives market move up or down relative to $p_1$, outsiders, in a historical cost regime, rationally believe that the firm must have earned positive speculative profits. Furthermore, the greater the change in price the higher is outsiders’ assessments of speculative profits. The intuition underlying this result is that on average an informed trader is on the right side of the market and makes profits from speculation. Thus, a historical cost regime would induce overly optimistic assessments of the firm’s financial condition if perchance the manager is caught on the wrong side of the market.

Continuing with expected value calculations, using standard Bayesian updating for Normally distributed variables,
\[ E(\bar{q}|x_i) = \beta_x x_i + (1 - \beta_x)\bar{q} \]  

(6)

where the regression coefficient \( \beta_x = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_\eta^2} \). Also,

\[ \text{Var}(\bar{q}|x_i) = (1 - \beta_x)\sigma_q^2 \]  

(7)

The calculation of outsider \( i \)'s expectation of \( \lambda \) is more subtle. Recall that \( \lambda \) is the proportion of outsiders that abandon the firm, so an assessment of \( \lambda \) requires a conjecture of the equilibrium abandon /stay decision rule. Given that the actions of all outsiders collectively affects the solvency of the firm, in deciding whether to abandon or stay with the firm, each individual outsider must not only take into account her own beliefs of the firm’s financial condition, but must also form beliefs about other outsiders’ beliefs. Suppose each individual outsider reasons as follows: Given the public observation of \( p_2 \) and given my private signal \( x_i \), if it is rational for me to abandon the firm then surely it must be rational for any other outsider who observes a signal lower than mine to also abandon the firm. Conversely, if it is rational for me to stay with the firm, then it must be rational for any outsider who has observed a higher signal than mine to also stay with the firm. The intuition suggested by such introspection, on the part of all outsiders, is that there is an equilibrium threshold \( x_h^* \) such that any outsider who observes a signal below \( x_h^* \) will abandon the firm, and any outsider who observes a signal above \( x_h^* \) will stay with the firm. The value of \( x_h^* \), in turn, would depend on the publicly observed price \( p_2 \). Below, we characterize such a threshold equilibrium. (A standard result in the global games literature is that if there is a unique threshold that constitutes an equilibrium, then there is no equilibrium of any other form.)

Given any threshold \( x_h^* \), an individual outsider \( i \), assesses \( \lambda \) as:

\[ E(\lambda|x_i) = \text{Probability}(x_j \leq x_h^*|x_i) \]  

(8)
where $x_j$ denotes the signal of any other outsider. Note that in the specification of (8), we have used the statistical fact that when there are a continuum of agents, the fraction of agents receiving signals below $x_h^*$ equals the probability of any agent receiving a signal below $x_h^*$.

Now, since $x_j = q + \eta_j$, the distribution of $x_j$ conditional on agent $i$'s private signal is Normal with,

$$E(x_j|x_i) = E(q|x_i), \text{ and}$$

$$Var(x_j|x_i) = Var(q|x_i) + \sigma^2$$

Using these facts,

$$E(\lambda|x_i) = \Phi \left( \frac{x_h^* - E(q|x_i)}{\sqrt{Var(q|x_i) + \sigma^2}} \right)$$

where $\Phi$ is the distribution function of the standard Normal.

Recall that outsider $i$ abandons the firm if $E(\tilde{w}|p_2, x_i) - \frac{1}{2} \rho \ Var(\tilde{w}|p_2, x_i) < 0$, and stays with the firm otherwise. Now,

$$Var(\tilde{w}|p_2, x_i) = (p_1 - p_2)^2 Var(\tilde{s}|p_2) + p_1^2 Var(\tilde{q}|x_i)$$

$$= (p_1 - p_2)^2 \beta \rho^2 \sigma^2 + p_1^2 Var(\tilde{q}|x_i)$$

Therefore, given $x_h^*$, outsider $i$ abandons the firm if,
\[ \frac{1}{2} \frac{1}{\rho \sigma^2_\epsilon} \beta_p (p_1 - p_2)^2 + p_1 E(q|x_i) - \frac{1}{2} \rho p^2_1 \text{Var}(q|x_i) - p_2 b \Phi \left( \frac{x_h^* - E(q|x_i)}{\sqrt{\text{Var}(q|x_i) + \sigma^2_\eta}} \right) < 0 \]  

Since \( E(q|x_i) \) is strictly increasing in \( x_i \) and \( \text{Var}(q|x_i) \) is constant, the left hand side of (10) is strictly increasing in \( x_i \). This implies that \( x_h^* \) is an equilibrium threshold if an individual outsider who observes \( x_i = x_h^* \) is indifferent between abandoning the firm and staying with the firm. Thus, we have established that an equilibrium in the historical cost regime is described by a threshold signal \( x_h^* \) that satisfies:

\[ \frac{1}{2} \frac{1}{\rho \sigma^2_\epsilon} \beta_p (p_1 - p_2)^2 + p_1 E(q|x_h^*) - \frac{1}{2} \rho p^2_1 \text{Var}(q|x_h^*) - p_2 b \Phi \left( \frac{x_h^* - E(q|x_h^*)}{\sqrt{\text{Var}(q|x_h^*) + \sigma^2_\eta}} \right) = 0 \]  

Within the setting of our model, a comparison of the economic consequences of the historical cost regime with the mark-to-market regime requires a comparison of (11) to a similar equilibrium in the mark-to-market regime.

5. Equilibrium in the Mark-to Market Regime

When accounting standards require derivatives to be marked-to-market, the accountant collects information on the firm’s derivatives trades that were actually made at date 1, observes the date 1 and date 2 prices in the derivatives market and reports the gain or loss on the firms derivatives position. This gain or loss is calculated on the aggregate derivative holdings \( z \), since the hedge and speculative components of these holdings cannot be disentangled. Thus, the accounting report released to the public at date 2 is:
\[ L = (p_1 - p_2)z \]

Since prices in the derivatives market are public information, the accounting report reveals the aggregate derivatives position \( z \) of the firm. However, this is not the only informational effect of releasing such a report. The report has a \textit{structural} effect on the manner in which outsiders assess the terminal income of the firm. We develop this effect below.

In the historical cost regime, where the firm’s derivatives position of \( z \) is unobserable, outsiders perceive the firm’s terminal wealth as \( \tilde{w} = (p_1 - p_2)(\tilde{q} + \tilde{s}) + p_2(\tilde{q} - b\lambda) \). Since the hedge component of \( z \) is perceived as a random variable and is assessed in exactly the same way as the real transaction, the term \(-p_2\tilde{q}\) is exactly offset by \(+p_2\tilde{q}\), and the assessment of terminal wealth collapses to

\[
(p_1 - p_2)E(\tilde{s}|p_2) + p_1E(\tilde{q}|x_i) - p_2bE(\lambda|p_2, x_i).
\]

However, in the mark-to-market regime, the accounting report \( L = (p_1 - p_2)z = (p_1 - p_2)q + (p_1 - p_2)s \) incorporates the \textit{actual} hedge and \textit{actual} speculative position of the firm, without revealing the separate values of \( q \) and \( s \). The term \(-p_2q\), contained in \( L \), does not exactly offset \(+p_2\tilde{q}\), because one expression incorporates the actual value of \( q \) while the other is a random variable whose expectation will be assessed. Given that \( L \) has been reported, an individual investor must assess the expected terminal wealth of the firm as:

\[
E(\tilde{w}|L, p_2, x_i) = L + p_2E(\tilde{q} - b\lambda|z, p_2, x_i)
\]

This calculation reflects a correction of the understatement contained in the accounting report.

This is clearly seen by rewriting (12) as,
\[ E(\tilde{w} | L, p_2, x_i) = [(p_1 - p_2)s + p_1q] - p_2q + p_2E(\tilde{q} | z, p_2, x_i) - p_2E(b\lambda | z, p_2, x_i) \]

The expression \([(p_1 - p_2)s + p_1q]\) is the firm’s true income, gross of the damage from abandonment, and this quantity is already contained in \(L\). However, \(L\) is obtained after subtracting \(p_2q\) from true income, and the expression \(+p_2E(\tilde{q} | z, p_2, x_i)\) is a correction of this accounting error. The correction is never perfect. If \(p_2 > p_1\), the estimation error \(\{E(\tilde{q} | z, p_2, x_i) - q\}\) has a larger effect on the assessed terminal wealth of the firm in the mark-to-market regime than in the historical cost regime, because in the mark-to-market regime the estimation error is multiplied by \(p_2\) while in the historical cost regime it is multiplied by \(p_1\).

Reasoning exactly as described in the historical cost regime, the equilibrium in the mark-to-market regime is described by a threshold \(x^*_m\) such that an individual outsider who privately observes \(x_i\) abandons the firm if,

\[
L + p_2 \left[ E(q | z, p_2, x_i) - b\Phi \left( \frac{x^*_m - E(q | z, p_2, x_i)}{\sqrt{\text{Var}(q | z, p_2, x_i) + \sigma^2_\eta}} \right) \right] - \frac{1}{2} \rho p_2^2 \text{Var}(q | z, p_2, x_i) < 0 \quad (13)
\]

and stays with the firm otherwise. Since the left hand side of the above expression is strictly increasing in \(x_i\), the equilibrium value of the threshold \(x^*_m\) is characterized by the condition that an investor receiving signal \(x_i = x^*_m\) is indifferent between abandoning and staying:

\[
L + p_2 \left[ E(q | z, p_2, x^*_m) - b\Phi \left( \frac{x^*_m - E(q | z, p_2, x^*_m)}{\sqrt{\text{Var}(q | z, p_2, x^*_m) + \sigma^2_\eta}} \right) \right] - \frac{1}{2} \rho p_2^2 \text{Var}(q | z, p_2, x^*_m) = 0 \quad (14)
\]

We proceed now to the calculation of \(E(q | z, p_2, x)\) for any arbitrary private signal \(x\). We use
repeated Bayesian updating to obtain insights into the nature of this expectation.

\[
E(q | z, p_2) = \bar{q} + \frac{\text{cov}(q, z | p_2)}{\text{var}(z | p_2)} [z - E(z | p_2)]
\]

Let,

\[
\beta_z \equiv \frac{\text{cov}(q, z | p_2)}{\text{var}(z | p_2)} = \frac{\sigma_q^2}{\sigma_q^2 + \text{var}(s | p_2)},
\]

where we have used \( \bar{z} = \bar{q} + \bar{s} \) and the fact that \( \bar{q} \) is independent of \( \bar{p}_2 \). Since \( E(z | p_2) = \bar{q} + E(s | p_2) \), we have:

\[
E(q | z, p_2) = \beta_z [z - E(s | p_2)] + (1 - \beta_z)\bar{q}
\]

(15)

The above expression characterizes the expectation of \( q \) conditional on the information that is publicly known. Notice that the availability of \( z \) forces outsiders to assess the firm’s speculative position, even though the income from the actual speculative position of the firm is already reflected in the accounting report of \( L \). However, this assessment enters outsiders’ calculation of the firm’s terminal income in a manner that is quite different from the historical cost regime. Using (15) as the prior expectation of \( \bar{q} \) and conditioning additionally on an individual outsider’s private signal gives,

\[
E(q | z, p_2, x) = \delta_x x + (1 - \delta_x)E(q | z, p_2)
\]

(16)

where,

\[
\delta_x = \frac{\text{cov}(q, x | z, p_2)}{\text{var}(x | z, p_2)} = \frac{\text{var}(q | z, p_2)}{\text{var}(q | z, p_2) + \sigma_n^2}
\]

(17)

Inserting the expression for \( E(q | z, p_2) \), derived in (15), gives:
The conditional expectation derived in (18) indicates that outsiders’ assessment of the firm’s future real transaction is a weighted average of three independent estimates of $q$, viz. $x$, $[z - E(s|p_2)]$, and the prior mean $\bar{q}$. The weights on each of these estimates is positive and sum to unity.

We now investigate the existence and uniqueness of threshold equilibria in the mark-to-market regime. Our findings here are important to the subsequent comparison of the two accounting regimes under consideration. Hereafter, we assume:

**Assumption (A)**

\[
\frac{\sigma_q^2 (v + \sigma_\eta^2)}{v^2 (2v + \sigma_\eta^2)} < \frac{2\pi}{b^2}
\]  

(19)

where,

\[
v \equiv \text{var}(q|z,p_2) = \sigma_q^2 \left(1 - \frac{\sigma_q^2}{\sigma_q^2 + \text{var}(s|p_2)}\right) = (1 - \beta_z)\sigma_q^2
\]  

(20)

Since $v$ does not depend on $\sigma_\eta^2$, the left hand side of (19) is strictly increasing in $\sigma_\eta^2$, and converges to zero as $\sigma_\eta^2 \to 0$. Therefore Assumption (A) is satisfied if $\sigma_\eta^2$ is small enough, i.e. if the private signals $x_i$ of individual outsiders is sufficiently precise.

**Lemma 1**

There exists a unique equilibrium threshold signal $x_m^*$ satisfying (14) if the private signals $x_i$ of individual outsiders is sufficiently precise, in the sense of Assumption (A).

**Proof**
For purposes of this proof, it is convenient to work in the space of posterior expectations of $q$.

Let,

$$
\xi^*_m \equiv E(q|z, p_2, x^*_m) = \delta_x x^*_m + (1 - \delta_x)E(q|z, p_2)
$$

Clearly, there is a one-to-one relationship between $\xi$ and $x$. Solving for $x^*_m$ gives,

$$
x^*_m = \frac{\xi^*_m}{\delta_x} - \left(\frac{1 - \delta_x}{\delta_x}\right) E(q|z, p_2)
$$

Stated in terms of $\xi^*_m$, the equilibrium condition described in (14) is:

$$
L + p_2 \left[ \xi^*_m - b\Phi \left( \left[ \frac{1 - \delta_x}{\delta_x} \right] [\xi^*_m - E(q|z, p_2)] \right) \right] - \frac{1}{2} \rho p_2^2 \text{Var}(q|z, p_2, x^*_m) = 0
$$

(21)

where $\gamma_m \equiv \sqrt{\text{Var}(q|z, p_2, x^*_m) + \sigma^2_\eta}$.

Equation (21) is uniquely satisfied if $\left[ \xi^*_m - b\Phi \left( \left[ \frac{1 - \delta_x}{\delta_x} \right] [\xi^*_m - E(q|z, p_2)] \right) \right]$ is strictly increasing in $\xi^*_m$. Differentiating with respect to $\xi^*_m$, we find that strict monotonicity is guaranteed if $1 - b\phi(.) \left[ \frac{1 - \delta_x}{\delta_x} \right] > 0$, where $\phi$ is the density function of the standard Normal distribution. Since $\phi$ is bounded above by $\frac{1}{\sqrt{2\pi}}$, it is sufficient that $\frac{1 - \delta_x}{\delta_x} < \frac{\sqrt{2\pi}}{b}$. Now, from (17), $\delta_x = \frac{v}{v + \sigma^2_\eta}$, so that $\frac{1 - \delta_x}{\delta_x} = \frac{\sigma^2_\eta}{v}$. Additionally,

$$
\text{var}(q|z, p_2, x^*_m) = v - \frac{\text{cov}^2(q, x|z, p_2)}{\text{var}(x|z, p_2)}
$$

$$
= v - \frac{v^2}{v + \sigma^2_\eta}
$$

$$
= \frac{v\sigma^2_\eta}{v + \sigma^2_\eta}
$$

Therefore,
\[
\gamma^2_m = \frac{v\sigma^2}{v + \sigma^2} + \sigma^2 = \frac{\sigma^2_\eta(2v + \sigma^2)}{v + \sigma^2_\eta}
\]  
(22)

which yields,

\[
\frac{1 - \delta}{\delta x m} = \left( \frac{\sigma^2_\eta(v + \sigma^2)}{v^2(2v + \sigma^2)} \right)^{\frac{1}{2}}
\]

Thus, Assumption (A) guarantees the strict monotonicity property that is sufficient for the existence and uniqueness of \( \xi^*_m \), and therefore for the existence and uniqueness of \( x^*_m \).

6. Comparison of the Historical Cost Regime to the Mark-to-Market Regime

A comparison of the two accounting regimes based on a rank ordering of the two thresholds, \( x^*_h \) and \( x^*_m \) is inappropriate. When \( x^*_h < x^*_m \) the probability of an individual outsider abandoning the firm is lower under the historical cost regime than under the mark-to-market regime, for every fixed \( q \). However, this probability may be inappropriately lower, so it would not follow that historical cost is "better" than mark-to-market accounting. What is more important is the change in the behavior of individuals and changes in the equilibrium threshold in relation to variations in the true income of the firm. From the perspective of date 2, which is the date at which individual outsiders make their decisions, the firm's true income is \( w_T = (p_1 - p_2)s + p_1q \). Here \( q \) is not a random variable, but rather the realized value of \( \tilde{q} \) perceived by the firm's manager. Treating \( p_1 > 0 \) as an exogenous constant, and noting that \( p_2 \) is a function of \( \theta \) and \( \epsilon \) and \( s \) is a function of \( \theta \), the primitive variables underlying the firm's true income are \( q, \theta \) and \( \epsilon \). These variables constitute the "fundamentals" or "states" underlying the firm's financial condition. An increase in \( p_2 \) caused by an increase in \( \theta \) is an increase in \( p_2 \) that was anticipated by the manager, while price changes caused by variations in \( \epsilon \) represent unanticipated changes in \( p_2 \). We compare the historical cost regime to the mark-to-market regime, by examining the change in outsiders' decisions and the change in equilibrium thresholds as the the fundamentals vary.
Variations in \( q \)

**The Historical Cost Regime:**

We first examine how variations in \( q \) affect the equilibrium in the historical cost regime. Given that \( p_1 > 0 \), the true income of the firm is strictly increasing in \( q \). However, in the historical cost regime, the decisions of outsiders and the equilibrium threshold \( x_h^* \) are invariant to changes in \( q \), as is apparent from (10) and (11). This does not mean that nothing changes in the historical cost regime. Let \( \lambda_h(q) \) be the equilibrium fraction of outsiders that abandon the firm given the state variable \( q \). Clearly,

\[
\lambda_h(q) = \Phi \left( \frac{x_h^*-q}{\sigma}\right),
\]

so that \( \lambda_h'(q) = -\frac{1}{\sigma} \phi \left( \frac{x_h^*-q}{\sigma}\right) < 0 \). A larger value of \( q \) decreases the probability that outsiders receive signals below \( x_h^* \), implying that a smaller fraction of outsiders abandon the firm as \( q \) increases.

**The Mark-to-Market Regime:**

We now examine variations in \( q \) in the mark-to-market regime. Since the firm’s derivatives position is \( z = q + s \), an increase in \( q \) causes an increase in \( z \). In turn, this leads to a change in the accounting report \( L \) and a change in outsider’s expectation of \( q \) through \( E(q|z,p_2,x_i) \). We first calculate the change in an individual outsider’s assessment of the firm’s terminal income induced by an increase in \( q \), then we derive the implications of these assessments for the change in the equilibrium threshold \( x_m^* \), and finally we examine the change in the equilibrium fraction of outsiders who abandon the firm.

**Proposition 1**

In the mark-to-market regime, given any threshold \( x_m^* \) and any private signal \( x_i \), an outsider’s assessment of the firm’s terminal income is strictly decreasing in \( q \), if \( p_2 \) is sufficiently greater than \( p_1 \).
Proof

For fixed $x_m^*$ and $x_i$, an individual outsider’s assessment of the firm’s risk adjusted terminal income is described by the left hand side of (13). Call this expression $\Omega_m$. Differentiating with respect to $q$ gives,

$$\frac{\partial \Omega_m}{\partial q} = \frac{\partial z}{\partial q} \left[ \frac{\partial L}{\partial z} + p_2 \frac{\partial \{ E(q|z,p_2,x_i) \} }{\partial z} \right] + p_2 b \phi(.) \frac{\partial \{ E(q|z,p_2,x_i) \} }{\partial z} $$

Inserting $\frac{\partial z}{\partial q} = 1$, $\frac{\partial L}{\partial z} = (p_1 - p_2)$, and $\frac{\partial \{ E(q|z,p_2,x_i) \} }{\partial z} = (1 - \delta_x) \beta_z$ yields,

$$\frac{\partial \Omega_m}{\partial q} = p_1 - p_2 \left[ 1 - (1 - \delta_x) \beta_z \left( 1 + b \phi(.) \frac{1}{\gamma_m} \right) \right]$$

Thus if $\left[ 1 - (1 - \delta_x) \beta_z \left( 1 + b \phi(.) \frac{1}{\gamma_m} \right) \right] > 0$ it follows that $\frac{\partial \Omega_m}{\partial q} < 0$ if $p_2$ is sufficiently large. We show below that the former inequality is implied by Assumption (A).

Since $\phi(.)$ is bounded above by $\frac{1}{\sqrt{2\pi}}$, it suffices to prove that $\left( 1 - \delta_x \right) \beta_z \left( 1 + \frac{b \phi(.)}{\gamma_m} \right) < 1$, or equivalently

$$\frac{2\pi}{b^2} > \left( \frac{(1 - \delta_x) \beta_z}{1 - (1 - \delta_x) \beta_z} \right)^2 \frac{1}{\gamma_m^2}$$

Inserting the value of $\gamma_m^2$ calculated in (22), the above inequality is equivalent to,

$$\frac{2\pi}{b^2} > \left( \frac{(1 - \delta_x) \beta_z}{1 - (1 - \delta_x) \beta_z} \right)^2 \frac{v + \sigma^2}{\sigma^2 (2v + \sigma^2)}$$

Given Assumption (A) the above inequality is true if,

$$\frac{(1 - \delta_x) \beta_z}{1 - (1 - \delta_x) \beta_z} \leq \frac{\sigma^2}{v}$$

Inserting $\beta_z = \frac{\sigma^2}{\sigma^2 + \text{var}(s|p_2)}$, the above inequality is equivalent to,
\[
\frac{(1 - \delta_x)\sigma_q^2}{\delta_x \sigma_q^2 + \text{var}(s|p2)} \leq \frac{\sigma^2}{v}
\]

(23)

Now, from the definition of \(v\) in (20), \(\text{var}(s|p2) = \frac{v\sigma_q^2}{\sigma_q^2 - v}\). Also \(\delta_x = \frac{v}{v + \sigma_q^2}\), and \(1 - \delta_x = \frac{\sigma_q^2}{v + \sigma_q^2}\).

Therefore,

\[
\frac{(1 - \delta_x)\sigma_q^2}{\delta_x \sigma_q^2 + \text{var}(s|p2)} = \frac{\frac{\sigma_q^2 \sigma_q^2}{v + \sigma_q^2}}{\frac{\sigma_q^2}{v + \sigma_q^2} + \frac{\sigma_q^2}{\sigma_q^2 - v}} \\
= \frac{\sigma_q^2}{v(v + \sigma_q^2 + v(\sigma_q^2 - v))} \\
= \frac{\sigma_q^2 \sigma_q^2 - v}{v(\sigma_q^2 + \sigma_q^2)}
\]

Therefore, the desired inequality (23) is equivalent to,

\[
\frac{\sigma_q^2 - v}{\sigma_q^2 + \sigma_q^2} < 1
\]

which is obviously true since \(v > 0\). This completes the proof of the Proposition.

Proposition 1 describes a perverse consequence of mark-to-market accounting. Even though the firm’s true income is strictly increasing in \(q\), the income assessed by each individual investor is strictly \textit{decreasing} in \(q\). This is because mark-to-market accounting calculates the loss on the entire derivative position \(z = q + s\) even though there really is no loss on the \(q\) component of \(z\).

On the other hand, the expected value of \(q\), \(E(q|z, p_2, x_i)\), increases at the rate \((1 - \delta_x)\beta_z < 1\). The reported loss increases more steeply with \(q\) than the offsetting increase in \(p_2E(q|z, p_2, x_i)\) if \(p_2\) is sufficiently bigger than \(p_1\).

The perverse assessments of individual outsiders has a perverse effect on the equilibrium threshold \(x_{m}^s\), as shown in the next proposition.
Proposition 2

In the mark-to-market regime the equilibrium threshold \( x_m^* \) is strictly increasing in \( q \) if \( p_2 \) is sufficiently greater than \( p_1 \).

Proof

The threshold \( x_m^* \) is characterized by the equilibrium condition:

\[
L + p_2 \left[ E(q|z, p_2, x_m^*) - b \Phi \left( \frac{x_m^* - E(q|z, p_2, x_m^*)}{\sqrt{Var(q|z, p_2, x_m^*) + \sigma_2^2}} \right) \right] = \frac{1}{2} \rho p_2^2 V ar(q|z, p_2, x_m^*)
\]

The right hand side of the above equation is a constant that does not vary with \( z \) and \( x_m^* \). In Proposition 1, we established that if if \( p_2 \) is sufficiently greater than \( p_1 \) and assumption (A) is satisfied,

\[
\frac{\partial}{\partial q} \left\{ L + p_2 \left[ E(q|z, p_2, x_i) - b \Phi \left( \frac{x_m^* - E(q|z, p_2, x_i)}{\sqrt{Var(q|z, p_2, x_i) + \sigma_2^2}} \right) \right] \right\} < 0
\]

Since this derivative is negative for every \( x_i \), it is also negative at \( x_i = x_m^* \). In the proof of Lemma 1, we established that

\[
\frac{\partial}{\partial x_m^*} \left\{ L + p_2 \left[ E(q|z, p_2, x_m^*) - b \Phi \left( \frac{x_m^* - E(q|z, p_2, x_m^*)}{\sqrt{Var(q|z, p_2, x_m^*) + \sigma_2^2}} \right) \right] \right\} > 0
\]

It follows that when \( q \) is increased \( x_m^* \) must also increase to restore the equilibrium.

The above results do not, by themselves, imply that in the mark-to-market regime the equilibrium fraction of outsiders abandoning the firm increases with \( q \). This is because even though the threshold increases with \( q \) the probability of receiving a signal below the threshold decreases with \( q \). We establish below that the first force dominates the second when \( p_2 \) is sufficiently bigger
Proposition 3

In the mark-to-market regime, the equilibrium fraction of outsiders that abandon the firm is strictly increasing in $q$ if $p_2$ is sufficiently greater than $p_1$.

Proof

Let $\lambda_m(q) \equiv \text{Prob}(x \leq x^*_m|q) = \Phi\left(\frac{x^*_m - q}{\sigma_m}\right)$ be the equilibrium fraction of outsiders who abandon the firm in state $q$. Since $\Phi(.)$ is strictly increasing in its argument, $\lambda'_m(q) > 0$ iff $\frac{\partial x^*_m}{\partial q} > 1$. Differentiating the equilibrium condition (14) with respect to $q$ gives,

$$\frac{\partial \Omega_m}{\partial q} + \frac{\partial x^*_m}{\partial q} p_2 \left[ \frac{\partial \{E(q|z,p_2,x^*_m)\}}{\partial x^*_m} - b \frac{\phi(.)}{\gamma_m} \left(1 - \frac{\partial \{E(q|z,p_2,x^*_m)\}}{\partial x^*_m}\right) \right] = 0$$

Inserting $\frac{\partial \{E(q|z,p_2,x^*_m)\}}{\partial x^*_m} = \delta_x$ and $\frac{\partial \Omega_m}{\partial q} = p_1 - p_2 \left[1 - (1 - \delta_x)\beta_z(1 + b \frac{\phi(.)}{\gamma_m})\right]$ into the above equation yields,

$$\frac{\partial x^*_m}{\partial q} p_2 \left[\delta_x - (1 - \delta_x) b \frac{\phi(.)}{\gamma_m}\right] + p_1 - p_2 \left[1 - (1 - \delta_x)\beta_z(1 + b \frac{\phi(.)}{\gamma_m})\right] = 0 \quad (24)$$

In Propositions 1 and 2, we established that when $p_2 > 0$ and $p_1 - p_2 \left[1 - (1 - \delta_x)\beta_z(1 + b \frac{\phi(.)}{\gamma_m})\right] < 0$ then $\frac{\partial x^*_m}{\partial q} > 0$. Under these conditions (24) implies that $\delta_x - (1 - \delta_x) b \frac{\phi(.)}{\gamma_m} > 0$. Solving from (24) gives,

$$\frac{\partial x^*_m}{\partial q} = \frac{1 - (1 - \delta_x)\beta_z(1 + b \frac{\phi(.)}{\gamma_m})}{\delta_x - (1 - \delta_x) b \frac{\phi(.)}{\gamma_m}} - \frac{p_1}{p_2 [\delta_x - (1 - \delta_x) b \frac{\phi(.)}{\gamma_m}]}$$

Now, $1 - (1 - \delta_x)\beta_z = \delta_x + (1 - \delta_x)(1 - \beta_z) > \delta_x$ and $\beta_z < 1$. Therefore,
$$\frac{1 - (1 - \delta_x)\beta_z(1 + \varphi_{\gamma m})}{\delta_x - (1 - \delta_x)\varphi_{\gamma m}} > 1$$

From these facts, it follows that $\frac{\partial w_T}{\partial q} > 1$ when $\frac{p_1}{p_2}$ is sufficiently small. This completes the proof.

Ideally, when the true income of the firm increases due to increases in its future real transaction $q$, outsiders’ assessments of the firm’s financial position ought to improve and the probability of abandoning the firm ought to decrease. Neither accounting regime has both of these desirable attributes. However, when there is a sufficiently large price increase in the derivatives market, mark-to-market accounting behaves perversely while historical cost accounting is directionally consistent with the improving profitability of the firm. Under mark-to-market accounting, when true income is increasing due to increases in the firm’s future real transaction, outsiders are rationally deceived into believing that the firm’s financial position is deteriorating, and the firm is increasingly damaged due to increasing abandonment, ultimately throwing the firm into insolvency. This perverse characteristic of mark-to-market accounting is entirely due to its reporting an increasingly larger fictitious loss on the firm’s derivatives position and the inability of outsiders to perfectly undo this error.

**Variations in $p_2$ that are unanticipated by the manager:**

An increase in $p_2$ caused by an increase in $\epsilon$ is unanticipated by the manager, since the manager obtains information about $\theta$ but not about $\epsilon$. This implies that the firm’s derivatives position $z$ remains fixed as $p_2$ increases. Since the firm’s true income $w_T = (p_1 - p_2) s + p_1 q$, $\frac{\partial w_T}{\partial p_2} = -s$. Therefore, the firm’s true income increases with $p_2$ if the firm has taken a long speculative position in the market, i.e. if it has observed $\theta > 0$. Conversely, the firm’s true income decreases with $p_2$ if the firm has taken a short speculative position, i.e. if it is on the wrong side of the market.

**The mark-to-market regime:**
We now examine how outsiders’ assessments of the firm’s income changes with \( p_2 \), in the mark-
to-market regime. Consider the marginal outsider, i.e. an outsider who receives signal \( x_i = x^*_m \), and
who is consequently indifferent between abandoning or staying with the firm. The risk adjusted
income of the firm, as assessed by this marginal outsider, satisfies the equilibrium condition:

\[
\Omega_m(x^*_m) = L + p_2 \left[ E(q \mid z, p_2, x^*_m) - b\Phi \left( \frac{x^*_m - E(q \mid z, p_2, x^*_m)}{\sqrt{Var(q \mid z, p_2, x^*_m) + \sigma_n^2}} \right) \right] - \frac{1}{2} \rho p_2^2 Var(q \mid z, p_2, x^*_m) = 0
\]  

(25)

Holding \( x^*_m \) fixed and differentiating with respect to \( p_2 \) gives,

\[
\frac{\partial \Omega_m}{\partial p_2} = -z + E(q \mid z, p_2, x^*_m) - b\Phi(.) - \rho p_2 Var(q \mid z, p_2, x^*_m) + p_2 \left( 1 + \frac{b\phi}{\gamma_m} \right) \frac{\partial E(q \mid z, p_2, x^*_m)}{\partial p_2} 
\]

Since, \( \frac{\partial E(q \mid z, p_2, x^*_m)}{\partial p_2} = (1 - \delta_x)\beta_z \frac{\partial E(s \mid p_2)}{\partial p_2} = (1 - \delta_x)\beta_z \rho \text{var}(s \mid p_2) \),

\[
\frac{\partial \Omega_m}{\partial p_2} = -z + E(q \mid z, p_2, x^*_m) - b\Phi(.) - \rho p_2 Var(q \mid z, p_2, x^*_m) + p_2 \left( 1 + \frac{b\phi}{\gamma_m} \right) (1 - \delta_x)\beta_z \rho \text{var}(s \mid p_2) \]  

(26)

Now, from the equilibrium condition (25),

\[
-z + E(q \mid z, p_2, x^*_m) - b\Phi(.) - \rho p_2 Var(q \mid z, p_2, x^*_m) = -\frac{p_2}{2} p_2 Var(q \mid z, p_2, x^*_m) - \frac{p_1}{p_2} z
\]

Inserting this into (26) gives,
\[ \frac{\partial \Omega_m}{\partial p_2} = p_2 \left[ \left( 1 + \frac{b\phi}{\gamma_m} \right) (1 - \delta_z) \beta_z \rho \text{var}(s \mid p_2) - \frac{\rho}{2} \text{var}(q \mid z, p_2, x_m^*) \right] - \frac{p_1}{p_2} z \]  

(27)

Calculations yield,

\[(1 - \delta_z) \beta_z = \left( \frac{\sigma^2_{\eta}}{v + \sigma^2_{\eta}} \right) \left( \frac{\sigma^2_q - v}{\sigma^2_q} \right), \]

\[\text{Var}(q \mid z, p_2, x_m^*) = \frac{v \sigma^2_{\eta}}{v + \sigma^2_{\eta}},\]

\[\text{Var}(s \mid p_2) = \frac{v \sigma^2_q}{\sigma^2_q - v}.\]

Inserting these calculations into (27),

\[\frac{\partial \Omega_m(x_m^*)}{\partial p_2} = p_2 \left[ \left( \frac{\sigma^2_{\eta}}{v + \sigma^2_{\eta}} \right) \left( \frac{\sigma^2_q - v}{\sigma^2_q} \right) \rho \left( \frac{v \sigma^2_q}{\sigma^2_q} \right) \left( 1 + \frac{b\phi}{\gamma_m} \right) - \frac{\rho}{2} \left( \frac{v \sigma^2_{\eta}}{v + \sigma^2_{\eta}} \right) \right] - \frac{p_1}{p_2} z \]

\[= p_2 \rho \left( \frac{v \sigma^2_{\eta}}{v + \sigma^2_{\eta}} \right) \left( \frac{1}{2} + \frac{b\phi}{\gamma_m} \right) \text{Var}(q \mid z, p_2, x_m^*) - \frac{p_1}{p_2} z \]

The following results follow from the last expression derived above.

**Proposition 4**

In the mark-to-market regime:

(i) If \( z < 0 \) then \( \frac{\partial(x_m^*)}{\partial p_2} < 0 \), i.e. the equilibrium threshold decreases with \( p_2 \).

(ii) If \( z > 0 \) there exists a critical value \( \hat{p}_2 \) such that \( p_2 < \hat{p}_2 \Rightarrow \frac{\partial(x_m^*)}{\partial p_2} > 0 \), and \( p_2 > \hat{p}_2 \Rightarrow \frac{\partial(x_m^*)}{\partial p_2} < 0 \). The critical value \( \hat{p}_2 \) is strictly decreasing in \( \sigma^2_{\eta} \).
When the firm is in a speculative long position (i.e. $s < 0$) and the price in the derivatives market rises, it is on the right side of the market so its financial condition improves. Proposition 4 indicates that, under these circumstances, mark-to-market accounting is directionally consistent with the firm’s true financial condition if either its aggregate position in the derivatives market is a long position (i.e. $q + s < 0$), or if the price in the derivatives market increases beyond a critical point ($p_2 > \hat{p}_2$). But, when the firm’s aggregate position in the derivatives market is a short position and its speculative position is long (i.e. $s < 0, q > -s$), mark-to-market accounting deceives outsiders into behaving perversely when the rise in price is modest ($p_2 < \hat{p}_2$). Conversely when the firm has acquired a short speculative position ($s > 0, q + s > 0$) so that its true financial condition deteriorates due to price increases in the derivatives market, mark-to-market accounting deceives outsiders into behaving perversely when there is a big run up in prices ($p_2 > \hat{p}_2$).

Authors Note: For lack of time, we provide here only a summary of the remaining results. These results will be fleshed out in a more complete version of the paper, which will follow shortly.

The historical cost regime:

In our discussion of the historical cost regime, we derived the result that any movement in $p_2$ away from $p_1$ increases outsiders’ assessments of the profits from speculation, while leaving outsiders’ expectation of $q$ unchanged. Differentiating the left hand side of (10), which describes the risk adjusted income of the firm $\Omega_h$, as assessed by outsiders in the historical cost regime, we obtain:

\[
\frac{\partial \Omega_h}{\partial p_2} = -E(s\mid p_2) - b\Phi \left( \frac{x_h^* - E(q\mid x_i)}{\gamma_h} \right)
\]

where $\gamma_h = \sqrt{Var(q\mid x_i) + \sigma_q^2}$ and $-E(s\mid p_2) = (p_2 - p_1) \frac{\delta p}{\delta p_2^2} > 0$ when $p_2 > p_1$.

The above implies that any increase in $p_2$ above $p_1$ increases outsiders’ assessments of the
firm’s financial position provided that the estimated long speculative position of the firm exceeds the expected damage to the firm’s future transaction. This assessment is directionally consistent with the firm’s true income if the firm has indeed taken a long speculative position in the market, but is directionally inconsistent with the firm’s true income if the firm has taken a short speculative position. In general, historical cost accounting is misleading when the manager is inadvertently caught on the wrong side of the market.

**Variations in \( p_2 \) that are anticipated by the manager:**

To be added.
References


