Cost of Capital in Imperfect Competition Settings

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Abstract

This paper analyzes the role of information in pricing and cost of capital in securities markets characterized by imperfect competition among investors. Imperfect competition results in markets being less than perfectly liquid. Our analysis demonstrates that the interaction between illiquid markets and asymmetric information gives rise to a role for information in cost of capital that is absent in perfect competition settings such as the Capital Asset Pricing Model. Our results are relevant to a large empirical literature that examines the relation between various information attributes and the cost of capital.

Keywords: Cost of Capital, imperfect competition, information asymmetry

1. INTRODUCTION

This paper analyzes the role of information in pricing and cost of capital in security markets characterized by imperfect competition: that is, in markets that are less than perfectly liquid. Standard setters and policy makers often claim that high-quality information and a level playing field are essential to the efficient allocation of capital in the economy. For example, Robert Herz, chairman of the Financial Accounting Standards Board states: It's about lowering the cost of capital, lowering the cost of preparation, and lowering the cost of using it, (see Wild, 2004). Similarly, Arthur Levitt, former chairman of the Securities and Exchange Commission, argues that high quality accounting standards ... improve liquidity [and] reduce capital costs, (see Levitt, 1997).

Yet information issues are largely absent in conventional models of asset pricing and cost of capital. For example, in a Capital Asset Pricing Model (CAPM) framework, all investors are presumed to have homogeneous information, so issues that arise from information asymmetry are precluded from occurring. Moreover, only non-diversifiable risk is priced, and the relevant non-diversifiable risk is the covariance of the firm's cash flows with the market. Therefore, the only way information can affect cost of capital is through its impact on this covariance. In noisy rational expectations models (e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981), heterogeneous information plays a prominent role, and the aggregation of this information through price is an important part of the analysis. The literature finds, however, that only the *average* precision of investors' information matrices (e.g., the inverse of their assessed covariance matrices) is relevant in deriving the equilibrium cost of capital (see, e.g., Lintner, 1969; Admati, 1985; and Lambert, Leuz, and Verrecchia, 2010). Information asymmetry across investors can affect cost of capital, but only through its effect on investors' average precision.

Controlling for average precision, the degree of information asymmetry across investors (e.g., the amount that investors' information precisions differ from the average) does not affect cost of capital.

As discussed in Merton (1989) and O'Hara (2003), one feature that the above theoretical models have in common is that they assume market prices are based on perfect competition. That is, all investors act as price takers, and they can buy and sell any quantity at the market price: markets are perfectly liquid.[1] In contrast, imperfect competition and asymmetric information are common features of market microstructure models going back to Kyle (1985) and Glosten and Milgrom (1985). Moreover, these models find that information asymmetry can affect market features such as bid-ask spreads.

Our paper contributes to the extant literature in a variety of ways. First, we offer a capital market model that generalizes the single-firm, imperfect competition setting of Kyle (1989) to multiple firms. Specifically, we consider a market that consists of J risky assets and one risk-free asset. In addition, we assume that there are two types of investors: a more-informed type that has the ability to condition its demands on private information, and a less-informed type that has no information beyond that which it can glean from price. We model imperfect competition by assuming that investors understand that the magnitude of their demand for firm shares can affect the price at which their demand is fulfilled, and this introduces a cost to trading.[2] Moreover, the

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[1]While securities markets for heavily followed, heavily traded firms are often held out as prime examples of settings that approximate perfect competition, there is growing empirical evidence that suggests that liquidity concerns can be important even for these cases (witness the recent market meltdown). For example, general asset pricing models such as Pastor and Stambaugh (2003) and Acharya and Pederson (2005) find that liquidity factors explain cross-sectional variation in expected returns not accounted for by conventional CAPM-betas. Other empirical papers that allude to liquidity risk include Chordia, Roll, and Subrahmanyam (2000) and Sadka (2006). Moreover, there are large numbers of firms with relatively light and infrequent trading volume, and firms where concerns about insider trading are large.

[2]For example, the literature on block trades shows positive (negative) price effects arising from large purchases (sales) by investors: see Kraus and Stoll (1972), Holthausen, Leftwich, and Mayers (1990), Chan and Lakonishok (1995), and

degree to which the market is illiquid (or the magnitude of the cost of trading) is endogenously determined, not exogenously imposed.[3] While our model derives from prior work, it is important to point out that Kyle (1989) does not discuss the cost of capital or its determinants; instead, Kyle focuses on issues of the existence and uniqueness of an equilibrium, and price efficiency.

Second, we show that our generalization to *J* risky assets and one risk-free asset extends the CAPM to an imperfect competition setting where investors have rational expectations about the market process. As with the CAPM, the multiple-firm dimension of our analysis is essential in that it allows us to distinguish between diversifiable and non-diversifiable risk.

Third, we compare and contrast the behavior of cost of capital in our imperfect competition, multi-asset setting with its behavior in a CAPM setting. Here, as is standard in the literature, we define cost of capital to be the extent to which investors discount firms' share prices relative to the expected value of firms' cash flows. To highlight the impact of asymmetric information and imperfect competition, we compare cost of capital in settings where we hold investors' average precision constant. We ask three questions: 1) how does information asymmetry affect the cost of capital; 2) how does the level of competition among informed investors affect cost of capital; and 3) how do information asymmetry and competition interact? Consistent with the results in Lambert, et al. (2010), our analysis demonstrates the importance of distinguishing between the precision of investors' information from the degree of information asymmetry across

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Keim and Madhavan (1997). While part of this effect is temporary and attributed to liquidity issues, there is also a permanent component that is attributed to the possible information-based motivation for the trade. In addition, a large literature has found positive price effects associated with index inclusion. See for example Harris and Gurel (1986), Shleifer (1986), Jain (1987), Dhillon and Johnson (1991), Beneish and Whaley (1996), Lynch and Mendenhall (1997), and Wurgler and Zhuravskaya (2002). These effects, which are generally found to be permanent, are attributed to the increased demand for shares of these firms by institutional investors whose investment policies lead them to mimic the composition of the index.

[3]In contrast, other papers incorporate illiquidity by imposing exogenous trading costs in their analysis. See Acharya and Pederson (2005) for an example.

investors. Controlling for average precision, the degree of information asymmetry and level of competition have no effect on cost of capital in settings such as the CAPM, which is premised on the assumption that markets are (approximately) perfectly competitive. In more illiquid markets, however, where perfect competition is less descriptive, information asymmetry and competition *can* affect the liquidity of the market and therefore the cost of capital.

Our results are relevant to a large empirical literature in accounting that examines the relation between various information attributes and the cost of capital.[4] Specifically, our results demonstrate that for empirical work that attempts to assess the effect of information in general, and information asymmetry in particular, on the cost of capital, the degree of competition among investors *matters*. Claims about the behavior of cost of capital that may be true in an imperfect competition setting may not hold in the CAPM (and vice versus). In other words, we believe that it is incumbent upon empirical work that investigates the relation between information and cost of capital to control for the level of competition. This is consistent with a forthcoming paper, Armstrong, Core, Taylor, Verrecchia, (2010), that shows that information asymmetry has a positive relation with firms' cost of capital in excess of standard risk factors when markets are imperfect, and no relation when markets approximate perfect competition. In other words, the degree of market competition is an important conditioning variable when examining the relation between information asymmetry and cost of capital.

In Section 2 we extend the CAPM to an imperfect competition, asymmetric information setting. In Section 3 we compare and contrast cost of capital in our imperfect competition

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[4]For recent examples, see: Botosan, Plumlee, and Xie (2004); Francis, La Fond, Olsson, and Schipper (2004, 2005); Botosan and Plumlee (2007); Liu and Wysocki (2007); Cohen (2008); Core, Guay and Verdi (2008); Mohanram and Rajgopal (2008); Ng (2008); Ogneva (2008); Ashbaugh-Skaife, Collins, Kinney, Lafond (2009); and Bhattacharya, Ecker, Olsson and Schipper (2009).

extension to the cost of capital in the CAPM. In a concluding section we summarize our results.

2. AN IMPERFECTLY COMPETITIVE CAPITAL MARKET

In this section, we extend the CAPM into an imperfect competition, asymmetric information setting. We consider a one-period capital market with J firms and a risk-free asset whose price is normalized to 1. Let $\tilde{\mathbf{V}}$ denote the $J \times 1$ vector of end-of-period firms' cash flows, where the j-th element of the vector, $\tilde{V}_j$, is the cash flow of the j-th firm. Let $E[\tilde{\mathbf{V}}]$ denote the *a priori* vector of expected values of firms' cash flows, and $\mathbf{Cov}_v$ and $\mathbf{\Pi}_v$ the *a priori* covariance and precision matrices for firms' cash flows, respectively.[5] Let $\tilde{\mathbf{P}}$ denote the $J \times 1$ vector of beginning-of-period firms' prices, or market values, where $\tilde{P}_j$ is the price of the j-th firm. This implies that purchasing a single share of each firm yields an end-of-period return of $(\tilde{\mathbf{V}} - \tilde{\mathbf{P}})^T \cdot \mathbf{1}$, where $\mathbf{1}$ is a $J \times 1$ vector of 1's and T denotes the transpose operator.

We incorporate information asymmetry into the capital market by having two types of investors: N informed investors and M uninformed investors. To distinguish between the two types, henceforth we subscript parameters and activities associated with informed and uninformed investors by I and U, respectively. We assume that each investor has a negative exponential utility function with constant absolute risk tolerance r_t, where t distinguishes an investor's type, i.e., $t \in \{I, U\}$. Let $\mathbf{\Phi}_t$, $t \in \{I, U\}$, represent the information available to an investor of type t. Along with prices, each informed investor observes the same $J \times 1$ vector of private information $\tilde{\mathbf{X}} = \tilde{\mathbf{V}} + \tilde{\boldsymbol{\varepsilon}}$, where $\tilde{\boldsymbol{\varepsilon}}$ is a $J \times 1$ vector of error terms whose expected value is $\mathbf{0}$ and whose

[5]Henceforth we use a tilde (i.e., $\sim$) to distinguish a random variable from a fixed element, and put vectors and matrices in bold to distinguish them from scalars.

precision matrix is $\mathbf{\Pi}_{\varepsilon}$. In short, $\mathbf{\Phi}_I = \{\tilde{\mathbf{P}} = \mathbf{P}, \tilde{\mathbf{X}} = \mathbf{X}\}$. Henceforth let $\mathbf{\Pi}_t$ denote the $J \times J$ *posterior* precision matrix an investor of type $t \in \{I, U\}$ associates with cash flows. When an informed investor observes $\tilde{\mathbf{X}} = \tilde{\mathbf{V}} + \tilde{\boldsymbol{\varepsilon}}$, the precision of his beliefs about firms' cash flows is $\mathbf{\Pi}_I = \mathbf{\Pi}_v + \mathbf{\Pi}_{\varepsilon}$. Moreover, a straightforward application of Bayes' Theorem implies that the expected value an informed investor assigns cash flows based on his information is $E[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I] = \mathbf{\Pi}_I^{-1}[E[\tilde{\mathbf{V}}] + \mathbf{\Pi}_{\varepsilon}(\mathbf{X} - E[\tilde{\mathbf{V}}])]$.

Imperfect competition settings are complex. In an attempt to grapple with this complexity, in this section we break down the derivation of firms' cost of capital into a series of steps. The first step provides an expression for an informed investor's demand for firms' shares based on a belief that his demand affects the price he pays to buy shares. The second step derives an uninformed investor's demand for firms' shares. The third step provides an expression for the price vector based on a requirement that the total demand for firms' shares equals the supply of those shares. The four step establishes that an equilibrium in the capital market relies on the solution to two endogenous variables: the element of illiquidity that informed investors face, and precision of the information uninformed investors glean from price. Having established the conditions for an equilibrium, in the final step we derive an expression for the cost of capital based on the extent to which investors discount price at the beginning of the period (i.e., in expectation) relative to the expected value of the firm's cash flow.

2.1 An informed investor's demand for firms' shares

We represent an informed investor's demand for firms' shares by the $J \times 1$ vector $\mathbf{D}_I$. Following Kyle (1989), we characterize imperfect competition as the self-sustaining belief by each investor that he faces an upwardly-sloping price curve for firm shares. In particular, we assume each informed investor believes that his demand is related to the price vector through the expression

$$\mathbf{P} = \mathbf{p}_0 + \mathbf{\Lambda} \cdot \mathbf{D}_I, \tag{1}$$

where $\mathbf{p}_0$ is a $J \times 1$ intercept vector that incorporates all elements of the price vector that are *not* related to an investor's demand, and $\mathbf{\Lambda}$ is a $J \times J$ matrix of coefficients. In effect, each informed investor believes that prices result from a factor this is *unrelated* to his demand, $\mathbf{p}_0$, and a factor that is related to his demand through the matrix coefficient $\mathbf{\Lambda}$. The goal of our series of steps is to solve for $\mathbf{P}$, $\mathbf{p}_0$, $\mathbf{\Lambda}$, and $\mathbf{D}_I$, all of which are endogenous variables that must be derived to achieve an expression for cost of capital. As is standard in a model of imperfect competition, we interpret $\mathbf{\Lambda}$ as the degree of illiquidity associated with an individual informed investor's demand. For example, when the row/column elements of $\mathbf{\Lambda}$ are small, an informed investor's demand moves price less, and thus the market for firm shares is more liquid with respect to demand; when the row/column elements of $\mathbf{\Lambda}$ are large, an informed investor's demand moves price more, and thus the market is less liquid for firm shares.

When investors have negative exponential utility functions and random variables have multi-variate normal distributions, the certainty equivalent of each investor's expected utility simplifies into the familiar expression of the expected value of his end-of-period wealth minus a term that is proportional to the variance of his wealth. Let $\mathbf{\Phi}_I$ represent the information available to each identically informed investor. We assume that each investor believes that the vector of firms' cash flows, $\tilde{\mathbf{V}}$, has a $J \times 1$ expected value of $E[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I]$ and a $J \times J$ covariance matrix of $\mathbf{Cov}_I$. In other words, each investor believes the expected cash flow of firm j is $E[\tilde{V}_j \mid \mathbf{\Phi}_I]$, and assesses the covariance between the cash flows of firms j and k to be $Cov_I[\tilde{V}_j, \tilde{V}_k]$, where

$Cov_I\left[\tilde{V}_j, \tilde{V}_k\right]$ is the j -th row, k -th column element of the matrix $\mathbf{Cov}_I$. Thus, based on his belief as to how his demand affects prices, an investor chooses $\mathbf{D}_I$ to maximize the following objective function

$$[E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right] - \left(\mathbf{p}_0 + \mathbf{\Lambda}\mathbf{D}_I\right)]^T \mathbf{D}_I - \frac{1}{2r_I}\mathbf{D}_I^T \mathbf{Cov}_I \mathbf{D}_I. \tag{2}$$

Taking the derivative with respect to $\mathbf{D}_I$ and re-arranging terms yields

$$\begin{aligned} \mathbf{D}_I &= \left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1}\left(E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right] - \left(\mathbf{p}_0 + \mathbf{\Lambda}\mathbf{D}_I\right)\right) \\ &= \left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1}\left(E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right] - \mathbf{P}\right), \end{aligned} \tag{3}$$

where the last equality follows from the relation $\mathbf{P} = \mathbf{p}_0 + \mathbf{\Lambda}\mathbf{D}_I$. Here, note that if every j -th row, k -th column element of $\mathbf{\Lambda}$ is 0, eqn. (3) reduces to the standard expression for demand for firms' shares in perfect competition settings.[6] If every j -th row, k -th column element of $\mathbf{\Lambda}$ is *not* 0, then higher (non-negative) row/column elements within $\mathbf{\Lambda}$ lower an investor's demand for shares (*ceteris paribus*). Intuitively, this is because the more an investor demands, the higher the price he pays, not just for the next share, but for all the shares he demands of that firm. This is easiest to see for the special case where the shares of only one firm is priced, or, equivalently, if the cash flows of all firms are uncorrelated and the $\mathbf{\Lambda}$ matrix is diagonal: in this circumstance eqn. (3) reduces to $D_{Ij} = \left(\frac{1}{r_I} Cov_I\left[\tilde{V}_j, \tilde{V}_j\right] + \Lambda_j\right)^{-1}\left(E\left[\tilde{V}_j \mid \mathbf{\Phi}_I\right] - P_j\right)$, where D_{Ij} is an investor's demand for the shares of firm j, $Cov_I\left[\tilde{V}_j, \tilde{V}_j\right]$ is the variance of the cash flow of firm j, and Λ_j is the j -th diagonal element of $\mathbf{\Lambda}$. In this case, the investor's demand is: 1) an increasing function of the

[6] See, for example, the discussion in Sections 2.2 and 2.3 of Verrecchia (2001).

extent to which he assesses the expected cash flow of the j-th firm to be higher than the price for that firm; 2) an increasing function of his risk tolerance; 3) a decreasing function of his assessed variance of the j-th firm's cash flow; and 4) a decreasing function of his beliefs as to the impact of his demand on share price.

For example, let $\boldsymbol{\beta}$ denote the sensitivity of the informed investor's demand as a function of his information; here, $\boldsymbol{\beta}$ is defined as

$$\boldsymbol{\beta} = \frac{\partial}{\partial \mathbf{X}} \mathbf{D}_I = \left(\frac{1}{r_I} \mathbf{Cov}_I + \boldsymbol{\Lambda}^T \right)^{-1} \boldsymbol{\Pi}_I^{-1} \boldsymbol{\Pi}_\varepsilon . \tag{4}$$

2.2. An uninformed investor's demand

Uninformed investors have no private information. Nonetheless, as is standard in any rational expectations setting, uninformed investors glean some of the informed investors' private information about firms' cash flows by conditioning their expectations on price.[7] With this in mind, let $\boldsymbol{\Phi}_U = \{\tilde{\mathbf{P}} = \mathbf{P}\}$ represent the information available to uninformed investors. We denote uninformed investors' beliefs about the covariance of cash flows as $\mathbf{Cov}_U$, and the precision of their beliefs by $\boldsymbol{\Pi}_U$. We express the precision of their posteriors as $\boldsymbol{\Pi}_U = \boldsymbol{\Pi}_v + \boldsymbol{\Pi}_\delta$, where $\boldsymbol{\Pi}_\delta$ is the precision of the information that uninformed investors gleans from price. The precision of the information that uninformed investors glean from price is endogenous; we discuss it in more detail below.[8]

Uninformed investors in our model represent an investor class that is made up of a very large number of very small investors. Because they are large in number but individually carry little

[7]See, for example, Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981).

[8]In the Appendix we are specific as to how uninformed investors rely on price to infer informed investors' private information about cash flows.

weight in trade in firms' shares, uniformed investors behave as price takers.[9] Price-taking behavior implies that each uninformed investor believes that his demand has no effect on prices, and in equilibrium this belief is sustained. Based on his belief that his demand has no effect on prices, an uninformed investor takes the price vector as a given and chooses $\mathbf{D}_U$ to maximize the following objective function:

$$[E[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U] - \mathbf{P}]^T \mathbf{D}_U - \frac{1}{2r_U} \mathbf{D}_U^T \mathbf{Cov}_U \mathbf{D}_U, \tag{5}$$

where $\mathbf{Cov}_U$ represents an uninformed investor's beliefs about the covariance of firms' cash flows. Solving for $\mathbf{D}_U$ yields

$$\mathbf{D}_U = r_U \mathbf{Cov}_U^{-1}\left(E[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U] - \mathbf{P}\right) = r_U \mathbf{\Pi}_U \left(E[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U] - \mathbf{P}\right). \tag{6}$$

2.3 Market clearing

Market clearing requires that the total demand for firms' shares equals the supply of those shares. Recall that $\tilde{\mathbf{Z}}$ represents the (random) supply vector of shares in the J firms. In our asymmetric information economy, market clearing requires that

$$N \cdot \mathbf{D}_I(\tilde{\mathbf{P}}, \tilde{\mathbf{X}}) + M \cdot \mathbf{D}_U(\tilde{\mathbf{P}}) - \tilde{\mathbf{Z}} = \mathbf{0}.$$

Uninformed investors are large number but carry little weight in trade in firms' shares. We capture the notion of a large number of investors with little weight by assuming that M is large (i.e., M is countably infinite) and r_U is small, and the product of M and r_U converges to an arbitrary (non-negative) constant, π: specifically, $\lim_{M \to \infty} M \cdot r_U \to \pi$. This, in turn, implies

[9] Uninformed investors need not be price takers: competition among uninformed investors could also be imperfect. An analysis along these lines is considerably more cumbersome, however, because here each investor-type faces a different illiquidity matrix (i.e., $\mathbf{\Lambda}_I$ versus $\mathbf{\Lambda}_U$). All of the qualitative insights discussed in the paper carry over to this setting.

$$\lim_{M\to\infty} M \cdot \mathbf{D}_U\left(\tilde{\mathbf{P}}\right) \to \pi \cdot \mathbf{\Pi}_U\left(E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \mathbf{P}\right).$$

The parameter π represents the *proportional weight* of the uninformed investors in aggregate. For example, π small is tantamount to an economy where uninformed investors carry proportionally little weight in aggregate and thus the market is primarily imperfect. Alternatively, π large is tantamount to an economy where uninformed investors carry proportionally considerable weight in aggregate - sufficient weight that competition is approximately perfect. We regard the appropriate value of π for any specific economy as largely an empirical issue.

2.4 A capital market equilibrium

To characterize the equilibrium to our asymmetric information setting, we must solve for the two endogenous variables: the element of illiquidity that informed investors face, $\mathbf{\Lambda}$, and precision of the information uninformed investors glean from price, $\mathbf{\Pi}_\delta$. We show in the Appendix that these two variables must satisfy the following set of equations.

Proposition 1. *In a setting where information is asymmetric and competition is imperfect, informed investors' illiquidity matrix,* $\mathbf{\Lambda}$*, and precision matrix of information conveyed by price to uninformed investors,* $\mathbf{\Pi}_\delta$*, will satisfy the following two equations:*

$$\begin{gathered}\pi\mathbf{\Lambda}\mathbf{\Pi}_U\mathbf{\Lambda} + \mathbf{\Lambda}\left(\left(N-2\right)\mathbf{I} + \frac{\pi}{r_I}\mathbf{\Pi}_U\mathbf{Cov}_I\right) - \frac{1}{r_I}\mathbf{Cov}_I = \mathbf{0} \text{ and} \\ \mathbf{\Pi}_\delta - \left(\mathbf{\Pi}_\varepsilon^{-1} + \left(N\boldsymbol{\beta}\right)^{-1}\mathbf{Cov}_z\left[\left(N\boldsymbol{\beta}\right)^{-1}\right]^T\right)^{-1} = \mathbf{0},\end{gathered} \tag{8}$$

where $\mathbf{I}$ *is the* $J \times J$ *identity matrix.*

These two equations, and the two unknowns $\mathbf{\Lambda}$ and $\mathbf{\Pi}_\delta$, characterize the equilibrium. The first equation is a quadratic in $\mathbf{\Lambda}$, the informed investor's illiquidity matrix. But this equation also depends on the endogenous variable $\mathbf{\Pi}_\delta$, which is a component of $\mathbf{\Pi}_U$. Similarly, the second equation specifies the precision of the information uninformed investors glean from price; here, the endogenous variable $\mathbf{\Lambda}$ is part of the definition of the term $\boldsymbol{\beta}$ on the left-hand-side. Recall that $\boldsymbol{\beta}$ represents the sensitivity of an informed investor's demand to his private information. In principle, one could solve the second equation to determine an expression for $\boldsymbol{\Pi}_{\delta}$ as a function of $\boldsymbol{\Lambda}$, and this could be substituted into the first equation to solve for $\boldsymbol{\Lambda}$. Because of the non-linearity of the equations, however, one cannot obtain a closed-form solution for $\boldsymbol{\Lambda}$ and $\boldsymbol{\Pi}_{\delta}$. Nevertheless, we can use these equations to develop insights into the properties of the equilibrium.

In particular, Proposition 1 provides conditions where the market approaches a perfect competition setting, in which case $\boldsymbol{\Lambda}$ approaches $\mathbf{0}$; it also provides conditions where $\boldsymbol{\Lambda}$ is non-zero.

Corollary 1. *Informed investors' illiquidity matrix,* $\boldsymbol{\Lambda}$, *approaches* $\mathbf{0}$ *as either N or* π *approaches infinity; otherwise* $\boldsymbol{\Lambda} \neq \mathbf{0}$.

Corollary 1 establishes that when the number of informed investors in the market becomes large, each informed investor becomes sufficiently atomistic so as to behave as a price taker.

Alternatively, if the aggregate weight of uninformed investors becomes large (i.e., π becomes large), uninformed investors dominate the market. As a result, they provide sufficient liquidity to absorb the trading activities of informed investors, and thus informed investors behave as price-takers. Absent these two conditions, the market is less than perfectly liquid. We discuss the determinants of illiquidity later.

2.5 Cost of capital

Finally we turn to the cost of capital. A standard definition of a firm's cost of capital is the extent to which investors discount price at the beginning of the period (i.e., in expectation) relative to the expected value of the firm's cash flow.[10] We use the investors' demands as characterized by

[10]See, for example, Lambert, et al., (2007).

eqns. (3) and (6), along with market clearing conditions to derive the cost of capital.

Proposition 2. *Cost of capital in an imperfect competition setting with asymmetrically informed investors reduces to*

$$E\left[\tilde{\mathbf{V}}\right]-E\left[\tilde{\mathbf{P}}\right]=\left(\frac{\left(\mathbf{I}+r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T\right)^{-1}Nr_I\mathbf{\Pi}_I+Mr_U\mathbf{\Pi}_U}{Nr_I+Mr_U}\right)^{-1}E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I+Mr_U}\right]. \qquad (9)$$

Proposition 2 demonstrates that cost of capital can be expressed as function of the precision matrix of each type of investor (i.e., $\mathbf{\Pi}_I$ and $\mathbf{\Pi}_U$), the aggregate weight of each type (i.e., Nr_I and Mr_U), and informed investors' illiquidity matrix (i.e., $\mathbf{\Lambda}$).

Equation (9) indicates that these features of the model interact in complex ways. In order to understand the nature of these interactions, we begin our analysis by considering a number of special cases of the model in which we can vary one aspect of the model at a time.

First, consider the special case in which there are no information asymmetries (M = $\pi = 0$) and where we exogenously set the illiquidity matrix $\mathbf{\Lambda}$ equal to zero, so that there is perfect competition among the homogenously informed investors. In this case equation (9) reduces simply to

$$E\left[\tilde{\mathbf{V}}\right]-E\left[\tilde{\mathbf{P}}\right]=\mathbf{Cov}_I E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I}\right]$$

This is simply the Capital Asset Pricing Model (CAPM), in which the covariance matrix of the firm's cash flows is what determines the firm's cost of capital.

Next, we continue to assume there are no information asymmetries (M=0), but allow for imperfect competition in the capital markets. In this case , cost of capital in equation (9) can be expressed as

$$E\left[\tilde{\mathbf{V}}\right]-E\left[\tilde{\mathbf{P}}\right]=\left[Cov_I+r_I\Lambda\right]E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I}\right]$$

This equation shows that cost of capital now contains both a covariance effect as in the CAPM, but also an illiquidity effect through the matrix $\boldsymbol{\Lambda}$. Higher market illiquidity will increase the firm's cost of capital, ceteris paribus. This is because investor's decrease their demand for firm's shares when illiquidity is high in order to avoid the adverse price effect. The decrease in the demand for shares lowers the price relative to the expected end-of-period cash flow, which raises the cost of capital. Of course, the illiquidity matrix is an endogenous variable, and for this special case, the system of equations that determines the illiquidity matrix (e.g., equations (7) and (8) simply reduce to $\boldsymbol{\Lambda}=\frac{1}{N-2}\frac{1}{r_I}\mathbf{Cov}_I$. Therefore, under this special case, the illiquidity matrix is proportional to the firm's covariance matrix. That is, there is more illiquidity in firm's stocks whose cash flows are more highly correlated with the market. Moreover, as the number of investors (N) or their risk tolerance gets larger, the illiquidity component of the cost of capital will become smaller compared to the covariance of cash flow component of cost of capital. At the extreme, as the number of investors gets larger, the market approaches conditions of perfect competition, and the pricing model approaches the CAPM.

Alternatively, consider the special case where we allow for information asymmetries (M and N are both nonzero), but impose perfection competition by setting $\boldsymbol{\Lambda}=\mathbf{0}$. In this case, cost of capital in equation (9) reduces to

$$E\left[\tilde{\mathbf{V}}\right]-E\left[\tilde{\mathbf{P}}\right]=\left(\frac{Nr_I\mathbf{\Pi}_I+Mr_U\mathbf{\Pi}_U}{Nr_I+Mr_U}\right)^{-1}E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I+Mr_U}\right];$$

Here, cost of capital depends *solely* on the inverse of the average precision matrix of information across investors, where the average precision is defined as

$$\mathbf{\Pi}_{avg} = \frac{Nr_I \mathbf{\Pi}_I + Mr_U \mathbf{\Pi}_U}{Nr_I + Mr_U}.$$

In other words, $\mathbf{\Pi}_{avg}$ is defined as the precision matrix of each investor type averaged over the aggregate weight of that type, i.e., Nr_I and Mr_U, respectively. This implies that under perfect competition (i.e., perfect liquidity), the degree to which each investor-type's precision differs from the average has *no* effect on the cost of capital once one controls for the average. That is, a change in the information environment (including a change that impacts the degree of information asymmetry across investors) will only affect the cost of capital if it impacts the average precision of investors' information. Stated somewhat differently, under perfect competition the degree of information asymmetry across the two investor types has no additional explanatory power in explaining the behavior of cost of capital once one controls for the average. Decreases in information asymmetry which increase the average precision of investors' information will decrease cost of capital, whereas decreases in information asymmetry which reduce the average precision will increase cost of capital.

Finally, consider the general case where the illiquidity matrix $\mathbf{\Lambda}$ is not $\mathbf{0}$, Proposition 2 indicates that illiquidity can affect cost of capital in two ways. First, illiquidity affects the average precision of investors in the economy. In particular, the greater the degree of illiquidity faced by informed investors, the less aggressively they trade on the basis of their private information, and the less uninformed investors learn by conditioning their expectations on prices. Therefore, *ceteris paribus*, greater market illiquidity lowers the precision of information held by the less informed type. This lowers investors' average precision of information, and thus raises the cost of capital.

Second, when $\mathbf{\Lambda} \neq \mathbf{0}$, the weighting scheme applied to investors in the cost of capital equation is more complex; it is not the simple weighted average of the precisions of the two investor types. While the weight assigned the precision of the uninformed type continues to be the aggregate weight of that type, Mr_U, illiquidity reduces the weight assigned to the precision of the informed type. Specifically, the weight assigned to the precision of informed investors is now $\left(\mathbf{I}+r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T\right)^{-1}Nr_I$; one can think of the term $\left(\mathbf{I}+r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T\right)^{-1}$ as a reduction factor arising from market illiquidity. This reduction arises because informed investors have to curb the aggressiveness of their demands because of market illiquidity. Thus, informed investors' private information does not get impounded or reflected in price as fully as the precision of their information (and their aggregate weight) would suggest. Because the informed type has greater precision, and this precision is reduced because of market illiquidity, this also suggests that market illiquidity leads to higher cost of capital, *ceteris paribus*.

Recall that in a market characterized by perfect competition, once one controls for average precision information asymmetry has no additional explanatory power in explaining the behavior of cost of capital. That said, observe that the precisions of the two investor-types are not equally weighted in Proposition 2. This suggests that the degree of information asymmetry in the imperfect competition economy *can* affect the cost of capital through its effect on market illiquidity. Intuitively, an increase in how far each type's precision deviates from the average precision will lead to a lower *liquidity-adjusted* weighted average, and a higher cost of capital. The occurs because the precision of the more informed type gets less weight in determining cost of capital. To properly analyze this conjecture, it is important that any comparative statics analysis hold constant investors' average precision of information. This is not a trivial exercise, because the average precision and degree of information asymmetry are both endogenous variables, and thus changes in parameters will affect them simultaneously. For example, increasing the precision of information for the informed type increases both average precision and information asymmetry.

On the other hand, increasing the precision of the information of the uninformed type will increase average precision but decrease information asymmetry.

To make the distinction between average precision and information asymmetry more transparent, we recast the equation for cost of capital as follows:

$$E\left[\tilde{\mathbf{V}}\right]-E\left[\tilde{\mathbf{P}}\right]=\left(\mathbf{\Pi}_{avg}-\frac{r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T(\mathbf{I}+r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T)^{-1}Nr_I\mathbf{\Pi}_I}{Nr_I+Mr_U}\right)^{-1}E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I+Mr_U}\right].$$

In this specification, cost of capital can be expressed as a function of the average precision of investors' information, $\mathbf{\Pi}_{avg}$, minus a term that reflects the discount that is attributable to the interaction between information asymmetry and illiquidity. In particular, holding the average precision constant, an increase in the precision of the information of informed investors, $\mathbf{\Pi}_I$, is a proxy for the degree of information asymmetry in the economy. That is, a higher value of $\mathbf{\Pi}_I$ has to be offset by a lower value of $\mathbf{\Pi}_U$ to preserve the average precision in the economy. This proxy for information asymmetry is then multiplied by a fractional amount, $r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T(\mathbf{I}+r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T)^{-1}$, that depends on both the degree of illiquidity and the precision of informed investors' information. When there is no illiquidity, i.e., $\mathbf{\Lambda}=\mathbf{0}$, both the fractional term and the reduction relative to the average precision are zero. Examining the equation above, any combination of parameter changes that holds the average precision constant and weakly increases the degree of information asymmetry (as proxied by the precision matrix $\mathbf{\Pi}_I$) and the illiquidity matrix $\mathbf{\Lambda}$, while strictly increasing at least one, is sufficient to increase the cost of capital. That is, holding the illiquidity matrix $\mathbf{\Lambda}$ constant, a higher precision for informed investors (e.g., more information asymmetry) will increase this discount and increase cost of capital. Similarly, holding the precision of the informed investors' information constant, an increase in the degree of illiquidity will also increase this discount and increase the cost of capital. Of course, these are sufficient conditions, not necessary ones.

3. NUMERICAL COMPARISONS

In this section we explore further the distinctions between imperfect and perfect competition, and how these distinctions affect cost of capital. In general, changes in the exogenous parameters of the model can impact the average level of precision, the degree of information asymmetry, and the level of competition. In order to highlight the effects of information asymmetry and the level of competition (the degree of market illiquidity), we will hold the average precision of information constant in our comparisons. This means, for example, that cost of capital will not be affected in a CAPM-type setting in any of our comparisons. Nonetheless, our examples will show that cost of capital does change in settings of imperfect competition.

3.1 Information asymmetry

In order to hold average precision constant, at least two exogenous variables must change simultaneously while also calibrating the magnitude of these changes. As this is computationally challenging, we impose more structure on the model. Specifically, henceforth we assume that the precision in the error terms in informed investors' information about those cash flows, $\mathbf{\Pi}_{\varepsilon}$, and the covariance of the supply of firms' shares, $\mathbf{Cov}_z$, are proportionate to the *a priori* precision of cash flows, $\mathbf{\Pi}_v$, through the relations $\mathbf{\Pi}_{\varepsilon} = \sigma_{\varepsilon}^{-2}\mathbf{\Pi}_v$, and $\mathbf{Cov}_z = \sigma_z^2\mathbf{\Pi}_v$, where σ_{ε}^2, and σ_z^2 are arbitrary, positive parameters. Note that this imposed structure is without loss of generality in a single-firm economy because in a single-firm economy matrices become scalars, and scalars are always proportionate to other scalars.

With this structure and some tedious calculations, one can show that eqn. (7) in Proposition

1 reduces to the requirement that $\mathbf{\Lambda} = \lambda \mathbf{Cov}_v$, where λ solves the following 4^{th}-order polynomial

$$\begin{aligned}
&\sigma_z^2 r_I^3 \rho \left(1+\sigma_\varepsilon^2\right)^3 \lambda^4 + r_I^2 \sigma_z^2 \left(1+\sigma_\varepsilon^2\right)^2 \left(r_I \left(1+\sigma_\varepsilon^2\right)(N-2) + 3\rho\sigma_\varepsilon^2\right)\lambda^3 \\
&+ r_I \left(1+\sigma_\varepsilon^2\right)\left(\left(3\sigma_z^2\sigma_\varepsilon^4 + N^2 r_I^2 \left(1+\sigma_\varepsilon^2\right)\right)\rho + r_I \sigma_z^2 \sigma_\varepsilon^2 \left(1+\sigma_\varepsilon^2\right)(2N-5)\right)\lambda^2 \\
&+ \sigma_\varepsilon^2 \left(\left(N^2 r_I^2 \left(1+\sigma_\varepsilon^2\right) + \sigma_z^2 \sigma_\varepsilon^4\right)\rho + r_I \left(1+\sigma_\varepsilon^2\right)\left(\sigma_z^2\sigma_\varepsilon^2 (N-4) + N^2 r_I^2 (N-2)\right)\right)\lambda \\
&- \sigma_\varepsilon^4 \left(\sigma_z^2 \sigma_\varepsilon^2 + N^2 r_I^2\right) = 0.
\end{aligned}$$

Here, λ measures the degree of market illiquidity: as λ increases (decreases), the market is less (more) liquid. It is straightfoward to show using Descartes' Rule of Signs that there exists a unique, positive λ that satisfies eqn. (12), provided that $N \geq 3$.[11] Having solved for λ, eqn. (8) in Proposition 1 reduces to the following requirement for $\mathbf{\Pi}_\delta$:

$$\mathbf{\Pi}_\delta = \frac{N^2 r_I^2}{r_I^2 \sigma_z^2 \left(1+\sigma_\varepsilon^2\right)^2 \lambda^2 + 2 r_I \sigma_z^2 \sigma_\varepsilon^2 \left(1+\sigma_\varepsilon^2\right)\lambda + \sigma_\varepsilon^2 \left(N^2 r_I^2 + \sigma_z^2 \sigma_\varepsilon^2\right)} \mathbf{\Pi}_v.$$

A benchmark case. Using the additional structure, first we establish a benchmark case. Specifically, consider a market setting with four informed investors, $N = 4$, who have a risk tolerance of 1 (i.e., $r_I = 1$). Let the proportionate weight of the uninformed investors converge to 1 as their number becomes large (i.e., $\lim_{M\to\infty} M r_U = \pi \to 1$). We begin with the assumptions that $\sigma_\varepsilon^2 = 1$ and $\sigma_z^2 = 1$. These assumptions imply that the precision of the information available to informed investors is $\mathbf{\Pi}_I = \mathbf{\Pi}_v + \mathbf{\Pi}_\varepsilon = 2 \cdot \mathbf{\Pi}_v$ and in equilibrium $\lambda = 0.15409$, and thus the precision of the information available to uninformed investors is $\mathbf{\Pi}_U = \mathbf{\Pi}_v + \mathbf{\Pi}_\delta = 1.9034 \cdot \mathbf{\Pi}_v$

[11]This is consistent with Kyle (1989), who also requires that the participation of at least three investors (i.e., $N \geq 3$) so as to eliminate the possibility of one investor, or a pair of investors, having too much monopoly power: see the discussion on p. 329 of Kyle (1989).

and $\mathbf{\Lambda} = 0.15409 \cdot \mathbf{Cov}_v$. Here, investors' average precision computes to $\mathbf{\Pi}_{avg} = 1.9807 \cdot \mathbf{\Pi}_v$. In addition, firms' cost of capital computes to $0.12471 \cdot \mathbf{Cov}_v \cdot E[\tilde{\mathbf{Z}}]$.

Illustrating the effect of information asymmetry. Our next goal is show the effect of increasing the degree of information asymmetry while holding average precision to the level in the benchmark case. To achieve this goal, we simultaneously increase the precision of informed investors' private information (through a decrease in the parameter σ_ε^2) and the variance of the liquidity shock (through an increase in the parameter σ_z^2). Increasing the precision of private information increases the precision of information available to informed investors. The increase in variance of the liquidity shock injects more noise into the information conveyed by price; this, in turn, ensures that the precision of the uninformed investors' information is reduced. As a result, the degree of information asymmetry between the two types is increased. We calibrate these two changes so as to hold the average precision of the two types constant. We examine the effect of these changes on the illiquidity factor λ and on cost of capital.

Specifically, consider a market setting where an informed investor acquires more private information, but the *average* precision of information remains at the same level. For example, suppose σ_ε^2 falls from $\sigma_\varepsilon^2 = 1$ to $\sigma_\varepsilon^2 = 0.9$, which implies that the precision of an informed investor's private information increases: specifically, $\mathbf{\Pi}_I = 2.1111 \cdot \mathbf{\Pi}_v$. In order for average precision of information to remain at the same level, we now require the variance of the liquidity shock to increase such that σ_z^2 rises to $\sigma_z^2 = 14.034$. This causes λ to increase to 0.16183 and the precision of the information an uninformed investor gleans from the price vector drops to $\mathbf{\Pi}_U = 1.459 \cdot \mathbf{\Pi}_v$. Here, investors' average precision remains at the same level: $\mathbf{\Pi}_{avg} = 1.9807 \cdot \mathbf{\Pi}_v$.

Despite the fact that the average precision does not change, the increase in information asymmetry between the two investor types manifests in greater illiquidity: specifically, λ increases from 0.15409 to 0.16183. This, in turn, results in higher cost of capital: here, firms' cost of capital computes to $0.12898 \cdot \mathbf{Cov}_v \cdot E[\tilde{\mathbf{Z}}]$.

This demonstrates that in our imperfect competition setting cost of capital increases as information asymmetry increases, despite the fact that average precision remains unchanged. Thus, we find a role for information asymmetry in cost of capital through its effect on market illiquidity - this role does not exist in perfect competition settings. This leads to the following observation.

Observation 1. *Holding investors' average precision constant, a change in the extent of information asymmetry between informed and uninformed investors can only affect cost of capital in an imperfect competition setting, not the benchmark setting of perfect competition.*

Observation 1 has the following important implication for empirical studies: information asymmetry can only affect cost of capital in imperfect competition settings.

3.2 Market competition

Our next goal is to show the effect of market competition while holding average precision to the level in the benchmark case. To achieve this, we simultaneously increase the number of informed investors (through an increase in the parameter N) and the variance of the liquidity shock (through an increase in the parameter σ_z^2). Increasing the number of informed investors creates more competition, but also serves to increase average precision because a higher percentage of investors are informed (when N increases from 4 to 5). As before, the increase in variance of the liquidity shock injects more noise into the information conveyed by price; this, in turn, ensures that the precision of the uninformed investors' information is reduced. We calibrate these two changes so as to hold the average precision of the two types constant. Once again we examine the effect of these changes on the illiquidity factor λ and the cost of capital.

Specifically, consider a market setting with more competition among informed investors, but the *average* precision of information remains at the same level. For example, suppose N increases from $N = 4$ to $N = 5$, which implies that there is more competition among informed investors. In order for average precision of information to remain at the same level, we now require the variance of the liquidity shock to increase such that σ_z^2 rises to $\sigma_z^2 = 2.1297$. Relative to the benchmark case, this causes λ to fall to 0.11996 and the precision of the information an uninformed investor gleans from the price vector drops to $\mathbf{\Pi}_U = 1.8842 \cdot \mathbf{\Pi}_v$. Here, investors' average precision remains at the same level: $\mathbf{\Pi}_{avg} = 1.9807 \cdot \mathbf{\Pi}_v$.

Despite the fact that the average precision does not change, the increase in competition among informed investors manifests in greater liquidity: specifically, λ decreases from 0.15409 to 0.11996. This, in turn, results in lower cost of capital: here, firms' cost of capital computes to $0.10051 \cdot \mathbf{Cov}_v \cdot E[\tilde{\mathbf{Z}}]$.

This demonstrates that in our imperfect competition setting cost of capital falls as competition among informed investors increases, despite the fact that average precision remains unchanged. Thus, we find a role for competition in cost of capital through its effect on market illiquidity - this role does not exist in perfect competition settings. This leads to our next observation.

Observation 2. *Holding investors' average precision constant, a change in the extent of competition among informed investors can only affect cost of capital in our imperfect competition setting, not a perfect competition such as the CAPM.*

Observation 2 has the following implication for empirical studies. Observation 1 establishes that information asymmetry can only affect cost of capital in imperfect competition settings holding investors' average precision constant. Obsercation 2 goes further to point out that the relation between information asymmetry and cost of capital is conditional on the level of market competition. In other words, more (less) competition ameliorates (exacerbates) the effect of information asymmetry on cost of capital in imperfect competition settings. This is consistent with Armstong, et al. (2010), a paper that shows that information asymmetry has a positive relation with firms' cost of capital in excess of standard risk factors when markets are imperfect, and no relation when markets approximate perfect competition.

3.3 The interaction between information asymmetry and competition

Our last comparison is to illustrate how information asymmetry and market competition *interact* while holding average precision constant. This case is interesting for the following reason. Observations 1 and 2 suggest that information asymmetry and market competition have countervailing effects on cost of capital: more (less) information asymmetry exacerbates cost of capital, while more (less) competition ameliorates cost of capital. But if we increase information asymmetry by increasing informed investors' private information, then this serves to increase average precision. Thus, to hold the average constant we need to reduce the number of informed investors in the economy: in other words, as informed investors become better informed, we need fewer of them to hold average precision constant. But fewer informed investors implies less competition, and so here information asymmetry and competition have reinforcing effects on cost of capital.

To show this, consider the following benchmark case: $N = 10$, $r_I = 1$, $\pi = 1$, $\sigma_\varepsilon^2 = 1$, and $\sigma_z^2 = 1$. Here, market illiquidity is $\lambda = 0.05679$, investors' average precision is 1.9497, and cost of capital computes to $0.051529 \cdot Cov_v \cdot E[\tilde{Z}]$.

Now suppose that we increase the precision of informed investors' private information through a decrease in σ_ε^2: specifically, now let $\sigma_\varepsilon^2 = 0.9$. When σ_ε^2 falls to 0.9, informed investors are better informed (i.e., Π_I increases from $2 \cdot \Pi_v$ to $2.1111 \cdot \Pi_v$); this, in turn, creates the potential for uninformed investors to be better informed because some of informed investors' private information is communicated, or transferred, to uninformed investors. Thus, to hold investors' average precision constant at 1.9497, we need to decrease the number of informed investors from $N = 10$ to $N = 4.91181$. But when informed investors are both better informed (because σ_ε^2 falls from 1 to 0.9) and fewer in number (because N falls from 10 to 4.91181), then greater information asymmetry and less competition have reinforcing effects, and thus here market illiquidity and cost of capital both rise to $\lambda = 0.13116$ and $0.10779 \cdot Cov_v \cdot E\left[\tilde{Z}_M\right]$, respectively. In other words, cost of capital approximately doubles, despite the fact that investors' average precision remains fixed.

This illustration makes the following interesting point. In two perfect competition settings where cost of capital does not change because investors' average precision remains fixed, cost of capital approximately doubles when competition is imperfect! This implies that our understanding of how features of an economy affect cost of capital, such as information asymmetry or the level of competition among informed investors, is very sensitive to the level of competition for firms' shares. Despite this, empirical studies of the effect of information on the cost of capital rarely acknowledge the role of competition.

4. CONCLUSION

This paper extends the CAPM to imperfect competition settings where investors have rational expectations about the market process. In an imperfect competition setting, investors understand that the magnitude of their demand for firm shares can affect the price at which their demand is fulfilled. Thus, markets are not perfectly liquid and there are costs to trading. The cost is endogenously determined as part of the equilibrium, and it occurs through an increase in price that must be paid when investors wish to buy more shares. Similarly, investors understand that attempts to sell more shares will lower the price they receive. As Kyle (1989) discusses, imperfect competition resolves the ``schizophrenia" manifest in perfect competition settings, where no single investor believes his demand influences price but yet in equilibrium prices do reflect aggregate demand.

Our analysis demonstrates the importance of distinguishing between the precision of investors' information from the degree of information asymmetry across investors. Controlling for average precision, the degree of information asymmetry and level of competition have no effect on cost of capital in settings such as the CAPM, which is premised on the assumption that markets are (approximately) perfectly competitive. In more illiquid markets, however, where perfect competition is less descriptive, information asymmetry and competition can affect the liquidity of the market and therefore the cost of capital.

Our results are relevant to a large empirical literature in accounting that examines the relation between various information attributes and the cost of capital. Specifically, we show that in discussions of the relation between information asymmetry and cost of capital, the degree of competition among investors matters. Claims about the behavior of cost of capital that may be true in an imperfect competition setting may not hold in the CAPM (and vice versus). In other words, the degree of market competition is an important conditioning variable when examining the relation between information asymmetry and cost of capital.

APPENDIX

Proof of Proposition 1. In competing with other investors each informed investor adopts the strategy

$$\mathbf{D}_I\left(\tilde{\mathbf{P}}, \tilde{\mathbf{X}}\right) = \boldsymbol{\alpha} + \boldsymbol{\beta} \cdot \mathbf{X} - \boldsymbol{\Gamma} \cdot \mathbf{P},$$

where $\boldsymbol{\alpha}$ is a $J \times 1$ vector of intercept terms, $\boldsymbol{\beta}$ is a $J \times J$ matrix of weights an investor places on the realization of his information $\tilde{\mathbf{X}} = \mathbf{X}$, and $\boldsymbol{\Gamma}$ is a $J \times J$ matrix of weights an investor places on $\mathbf{P}$. For this strategy to be rational based on the computation of $\mathbf{D}_I$ in eqn. (3), it must be the case that

$$\boldsymbol{\Gamma} = \left(\frac{1}{r_I} \mathbf{Cov}_I + \boldsymbol{\Lambda}^T \right)^{-1}.$$

In addition, substituting an informed investor's conditional expectation into his demand function in eqn. (A1), it must also be the case that

$$\boldsymbol{\alpha} = \boldsymbol{\Gamma}\left[\boldsymbol{\Pi}_v + \boldsymbol{\Pi}_\varepsilon\right]^{-1} \boldsymbol{\Pi}_v \mathbf{E}\left[\tilde{\mathbf{V}}\right] \text{ and } \boldsymbol{\beta} = \boldsymbol{\Gamma}\left[\boldsymbol{\Pi}_v + \boldsymbol{\Pi}_\varepsilon\right]^{-1} \boldsymbol{\Pi}_\varepsilon .$$

Market clearing requires that the total demand for firms' shares equals the supply of those shares. Recall that $\tilde{\mathbf{Z}}$ represents the (random) supply vector of shares in the J firms. In our asymmetric economy, market clearing requires that

$$N \cdot \mathbf{D}_I\left(\tilde{\mathbf{P}}, \tilde{\mathbf{X}}\right) + M \cdot \mathbf{D}_U\left(\tilde{\mathbf{P}}\right) - \tilde{\mathbf{Z}} = \mathbf{0}.$$

Substituting for $\mathbf{D}_I$ and $\mathbf{D}_U$ from eqns. (A1) and eqn. (6), we express market clearing as

$$N\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{X} - \boldsymbol{\Gamma}\mathbf{P}\right) + M r_U \boldsymbol{\Pi}_U \left(E\left[\tilde{\mathbf{V}} \mid \boldsymbol{\Phi}_U\right] - \mathbf{P}\right) - \tilde{\mathbf{Z}} = \mathbf{0}.$$

This allows the price vector to be expressed as

$$\tilde{\mathbf{P}} = \boldsymbol{\Delta}\left(N\boldsymbol{\alpha} + N\boldsymbol{\beta}\mathbf{X} + \pi\boldsymbol{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \boldsymbol{\Phi}_U\right] - \tilde{\mathbf{Z}}\right),$$

where $\boldsymbol{\Delta}$ is given by

$$\boldsymbol{\Delta} = \left(N\boldsymbol{\Gamma} + \pi\boldsymbol{\Pi}_U\right)^{-1}.$$

Equilibrium conditions rely on the solutions to $\boldsymbol{\Lambda}$ and $\boldsymbol{\Pi}_U$. We discuss the solution to $\boldsymbol{\Lambda}$ first and then turn our attention to the solution to $\boldsymbol{\Pi}_U$.

To reconcile the market-clearing condition in eqn. (A4) with an informed investor's strategy, it must be the case that $\tilde{\mathbf{p}}_0$ in the expression $\tilde{\mathbf{P}} = \tilde{\mathbf{p}}_0 + \mathbf{\Lambda D}$ is of the form

$$\tilde{\mathbf{p}}_0 = \mathbf{\Lambda}\left((N-1)\boldsymbol{\alpha} + (N-1)\boldsymbol{\beta}\mathbf{X} + \pi\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \tilde{\mathbf{Z}}\right),$$

and thus

$$\tilde{\mathbf{P}} = \mathbf{\Lambda}\left(N\boldsymbol{\alpha} + \mathbf{N}\boldsymbol{\beta}\mathbf{X} + \pi\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \tilde{\mathbf{Z}}\right) - \mathbf{\Lambda\Gamma}\tilde{\mathbf{P}}.$$

Eqn. (A5) implies

$$\tilde{\mathbf{P}} = (\mathbf{I} + \mathbf{\Lambda\Gamma})^{-1}\mathbf{\Lambda}\left(N\boldsymbol{\alpha} + \mathbf{N}\boldsymbol{\beta}\mathbf{X} + \pi\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \tilde{\mathbf{Z}}\right).$$

But eqn. (A6) implies the following identity:

$$(\mathbf{I} + \mathbf{\Lambda\Gamma})^{-1}\mathbf{\Lambda} = \mathbf{\Delta} = (N\mathbf{\Gamma} + \pi\mathbf{\Pi}_U)^{-1},$$

or

$$\mathbf{\Lambda}(N\mathbf{\Gamma} + \pi\mathbf{\Pi}_U) = \mathbf{I} + \mathbf{\Lambda\Gamma}.$$

Because $\mathbf{\Gamma} = \left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1}$, we can express eqn. (A7) as

$$\mathbf{\Lambda}\left(N \cdot \mathbf{I} + \pi\mathbf{\Pi}_U\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)\right) = \left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right) + \mathbf{\Lambda}.$$

The fact that $\mathbf{\Pi}_U$ and $\mathbf{Cov}_I$ are both symmetric matrices yields the following quadratic matrix equation that solves for $\mathbf{\Lambda}$:

$$\pi\mathbf{\Lambda\Pi}_U\mathbf{\Lambda} + \mathbf{\Lambda}\left(N - 2 + \frac{\pi}{r_I}\mathbf{\Pi}_U\mathbf{Cov}_I\right) - \frac{1}{r_I}\mathbf{Cov}_I = \mathbf{0}.$$

To derive the uninformed investors' beliefs, recall that the market clearing condition implies

$$\tilde{\mathbf{P}} = \mathbf{\Delta}\left(N\boldsymbol{\alpha} + N\boldsymbol{\beta}\mathbf{X} + \pi\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \tilde{\mathbf{Z}}\right).$$

As is standard in a rational expectations economy (see, e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981), we assume that each uninformed trader can manipulate the market clearing price $\tilde{\mathbf{P}}$ to obtain information about $\tilde{\mathbf{V}}$ through the statistic $\tilde{\mathbf{Q}}$, where

$$\tilde{\mathbf{Q}} = (N\mathbf{\Delta}\boldsymbol{\beta})^{-1}\left[\tilde{\mathbf{P}} - \mathbf{\Delta}\left(N\boldsymbol{\alpha} + \pi\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right]\right)\right] = \tilde{\mathbf{X}} - (N\boldsymbol{\beta})^{-1}\tilde{\mathbf{Z}} = \tilde{\mathbf{V}} + \tilde{\boldsymbol{\delta}},$$

where $\tilde{\boldsymbol{\delta}} = \tilde{\boldsymbol{\varepsilon}} - (N\boldsymbol{\beta})^{-1}\tilde{\mathbf{Z}}$. The statistic $\tilde{\mathbf{Q}}$ measures an informed investor's private information,

$\tilde{\mathbf{X}}$, with error; this makes $\tilde{\mathbf{Q}}$ a noisier measure of the cash flow vector $\tilde{\mathbf{V}}$ than $\tilde{\mathbf{X}}$. Stated somewhat differently, an uninformed investor's error about the realization of the cash flow vector $\tilde{\mathbf{V}} = \mathbf{V}$ is the sum of two terms: the error in $\tilde{\mathbf{X}}$ plus the additional error in $\tilde{\mathbf{Q}}$. Henceforth let $\mathbf{\Pi}_\delta$ represent the precision matrix for the covariance matrix $\tilde{\mathbf{Q}}$.

Conditional on the realization of the price vector $\tilde{\mathbf{P}} = \mathbf{P}$ and manipulating this vector to yield $\tilde{\mathbf{Q}} = \mathbf{Q}$, an uninformed investor's posterior beliefs about the vector of cash flows $\tilde{\mathbf{V}}$ is that it has an expected value of

$$E\left[\tilde{\mathbf{V}} \mid \tilde{\mathbf{Q}} = \mathbf{Q}\right] = E\left[\tilde{\mathbf{V}}\right] + \left(\mathbf{\Pi}_v + \mathbf{\Pi}_\delta\right)^{-1} \mathbf{\Pi}_\delta \left[\mathbf{Q} - E\left[\tilde{\mathbf{Q}}\right]\right];$$

in addition, an uninformed investor associates a precision of

$$\mathbf{\Pi}_U = \mathbf{\Pi}_v + \mathbf{\Pi}_\delta$$

to those beliefs. Thus, we have to solve for $\mathbf{\Pi}_\delta$ to determine $\mathbf{\Pi}_U$. Note, however, that from eqns. (A2) and (A3), we know that $\mathbf{\Gamma} = \left(\tfrac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1}$ and $\boldsymbol{\beta} = \mathbf{\Gamma}\left[\mathbf{\Pi}_v + \mathbf{\Pi}_\varepsilon\right]^{-1}\mathbf{\Pi}_\varepsilon$. Thus, the precision matrix for $\tilde{\mathbf{Q}}$, $\mathbf{\Pi}_\delta$, must equal

$$\mathbf{\Pi}_\delta = \left(\mathbf{\Pi}_\varepsilon^{-1} + \left(N\boldsymbol{\beta}\right)^{-1}\mathbf{Cov}_z\left[\left(N\boldsymbol{\beta}\right)^{-1}\right]^T\right)^{-1}.$$

Q.E.D.

Proof Of Corollary 1. Dividing eqn. (7) by N yields

$$\frac{1}{N}\pi\mathbf{\Lambda}\mathbf{\Pi}_U\mathbf{\Lambda} + \mathbf{\Lambda}\left(1 - \frac{2}{N} + \frac{1}{r_I}\mathbf{\Pi}_U\mathbf{Cov}_I\right) - \frac{1}{Nr_I}\mathbf{Cov}_I = \mathbf{0}.$$

Taking the limit as N approaches infinity implies $\mathbf{\Lambda} = \mathbf{0}$. Similarly, dividing eqn. (9) by π yields

$$\mathbf{\Lambda}\mathbf{\Pi}_U\mathbf{\Lambda} + \mathbf{\Lambda}\left(\frac{N-2}{\pi} + \frac{1}{r_I}\mathbf{\Pi}_U\mathbf{Cov}_I\right) - \frac{1}{\rho r_I}\mathbf{Cov}_I = \mathbf{0}.$$

Taking the limit as π approaches infinity yields $\mathbf{\Lambda}\mathbf{\Pi}_U\mathbf{\Lambda} + \mathbf{\Lambda}\left(\tfrac{1}{r_I}\mathbf{\Pi}_U\mathbf{Cov}_I\right) = \mathbf{0}$, which implies

$\mathbf{\Lambda} = \mathbf{0}$. Finally, suppose N and π are both finite. Then if $\mathbf{\Lambda} = \mathbf{0}$, eqn. (7) becomes $-\frac{1}{r_I}\mathbf{Cov}_I = 0$, which is a contradiction. Q.E.D.

Proof of Proposition 2. Market clearing implies

$$N \cdot \mathbf{D}_I\left(\tilde{\mathbf{P}}, \tilde{\mathbf{X}}\right) + M \cdot \mathbf{D}_U\left(\tilde{\mathbf{P}}\right) - \tilde{\mathbf{Z}} = \mathbf{0}.$$

Substituting for D_I and D_U implies

$$N \cdot \left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1}\left(E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right] - \tilde{\mathbf{P}}\right) + M \cdot r_U \mathbf{\Pi}_U\left(E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \tilde{\mathbf{P}}\right) - \tilde{\mathbf{Z}} = \mathbf{0}.$$

Re-arranging terms yields

$$N\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right] + Mr_U\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \left[N\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} + Mr_U\mathbf{\Pi}_U\right]\tilde{\mathbf{P}} - \tilde{\mathbf{Z}} = \mathbf{0}.$$

Solving for $\tilde{\mathbf{P}}$ results in

$$\tilde{\mathbf{P}} = \left[N\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} + Mr_U\mathbf{\Pi}_U\right]^{-1}$$
$$\times\left[N\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right] + Mr_U\mathbf{\Pi}_U E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right] - \tilde{\mathbf{Z}}\right].$$

Taking expected values and using the law of iterated expectations implies

$E\left[E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_I\right]\right] = E\left[E\left[\tilde{\mathbf{V}} \mid \mathbf{\Phi}_U\right]\right] = E\left[\tilde{\mathbf{V}}\right]$, or

$$\tilde{\mathbf{P}} = E\left[\tilde{\mathbf{V}}\right] - \left[N\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} + Mr_U\mathbf{\Pi}_U\right]^{-1} E\left[\tilde{\mathbf{Z}}\right]$$
$$= E\left[\tilde{\mathbf{V}}\right] - \left[\frac{N\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} + Mr_U\mathbf{\Pi}_U}{Nr_I + Mr_U}\right]^{-1} E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I + Mr_U}\right].$$

Finally we substitute $\left(\frac{1}{r_I}\mathbf{Cov}_I + \mathbf{\Lambda}^T\right)^{-1} = \left(\mathbf{I} + r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T\right)^{-1} r_I\mathbf{\Pi}_I$ to get

$$\tilde{\mathbf{P}} = E\left[\tilde{\mathbf{V}}\right] - \left[\frac{\left(\mathbf{I} + r_I\mathbf{\Pi}_I\mathbf{\Lambda}^T\right)^{-1} Nr_I\mathbf{\Pi}_I + Mr_U\mathbf{\Pi}_U}{Nr_I + Mr_U}\right]^{-1} E\left[\frac{\tilde{\mathbf{Z}}}{Nr_I + Mr_U}\right].$$

Q.E.D.

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