INTER-TEMPORAL COST ALLOCATION AND INVESTMENT DECISIONS

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ABSTRACT

This paper considers the profit maximization problem of a firm that must make sunk investments in long-lived assets to produce output. It is shown that if per period accounting income is calculated by using a particular allocation rule for investment called the relative replacement cost (RRC) rule, that, in a broad range of plausible circumstances, the fully optimal sequence of investments over time can be achieved simply by choosing a level of investment each period to maximize next period's accounting income. Furthermore, in a model where shareholders delegate the investment decision to a better-informed manager, it is shown that if accounting income based on the RRC allocation rule is used as a performance measure for the manager, robust incentives are created for the manager to choose the profit maximizing level of investment regardless of the manager's own personal discount rate or other aspects of the manager's personal preferences.

INTRODUCTION

In a variety of industries, firms must make sunk investments in long-lived assets in order to produce output. Calculation of profit maximizing investment levels and evaluation of the firm's performance in such a situation is inherently complicated because of the need to consider implications for cash flows over multiple future periods. One technique that firms routinely use to create simplified single period "snapshots" of their performance is to calculate per period accounting income using accounting measures of cost that allocate the costs of purchasing long-lived assets over the periods that the assets will be used. Firms use these single period snapshots of performance both to directly guide their investment decisions and to evaluate the performance of managers who make investment decisions. Given the widespread use of these accounting practices to guide investment decisions, it is perhaps surprising that there has been almost no formal analysis in the economics, finance, or accounting literature that attempts to investigate whether there is any basis for these practices and, if so, how the choice of an allocation rule ought to be affected by factors such as the pattern of depreciation of the underlying asset, the firm's discount rate, the rate at which asset prices are changing over time, and the manager's own rate of time preference. This paper provides a theory which addresses these questions. It shows that, in a broad range of plausible circumstances, a particular allocation rule which will be called the relative replacement cost (RRC) rule, can be used both to create simple rules for calculating the fully optimal investment level and to create robust incentives for managers to choose this level of investment when the decision is delegated to them.

Although the literature has devoted very little attention to the question of how cost allocation rules can be used to guide investment decisions, there is, of course, an enormous literature on the more general subject of characterizing the optimal investment decision of a profit maximizing firm.[1] For purposes of investigating the role of cost allocation rules, a significant shortcoming of this literature is that is almost entirely focused on the special case of exponential depreciation where it is assumed that depreciation is equal to a constant share of the capital stock regardless of the age profile of the existing capital stock. This assumption considerably simplifies the analysis because the age profile of existing assets can be ignored. However this simplifying assumption is clearly inappropriate for purposes of investigating the role of cost allocation rules, since one of the most natural questions to investigate regarding cost allocation rules is how and why the choice of an allocation rule ought to be affected by the depreciation pattern of the underlying asset whose costs are being allocated. Obviously, the pattern of depreciation needs to be a factor which can be varied in order to consider this question. Furthermore, the case of exponential depreciation is not a particularly natural case to consider for purposes of applications. Even if there is some sense in which a certain share of a firm's over-all capital stock tends to needs replacement in any given year,[2] it is most certainly the case that almost no individual asset exhibits an exponential pattern of depreciation, even in a stochastic sense. Real firms, of course, must decide how to allocate the cost of individual assets. For such purposes, a much more natural simple case to consider is the case of "one hoss shay" depreciation, where it is assumed that assets have a known lifetime and remain equally productive over their entire lifetime. This paper will consider the case of a perfectly general

[1]See Abel (1990), Dixit and Pindyck(1994), and Nickel(1978) for excellent surveys of the literature and further references.

[2]And even this point is subject to controversy. See Feldstein and Rothschild (1974).

pattern of depreciation which includes both exponential depreciation and one hoss shay depreciation as special cases.

A well-known early result in the literature studying the case of exponential depreciation due to Jorgensen (1963) is that, if demand is weakly increasing over time so that the constraint that investment must be non-negative can be ignored,[3] the seemingly complex multi-period investment problem actually decomposes into a series of simple single-period problems. In particular, a vector of hypothetical perfectly competitive rental prices can be calculated for capital stock, and the solution to the true multi-period investment problem can be calculated as the solution to this series of time-separable hypothetical rental problems. This paper begins by showing that this decomposition result can be generalized to the case of arbitrary patterns of depreciation and derives a very simple formula for calculating the hypothetical perfectly competitive rental prices for the general case.[4] It is then shown that a particular allocation rule called the relative replacement cost (RRC) allocation rule has the property that the cost of an asset allocated to any period is equal to the hypothetical perfectly competitive rental cost of the asset in that period. It then follows immediately that the level of investment that maximizes next period's accounting income calculated using the RRC allocation rule is also the fully optimal level of investment.

The above results are derived in a model where it is assumed that there is a single centralized decision-maker that owns the firm. Therefore the role of the cost allocation rule is

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[3]A precise statement of the sufficient condition to ignore the non-negativity constraint on investment will be given below in the body of the paper.

[4]Related versions of this result already exist in the literature although not in the specific form derived in this paper. See the discussion in Section 3G.

that it simplifies the centralized decision-maker's problem of calculating the optimal level of investment. An extension of the basic model is constructed that shows how this allocation rule can also be used to create desirable investment incentives for managers when the investment decision is delegated to them. In the extension, it is assumed that shareholders delegate the investment decision to a manager because the manager has better information about demand. It is shown that shareholders can use the RRC allocation rule to create robust incentives for the manager to choose the profit-maximizing level of investment each period in the following sense. Suppose that per period accounting income is calculated using the RRC allocation rule and the manager is paid a wage each period that depends on current and past periods' accounting income. Then, it is shown that the manager will have the incentive to choose the profit maximizing level of investment in every period for any wage function that satisfies the property that the wage paid in any period is a weakly increasing function of current and past periods' accounting income. Furthermore, shareholders do not need to know the manager's own personal rate of time preference or anything else about the manager's preferences in order to calculate the RRC allocation rule. Therefore, the investment incentive problem is solved is a robust way and the firm is left with considerable degrees of freedom to address any other incentive problems that may exist, such as a moral hazard problem within each period, by choosing the precise functional form of the wage function each period.

In the formal model of this paper, it is assumed that assets have a known but arbitrary depreciation pattern and that the purchase price of new assets changes at a known constant percentage rate over time. The RRC allocation rule is defined to be the unique allocation rule that satisfies the two properties that: (i) the cost of purchasing an asset is allocated across periods of its lifetime in proportion to the relative cost of replacing the surviving amount of the asset with new assets and (ii) the present discounted value of the cost allocations using the firm's discount rate is equal to the initial purchase price of the asset.

Property (i) is related to the "matching principal" from accrual accounting that states that investment costs should be allocated across periods so as to match costs with benefits. However, the particular definition of "benefit" that it is appropriate to use requires some explanation. In order to interpret the RRC allocation rule as allocating cost in proportion to benefit, the benefit that an asset contributes to any future period must be defined to be the avoided cost of purchasing new capacity in that period. In particular, then, the relative benefit is determined solely by the avoided replacement cost and is not determined by any demand-side considerations such as the relative price that the firm will be able to sell output for in future periods.[5] Thus in the case considered by this paper where a firm engages in ongoing investment to maintain and possibly expand a stock of capital used to produce output over multiple periods, this paper shows that, for purposes of applying the matching principle to allocate investment costs, the appropriate notion of benefit to consider is the avoided cost of purchasing replacement assets at a later date. As will be discussed further below, this result stands in contrast to an earlier result by Rogerson (1997) that considers the case of a one-time investment.

Property (ii) can be viewed as stating that the investment is fully allocated taking the time

[5]Of course demand side considerations play an important role in determining whether or not the sufficient condition for using the RRC rule is satisfied. That is, whether the non-negativity constraint on investment is not binding and can therefore be ignored depends on whether demand is weakly growing or not. However, given that demand is such that this condition is satisfied, the precise manner in which demand varies over time plays no further role in determining the correct allocation rule.

value of money into account. Most traditional accounting systems ignore the time value of money when allocating investment costs over time. The term "residual income" is generally used in the accounting literature to describe income measures that are calculated using an allocation rule for investment that takes the time value of money into account (Horngren and Foster 1987, pp. 873-74). Recently there has been an explosion of applied interest in using residual income both to directly guide capital budgeting decisions and as a performance measure for managers that make capital budgeting decisions. Management consulting companies have renamed this income measure "economic value added" (EVA) and very successfully marketed it as an important new technique for maximizing firm value. *Fortune*, for example, has run a cover story on EVA, extolling its virtues and listing a long string of major companies that have adopted it (Tully 1993).[6] This paper provides an explicit formal model which justifies the use of residual income in the capital budgeting process and also specifically identifies the particular allocation rule that should be used to calculate residual income.

There are two recent groups of papers in the literature that have attempted to provide a more formal analysis of whether or not allocation rules for investment can be identified such that the resulting measures of per period accounting income can play a useful role in the capital budgeting process. One approach is due to Anctil (1996) and Anctil, Jordan and Mukherji (1998). They consider the traditional case of exponential depreciation and assume that the environment is completely stationary so that a stationary equilibrium level of capital stock exists in the sense that, if the firm begins at the stationary level of capital stock, then the firm optimally

[6]See the roundtable discussion in the *Continental Journal of Applied Corporate Finance* (Stern and Stweart 1994) and the associated articles (Sheehan 1994, Stewart 1994).

maintains a constant capital stock by incurring a constant level of investment. They also assume that there is an adjustment cost to changing the size of the capital stock and that the firm begins the first period with a capital stock that is not equal to the stationary value. The papers show that the time path of capital stock when fully optimal investments are chosen and the time path of capital stock when the firm simply attempts each period to maximize that period's residual income both converge to the stationary capital stock and thus converge to one another. This means that the policy of attempting to maximize residual income on a period-by-period basis yields a policy that converges to the optimal policy. While this is an interesting result, it only shows convergence to the optimal policy in the limit and only applies to the case of a stationary environment with exponential depreciation.

The other approach is due to Rogerson (1997).[7] Rogerson(1997) derives similar sorts of results to the results of this paper for the simpler case where it is assumed that the firm engages in a one-time investment at the beginning of the first period. This paper considers the more complex case where the firm engages in ongoing investment to create and maintain a stock of capital that it uses for ongoing production, and a different method of proof is required for this case. Furthermore, the nature of the allocation rule that creates desirable investment incentives turns out to be somewhat different in each case. In particular, while the allocation rules identified by both papers can be interpreted as allocating investment costs across periods in proportion to the relative benefit that the investment creates across periods, the relevant notion of

[7]See Rogerson (1992) for an earlier, related, result. Papers that have generalized Rogerson's(1997) result and applied it in a number of different settings include Baldenius and Reichelstein (2005), Baldenius and Ziv (2003), Dutta and Reichelstein (1999, 2002), and Reichelstein (1997, 2000).

benefit turns out to be different in each case. In particular, demand side considerations play a role in determining the relevant notion of benefit for the case considered in Rogerson (1997) but, as described above, they do not play any role in the case considered by this paper. It will be shown that there is a natural and intuitive explanation for this difference. Therefore, the two papers can be interpreted as conducting the same sort of analysis on two different types of investment problems and showing that the nature of the cost allocation rule that it is appropriate to use depends in a natural and intuitive manner on the type of investment problem being considered.

The result of this paper is also relevant to the large literature on cost allocation in both the accounting and economics literature that generally finds that there is no economically meaningful way to allocate a joint cost between products.[8] The contribution of this paper is to show that this principle does not necessarily apply to the case of "over-lapping joint costs" that is naturally created when a firm engages in ongoing investment to produce output over multiple periods.[9] When there is a single joint cost that applies to every product, the only way that the firm can increase the output of any product is by increasing its investment in the single joint cost, and this results in increased output of all of the products. However, it will be shown that in the model of this paper where there are overlapping joint costs, it is possible to adjust the entire vector of planned investments to increase output in the current period while holding output in all other periods constant. It is this feature that creates the rather surprising result that a formula

[8] See, for example, Demski(1981), Thomas(1978) and Young(1985).

[9] That is, if assets last T years, then assets purchased in period 0 are a joint cost of production for output in periods 1 through T, assets purchased in period 1 are a joint cost of production for output in periods 2 through T+1, etc.

which can be interpreted as a cost allocation rule can be used to calculate the true marginal cost of using capital in individual periods.

In a companion paper to this paper (Rogerson 2005), it is shown that the decomposition result can also be applied to the case of calculating welfare maximizing prices for a regulated firm. In particular, it is shown that when there are constant returns to scale within each period (i.e., when output in each period is proportional to the capital stock in each period), that the fully allocated unit cost of production calculated using the RRC rule is equal to the true long run marginal cost of production. Therefore setting price equal to fully allocated accounting cost calculated using the RRC allocation rule both induces efficient consumption decisions and allows the firm to break even.[10]

The paper is organized as follows. Section I presents the model under the assumption there is a single central decision-maker. Section II presents the decomposition result which shows that a simple formula can be used to calculate a hypothetical perfectly competitive rental price for capital stock each period, and the solution to the true multi-period investment problem can be calculated as the solution to this series of time-separable hypothetical rental problems. Section III introduces notation to define allocation rules and shows that the if accounting income is calculated using the RRC allocation rule, that the fully optimal sequence of investments can be calculated by choosing a level of investment each period that maximizes next period's accounting income. Section IV considers the extension to the basic model in which it is assumed that shareholders delegate the investment decision to a better-informed manager and shows that

[10]See Hausman (1997), Laffont and Tirole (2000) and Mandy and Sharkey (2003) for other related results on cost-based regulation.

robust incentives for the manager to choose the fully optimal sequence of investments are created if accounting income calculated based on the RRC allocation rule is used as a performance measure for the manager. Section V discusses the implications of this paper's theory for the inter-temporal cost allocation practices of firms. Finally, section VI draws a brief conclusion.

I. THE MODEL

Suppose that there are an infinite number of periods indexed by $t \in \{0, 1, 2, \ldots\}$ where period 0 is the current period.[11] Let $I_t \in [0, \infty)$ denote the number of assets that the firm purchases in period $t \in \{0, 1, \ldots\}$. This will also be referred to as the level of investment in period t. Let $\mathbf{I} = (I_0, I_1, \ldots)$ denote the entire vector of asset purchases. For simplicity it will be assumed that there are no other inputs.[12] Assume that an asset becomes available for use one period after it is purchased,[13] and that in each period of its life a certain fraction of the asset "depreciates" or becomes permanently unavailable for further use. It will be convenient to use notation that directly defines the share of the asset that survives and is thus available for use in each period rather than the share that depreciates. Let s_t denote the share of an asset that survives until at least period t and is therefore available for use in period t. This will be referred to as the period t survival share. Let $\mathbf{s} = (s_1, s_2, \ldots,)$ denote the entire vector of survival shares. The vector $\mathbf{s}$ will

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[11]It is possible to conduct the entire analysis in continuous time without any significant changes.

[12]It is straightforward to allow for the existence of a variable input each period, in addition to the capital input.

[13]The analysis generalizes to any lag length between the time that assets are purchased and they first enter into service.

also be referred to as the "depreciation pattern" of assets. It will be assumed that $s_t \in [0,1]$ for every t, $s_1 = 1$, and that s_t is weakly decreasing in t. For most applications it would also be natural to assume that assets are finite-lived in the sense that there is some $T \in \{1, 2, \ldots\}$ such that s_t is zero for every $t > T$. However, in order to include the case that the traditional optimal investment literature focuses on of an infinite lived asset that decays exponentially, notation that potentially allows for an infinite lifetime will be used. Let K_t denote the number of assets that the firm has available for use in period $t \in \{1, 2, \ldots\}$ due to assets purchased in period 0 or later. This will also be referred to as the capital stock in period t. Let $\mathbf{K} = (K_1, K_2, \ldots)$ denote the entire vector of capital stocks. The variable K_t is formally defined by

$$(1) \qquad K_t = \sum_{i=1}^{t} I_{t-i}\, s_i$$

Note that the above formulation does not exclude the possibility that the firm enters period 0 with existing assets that were purchased in earlier periods. However, the capital stock variable, K_t, is defined to only include assets purchased in period 0 or thereafter. It could therefore be thought of as the "incremental" capital stock created by the firm's investment decisions beginning in the current period. As will be seen, there will be no need in this paper to formally introduce notation to describe the capital stock created by assets that the firm enters period 0 with. That is, the only notion of capital stock that it will be necessary to formally model will be the "incremental" capital stock as defined above by equation (1). Therefore, for purposes of this paper, unless otherwise indicated, the terms "capital stock" and "incremental capital stock" will be used as interchangeable synonyms. The term "legacy assets" will be used to refer to assets that the firm already owns at the beginning of period 0 and the term "legacy capital stock" will be used to refer to the capital stock that a firm owns in a given period due to legacy assets when it is necessary to discuss these concepts.

It will be useful to define two special simple cases of depreciation patterns. The case of "exponential depreciation" occurs when a constant share of the asset depreciates each period. Formally this corresponds to the case where s_t is of the form

$$s_t = \beta^{t-1} \tag{2}$$

for every $t \in \{1, 2, . . \}$ where $\beta \in (0,1)$. The case of "one hoss shay depreciation" occurs when assets have a finite lifetime of T years and remain equally productive over their entire lifetime. Formally this corresponds to the case where s_t is of the form

$$s_t = \begin{cases} 1, & t \in \{1, 2, . . ., T\} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where T is a positive integer.

Let $\delta \in (0,1)$ denote the firm's discount rate. Let p_t denote the price of purchasing a new unit of the asset in period t. Assume that p_t is determined by

$$p_t = \alpha^t p_0 \tag{4}$$

where $p_0 \in (0, \infty)$ and $\alpha \in (0, 1/\delta)$. That is, the price of a new unit of the asset in period 0 is a known positive number and the price of purchasing a unit of the asset either stays constant over time or changes at a known constant percentage rate. The assumption that $\alpha < 1/\delta$ guarantees that the firm does not find it profitable to "stockpile" assets ahead of time.

It will turn out to be convenient to formally view the firm as directly choosing the vector of capital stocks **K** instead of as directly choosing the vector of asset purchases, **I**.[14] To do this, it will be necessary to describe how the firm chooses a vector of asset purchases to minimize the discounted cost of producing any vector of capital stocks **K.** Since the firm discounts future cash flows and since $\alpha < 1/\delta$, it is clear that the optimal policy is to simply purchase new assets as they are needed on a period-by-period basis and to never purchase new assets before they are needed. That is, beginning with period 0, the firm considers each period sequentially and plans to purchase the minimum number of assets necessary to produce the desired amount of capital stock in the next period given its existing assets. If sufficient capital stock will be available without any new investment, then the firm purchases no assets. Therefore the number of assets that the firm purchases in period t will depend on capital stock levels up to and including period t+1. Let $\varphi_t(K_1, \ldots, K_{t+1})$ denote the function giving the number of units of assets the firm purchases in period t to produce the vector of capital stocks $(K_1, \ldots, K_{t+1})$. Let $C(\mathbf{K})$ denote the present discounted cost of producing the vector of capital stocks **K.** It is equal to the discounted cost of purchasing the required assets.

$$(5) \qquad C(\mathbf{K}) \quad = \quad \sum_{t=0}^{\infty} \delta^t \, \varphi_t(K_1, \ldots, K_{t+1}) p_t.$$

If a firm invests efficiently to produce a given vector of capital stocks it is still possible

[14]For purposes of verbally discussing the model and its results, the firm's decision will be interchangeably referred to as either as investment decision of a capital stock decision since these are formally equivalent. For purposes of interpreting the model, it will often be more natural to refer to the firm's decision as an investment decision.

that the firm may have "excess capacity" in some periods. In any given period t, there is a certain amount of capital stock that will exist in period t+1 even if the firm engages in no new investment in period t. If the required level of capital stock in period t+1 drops below this level, then the firm will have excess capacity in period t+1 even if it purchases no new assets in period t. A vector of capital stocks that never induces such excess capacity in any period will be said to satisfy the fully utilized new investment (FUNI) property. Formally, let $\xi_t(K_1, \ldots, K_t)$ denote the amount of capital that would be available for use in period t if the firm invested efficiently in order to produce the vector of capital stocks $(K_1, \ldots, K_t)$. It is defined by

$$(6) \qquad \xi_t(K_1, \ldots, K_t) = \sum_{i=1}^{t} \varphi_{t-i}(K_0, \ldots, K_{t-i+1})\, s_i$$

Then a vector of capital stocks, **K**, will be said to satisfy the FUNI property if it satisfies

$$(7) \qquad K_t = \xi_t(K_1, \ldots, K_t)$$

for every $t \in \{0, 1, \ldots\}$.[15] Since existing assets weakly decay over time, it is clear that a sufficient condition for a vector of capital stocks to satisfy the FUNI property is that it be weakly increasing in t. For future reference this observation will be stated as a lemma.

[15] The FUNI property requires that available capital be less than or equal to the required capital. However, the functions φ_t are constructed to satisfy the requirement that the available capital is greater than or equal to the required capital. Therefore, when the FUNI property is satisfied, the required amount of capital will be precisely equal to the available amount of capital.

Lemma 1:

Consider any vector of capital stocks $\mathbf{K} = (K_1, K_2, \ldots)$. If K_t is weakly increasing in t then $\mathbf{K}$ satisfies FUNI.

Proof:

Straightforward. QED

To complete the description of the model, notation to describe the revenue that the firm earns by selling output produced by its capital stock must be introduced. Let R(K, t) denote the revenue that the firm would earn in period t if its capital stock was K, it produced at full capacity,[16] and charged as high a price as possible consistent with selling all of its output. The revenue function is of course determined by the underlying demand functions and production function. However, there will be no need to explicitly define these underlying functions. Let $R_K(K, t)$ denote the marginal revenue function. It will be assumed that the revenue function is "well-behaved" in the sense that for every t, $R_K(K, t)$ is a strictly decreasing continuous function that becomes negative for large enough values of K. Let R(**K**) denote the present discounted value of revenues produced by the vector of capital stocks **K.** It is defined by

$$(8) \qquad R(\mathbf{K}) = \sum_{t=1}^{\infty} R(K_t, t)\, \delta^t$$

[16]Producing at full capacity means that the firm uses not only all of its "incremental" capital stock captured by the variable K but also all of its "legacy" capital stock consisting of assets that were originally purchased in some period before period 0. Thus, the revenue functions as defined above will depend on the vector of legacy assets that the firm enters period 0 with. However, since the vector of legacy assets that the firm enters period 0 with is held completely fixed for the entire analysis, there is no need to introduce notation to explicitly describe this effect.

The optimization problem of the firm can now be formally defined. The firm's problem is to choose a vector of capital stocks **K** to maximize the present discounted value of its cash flows which is given by

(9) $$R(\mathbf{K}) - C(\mathbf{K})$$

As discussed in the introduction, it will also be necessary to assume that one additional condition with real economic content is satisfied in order to guarantee that the non-negativity constraint on investment does not bind. The assumption that will be made to guarantee this property is that the marginal revenue function is weakly increasing over time when marginal revenue is calculated in "real" dollars using the cost of purchasing one new unit of the asset as a numeraire. This will be referred to as the monotonicity (M) condition since its role is to guarantee monotonicity of the capital stock. It is formally stated below.

The Monotonicy (M) Condition:

$[R_K(K, t)/ \alpha^t]$ is weakly increasing in t for every K

To interpret the M condition, note that we would generally expect marginal revenue to weakly shift out over time so long as the demand curve was weakly shifting out over time. Therefore, we would generally expect the M condition to be satisfied so long as demand was weakly growing over time for any constant level of "real" prices. That is, so long as the market for the good was not shrinking because of exogenous changes in the number of consumers for the product, exogenous changes in the preferences of consumers, or exogenous changes in the real price of substitute products, we would generally expect the M condition to be satisfied so long as the rate of inflation in asset prices was less than or equal to the rate of over-all level of inflation in the economy. To the extent that the level of inflation of asset prices was strictly less than the over-all inflation rate, the M condition would tend to be satisfied even if the market was shrinking to some extent. Similarly, to the extent that the market was strictly growing, the M condition would tend to be satisfied even if the inflation level of asset prices was greater than the over-all inflation rate to some extent. Therefore, while the M condition will not always be satisfied, it is clear that it will be satisfied in a broad range of plausible circumstances.[17] For the remainder of this paper, it will be assumed that the M condition is satisfied.

II. THE DECOMPOSITION RESULT

The firm's decision-making problem would be considerably simpler if was able to rent capital stock on a period-by-period basis, instead of having to purchase assets, since the multi-period investment problem would then decompose into a series of simple single-period problems. Of course, in the model of this paper it is assumed that no such rental market for assets exists. Rental markets fail to exist in many real economic situations because it can be very expensive to install or move assets and/or because the assets used by individual firms must be specialized in ways that make the assets less useful for other firms. This section will show that, even though no such rental market exists, that it is still possible to solve the firm's decision problem by calculating the solution to the purely hypothetical problem that the firm would face if it was able

[17]Furthermore, it will clear that while the M condition is a sufficient condition for the decomposition result to hold, it is not necessary.

to rent capital stock at a vector of rental prices which can be interpreted as being "perfectly competitive prices" in the sense that they satisfy a certain simple zero-profit condition.

Specifically, the hypothetical cost function will be defined to be the function that gives the present discounted cost of providing a vector of capital stocks under the assumption that capital stock can be rented each period at perfectly competitive rental prices. The cost function defined by (5) which gives the actual cost of providing a vector of capital stocks given that assets must be purchased will sometimes be referred to as the "true" cost function to distinguish it from the hypothetical cost function. The main result is to show that the hypothetical cost of producing a vector of capital stocks is equal to the true cost of producing the vector of capital stocks if the vector of capital stocks satisfies the FUNI property and that the hypothetical cost is always less than or equal to the true cost for any vector of capital stocks. It follows immediately from this result, that the solution to the hypothetical profit maximization problem is also a solution to the true optimization problem if the solution to the hypothetical problem satisfies the FUNI property. The proof is then completed by showing that the M condition is sufficient to guarantee that the solution to the hypothetical profit maximization problem is weakly increasing which in turn implies (by Lemma 1) that it satisfies the FUNI property.

A. The Hypothetical Perfectly Competitive Rental Prices

Let w_t denote the price of renting one unit of capital stock in period t and let $\mathbf{w} = (w_1, w_2, \ldots)$ denote an entire vector of rental prices. Suppose that a hypothetical supplier of rental services can enter the market in any period by purchasing one unit of the asset and then renting out the available capital stock over the asset's life. Then under the assumptions that suppliers incur no extra costs besides the cost of purchasing the asset, they can rent the full remaining amount of the asset to a customer every period, and they have a discount rate equal to δ, the zero profit condition that must be satisfied by a perfectly competitive equilibrium is

$$(10) \qquad p_t = \sum_{i=1}^{\infty} w_{t+i}\, s_i \delta^i \qquad \text{for every } t \in \{0, 1, 2, \ldots\}$$

Let w_t* denote the period t rental price given by [18]

$$(11) \qquad w_t^* = k^*\, p_{t-1}$$

where the constant k* is defined by

$$(12) \qquad k^* = 1/\left[\sum_{i=1}^{\infty} s_i\, (\delta\alpha)^i\right]$$

and let **w*** = (w_1*, . . .) denote the entire vector of these values for rental prices. It is straightforward to verify that these rental prices satisfy the zero profit condition (10).

Lemma 2:

The vector of rental prices **w*** satisfies

$$(13) \qquad p_t = \sum_{i=1}^{T} w_{t+i}^*\, s_i\, \delta^i \qquad \text{for every } t \in \{0, 1, 2, \ldots\}$$

Proof:

Straightforward algebra. QED

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[18]Since $p_t = \alpha\, p_{t-1}$, equation (11) is equivalent to $w_t^* = k^*\alpha p_t$. It will turn out to be more convenient to express the current period rental price as a function of the previous period asset price.

The vector **w*** will therefore be called the vector of perfectly competitive rental prices.

B. The Hypothetical Profit Maximization Problem

Define the hypothetical profit maximization problem to be the problem of choosing a vector of capital stocks **K** to maximize the present discounted value of cash flows under the assumption that the firm can rent capital stock each period at the perfectly competitive rental price. Formally, let H(**K**) denote the total discounted hypothetical cost of producing the vector of capital stocks **K**. This will often be referred to as the hypothetical cost function. It is defined by

$$(14) \qquad H(\mathbf{K}) = \sum_{t=1}^{\infty} \delta^t w_t^* K_t$$

The hypothetical profit maximization problem is then to choose **K** to maximize

$$(15) \qquad R(\mathbf{K}) - H(\mathbf{K}).$$

Substitution of (8) and (14) into (15) yields

$$(16) \qquad \sum_{t=1}^{\infty}[R(K_t, t) - w_t^* K_t]\, \delta^t$$

Note that K_t appears only in the t^{th} term of equation (16). Therefore the solution to the hypothetical problem decomposes into a series of single period problems. Namely, K_t is the value of K that maximizes

$$(17) \qquad R(K, t) - wK.$$

where w is set equal to w_t^*. The first order condition for a maximum to (17) is

(18) $\{R_K(K, t) = w \text{ and } K > 0\}$ or $\{R_K(K, t) \leq w \text{ and } K = 0\}$.

The assumption that marginal revenue is a strictly decreasing continuous function that eventually becomes negative for large enough values of K obviously implies that for every $w \in [0, \infty)$ and $t \in \{1, 2, \ldots\}$ there is a unique solution to (17) and that the sufficient conditions for maximization are also satisfied by this solution. Let K_t^* denote the unique solution to (18) for $w=w_t^*$ and let $\mathbf{K}^* = (K_1^*, K_2^*, \ldots)$ denote the entire vector of such capital stocks.

Lemma 3:

The unique solution to the hypothetical profit maximization problem is $\mathbf{K}^* = (K_1^*, K_2^*, \ldots)$

Proof:

Straightforward. QED

C. Comparing the Hypothetical Cost Function and the True Cost Function

Lemma 4 shows that the true cost function is equal to the hypothetical cost function over the set of vectors of outputs satisfying FUNI and that the true cost is strictly greater than the hypothetical cost for all other vectors of outputs.

Lemma 4:

(19) $C(\mathbf{K}) = H(\mathbf{K})$ if and only if $\mathbf{K}$ satisfies FUNI

(20) $C(\mathbf{K}) > H(\mathbf{K})$ for every $\mathbf{K}$ which does not satisfy FUNI

proof:

See Appendix. QED

The intuition for Lemma 4 is as follows. The rental prices are constructed to have the property that the cost of purchasing an asset is exactly equal to the cost of renting the same asset if the asset is rented for every period of the asset's life. By definition, when **K** satisfies FUNI, the rental services the firm would choose in the hypothetical case where it can rent assets could be produced by purchasing assets and fully utilizing them. Therefore the cost should be the same. When **K** does not satisfy FUNI, the firm would take advantage of a rental market to use fewer assets in some periods than would be possible in the case where it had to purchase its own assets. This means that the hypothetical cost is then lower than the true cost. Intuitively, the fact that firm must purchase its own assets and cannot resell them places limits on the extent to which it can reduce its costs by reducing asset usage from period to period. However, when FUNI is satisfied, this means that the constraint is not binding, and the true cost of production is therefore equal to the hypothetical cost of production.

D. Directly Calculating the Marginal Cost of Increasing Capital Stock in a Single Period

According to Lemma 4, the marginal cost of increasing the capital stock in a given period is a constant so long as the FUNI property is satisfied. An intuition for this result can be developed by directly calculating marginal cost. For any given period, one can construct the entire series of incremental changes in investment levels that would result in a small increase in the capital stock in that period while holding the level of capital stock constant in all other periods. The present discounted value of these changes is by definition the marginal cost of increasing capital usage in the given period. So long as the level of investment in all future periods is positive so that small adjustments either up or down are possible in any period, the pattern of incremental adjustments is exactly the same regardless of the particular levels of capital stock in the given period or future periods, and this in turn implies that the marginal cost is the same. Capital stock vectors in the interior of the set of capital stock vectors satisfying the FUNI property of course satisfy the property that investment in every period is strictly positive. (It turns out that the result also holds on the edges of the set.)

For example, consider the case of one hoss shay depreciation so that each asset last with undiminished productivity for T years. Suppose that the firm plan plans to purchase at least one unit of the asset every period.[19] Then the firm can increase its capital stock in period 1 by one unit while holding the capital stock in all other periods constant by implementing the following series of adjustments to its investment plans. The firm must purchase an additional unit of the asset in period 0 to increase the capital stock by one unit in period 1. However, it will now be able to reduce its asset purchases by one unit in period 1. Now when period T arrives, the extra asset that the firm purchased in period 0 will no longer be available in the next period, so the firm will have to purchase an extra unit of the asset in period T to maintain its level of capital stock at the previously planned level. However, as before, it will now be able to reduce its asset purchases by one unit in period T+1. This process continues indefinitely. That is, the firm can

[19]The assumption that the firm is purchasing at least one unit of the asset every period will be made use of to calculate the cost of increasing the capital stock by one unit. Correspondingly smaller increases in the capital stock require correspondingly smaller levels of investment every period.

increase its capital stock in period 1 by exactly one unit and hold the capital stock in all other periods constant by shifting the purchase of one unit of the asset forward in time from period 1 to 0, T+1 to T , 2T+1 to 2T, etc. The present discounted value of the cost of these adjustments calculated in period 1 dollars is, by definition, the marginal cost of increasing the capital stock by one unit in period 1. It is straightforward to directly calculate this value and show that it is equal to w_1*.

For more complex patterns of depreciation, the pattern of shifts in asset purchases over all future periods required to increase the capital stock in the next period by one unit while holding the capital stock in all future periods constant can become very complicated and difficult to directly calculate. Lemma 4 essentially shows that there is a very simple formula which can be used to calculate the present discounted value of the required shifts in asset purchases even if the actual series of shifts in asset purchases becomes very complicated and difficult to calculate.

E. Sufficient Conditions for the Solution to the Hypothetical Problem to Also be the Solution to the True Problem

Proposition 1, stated below, follows immediately from Lemma 4.

Proposition 1:

(i) Suppose that **K** is a solution to the hypothetical profit maximization problem. Then if **K** satisfies FUNI, it is also a solution to the true profit maximization problem.

(ii) If a unique solution exits to the hypothetical profit maximization problem and satisfies FUNI, then it is also the unique solution to the true profit maximization problem.

proof:

See Appendix QED

Lemma 3 has already established that **K*** is the unique solution to the hypothetical profit maximization problem. It is straightforward to show that the M condition is sufficient to guarantee that K_t* is weakly increasing in t. It then follows immediately from Lemma 1 that the M condition is sufficient to guarantee that **K*** is the unique solution to the true profit maximization problem. This result is stated as Proposition 2.

Proposition 2:

If Condition M holds, then the vector of capital stocks **K*** is the unique solution to the true profit maximization problem.

Proof:

As above QED

F. Discussion

The above results show that the optimal number of additional assets for the firm to purchase in period 0 is equal to the number of additional assets it would be optimal to rent in period 1 at the rental price w_1*. Equation (11) shows that w_1* is equal to k*p_0. Therefore in order to calculate the optimal investment level in the current period, the firm only needs to calculate the number of assets that it would be profit maximizing to rent next period if the rental rate was equal to the constant k* multiplied by the current purchase of assets, p_0.

In addition to being very simple, this problem also has low informational requirements about the nature of future demand in the sense that the precise functional forms of future demand curves beyond period 1 do not need to be known. That is, so long as the firm knows that future demand satisfies condition M, there is no need for the firm to know the precise functional forms of the future demand curves beyond the immediate next period in order to calculate the fully optimal investment decision in the current period.

G. Previous Literature on the Decomposition Result

As mentioned in the introduction, the traditional optimal investment literature has almost exclusively studied the special case of exponential depreciation because it greatly simplifies the analysis. The decomposition result for this special case is well-known and is due to Jorgesen (1963). Versions of the decomposition result have also been derived for more general depreciation patterns. Most notably, Arrow(1964) proved a very general version of this result in an early but now somewhat forgotten paper. Arrow also provides references to yet-earlier papers that contain this result for the special case of one hoss shay depreciation. Therefore the main incremental contribution of this paper is to show how the decomposition result can be applied to the analysis of cost allocation rules.

However, the version of the decomposition result derived in this paper differs from Arrow's in a number of respects, in ways that allow it to be applied to the issue of cost allocation rules. Most importantly, Arrow bases his derivation on calculating the so-called "renewal function" which is a relatively complicated function describing the infinite series of replacements of an asset as it depreciates, replacements of the replacement assets as they in turn depreciate, etc. that would be required to maintain a constant stock of capital. This paper shows that it is possible to directly calculate the hypothetical rental prices based on a much simpler calculation that simply requires the present discounted value of the stream of hypothetical rental payments from renting any given asset be equal to the cost of purchasing the asset. The result that the RRC allocation rule is a residual income measure follows directly from this alternate formula while there does not appear to be any simple way to directly establish this result using Arrow's formula based on the renewal function. Related to this point, instead of analyzing the optimization problem using control theory, this paper shows that it is possible to use the simpler and more direct approach of directly proving that the present discounted cost of providing a vector of capital stocks is a simple linear function of the vector of capital stocks (over the relevant range of capital stocks) so that the optimization problem can be directly solved by point-wise maximization of profit at every point in time. That is, the result of Lemma 4 is new to this paper.

III. USING COST ALLOCATION RULES TO CALCULATE OPTIMAL INVESTMENT LEVELS

The previous section has shown that the optimal number of assets for the firm to purchase in the current period (period 0) is equal to the number of additional assets that it would be optimal to rent in the next period (period 1) if additional assets could be rented at a rental rate equal to the constant k* multiplied by the current purchase price of assets, p_0. As will be explained below, any cost allocation rule for assets can be characterized as a vector of real numbers $\mathbf{a} = (a_1, a_2, \ldots)$ where a_i is interpreted as the share of the original purchase price of the asset allocated to the i^{th} period of the asset's life. It therefore follows immediately that if accounting income is calculated using an allocation rule such that $a_1 = k^*$, that the level of period 0 investment that maximizes next period's accounting income will also be the optimal level of investment. This section will introduce definitions and notation to formally describe allocation rules and formally demonstrate this point. It will also identify one particularly natural allocation rule that has the property that $a_1= k^*$ which will be called the relative replacement cost (RRC) allocation rule.

A. Allocation and Depreciation Rules

Define a depreciation rule to be a vector $\mathbf{d} = (d_1, d_1, \ldots)$ such that $d_i \geq 0$ for every i and

$$\sum_{i=1}^{\infty} d_i = 1 \tag{21}$$

where d_i is interpreted as the share of depreciation allocated to the i^{th} period of the asset's life.[20] Let Ddenote the set of all depreciation rules. Define an allocation rule to be a vector $\mathbf{a} = (a_1, a_2, \ldots)$ that satisfies $a_i \geq 0$ where a_i is interpreted as the share of the asset's purchase cost that is allocated to the i^{th} period of the asset's life. Let Adenote the set of all allocation rules. For any $\gamma \in [0, \infty)$ an allocation rule will be said to be "complete given γ" if the discounted sum of the allocation shares using γ as a discount rate is equal to 1, i.e., if

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[20]Since the notation of this paper potentially allows for the case where an asset is infinitely lived, the notation used for depreciation rules and allocation rules will also potentially allow a positive share of depreciation or cost to allocated to periods infinitely far in the future. For the typical case of a finite lived asset all of the depreciation and allocation shares will be zero for periods beyond the end of the asset's lifetime.

(22) $$\sum_{i=1}^{\infty} a_i \gamma^i = 1$$

It is clear that for any $\mathbf{a} \in A$ there exists a unique value of $\gamma \in [0, \infty)$ such that $\mathbf{a}$ is complete given γ. Let $\Gamma(\mathbf{a})$ denote the unique value of γ that $\mathbf{a}$ is complete with respect to.

Firms generally think of themselves as directly choosing a depreciation rule and a discount rate instead of as directly choosing an allocation rule. The cost allocated to each period is then calculated as the sum of the depreciation allocated to that period plus imputed interest on the remaining (non-depreciated) book value of the asset. Formally, for any depreciation rule, $\mathbf{d}$, and discount rate γ the corresponding allocation rule is given by

(23) $$a_i = d_i + \{(1-\gamma)/\gamma\} \sum_{j=i}^{\infty} d_j$$

It is straightforward to verify that the resulting allocation rule determined by (23) is complete given γ. It is also straightforward to verify that for any $\mathbf{a} \in A$, there is a unique $(\mathbf{d}, \gamma) \in D\mathrm{x}[0, \infty)$ such that (23) maps $(\mathbf{d}, \gamma)$ into $\mathbf{a}$. It is defined by $\gamma = \Gamma(\mathbf{s})$ and

(24) $$d_i = \sum_{j=i}^{\infty} \gamma^{j-i} a_j - \sum_{j=i+1}^{\infty} \gamma^{j-i-1} a_j$$

Therefore one can equivalently think of the firm as choosing either a depreciation rule and a discount rate or as choosing an allocation rule. For the purposes of this paper, it will be more convenient to view the firm as directly choosing an allocation rule.[21]

B. The Result

It will now be formally shown that if an allocation rule that satisfies $a_1 = k^*$, the fully optimal level of investment in period 0 is also the unique level of investment that maximizes period 1 accounting income. To do this it will be necessary to introduce notation to formally define accounting cost and accounting income. The accounting cost in any period is by definition equal to the sum of all historic investment costs allocated to that period including the cost of legacy investments made before period 0.[22] Let $L_t \in [0, \infty)$ denote the cost of legacy assets allocated to period t for $t \in \{1, 2, \ldots\}$. Let $\mathbf{L} = (L_1, L_2, \ldots)$ denote the entire vector of legacy cost allocations. Let $A_t(K_1, \ldots, K_t, \mathbf{a})$ denote the total accounting cost allocated to period t if the firm produces the vector of capital stocks $(K_1, \ldots, K_t)$ and uses the allocation rule **a.**[23] This will be called the period t accounting cost and if given by

[21]See Rogerson (1992) for a fuller discussion of the relationship between depreciation and allocation rules and their properties.

[22]As will be seen, the costs of legacy assets are additively separable from the costs of assets purchased in period 0 and beyond and therefore legacy costs will be irrelevant to determining the values of current and future investments that maximize accounting income. Notation to formally defined legacy costs will be introduced essentially only to make it clear that these costs can be ignored.

[23]Accounting cost also obviously depends on the vector of legacy costs, L. However, since the vector of legacy costs remains fixed for the entire analysis and is additively separable from the accounting cost of investment made in period 0 and onwards, it will have no effect on the analysis and it is convenient to use notation that suppresses legacy costs.

(25) $$A_t(K_1, \ldots, K_t, \mathbf{a}) = \sum_{i=1}^{t} I_{t-i}\, p_{t-i}\, a_i + L_t$$

where

(26) $$I_j = \begin{cases} \phi_j(K_1, \ldots, K_{j+1}), & j \geq 0 \\ 0 & j \leq -1. \end{cases}$$

Equations (25)-(26) can be explained as follows. In the RHS of equation (25), the term L_t is the accounting cost of legacy assets allocated to period t and the term with the summation sign is the cost of investments made in period 0 onwards that is allocated to period t. The ith term of the summation is the cost of investment made in period t-i that is allocated to period t.

Let $Y_t(K_1, \ldots, K_t, \mathbf{a})$ denote the total accounting income in period t if the firm produces the vector of capital stocks $(K_1, \ldots, K_t)$ and the allocation rule **a** is used to calculate accounting cost. It is given by

(27) $$Y_t(K_1, \ldots, K_t, \mathbf{a}) = R(K_t, t) - A_t(K_1, \ldots, K_t, \mathbf{a})$$

Proposition 3 now states the desired result.

Proposition 3:

Suppose that **a** is an allocation rule such that $a_1 = k^*$. Then the optimal level of capital stock in period 1, K_1^*, is also the unique value of capital stock that maximizes period 1 accounting income calculated using **a**.

Proof:

See Appendix. QED

C. The RRC Allocation Rule

Let **a*** = (a_1*, a_2*, . . .) denote a particular allocation rule which will be called the relative replacement cost (RRC) allocation rule, defined by

(28) $$a_i^* = k^* s_i \alpha^{i-1}$$

If one unit of an asset is purchased in some period t, the price of purchasing new assets in the ith period of its lifetime is equal to $\alpha^i p_t$. Therefore the cost of purchasing new replacement assets for the surviving amount of the asset in the ith period of its life is equal to $\alpha^i p_t s_i$. Therefore a cost allocation rule **a** can be interpreted as allocating costs across periods in proportion to the cost of replacing the surviving amount of the asset if the allocation shares satisfy

(29) $$a_i = k\, s_i \alpha^i \qquad \text{for some } k \in (0, \infty).$$

The RRC allocation rule obviously satisfies (29). It is also straightforward to observe that the RRC allocation rule is complete with respect to δ and, furthermore, that it is the only allocation rule that is both complete with respect to δ and satisfies (29). Therefore the RRC allocation rule can be thought of as the unique allocation rule that satisfies the two properties that: (i) costs are allocated in proportion to replacement cost as specified in equation (29); and (ii) the present discounted value of the cost allocation shares is equal to 1.

This rule is very simple and easy to understand even for the case of a general pattern of depreciation. The case of one hoss shay depreciation is particularly simple and likely to be a natural case to consider in many real situations. In this case, the asset has a finite lifetime of T years and remains equally productive over its entire lifetime. The replacement cost of one unit of the asset in any period is of course simply the price of a unit of the asset in that period. Therefore allocation shares are chosen to satisfy the two requirements that (i) the allocation shares change at the same percentage rate over time that the purchase price of new assets is changing at; and (ii) the present discounted value of the allocation shares using the firm's discount rate is equal to one.

Finally, from (28) it is also immediate that the RRC allocation rule satisfies $a_1^*= k^*$. Therefore by Proposition 3, the level of investment that maximizes period 1 accounting income is also the optimal level of investment.

Corollary 1:

The optimal level of capital stock in period 1, K_1^*, is also the unique value of capital stock that maximizes period 1 accounting income calculated using the RRC allocation rule.

Proof:

As above. QED

Since the requirement that a_1 be equal to k^* only determines the first period allocation share, there are of course a continuum of other allocation rules that also satisfy this requirement. However, the RRC allocation rule is a particularly natural and simple rule meeting this requirement and it is not clear that other equally simple or natural rules satisfying this requirement can be identified. Furthermore, as will be shown in the next section, the RRC allocation rule is actually the unique allocation rule satisfying a more stringent set of conditions that create additional desirable properties.

IV. MANAGERIAL INVESTMENT INCENTIVES

The above results are derived in a model where it is assumed that there is a single centralized decision-maker that owns the firm. Therefore the role of the cost allocation rule is that it simplifies the calculation of the optimal investment level. This section will consider a situation where shareholders delegate the investment decision to the manager of the firm and show that there is a sense in which using accounting income based on the RRC allocation role as a performance measure can create robust incentives for the manager to choose the profit maximizing investment level. Specifically, it will be assumed that shareholders create a compensation scheme for the manager by choosing an allocation rule to calculate accounting income, and then base the manager's wage each period on current and possibly historic values of accounting income. It will be shown that, if the RRC allocation rule is used to calculate accounting income, that so long as the wage function each period is weakly increasing in current and past periods' accounting income, that the manager will have the incentive to choose the profit maximizing investment plan regardless of the manager's own personal rate of time preference or any other aspect of the manager's own preferences. Therefore, by restricting themselves to using a compensation scheme where the RRC allocation rule is used to calculate accounting income and the manager's wage each period is determined by some weakly increasing function of current and past periods accounting incomes, shareholders can guarantee in a robust way that the manager will choose the profit maximizing investment level. Furthermore, shareholders are left with considerable degrees of freedom to address other managerial incentive issues by choosing the particular form of the wage function each period. For example, if there was a moral hazard problem within each period so that the manager must be given incentives to exert effort each period, the wage function could be tailored to address this problem while use of the RRC allocation rule simultaneously guaranteed that the manager always chose the profit maximizing investment level.

A. The Basic Result

Assume that the production/demand environment is as described in the previous sections. Assume that shareholders know the parameters of model necessary to calculate the RRC allocation rule, i.e., they know the depreciation pattern of assets, **s**, and they know the future rate of change of asset prices, α. Finally, assume that while shareholders know that the revenue functions for future periods satisfy the M condition, assume that this is all they know about the future revenue functions. This means that they do not have sufficient information to calculate the optimal investment level. Assume that the manager knows all of the functions and parameters in the model so that the manager is able to calculate the optimal investment level.

Suppose that shareholders delegate the production decision to the manager in order to take advantage of his private information and that they create a compensation scheme for the manager by choosing an allocation rule and wage function. The allocation rule is used to calculate each period's accounting income. The wage function determines the wage the manager receives each period as a function of the accounting income in the current period and past periods. Assume that the manager has a utility function defined over vectors of wage payments over time. For any given wage function, one can therefore define an indirect utility function of the manager over vectors of accounting income. It will turn out to be useful to employ notation that suppresses the manager's utility function over wage payments and the wage function chosen by shareholders and instead focuses on the indirect utility function over income created by the composition of these two functions. Let y_t denote period t accounting income and let $\mathbf{y} = (y_1, y_2, \ldots)$ denote an entire vector of accounting incomes. Let $U(\mathbf{y})$ denote the manager's indirect utility function over vectors of accounting incomes that is created by the choice of a wage function and the manager's own direct utility function over wage payments.

For any given allocation rule **a**, the manager will choose the vector of capital stocks **K** to maximize

$$U(Y_1(K_1, \mathbf{a}), Y_2(K_1, K_2, \mathbf{a}), \ldots). \tag{30}$$

In general this would appear to be a very complex problem whose solution depends on the precise functional form of U which in turn depends on both the wage function and manager's own underlying preferences over vectors of wage payments, including the manager's own personal discount rate. Therefore, even if it was possible to identify a wage function and allocation rule that induced the manager to choose the efficient level of investment each period, it appears that it would be necessary to have detailed information about the manager's preferences, including his own personal discount rate, in order to perform such a calculation.

This section will show that almost all of this apparent complexity can be avoided by the appropriate choice of an allocation rule. In particular, most of the apparent complexity is created by the fact that, in general, the manager faces trade-offs between maximizing accounting income in different periods. That is, an investment plan that would maximize accounting income in any given period is unlikely to also maximize accounting income in all other periods. Therefore, selecting the optimal investment plan requires the manager to make complex tradeoffs between periods. The wage function and the manager's own underlying utility function over wage payments both will have significant and potentially complex effects on how the manager trades off accounting income between periods. However, suppose that there was a vector of capital stocks that simultaneously maximized the accounting income in *every* period. Then, so long as the indirect utility function U was weakly increasing in all of its arguments, this vector of capital stocks would also obviously maximize the managers's utility regardless of the precise functional form of U. The fairly modest condition that each period's wage is weakly increasing in current and past periods' accounting income is clearly sufficient to guarantee that U is weakly increasing in each of its arguments.[24] Therefore if there was a vector of capital stocks that simultaneously maximized the accounting income in every period, this vector of capital stocks would maximize the manager's utility for the entire class of wage functions satisfying the modest condition that each period's wage is a weakly increasing function of current and past periods' accounting income. The result of this section is that the RRC allocation rule has this property with respect to the profit maximizing vector of capital stocks **K*.**

Formally, an allocation rule **a** will be said to create robust incentives for the manager to choose the capital stock vector **K** if **K** maximizes each period's accounting income calculated

[24]Of course, it must also be assumed that the agent's direct utility function over wages is weakly increasing in each period's wage.

using **a.**

Definition:

An allocation rule $\mathbf{a}' = (a_1', a_2', \ldots)$ will be said to create robust incentives for the manager to choose the capital stock vector $\mathbf{K}' = (K_1', K_2', \ldots)$ if

$$(31) \qquad (K_1', K_2', \ldots K_t') \in \underset{(K_1, K_2, \ldots K_t)}{\operatorname{argmax}} \; Y_t(K_1, K_2, \ldots K_t, \mathbf{a}') \text{ for every } t \in \{1, 2, \ldots\}.$$

As discussed above, if an allocation rule creates robust incentives for the manager to choose a vector of capital stocks, this means that the manager will find it optimal to choose this vector of capital stocks when his wage each period is a weakly increasing function of current and past periods' accounting income caclulated using this allocation rule.

The main step in showing that the RRC allocation rule creates robust incentives for the manager to choose the profit maximizing vector of capital stocks, **K*,** is presented in Lemma 5. One additional piece of notation must be introduced to state this lemma. In any period t, the firm may own and operate units of capital that were originally purchased i periods earlier for every $i \in \{1, 2, \ldots, t\}$ such that s_i is positive. An allocation rule **a** will determine a potentially different accounting cost for each vintage of capital. Let $c(t, i, \mathbf{a})$ denote the accounting cost of a unit of capital in period t calculated using the allocation rule **a**, given that it was purchased i periods earlier.

$$(32) \qquad c(t, i, \mathbf{a}) = a_i \, \alpha^{-i} \, p_t/s_i$$

This formula can be explained as follows. The price of purchasing a unit of the asset in period t-i is equal to $\alpha^{-i}\, p_t$. In order to provide one unit of capital stock in period t the firm must purchase $1/s_i$ units of the asset in period t-i. Therefore the total cost of purchasing sufficient assets in period t-i to create one unit of capital in period t is equal to $\alpha^{-i}\, p_t/s_i$. The share a_i of this cost is allocated to period t.

Lemma 5 now observes that the RRC allocation rule has the property that for any period t, the accounting cost of a unit of capital for any vintage of capital is the same and is equal to the hypothetical perfectly competitive rental rate for that period w_t*. Furthermore the RRC allocation rule is the only allocation rule satisfying this property.

Lemma 5:

For every $t \in \{1, 2, . . .\}$ and $i \in \{1, . . ., t\}$ such that s_i is positive, the RRC allocation rule, **a***, is the unique allocation rule that satisfies

$$c(t, i, \mathbf{a}) = w_t^*. \tag{33}$$

Proof:

See Appendix. QED

Lemma 5 states that the RRC allocation rule calculates accounting cost by imputing a cost of w_t* to every unit of capital that the firm owns in that period. However, this is precisely how cost is calculated in the hypothetical profit maximization problem. The result that **K*** maximizes each period's accounting income calculated using the RRC rule therefore follows directly.

Proposition 4:

The RRC allocation rule, **a***, creates robust incentives for the manager to choose the profit maximizing vector of outputs, **K***.

Proof:

See Appendix. QED

B. Discussion

The above result does not show that a contract using the RRC allocation rule to calculate accounting income is a fully optimal solution to a completely specified principal agent problem It is clear that such a result would be straightforward to prove in a model where it was assumed that the only incentive/information problem is that the manager is better informed than shareholders about some information necessary to calculate the fully optimal investment/production plan. However, there would be no need in such a model to base the manager's wage on any measure of the firm's performance. That is, one fully optimal contract would be for shareholders to simply pay the manager a constant wage each period sufficient to induce the manager to accept the job. Then the manager would be (weakly) willing to choose the profit maximizing investment plan.

Therefore, in reality, the result of this paper will only be useful in situations where there is some additional incentive problem which requires shareholders to base the manager's wage on some measure of the firm's performance. A natural candidate would be to assume that there is a moral hazard problem within each period, i.e., that each period the manger can exert unobservable effort which affects the firm's cash flow that period. The modeling problem this creates is that calculating optimal contracts that are fully optimal solutions to multi-period principal agent problems is exceedingly complex. Furthermore, an essential aspect of the situation being considered in this paper is that the manager must have better information than shareholders about some aspect of the environment in order to justify the assumption that shareholders delegate the investment decision to management in the first place. Adding the problem of incomplete information to a multi-period moral hazard problem increases the complexity and difficulty of the problem by another order of magnitude. It is generally impossible to solve such problems using existing techniques unless extreme simplifying assumptions are made about the nature of the incomplete information. Furthermore, even when sufficient assumptions are made to allow calculation of a solution, it is often the case that the nature of the solution depends in extreme ways on particular aspects of the environment such as particular aspects of the agent's preferences that the principal is unlikely to have reliable information about. Thus, it is not clear that such contracts would be suitable for use in the real world where robustness to small changes in the environment is likely to be important.

In light of these difficulties, the result of this paper can be interpreted as offering a useful alternative approach. In particular, this paper shows that, by restricting themselves to choosing a compensation scheme where accounting income is calculated using the RRC allocation rule and each period's wage is a weakly increasing function of current and past periods' accounting income, shareholders can guarantee in a robust way that the investment incentive problem will be completely solved and still leave themselves considerable degrees of freedom to address remaining incentive issues. By using accounting income based on the RRC allocation rule as a performance measure, shareholders could thereby guarantee that the investment incentive problem was completely solved and then use a "trial and error" process over time to identify a wage function that appeared to create the appropriate level of effort incentives.[25]

This paper's approach of showing that a certain allocation rule induces the manager to make first-best investment decisions so long as each period's wage is weakly increasing in current and past periods' accounting income was first used in Rogerson (1997) to analyze a model where it is assumed that investment only occurs once, in the initial period. The current paper essentially provides a similar sort of result for the more complex case where investment occurs every period. The reader is referred to Rogerson(1997) for a more detailed discussion of the rationale for this approach. Dutta and Riechelstein (2002) have shown that Rogerson's (1997) allocation rule can be made part of a fully optimal contractual solution to a fully specified principal agent model that includes a moral hazard component in what they refer to as a LEN model - which means that contracts are assumed to be linear, utility is assumed to be exponential, and noise is assumed to be normal. An interesting project for future research would be to determine if the Dutta/Reicheltein(2002) approach could be adapted to the model of this paper where investment occurs every period, and, more generally, whether the RRC allocation

[25]Note that in cases where it is possible to calculate a fully optimal contract it may be that the fully optimal contract will not necessarily induce the agent to choose the profit maximizing level of investment. A general lesson from the incentives literature is that when one calculates the fully optimal contractual solution to a situation involving multiple interacting incentive problems, it is often the case that it is optimal to purposely distort the solution to one problem away from the first best in order to get extra leverage on the other problem. However, it precisely these sorts of calculations that are exceedingly complex and that are unlikely to be robust to small changes in the contracting environment. Therefore, if shareholders have an opportunity to guarantee that a first best solution is created to one of the two incentive problems while still leaving themselves considerable degrees of freedom to address the second incentive problem through a trial and error process, this may be a very attractive alternative in the real world.

rule could be shown to be part of a fully optimal contract in more general circumstances.

V. IMPLICATIONS FOR THE INTER-TEMPORAL COST ALLOCATION PRACTICES OF REAL FIRMS

The results of this paper suggest that firms should use the RRC allocations rule to allocate investment costs for purposes of creating measures of accounting income that can be used to guide investment decisions. This section will be discuss some of the characteristics of the RRC allocation rule and discuss how the normative theory of cost allocation suggested by this paper addresses some of the cost allocation issues traditionally considered in the economics and accounting literature.

A. The Matching Principal

The "matching principal" from accrual accounting states that investment costs should be allocated across periods in proportion to the benefit that the asset contributes to each period. The property of the RRC allocation rule that costs are allocated across periods in proportion to replacement cost can therefore be interpreted as satisfying a version of the matching principal when the benefit that an asset contributes to any future period is defined to be the avoided cost of purchasing new capacity in that period. Note that, under this definition, the relative benefit is determined solely by the avoided replacement cost and is not determined by any demand-side considerations such as the relative price that the firm will be able to sell output for in future periods.[26]

This result stands in sharp contrast to the result from Rogerson (1997) which considers a model where there is only a single investment opportunity at the beginning of the first period[27] Rogerson (1997) shows that an allocation rule which it calls the relative benefit (RB) rule has the property that the level of investment that maximizes the discounted present value of cash flows also maximizes accounting income on a period-by-period basis. This, in turn, creates robust incentives for the manager to choose the profit-maximizing investment level in the same sense as the term is used in this paper. Just as for the RRC rule in this paper, the RB rule can be interpreted as the unique allocation rule that allocates costs across periods in proportion to the "benefit" the asset induces across periods such that the present discounted value of the allocations is equal to the original purchase cost of the asset. However, the relevant notion of

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[26]Of course demand side considerations play an important role in determining whether or not the sufficient condition for using the RRC rule is satisfied. That is, whether the non-negativity constraint on investment is not binding and can therefore be ignored depends on whether demand is weakly growing or not. However, given that demand is such that this condition is satisfied, the precise manner in which demand varies over time plays no further role in determining the correct allocation rule.

[27]More precisely, the distinction between the case considered in Rogerson (1997) and the case considered in the current paper is that Rogerson (1997) considers an investment project whose returns are additively separable from any other investment project that the firm may undertake in the future. That is, it is clear that Rogerson's(1997) result would continue to hold in a situation where the firm invests every period so long as the investment projects are all additively separable from one another (i.e., if the firm makes an investment each period which affects cash flows in future periods and the cash flows from each project are additively separable from the cash flows of every other project on a period-by-period basis.) However, in the case where the firm invests to maintain and possibly increase a stock of capital that it uses to produce units of the same good over multiple periods, the investment problems are inherently not separable in this sense. That is, the marginal productivity of a new unit of investment in a given period is obviously affected by the existing level of capital stock which is in turn affected by the level of investment in previous periods.

benefit in this case is the cash flow that the investment would have generated in that period. In particular, then, demand side considerations affect the relevant notion of benefit.

The difference in results for the two different cases is intuitively reasonable. In the case of a one-time investment that can either be undertaken in the current period or not undertaken at all, the relevant alternative to investing is not investing at all. Relative to this alternative the benefit of the investment is the entire cash flow that it will generate. In the case of ongoing investment associated with producing the same product over time, if it is profitable to produce output in future periods, the firm can make investments in the future regardless of whether or not it invests today. Therefore the relevant alternative to investing today is to wait and invest tomorrow instead. Relative to this alternative, the benefit of the investment is the avoided replacement cost. In summary, the current paper and Rogerson (1997) can be interpreted as conducting the same sort of analysis on two different types of investment problems and showing that the nature of the cost allocation rule that it is appropriate to use depends in a natural and intuitive manner on the type of investment problem being considered.

It is also important to note that the matching principle is applied directly to *cost allocation* rules and that is NOT applied directly to *depreciation* rules. For example, consider the case of one hoss shay depreciation and suppose that the replacement cost of assets is expected to stay remain constant over time. Then the RRC allocation allocates the same cost to every period subject to the constraint that the present discounted value of the allocations is equal to the original purchase price of the asset. It is straightforward to see that the depreciation rule that induces this allocation rule will have depreciation shares that *increase* over time.[28] In particular, the RRC rule is NOT induced by using the so-called straight-line depreciation rule that allocates the same share of depreciation to every period. This suggests that if firms began to employ the approach suggested by this paper to inter-temporal cost allocation, it would become much more natural and simpler for them to directly think of themselves as choosing a cost allocation rule instead of as directly choosing depreciation rule and interest rate.

B. Residual Income

Most traditional accounting systems ignore the time value of money when allocating investment costs over time. That is, they simply choose depreciation shares that sum to one and define the share of cost allocated to each period to be equal to that period's depreciation. The term "residual income" is generally used in the accounting literature to describe income measures that are calculated using an allocation rule for investments that takes the time value of money into account (Horngren and Foster 1987, pp. 873-74). Recently there has been an explosion of applied interest in using residual income both to directly guide capital budgeting decisions and as a performance measure for managers that make capital budgeting decisions. Management consulting companies have renamed this income measure "economic value added" (EVA) and very successfully marketed it as an important new technique for maximizing firm value. *Fortune*, for example, has run a cover story on EVA, extolling its virtues and listing a long string

[28]Since the non-depreciated book value of an asset must be decreasing over time, the only way for the total cost allocated to each period to remain constant is for the amount of depreciation allocated to each period to increase.

of major companies that have adopted it (Tully 1993).[29]

It is intuitively clear that any measure of accounting income that can be used to sensibly guide decisions with implications for cash flows across multiple periods must somehow take the time value of money into account. The contribution of this paper is to provide an explicit theory that justifies this intuition and, to identify the particular allocation rule that should be used to calculate residual income.

C. "Overlapping Joint Costs" and Calculating Marginal Cost by Allocation of Joint Costs

There is large literature on cost allocation in both the accounting and economics literature that generally finds that there is no economically meaningful way to allocate a joint cost between products.[30] The contribution of this paper is to show that this principle does not necessarily apply to the case of "over-lapping joint costs" that is naturally created when a firm engages in ongoing investment to produce output over multiple periods.[31] When there is a single joint cost that applies to every product, the only way that the firm can increase the output of any product is by increasing its investment in the single joint cost, and this results in increased output of all of the products. However, in the model of this paper where there are overlapping joint costs, it is possible to adjust the entire vector of planned investments to increase output in the

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[29]See the roundtable discussion in the *Continental Journal of Applied Corporate Finance* (Stern and Stweart 1994) and the associated articles (Sheehan 1994, Stewart 1994).

[30]See, for example, Demski(1981), Thomas(1978) and Young(1985).

[31] That is, if assets last T years, then assets purchased in period 0 are a joint cost of production for output in periods 1 through T, assets purchased in period 1 are a joint cost of production for output in periods 2 through T+1, etc.

current period while holding output in all other periods constant. It is this feature that creates the two rather surprising related results that: (i) the formula for calculating the present discounted cost of producing a vector of capital stocks over time is linear and additively separable in each period's capital stock over a broad range of capital stocks and (ii) a formula which can be interpreted as a cost allocation rule can be used to calculate the true marginal cost of using capital in a single period.

D. The Effect of Technical Progress on Cost Allocation

If technical progress is causing the purchase price of assets to fall over time, the RRC allocation rule will allocate less cost to early periods of the assets life and more cost to later periods of the assets life. Thus, holding all other factors constant, an increase in the rate of technical progress will increase next period's accounting cost of using capital when accounting cost is calculated using the RRC rule. At first glance, the result that an increase in the rate of technical progress, which should presumably result in lower costs in the future, causes the accounting cost of using assets in the next period to rise might seem counter-intuitive. The explanation for this result is that the firm increases capital stock in the next period while holding the level of capital stock in all future periods constant by essentially shifting asset purchases forward in time. When there is a higher rate of technical progress, the opportunity cost of shifting asset purchases forward in time becomes higher. Therefore, while a higher rate of future technical progress will eventually cause production costs to drop, it will actually increase the marginal cost of providing capital in the next period and the RRC allocation rule reflects this.

V. CONCLUSION

This paper shows that, in a surprisingly broad range of plausible circumstances, the seemingly complex multi-period profit maximization problem faced by a firm that must invest in long-lived assets to produce output can be decomposed into a series of much simpler single period problems. Namely, it is shown that a simple formula exists to calculate hypothetical perfectly competitive rental prices for assets and that the capital stock that the firm would choose to employ each period if it could rent assets at the hypothetical perfectly competitive price turns out to also be the capital stock that maximizes the firm's profits given that it must purchase assets. Furthermore, it is shown that a simple cost allocation rule called the relative replacement cost (RRC) rule has the property that the cost of purchasing an asset allocated to any period is exactly equal to the hypothetical perfectly competitive rental price for that period. This implies that accounting income calculated using the RRC allocation rule has two desirable properties. First, it can be used to simplify the calculation of the optimal investment level, because in any given period the firm can calculate the fully optimal level of investment for that period simply by calculating the level of investment that maximizes next period's accounting income. Second, robust incentives for management to choose the profit maximizing level of investment can be created by using this measure of accounting income as a performance measure for management.

APPENDIX

PROOFS OF PROPOSITIONS

Proof of Lemma 4:

The following two conditions are satisfied by construction and by the definition of FUNI

(A.1) $\xi_t(K_1, \ldots, K_t) \geq K_t$ for every $t \in \{1, 2, \ldots\}$

(A.2) $\xi_t(K_1, \ldots, K_t) = K_t$ for every $t \in \{1, 2, \ldots\}$ if and only if **K** satisfies FUNI.

Substitute equation (13) into equation (5) and reorganize the summation to yield

(A.3) $$C(\mathbf{K}) = \sum_{t=1}^{\infty} w^*_t \delta^t \sum_{i=1}^{t} s_i \delta^t \phi_{t-i}(K_1, \ldots, K_{t-i+1})$$

Substitution of (6) into (A.3) yields

(A.4) $$C(\mathbf{K}) = \sum_{t=1}^{\infty} w^*_t \delta^t \xi_t(K_1, \ldots, K_t)$$

Note that the hypothetical cost function, given by (14), is created by substituting K_t for $\xi_t(K_1, \ldots, K_t)$ in (A.4). The result then follows from (A.1) and (A.2). QED

Proof of Proposition 1:

Part (i):

Suppose that $\mathbf{K}'$ is a solution to the hypothetical problem and satisfies FUNI. Now suppose for contradiction that $\mathbf{K}'$ is not a solution to the true problem. Then there exists a vector of capital stocks $\mathbf{K}''$ such that

(A.5) $R(\mathbf{K}'') - C(\mathbf{K}'') > R(\mathbf{K}') - C(\mathbf{K}')$.

Since $\mathbf{K}'$ satisfies FUNI,

(A.6) $$C(\mathbf{K}') = H(\mathbf{K}').$$

All capital stock vectors, including $\mathbf{K}''$ must satisfy

(A.7) $$C(\mathbf{K}'') \geq H(\mathbf{K}'')$$

Substitution of (A.6) and (A.7) into (A.5) yields

(A.8) $$R(\mathbf{K}'') - H(\mathbf{K}'') > R(\mathbf{K}') - H(\mathbf{K}')$$

which contradicts the assumption that $\mathbf{K}'$ is a solution to the hypothetical problem. QED

Part (ii):

Suppose that $\mathbf{K}'$ is a the unique solution to the hypothetical problem and satisfies FUNI. By part (i), $\mathbf{K}'$ is also a solution to true problem. Now suppose for contradiction that $\mathbf{K}'$ is not the unique solution to the true problem. Then there exists a $\mathbf{K}'' \neq \mathbf{K}'$ such that

(A.9) $$R(\mathbf{K}'') - C(\mathbf{K}'') = R(\mathbf{K}') - C(\mathbf{K}').$$

Substitution of (A.6) and (A.7) into (A.9) yields

(A.10) $$R(\mathbf{K}'') - H(\mathbf{K}'') \geq R(\mathbf{K}') - H(\mathbf{K}')$$

which implies that $\mathbf{K}''$ is a solution to the hypothetical problem, which is a contradiction. QED

Proof of Proposition 3:

Accounting income in period 1 is given by

(A.11) $R(K_{1,} 1) - p_0 a_1 K_1 - L_1$

Substitution of $a_1 = k^*$ and $w_1{}^* = k^* p_0$ into (A.11) yields

(A.12) $R(K_1, 1) - w_1{}^* K_1 - L_1$

Equation (A.12) is obviously maximized by $K_1{}^*$. QED

Proof of Lemma 5:

From equations (33) and (11), an allocation rule **a** satisfies c(t, i, **a) =** $w_t{}^*$ if and only if

(A.13) $a_i\, \alpha^{-i}\ p_t/s_i = (k^* p_t / \alpha)$

Equation (A.13) can be rewritten as

(A.14) $a_i = s_i \alpha^{i-1}\, k^*$

By equation (28), the RHS of (A.14) is the definition of $a_i{}^*$. QED

Proof of Proposition 4:

Lemma 5 states that each unit of capital stock will create an accounting cost of $w_t{}^*$. Therefore the firm will maximize its period t accounting income by choosing K_t to maximize

(A.15) $R_t(K_t) - w_t{}^* K_t - L_t$

and by choosing the entire vector **K** to make sure that the capital stock in period t is fully utilized The vector **K*** obviously satisfies these conditions. QED

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