Optimal Managerial Tenure
in Dynamic Agency with Renegotiation

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This paper examines the impact of repeated renegotiation on incentives and managerial tenure when performance information is serially correlated. In addition to providing a general solution to a multi-period agency problem with serially correlated performance measures, the paper characterizes optimal managerial tenure/turnover policies as a function of the time-series properties of performance measures. With negatively correlated performance measures, the principal prefers longer managerial tenure, and no turnover is optimal. With positively correlated performance measures, absent a switching cost, turnover every period is optimal. In the presence of a fixed switching cost, interior optimal turnover policies exist if the performance measures are positively correlated. The optimal turnover policies present an alternative to theories of performance-driven managerial turnover and are consistent with evidence that a majority of managerial turnovers are (age-related) normal retirements.

Key words: dynamic agency, renegotiation, managerial tenure, LEN models

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1. Introduction
This paper examines the impact of repeated renegotiation on incentives and managerial tenure when performance information is serially correlated.¹ The paper makes three contributions to existing research on dynamic agency. First, it provides a general proof that the analysis can be restricted without loss of generality to renegotiation-proof contracts in a dynamic LEN model with renegotiation that generalizes Christensen, Feltham, and Şabac 2005. Second, it provides a general recursive solution for an N-period agency with renegotiation and (arbitrarily) correlated performance measures that generalizes Gibbons and Murphy 1992, Christensen, et al. 2003, 2005, and Dutta and Reichelstein 1999. Third, the paper characterizes optimal managerial tenure/turnover policies as a function of the time-series properties of performance measures.

Declining managerial performance prior to managerial turnover is usually interpreted in support of the theory that poor performance leads to forced turnover, see Murphy and Zimmerman 1993, ¹ Limited commitment and/or inter-temporal dependencies are frequently assumed away in repeated agency models, as they tend to make the analysis intractable by introducing information asymmetries at contracting time. For example, Radner 1985 assumes independent periods, Rogerson 1985 and Spear and Srivastava 1987 assume commitment. Rey and Salanie 1990 and Fudenberg et al. 1990 show that for commitment to long-term contracts not to have value, it is necessary that no information asymmetries exist at contracting time.
Huson, Parrino, and Starks 2001, Engel, Hayes, and Wang 2003, Huson, Malatesta, and Parrino 2004, and the references therein. This view of managerial tenure is based on the “common sense […] that when an organization is performing poorly, replacement of the manager might be expected”, Salancick and Pfeffer 1980, p. 653. Others argue that CEO power determines the link between pay and performance, and tenure in office. For example, Hill and Phan 1991 present evidence that the pay/performance link for a CEO weakens with increased CEO tenure and attribute this to an increase in the CEO’s power. Hermalin and Weisbach 1998 develop an analytical model of CEO performance, board of directors selection, and CEO turnover in which endogenous turnover is associated with poor performance—given that the CEO’s power is based on perceived ability, which in turn is based on observed past performance.

Commenting on the existing empirical evidence on managerial turnover and firm performance, Brickley 2003, p. 232 writes: “[…] we have probably reached a point of diminishing returns in estimating logit models that focus on the relation between CEO turnover and firm performance measures.” In the same discussion paper, Brickley points to the fact that age is a better explanatory variable for CEO turnover than firm performance. Moreover, Huson et al. 2004 find that firm performance has a V shape in the years around CEO turnover, while a majority of turnovers in their sample (almost three quarters) are normal retirements. These empirical facts raise the question of what may determine these non-forced turnovers.

This paper offers optimal managerial tenure as a possible alternative to performance-driven turnover. Thus, the proposed explanation to why age-related turnover dominates is that it is optimal in those cases to retain the manager until retirement age. But what is optimal for a manager’s tenure, and what determines it? In a qualitative model of CEO tenure, Hambrick and Fukutomi 1991 raise the question of optimal managerial tenure, but do not pursue the issue other than discussing some tradeoffs which are expected to determine optimal CEO tenure with a firm. In this paper, I use an $N$-period LEN (Linear contracts, Exponential utility, Normal distributions) moral hazard model in which the time-series properties of performance measures used in incentive contracts determine optimal managerial turnover policies.

The LEN model is based on Dutta and Reichelstein 1999, and extends Christensen, Feltham, and Şabac 2003, 2005 to $N$ periods. Developing a multi-period moral hazard model is necessary.
both for the analysis of managerial tenure and turnover, and for the analysis of the related question of incentives and performance dynamics.\textsuperscript{4}

The paper contributes to the literature on multi-period LEN models first by providing a proof that renegotiation-proof contracts are sufficient in characterizing equilibria in the “standard” LEN model. This result is a generalization to multiple periods and time-additive agent utility of Christensen et al. 2005, who prove the sufficiency of renegotiation-proof contracts in a two-period LEN model with single consumption date multiplicative agent utility. Gibbons and Murphy 1992, Sliwka 2002, and Dutta and Reichelstein 2003 all use renegotiation-proof contracts, but do not provide a proof for their sufficiency given the linear contract restriction. The second contribution is to provide a general solution to the agency problem under renegotiation, given performance measures with arbitrary time-series properties. The general solution to the agency problem, allows for considering a variety of performance measurement systems, in particular negatively auto-correlated accounting-based performance measures, and highlights the different implications of using different performance measurement systems. By contrast, Gibbons and Murphy 1992 and other related papers are restricted to a single performance measure structure corresponding to the career concerns model of Holmström 1999.

Inter-temporal correlation of the performance measures implies that the solution to the agency problem is not a simple repetition of the single-period contract, in particular the sufficient conditions of Fudenberg, Holmström, and Milgrom 1990 for short-term contracts to replicate long-term contracts are not satisfied. Institutional restrictions on contract form or duration, or the inability of parties to commit to not renegotiate has been recognized as a key feature of dynamic agency (see for example Hart and Tirole 1988, Dewatripont 1989, Fudenberg and Tirole 1990, Hermelin and Katz 1991, Ma 1994, and Dewatripont and Maskin 1995). Demski and Frimor 2001 also examine the impact of repeated renegotiation in more than two periods and find that the negative effects of renegotiation are reduced by adding more periods. However, they are mainly concerned with “earnings management” and assume independent periods, while I exogenously assume the agent has no control over the reporting of performance measures and I focus on their inter-temporal correlation.\textsuperscript{5}

\textsuperscript{4} Within a pure moral hazard agency theory framework, Gibbons and Murphy 1992 extend the career concerns model of Holmström 1999 to include endogenously determined managerial performance and incentives as a function of time, given explicit contracts in anticipation of retirement. Gibbons and Murphy do not consider varying managerial tenure and their model is based on a specific performance measurement system. Thus, they cannot examine the link between the performance measurement system on one hand, and managerial tenure and performance on the other hand.

\textsuperscript{5} In a series of multi-period papers on the design of accounting-based performance measures and residual income that use LEN models, Dutta and Reichelstein 1999, 2002 and Dutta and Zhang 2002 assume full commitment to an
To determine optimal managerial tenure, I consider three specific cases: an auto-regressive structure, a career concerns model with learning of a productivity parameter, and an accounting model that allows for reversible accruals. The main findings on managerial tenure are as follows. First, the principal’s welfare is increasing in managerial tenure if the performance measures are negatively correlated (in the accounting model). This finding suggests that normal retirement is optimal given negatively correlated performance measures, consistent with the view that negatively correlated performance measures are generally preferred in a dynamic agency (see Christensen et al. 2005), with the use of accounting-based performance measures, and with normal retirement as a dominant form of managerial turnover. Second, in all models with positive correlation, within a certain range of managerial switching costs, there exists interior optimal tenure. If the switching costs are high, the principal’s preferences are for retaining the manager indefinitely, while if they are small enough, switching agents every period dominates.

The issue of optimal turnover has been addressed in two period models; however, this restricts optimal turnover policies to corner solutions and rules out the consideration of more complex information structures. Dutta and Reichelstein 2003 compare contracting with a single agent over two periods under full commitment and contracting with two agents over the same two periods in a slightly different LEN model with long-term investments. They characterize conditions under which long-term/short-term contracts are preferred by the principal. In a two-period version of the setting considered in this paper, Christensen et al. 2003, footnote 2, point out that the principal prefers agent turnover every period, if, and only if the performance measures are positively correlated.

The remainder of the paper is organized as follows. Section 2 presents the model, and Section 3 presents the result on renegotiation-proof contracts, together with the optimal linear renegotiation-proof contract. Section 4 presents the analysis for managerial tenure. Section 5 concludes the paper. Appendix A gives a detailed analysis of the information structures used, and Appendix B contains the proofs.

N-period contract. Moreover, the residual income performance measures in Dutta and Reichelstein 1999 and Dutta and Zhang 2002 reduce the agency problem to one with independent periods in which the performance measures are statistically independent and the agent’s effort affects only current period performance. By contrast, while the performance measures in this paper can be transformed to an equivalent set that are serially uncorrelated, the agent’s effort affects these independent measures over several periods. Christensen et al. 2004 provide a detailed discussion of statistical and technological independence of performance measures, see also Şabac 2005.

2. The principal-agent model

A risk-neutral principal owns a production technology that requires effort \( a_t \) from an agent in each of \( N \) periods \( t = 1, \ldots, N \). The agent’s utility is time additive with multiplicatively separable effort cost \( u_t(q) = -\sum_{k=1}^{\infty} \gamma^{k-t} \exp(-\hat{\gamma}(q_k - \frac{1}{2} a_k^2)) \), for a consumption stream \( q = (q_t, q_{t+1}, \ldots) \), where \( q_t \) represents the agent’s consumption at date \( t \), the start of period \( t+1 \), \( \frac{1}{2} a_t^2 \) is the agent’s personal effort cost in period \( t \), and \( \hat{\gamma} \) is the agent’s risk aversion. The discount rate \( \gamma = (1 + R)^{-1} \) is the same for the principal and the agent and the agent can freely borrow or lend at rate \( R \). The output from agent’s effort \( a_t \in \mathbb{R} \) is, for \( t = 1, \ldots, N \), \( z_t = b_t a_t + \lambda_t \), where \( \lambda_t \) is a mean zero noise term which does not depend on \( a_t \). Neither the outcomes \( z_t \) nor the agent’s actions \( a_t \) are observable, hence neither is contractible.\(^7\) A contractible performance measure \( x_t \) is observed at the end of each period. The agent’s effort in period \( t \) affects only the mean of the performance measure in that period, \( x_t = m_t a_t + \varepsilon_t \), where \( \varepsilon_t \) are mean zero noise terms that are joint normally distributed.

For each \( 1 \leq t \leq N \), let \( a_t = (a_1, \ldots, a_t) \), and \( x_t = (x_1, \ldots, x_t) \) denote the histories of actions and performance for the first \( t \) periods. I use the notation \( E_t[\cdot] \) for the conditional expectation given history \( x_t \) and \( \text{cov}_t(\cdot, \cdot) \) for the conditional covariance given history \( x_t \). The conditional variance of \( x_t \) given history \( x_{t-1} \) is denoted \( \sigma^2_t = \text{var}(x_t|x_{t-1}) \).\(^8\)

Let \( w_t \) denote the agent’s compensation at date \( t \) (in date \( t \) currency at the end of period \( t \)). After date \( N \), the agent retires, provides no more productive effort, and receives no further compensation. Let \( W_t \) represent the NPV (net present value) of future compensation discounted to date \( t \) for each employment date \( t = 1, \ldots, N \): \( W_t = \sum_{k=t}^{N} \gamma^{k-t} w_k \). Similarly, let \( K_t \) represent the present value of future effort cost at date \( t \): \( K_t = \frac{1}{2} \sum_{k=t}^{N} \gamma^{k-t} a_k^2 \). Since I assume both the principal and the agent can borrow and lend at the same rate, the timing of the agent’s compensation does not affect either the principal’s or the agent’s utility, provided the present value of total compensation at a fixed date is constant.

The agent’s compensation is determined by long-term linear contracts. A long-term linear contract at date \( t-1 \) is a take-it-or-leave-it offer by the principal \( c_t = (c_{t-1}, \beta_2, \ldots, \beta_N) \) where \( c_{t-1} \)

\(^7\) I assume the output is not observable because of measurement problems, which creates demand for accounting-based performance measures; these are correlated with output, but the conditional expectations of the random components of output given the observed performance measures do not enter the principal’s decision problem. Similarly, the change in market value over a period is based on market participants’ information and is, like the accounting, another performance measure correlated with unobservable output. Contracting on output or output measured with error can be treated as particular cases in which \( b_t = m_t \).

\(^8\) The conditional variances do not depend on the agent’s actions and represent the common posterior beliefs of the principal and the agent about the variance of future performance measures given past observations of the performance measures. The conditional expectations depend on past observed values of the performance measures, but conditional variances do not.
specifies a fixed payment of $\alpha_{t-1}$ in date $t-1$ currency, and $\beta_k$ specifies a variable payment $\beta_k x_k$ in date $k$ currency, $t \leq k \leq N$. Since the principal and the agent can freely borrow and lend, there is flexibility in the timing of compensation, and the above specification of a linear compensation scheme is without loss of generality. In other words, any sequence of payments linear in the history of reported performance measures $w_t(x_t)$ can be rearranged so that a fixed amount is paid upfront, and the portion that is variable with respect to $x_t$ is paid after $x_t$ is reported. I assume there is no other contractible information. The time line for period $t$ is presented in Figure 1.

![Figure 1](https://example.com/figure1.png)

**Figure 1** Time line of events in period $t$ from date $t-1$ to date $t$.

Given linear contracts, the agent’s compensation is normally distributed and the agent’s expected utility at date $t-1$ is characterized by the following certainty equivalent, where $r = (1 - \gamma)\hat{r}$ is the agent’s effective risk aversion, see Dutta and Reichelstein 1999 and Christensen et al. 2004:

$$ACE_{t-1} = \gamma E_{t-1}[W_t] - \gamma K_t - \frac{1}{2}\sum_{k=t}^{N} \gamma^{k-t+1} \text{var}_{k-1}(E_k[W_k]).$$

(1)

The agent’s reservation certainty equivalent is normalized to zero at the start of the first period. Utility functions, discount rates, reservation wages, the nature of the production technology, and the information structure are common knowledge.

### 3. Renegotiation of a long-term contract

I make two key commitment assumptions. First, if the agent accepts the contract offer $c^0$, both the principal and the agent commit to the employment relationship for $N$ periods. Second, I assume the principal and the agent cannot commit not to renegotiate after the performance measures are observed. The existing contract can only be replaced by a new contract if both parties agree to

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9 Throughout the paper I consider the actions $a_{t-1}$ a sunk cost at the start of period $t$, and therefore not directly included in the agent’s certainty equivalent (1). Past actions affect the agent’s welfare at the start of period $t$ only through the means of past performance measures in the conditional expectation $E_{t-1}[W_t]$. 
The renegotiation takes the form of a take-it-or-leave-it offer by the principal. By contrast, full commitment is the case in which the principal and the agent commit not to renegotiate the initial contract at any later date.

An initial contract $\theta_0 = (\alpha_0, \beta_1, \ldots, \beta_N)$, unless renegotiated, becomes at date $t$ a contract $c^t = (\alpha_t, \beta_{t+1}, \ldots, \beta_N)$, at dates $t, \ldots, N$, with $\alpha_t = \beta_t x_t$ the fixed payment at date $t$. The principal’s renegotiation offer at any date $t$ is $c^{Rt+1} = (\alpha_{t+1}^{Rt+1}, \beta_{t+2}^{Rt+1}, \ldots, \beta_N^{Rt+1})$, that is a date $t$ fixed payment and a sequence of variable payments at subsequent dates, such that the agent’s compensation discounted to any date $t, \ldots, N$ is linear in the agent’s future performance. At date $t+1$, assuming the renegotiation offer at date $t$ is accepted, the contract in effect is of the form $c^{It+2} = (\gamma^{-1} \alpha_{t+1}^{Rt+1} + \beta_{t+1}^{Rt+1} x_{t+1}, \beta_{t+2}^{Rt+1}, \ldots, \beta_N^{Rt+1})$, that is a date $t+1$ fixed payment and a sequence of variable payments at subsequent dates. Thus, the renegotiation offer $c^{Rt+2}$ is restricted to the same linear form as the existing contract in effect at that date, $c^{It+2}$.

Renegotiation-proof contracts It is a well-known result, when no restriction is imposed on the contract form, that there is no loss of generality in restricting the analysis to renegotiation-proof contracts, see Fudenberg and Tirole 1990. Christensen et al. 2003, 2005 extend this result to a two-period LEN model with a single consumption date and multiplicative exponential agent utility. In what follows, I extend this result to a LEN model with multiple consumption dates and time-additive agent utility.

An equilibrium in the principal-agent renegotiation game consists of a sequence of contracts $(c^{I1}, c^{R1}, \ldots, c^{IN}, c^{RN})$, the agent’s actions $a_1, \ldots, a_N$, and principal’s conjectures about the agent’s actions $\hat{a}_1, \ldots, \hat{a}_N$ such that: (i) the agent accepts both the initial contract and all the renegotiation offers, and rationally anticipates all renegotiation offers and their acceptance when selecting his actions; (ii) the principal’s conjectures about the agent’s actions are correct, that is $\hat{a}_t = a_t$ for all $t$; (iii) the sequence $(c^{I1}, c^{R1}, \ldots, c^{IN}, c^{RN})$ is ex-ante (start of the first period) optimal and $(c^{I1}, c^{R1}, \ldots, c^{IN}, c^{RN})$, is ex-post (start of period $t$, for all $t$, conditional on $x_{t-1}, \hat{a}_{t-1}$) optimal from the principal’s point of view. Note that in an equilibrium with renegotiation, the acceptance of all renegotiation offers means that $c^{It+1} = c^{Rt}$.

10 A commitment to a contract that both parties agree to renegotiate would be hard to enforce, by a third party, in court. On the other hand, as long as terminating the employment relationship before the initially agreed date is costly to either party, commitments to the duration of contract are likely enforceable.

11 The renegotiation concept I use here is the same as that of Fudenberg and Tirole 1990 in that both parties must agree to the renegotiated contract, but the timing is different. In my model, renegotiation takes place after the performance measure $x_t$ is observed, while Fudenberg and Tirole have the renegotiation take place between the time the agent takes the action and the time the performance measure is observed in a single period model. Having the renegotiation take place after $x_t$ is observed and before $a_{t+1}$ is taken avoids the insurance/adverse selection problem of Fudenberg and Tirole.
A linear renegotiation-proof contract is a contract such that, once agreed upon at the start of the first period, there does not exist a contract at any later renegotiation stage which is weakly preferred by both parties and at least one party strictly prefers. If an initial contract \( c^I \) is renegotiation-proof, then \( c^I_t = c^R_t = c^I_{t+1} \) for all \( t \). The fact that a linear contract can be renegotiation-proof is due to the additive structure of the LEN framework. It turns out that, for every initial contract that is linear in \( x_1, \ldots, x_N \), the optimal renegotiation offers (which are restricted to be linear in \( x_t, \ldots, x_N \) ) have period \( t, \ldots, N \) incentives independent of \( x_{t-1} \) and a fixed wage that is linear in \( x_{t-1} \). In other words, the renegotiation offers are also linear in the \( N \) performance measures from an ex-ante (start of the first period) perspective, and thus can be offered in an initial linear contract.\(^{12} \)

The following proposition shows that the analysis of the equilibrium can be restricted without loss of generality to renegotiation-proof contracts.

**Proposition 1.** If the sequence \( (c^I_1, c^R_1, \ldots, c^I_N, c^R_N, a_1, \ldots, a_N) \) of contracts and actions is an equilibrium in the principal-agent renegotiation game, then \( c^R_t \) is also ex ante (at the start of period \( 1, \ldots, t - 1 \) ) linear in \( x_1, \ldots, x_N \) and offering the contract \( c^R_N \) in every period \( (c^R_N, \ldots, c^R_N, a_1, \ldots, a_N) \) is an equivalent equilibrium with a single renegotiation-proof contract. Moreover, a date \( t \) contract \( c^t = (\alpha_t, \beta_{t+1}, \ldots, \beta_N) \) is renegotiation-proof at all later dates if, and only if, \( \beta_k \) is the ex post optimal incentive rate at date \( k - 1 \) for all dates \( k = t + 1, \ldots, N \).

The main idea in the proposition is that if the contract is renegotiated in equilibrium, the agent’s actions are completely determined by the renegotiated contract. The principal cannot gain by offering the agent a contract that will later be renegotiated as long as the agent anticipates the renegotiation.

Because the principal and the agent rationally anticipate future renegotiation encounters, the agent’s actions in periods \( 1, \ldots, t - 1 \) are influenced by the rationally anticipated future contractual terms in periods \( t, \ldots, N \). At the same time, past actions \( a_1, \ldots, a_{t-1} \) are sunk and ignored at renegotiation time in period \( t \). As a result, renegotiation imposes constraints that make the renegotiation-proof contract suboptimal relative to a full commitment contract. In other words, the renegotiation-proof contract is time consistent but not optimal in the sense used by Kydland and Prescott 1977. If the performance measures in the \( N \) periods are independent, there is no difference between the two contracts, which take the form of repeatedly inducing the optimal action from the one period problem.

\(^{12} \) Note that, in general, linear renegotiation offers do not allow for linear renegotiation-proof contracts, see Christensen and Feltham 2005.
The optimal linear renegotiation-proof contract Since I assumed the principal has all the bargaining power, the fixed wage offered in equilibrium to the agent at the initial date is determined by the agent’s participation constraint and compensates the agent for effort cost and the risk premium, both of which are determined by the variable incentive rates. Thus, the following proposition completely characterizes the optimal linear renegotiation-proof contract.

PROPOSITION 2. The actions induced by the optimal linear renegotiation-proof contract and the optimal incentives are given by the following recursive relations:

\[ a_N = m_N \beta_N = \frac{m_N^2 b_N}{m_N^2 + r \sigma_N^2}, \]  
\[ a_t = \beta_t m_t, \]  
\[ \beta_t = \frac{m_t b_t}{m_t^2 + r \sigma_t^2} - \frac{r \sigma_t^2}{m_t^2 + r \sigma_t^2} \sum_{k=t+1}^{N} \gamma^{k-t} \beta_k H_{kt}, \]

where \( H_{kt} \) characterizes the conditional expectations operator, \( E_t[\varepsilon_k] = H_{kt} \cdot \varepsilon_t \), with components \( H_{kt} = (H_{kt1}, \ldots, H_{ktN}) \), and \( \sigma_t^2 = \text{var}_{t-1}(x_t) \).

The agent’s effort in period \( t \) is uniquely determined by the period \( t \) incentive rate \( \beta_t \) since \( a_t \) impacts only \( x_t \) and the contract is renegotiation-proof. The last period action (and incentive) are the familiar ones from the single period LEN agency problem, where the performance measure variance is the posterior variance \( \sigma_N^2 \) in period \( N \). In earlier periods, the optimal incentive has two components. The first component is the myopic optimal incentive for that period, \( (m_t b_t)/(m_t^2 + r \sigma_t^2) \), based on the marginal benefit to the principal \( b_t \) from period \( t \) productive effort \( a_t \) and the posterior variance of the period \( t \) performance measure. The second component is the compensation risk insurance adjustment the principal makes because of covariance between current and future performance measures.

To highlight the insurance component of the incentive rate, consider as a simple example the case \( b_t = m_t = 0 \), that is the principal has no benefit from period \( t \) agent effort and the period \( t \) performance measure is not informative with respect to period \( t \) effort (the parameters for the remaining periods \( t+1, \ldots, N \) are arbitrary, and generally non-zero). Since for any incentive rate

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13 The principal’s conjectures of the agent’s actions are independent of the observed performance measures and of the agent’s actual actions. The agent, regardless of chosen effort, always effectively observes the performance measure noise \( \varepsilon_t \). Thus, by deviating from equilibrium, the agent benefits neither from better information on performance measure noise, nor from changing the principal’s beliefs. Moreover, if an equilibrium exists, then \( a_t = \bar{a}_t \) and the principal can always offer an equivalent renegotiation-proof contract by Proposition 1. Having offered the optimal linear renegotiation-proof contract, regardless of the agent’s actions and the observed performance measures, the principal will not renegotiate the contract, the key being that the optimal incentive rates are independent of performance measure realizations and conjectured agent’s actions. The agent is precluded from making renegotiation offers in the model, but having deviated from equilibrium in the past does not change his current optimal action choice.
β_t, the induced period t action is a_t = 0, the principal only sets β to minimize risk premium given β_{t+1},...,β_N. If b_t = 0, but m_t ≠ 0, the principal’s problem is the same except for agent effort a_t = m_tβ_t being induced in period t, which is costly to the principal. As a result, compared to the previous case, the absolute value of the insurance adjustment is reduced by a factor of \( r \sigma_t^2 / (m_t^2 + r \sigma_t^2) < 1 \). See also Christensen et al. 2005, where the two cases discussed above correspond to their basic insurance model and basic window dressing model, respectively.

The insurance adjustment in the incentive rates takes into account future optimal incentive rates (which are fixed by the requirement that the contract is renegotiation proof) and the covariance between current period performance and future periods’ performance.\(^{14}\) Thus, if \( x_t \) is uncorrelated with future performance measures, the optimal incentive rate is the myopic one. In this case, the performance measure plays a pure effort incentive role since there is no covariance compensation risk. Otherwise, a positive correlation between period t performance and period k performance results in a downward adjustment to the period t incentive rate, assuming future incentive rates to be positive, because the principal trades off myopic incentives for period t against increased covariance risk cost. Conversely, a negative correlation between period t performance and period k performance results in an upward adjustment to the period t incentive, because the negative covariance further reduces period t risk cost.

4. Managerial tenure

In this section I consider the principal’s preferences for different contracting horizons and the possibility of optimal tenure (contract duration) for the agent. Since I need to compare different lengths of managerial tenure from the point of view of a long-lived firm, I assume the problem is stationary: that is, the firm is infinitely lived and identical managers are replaced every N periods, and historical performance information from previous agents is such that the information structure is the same each time a new agent is hired.\(^{15}\) The first task is to construct a measure of the principal’s welfare.

The principal’s surplus Since the principal has all the bargaining power, the expected agent compensation is equal to the agent’s expected effort cost plus the risk premium. The principal’s expected surplus (expected benefit less expected agent compensation) is then completely determined by the optimally induced actions: directly through the expected benefit \( b_t a_t \) and the agent

\(^{14}\) The impact of \( x_t \) on the conditional expectation \( E_t[x_k] \) is proportional, with a positive proportionality constant, to \( \text{cov}(x_t, x_k) \).

\(^{15}\) The key assumption is the stationarity of the information environment over time, so that different contract durations can be compared. To the extent firm growth does not alter the information structure of performance measurement, the results should not differ for growing or declining firms.
effort cost $\frac{1}{2}a_t^2$ in each period, and indirectly through the risk premium, which depends on the incentive rates, which in turn are related to the induced actions by (3). The only complex term in the representation of the principal’s surplus is the risk premium for which the agent must be compensated, see (1). As the following proposition shows, it is possible to obtain a simple representation for the principal’s surplus in terms of the optimally induced actions.

**Proposition 3.** The principal’s expected surplus given the optimal linear renegotiation-proof contract is given by

$$U_p^t = \sum_{t=1}^N \gamma^t \left( b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} \frac{m_t^2}{r \sigma_t^2} (b_t - a_t)^2 \right) = \sum_{t=1}^N \gamma^t \left( \frac{1}{2} b_t^2 - \frac{1}{2} \frac{m_t^2}{r \sigma_t^2} (b_t - a_t)^2 \right) .$$ (5)

Following on the above expression for the principal’s expected surplus, I denote by $U_p^t$ the portion of the principal’s expected utility attributable to period $t$ agent effort:

$$U_p^t = \frac{1}{2} b_t^2 - \frac{1}{2} \frac{m_t^2}{r \sigma_t^2} (b_t - a_t)^2 .$$ (6)

The principal’s total expected utility is then simply the present value of period-specific expected utilities, $U_p = \sum_{t=1}^N \gamma^t U_p^t$ and the principal’s objective function is the annual equivalent value of expected surplus (AEV thereafter) $U_p(N) = U_p / A_N = \sum_{t=1}^N \gamma^{t-1} U_p^t / \sum_{t=1}^N \gamma^{t-1}$, which corresponds to a stationary policy of replacing agents every $N$ periods. The factor $1/A_N$ represents an annuity with an NPV of one over $N$ periods. For the remainder of the paper, I will analyze a repeated agency problem, for which it is natural to assume identical periods, $b_t = b$, $m_t = m$, for all $t$.

Note that the above decomposition of the principal’s expected utility (5) characterizes the principal’s welfare as a function of only the induced managerial effort. Since

$$\frac{\partial U_p^t}{\partial a_t} = \left( 1 + \frac{m_t^2}{r \sigma_t^2} \right) (b_t - a_t) \geq 0$$ (7)

for all $a_t \leq b_t$, it follows that the expected period $t$ profit is strictly increasing in managerial effort and attains a maximum at $a_t = b_t$ (which is first-best in this case). Over a manager’s tenure there are two main effects that determine incentive rates (and induced effort): first, decreasing posterior variances of the performance measure result in increasing myopic rates, and thus increasing incentive rates; I refer to this as a learning effect, as it is due to uncertainty resolution based on observing the past history of performance measures. Second, the insurance adjustment can be increasing or decreasing over time, depending on the performance measure correlation, the remaining employment horizon, and anticipated future incentive rates; the impact on incentive rates thus varies, and I refer to it as a horizon effect, although decreasing posterior variances also play an
indirect role, mostly through anticipated future incentive rates. The combined effect on incentive rates is complex and cannot be characterized in general; see Şabac 2005 for a discussion of dynamic incentives in a multi-period LEN model.

To pursue the issue of managerial tenure, and to provide a link to common characteristics of performance measures, I further narrow the analysis to three performance measure models: an auto-regressive model, a career concerns model, and an accounting model. The three special cases considered below are highly stylized descriptions for performance measures used in contracting. Each highlights one important feature that is likely to be present to a differing extent in any performance measurement system. The auto-regressive model has unpredictable uncertainty in the environment as the main feature, and no learning about that uncertainty is possible; the career concerns model captures primarily the idea of an uncertain persistent parameter that can be learned over time, be it technological uncertainty, or managerial ability; the accounting model ignores all other issues and captures the reversible accrual noise (such as under/over estimation of bad debts or adjustments to conservative estimates of future gains/losses). In general, performance measure uncertainty will likely include a combination of external uncertainty, managerial ability, and reversible accrual estimation noise, and if one of these components dominates, the corresponding model provides a good approximation. For example, if there is little uncertainty regarding managerial ability for a CEO, the negative auto-correlation in accounting measures should dominate, while for a junior manager, uncertainty about ability will likely dominate.

The auto-regressive model

The first information system, denoted $\eta^1$, is a simple auto-regressive noise process characterized by

$$
\varepsilon_t = \omega \varepsilon_{t-1} + \delta_t,
$$

where $\omega \in [0, 1]$ and $\delta_t$ are independent, identically distributed mean zero terms with $\text{var}(\delta_t) = \sigma_\delta^2$, and $\varepsilon_1 = \delta_1$. The key characteristic of this information system is that posterior variances are constant since $E_{t-1}[\varepsilon_t] = \omega \varepsilon_{t-1}$ and $\sigma_\varepsilon^2 = \text{var}_{t-1}(\varepsilon_t) = \sigma_\varepsilon^2$. In particular, for $\omega = 1$, the performance measure noise is a random walk. The performance measures in the auto-regressive model are positively correlated, so (with positive incentive rates) the insurance adjustment always reduces the current incentive rate below its myopic level, which in turn is constant since there is no learning effect in this model.

---

16 I am grateful to Peter Christensen for suggesting this particular case. See also Holmström 1999 for a related model in the context of career concerns.
The career concerns model The second correlation structure corresponds to the career concerns model of Gibbons and Murphy 1992. The information system for the career concerns model, denoted $\eta^2$, is characterized by

$$\varepsilon_t = \delta_t + \theta,$$

(9)

where the terms $\delta_t$ are independent identically distributed with mean zero and variance $\sigma^2_\delta$, while $\theta$ is independent of the $\delta_t$, has mean zero and variance $\sigma^2_\theta$. Thus, $\delta_t$ represents period-specific noise that is independent of all other noise components, while $\theta$ represents managerial ability or the match between manager and job, and is the same in all periods. The manager’s ability or job match is not known to either the principal or the manager prior to contracting. The presence of $\theta$ implies that any two performance measures are positively correlated since $\text{cov}(x_k, x_l) = \sigma^2_\theta$ for $k \neq l$ and the insurance adjustment reduces the current incentive below its myopic level.

The accounting model The third correlation structure limits inter-period correlation to adjacent periods and I refer to it as the accounting model. The information system corresponding to the accounting model, denoted $\eta^3$, is characterized by

$$\varepsilon_t = \delta_t + \rho \theta_{t-1} + \theta_t,$$

(10)

where $\rho = \pm 1$, $\delta_t$ are period-specific, and $\theta_t$ is a common component in adjacent periods. The $\delta_t$, $\theta_t$ terms are mutually independent, have mean zero, $\text{var}(\delta_t) = \sigma^2_\delta$, and $\text{var}(\theta_t) = \sigma^2_\theta$. Note that $\rho$ only indicates the sign of the correlation, and not the actual correlation.

With negative correlation, $\rho = -1$, and the common (between adjacent periods) component $\theta_t$ can be thought of as period $t$ accrual estimation errors that are reversed in the next period. Thus, negatively correlated noise in accounting-based performance measures reflects the nature of the accrual process. In this model, however, the accrual estimation errors, as with all the other components of the noise in the performance measure, are outside the manager’s control.$^{17}$

With positive correlation, $\rho = 1$, and the common (between adjacent periods) component $\theta_t$ can be thought of as a shock that persists from one period to the next period. The noise component $\theta_t$ first appears in period $t$, persists for one period and then disappears. In period $t + 1$, a new noise term $\theta_{t+1}$ appears, and so on. Note that the career concerns model and the accounting model coincide in the two periods case for positive correlation.$^{18}$

$^{17}$ Christensen, Demski, and Frimor 2002 and Demski and Frimor 1999, 2001 examine the role played by income smoothing, or earnings management in dynamic incentives with renegotiation.

$^{18}$ In the two periods case, Christensen et al. 2005 show that the principal’s surplus decreases in the correlation of the performance measures, and that accrual estimation errors may counteract the negative effects of having a high variance of managerial ability or job matching. Their findings suggest the correlation of performance measures may be endogenously determined. By contrast, in this paper, I assume the correlation structure to be exogenously given.
By contrast with the other models with positively correlated performance measures, in the accounting model with negative correlation, the insurance adjustment increases the current incentive above its myopic level.

**Managerial tenure absent a switching cost** I begin examining the question of managerial tenure by assuming there is no cost to switching agents. The optimal managerial turnover policies depend in this case on whether the performance measures are positively or negatively correlated. Broadly speaking, the annual equivalent value of the principal’s surplus decreases with managerial tenure when the performance measures are positively correlated and increases with managerial tenure when the performance measures are negatively correlated.

The following proposition summarizes three cases in which it can be shown analytically that switching agents every period is optimal.

**Proposition 4.** Assume no switching costs and identical periods, \( b_t = b, m_t = m \).

1. In the auto-regressive model, \( U^p(N) \) is strictly decreasing in \( N \).
2. In the career concerns model, if \( \sigma_2^2 = 0 \), then \( U^p(N) \) is strictly decreasing in \( N \).
3. In the accounting model, if \( \rho = 1 \) and if \( \sigma_1^2 = \sigma_\infty^2 \), then \( U^p(1) > U^p(N) \) for all \( N > 1 \).

The key fact underlying Proposition 4 is that, in replacing agents every period, the principal avoids the positive covariance risk associated with contracting with a single agent over multiple periods. This is the only effect in cases 1 and 3, as posterior variances are constant and equal to the prior variance of the first-period performance measure, in other words when there is no learning in the model. In case 2, however, all learning takes place after the first period, so that first-best is attained in all subsequent periods. Despite the high gain from first-best effort in all periods other than the first, the agent’s compensation risk is also high and erases these gains from higher effort and the principal is still better off switching agents every period. Renegotiation is critical here, as the incentive rates in periods \( 2, \ldots, N \) are based on the (zero) posterior variances of the performance measures, while ignoring the total compensation risk the agent bears, and are thus set too high. While in this case it is possible that \( a_1 \) is negative, the result holds for all parameter values and is not driven by cases where \( a_1 \) is negative.

Outside the cases that are tractable analytically, numerical analysis over a broad range of parameter values indicates that the principal’s AEV is decreasing in \( N \) in all three models with positive correlation, see Figure 2, (a), (c), and (e).

While longer tenure increases the agent’s compensation risk when the performance measures are positively correlated, the effect is reversed with negatively correlated performance measures. In the
accounting model with negative correlation, the following proposition shows that the principal’s AEV is increasing with agent tenure.

**Proposition 5.** Assume identical periods. In the accounting model with negative correlation, if $\sigma_1^2 \geq \sigma_\infty^2$, then $U_p(N)$ is increasing in managerial tenure $N$.

The key factor in the accounting model with negative correlation is that the negative performance measure covariance reduces the agent’s compensation risk, so that higher incentive rates are optimal, higher effort is induced, which in turn increases the principal’s surplus. In particular, with longer managerial tenure, higher agent effort is induced in every period starting either with the first period of the agent tenure and moving forward, or starting with the agent’s last period and moving backwards. As a result, the principal’s AEV of expected surplus is increasing in agent tenure.

**Managerial tenure in the presence of switching costs** The analysis so far ignores managerial switching costs. Switching costs arise as recruiting costs (either in direct compensation to a new manager or in opportunity costs during the transition to a new managerial team), or as learning costs (arising from a new manager adjusting to the particulars of a new environment). I incorporate switching costs in the model as an exogenous cost $c > 0$ incurred by the principal in the first period of a manager’s tenure.

In the accounting model with negative correlation, adding a switching cost does not alter the principal’s preference for managerial retention, since the principal’s AEV of expected surplus is reduced by $c/A_N$, which is decreasing with tenure. Thus, the conclusion of Proposition 5 remains the same after including a switching cost: the principal’s AEV is increasing in managerial tenure.

In all the other models with positive correlation, there is a tradeoff between the decreasing principal’s AEV of expected surplus $U_p/A_N$ and the benefit of a decreasing AEV of the switching cost $c/A_N$. Depending on the parameter values, this tradeoff can result in optimal interior tenure for the manager. The following proposition characterizes the principal’s preferences for managerial tenure in the presence of a switching cost for a particular case of the career concerns model.

**Proposition 6.** In the career concerns model, if $\sigma^2 = 0$, a sufficient condition for interior optimal managerial tenure is that

$$
\frac{1}{2} \frac{b^2 r \sigma_\theta^2}{m^2 + r \sigma_\theta^2} < c < \frac{1}{2} (1 - \gamma)^{-2} \frac{b^2 r \sigma_\theta^2}{m^2 + r \sigma_\theta^2}
$$

(11)
Figure 2  Principal’s AEV, \( b_t = m_t = r = 1, R = 5\%\), and \( N = 1, \ldots, 35 \).
The above proposition demonstrates what numerical analysis reveals as the typical behavior of models with positively correlated performance measures: there is interior optimal tenure for switching costs in a given range that depends on the principal’s marginal benefit from agent effort, performance measure sensitivity, managerial risk aversion, and the discount rate. Given that closed-form expressions of the principal’s surplus as a function of agent tenure are generally not tractable, additional results are illustrated by a few numerical examples. Figure 2, (b), (d), and (f), provides examples for the switching cost that give internal optima. By continuity, all results hold in a neighborhood of the given parameter values. In all cases, an interior optimum is obtained as a tradeoff between the benefit of switching managers as often as possible and the switching costs (such as learning and recruiting).

5. Conclusions

In this paper, I examine an alternative to the view that managerial turnover is driven by managerial performance, namely that the principal’s preferences for managerial tenure optimally determine turnover. The principal’s preferences for managerial tenure then depend on the time-series properties of performance measures.

The analysis is based on a multi-period LEN model with renegotiation. Considering more than two (or three) periods allows for more covariance structures and a separation between learning effects at the start of a manager’s tenure and horizon effects at the end of the manager’s tenure; by contrast the related papers of Christensen et al. 2003, 2005 and Dutta and Reichelstein 2003 only consider two-period models. The first part of the paper presents two general results for multi-period LEN models with time additive agent utility which are necessary for the managerial tenure analysis in the second part. First, I show the sufficiency of renegotiation-proof contracts in characterizing equilibria with long-term contracts subject to renegotiation; this generalizes Christensen et al. 2005 who prove the result in a two-period LEN model with single consumption date, multiplicative agent utility. Second, the main result in the first part of the paper provides a general recursive solution to the (renegotiation) agency problem that determines both the dynamic of managerial actions and contractual incentives by backwards induction as a function of posterior variances of the performance measures, future anticipated incentives, and the impact of current actions on future performance expectations. The two main effects determining incentives and managerial actions are: a forward-looking horizon effect due to the anticipation of future contractual terms and a backward-looking learning effect due to observing the history of performance measures (which leads to changes in the posterior variances of the performance measures).
In the second part of the paper, optimal turnover policies are examined by considering three special cases: an auto-regressive model in which posterior variances are constant; a career concerns model similar to the one of Gibbons and Murphy 1992 in which uncertainty about a persistent productivity parameter is gradually resolved; and an accounting model in which early uncertainty resolution is accompanied by reversible accrual estimation errors. In all cases, I include exogenous switching costs incurred by the principal when replacing managers (direct recruiting costs or indirect managerial learning costs). The main results on managerial tenure can be summarized as follows. First, the principal’s welfare is increasing in managerial tenure with negatively correlated performance measures regardless of switching costs. Second, absent a switching cost, or if the switching cost is small enough when hiring a new manager, there is no interior optimal managerial tenure, and turnover every period is preferred in all three models with positively correlated performance measures. Third, there is a range of switching costs for which there is interior optimal tenure in all three models with positively correlated performance measures. Fourth, if the switching cost is large enough, maximum possible tenure is preferred in all three models with positively correlated performance measures.

The principal’s welfare is decreasing in the switching cost, and increasing in the optimally induced effort, holding tenure constant. The optimally induced effort in turn is increased by declining posterior variances of the performance measures (learning effect) and either decreased or increased by the insurance adjustment depending on whether the performance measures are positively or negatively correlated (retirement horizon effect). While the switching cost and the learning effect make the principal prefer longer managerial tenure in all cases, the retirement horizon’s impact depends on the correlation of performance measures.

For negatively correlated performance measures, covariance incentive risk increases, while for positively correlated performance measures covariance incentive risk decreases over a manager’s tenure. Thus, all else equal, covariance incentive risk is lower for longer tenure with negative correlation, and higher with positive correlation. With positive correlation, the principal’s tradeoff is between the higher covariance incentive risk associated with a longer retirement horizon on one hand, and the ability to spread the switching cost over several periods together with the reduction in posterior variances of the performance measures on the other hand.

The main implication for empirical research is that managerial retention policies are related to the time-series properties of the performance measures used in contracting. If managerial tenure is optimally chosen, negatively correlated performance measures should be associated with longer tenure, while positively correlated performance measures should be associated with shorter tenure.
If incentives are based on negatively correlated performance measures, such as accrual accounting measures subject to accrual estimation errors, age-related retirement should be the dominant form of managerial turnover, because it is optimal to retain managers as long as possible. On the other hand, for the career concerns model, or when performance measures follow an auto-regressive process, more frequent managerial turnover is optimal if the switching costs are not too high. If the managerial switching costs are very large relative the principal’s benefit of employing an agent, then age-related turnover again dominates.

The above predictions have to be considered with some caution, as the model on which they are based is highly stylized: the information environment is stationary, the board of directors does not play an active role and multiple managerial tasks or long-term managerial actions are not considered.\(^{19}\)

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**Appendix A: Information environment, special cases**

In this section, I characterize the posterior beliefs for the special cases considered in the paper. Since the performance measures are joint normally distributed, the conditional expectations, given past observations of the performance measures and conjectured past actions are linear in the available history \(E_t[x_k] = m_t a_k + H_{kt} \cdot (x_t - m_t a_t)\), where the vector \(H_{kt}\) characterizes the conditional expectation operator and \(m_t a_t\) refers to multiplication component by component, that is \(m_t a_t = (m_1 a_1, \ldots, m_t a_t)\).

**The auto-regressive model**

The key characteristic of this information system is that posterior variances are constant since \(E_{t-1}[\varepsilon_t] = \omega \varepsilon_{t-1} \) and \(\sigma_t^2 = \text{var}_{t-1}(\varepsilon_t) = \sigma_0^2\). In particular, for \(\omega = 1\), the performance measure noise is a random walk.

\(^{19}\) An important distinguishing feature of my model is that managerial performance is endogenously determined given rationally anticipated turnover and is not an exogenous determinant of managerial turnover as in Hermelin and Weisbach 1998 (see also Murphy and Zimmerman 1993, Engel et al. 2003, and Huson et al. 2004). Thus, the principal’s welfare depends only on managerial tenure, given optimal incentives. However, a direct comparison between the implications of Hermelin and Weisbach 1998 and this paper is difficult. Hermelin and Weisbach do not consider managerial tenure as a choice variable; their results regarding managerial performance are based on the bargaining between the CEO and the board without considering moral hazard. By contrast, my model is driven by pure moral hazard and the board plays no role in it (other than being “the principal”).
Since $E_t[\varepsilon_k] = \omega^{k-t} \varepsilon_t$, it follows that $H_{t+1} = (0, \ldots, 0, \omega^{k-t})$ and $H'_{t+1} = \omega^{k-t}$. Substituting in the optimal incentive recursive relation (4) yields

$$\beta_t = \frac{mb}{m^2 + r \sigma^2_t} - \frac{r \sigma^2_t}{m^2 + r \sigma^2_t} \sum_{k=t+1}^{N} (\gamma \omega)^{k-t} \beta_k.$$  

(12)

The career concerns model

For information system $\eta^2$, let $\zeta_k = (\varepsilon_k - E_{k-1}[\varepsilon_k]) / \text{var}(\varepsilon_k - E_{k-1}[\varepsilon_k])^{1/2}$. The random variable $\zeta_k$ is independent of $\varepsilon_1, \ldots, \varepsilon_{k-1}$ and normalized to have variance one. The conditional expectation of $\varepsilon_{k+1}$ can then be calculated as

$$E_k[\varepsilon_{k+1}] = E_{k-1}[\varepsilon_k] + \text{cov}(\varepsilon_{k+1}, \zeta_k) \zeta_k,$$  

(13)

where I have used $E_{k-1}[\varepsilon_{k+1}] = E_{k-1}[\varepsilon_k]$ due to the fact that the correlation structure is the same in both cases. I now show that, for $k \geq 2$,

$$E_{k-1}[\varepsilon_k] = \frac{\sigma^2_\theta}{(k-1) \sigma^2_\theta + \sigma^2_s} (\varepsilon_1 + \cdots + \varepsilon_{k-1})$$  

(14)

For $k = 2$, equation (14) follows from

$$E_1[\varepsilon_2] = \text{cov}(\varepsilon_2, \zeta_1) \zeta_1 = \text{cov} \left( \varepsilon_2, \frac{\varepsilon_1}{\sigma_1} \right) \frac{\varepsilon_1}{\sigma_1} = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_s} \varepsilon_1.$$  

(15)

Assuming (14) to hold for $k - 1$, I will prove that it holds for $k$. From the induction hypothesis (14) it follows that

$$\varepsilon_k - E_{k-1}[\varepsilon_k] = \theta + \delta_k - \frac{\sigma^2_\theta}{(k-1) \sigma^2_\theta + \sigma^2_s} ((k-1) \theta + \delta_1 + \cdots + \delta_{k-1})$$

$$= \frac{\sigma^2_\theta}{(k-1) \sigma^2_\theta + \sigma^2_s} (\theta_1 + \cdots + \delta_{k-1}) + [(k-1) \sigma^2_\theta + \sigma^2_s] \delta_k.$$  

(16)

Consequently, the variance is given by

$$\sigma^2_k = \text{var}(\varepsilon_k - E_{k-1}[\varepsilon_k]) = \frac{\sigma^2_\theta}{k \sigma^2_\theta + \sigma^2_s}.$$  

(17)

From (13) it follows that

$$E_k[\varepsilon_{k+1}] = E_{k-1}[\varepsilon_k] + \text{cov}(\varepsilon_{k+1}, \zeta_k) \zeta_k = E_{k-1}[\varepsilon_k] + \frac{\sigma^2_\theta}{k \sigma^2_\theta + \sigma^2_s} (\varepsilon_k - E_{k-1}[\varepsilon_k])$$

$$= \frac{\sigma^2_\theta}{k \sigma^2_\theta + \sigma^2_s} (\varepsilon_1 + \cdots + \varepsilon_k).$$  

(18)

The conditional expectation operator is then determined by

$$H'_{k-1} = \frac{\sigma^2_\theta}{(k-1) \sigma^2_\theta + \sigma^2_s} \text{ for } 1 \leq t \leq k - 1,$$

(19)

and the conditional variance is

$$\sigma^2_k = \sigma^2_\theta \frac{k \sigma^2_\theta + \sigma^2_s}{(k-1) \sigma^2_\theta + \sigma^2_s} = \sigma^2_\theta + \sigma^2_\theta \sigma^2_s$$

$$= \frac{\sigma^2_\theta}{(k-1) \sigma^2_\theta + \sigma^2_s}.$$  

(20)

The conditional variances converge in the limit to $\sigma^2_\infty = \sigma^2_s$ as the uncertainty over $\theta$ is eliminated.

For the career concerns model, $E_t[\varepsilon_k] = E_t[\varepsilon_{t+1}]$, so that $H_{kt} = H_{t+1t}$, and thus

$$H'_{kt} = \frac{\sigma^2_\theta}{t \sigma^2_\theta + \sigma^2_s}.$$  

(21)

Substituting in the recursive relation for optimal incentives yields

$$\beta_t = \frac{mb}{m^2 + r \sigma^2_t} - \frac{r \sigma^2_t}{m^2 + r \sigma^2_t} \frac{\sigma^2_\theta}{t \sigma^2_\theta + \sigma^2_s} \sum_{k=t+1}^{N} (\gamma \omega)^{k-t-1} \beta_k.$$  

(22)
The accounting model

For information system ηt, the period k + 1 noise term εk+1 is independent of the history ηk−1. It follows that 

\[ E_k[ε_{k+1}] = E[ε_{k+1}|ε_k - E_{k-1}[ε_k]]. \]

Since \( σ_k^2 = \text{var}_k(ε_k) = \text{var}(ε_k - E_{k-1}[ε_k]) \) and \( \text{cov}(ε_{k+1}, ε_k - E_{k-1}[ε_k]) = \text{cov}(ε_{k+1}, ε_k) = ρσ_κ^2 \), it follows that 

\[ E_k[ε_{k+1}] = \frac{ρσ_κ^2}{σ_k^2}(ε_k - E_{k-1}[ε_k]). \quad (23) \]

Recall that \( E_k[ε_{k+1}] = H_{k+1}|ε_k \) and \( E_{k-1}[ε_k] = H_{k-1}|ε_k \). Substituting in the above equation gives the following recursive relation for the vector \( H_{k+1|k} \):

\[ H_{k+1|k} = \frac{ρσ_κ^2}{σ_k^2}(-H_{k-1|k}, 1). \quad (24) \]

Since \( H_0 = ρσ_κ^2/σ_k^2 \), iterating the above recursive relation determines the conditional expectation operator

\[ H_{k-1|k} = -\frac{(ρσ_κ^2)^{k-τ}}{σ_k^2 \cdots σ_{k-1}^2}, \quad (25) \]

where \( H_{k-1|k} = (H_{k-1|k-1}, \ldots, H_{k-1|k-k}) \). The conditional variances are determined following a similar calculation, as follows:

\[ \sigma_{k+1}^2 = \text{var}_k(ε_{k+1}) = \text{var}(ε_{k+1} - E_k[ε_{k+1}]) = \text{var}(ε_{k+1} - \frac{ρσ_κ^2}{σ_k^2}(ε_k - E_{k-1}[ε_k])) \\
= \text{var}(ε_{k+1}) - 2ρσ_κ^2cov(ε_{k+1}, ε_k) + \left(\frac{σ_κ^2}{σ_k^2}\right)^2 σ_k^2 = \text{var}(ε_{k+1}) - \frac{σ_κ^2}{σ_k^2}. \quad (26) \]

Thus, the conditional variances are determined by the recursive relation

\[ σ_{k+1}^2 = σ_k^2 + 2σ_κ^2 - \frac{σ_κ^2}{σ_k^2}, \quad (27) \]

for any given \( σ_τ^2 \). Identical prior variances require \( σ_τ^2 = σ_k^2 + 2σ_κ^2 \).

Since \( σ_{k+1}^2 - σ_k^2 = -σ_κ^2(σ_k^2 - σ_{k-1}^2) \), the sequence \( σ_k^2 \) is decreasing if, and only if, \( σ_τ^2 > σ_2^2 \). Note that the posterior variances rapidly converge to the limit value

\[ σ^2_∞ = \frac{1}{2} \left( σ_κ^2 + 2σ_κ^2 + √{σ_κ^2 + 4σ_κ^2σ_κ^2} \right). \quad (28) \]

It is then straightforward to show that \( σ_τ^2 > σ_2^2 \) if, and only if, \( σ_τ^2 > σ^2_∞ \).

For the accounting model, the performance measures from two non-adjacent periods are uncorrelated. Consequently, \( E[ε_k] = 0 \) for \( k ≥ t + 2 \) and

\[ H_{kt} = \frac{ρσ_κ^2}{σ_τ^2} \quad \text{if } k = t + 1, \text{ and } H_{kt} = 0 \quad \text{if } k ≥ t + 2. \quad (29) \]

Substituting in the recursive relation for optimal incentives yields

\[ β_t = \frac{mb}{m^2 + rσ_τ^2} - γρ \frac{σ_κ^2}{m^2 + rσ_τ^2} β_{t+1}. \quad (30) \]
Appendix B: Proofs

The conditional expectations of performance measures depend on either observed or conjectured past managerial effort: the manager knows his past actions ̂a_t, so from his perspective, \( E_t[\cdot] = E[\cdot | x_t, ̂a_t] \); the principal, on the other hand, does not observe the manager’s actions, but infers the agent’s past actions ̂a_t = ( ̂a_1, . . . , ̂a_t ). Thus, from the principal’s perspective, \( E_t[\cdot] = E[\cdot | x_t, ̂a_t] \). In addition, the conditional expectations at time t for any future performance measures \( E_t[x_{t+k}] \) implicitly assume conjectured future actions ̂a_{t+k} both from the principal’s and the agent’s perspective.

**Proof of Proposition 1.** For simplicity, I assume that for any contract, the specified payments are accruals in the manager’s bank account, and there is a final settlement between the principal and the agent at the end of period \( N \). The proof proceeds by backwards induction and has two key components: the optimal linear renegotiation offer at date \( t - 1 \) is linear in \( x_{t-1} \) at date \( t - 2 \); and the optimal linear renegotiation offer at date \( t - 2 \) can be set to be renegotiation-proof at dates \( t - 1, . . . , N \).

Let \( \beta_N^{RN} \) denote the optimal (equilibrium) incentive rate for period \( N \) from the optimal linear contract \( c^{RN} = (\alpha_{N-1}^{RN}, \beta_N^{RN}) \). The contract in effect at renegotiation date \( N - 1 \) is determined by the renegotiation offer at date \( N - 2 \), \( c^{RN-1} = (\alpha_{N-2}^{RN-1}, \beta_{N-1}^{RN-1}, \beta_N^{RN-1}) \),

\[
\begin{align*}
  c^{IN} &= (\alpha_{N-1}^{IN}, \beta_N^{IN}) = (\gamma^{-1}a_{N-2}^{RN-1} + \beta_{N-1}^{RN-1} x_{N-1}, \beta_N^{RN-1}) \quad (31)
\end{align*}
\]

Since \( c^{RN} \) is accepted by the agent in equilibrium, \( a_N \) is determined by \( c^{RN} \). At the time the agent selects his period \( N \) action \( a_N \), his certainty equivalent is

\[
\text{ACE}_{N-1}(c^{RN} | a_N) = \gamma E_{N-1}[W_N(c^{RN})] - \frac{1}{2} \gamma a_N^2 - \frac{1}{2} r \gamma \text{var}_{N-1}(W_N(c^{RN}))
\]

\[
= \alpha_{N-1}^{RN} + \gamma \beta_N^{RN} E_{N-1}[x_N] - \frac{1}{2} \gamma a_N^2 - \frac{1}{2} r \gamma \text{var}_{N-1}(W_N(c^{RN}))) \quad (32)
\]

Since \( a_N \) does not impact compensation variance, the agent’s problem reduces to

\[
a_N \in \text{argmax}(\alpha_{N-1}^{RN} + \gamma \beta_N^{RN} E_{N-1}[x_N]) \quad (33)
\]

Since the fixed wage does not depend on action choice and since \( \partial / \partial a_N E_{N-1}[x_N] = m_N \), the first-order condition determines the induced action as \( a^{RN} = m_N \beta_N^{RN} \). Thus, \( a_N \) is determined by the period \( N \) incentive, and does not otherwise depend on past history or the principal’s beliefs.

Consider now the acceptance of the renegotiation offer at date \( t - 1 \). If ̂a_{IN} and ̂a_{RN} are the actions induced in period \( N \) by \( c^{IN} \) and \( c^{RN} \), respectively, the participation constraint is

\[
\text{ACE}_{N-1}(c^{IN} | ̂a_{IN}) \leq \text{ACE}_{N-1}(c^{RN} | ̂a_{RN}) \quad (34)
\]

The principal having all the bargaining power, the participation constraint is binding in equilibrium and determines the fixed wage \( \alpha_{N-1}^{RN} \):

\[
\alpha_{N-1}^{RN} = \alpha_{N-1}^{IN} + \gamma \beta_N^{IN} E_{N-1}[x_N | ̂a_{IN}] - \gamma \beta_N^{RN} E_{N-1}[x_N | ̂a_{RN}]
\]

\[
+ \frac{1}{2} r \gamma (\text{var}_{N-1}(W_N(c^{RN})) - \text{var}_{N-1}(W_N(c^{IN}))) + \frac{1}{2} \gamma ( ̂a_{RN}^2 - ̂a_{IN}^2 ) \quad (35)
\]
Since \( \alpha_{N-1}^{RN} = \gamma^{-1} \alpha_{N-2}^{RN-1} + \beta_{N-1}^{RN-1} x_{N-1} \), it follows that, viewed from date \( N - 2 \), the renegotiation offer at date \( N - 1 \), \( c^{RN} \) is linear in \( x_{N-1}, x_N \) conditional on date \( N - 2 \) information.

Moreover, from the same binding participation constraint at date \( N - 1 \) follows that

\[
\gamma E_{N-1}[W_N(c^{RN})] = A_C E_{N-1}(W_N(c^{LN})) + \frac{1}{2} \gamma a_{RN}^2 + \frac{1}{2} \gamma \text{var}_{N-1}(W_N(c^{RN})),
\]

so the principal compensates the agent for his effort cost and risk premium, plus an amount determined by the contract in effect at renegotiation time. It follows that the principal’s problem in choosing \( \beta_N^{RN} \) is equivalent to

\[
\max_{\beta_N^{RN}} \left[ \gamma b_N a_{RN} - \frac{1}{2} \gamma a_{RN}^2 - \frac{1}{2} \gamma \text{var}_{N-1}(W_N(c^{RN})) \right]
\]

subject to the agent rationality constraint

\[
a_{RN} = \beta_N^{RN} m_N.
\]

After accepting \( c^{RN-1} \) at date \( N - 2 \), the agent anticipates the acceptance of \( c^{RN} \) at date \( N - 1 \), so when choosing action \( a_{N-1} \), the agent’s certainty equivalent is determined by \( c^{RN} \), where \( \beta_N^{RN} \) is the optimal rate and the fixed wage \( \alpha_{N-1}^{RN} \) is constrained by (35). As before, since the agent’s actions have no impact on compensation variance, the induced action \( a_{N-1} \) maximizes

\[
ACE_{N-2}(c^{RN-1}, c^{RN}| a_{N-1}, a_{RN}) = \gamma E_{N-2}[W_{N-1}(c^{RN})] - \frac{1}{2} \gamma a_{N-1}^2
\]

\[
= \gamma E_{N-2}[a_{RN}^{LN} + \beta_N^{RN} \bar{E}_{N-1}[x_N|a_{LN}] - \beta_N^{RN} \bar{E}_{N-1}[x_N|a_{RN}] + \gamma \beta_N^{RN} x_N|a_{N-1}, a_{RN}] - \frac{1}{2} \gamma a_{N-1}^2,
\]

where in the last line I have used (35) and dropped the variance and effort cost terms that do not depend on \( a_{N-1} \). Furthermore, substituting for the conditional expectations \( \bar{E}_{N-1}[x_N|a_{LN}] = m_N a_{LN} + H_{N-1} \cdot (x_{N-1} - m_{N-1} \bar{a}_{N-1}) \) and \( \bar{E}_{N-1}[x_N|a_{RN}] = m_N a_{RN} + H_{N-1} \cdot (x_{N-1} - m_{N-1} \bar{a}_{N-1}) \), the agent’s problem becomes

\[
a_{N-1} \in \arg \max \left( \beta_N^{RN-1} m_{N-1} a_{N-1} + \gamma (\beta_N^{LN} - \beta_N^{RN}) H_{N-1}^{N-1} m_{N-1} a_{N-1} - \frac{1}{2} a_{N-1}^2 \right).
\]

The first-order condition determines the optimally induced action as

\[
a_{N-1} = \left[ \beta_N^{RN-1} + \gamma (\beta_N^{LN} - \beta_N^{RN}) H_{N-1}^{N-1} \right] m_{N-1}.
\]

At date \( N - 2 \), the principal sets the fixed wage given the incentive rates to compensate the agent for effort cost and the risk premium, so the principal’s problem is equivalent to maximizing over the variable incentive rates in \( c^{RN-1} \):

\[
\max_{\beta_N^{RN-1}, \beta_N^{RN}} \left[ \gamma b_{N-1} a_{RN-1} - \frac{1}{2} \gamma a_{RN-1}^2 - \frac{1}{2} \gamma \text{var}_{N-2}(\bar{E}_{N-1}[W_{N-1}(c^{RN})]) + \gamma \text{var}_{N-2}(\bar{E}_{N-1}[W_N(c^{RN})]) \right],
\]

subject to the induced actions \( a_{RN-1}, a_{RN} \) being constrained by (41) and (38), respectively. Note that the principal’s renegotiation offer has no impact on \( a_{RN} \) and \( \text{var}_{N-1}(\bar{E}_{N}[W_N(c^{RN})]) \) (which depend only on the anticipated \( c^{RN} \)), and thus the principal’s problem reduces to

\[
\max_{\beta_N^{RN-1}, \beta_N^{RN}} \left[ \gamma b_{N-1} a_{RN-1} - \frac{1}{2} \gamma a_{RN-1}^2 - \frac{1}{2} \gamma \text{var}_{N-2}(\bar{E}_{N-1}[W_{N-1}(c^{RN})]) \right].
\]
Calculating the relevant portion of the risk premium yields
\[
\text{var}_{N-2}(\mathcal{E}_{N-1}[W_{N-1}(c^\text{RT})]) = \text{var}_{N-2}(\mathcal{E}_{N-1}[\alpha_{N-1}^N + \gamma \beta_N^{RN} x_N])
\]
\[
= \text{var}_{N-2}(\mathcal{E}_{N-1}[\alpha_{N-1}^N + \gamma \beta_N^{RN} \mathcal{E}_{N-1}[x_N] - \gamma \beta_N^{RN} \mathcal{E}_{N-1}[x_N] + \gamma \beta_N^{RN} x_N])
\]
\[
= \text{var}_{N-2}(\beta_N^{RN-1} + \gamma \beta_N^{RN-1} H_{N-1}^{N-1} x_{N-1}),
\]
where I have used \( \alpha_{N-1}^N = \gamma^{-1} a_{N-2}^{RN-1} + \beta_N^{RN-1} x_{N-1} \) and \( \beta_N = \beta_N^{RN-1} \). From the above, it follows that both the induced action \( a_{N-1} \) and the relevant part of the risk premium that enter the principal’s problem in choosing incentive rates, depend on the terms of the renegotiation offer only through \( \beta_N^{RN-1} + \gamma \beta_N^{RN-1} H_{N-1}^{N-1} \).

It follows that the principal’s problem is over-determined and \( c^{RN-1} \) can be set without loss of generality by fixing \( \beta_N^{RN-1} \) to any value, in particular by setting \( \beta_N^{RN-1} = \beta_N^{RN} \). Finally, if \( \beta_N^{RN-1} = \beta_N^{RN} \), then \( a_{N-1} = \beta_N^{RN-1} m_{N-1} \), \( c^{RN-1} \) is renegotiation-proof at date \( N - 1 \), and \( c^{RN-1} = c^\text{RN} \). This concludes the proof that \( c^{RN-1} \) can be assumed without loss of generality to be renegotiation-proof at date \( N - 1 \).

For the induction step of the proof, consider renegotiation at date \( t - 2 \), assuming that \( c^\text{RT} \) is renegotiation-proof at all subsequent dates. The actions \( a_{t-1}, \ldots, a_N \) are determined by the incentives in \( c^\text{RT} \), so it remains to show that there is no loss of generality from assuming that \( c^{RT-1} \) is renegotiation-proof at \( t - 1, \ldots, N \).

The participation constraint at \( t - 1 \), binding in equilibrium, determines \( \alpha_{t-1}^R \):
\[
\alpha_{t-1}^R = \alpha_{t-1}^T + \mathcal{E}_{t-1} \left( \beta^{RN}_{t-1} + \sum_{k=0}^{N-t} \gamma^{k+1} \beta_{t+k}^R x_{t+k} | a_{t1}, a_{t+1}, \ldots \right) - \mathcal{E}_{t-1} \left( \sum_{k=0}^{N-t} \gamma^{k+1} \beta_{t+k}^R x_{t+k} | a_{t1}, a_{t+1}, \ldots \right)
\]
\[
+ \frac{1}{2} \left( \sum_{k=0}^{N-t} \gamma^{k+1} a_{t+k}^2 \right) - \frac{1}{2} \left( \sum_{k=0}^{N-t} \gamma^{k+1} a_{t+k} \right) + \text{RP}(c^\text{RT}) - \text{RP}(c^\text{RT-1}),
\]
where \( \text{RP}(\cdot) \) denotes the risk premium corresponding to each contract.

The agent’s action choice \( a_{t-1} \) is determined by
\[
a_{t-1} \in \arg \max \mathcal{A} \mathcal{C} \mathcal{E}_{t-1}(c^{RT-1}, c^\text{RT} | a_{t-1}, a_{t}, a_{t+1}, \ldots) = \arg \max (\gamma \mathcal{E}_{t-1}[W_{t-1}(c^\text{RT})] - \frac{1}{2} a_{t-1}^2)
\]
\[
= \arg \max (\beta_N^{RT-1} m_{t-1} a_{t-1} + \sum_{k=0}^{N-t} \gamma^{k+1} (\beta_{t+k}^R - \beta_{t+k}^R) H_{t+k-1}^{T-1} m_{t-1} a_{t-1} - \frac{1}{2} a_{t-1}^2). \]
(46)

Thus, the agent’s rationality constraint determines the induced action \( a_{t-1} \) from the first-order condition as follows:
\[
a_{t-1} = \left( \beta_N^{RT-1} + \sum_{k=0}^{N-t} \gamma^{k+1} (\beta_{t+k}^R - \beta_{t+k}^R) H_{t+k-1}^{T-1} \right) m_{t-1}.
\]
(47)

At date \( t - 2 \), the principal sets the fixed wage to compensate the agent for effort cost and the risk premium.

Since effort in periods \( t, t + 1, \ldots, N \) is determined by the renegotiation-proof contract \( c^\text{RT} \), the principal’s problem reduces to choosing variable incentive rates in the following objective function:
\[
\max_{\beta_{t-1}^{RT}, \ldots, \beta_N^{RT-1}} \left( \beta_{t-1}^{RT} b_{t-1} a_{t-1} - \frac{1}{2} a_{t-1}^2 - \frac{1}{2} \sum_{k=0}^{N-t} \gamma^{k+1} \text{var}_{t+k} \left( \mathcal{E}_{t+k}[W_{t+k}(c^\text{RT})] \right) \right).
\]
(48)

The variance terms \( \text{var}_{t+k} \left( \mathcal{E}_{t+k}[W_{t+k}(c^\text{RT})] \right) \) only depend on \( \beta_{t+k}^R \) for \( k = 1, \ldots, N - t + 1 \), and do not depend in any way on \( \alpha_{t-1}^R \), which is the only way \( c^{RT-1} \) impacts \( c^\text{RT} \). Therefore, the principal’s problem at date \( t - 2 \) reduces to
\[
\max_{\beta_{t-1}^{RT}, \ldots, \beta_N^{RT-1}} \left( b_{t-1} a_{t-1} - \frac{1}{2} a_{t-1}^2 - \frac{1}{2} \sum_{k=0}^{N-t} \text{var}_{t+k} \left( \mathcal{E}_{t+k}[W_{t+k}(c^\text{RT})] \right) \right).
\]
(49)
From the expression for the fixed wage (45), it follows that

\[
\text{var}_{t-2}(\hat{E}_t|W_t(c))] = \text{var}_{t-2} \left( \left( \beta_t^{Rt-1} + \sum_{k=0}^{N-t} \gamma^k \beta_t^{Rt+k} R_{t+k-1} \right) x_{t-1} \right). \tag{50}
\]

Therefore, the variable incentive rates for periods \( t, \ldots, N \) enter the principal’s problem only through the term \( \beta_t^{Rt-1} + \sum_{k=0}^{N-t} \gamma^k \beta_t^{Rt+k} R_{t+k-1} \). This concludes the proof, since the principal’s problem is over-determined with respect to these incentive rates, and thus there is no loss of generality in setting \( \beta_t^{Rt+k} = \beta_t^{Rt+k} \) for all \( k = 0, \ldots, N - t \). In particular, this implies \( c^{Rt-1} \) is renegotiation proof at all subsequent contracting dates and that \( a_{t-1} = \beta_t^{Rt-1} m_{t-1} \).

**Proof of Proposition 2.** The optimal incentive and induced action for the last period (2) have been determined in Proposition 1. It remains to prove (3) and (4). For a renegotiation-proof contract, it follows from the expression for the optimally induced action (47) that \( a_t = \beta_t m_t \). The principal’s problem at date \( t - 1 \) is to choose \( \beta_t \) anticipating future optimal incentive rates \( \beta_{t+1}, \ldots, \beta_N \), so from (49) it follows that the principal’s problem reduces to

\[
\max_{\beta_t} \left( b_t \beta_t m_t - \frac{1}{2} (\beta_t m_t)^2 - \frac{1}{2} \text{var}_{t-1}(\hat{E}_t|W_t(c)) \right). \tag{51}
\]

The risk premium, see (50), can be written as

\[
\text{var}_{t-1}(\hat{E}_t|W_t(c)) = \text{var}_{t-1} \left( \left( \beta_t + \sum_{k=t}^{N} \gamma^{k-t} \beta_k H_{kt}^t \right) x_t \right) = \left( \beta_t + \sum_{k=t}^{N} \gamma^{k-t} \beta_k H_{kt}^t \right)^2 \sigma_t^2. \tag{52}
\]

Substituting back in the principal’s problem (49), and solving the first-order condition with respect to \( \beta_t \) proves the proposition. \( \square \)

**Proof of Proposition 3.** For any \( t \), and any set of principal’s beliefs regarding the agent’s actions, define the adjusted performance measure \( \hat{x}_t = x_t - \hat{E}_t[x_t] = x_t - m_t \hat{a}_t - H_{tt-1} \cdot (x_t - m_t \hat{a}_t) \). The adjusted performance measures are statistically independent since \( \hat{x}_t \) represents the unexpected component of \( x_t \), is independent of \( x_1, \ldots, x_{t-1} \), and \( \hat{x}_1, \ldots, \hat{x}_{t-1} \) are linear functions of \( x_1, \ldots, x_{t-1} \). The statistical independence of the adjusted performance measures allows for a very simple expression for the agent’s risk premium for any linear contract in \( \hat{x}_1, \ldots, \hat{x}_N \). For any linear contract \( c = (\hat{a}_0, \hat{1}_1, \ldots, \hat{1}_N) \), the risk premium in the agent’s certainty equivalent at date \( t - 1 \) is given by, see (1)

\[
RP_{t-1}(c) = \frac{1}{2} \sum_{k=t}^{N} \gamma^{k-t+1} \text{var}_{k-1}(E_k[W_k(c)]). \tag{53}
\]

For any information/beliefs about past/future actions of the agent and the principal, and for any \( p \geq k + 1 \),

\[
E_k[\hat{x}_p] = E_k[x_p - \hat{E}_{p-1}[x_p]] = E_k[x_p - E_{p-1}[x_p] + E_{p-1}[x_p] - \hat{E}_{p-1}[x_p]] = E_k[x_p - E_{p-1}[x_p] + m_p(a_p - \hat{a}_p) - H_{pp-1} \cdot (m_{p-1} \hat{a}_{p-1} - m_{p-1} \hat{a}_{p-1})] = m_p(a_p - \hat{a}_p) - H_{pp-1} \cdot (m_{p-1} \hat{a}_{p-1} - m_{p-1} \hat{a}_{p-1}) \tag{54}
\]

The conditional variances then simplify to

\[
\text{var}_{k-1}(E_k[W_k(c)]) = \text{var}_{k-1} \left( E_k \left[ \sum_{p=k}^{N} \gamma^{p-k} \hat{\beta}_p \hat{x}_p \right] \right) = \text{var}_{k-1} \left( E_k \left[ \hat{\beta}_k \hat{x}_k \right] \right) = \hat{\beta}_k^2 \sigma_k^2, \tag{55}
\]
and the risk premium expression (53) simplifies to
\[
RP_{t-1}(c) = \frac{1}{2} \sum_{k=t}^{N} \gamma^{k-t+1} \beta_k^2 \sigma_k^2.
\] (56)

Consider now the equivalent representation of the optimal linear renegotiation-proof contract in terms of the adjusted performance measures \( \hat{x}_t \) with corresponding incentive rates \( \hat{\beta}_t \). Using the characterization of the risk premium in (56), the agent’s certainty equivalent at date \( t-1 \) can be written as
\[
ACE_{t-1}(c) = \gamma E_{t-1}[W_t(c)] - \gamma K_t - RP_{t-1}(c) = \gamma E_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} \hat{\beta}_k \hat{x}_k \right) - \gamma K_t - \frac{1}{2} \sum_{k=t}^{N} \gamma^{k-t+1} \beta_k^2 \sigma_k^2.
\] (57)
The action induced by the contract satisfies
\[
a_t = \arg\max \left( \gamma E_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} \hat{\beta}_k \hat{x}_k \right) - \gamma K_t \right) = \arg\max \left( E_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} \hat{\beta}_k (x_k - \hat{E}_{k-1}[x_k]) \right) - \frac{1}{2} a_t^2 \right).
\] (58)
Differentiating with respect to \( a_t \) gives the first-order condition that determines \( a_t \):
\[
\hat{\beta}_t m_t - \sum_{k=t+1}^{N} \gamma^{k-t} \hat{\beta}_k \frac{\partial}{\partial a_t} E_{t-1} \hat{E}_{k-1}[x_k] - a_t = 0.
\] (59)
The action induced by the incentives \( \hat{\beta}_t \) is
\[
a_t = \hat{\beta}_t m_t - \sum_{k=t+1}^{N} \gamma^{k-t} \hat{\beta}_k \frac{\partial}{\partial a_t} E_{t-1} \hat{E}_{k-1}[x_k].
\] (60)

Since the participation constraint is binding in equilibrium, I can replace the agent’s expected compensation by the expected effort cost plus the expected risk premium in the principal’s problem at date \( t-1 \):
\[
\max_{\hat{\beta}_t} \hat{E}_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} z_k - W_{t-1}(c) \right) = \max_{\hat{\beta}_t} \hat{E}_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} z_k - K_t - \frac{1}{2} \sum_{k=t}^{N} \gamma^{k-t} \beta_k^2 \sigma_k^2 \right).
\] (61)
The first-order condition with respect to \( \hat{\beta}_t \) gives
\[
b_t \frac{\partial a_t}{\partial \hat{\beta}_t} - a_t \frac{\partial a_t}{\partial \hat{\beta}_t} - r \hat{\beta}_t \sigma_t^2 = 0.
\] (62)
Using the fact that \( \frac{\partial a_t}{\partial \hat{\beta}_t} = m_t \) and substituting in the principal’s first-order condition yields \( b_t m_t - a_t m_t - r \hat{\beta}_t \sigma_t^2 = 0 \), and solving for \( a_t \), I obtain
\[
a_t = b_t - r \hat{\beta}_t \sigma_t^2 / m_t.
\] (63)

Since the principal compensates the agent in equilibrium for effort cost and the risk premium, the principal’s expected surplus discounted to date \( t-1 \) can be written as
\[
E_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} z_k - W_{t-1}(c) \right) = E_{t-1} \left( \sum_{k=t}^{N} \gamma^{k-t} \left( z_k - \frac{1}{2} a_t^2 - \frac{1}{2} r \hat{\beta}_t^2 \sigma_t^2 \right) \right).
\] (64)
Solving for \( \hat{\beta}_t \) from (63), \( \hat{\beta}_t = m_t (b_t - a_t) / (r \sigma_t^2) \), allows expressing the principal’s expected surplus only in terms of the optimally induced actions, without reference to the adjusted performance measures:
\[
U^n = \sum_{t=1}^{N} \gamma^t \left( b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} r \hat{\beta}_t^2 \sigma_t^2 \right) = \sum_{t=1}^{N} \gamma^t \left( b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} m_t^2 / (r \sigma_t^2) (b_t - a_t)^2 \right).
\] (65)
The following lemma is used in the proofs of Propositions 4 and 5. A proof is available from the author.
LEMMA 1. If \( \zeta_1, \ldots, \zeta_N \) and \( \xi_1, \ldots, \xi_{N+1} \) satisfy \( \zeta_t \leq \xi_t \) and \( \zeta_t \geq \xi_{t+1} \) for all \( 1 \leq t \leq N \), then, for any \( \gamma > 0 \),

\[
\frac{\sum_{s=1}^{N} \gamma^{s-1} \zeta_s}{\sum_{s=1}^{N} \gamma^{s-1}} < (\frac{\sum_{s=1}^{N+1} \gamma^{s-1} \xi_s}{\sum_{s=1}^{N} \gamma^{s-1}}).
\]

(66)

Proof of Proposition 4.

In case 1, the auto-regressive equation for managerial effort yields

\[
a_t = \frac{(1 - \gamma \omega)m^2 b}{(1 - \gamma \omega)m^2 + \sigma^2} + \frac{r \sigma^2 b}{(1 - \gamma \omega)m^2 + \sigma^2} \left( \frac{\gamma \omega m^2}{m^2 + \sigma^2} \right)^{N+1}.
\]

Thus, agent effort is decreasing in managerial tenure \( N \), holding \( t \) constant. Since \( U_t^p \) is increasing in agent effort \( a_t \), it follows that \( U_t^p \) is decreasing in \( N \) holding \( t \) constant. Moreover, since \( a_t^N = a_{t+1}^{N+1} \), it follows that \( U_t^p(N+1) = U_t^p(N) \). This proves case 1 using Lemma 1.

In case 2, \( \sigma_0^2 > 0 \) and \( \sigma_1^2 = 0 \) imply that \( \sigma_1^2 = \sigma_0^2 \) and \( \sigma_2^2 = 0 \) for all \( t \geq 2 \). Thus, first-best is attained in all periods other than the first, and \( a_t = b \) for \( t \geq 2 \). In addition, I can simplify the analysis by normalizing the performance measure variance in the first period to one (all expressions depend on the ratio \( m/\sigma_0 \)). The first-period action is determined by

\[
a_1 = \frac{b m^2}{m^2 + r} - \frac{r \sigma^2}{m^2 + r} \sum_{k=2}^{N} \gamma^{k-1} H_k a_k = \frac{b m^2}{m^2 + r} - \frac{r \sigma^2}{m^2 + r} \sum_{k=2}^{N} \gamma^{k-1} b.
\]

(68)

Substituting in the expression for the principal’s first-period utility and simplifying yields

\[
U_t^p = \frac{1}{2} b^2 - \frac{1}{2} \left( 1 + \frac{m^2}{\sigma_0^2} \right) (b - a_1)^2 = \frac{1}{2} b^2 \left( 1 - \frac{r A^2}{m^2 + r} \right),
\]

where I have used the normalized variance \( \sigma^2 = \sigma_0^2 = 1 \) and

\[
b - a_1 = b - \frac{b m^2}{m^2 + r} + \frac{b r}{m^2 + r} \sum_{k=2}^{N} \gamma^{k-1} = \frac{b r}{m^2 + r} A_N
\]

(70)

Since in periods \( t = 2, \ldots, N, U_t^p = \frac{1}{2} b^2 \), the total AEV principal’s surplus reduces to

\[
U^p(N) = \frac{1}{2} b^2 \left( 1 - \frac{r A_N}{m^2 + r} \right),
\]

(71)

and is strictly decreasing in \( N \) because \( A_N \) is strictly increasing in \( N \).

In case 3, posterior variances are constant, and equal to the prior variance the first-period performance measure when a new agent is hired. It follows that the optimal incentive rate offered in a single period under the policy of switching agents every period is the same as the myopic incentive rate which is the first term in expression (4). Given the recursive relations for the incentive rates in the accounting model, it follows that the induced action in every period with a single agent hired for \( N \) periods is less than the induced action with an agent hired for a single period. Given that \( \sigma^2 = \sigma_1^2 \), it follows that \( U_t^p < U^p(1) \) for all \( 1 \leq t \leq N \) and, consequently, \( U^p(N) < U^p(1) \).

Proof of Proposition 5. The proof is straightforward calculation based on three steps. First, \( U_t^p(N) < U_t^p(N+1) \) for all \( 1 \leq t \leq N \). Second, \( U_t^p(N) < U^p_t(N+1) \) for all \( 1 \leq t \leq N \). Third, Lemma 1 proves the proposition by setting \( \xi_t = U_t^p(N+1) \) and \( \zeta_t = U_t^p(N) \). Returning to the proof of the proposition, note that

\[
U_t^p(a_t) = \frac{1}{2} b^2 - \left( 1 + \frac{m^2}{\sigma_0^2} \right) (b - a_t)^2
\]

(72)
is an increasing function of $a_t$. Thus, if $a_{Nt} < a_{N+1t}$, it follows that $U_p^t(a_{Nt}) < U_p^t(a_{N+1t})$. To complete the first step, it remains to show that $a_{Nt} < a_{N+1t}$. This is proved by backwards induction. For $t = N$,

$$a_{NN} = \frac{bm^2}{m^2 + r\sigma_N^2} < \frac{bm^2}{m^2 + r\sigma_N^2} + \frac{r\gamma\sigma^2}{m^2 + r\sigma_N^2} a_{N+1N+1} = a_{N+1N}.$$  \hfill (73)

For any other $t$,

$$a_{Nt} = \frac{bm^2}{m^2 + r\sigma_t^2} + \frac{r\gamma\sigma^2}{m^2 + r\sigma_t^2} a_{Nt+1} < \frac{bm^2}{m^2 + r\sigma_t^2} + \frac{r\gamma\sigma^2}{m^2 + r\sigma_t^2} a_{N+1t+1} = a_{N+1t},$$  \hfill (74)

using the induction hypothesis $a_{Nt+1} < a_{N+1t+1}$. For the second part of the proof, I start by showing that $a_{Nt} < a_{N+1t+1}$, again by backwards induction. For $t = N$,

$$a_{NN} = \frac{bm^2}{m^2 + r\sigma_N^2} = a_{N+1N+1}.$$  \hfill (75)

For any other $t$, using $\sigma_t > \sigma_{t+1}$ and the induction hypothesis $a_{Nt+1} < a_{N+1t+2}$,

$$a_{Nt} = \frac{bm^2}{m^2 + r\sigma_t^2} + \frac{r\gamma\sigma^2}{m^2 + r\sigma_t^2} a_{Nt+1} < \frac{bm^2}{m^2 + r\sigma_{t+1}^2} + \frac{r\gamma\sigma^2}{m^2 + r\sigma_{t+1}^2} a_{N+1t+2} = a_{N+1t+1}.$$  \hfill (76)

To show that $U_p^t(a_{Nt}) < U_p^t(a_{N+1t+1})$, it suffices to prove the following inequality:

$$\left(1 + \frac{m^2}{r\sigma_{t+1}^2}\right)(b - a_{Nt+1})^2 < \left(1 + \frac{m^2}{r\sigma_t^2}\right)(b - a_{Nt})^2.$$  \hfill (77)

Substituting the recursive expressions for the agent’s actions and simplifying yields the equivalent inequality:

$$\frac{\sigma_{t+1}^2}{m^2 + r\sigma_{t+1}^2} \left( b - \frac{\gamma\sigma \sigma_{Nt+2}^2}{\sigma_{t+1}^2} a_{N+1t+2} \right) < \frac{\sigma_t^2}{m^2 + r\sigma_t^2} \left( b - \frac{\gamma\sigma \sigma_{Nt+1}^2}{\sigma_t^2} a_{Nt+1} \right)^2.$$  \hfill (78)

The above inequality holds because $a_{Nt+1} < a_{N+1t+2}$ and $\sigma_{t+1}^2 < \sigma_t^2$, which imply that

$$\frac{a_{Nt+1}}{\sigma_{t+1}^2} < \frac{a_{Nt+1+2}}{\sigma_{t+1}^2} \quad \text{and} \quad \frac{\sigma_{t+1}^2}{m^2 + r\sigma_{t+1}^2} < \frac{\sigma_t^2}{m^2 + r\sigma_t^2}. \hfill (79)$$

**Proof of Proposition 6.** With a switching cost $c > 0$, the principal’s net AEV is given by

$$U_p^p(N) = \frac{1}{2} b^2 \left( 1 - \frac{r\sigma_N^2}{m^2 + r\sigma_N^2} A_N \right) - \frac{c}{A_N} = \frac{1}{2} b^2 - \left( \frac{1}{2} b^2 \frac{r\sigma_N^2}{m^2 + r\sigma_N^2} A_N + \frac{c}{A_N} \right). \hfill (80)$$

Since $A_N = (1 - \gamma^N)/(1 - \gamma)$ and $d\gamma^N/dN = (\ln \gamma)\gamma^N$, I have that

$$\frac{d}{dN} U_p^p(N) = (\ln \gamma)\gamma^N \left( \frac{1}{2} b^2 \frac{r\sigma_N^2}{m^2 + r\sigma_N^2} - \frac{1}{1 - \gamma} - c \frac{1 - \gamma}{(1 - \gamma^N)^2} \right). \hfill (81)$$

A sufficient condition for an interior maximum with respect to $N$ is that

$$\frac{d}{dN} U_p^p(N) \bigg|_{N=1} > 0 \quad \text{and} \quad \frac{d}{dN} U_p^p(N) \bigg|_{N=\infty} < 0.$$  \hfill (82)

Solving the above inequalities yields the condition in the proposition. □
References


