Asset Substitution, Debt Overhang, and Prudential Regulation*

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October 16, 2013

Abstract

We develop a theory to show how market-based regulation interacts with shareholder–debt holder agency conflicts to affect the prudential capital regulation of a financial institution. Relative to a benchmark regime with no regulation, regulation based on market prices could alleviate the inefficiencies arising from asset substitution, but exacerbate those arising from debt overhang. An increase in the propensity for asset substitution mitigates debt overhang, and this tradeoff is especially pronounced for highly levered financial institutions. The prudential capital requirement that maximizes the total ex ante value of the institution balances the tradeoff between debt overhang and asset substitution. Capital requirements should be higher in booms than in recessions, and a uniform capital requirement across institutions could be sub-optimal. While strict capital requirements could mitigate or eliminate asset substitution, they could exacerbate debt overhang, which sounds a note of caution given the recent proposals to require both mark-to-market accounting and stricter capital requirements in the Basel III accords.

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*This paper was previously titled: "Agency Conflicts, Prudential Regulation and Marking to Market." We thank Patrick Bolton, Pingyang Gao, Frank Gigler, Christopher Hennessy, Robert McDonald, and workshop participants at the Western Finance Association Meetings, the C.R.E.D.I.T Conference on Stability and Risk Control in Banking, Insurance, and Financial Markets (Venice, Italy), the Fifth Interdisciplinary Accounting Symposium held by the Danish Center for Accounting and Finance (Copenhagen, Denmark), Emory University, Georgia State University, London School of Economics, Northwestern University, Stanford University, Temple University, University of Houston, University of Minnesota, University of North Carolina at Chapel Hill, and Washington University at St. Louis for valuable comments. Haresh Sapra is grateful to the University of Chicago Booth School of Business for financial support.
1 Introduction

We develop a theory of how regulation based on market prices interacts with agency conflicts between the shareholders and debt holders of a financial institution to affect the design of prudential capital regulation. We show that, relative to a benchmark autarkic regime with no regulation, market–based regulation could mitigate inefficiencies arising from asset substitution or risk-shifting (the choice of risky, negative NPV projects), but exacerbate inefficiencies due to debt overhang (the avoidance of risky, positive NPV projects). The inefficiencies due to debt overhang and asset substitution work in opposing directions in that an increase in the propensity for asset substitution alleviates the debt overhang problem. The choice of capital regulation that maximizes the total/enterprise value of the institution balances the trade-off between debt overhang and asset substitution. We show that capital requirements should be higher in booms than in recessions, and that a uniform capital requirement across institutions could be sub-optimal. Although strict capital requirements could mitigate or eliminate asset substitution, they could greatly exacerbate underinvestment, which sounds a note of caution given the recent proposals to require both mark–to–market accounting and stricter capital requirements in the Basel III accords.\footnote{Mark-to-Market accounting is the use of market prices to measure the claims of a firm.}

Our theory focuses on financial institutions such as insurance firms and commercial banks that are subject to prudential regulation. Financial institutions differ from non-financial institutions in two important aspects. First, financial institutions are much more highly leveraged than non-financial firms. Second, in contrast to industrial firms, a relatively large proportion of the debt of a financial institution is held by uninformed and widely dispersed debt holders. It is difficult (if not impossible) for such debt holders to coordinate their actions to intervene in the institution’s operations when it is in their interests to do so. In other words, the relevant externality that could be mitigated by regulatory intervention arises from a coordination problem among debt holders. As Dewatripont and Tirole (1994) argue, prudential capital regulation plays an important role of an ex ante commitment and coordination mechanism that enforces an ex post transfer of control from shareholders to creditors by imposing a capital requirement that is ex ante optimal. By their “representation hypothesis,” the regulator can serve as a representative of dispersed and uninformed debt holders by effecting such a transfer of control.
We capture the aforementioned distinguishing features of financial institutions in a two-period model in which a representative institution finances a long-term project (or a pool of projects) through a combination of debt and equity. As in a number of prior studies (e.g. Gorton and Pennacchi (1990), Giammarino, Lewis and Sappington (1993), Heaton, Lucas and McDonald (2010), Hanson, Kashyap and Stein (2011), Stein (2012)), there are excess costs of equity relative to debt financing that create an incentive for the institution to choose a high leverage. The project’s cash flows are realized at the end of period 2. The cash flows depend stochastically on the project’s quality that is privately chosen by shareholders with a higher quality project entailing a higher expected investment cost. At the end of period 1, there is an observable, but non-verifiable signal about the terminal cash flows of the project that indicates either a good or bad interim state. A higher project quality increases the likelihood of the good state.

The institution faces a capital constraint imposed by a regulator at date 1, which ensures that the market value of its assets is sufficiently high relative to the market value its liabilities (Dewatripont and Tirole (1994)). If the institution meets the capital constraint at date 1, its shareholders may act opportunistically by engaging in asset substitution in period 2. If the institution violates the constraint, control transfers to the regulator, who closely monitors the institution’s operations and ensures that the ex post efficient continuation strategy—no asset substitution—is chosen in period 2. Consequently, the project’s terminal cash flows are affected by the project’s quality in period 1 and potential asset substitution or control transfer in period 2. Given the institution’s capital structure and the capital constraint, we first examine the institution’s optimal choices of project quality and asset substitution. We then derive the optimal capital structure of the institution given a capital constraint. Finally, we examine the choice of the constraint that is ex ante efficient in that it maximizes the total surplus that is equivalent to maximizing the total ex ante value of the institution in our environment with a representative institution.

There are two well known inefficiencies—asset substitution and debt overhang—that arise from agency conflicts between shareholders and debt holders. First, the higher the leverage, the greater are shareholders’ incentives to increase risk by engaging in asset substitution in the second period.

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2Our analysis does not change if we instead assume that the regulator liquidates the institution’s assets where the liquidation payoff equals the institution’s value under no asset substitution. Stated differently, our results hold even if the institution’s claims are traded in frictionless, competitive markets so that prices fully reflect fundamental values. Our results are also qualitatively unaltered if we allow for a recapitalization of the institution, where its leverage is lowered through a debt-equity swap.
Second, the higher the leverage, the lower are shareholders’ incentives to make a costly investment to increase project quality in the first period because a larger proportion of the increased total payoffs from a higher project quality accrues to debt holders. The novel and interesting outcome of our analysis, however, is that, as leverage increases, there is a subtle trade-off between asset substitution and debt overhang. An increase in the propensity for asset substitution in the second period alleviates the debt overhang problem in the first period.

The intuition for the trade-off between asset substitution and debt overhang is as follows. At low leverage levels, asset substitution occurs (if at all) only in the low state, i.e., when the interim signal is low. At low leverage levels, however, the debt overhang problem is also insignificant in that the institution chooses high project quality in the first period. Consequently, at low leverage levels, eliminating the possibility of asset substitution through a capital constraint has little impact on the ex ante project quality choice. At high leverage levels, however, asset substitution is pervasive in that it occurs in both the low and high interim states. The reason is that, as leverage increases, the call option in the low state becomes more out of the money than the call option in the high state. Further, at higher leverage levels, the payoffs from asset substitution are much greater for the high state relative to the low state because the good outcome for the project is realized. Because the high state is more likely for the high quality project, this, in turn, increases the institution’s incentives to choose the higher project quality. Consequently, a capital constraint that shuts down asset substitution in the good state by transferring control to the regulator eliminates the incremental rents from asset substitution in the good state relative to the bad state. Consequently, shareholders’ ex ante incentives to invest in the higher quality project decline, that is, the debt overhang problem worsens.

By triggering a change in control, the capital constraint plays an important role in mediating the inefficiencies arising from asset substitution and debt overhang. The choice of the constraint that maximizes the total ex ante value of the institution reflects the trade-off between these inefficiencies. A tighter constraint mitigates asset substitution by increasing the likelihood of a change in control, but potentially dampens incentives to invest in higher project quality. A tighter constraint also implies that the institution must tilt its capital structure towards costlier equity capital. Thus, the total value-maximizing capital constraint reflects the interplay among the inefficiencies arising from asset substitution and debt overhang as well as the cost of financing.
We demonstrate that the optimal capital constraint does not eliminate either inefficiency, that is, both debt overhang and asset substitution are possible at the optimum. Further, the constraint becomes more stringent when the excess cost of equity capital increases. As the excess cost of equity capital increases, the institution’s incentives to use debt financing increase so that both asset substitution and debt overhang inefficiencies are more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the constraint becomes tighter to mitigate asset substitution at the expense of potentially curbing incentives to invest in project quality. The excess cost of equity capital relative to debt capital is likely to vary across the business cycle. During an upswing in the business cycle, credit becomes cheaper/easier to obtain so that it is plausible that the excess cost of equity capital increases with the reverse being true during a downswing in the business cycle. The result that the optimal capital requirement should increase with the excess cost of equity capital is, therefore, consistent with the proposal for higher capital requirements during booms compared with recessions that has been made by several academics and policy makers.

The optimal capital constraint is *institution-specific* in that it depends on parameters that determine the payoff distribution of the institution’s projects. These parameters are likely to vary across institutions even if they belong to a particular category such as commercial banks or insurance firms. A uniform capital requirement across institutions could, therefore, be suboptimal. In fact, if the capital constraint is too tight, then the basic trade-off between asset substitution and debt overhang that we highlight suggests that there could be excessive underinvestment. The proposals to require stricter capital requirements based on market prices in the Basel III accords should, therefore, be implemented with caution.

Our model and results are particularly pertinent to the prudential regulation of highly levered financial institutions that has become a hotly debated issue in the aftermath of the financial crisis. As discussed above, the key trade-off between asset substitution and debt overhang is particularly pronounced at high leverage levels when both problems are severe, and asset substitution is pervasive in that it occurs in both “good” and “bad” states. Indeed, one of the primary causes of the financial crisis was risky subprime mortgage lending by banks during a period when the economy was booming and credit was cheap. Subprime mortgage lending could be more generally viewed as asset substitution that occurred in “good” states. Our analysis highlights the fact that, at higher
leverage levels that are more typical of financial institutions and where prudential regulation potentially plays a role, the option value of asset substitution is significantly higher in good states. Consequently, shutting down asset substitution through a prudential capital constraint and the transfer of control to a regulator could have a much bigger negative impact on \textit{ex ante} investment in project quality. Our study therefore sheds light on the trade-off between asset substitution and debt overhang problems, and the role that prudential capital regulation based on market values plays in balancing this trade-off.

2 Related Literature

We contribute to the growing stream of literature that theoretically analyzes the real effects of mark-to-market accounting. O’Hara (1993) investigates the effect of market value accounting on project maturity and finds that market prices results in a preference for short-term projects over long-term projects. Allen and Carletti (2008) (hereafter, AC) and Plantin, Sapra, and Shin (2008) (hereafter, PSS) are two recent studies that show how market prices may have detrimental consequences for financial stability. In both studies, markets are illiquid and incomplete and therefore a reliance on price signals may lead to inefficiencies. We complement these studies in a number of ways. First, in contrast to the above studies, we analyze the effects of using market prices to measure the capital regulation of financial institutions. Because capital constraints depend on how the values of assets and liabilities are measured, measurement rules naturally have \textit{real} effects. PSS, instead, assume that managers maximize expected accounting earnings so that accounting has real effects. Second, because the issues we examine are different, there are important distinctions in the tensions identified. In our setup, markets are frictionless and competitive so that price signals perfectly impound information about future cash flows. We focus on the effects of agency conflicts between a financial institution’s shareholders and its debt holders. We show that, even in the absence of liquidity risk so that prices fully reflect fundamentals, mark-to-market accounting curbs inefficient risk shifting, it could reduce incentives to invest in high quality projects.

Burkhart and Strausz (2009) (BS) and Heaton, Lucas, and McDonald (2010) (HLM) model the effects of market-based regulation on financial institutions and also assume frictionless and competitive markets so that prices fully reflect fundamentals. BS show that mark-to-market accounting
increases the liquidity of a financial institution's assets, which, in turn, increases the institution's asset substitution incentives. Our analysis identifies different frictions, and therefore generates very different conclusions. BS focus on the information asymmetry between the institution’s current shareholders and prospective shareholders, while we examine conflicts between debt holders and shareholders. In their environment, mark–to–market accounting reduces information asymmetry that induces asset substitution. In our environment, regulation based on market prices curbs asset substitution through the intervention of the regulator but unfortunately, the debt overhang problem is exacerbated. HLM build a general equilibrium model of an institution and study how mark-to-market accounting interacts with an institution’s capital requirements to affect the social costs of regulation. In their model, financial institutions invest in firms whose technologies are exogenous and fixed. In contrast, our analysis centers on how the optimal choice of the capital constraint anticipate the financial institution’s endogenous project choices.

Our study is also related to the literature on the capital regulation of banks and, more generally, financial institutions (see Dewatripont and Tirole (1995) and Santos (2001) for surveys). We adopt the perspective in Dewatripont and Tirole (1995) who argue that the main concern of prudential regulation is the solvency of financial institutions that, in turn, is related to their capital structure. Capital structure is relevant because it implies an allocation of control rights (Aghion and Bolton (1992)) between shareholders and debt holders. Further, the importance of regulation stems from the fact that small, uninformed debt holders of institutions need a representative to effect an ex post transfer of control from shareholders to debt holders that is ex ante optimal. In early studies, Merton (1978) and Bhattacharya (1982) show that capital requirements curb inefficient risk-shifting. However, studies such as Koehn and Santomero (1980), Kim and Santomero (1988), Gennette and Pyle (1991), and Rochet (1991) argue that capital requirements could alter the equilibrium scale of operations of an institution and, therefore, its optimal asset composition in ambiguous ways. Besanko and Kanatas (1996) show that conflicts of interest between a bank’s management and its shareholders could lower, and sometimes even reverse, the beneficial effects of capital regulation in curbing asset substitution. Kahn and Winton (2004) emphasize that risk-shifting incentives are particularly important for financial institutions. Gorton and Winton (1995) examine the design of prudential regulation in a general equilibrium model. We contribute to this literature by showing how capital constraints optimally balance the inefficiencies arising from asset substitution and debt
overhang.

3 Model

3.1 Environment

A financial institution finances a long-term project through a combination of debt and equity. The term “project” should be viewed as a metaphor for the institution’s “pool” of projects. Because our theory is broadly applicable to institutions that are subject to regulation such as insurance firms and commercial banks, we deliberately do not model a specific type of institution. Our focus is on agency conflicts between shareholders and debt holders so we assume that the institution’s managers/insiders behave in the interests of shareholders.

The project’s payoff increases stochastically (in the sense of first-order stochastic dominance) in the project’s quality. The institution chooses the project’s quality through careful analysis and selection. The cost incurred by the institution’s shareholders in choosing the quality of the project is a nonnegative random variable whose expected value increases with project quality. At some interim date, there is a publicly observable, but non-verifiable, signal about the performance of the project. At this date, shareholders may act opportunistically by engaging in asset substitution or risk-shifting that results in a transfer of wealth from debt holders to shareholders, but lowers the value of the overall project.

The institution operates in a regulated environment. There is a regulator who imposes a capital requirement or constraint to ensure that the value of the institution’s assets is sufficiently high relative to the value its liabilities (Dewatripont and Tirole (1994)). If the constraint is violated at the interim date, control transfers to the regulator who closely monitors the institution and ensures that it chooses the efficient continuation strategy—no asset substitution—in the second period. Our analysis does not change in any way if we, instead, assume that the regulator sells or liquidates the institution’s assets where the total payoff is the market value of the assets assuming the efficient continuation strategy—no asset substitution—is chosen in the second period. In other words, our

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3 If the institution is an insurance firm, its “creditors” include insurance policyholders. With the pooling of insurance risks, the insurance firm’s liabilities arising from insurance claims are similar to a debt obligation. If the institution is a bank, its creditors include depositors and other debt holders.

4 Asset substitution could be achieved by either engaging in off-balance sheet derivative transactions and/or altering the characteristics of the existing project.
results hold even if the institution’s claims are traded in frictionless, competitive markets, that is, there are no deadweight costs arising from the early sale or liquidation of the institution’s assets. Our results are also qualitatively unaltered if we assume that the regulator intervenes and forces a recapitalization of the institution, that is, a lowering of the institution’s leverage through a debt-equity swap.

It is important to note that, in our environment, regulation plays the important role of serving as an \textit{ex ante} commitment mechanism that enforces an \textit{ex post} transfer of control that is optimal from an \textit{ex ante} standpoint, that is, from the standpoint of maximizing the total enterprise value of the institution. The debt issued by non-financial firms typically has associated covenants that also play the role of effecting a transfer of control if they are violated. As discussed by Dewatripont and Tirole (1994), from a “high level” perspective, prudential regulation of financial firms and debt covenants for non-financial ones are “isomorphic” in that they are both mechanisms that achieve a transfer of control. As such, the main economic rationales for their presence are similar. As they emphasize, however, the debt issued by financial firms differs significantly from that issued by non-financial ones in that the former is held by widely dispersed, uninformed investors. Covenants are difficult (if not impossible) to enforce for such investors because it is costly for them to monitor the institution and coordinate to effect a transfer of control when covenants are violated. In other words, the relevant externality arises from a coordination problem among the institution’s debt holders. In this respect, regulation serves as a commitment and coordinating mechanism. Consequently, the mechanisms through which a transfer of control is achieved differ for financial and non-financial firms. Further, financial institutions have much higher leverage levels than non-financial firms (Gropp and Heider (2010), DeAngelo and Stulz (2013)), and the trade–off we identify between asset substitution and debt overhang is particularly pronounced at high leverage levels. Consequently, the main implications of our study are much more pertinent to financial institutions. We next describe the ingredients of the model in more detail.

### 3.2 Project and Capital Structure

There are three dates \( t \in \{0,1,2\} \) and two periods. We denote period 1 as the period between dates 0 and 1 and period 2 as the period between dates 1 and 2. All agents are risk-neutral and the risk-free rate is normalized to zero. At \( t = 0 \), the institution makes an investment \( A_0 \) in a long-
term project that is financed through a combination of debt and equity. Our objective is to study shareholder-debt holder conflicts, especially when the institution’s leverage may be high. Similar to studies such as Gorton and Pennacchi (1990), Giammarino et al. (1993), Heaton et al. (2010), Mehran and Thakor (2010), Hanson et al. (2011), and Stein (2012), there are deadweight excess costs of equity financing that we model by assuming that equity holders demand a higher (risk-adjusted) expected return on their investment than debt holders. For example, if the institution is an insurance firm, the lower cost of debt capital could arise from the fact that agents have a demand for insurance. The insurance firm’s core business is the provision of insurance so that it has a comparative advantage in supplying insurance that it does not possess in raising equity capital. If the institution is a bank, the lower cost of debt could arise from the fact that investors have a demand for information insensitive and liquid securities such as demand deposits that the bank has a comparative advantage in providing.

Prior literature suggests a number of reasons why financial institutions issue debt and, more importantly why financial institutions have relatively high leverage levels (e.g., see Dewatripont and Tirole (1994), Allen and Gale (1999), Santos (2001) and more recently DeAngelo and Stulz (2013)). All these mechanisms have the effect of lowering the “effective” cost of debt relative to equity. Because the economic insights we focus on in this study do not hinge on the particular frictions that give rise to the excess cost of equity, we follow the aforementioned studies by not modeling such frictions to simplify the exposition and analysis.

We normalize the cost of debt to $1$ and the cost of equity to $1 + \lambda$ over the period between date $0$ and date $2$, where $\lambda$ denotes the excess cost of equity. To simplify notation, the two periods are of equal length so that the cost of equity over the period between date $1$ and date $2$ is $\sqrt{1+\lambda}$. Note that the parameter $\lambda$ is not a risk premium: it represents the additional deadweight cost of equity financing relative to debt financing.

Because financial institutions have substantially higher leverage levels relative to industrial firms, their “effective” $\lambda$ is significantly higher in the context of our model. Gropp and Heider (2010) conduct an empirical analysis of the determinants of the capital structures of large U.S. and European banks. They document that the median book and market leverage ratios of banks in their sample are 92.6% and 87.3%, respectively, while the corresponding median ratios for non-financial firms are 24% and 23%, respectively. They argue that their findings are consistent with banks
facing significantly higher excess costs of equity financing compared with non-financial firms, which could explain their substantially higher leverage levels.

For simplicity, we consider debt that matures at $t = 2$ with no intermediate payments. The amount of debt, $D_0$, the institution chooses to issue at $t = 0$ is determined by its face value, $M$, at maturity. We later endogenize $M$ when we analyze the institution’s capital structure choice. The institution finances the remaining amount $E_0 = A_0 - D_0$ of the project through equity. Capital markets are competitive.

If the institution is a bank, its depositors are protected by deposit insurance in practice. We do not incorporate the presence of deposit insurance in our analysis because, as mentioned earlier, we intend our theory to be applicable to a general financial intermediary whose liabilities need not be protected by deposit insurance. Further, even in the case of banks, a substantial portion of their debt is long-term and uninsured.

It turns out that, even if we restrict ourselves to the specific case in which the institution is a bank and all its debt comprises of insured demand deposits, our implications are unaffected as long as deposit insurance is fairly priced. The reason is that fairly priced deposit insurance—that is, the deposit insurance premium rationally incorporates the institution’s optimal choices of capital structure, project quality, and asset substitution—is merely a transfer of funds from shareholders to debt holders. Shareholders pay the deposit insurance premium to the deposit insurer who, in turn, compensates debt holders if the institution defaults. Consequently, although debt is risk-free due to deposit insurance, the deposit insurance premium lowers the value of equity so that the value of the institution—the size of the total pie—is unchanged. Furthermore, the deposit insurance premium is a sunk cost that is incurred \textit{ex ante}. Consequently, the \textit{ex post} value of equity—that is, after deposit insurance and capital structure are in place—is identical to its value in the scenario in which there is no deposit insurance. The upshot of these implications is that none of the institution’s decisions—capital structure, project quality, and asset substitution—is affected by the presence of deposit insurance. Because the size of the total pie is unchanged by deposit insurance, the regulator’s objective function is also unaltered. The only result that changes is the magnitude of the optimal capital constraint which increases with deposit insurance because the value of insured debt is higher than that of uninsured debt.\footnote{An analysis of the model with deposit insurance is available upon request. Chan et al. (1992), Giammarino et al.}
3.3 Project Quality

The terminal cash flows of the project, which are realized at date \( t = 2 \), are affected by both the quality of the project chosen in period 1 and potential asset substitution chosen in period 2. The quality \( q \) of the institution’s project can be either low (\( q = q_L \)) or high (\( q = q_H \)) where \( 0 \leq q_L < q_H \leq 1 \). Without loss of generality, we simplify notation by normalizing the low project quality \( q_L \) to 0. The project quality \( q \) is the shareholders’ hidden action. In period 1, the institution can always invest in a default long-term project, i.e., in a project with a low quality level 0. However, by investing additional costly resources, the institution can raise the quality of its project from 0 to \( q_H \). The resources invested by the shareholders in choosing a project of quality \( q \in \{0, q_H\} \) are represented by a nonnegative random variable \( \tilde{C}(q) \) with support \([0, \infty)\). The expected cost of choosing a project is increasing in its quality. Enhancing the project quality from 0 to \( q_H \) requires the institution’s shareholders to incur an additional expected cost of \( E[\tilde{C}(q_H) - \tilde{C}(0)] = kq_H \). We alternately refer to the additional expected cost \( kq_H \) as the additional investment in project quality and to the parameter \( k \) as the marginal cost of investment in project quality.

3.4 Interim Signal and Terminal Cash Flows

At \( t = 1 \), there is an observable, but non-verifiable, signal \( X \) that provides information about the terminal cash flows of the institution. The signal \( X \in \{x_L, x_H\} \) where \( 0 < x_L < A_0 < x_H \). If the quality of the project is \( q \in \{0, q_H\} \), then

\[
\Pr(X = x_H) = q \quad \text{and} \quad \Pr(X = x_L) = 1 - q.
\]

As we describe shortly, if the interim signal is \( X \in \{x_L, x_H\} \), then the terminal cash flows are equal to \( X \) provided the institution engages in no asset substitution in period 2. The signal, \( X \), therefore potentially resolves some of the uncertainty associated with the project’s terminal cash flows. By (1), the high quality project first-order stochastically dominates the low quality project, that is, the probability of a high interim signal is greater with the higher quality project. Note that, because the signal \( X \) is non-verifiable, regulatory intervention cannot be directly contingent on it, but only

(1993) and Freixas and Rochet (1995) examine the feasibility of fairly priced deposit insurance when there is adverse selection regarding the bank’s projects.
indirectly through a verifiable metric such as a capital constraint.

At the beginning of period 2, the shareholders decide whether to engage in asset substitution. In particular, given the signal \( X = x_i \), where \( i \in \{L, H\} \), the shareholders take a hidden action that is represented by the ordered pair \((r, z) \in \{(0, 0), (r_H, z_H)\}\) that alters the distribution of terminal cash flows of the institution. Given \( X \), the terminal payoff of the institution, \( \tilde{V} \), takes two possible values, either \((1 + z)X\) or \((1 - z)X\), where

\[
\text{Pr}(\tilde{V} = (1 + z)X) = \frac{1}{2} - r; \quad \text{Pr}(\tilde{V} = (1 - z)X) = \frac{1}{2} + r.
\]  

(2)

We assume that \( 0 < r_H \leq \frac{1}{2} \) and \( 0 < z_H \leq 1 \). Given the asset substitution strategy \((r, z)\) and signal \( X \), the expected value of the terminal cash flows of the institution is

\[
E(\tilde{V} | X) = (1 - 2rz)X
\]

From the above discussion, it immediately follows that the action \((r, z) = (0, 0)\) captures “no asset substitution” because the terminal payoff conditional on the intermediate signal is risk-free and equals the value of the signal. On the other hand, the action \((r, z) = (r_H, z_H)\) captures asset substitution because it injects uncertainty in the terminal payoffs, while simultaneously reducing the expected terminal cash flows of the institution from \( X \) to \((1 - 2r_Hz_H)X\). In order to sharpen the analysis, we choose binary asset substitution strategies.

To simplify the algebra, we assume a “recombining” binomial tree when asset substitution is chosen in the high and low states. More precisely, the best possible terminal payoff from asset substitution when the intermediate signal is low (when \( X = x_L \)) equals the worst possible terminal payoff from asset substitution when the intermediate signal is high (when \( X = x_H \)) so that

\[
(1 - z_H)x_H = (1 + z_H)x_L.
\]  

(3)

We also make the following standing assumption on project parameters:

\[
\frac{1}{1 + \lambda}(1 + z_H)x_L < A_0 < \frac{1}{1 + \lambda}x_H - kg_H.
\]  

(4)
The first inequality implies that, conditional on a low intermediate signal at date 1, even the best possible outcome under asset substitution is not sufficient to recover the initial investment $A_0$. The second inequality ensures that, conditional on a high intermediate signal at date 1, engaging in no asset substitution has a positive net payoff in the sense that the corresponding terminal payoff $x_H$ is greater than the sum of the initial investment $A_0$ and the expected incremental cost $kq_H$ of choosing high project quality. Assumption (4) ensures that the inefficiencies due to asset substitution are severe enough for prudential regulation to be relevant.

### 3.5 Capital Constraint

At any date $t$, the institution faces a capital constraint imposed by a regulator, which requires that the value of the institution’s assets be high enough relative to the value of its liabilities, that is,

$$\frac{D_t}{F_t} \leq c \text{ where } c \in [0, 1] \text{ and } t \in \{0, 1\}. \quad (5)$$

In the above, $F_t = D_t + E_t$ is the market value of the institution’s total assets at date $t$, which is the sum of the market value $D_t$ of the institution’s debt and the market value $E_t$ of the institution’s equity. The institution’s leverage ratio must be below a threshold or ceiling $c$ where a smaller (larger) value of $c$ implies a tighter (looser) constraint. If the constraint is satisfied at date 1 ($\frac{D_1}{F_1} \leq c$), the institution’s shareholders maintain control for the second period. However, if it is violated ($\frac{D_1}{F_1} > c$), control transfers to the regulator who closely monitors the institution and ensures that it does not engage in asset substitution.\(^6\)

### 3.6 Payoffs

By (1) and (2), the distribution of terminal cash flows $\tilde{V}$ depends on both the unobservable project quality $q \in \{0, q_H\}$ chosen in period 1 and on the unobservable asset substitution strategy $(r, z) \in \{(0, 0), (r_H, z_H)\}$ chosen in period 2. We refer to period 1 as the quality investment stage and to period 2 as the asset substitution stage.

Figure 1 summarizes the sequence of events. Figure 2 illustrates how the distribution of terminal cash flows $\tilde{V}$ depends on the institution’s investment $q$ in period 1 and its asset substitution choice

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\(^6\)Our results would not qualitatively change if the incidence of asset substitution is lower under the regulator’s control.
(r, z) in period 2.

The payoffs of the shareholders and debt holders depend on whether the prudential constraint is violated at the end of period 1. If the constraint is violated, the regulator ensures that the institution chooses the ex post efficient strategy of no asset substitution so that the institution chooses (r, z) = (0, 0) in period 2. The debt holders’ payoffs equal the lower of the face value M of the debt or the total project payoffs \( \tilde{V}(q; (r, z)) \) of the institution, where we explicitly indicate the dependence of the terminal cash flows on the project quality q and the asset substitution strategy (r, z). Shareholders receive the total project cash flows net of payments to debt holders and net of the cost of investment in project quality. The following table summarizes the payoffs of the shareholders and the debt holders:
Table 1: Payoffs of Debt Holders and Shareholders

<table>
<thead>
<tr>
<th></th>
<th>Institution Maintains Control</th>
<th>Regulator Takes Control</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Date 0</strong></td>
<td></td>
<td></td>
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<tr>
<td>Debt holders’ Payoff</td>
<td>$\min{M, \tilde{V}(q; (r, z))}$</td>
<td>$\min{M, \tilde{V}(q; (0, 0))}$</td>
</tr>
<tr>
<td>Shareholders’ Payoff</td>
<td>$-\tilde{C}(q)$</td>
<td>$\max{\tilde{V}(q; (r, z)) - M, 0}$</td>
</tr>
<tr>
<td>Total Payoff</td>
<td>$-\tilde{C}(q)$</td>
<td>$\tilde{V}(q; (r, z))$</td>
</tr>
</tbody>
</table>

In the first best scenario, all decisions are made to maximize the total value of the institution rather than just shareholder value, and the excess cost of equity $\lambda$ is zero. In this scenario, it is easy to show that the institution always chooses the high quality project and does not engage in asset substitution. Because the incentives of shareholders are aligned with those of creditors, the institution’s capital structure and therefore its leverage play no role.

In the second-best world, maximizing shareholder value is not necessarily equivalent to maximizing the total value of the institution (that is, the equity value plus the debt value). We analyze the model using backward induction. We start at the beginning of period 2 when the public signal has been released. For a given capital constraint, capital structure, and a given public signal, we first derive the transfer of control decision and asset substitution decision in period 2. We next derive the project quality decision in period 1, which anticipates the transfer of control and asset substitution decisions in period 2. We then determine the capital structure decision at date 0, which is determined by the choice of the face value of debt. Finally, we derive the capital constraint that maximizes the total enterprise value of the institution rationally anticipating its choices of capital structure, project quality and asset substitution.

4 Benchmark Regime: No Regulation

We begin by analyzing a hypothetical environment in which there is no capital constraint so that transfer of control is ruled out. This environment serves as a useful benchmark to sharpen the intuition behind the main results by highlighting the role that the capital constraint plays in mediating the distortions created by shareholder-debt holder agency conflicts.
4.1 Asset Substitution

In the absence of a capital constraint at date 1, the shareholders’ asset substitution decision is straightforward. Given the interim signal $X$, they compare their incremental discounted expected payoffs from engaging in asset substitution. By Table 1, an action of $(r_H, z_H)$ results in shareholders’ period 2 discounted expected payoffs of

$$
\frac{1}{\sqrt{1 + \lambda}} \left[ \left( \frac{1}{2} - r_H \right) \max\{(1 + z_H)X - M, 0\} + \left( \frac{1}{2} + r_H \right) \max\{(1 - z_H)X - M, 0\} \right]
$$

while an action of $(r_L, z_L)$ results in expected discounted payoffs of

$$
\frac{1}{\sqrt{1 + \lambda}} \max\{X - M, 0\}.
$$

Using expressions (6) and (7) yields the following result.

**Proposition 1 (Asset Substitution in Benchmark Regime)** Given no capital constraint, asset substitution $(r_H, z_H)$ occurs if and only if $M \geq c_0$, where

$$
c_0 = 1 - \frac{1}{2} - \frac{r_H}{2 + \lambda} z_H.
$$

Proposition 1 illustrates the well-known result that shareholders engage in asset substitution when the institution’s leverage is sufficiently high. Shareholders’ claim is a call option whose strike price equals the face value of debt. Consequently, it is optimal for them to increase risk by choosing asset substitution when the intermediate signal is sufficiently low relative to the face value of debt.

Note that asset substitution is feasible in both in the low and the high states, i.e., when $X = x_L$ and $X = x_H$. As $\frac{1}{2} - r_H$ (the probability of a good outcome given asset substitution) and/or $z_H$ (the spread of outcomes resulting from asset substitution) increases, asset substitution becomes more attractive to shareholders in period 2. Consequently, the threshold value $c_0X$ of the debt face value above which asset substitution takes place decreases, that is, asset substitution occurs for a larger range of debt face values.

It follows from the proposition that the propensity to choose asset substitution depends on the leverage level of the financial institution which is endogenous. Furthermore, for high leverage levels,
Asset substitution is likely in both the good state \((X = x_H)\) and in the bad state \((X = x_L)\); an observation that is important for our subsequent analysis.

### 4.2 Project Quality

In period 1, given the debt face value, \(M\), shareholders choose the project quality \(q\) anticipating the asset substitution decision described in Proposition 1. In choosing the project quality, shareholders trade off their expected payoff incorporating the period 2 asset substitution decision against the expected investment in project quality. The following result characterizes shareholders’ optimal choice of project quality in period 1.

**Proposition 2 (Project Quality in Benchmark Regime)** Shareholders choose low project quality in period 1 if and only if the maturity value \(M\) of debt is sufficiently high. (i) For \(k \leq k^*\), \(q_L\) is chosen if and only if \(M > c_2 x_H\). (ii) For \(k > k^*\), \(q_L\) is chosen if and only if \(M > c_1 x_H\). In the above,

\[
c_1 = 1 - \frac{k(1 + \lambda)}{x_H}; \quad c_2 = (1 + z_H) - \frac{k(1 + \lambda)}{(2 - r_H) x_H}; \quad k^* = \frac{1}{2} - \frac{r_H z_H x_H}{z_H x_H / (1 + \lambda)}. \tag{9}
\]

Proposition 2 illustrates the well-known debt overhang problem. When leverage is sufficiently high, shareholders underinvest in project quality. If the amount of debt in the institution’s capital structure is sufficiently high, shareholders’ incentives to make a costly investment in the higher quality project are curbed because a larger proportion of the increased total payoff from such an investment accrues to debt holders.

The novel and interesting outcome of our analysis is that, taken together, Propositions 1 and 2 imply that the debt overhang problem in period 1 is alleviated by the possibility of asset substitution in period 2. To see the beneficial impact of asset substitution on the debt overhang problem, suppose asset substitution were exogenously ruled out. Then it is relatively straightforward to show that regardless of the value of \(k\), underinvestment occurs if and only if:

\[M > c_1 x_H\]
When $k \leq k^*$, $c_1 < c_2$, so that Proposition 2 implies that the underinvestment problem is alleviated in the presence of asset substitution. The next corollary illustrates how large *ex post* rents from asset substitution increase the institution’s incentives to choose the high quality project in the first period, thereby reducing the debt overhang problem.

**Corollary 1 (Asset Substitution and Debt Overhang in Benchmark Regime)** If $r_H$ decreases and/or $z_H$ increases (i) the threshold level of the debt face value above which asset substitution occurs decreases for any value of the intermediate signal $X$; (ii) for given $k$, the threshold level of the debt face value above which the low project quality is chosen increases; and (iii) the threshold level $k^*$ in Proposition 2 increases.

By the discussion following Proposition 1, a decrease in $r_H$ and/or an increase in $z_H$ increases the incentives for asset substitution in period 2, that is, the range of debt levels for which asset substitution occurs increases for any value of the intermediate signal. The corollary, however, shows that a decrease in $r_H$ and/or an increase in $z_H$ causes the range of debt levels for which the low quality project is chosen to shrink. In other words, an increase in the propensity for asset substitution in the second period *increases* the likelihood of choosing high project quality in the first period, that is, it *alleviates* the debt overhang problem by providing incentives to invest in higher project quality. Furthermore, as $r_H$ decreases and/or $z_H$ increases, the threshold $k^*$ in Proposition 2 increases so that the region $k \leq k^*$ expands while the region $k > k^*$ shrinks. Therefore, not only does the range of debt levels for which the low quality project is chosen shrink in the presence of asset substitution, but as asset substitution becomes more attractive, the latter effect also persists for larger values of $k$.

Figure 3 illustrates the corollary via a numerical example. It demonstrates how an increase in the propensity of asset substitution via a decrease in the value of $r_H$ from $\frac{1}{10}$ to $\frac{1}{16}$ affects the incentives to invest in project quality in the $HC$ regime. The top half of Figure 3 illustrates that, as $r_H$ declines from $\frac{1}{10}$ to $\frac{1}{16}$, the range of debt face values $M$ for which asset substitution occurs *expands* from the interval $M > 0.4X$ to the interval $M > 0.3X$. The bottom half of Figure 3 shows that, following a decline in $r_H$, the corresponding range of values of $M$ for which the low quality project is chosen *shrinks* from $M > 40$ to $M > 53$. Hence, an *increase* in the propensity for asset substitution *enhances* incentives to invest in higher project quality.
The intuition for these results is as follows. At low leverage levels, asset substitution is either non-existent or occurs only in the low state $x_L$. At low leverage levels, however, the debt overhang problem is also nonexistent in that the high project quality is chosen in the first period as shown by Proposition 2. Since asset substitution occurs (if at all) only in the low state where payoffs are low, a change in the incentives for asset substitution triggers little distortion from an ex ante perspective so that the project quality choice in the first period is unaffected. As leverage increases, however, the option to engage in asset substitution becomes more valuable in the high state relative to the low state because the call option in the low state becomes more out of the money relative to the high state. Further, at higher leverage levels, the payoffs from asset substitution are much greater for the high state relative to the low state because the good outcome for the project is realized. Given that the high state is more likely for the high quality project, an increase in the propensity for asset substitution in the second period increases the institution’s incentives to choose the higher project quality in the first period. Furthermore, as asset substitution becomes more profitable in period 2 (i.e., when $r_H$ decreases and/or $z_H$ increases), the call option in the high state becomes even more valuable so that the incentives to choose the higher quality project persist even for large values of $k$.

To summarize, at low leverage levels, the debt overhang and asset substitution problems are both minor so that the low ex post rents from asset substitution have little or no impact on the ex ante project quality choice. At high leverage levels, however, asset substitution also occurs in the good state. The potentially large ex post rents from asset substitution in the good state increase
the institution’s incentives to choose the high quality project in the first period, thereby reducing the debt overhang problem.

Note that when \( k > k^* \), asset substitution has essentially no impact on the *ex ante* underinvestment problem because the parameter \( c_1 \)–which determines the threshold for underinvestment in Proposition 2–does not depend on the asset substitution parameters, \( r_H \) and \( z_H \). The intuition for the latter result is that when the marginal cost of investment is relatively high, the debt overhang problem is not severe in the first period implying that asset substitution plays essentially no role in alleviating underinvestment. For the rest of the paper, in order to focus on the interplay between asset substitution and debt overhang inefficiencies, we will focus the discussion of our results on environments in which the debt overhang problem is severe enough so that the *ex post* rents from asset substitution have the potential of alleviating the *ex ante* debt overhang problem, i.e., environments in when \( k \leq k^* \).

### 4.3 Capital Structure

We now analyze the institution’s optimal choice of its capital structure. The institution’s *original* shareholders (that is, before capital structure is in place) optimally finance the project by issuing debt and equity rationally anticipating the *ex post* project quality and asset substitution choices. In particular, prior to the initial date, \( t = 0 \), the original shareholders choose its capital structure to maximize their value. The value of original shareholders at date zero equals the market value of equity plus the market value of debt. Recall that the cost of debt is normalized to 1 and that of equity to \( 1 + \lambda \). Consequently, the debt face value, which determines the institution’s capital structure, solves

\[
M^{benchmark} = \arg \max_M \left\{ \frac{E(\max\{\hat{V} - M, 0\})}{1 + \lambda} + E\left( \min\{M, \hat{V}\} \right) - \right. \\
\left. \frac{E[C(q^{benchmark})]}{A_0} \right\} - (10)
\]

where \( D_0 \) is the market value of debt at date 0. In (10), \( q^{benchmark} \) and \( M^{benchmark} \) respectively denote the optimal project quality and the optimal debt face value in the benchmark regime.
By (10), $M_{\text{benchmark}}$ balances the trade-off between the excess cost of equity represented by $\lambda$ and the inefficiencies arising from debt overhang and asset substitution due to the presence of debt in the institution’s capital structure. The following proposition characterizes the institution’s optimal capital structure.

**Proposition 3 (Optimal Capital Structure for Given Constraint)** There exists a threshold $\lambda_1$ of the excess cost of equity such that the institution’s optimal choice $M_{\text{benchmark}}$ of the debt face value, which determines its capital structure, is as follows:

$$
\lambda < \lambda_1 : M_{\text{benchmark}} = c_0 x_H \quad \lambda > \lambda_1 : M_{\text{benchmark}} = c_2 x_H
$$

where $\lambda_1 \equiv \frac{2r_Hx_H}{(k^*-k)}$ while $c_0$, $c_2$, and $k^*$ are defined in (8) and (9).

From Proposition 3, it is obvious that the optimal face value of debt, $M_{\text{benchmark}}$, depends on the underlying parameters of the institution’s environment such as the excess cost of equity capital, $\lambda$. When $k \leq k^*$, $c_0 \leq c_2$ so that the institution finances the project with more debt when the cost of equity is relatively high (i.e., $\lambda > \lambda_1$). We can also use Proposition (4.3) to investigate the impact of the optimal leverage on the shareholder-debt holder conflicts. To see the interplay between asset substitution and debt overhang inefficiencies, we will focus on the interesting region of $k \leq k^*$.

When $\lambda < \lambda_1$, $M_{\text{benchmark}} = c_0 x_H$ so that using Propositions 1 and 2, it is straightforward to show that asset substitution occurs only in the low interim state. However, the high project quality is chosen in the first period. If $\lambda > \lambda_1$, $M_{\text{benchmark}} = c_2 x_H$. In this case, it follows from Proposition 1 that asset substitution occurs in both high and low interim states. By Proposition 2, the possibility of asset substitution in the high interim state eliminates debt overhang in the first period.

In summary, if the excess cost of equity is below a threshold, the optimal leverage is low enough that asset substitution and underinvestment are both eliminated. But, if the excess cost of equity is above a threshold, the optimal leverage is high so that it is not possible to completely eliminate asset substitution in the second period. The institution therefore exploits the benefits of asset substitution in the first period to eliminate the debt overhang problem in the first period. In the next section, we examine the role that a capital constraint plays in regulating the trade-off between asset substitution and debt overhang.
5 Regulation

We now proceed to examine the effects of regulation. By (5), the capital constraint is expressed as

\[ \frac{D_0}{F_0} \leq c \text{ at } t = 0 \text{ and } \frac{D_1}{F_1} \leq c \text{ at } t = 1, \]  

(11)

where \( D_t \) and \( F_t \), respectively, denote the market values of the institution’s debt and assets at \( t \). If \( \frac{D_1}{F_1} > c \), the regulator takes control and closely monitors the institution to ensure that there is no asset substitution in period 2.

5.1 Asset Substitution

The analysis of shareholders’ asset substitution strategy in the presence of a capital constraint is more subtle because the constraint at date 1 depends on the market values of the institution’s debt and assets. However, these market values are determined in equilibrium along with the institution’s asset substitution strategy. More precisely, at \( t = 1 \), the institution’s asset substitution decision \((r, z)\) is unobservable. Consequently, in order to value the institution’s debt, the capital market forms a conjecture \((\hat{r}, \hat{z})\) about \((r, z)\). Given the date \( t = 1 \) signal \( y \), the capital market values the institution’s debt at

\[ D_1(X, (\hat{r}, \hat{z})) = E[\min\{M, \bar{V}\} | X, (\hat{r}, \hat{z})] \]

\[ = \left( \frac{1}{2} - \hat{r} \right) \min\{M, (1 + \hat{z})X\} + \left( \frac{1}{2} + \hat{r} \right) \min\{M, (1 - \hat{z})X\}. \]  

(12)

Similarly, at date \( t = 1 \), the market value of equity is

\[ E_1(X, (\hat{r}, \hat{z})) = E[\max\{\bar{V}(X, (\hat{r}, \hat{z})) - M, 0\}] \]

\[ = \left( \frac{1}{2} - \hat{r} \right) \max\{(1 + \hat{z})X - M, 0\} + \left( \frac{1}{2} + \hat{r} \right) \max\{(1 - \hat{z})X - M, 0\} \]

\[ \frac{1}{\sqrt{1 + X}}. \]  

(13)

These date \( t = 1 \) market prices along with the prudential constraint determine whether transfer of control occurs. Given the continuation or control transfer outcome, the institution chooses \((r, z) \in \{(0, 0), (r_H, z_H)\}\) in period 2. If transfer of control occurs at date \( t = 1 \), the regulator ensures that the ex post efficient continuation strategy, i.e., \((r, z) = (0, 0)\) in period 2 is chosen.
so that the payoff at date 2 is $X$. If transfer of control does not occur at date $t = 1$, then shareholders could choose whether or not to engage in asset substitution. In a rational expectations equilibrium, the market’s conjecture regarding the chosen asset substitution strategy is correct. In other words, given $D_1(X, (\hat{r}, \hat{z}))$ and $F_1(X, (\hat{r}, \hat{z}))$ and the prudential constraint $c$, the institution’s optimal asset substitution strategy $(r, z)$ is indeed $(\hat{r}, \hat{z})$. The following Proposition characterizes the optimal continuation/transfer of control and asset substitution decisions given the debt face value $M$ and constraint $c$.

**Proposition 4 (Asset Substitution)** Shareholders choose asset substitution if and only if the capital constraint is less than a threshold and the debt level lies in an intermediate interval. That is, asset substitution is chosen if and only if $M \in [c_0X, T(c)X]$ and $c_0 < T(c)$, where $c_0$ is defined in (8) and

$$T(c) \equiv \frac{c}{\sqrt{1+\lambda} - c(\sqrt{1+\lambda} - 1)}.$$  \hspace{1cm} (14)

For $M < c_0X$, shareholders choose no asset substitution voluntarily. For $M > T(c)X$, no asset substitution is chosen because the capital constraint is violated and transfer of control occurs.

As in the benchmark regime, when the debt level, $M$, is below a threshold relative to $X$, shareholders voluntarily do not engage in asset substitution because the level of debt is low. As $M$ increases above the threshold, asset substitution becomes attractive to shareholders. However, unlike the benchmark regime, transfer of control occurs if $M$ is sufficiently large relative to $X$, and such transfer of control prevents asset substitution in period 2. Consequently, asset substitution only occurs (if at all) for intermediate values of $M$ relative to $X$. Note, however, that if the capital constraint is tighter than a threshold (i.e., $T(c) < c_0$), asset substitution is completely eliminated either (i) because transfer of control occurs or (ii) because shareholders retain control and voluntarily do not choose asset substitution in period 2. As $\frac{1}{2} - r_H$ (the probability of a good outcome given asset substitution) and/or $z_H$ (the spread of outcomes resulting from asset substitution) increases, $c_0$ decreases. Therefore as shareholders find asset substitution more enticing in period 2, an even tighter constraint is necessary to eliminate asset substitution.
5.2 Project Quality

In period 1, given the debt face value $M$, shareholders choose the project quality $q$ anticipating the transfer of control/continuation and asset substitution decisions described in Proposition 4.

**Proposition 5 (Project Quality)** Shareholders’ choice of project quality is as follows:

(i) For $k \leq k^*$:

- If $T(c) > c_2$, $q_L$ is chosen if and only if $M > c_2x_H$.
- If $T(c) \in [c_1, c_2]$, $q_L$ is chosen if and only if $M > T(c)x_H$.
- If $T(c) < c_1$, $q_L$ is chosen if and only if $M > c_1x_H$.

(ii) For $k > k^*$: $q_L$ is chosen if and only if $M > c_1x_H$.

In the above, $c_1, c_2$ and $k^*$ are as defined in (9), and $T(c)$ is defined in (14).

Analogous to Proposition 2, shareholders underinvest in project quality when leverage is relatively high. Proposition 5 shows that the incidence of debt overhang explicitly depends on $c$, the level of the capital constraint.

For low values of $c$ ($T(c) < c_1$), the capital constraint is relatively tight so that the institution is very likely to exceed it. A high likelihood of transfer of control implies that the incidence of asset substitution is very low. Not surprisingly, this scenario boils down to one in which asset substitution is exogenously ruled out so that shareholders choose a lower project quality (that is, choose $q = 0$) if and only if $M > c_1x_H$.

For high values of $c$ ($T(c) > c_2$), the constraint is relatively loose so that transfer of control is highly unlikely. We, therefore, recover the same result obtained in the benchmark regime with no constraint: shareholders choose a lower project quality if and only if $M > c_2x_H$.

For intermediate values of $c$ ($T(c) \in [c_1, c_2]$), the threshold $T(c)x_H$ of the face value of debt triggering a low project quality increases in $c$. Recall from Proposition 4 that, the smaller $c$ is, the higher the likelihood of transfer of control. Proposition 5 implies that, the smaller $c$ is, the higher the likelihood of choosing the low quality project $q = 0$, that is, the debt overhang problem is more severe. Taken together, these two propositions imply a positive relationship between transfer of control and the debt overhang problem. To understand this result, note that transfer of control
in the regulation regime shuts down asset substitution and such transfer of control is more likely the higher the leverage of the bank. But as we discussed earlier, this is precisely when the option value of asset substitution is greater for the high state than for the low state. Consequently, shutting down asset substitution via a change in control in the regulation regime has a significant negative impact on the project quality choice in the first period. Furthermore, as shareholders find asset substitution more attractive in period 2, i.e., as $\frac{1}{2} - r_H$ and/or $z_H$ increases, both $c_2$ and $k^*$ increase. Put differently, as the \textit{ex post} rents from asset substitution increase, shareholders find asset substitution in the high state even more valuable so that the positive relationship between transfer of control and the debt overhang problem becomes more pervasive as it applies to a larger set of values of the ceiling $c$ and the marginal cost $k$. The following corollary makes this precise.

\textbf{Corollary 2 (Asset Substitution and Debt Overhang)} As $r_H$ decreases and/or $z_H$ increases, (i) the range of debt face values for which asset substitution occurs increases for each value of the intermediate signal; (ii) for given $k$, the range of debt face values for which the low project quality is chosen shrinks; (iii) the threshold $k^*$ in Proposition 5 increases.

Figure 4 illustrates the corollary using the same numerical example introduced earlier in Figure 3. We set the value of the constraint, $c = 0.6$. For the chosen parameter values, when $r_H$ declines from $\frac{1}{10}$ to $\frac{1}{16}$, asset substitution incentives increase. For $r_H = \frac{1}{16}$, then asset substitution occurs for $M \in [0.40X, 0.58X]$. For $r_H = \frac{1}{16}$, asset substitution occurs over a larger intermediate range of $M \in [0.30X, 0.58X]$. For $M < 0.30X$, no asset substitution occurs because the debt face value
is relatively low so that shareholders have no incentives to asset substitute regardless of the value of $X$. For $M > 0.58X$, the leverage is so high that the prudential solvency constraint is violated. Control transfers to the regulator and asset substitution is shut down. The bottom half of the figure shows that the decline in $r_H$ from $\frac{1}{10}$ to $\frac{1}{16}$ shrinks the range of debt face values for which the debt overhang problem occurs.

The discussion above along with the intuition for Proposition 4 suggests that, while transfer of control at date $t = 1$ mitigates inefficiencies created by asset substitution in period 2, it exacerbates inefficiencies arising from debt overhang due to which the likelihood of choosing low quality project in period 1 increases. As we discuss shortly, the optimal choice of the capital constraint balances the trade-off between these two sources of inefficiencies while also incorporating the fact that equity capital is costlier than debt capital.

5.3 Optimal Capital Structure and Capital Constraint

For a given constraint $c$, the institution’s original shareholders choose its capital structure to maximize their value subject to the constraint (11). The shareholders’ payoffs are affected by the potential transfer of control at $t = 1$. The following proposition characterizes the optimal capital structure given a constraint $c$.

**Proposition 6 (Optimal Capital Structure for Given Constraint)** For a given level $c$ of the capital constraint, there exist thresholds, $\lambda_1$ and $\lambda_2$, of the excess cost of equity with $\lambda_1 < \lambda_2$ such that the institution’s optimal choice, $M^*$, of the debt level is as follows:

<table>
<thead>
<tr>
<th>$T(c)$</th>
<th>$\lambda &lt; \lambda_1$</th>
<th>$\lambda &gt; \lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_2$</td>
<td>$M^* = c_0 x_H$</td>
<td>$M^* = c_2 x_H$</td>
</tr>
<tr>
<td>$[c_1, c_2]$</td>
<td>$\lambda &lt; \lambda_2$</td>
<td>$\lambda &gt; \lambda_2$</td>
</tr>
<tr>
<td></td>
<td>$M^* = c_0 x_H$</td>
<td>$M^* = T(c) x_H$</td>
</tr>
<tr>
<td>$&lt; c_1$</td>
<td>$M^* = c_1 x_H$</td>
<td></td>
</tr>
</tbody>
</table>

where $c_0$, $c_1$, $c_2$ and $T(c)$ are defined in (8), (9) and (14).

Unlike the benchmark regime, Proposition 6 shows that, in addition to the excess cost of equity, $\lambda$, the optimal capital structure now depends on the capital constraint $c$. As shown by
Propositions 4 and 5, the constraint determines the project quality and asset substitution choices in periods 1 and 2. The capital structure decision rationally anticipates the ex post project quality and asset substitution choices that, in turn, depend on the capital structure and prudential constraint. For high values of c \((T(c) > c_2)\), the constraint is relatively loose so that transfer of control is highly unlikely. We, therefore, recover the same result obtained in the benchmark regime with no constraint (see Proposition 3). For low values of c \((T(c) < c_1)\), the capital constraint is relatively tight so that the institution is very likely to exceed it. A high likelihood of transfer of control implies that the incidence of asset substitution is very low. From Proposition (5), we know that shareholders underinvest if and only if \(M > c_1 x_H\). Therefore, in order to maximize their expected payoffs, the original shareholders choose the highest value of \(M\) that minimizes the underinvestment inefficiency, i.e., \(M^* = c_1 x_H\).

For intermediate values of c \((T(c) \in [c_1, c_2])\), when \(k \leq k^*\), \(c_0 \leq T(c)\) implies that the institution’s leverage is higher when the cost of equity is relatively high (i.e., \(\lambda > \lambda_2\)) as expected. More interestingly, when \(\lambda < \lambda_2\), \(M^* = c_0 x_H\) so that using Propositions 4 and 5, it is straightforward to show that there are no asset substitution and underinvestment inefficiencies. Conversely, when \(\lambda > \lambda_2\), it is relatively straightforward to show that the optimal face value of debt \(M^* = T(c) x_H\) induces asset substitution in the second period but the high project quality is chosen in the first period. The intuition for these results are similar to that in the benchmark regime. If the excess cost of equity is below a threshold, the optimal leverage is low enough that asset substitution and underinvestment inefficiencies are non-existent. But, if the excess cost of equity is above a threshold inducing the institution to choose a high leverage, then it is not possible to completely eliminate asset substitution in the second period. In this case, the institution exploits the benefits of ex post asset substitution to eliminate the ex ante debt overhang problem. In the next section, we examine the role that a capital constraint plays in regulating the trade-off between asset substitution and debt overhang.

We now turn to the derivation of the capital constraint that maximizes the total ex ante value of the institution rationally anticipating the institution’s capital structure, project quality, and asset substitution decisions. A high value of c (a loose constraint) aggravates the asset substitution problem in period 2 while a low value of c (a tight constraint) reduces incentives to invest in higher project quality in period 1. Further, low values of c imply that the institution must tilt its capital
structure more towards costlier equity capital instead of debt capital.

To illustrate the above trade-offs, Figure 5 uses the same numerical example illustrated in Figure 4 except that we now fix \( r_H \) at \( \frac{1}{10} \) and change \( c \) from 0.5 to 0.6. The top half of Figure 5 shows that as the prudential constraint \( c \) increases from 0.5 to 0.6 (i.e., the constraint becomes looser), the range of debt face values for which asset substitution occurs expands. The bottom half of Figure 5 shows that such an increase in \( c \) shrinks the range of values of \( M \) for which the debt overhang problem occurs. The next result explicitly characterizes the optimal capital constraint.

**Proposition 7 (Optimal Capital Constraint)**  The capital constraint that maximizes the total value of the institution is given by

\[
c^* = \frac{1}{1 + \frac{k \sqrt{T + X}}{x_H} - k(1 + \lambda)}.
\]

As discussed above, the capital constraint, \( c^* \), optimally balances the distortions created by asset substitution and debt overhang. From Propositions 4 and 5, we can show that neither inefficiency is completely eliminated in general, that is, debt overhang and asset substitution both occur at the optimum. This is because the optimal constraint incorporates the fact that allowing for some asset substitution in the second period alleviates the debt overhang problem in the first period.

Proposition 7 also shows that the optimal capital constraint becomes tighter as the excess cost of equity \( \lambda \) or the marginal cost of investment in project quality \( k \) increase. As \( \lambda \) increases, the institution’s incentives to use debt financing increase so that asset substitution and debt overhang inefficiencies both become more likely. Nevertheless, it turns out that the asset substitution problem
is relatively more pernicious than the debt overhang problem. Consequently, the capital constraint becomes tighter to mitigate asset substitution at the expense of potentially reducing incentives to invest in higher project quality. As \( k \) increases, the debt overhang problem becomes less severe because the NPV of the project decreases. The optimal capital constraint, therefore, again becomes tighter to mitigate asset substitution.

Our analysis and results suggest that the key trade-off between asset substitution and debt overhang, and the role that regulation plays in mediating the distortions arising from them, are particularly pronounced at high leverage levels where both problems are significant, and regulation plays a role. As we mentioned in Section 3.2, financial institutions are characterized by much higher leverage levels (on average) than non-financial firms. Indeed, Gropp and Heider (2010) document that the average leverage ratio of banks is approximately 90%, while that of non-financial firms is only around 25%. In the context of our model, financial institutions have greater effective costs of equity capital \( \lambda \) that induces them to choose higher leverage levels, which is consistent with the empirical findings and discussion in Gropp and Heider (2010). Consequently, even though asset substitution and debt overhang are also relevant for non-financial firms, our results are especially pertinent to financial institutions. We further discuss the relevance of our results in the context of financial and non-financial firms in Section 6.

6 Conclusions

In the aftermath of the 2007-2009 financial crisis, the merits and demerits of prudential regulation are being actively debated by academics, practitioners, and regulators. Our study contributes to the debate by showing how prudential regulation interacts with the agency conflicts between a financial institution’s shareholders and debt holders. Relative to a benchmark regime with no capital constraint, regulation based on market prices could alleviate the inefficiencies arising from asset substitution, but exacerbate those due to debt overhang. The subtle, but important, opposing effects of regulation on asset substitution and debt overhang inefficiencies are especially pronounced at high leverage levels that are typical of financial institutions.

The prudential capital constraint that maximizes the ex ante total value of the institution
balance the conflicts between shareholders and debt holders while also incorporating the fact that equity capital is costlier than debt capital. The optimal capital constraint declines with the marginal cost of investment in higher project quality and the excess cost of equity capital relative to debt capital. Our results suggest that a uniform capital constraint across institutions could be suboptimal. Overly strict capital requirements could exacerbate underinvestment, while mitigating asset substitution, which sounds a note of caution given the recent proposals to require both mark–to–market accounting and stricter capital requirements in the Basel III accords.

To sharpen the analysis and to highlight the main results in the paper, we developed a two-period binomial model with binary actions. However, we believe that the central trade-off between debt overhang and asset substitution would generalize to a setting with multiple states and multiple actions even though the analysis would be much more complicated. Even in a general setting, the debt overhang and asset substitution problems are either both absent or insignificant at very low leverage levels. At moderate leverage levels, asset substitution is present, but is not severe in that it occurs only in “bad” states. However, at these moderate leverage levels, the debt overhang problem is also not severe so that high quality projects are chosen anyway. Further, shutting down the possibility of asset substitution in bad states has only a minor impact from an \textit{ex ante} standpoint because payoffs in these states are low to begin with. Consequently, shutting down asset substitution would have only a minor impact on the project quality choice. But, at higher leverage levels, which are more typical of financial institutions and where prudential regulation is relevant, the asset substitution problem is more severe in that asset substitution also occurs in “good” states. Further, at these leverage levels, the expected payoff from asset substitution is much higher in the good states because the corresponding call option is deep out of the money in bad states. Consequently, shutting down asset substitution has a much bigger negative impact on the expected payoffs in the good states than in the bad states. Since payoffs are higher in the good states, this, in turn, has a significant negative impact on the \textit{ex ante} project quality choice.

As discussed above, the key trade-off between asset substitution and debt overhang is particularly pronounced at high leverage levels when both problems are severe, and asset substitution is pervasive in that it occurs in both “good” and “bad” states. The trade-off we identify is especially relevant in the context of the recent financial crisis. Indeed, one of the primary causes of the financial crisis was risky subprime mortgage lending by banks during a period when the economy
was booming and credit was cheap. Subprime mortgage lending could be more generally viewed as asset substitution that occurred in “good” states. Our study sheds light on the interactions between pervasive asset substitution and debt overhang inefficiencies and the role that prudential capital regulation based on market values plays in balancing the trade-off between these two inefficiencies.

Appendix

Proof of Proposition 1

If the shareholders choose \((r_H, z_H)\), it follows from (6) that their value is

\[
\frac{1}{2} - r_H \max\{(1 + z_H)X - M, 0\} + \frac{1}{2} + r_H \max\{(1 - z_H)X - M, 0\}. \tag{16}
\]

However, if they choose \((0, 0)\), it follows from (7) that their value is

\[
\max\{X - M, 0\}. \tag{17}
\]

Using expressions (16) and (17), the following table summarizes shareholders’ expected payoff from asset substitution (“AS”) and that from no asset substitution.

For example, for \(M \in [(1 - z_H)X, X]\), with probability \(\frac{1}{2} - r_H\), AS will produce \((1 + z_H)X\), which is larger than \(M\), so shareholders, as a residual claimant, will get \((1 + z_H)X - M\), and with probability \(\frac{1}{2} + r_H\), AS will produce \((1 - z_H)X\), which is smaller than \(M\), so shareholders will get nothing. Therefore, shareholders’ expected payoff from AS is \((\frac{1}{2} - r_H)[(1 + z_H)X - M]\). In contrast, no AS will always produce \(X\), which is larger than \(M\), so shareholders will get \(X - M\). Comparing the expected payoff from AS with that from no AS, \((\frac{1}{2} - r_H)[(1 + z_H)X - M]\) versus \(X - M\), yields the decision rule: AS if and only if \(M > c_0X\) for \(M \in [(1 - z_H)X, X]\).
In the rest of the paper, we make a tie-breaker assumption that the shareholders will choose no AS when $M > (1 + z_H)X$.

The above table implies the following:

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>payoff from decision</th>
<th>payoff from no AS</th>
<th>decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; (1 - z_H)X$</td>
<td>$(\frac{1}{2} - r_H)[(1 + z_H)X - M] + (\frac{1}{2} + r_H)[(1 - z_H)X - M]$</td>
<td>$X - M$</td>
<td>no AS</td>
</tr>
</tbody>
</table>
| $M \in [(1 - z_H)X, X]$ | $(\frac{1}{2} - r_H)[(1 + z_H)X - M]$ | $X - M$ | no AS if $M < c_0X$ | AS if $M > c_0X$
| $M \in [X, (1 + z_H)X]$ | $(\frac{1}{2} - r_H)[(1 + z_H)X - M]$ | 0 | AS |
| $M > (1 + z_H)X$ | 0 | 0 | no AS or AS |

**Proof of Proposition 2**

The shareholders’ expected payoff from $q$ at date $t = 0$ is their expected payoff from asset substitution decision (given in Table 2 in the proof of Proposition 1) minus the cost of investment in $q$. The shareholders’ expected payoffs from choosing $q$ are summarized below for all the feasible values of $M$.

For example, for $M \in [c_0x_H, (1 + z_H)x_H]$, we know from Table 2 that shareholders will engage in asset substitution both when $X = x_H$ and when $X = x_L$. If $X = x_H$ (which occurs with probability $q$), Table 2 tells us that the payoff is $(\frac{1}{2} - r_H)[(1 + z_H)x_H - M]$; if $X = x_L$ (which occurs with probability $1 - q$), Table 2 tells us that the payoff is 0. Therefore, shareholders’ expected payoff from asset substitution is $q(\frac{1}{2} - r_H)[(1 + z_H)x_H - M]$. Discounting this payoff to its present value at Date 0 by the cost of equity of $1 + \lambda$ and subtracting the cost of quality investment of $kq$ yields the shareholders’ expected payoff from $q$ for this range of $M$: $-kq + \frac{1}{1 + \lambda}q(\frac{1}{2} - r_H)[(1 + z_H)x_H - M]$. 

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<table>
<thead>
<tr>
<th>range of $M$</th>
<th>asset substitution decision</th>
<th>shareholders’ expected payoff from $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0x_L$</td>
<td>(0, 0) if $X = x_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[x_H - M]$</td>
</tr>
<tr>
<td></td>
<td>(0, 0) if $X = x_L$</td>
<td>$+(1-q)[x_L - M]$</td>
</tr>
<tr>
<td>$M \in [c_0x_L, (1+z_H)x_L]$</td>
<td>(0, 0) if $X = x_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[x_H - M]$</td>
</tr>
<tr>
<td></td>
<td>$(r_H, z_H)$ if $X = x_L$</td>
<td>$+(1-q)(r_H - x_H)(1+z_H)x_L - M]$</td>
</tr>
<tr>
<td>$M \in [(1+z_H)x_L, c_0x_H]$</td>
<td>(0, 0) if $X = x_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[x_H - M]$</td>
</tr>
<tr>
<td></td>
<td>$(r_H, z_H)$ if $X = x_L$</td>
<td>$+(1-q)(r_H - x_H)(1+z_H)x_L - M]$</td>
</tr>
<tr>
<td>$M \in [c_0x_H, (1+z_H)x_H]$</td>
<td>$(r_H, z_H)$ if $X = x_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[x_H - M]$</td>
</tr>
<tr>
<td></td>
<td>$(r_H, z_H)$ if $X = x_L$</td>
<td>$+(1-q)(r_H - x_H)(1+z_H)x_L - M]$</td>
</tr>
</tbody>
</table>

Using the above table, we can investigate the shareholders’ expected payoff from $q_H$ and that from $q_L$ and derive the following decision rules:

Case 1: $M < c_0x_L$: Shareholders choose $q_H$ if and only if $k < \frac{1}{1+\lambda}(x_H - x_L)$, which is true by assumption in (4). Therefore, shareholders will choose $q_H$.

Case 2: $M \in [c_0x_L, (1+z_H)x_L]$: Shareholders choose $q_H$ if and only if $M < \frac{x_H - (1/z_H)(1+z_H)x_L - k(1+\lambda)}{1+r_H}$. But even for the highest possible value of $M$ in Case 2, $(1+z_H)x_L$, that inequality always holds as long as $k < x_H - (1+z_H)x_L$. Because of the assumption of $(1+z_H)x_L = (1-z_H)x_H$, $k < x_H - (1+z_H)x_L \iff k < z_H x_H$, which is satisfied because $k < k^*$. So $q_H$ will be chosen. In Case 3, shareholders choose $q_H$ if and only if $M < c_1x_H$. But even for the highest possible value of $M$ in Case 3, $c_0x_H$, that inequality always holds as long as $k < k^*$, which is true by the assumption for $k$ in this scenario (i). So $q_H$ will be chosen. In Case 4, shareholders choose $q_H$ if and only if $M < c_2x_H$, which is exactly stated in the statements of this proposition.

(i) $k < k^*$: Shareholders choose $q_H$ if and only if $M < c_2x_H$.

Proof: In Case 1, shareholders will always choose $q_H$. In Case 2, shareholders choose $q_H$ if and only if $M < \frac{x_H - (1/z_H)(1+z_H)x_L - k(1+\lambda)}{1+r_H}$. But even for the highest possible value of $M$ in Case 2, $(1+z_H)x_L$, that inequality always holds as long as $k < x_H - (1+z_H)x_L$. Because of the assumption of $(1+z_H)x_L = (1-z_H)x_H$, $k < x_H - (1+z_H)x_L \iff k < z_H x_H$, which is satisfied because $k < k^*$. So $q_H$ will be chosen. In Case 3, shareholders choose $q_H$ if and only if $M < c_1x_H$. But even for the highest possible value of $M$ in Case 3, $c_0x_H$, that inequality always holds as long as $k < k^*$, which is true by the assumption for $k$ in this scenario (i). So $q_H$ will be chosen. In Case 4, shareholders choose $q_H$ if and only if $M < c_2x_H$, which is exactly stated in the statements of this proposition.

(ii) $k \in [k^*, (x_H - (1+z_H)x_L)/(1+\lambda)]$: shareholders choose $q_H$ if and only if $M < c_1x_H$. (The proof is analogous to that in case (i) and so is omitted.)

(iii) $k > (x_H - (1+z_H)x_L)/(1+\lambda)$: This case is infeasible by the assumption in (4).

We summarize the shareholders’ expected payoff from the optimal choice of $q$ in the following:
Proof of Proposition 3

We use (10) to derive the shareholders’ expected payoff at the time when they make capital structure decision. This payoff is the shareholders’ expected payoff from quality decision (given in Table 3 in the proof of Proposition 2) minus $E_0$, the shareholders’ equity investment, which equals $A_0 - D_0$. Therefore, in the following, we first derive $D_0$, the equilibrium debt price at Date 0, and then substitute this price into the shareholders’ expected payoff, and finally derive the shareholders’ optimal choice of $M$.

(i) The case where $k < k^*$:

Under the benchmark regime, taking into consideration of the optimal choices of $(r, z)$ and $q$, we first derive $D_0$, which is the debt holders’ expectation of their payoffs. Table 4 summarizes various values of $D_0$ for different ranges of $M$.

For example, for $M \in [(1 + z_H)x_L, c_0x_H]$, we know from Table 3 that shareholders will choose $q_H$ and therefore $D_0 = q_H E[M, X(q_H, (0, 0))] + (1 - q_H) E[M, X(q_H, (0, 0))]$. Because $X(q_H, (0, 0)) = x_H > M$, debt holders expect to receive $M$; because $X(q_H, (r_H, z_H)) = x_L < M$, debt holders expect to receive $x_L$. Therefore, $D_0 = q_H M + (1 - q_H)x_L$. 

---

Table 3 (the case of $k < k^*$)

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0x_L$</td>
<td>$q_H$</td>
<td>$-kq_H + \frac{1}{1+\lambda} {q_Hx_H + (1 - q_H)x_L - M}$</td>
</tr>
<tr>
<td>$M \in [c_0x_L, (1 + z_H)x_L]$</td>
<td>$q_H$</td>
<td>$-kq_H + \frac{1}{1+\lambda} {q_Hx_H + (1 - q_H)(\frac{1}{2} - r_H)(1 + z_H)x_L - [q_H + (1 - q_H)(\frac{1}{2} - r_H)]M}$</td>
</tr>
<tr>
<td>$M \in [(1 + z_H)x_L, c_0x_H]$</td>
<td>$q_H$</td>
<td>$-kq_H + \frac{1}{1+\lambda} q_H[x_H - M]$</td>
</tr>
<tr>
<td>$M \in [c_0x_H, c_2x_H]$</td>
<td>$q_H$</td>
<td>$-kq_H + \frac{1}{1+\lambda} q_H(\frac{1}{2} - r_H)[(1 + z_H)x_H - M]$</td>
</tr>
<tr>
<td>$M \in [c_2x_H, (1 + z_H)x_H]$</td>
<td>$q_L$</td>
<td>0</td>
</tr>
</tbody>
</table>

The payoff for $k > k^*$ is similar and so is omitted.
Substituting \(D_0\) in Table 4 into the shareholders' expected payoff, which is the shareholders' expected payoff from quality decision (given in Table 3 in the proof of Proposition 2) minus \(E_0 = A_0 - D_0\), yields the following:

<table>
<thead>
<tr>
<th>range of (M)</th>
<th>Shareholders' expected payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M \leq c_0 x_L)</td>
<td>(-A_0 - kq_H + \frac{1}{1+\lambda}[q_H x_H + (1 - q_H)x_L] + \frac{\lambda}{1+\lambda} M)</td>
</tr>
<tr>
<td>(M \in [c_0 x_L, (1 + z_H)x_L])</td>
<td>(-A_0 - kq_H + \frac{1}{1+\lambda}[q_H x_H + (1 - q_H)(\frac{1}{2} - r_H)(1 + z_H)x_L] + (1 - q_H)(\frac{1}{2} + r_H)(1 - z_H)x_L + \frac{\lambda}{1+\lambda} [q_H + (1 - q_H)(\frac{1}{2} - r_H)] M)</td>
</tr>
<tr>
<td>(M \in [(1 + z_H)x_L, c_0 x_H])</td>
<td>(-A_0 - kq_H + \frac{1}{1+\lambda} q_H x_H + (1 - q_H)x_L + \frac{\lambda}{1+\lambda} q_H M)</td>
</tr>
<tr>
<td>(M \in [c_0 x_H, c_2 x_H])</td>
<td>(-A_0 - kq_H + \frac{1}{1+\lambda} q_H(\frac{1}{2} - r_H)(1 + z_H)x_H + q_H(\frac{1}{2} + r_H)(1 - z_H)x_H + (1 - q_H)x_L + \frac{\lambda}{1+\lambda} q_H(\frac{1}{2} - r_H) M)</td>
</tr>
<tr>
<td>(M \in [c_2 x_H, (1 + z_H)x_H])</td>
<td>(-A_0 + x_L)</td>
</tr>
</tbody>
</table>

Because regional payoffs are increasing in \(M\), the optimal value of \(M\) for a given region is the upper bound of that region. Substituting the regional optimal \(M\) into the regional payoff function yields the regional maximal expected payoffs for shareholders at Date 0. For example, for the region of \(M \leq c_0 x_L\), the shareholders' expected payoff is increasing in \(M\) and so the optimal value of \(M\) for this particular region is \(c_0 x_L\). Inserting \(M = c_0 x_L\) into the payoff function for this region yields \(-A_0 - kq_H + \frac{1}{1+\lambda}[q_H x_H + (1 - q_H)x_L] + \frac{\lambda}{1+\lambda} c_0 x_L\). Similar analyses apply to other regions.

Note that the payoff in the third region is always larger than that in the second region and that in the first region, and therefore we combined those three regions. In addition, the last region is never optimal because the payoff of \(-A_0 + x_L\) is negative by the assumption that a project with a low quality and asset substitution is a negative NPV project. Therefore, the five regions above boil down to the following two regions:
Table 5

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>choice of $M$</th>
<th>Shareholders’ ex ante payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M \leq c_0 x_H$</td>
<td>$c_0 x_H$</td>
<td>$\pi^C_I = -A_0 - k q_H + \frac{1}{1 + \lambda} {q_H x_H + (1 - q_H) x_L}$ $+ \frac{\lambda}{1 + \lambda} {q_H (1 - \frac{t_H}{1 + t_H}) z_H x_H + (1 - q_H) x_L}$</td>
</tr>
<tr>
<td>$M \in [c_0 x_H, c_2 x_H]$</td>
<td>$c_2 x_H$</td>
<td>$\pi^C_{II} = -A_0 - k q_H$ $+ \frac{1}{1 + \lambda} {q_H (1 - 2 r_H z_H) x_H + (1 - q_H) x_L}$ $+ \frac{\lambda}{1 + \lambda} {q_H [(1 - 2 r_H z_H) x_H - k (1 + \lambda)] + (1 - q_H) x_L}$</td>
</tr>
</tbody>
</table>

Note that both $\pi^C_I$ and $\pi^C_{II}$ are decreasing in $\lambda$, and at $\lambda = 0$, $\pi^C_I > \pi^C_{II}$. Comparing those two payoffs demonstrates that shareholders prefer $\pi^C_I$ for low values of $\lambda$ and $\pi^C_{II}$ for high values of $\lambda$:

$$\pi^C_I > \pi^C_{II} \iff \lambda < \lambda_1 \equiv \frac{2 r_H z_H}{(k^* - k)/x_H}.$$ 

Therefore, the value of $\lambda$ dictates the choice of $M$. For example, when $\lambda < \lambda_1$, shareholders prefer $\pi^C_I$. To induce it, they set $M$ to be $c_0 x_H$.

(ii) The case where $k > k^*$: The analysis of this case is analogous to that of the preceding case and so is omitted.

**Proof of Proposition 4.** We first derive the market values of the institution’s assets and debt at Date 1 that depend on the market’s conjecture $(\hat{r}_H, \hat{z}_H)$ or $(\hat{r}_L, \hat{z}_L) \equiv (0, 0)$ of the Period 2 asset substitution decision. Given the markets’ conjecture $(\hat{r}_H, \hat{z}_H)$, by (13), the institution value is

$$F_1(\hat{r}_H, \hat{z}_H) = \left(\frac{1}{2} - \hat{r}_H\right)(1 + \hat{z}_H) + \left(\frac{1}{2} + \hat{r}_H\right)(1 - \hat{z}_H) = (1 - 2 \hat{r}_H \hat{z}_H) X,$$

and by (12), the debt value is

$$D_1(\hat{r}_H, \hat{z}_H) = \left(\frac{1}{2} - \hat{r}_H\right) \min\{M, (1 + \hat{z}_H) X\} + \left(\frac{1}{2} + \hat{r}_H\right) \min\{M, (1 - \hat{z}_H) X\}. $$

Given the markets’ conjecture $(\hat{r}_L, \hat{z}_L) \equiv (0, 0)$, by (13), the institution value is

$$F_1(\hat{r}_L, \hat{z}_L) = \left(\frac{1}{2} - \hat{r}_L\right)(1 + \hat{z}_L) + \left(\frac{1}{2} + \hat{r}_L\right)(1 - \hat{z}_L) = X$$

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and by (12), the debt value is

\[ D_1(\hat{r}_L, \hat{z}_L) = \left( \frac{1}{2} - \hat{r}_L \right) \min\{M, (1 + \hat{z}_L)X\} + \left( \frac{1}{2} + \hat{r}_L \right) \min\{M, (1 - \hat{z}_L)X\} = \min\{M, X\}. \]

The following table shows how the institution and debt values as well as the leverage ratio vary with \( M \):

<table>
<thead>
<tr>
<th>( \frac{M}{X} )</th>
<th>( D_1 = M )</th>
<th>( D_1 = (\frac{1}{2} - r_H)M + (\frac{1}{2} + r_H)(1 - z_H)X )</th>
<th>( D_1 = (1 - 2r_Hz_H)X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>for ( \frac{M}{X} &lt; 1 - z_H ),</td>
<td>( \frac{D_1}{F_1} &gt; c \Leftrightarrow \frac{M}{X} &gt; c(1 - 2r_Hz_H) )</td>
<td>( \frac{D_1}{F_1} &gt; c \Leftrightarrow \frac{M}{X} &gt; \frac{c(1 - 2r_Hz_H) - (\frac{1}{2} + r_H)(1 - z_H)}{\frac{1}{2} - r_H} )</td>
<td>( \frac{D_1}{F_1} = 1 &gt; c )</td>
</tr>
<tr>
<td>for ( \frac{M}{X} \in [1 - z_H, 1 + z_H] ),</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>for ( \frac{M}{X} &gt; 1 + z_H ),</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The two tables above imply the following:

(a) When \( \frac{M}{X} > \frac{c(1 - 2r_Hz_H) - (\frac{1}{2} + r_H)(1 - z_H)}{\frac{1}{2} - r_H} \), control is transferred to debt holders regardless of whether the market’s conjecture is \((\hat{r}_H, \hat{z}_H)\) or \((\hat{r}_L, \hat{z}_L)\). So no asset substitution will occur. (b) When \( \frac{M}{X} \in [c, \frac{c(1 - 2r_Hz_H) - (\frac{1}{2} + r_H)(1 - z_H)}{\frac{1}{2} - r_H}] \), shareholders voluntarily choose no asset substitution if the market’s conjecture is \((\hat{r}_L, \hat{z}_L)\), and choose asset substitution if the market’s conjecture is \((\hat{r}_H, \hat{z}_H)\).

(c) When \( \frac{M}{X} \in [c_0, c] \), shareholders choose asset substitution regardless of whether the market’s conjecture is \((\hat{r}_H, \hat{z}_H)\) or \((\hat{r}_L, \hat{z}_L)\). (d) When \( \frac{M}{X} < c_0 \), shareholders choose no asset substitution regardless of whether the market’s conjecture is \((\hat{r}_H, \hat{z}_H)\) or \((\hat{r}_L, \hat{z}_L)\).

Therefore, we can derive the shareholders’ optimal asset substitution decision and their period 2 expectation of payoffs as follows:
Table 6

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>$\frac{M}{X} &lt; c_0$</th>
<th>$\frac{M}{X} \in [c_0, c]$</th>
<th>$\frac{M}{X} \in [c, 1]$</th>
<th>$\frac{M}{X} &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>asset substitution choice</td>
<td>$(r_L, z_L)$</td>
<td>$(r_H, z_H)$</td>
<td>$(r_L, z_L)$</td>
<td>$(r_L, z_L)$</td>
</tr>
<tr>
<td>payoff</td>
<td>$\frac{1}{1+\lambda}(X - M)$</td>
<td>$\frac{1}{1+\lambda}(\frac{1}{2} - r_H)[(1 + z_H)X - M]$</td>
<td>$\frac{1}{1+\lambda}(X - M)$</td>
<td>0</td>
</tr>
</tbody>
</table>

Proof of Proposition 5. From Table 6 in the proof of Proposition 4 we have the following cases to consider that are defined by subintervals of the range of possible values of $M$:

Table 7

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $M$</td>
<td>$c_0x_L$</td>
<td>$[c_0x_L, cx_L]$</td>
<td>$[cx_L, x_L]$</td>
<td>$[x_L, c_0x_H]$</td>
<td>$[c_0x_H, cx_H]$</td>
<td>$[cx_H, x_H]$</td>
<td>$x_H$</td>
</tr>
<tr>
<td>$(r, z)$ given $x_H$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
<td>$(r_H, z_H)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>$(r, z)$ given $x_L$</td>
<td>$(0, 0)$</td>
<td>$(r_H, z_H)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
</tr>
</tbody>
</table>

At the time of choosing $q$, the shareholders incur an expected cost of $kq$ and expect a period 2 payoff described in Table 6. Thus, we have the following expected payoffs from $q$ in the seven cases of Table 7:

Table 8

<table>
<thead>
<tr>
<th>Case</th>
<th>shareholders' payoff from $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-kq + \frac{1}{1+\lambda}q(x_H - M) + (1 - q)(x_L - M)$</td>
</tr>
<tr>
<td>2</td>
<td>$-kq + \frac{1}{1+\lambda}q(x_H - M) + (1 - q)(\frac{1}{2} - r_H)[(1 + z_H)x_L - M]$</td>
</tr>
<tr>
<td>3</td>
<td>$-kq + \frac{1}{1+\lambda}q(x_H - M) + (1 - q)(x_L - M)$</td>
</tr>
<tr>
<td>4</td>
<td>$-kq + \frac{1}{1+\lambda}q(x_H - M)$</td>
</tr>
<tr>
<td>5</td>
<td>$-kq + \frac{1}{1+\lambda}q(\frac{1}{2} - r_H)[(1 + z_H)x_H - M]$</td>
</tr>
<tr>
<td>6</td>
<td>$-kq + \frac{1}{1+\lambda}q(x_H - M)$</td>
</tr>
<tr>
<td>7</td>
<td>$-kq$</td>
</tr>
</tbody>
</table>

Now we derive the shareholders’ optimal $q$ for each case.

Cases 1 and 3: Shareholders choose $q_H$ iff $-k + \frac{1}{1+\lambda}(x_H - x_L) > 0$, which is true by our assumption that $k \leq \frac{1}{1+\lambda} \frac{3}{2} x_H z_H x_H$. Case 2: Shareholders choose $q_H$ iff $M < \frac{x_H - \frac{1}{2} - r_H)(1 + z_H)x_L - \frac{1}{1+\lambda}k}{\frac{1}{2} + r_H}$.
which is true because $M \leq cx_L < (1+z_H)x_L$. Case 4: Shareholders choose $q_H$ iff $M \frac{M}{x_H} < c_1$, which is true because $M \frac{M}{x_H} < c_0 < c_1$. Case 7: Because the payoff is nonpositive, shareholders choose $q_L \equiv 0$.

So the transition from $q_H$ to $q_L$ must occur in Case 5 or 6. In Case 5, shareholders choose $q_H$ iff $M \frac{M}{x_H} < c_2$, and in Case 6, shareholders choose $q_H$ iff $M \frac{M}{x_H} < c_1$. If $c \in [c_1, c_2]$, shareholders choose $q_H$ in Case 5 and $q_L$ in Case 6, so the transition point is $M \frac{M}{x_H} = c$, the boundary between Case 5 and Case 6. If $c > c_2$, shareholders choose $q_L$ in Case 6, so the transition point is $M \frac{M}{x_H} = c_2$, which falls in Case 5. If $c < c_1$, shareholders choose $q_H$ in Case 5, so the transition point is $M \frac{M}{x_H} = c_1$, which falls in Case 6.

The analysis of the case where $c < c_0$ is similar to analysis for the case where $c > c_0$, and we get the following: Shareholders choose $q_H$ iff $M \frac{M}{x_H} < c_1$.

To summarize the results, $q_L$ is chosen if and only if (i) $M \frac{M}{x_H} > c$ when $c \in [c_1, c_2]$, (ii) $M \frac{M}{x_H} > c_2$ when $c > c_2$, and (iii) $M \frac{M}{x_H} > c_1$ when $c < c_1$. In other words, $q_L$ will be chosen if and only if $M \frac{M}{x_H} > \max \{c_1, \min(c, c_2)\}$.

We summarize the shareholders’ expected payoff from the optimal choice of $q$ in the following, where payoffs in Scenarios A, C, and D are expressed in terms of payoffs in Scenario B:

<table>
<thead>
<tr>
<th>Table 9 Scenario A:</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(c) \leq c_0$</td>
<td>$q_H$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td>$M &lt; X_L$</td>
<td>$q_H$</td>
<td>$\pi_6$</td>
</tr>
<tr>
<td>$M \in [X_L, c_1 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_6$</td>
</tr>
<tr>
<td>$M \in [c_1 X_H, (1+z_H) X_H]$</td>
<td>$q_L$</td>
<td>$\pi_7$</td>
</tr>
</tbody>
</table>
Table 9 Scenario B: 

<table>
<thead>
<tr>
<th>$T(c) \in [c_0, c_1]$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0 X_L$</td>
<td>$q_H$</td>
<td>$\pi_1 \equiv \frac{1}{1+X_H} {q_H X_H + (1 - q_H) X_L - M} - k q_H$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, T(c) X_L]$</td>
<td>$q_H$</td>
<td>$\pi_2 \equiv \frac{1}{1+X_H} {q_H X_H + (1 - q_H) \left(\frac{1}{2} - r_H\right) (1 + z_H) X_L - [q_H + (1 - q_H) \left(\frac{1}{2} - r_H\right)] M} - k q_H$</td>
</tr>
<tr>
<td>$M \in [T(c) X_L, X_L]$</td>
<td>$q_H$</td>
<td>$\pi_3 \equiv \frac{1}{1+X_H} {q_H X_H + (1 - q_H) X_L - M} - k q_H$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_4 \equiv \frac{1}{1+X_H} q_H [X_H - M] - k q_H$</td>
</tr>
<tr>
<td>$M \in [c_0 X_H, T(c) X_H]$</td>
<td>$q_H$</td>
<td>$\pi_5 \equiv \frac{1}{1+X_H} q_H \left(\frac{1}{2} - r_H\right) [(1 + z_H) X_H - M] - k q_H$</td>
</tr>
<tr>
<td>$M \in [T(c) X_H, c_1 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_6 \equiv \frac{1}{1+X_H} q_H [X_H - M] - k q_H$</td>
</tr>
<tr>
<td>$M \in [c_1 X_H, (1 + z_H) X_H]$</td>
<td>$q_L$</td>
<td>$\pi_7 \equiv 0$</td>
</tr>
</tbody>
</table>

Table 9 Scenario C: 

<table>
<thead>
<tr>
<th>$T(c) \in [c_1, c_2]$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0 X_L$</td>
<td>$q_H$</td>
<td>$\pi_1$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, T(c) X_L]$</td>
<td>$q_H$</td>
<td>$\pi_2$</td>
</tr>
<tr>
<td>$M \in [T(c) X_L, X_L]$</td>
<td>$q_H$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_4$</td>
</tr>
<tr>
<td>$M \in [c_0 X_H, T(c) X_H]$</td>
<td>$q_H$</td>
<td>$\pi_5$</td>
</tr>
<tr>
<td>$M \in [T(c) X_H, (1 + z_H) X_H]$</td>
<td>$q_L$</td>
<td>$\pi_7$</td>
</tr>
</tbody>
</table>

Table 9 Scenario D: 

<table>
<thead>
<tr>
<th>$T(c) &gt; c_2$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0 X_L$</td>
<td>$q_H$</td>
<td>$\pi_1$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, T(c) X_L]$</td>
<td>$q_H$</td>
<td>$\pi_2$</td>
</tr>
<tr>
<td>$M \in [T(c) X_L, X_L]$</td>
<td>$q_H$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_4$</td>
</tr>
<tr>
<td>$M \in [c_0 X_H, c_2 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_5$</td>
</tr>
<tr>
<td>$M \in [c_2 X_H, (1 + z_H) X_H]$</td>
<td>$q_L$</td>
<td>$\pi_7$</td>
</tr>
</tbody>
</table>

Proof of Proposition 6

We derive the shareholders’ expected payoff at the time when they make capital structure decision. This payoff is the shareholders’ expected payoff from quality decision (given in Table 9 in
the proof of Proposition 5) minus $E_0$, the shareholders’ equity investment, which equals $A_0 - D_0$. Therefore, in the following, we first derive $D_0$, the equilibrium debt price at Date 0, and then substitute this price into the shareholders’ expected payoff, and finally derive the shareholders’ optimal choice of $M$.

Table 9 identifies four scenarios, A, B, C, and D. We first analyze Scenario B.

Scenario B: Taking into consideration of the optimal choices of $(r, z)$ and $q$, we first derive $D_0 = E[\min\{M, \tilde{X}\}]$:

<table>
<thead>
<tr>
<th>Table 10: Scenario B: $T(c) \in [c_0, c_1]$</th>
<th>$D_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $M$</td>
<td>$D_0$</td>
</tr>
<tr>
<td>$M &lt; c_0x_L$</td>
<td>$M$</td>
</tr>
<tr>
<td>$M \in [c_0x_L, T(c)x_L]$</td>
<td>$[q_H + (1 - q_H)(\frac{1}{2} - r_H)]M + (1 - q_H)(\frac{1}{2} + r_H)(1 - z_H)x_L$</td>
</tr>
<tr>
<td>$M \in [T(c)x_L, x_L]$</td>
<td>$M$</td>
</tr>
<tr>
<td>$M \in [x_L, c_0x_H]$</td>
<td>$q_HM + (1 - q_H)x_L$</td>
</tr>
<tr>
<td>$M \in [c_0x_H, T(c)x_H]$</td>
<td>$q_H(\frac{1}{2} - r_H)M + q_H(\frac{1}{2} + r_H)(1 - z_H)x_H + (1 - q_H)x_L$</td>
</tr>
<tr>
<td>$M \in [T(c)x_H, c_1x_H]$</td>
<td>$q_HM + (1 - q_H)x_L$</td>
</tr>
<tr>
<td>$M \in [c_1x_H, (1 + z_H)x_H]$</td>
<td>$x_L$</td>
</tr>
</tbody>
</table>

We now substitute the equilibrium values of $D_0$ in Table 10 into the expected payoff at the time when shareholders make capital structure decision, which is the shareholders’ expected payoff from quality decision (given in Table 9) minus $E_0 = A_0 - D_0$. We do it region by region. As it turns out, the regional payoff increases in $M$, and so the regional optimal $M$ is the upper bound of the region. Evaluating the regional payoff at the regional optimal $M$ yields the regional maximal payoffs in the following:
It is straightforward to show the following results:

\[ \pi_{B3} > \pi_{B1}; \pi_{B3} > \pi_{B2}; \pi_{B4} > \pi_{B3}; \pi_{B6} > \pi_{B4}; \pi_{B6} > \pi_{B5}; \pi_{B6} > \pi_{B7}. \]

Therefore, \( \pi_{B6} \) is the highest payoff in Scenario B. To induce \( \pi_{B6} \), by Table 11, shareholders must choose \( M = c_1 x_H \).

Scenario A: By similar reasoning process demonstrated in Scenario B, we can derive \( \pi_{A1} \) to \( \pi_{A3} \) for Scenario A, using Table 9 Scenario A, where \( \pi_{A1} \) to \( \pi_{A3} \) are the regional maximal payoffs for the three regions in Scenario A. It is straightforward to show the following results:

\[ \pi_{A1} = \pi_{B3}; \pi_{A2} = \pi_{B6}; \pi_{A3} = \pi_{B7}. \]
Therefore, $\pi_{A2} = \pi_{B6}$ is the highest payoff in Scenario A. To induce $\pi_{A2}$, shareholders must choose $M = c_1 x_H$.

Scenario C: By similar reasoning process demonstrated in Scenario B, we can derive $\pi_{C1}$ to $\pi_{C6}$ for Scenario C, using Table 9 Scenario C, where $\pi_{C1}$ to $\pi_{C6}$ are the regional maximal payoffs for the six regions in Scenario C. It is straightforward to show the following results:

$$\pi_{C1} = \pi_{B1}; \pi_{C2} = \pi_{B2}; \pi_{C3} = \pi_{B3}; \pi_{C4} = \pi_{B4}; \pi_{C5} = \pi_{B5}; \pi_{C6} = \pi_{B7}.$$ 

Therefore, $\pi_{C4}$ and $\pi_{C5}$ dominate the other payoffs. $\pi_{C4} > \pi_{C5}$ if and only if $\lambda < \lambda_3$. To induce $\pi_{C4}$, shareholders must choose $M = c_0 x_H$; to induce $\pi_{C5}$, shareholders must choose $M = T(c) x_H$.

Scenario D: By similar reasoning process demonstrated in Scenario B, we can derive $\pi_{D1}$ to $\pi_{D6}$ for Scenario D, using Table 10 Scenario D, where $\pi_{D1}$ to $\pi_{D6}$ are the regional maximal payoffs for the six regions in Scenario D. It is straightforward to show the following results:

$$\begin{align*}
\pi_{D1} &= \pi_{B1}; \pi_{D2} = \pi_{B2}; \pi_{D3} = \pi_{B3}; \pi_{D4} = \pi_{B4}; \\
\pi_{D5} &= -A_0 - kq_H + \frac{1}{1 + \lambda} \left( q_H (1 - 2r_H z_H) x_H + (1 - q_H) x_L \right) \\
&\quad + \frac{\lambda}{1 + \lambda} \left( q_H [(1 - 2r_H z_H) x_H - k (1 + \lambda)] + (1 - q_H) x_L \right); \\
\pi_{D6} &= \pi_{B7}.
\end{align*}$$

Therefore, $\pi_{D4}$ and $\pi_{D5}$ dominate the other payoffs. $\pi_{D4} > \pi_{D5}$ if and only if $\lambda < \lambda_1$. To induce $\pi_{D4}$, shareholders must choose $M = c_0 x_H$; to induce $\pi_{D5}$, shareholders must choose $M = c_0 x_H$.

**Proof of Proposition 7**

The proof of Proposition 6 shows that $\pi_{B6}$ is the highest payoff in Scenario B and that $\pi_{A2} = \pi_{B6}$ is the highest payoff in Scenario A.

It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario C. Specifically, region 6 in Scenario B does not exist in Scenario C.

It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario D. Specifically, not only region 6 in Scenario B does not exist in Scenario D, but also the payoff in region 5 in B is larger than the payoff in region 5 in D.

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Therefore, the highest payoff among all the four scenarios of A, B, C, and D is \( \pi_{A2} = \pi_{B6} \), and so the regulator will choose \( c \) to induce Scenarios A and B.

Recall that Scenario A will be viable when \( T(c) \leq c_0 \). Recall also that Scenario B will be viable when \( T(c) \in [c_0, c_1] \). Therefore, to induce Scenario A and/or B, it suffice for the regulator to set \( c \leq c_1 \). However, any further reduction of \( c \) below \( c_1 \) will constrain shareholders’ choice of \( M \) at date 0 and therefore damage their \textit{ex ante} welfare. This tension gives rise to the socially optimal constraint, \( T(c) = c_1 \Leftrightarrow c = \frac{1}{\frac{k\sqrt{1+x}}{1+q}+1} \).

References


the Literature,” Financial Markets, Institutions, and Instruments, 10(2): 41–84.

[27] Stein, Jeremy C., 2012, Monetary policy as financial stability regulation, Quarterly Journal of
Economics 127, 57–95.