First Passage Times of Two-Dimensional Brownian Motion and Correlated Defaults

Abstract: First passage times (FPTs) of two-dimensional Brownian motion have been used to study correlated defaults under structural models. Despite various attempts since 1960’s, analytical expression for the joint distribution of the FPTs is still not available. By solving a PDE problem using finite Fourier transform, we obtain an analytical characterization for the joint Laplace transform of FPTs. We develop an inversion algorithm to compute the probability distribution and utilize the results to study default correlations. We also point out a link between the joint Laplace transform and a bivariate exponential distribution which is absolute continuous and does not have memoryless property. Compared to existing methods, our approach is more accurate, efficient, and has potentially broader applications in credit risk modeling, e.g. on calculating the probability that two firms' default time are close to each other.