Dynamic bidding

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Abstract

Consider a second-price auction with costly bidding in which bidders with i.i.d. private values have multiple opportunities to bid. If bids are publicly observable, the resulting dynamic-bidding game generates greater expected total welfare than when bids are sealed or, if the seller commits to an optimal reserve, greater expected revenue. If the seller cannot commit to a bid-revelation policy, however, equilibrium outcomes are the same as if bids cannot be revealed.

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1   Introduction

Famed investor Warren Buffett doesn't like to buy companies at auction. As he explained at Berkshire Hathaway's 1997 Annual Meeting: "If there's an auction going on, we have no interest in talking about it ... There's other things to do with your time." The participation costs that keep Buffett away from the auction block have long been recognized in the auction theory literature as a significant barrier to bidding.[1] For instance, in a study of eBay coin auctions, Bajari and Hortacsu (2003) estimate that bidders faced participation cost of \$3.20, a significant amount given that expected revenue in these auctions ranged from \$40 to \$50.

This paper explores how to encourage bidder participation in the face of such bidding costs, to increase expected welfare and/or expected revenue in an otherwise standard model with costly bidding and i.i.d. private values, by allowing bidders to choose *when* as well as *whether* to bid in an auction.

Samuelson (1985) provides a helpful benchmark analysis. Bidders there must simultaneously decide whether to incur cost $c > 0$ to bid in an auction. In the unique symmetric threshold equilibrium, each bidder submits a bid iff his value exceeds an entry threshold e^S. Ex post welfare losses result from both too much entry (if multiple bidders have values above e^S, all of them incur cost c) and too little entry (if the highest value lies in (c, e^S), no one bids even though entry would have been efficient).

This paper extends Samuelson's model by allowing for multiple bidding rounds, with bids in each round publicly observable before the next round, characterizing the unique symmetric threshold equilibrium (Theorem 1) and the limit of this equilibrium as the

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[1]Samuelson (1985) observed that "competing firms must bear significant bid-preparation and documentation costs." Buffett's comment suggests that the opportunity cost of managerial attention may also be significant.

number of rounds goes to infinity (Theorem 2).

Several observations about welfare and revenue emerge from this analysis. For any given reserve price, both interim expected bidder surplus and expected total surplus are higher under "dynamic bidding" when there are multiple bidding rounds than when bids are sealed (Propositions 2-3). The reason, intuitively, is that having multiple opportunities to bid facilitates bidder coordination, helping low-value bidders stay out of auctions they are bound to lose and allowing high-value bidders to deter low-value bidders from competing. Such bidder coordination might seem like bad news for the seller but, in fact, expected revenue is higher under dynamic bidding than under sealed bidding when the seller can commit to an optimal reserve price (Proposition 1). Why? Since dynamic bidding makes the auction more attractive to bidders at any given reserve price, the seller can raise the reserve without losing sales. Indeed, by raising the reserve price, the seller can extract all the welfare gains associated with better bidder coordination in the form of greater expected revenue.

What if the seller lacks the power to commit to a bid-revelation policy? When the seller reveals bids strategically, all bidders are silent until the final round, so that the K-round game ultimately reduces to a standard sealed-bid auction (Theorem 3). The reason, intuitively, is that the seller will only reveal bids when doing so spurs additional bidding competition in subsequent rounds. Anticipating this, bidders whose bids the seller might like to reveal will wait until the last bidding opportunity.

The rest of the paper is organized as follows. The introduction continues with some discussion of related literature. Section 2 presents the model of dynamic bidding. Section 3 provides the main analysis, while Section 4 considers an extension in which the seller decides which bids to reveal. Section 5 offers concluding remarks.

Related literature. This paper fits into the literature following Samuelson (1985) on auctions with costly bidding, the novel feature here being that bidders have multiple opportunities to enter the auction. A key finding is that bidders with higher values enter the auction earlier in order to signal their strength and deter others from entering the auction later. As such, the paper is similar in spirit to the jump-bidding literature (e.g. Avery (1998) and Horner and Sahuguet (2007)) and to the diverse literature that explores other mechanisms for bidder-to-bidder signaling in auctions.[2]

Another large related strand of literature features sequential bidder arrivals, where early entry also can serve to deter later entry. See e.g. McAfee and McMillan (1987) and Bulow and Klemperer (2009). A crucial difference is that, in such sequential-entry models, bidders who arrive earlier to the auction are typically assumed to be ex ante identical to those who arrive later. By contrast, because bidders here *choose* when to enter the auction, earlier entrants' values are drawn from a higher distribution.

Several other reasons have been explored in the literature for why bidders can have an incentive to bid early or wait until the last minute to bid, including: *Common values*: When the good has unknown quality, revealing one's interest may convey a positive signal about quality, prompting others to bid more aggressively. This gives bidders an incentive to shroud their interest, including by waiting until the very last moment to bid (Bajari and Hortacsu (2003), McAdams (2013)). *Endogenous information acquisition*: Fishman (1988) and Hirschliefer and Png (1989) provide an alternative explanation of jump bidding, that such early bids can deter others from acquiring information during the auction. On the other hand, Rasmusen (2006) shows that early bidding may also

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[2] See e.g. Eso and Schummer (2004), where higher bribes signal higher values; McAdams and Schwarz (2007), where waiting until the deadline to bid signals a high value; and Daley, Schwarz, and Sonin (2012), where bidders can burn money and/or make costly investments.

provoke others to invest in information acquisition, providing a possible explanation of the commonly-observed flurry of last-minute bids on online-auction sites such as eBay. Still more potential reasons for late bidding on sites like eBay include *overlapping auctions* of substitute goods with different deadlines (Wang (2003), Zeithammer (2006)), *unsophisticated bidders* (Ockenfels and Roth (2006), Ariely and Simonsohn (2008)), and *random delays* in bid transmission (Ockenfels and Roth (2006)).

My focus here is on comparing equilibrium auction outcomes under sealed vs dynamic bidding, not on characterizing the optimal sales mechanism. Several papers in the dynamic mechanism-design literature have characterized optimal dynamic mechanisms in somewhat related settings. See e.g. Board and Skrzypacz (2010), Ely, Garrett, and Hinnosaar (2012), Gershkov and Moldavanu (2009), Hinnosaur (2011), and Pai and Vohra (2013). A common theme of this literature is that bidders arrive to the auction according to an exogenous process. However, when participation is costly and bidders observe private information about their values before deciding whether/when to participate, the bidder-arrival process is inherently endogenous. This makes characterizing the optimal mechanism potentially a very challenging problem.

The most closely related paper is Levin and Peck (2003) (hereafter "LP"). LP considers a game in which two firms[3] with i.i.d. entry costs have multiple opportunities to enter a new market. Each firm enjoys monopoly revenue R_m if it is the only one to enter or duopoly revenue R_d if both enter. When $R_d = 0$, LP's game can be interpreted as a second-price auction with zero reserve price, where bidders have known common value $v = R_m$ and i.i.d. entry costs. The basic structure of equilibrium entry is similar here and in LP, but the papers take different (and complementary) analytical approaches. For instance, whereas LP use a contraction argument to establish equilibrium uniqueness, I

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[3]LP provide an extension to $n > 2$ firms, under the assumption that duopoly revenue $R_d = 0$.

provide a direct proof and a simple algorithmic method to compute the equilibrium.

2   Model

Potential bidders $i = 1, ..., n$ observe i.i.d. private values v_i with continuous c.d.f. $F(\cdot)$, $p.d.f. f(\cdot)$, and full support on $[0, \overline{V}]$. The seller then holds a second-price auction with reserve price r, with the novel feature that there are $K \geq 2$ "bidding rounds" at which bidders simultaneously decide whether to incur cost $c > 0$ to submit a bid. (For simplicity, I assume that each bidder submits at most one bid.)

Bids made in rounds $k = 1, ..., K - 1$ have the potential to influence bidding in later rounds, if they are revealed to other bidders. Three cases will be considered:

- *Sealed bidding.* Bids are not revealed until after bidding is closed.

- *Dynamic bidding.* Bids submitted in round k are publicly revealed at the end of round k.

- *Strategic bid-reporting.* The seller decides whether and when to reveal each bid.

Additional modeling of strategic bid-reporting will be delayed until Section 4. For now, I will focus on sealed bids and dynamic bids.

Definition 1 (Symmetric threshold equilibrium). A *"symmetric threshold equilibrium (STE)"* is a perfect Bayesian equilibrium in which each bidder i bids (and bids truthfully) in round $k = 1, ..., K$ if and only if $v_i \geq e^{k|K}$ and no one has bid previously, where $e^{1|K} \geq ... \geq e^{K|K}$ are entry thresholds.

Let $v_{-i} = \max_{j \neq i} v_j$ and let $G(\cdot)$ and $g(\cdot)$ denote the c.d.f. and p.d.f. of v_{-i}.

Benchmark case: Sealed bids. Samuelson (1985) characterized the unique STE when there is just one round of bids or, equivalently, when there are multiple rounds but bids are sealed. In this equilibrium, each bidder enters iff $v_i \geq e^{1|1}$, where the "simultaneous-entry threshold" $e^{1|1}$ is defined by the entry-indifference condition:

$$(e^{1|1} - r)G\left(e^{1|1}\right) = c. \tag{1}$$

Comments on the model. (a) The assumption that bidders submit at most one bid simplifies the analysis of strategic bid-reporting in Section 4, but is not essential to any of the results. (b) Bidders strictly prefer to bid truthfully in any STE. This is not immediately obvious, since bidding more than one's true value can deter others from entering the auction later. In any STE, however, all bids above the round-k entry threshold e^k (including truthful bids when $v_i \geq e^k$) are high enough to deter all future entry. (c) The fact that bids are observed is not essential. In particular, any STE in this paper's model remains an equilibrium in an alternative model in which only entry can be observed. The reason is that, in equilibrium, entering in round k is a credible signal that one's value (and hence one's bid) exceeds e^k.

3 Dynamic bidding

This section considers dynamic bidding, when all bids made in each round are publicly observable before the next round. Section 3.1 characterizes the unique symmetric threshold equilibrium (STE) when there are K bidding rounds (Theorem 1), as well as the limit as $K \to \infty$ (Theorem 2). Section 3.2 then explores the welfare and revenue effects of dynamic bidding.

3.1 Symmetric threshold equilibrium

Theorem 1. *The K-round bidding game has a unique symmetric threshold equilibrium, with entry thresholds $(e^{1|K}, ..., e^{K|K})$ such that $r + c < e^{K|K} < ... < e^{1|K} < e^{0|K} = \overline{V}$, characterized by the following system of equations:*

$$(e^{K|K} - r)G(e^{K|K}) = cG(e^{K-1|K}) \tag{2}$$

$$\int_{e^{k+1|K}}^{e^{k|K}} (v_{-i} - r)dG(x) = c(G(e^{k-1|K}) - G(e^{k|K})) \text{ for } k = 1, ..., K - 1 \tag{3}$$

Discussion: (2) is the equilibrium zero-profit condition for bidders having value $v_i = e^{K|K}$. To see why, note that bidder i only enters the auction when round K is reached with no prior bids, i.e. $v_{-i} < e^{K-1|K}$ which occurs with ex ante probability $G(e^{K-1|K})$, and then only wins the object (at the reserve price r) when $v_{-i} < e^{K|K}$. Similarly, for any $k = 1, ..., K - 1$, (3) is the equilibrium indifference condition for bidders having value $v_i = e^{k|K}$ between bidding in round k and waiting until round $k + 1$. To see why, note that entering in round k forces bidder i to incur a loss of c whenever others also enter in round k, i.e. when $v_{-i} \in (e^k, e^{k-1})$, while deterring others from entering – allowing bidder i to avoid competition and pay the reserve price r rather than v_{-i} – whenever others would have entered in round $k + 1$, i.e. $v_{-i} \in (e^{k+1}, e^k)$.

Corollary to Theorem 1. *In the unique STE, the ex ante probability that the first bid arrives in round k is decreasing in k.*

Proof. The first bid arrives in round k when $\max_i v_i \in [e^{k|K}, e^{k-1|K})$, i.e. with probability $F(e^{k-1|K})^N - F(e^{k|K})^N$. Since $e^{0|K} > e^{1|K} > ... > e^{K|K}$, $F(e^{k-1|K})^N - F(e^{k|K})^N$ is decreasing in k if and only if $G(e^{k-1|K}) - G(e^{k|K}) = F(e^{k-1|K})^{N-1} - F(e^{k|K})^{N-1}$ is decreasing in k. (Details for this step are straightforward and omitted.) Note that

conditions (2,3) can be rewritten as

$$\frac{G(e^{K-1|K})}{G(e^{K|K})} = \frac{e^{K|K} - r}{c} \tag{4}$$

$$\frac{G(e^{k-1|K}) - G(e^{k|K})}{G(e^{k|K}) - G(e^{k+1|K})} = \frac{E[v_{-i} - r|v_{-i} \in (e^{k+1|K}, e^{k|K})]}{c} \tag{5}$$

for all $k = 1, ..., K - 1$. Since $e^{k+1|K} > r + c$, the right-hand side of (14) is greater than one, implying $G(e^{k-1}) - G(e^k) > G(e^k) - G(e^{k+1})$ for all $k = 1, ..., K - 1$, as desired. $\square$

Theorem 2. $e^{k|K}$ *is increasing in* K *for all* k, *with* $\{e^{k|\infty} = \lim_{K \to \infty} e^{k|K} : k = 1, 2, ...\}$ *determined recursively by*

$$(e^{1|\infty} - r)G(e^{1|\infty}) - \int_{r+c}^{e^{1|\infty}} G(x)dx = c \tag{6}$$

$$(e^{k|\infty} - r)G(e^{k|\infty}) - \int_{r+c}^{e^{k|\infty}} G(x)dx = cG(e^{k-1|\infty}) \text{ for } k = 2, 3, ... \tag{7}$$

Discussion: The limit-thresholds $(e^{1|\infty}, e^{2|\infty}, ...)$ provide a lower bound of sorts on bidding activity in each round. In particular, since $e^{k|\infty} > e^{k|K}$ for all (k, K), any bidder with value $v_i > e^{1|\infty}$ always bid in the first round, no matter how many rounds there may be, anyone with value $v_i \in (e^{2|\infty}, e^{1|\infty})$ bids no later than the second round, and so on. Because the system (6,7) characterizes the limit-thresholds recursively, they are also easy to compute. For instance, suppose that there are two bidders with i.i.d. values uniformly distributed on $[0, 1]$, the reserve price is zero, and the bidding cost $c = \frac{1}{6}$. The simultaneous-entry threshold e^S solves (1): $(e^S)^2 - c = 0$, or $e^S = \sqrt{c} \approx .408$. By contrast, $e^{1|\infty}$ solves (6): $(e^{1|\infty})^2 - c = \int_c^{e^{1|\infty}} x dx$, or $e^{1|\infty} = \sqrt{2c - c^2} \approx .552$. Another interesting fact here is that $e^{2|\infty} = \sqrt{2e^{1|\infty}c - c^2} \approx .396 < e^S$. So, any bidder who waits until after the second round to bid in any K-round game would not bid at all if bids were sealed.

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3.2 Welfare and revenue effects of dynamic bidding

This section investigates the welfare and revenue effects of having multiple bidding rounds $(K > 1)$ compared to having sealed bids $(K = 1)$. The main findings are that having multiple bidding rounds (i) increases expected revenue under an optimal reserve price (Proposition 1) and (ii) increases both interim expected surplus and expected total welfare under any fixed reserve price (Propositions 2-3).

Proposition 1. *If the seller can commit to a reserve price, expected revenue is strictly higher when $K > 1$ than when $K = 1$.*

Proof. Let $e^S(r)$, $e^{K|K}(r)$ denote the simultaneous-entry threshold in the 1-round game and the K-th round entry threshold for some $K > 1$, respectively, viewed as functions of the reserve price r. By inspection and comparison of (1) and (2), $e^{K|K}(r) < e^S(r)$ for all $r < \overline{V}$. Moreover, it is straightforward to show that $e^{K|K}(r)$ is continuous and strictly increasing in r.

Let r^S denote the optimal reserve price when bids are sealed. Define $\hat{r} > r^S$ so that $e^{K|K}(\hat{r}) = e^S(r^S)$, and note that the K-round game with reserve $\hat{r}$ has strictly less equilibrium entry than the 1-round game with reserve r^S. (Bidders with values in $(e^{K|K}(\hat{r}), e^{1|K}(\hat{r}))$ always enter when bids are sealed, but stay out when $\max_i v_i > e^{1|K}(\hat{r})$ in the K-round game.) On the other hand, the object's allocation is identical (sold to the highest-value bidder iff $\max_i v_i > e^S(r^S)$) in both cases, as is interim expected surplus (by (15 since $e^S(r^S) = e^{K|K}(\hat{r})$). Expected revenue must therefore be strictly higher in the K-round game with reserve $\hat{r}$, by an amount equal to the expected cost savings of having less equilibrium entry. This completes the proof since seller expected revenue in the K-round game is even higher under an *optimal* reserve. $\square$
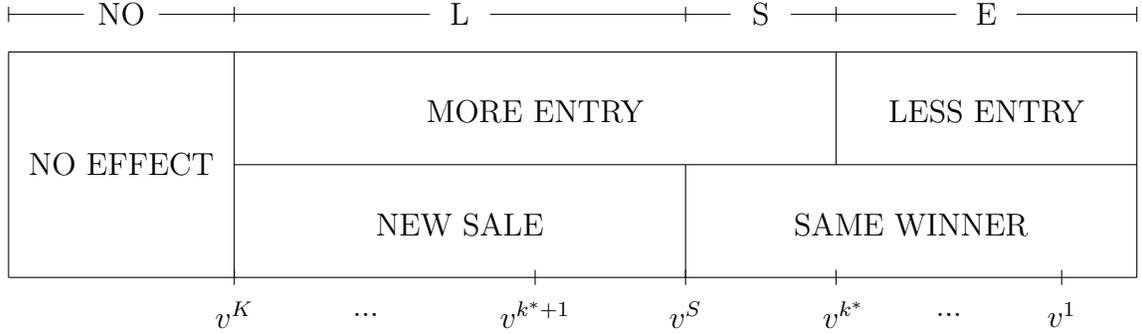
Figure 1: How having multiple bidding rounds changes equilibrium entry and sales.

Proposition 2. *For any given reserve price, bidders' interim expected surplus is strictly increasing in K.*

Proof. Interim expected surplus $\Pi^K(v_i) = \int_{e^{K|K}}^{v_i} G(x)dx$ in the K-round bidding game; see the derivation of (15) in the proof of Theorem 2. The desired result is therefore a direct corollary of the fact, also shown in the proof of Theorem 2, that $e^{K|K}$ is decreasing in K. □

Proposition 3. *For any given reserve price, expected total surplus is strictly higher when $K > 1$ than when $K = 1$.*

Proof. Let k^* be the last bidding round in the K-round game in which all entrants would have entered in the simultaneous-move game, i.e., $v^{k^*} \geq v^S > v^{k^*+1}$. Figure 1 illustrates the effect of moving from one to K rounds on equilibrium outcomes, in terms of entry and sales. For convenience, the value-space is divided into four regions $\{E, S, L, NO\}$ with mnemonic labels: when $\max_i v_i = v^{(1)} \in E = (v^{k^*}, \overline{V})$, some bidder enters Early (in rounds $1, ..., k^*$) in the K-round game and would enter simultaneously; when $v^{(1)} \in S = (v^S, v^{k^*})$, no one enters Early but at least one bidder would enter Simultaneously; when $v^{(1)} \in L = (v^{k^*-1}, v^S)$, no one enters Simultaneously but someone would enter Late (in rounds $k^* + 1, ..., K$); finally, when $v^{(1)} \in NO = [0, v^K)$, no one ever enters.

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Step 1: when $v^{(1)} \in E$, total welfare rises. When $v^{(1)} \in E$, the winner enters in round $\hat{k} \leq k^*$ and is the same as under simultaneous entry. Ex post total welfare is (weakly) higher since bidders with values in $(v^S, v^{\hat{k}})$ do not enter the auction.

Step 2: when $v^{(1)} \in S$, expected total welfare falls by the same amount as L-type bidders' surplus falls. When $v^{(1)} \in S$, the winner enters in round $k^* + 1$ and is the same as under simultaneous entry. Ex post total welfare is (weakly) lower, however, since bidders with values in $(v^{k^*+1}, v^S) \subset L$ enter and lose, when they would not have under simultaneous bidding. Since these L-type bidders do not win, their surplus falls and, importantly, falls by exactly the same amount in aggregate as total welfare falls.

Step 3: when $v^{(1)} \in L$, expected total welfare rises by more than L-type bidders' surplus rises. When $v^{(1)} \in L$, the seller's revenue is higher in the K-round game, obviously, since no entry would have occurred under simultaneous entry. The overall change in expected total welfare in this event is therefore strictly greater than the change in type-L bidders' surplus.

Step 4: when $v^{(1)} \in L \cup S$, expected total welfare rises. Combining Steps 2-3, the net welfare effect of having multiple bidding rounds when $v^{(1)} \in L \cup S$ is greater than the net effect on type-L bidders in this event. Note that type-L bidders earn zero surplus when $v^{(1)} \in E$, whether or not preemptive bidding is allowed. So, the net effect of having multiple bidding rounds on type-L bidders when $v^{(1)} \in L \cup S$ is positive iff its effect on type-L bidders' interim expected surplus is non-negative. As shown in Part One, however, bidders' interim expected surplus is greater when there are multiple bidding rounds.

All together, we conclude that expected total welfare is strictly higher when there are multiple bidding rounds, conditional on the event $v^{(1)} \in L \cup S \cup E$. This completes

the proof, since bidder surplus and revenue are zero in the remaining event $v^{(1)} \in NO$ in which no sales ever occur. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ □

4   Strategic bid-reporting

This section considers an alternative model with "strategic bid-reporting," in which the seller chooses what previously-submitted bids to publicly reveal after each bidding round and lacks the power to commit to a bid-reporting policy. The main result is that equilibrium outcomes are the same as if bids cannot be revealed.

Theorem 3. *In the K-round game with strategic bid-reporting, (i) a "silent symmetric equilibrium" exists in which no bids are ever submitted prior to round K and (ii) all symmetric equilibria are outcome-equivalent to the silent symmetric equilibrium.*

Proof. Part One: A silent symmetric equilibrium exists. Consider the following strategies. *For bidders:* (i) Do not bid prior to round K, no matter what the seller has revealed; (ii) In round K, if the seller has revealed $m \geq 0$ bids $b^1 \leq ... \leq b^m$, play according to the unique STE of the sealed-bidding game with $N - m$ bidders, reserve price b^m (or reserve r if $m = 0$), and i.i.d. values drawn from c.d.f. $F(\cdot)$. *For the seller:* (i) Do not reveal any bids prior to round $K - 1$; (ii) In round $K - 1$, reveal whatever combination of previously-submitted bids minimizes the subsequent round-K entry threshold.

This strategy specified here for the seller is clearly a best response, since it maximizes round-K participation. Note that the seller will only reveal a bid made by bidder i when doing so induces more entry in round K, which (at least weakly) harms bidder i. Waiting until round K to bid is therefore clearly a best response for each bidder.

Now, consider the bidding in round K. On the equilibrium path, because when no ever bids prior to round K, bidders have the same beliefs about others' values as if the

game only had one bidding round. Clearly, then, playing the unique STE of the 1-round game with N bidders, reserve price r, and values drawn from c.d.f $F(\cdot)$ is an on-path equilibrium for bidders in round K. In much the same way, when $m > 0$ bids $b^1 \leq ... \leq b^m$ have been revealed, playing the unique STE of the 1-round game with $N - m$ bidders, reserve price b^m, and values drawn from c.d.f $F(\cdot)$ is an off-path equilibrium in round K, given the off-path belief that all $N - m$ bidders whose bids have not been revealed are certain not to have bid.

All together, we conclude that the specified strategies and beliefs constitute a silent symmetric equilibrium, which itself is outcome-equivalent to the unique STE when there is only one round of bidding. (Moreover, any silent symmetric equilibrium must clearly be outcome-equivalent to the unique STE of the 1-round game, since all on-path bidding activity in any such equilibrium occurs in the final bidding round.)

Part Two: All symmetric equilibria are outcome-equivalent to the silent symmetric equilibrium from Part One. First, I will show that there must be "silence" in round $K - 1$ in any symmetric equilibrium, in the sense that equilibrium outcomes in any round-$(K - 1)$ subgame are always the same *as if* no one ever bids in round $K - 1$.

Round-$(K - 1)$ bids can only affect equilibrium outcomes if they are revealed, so suppose that a symmetric equilibrium exists in which, with positive probability, a bid (or bids) is submitted in round $K - 1$ and then revealed by the seller. For any given round-$(K - 1)$ subgame, let $\underline{b}$ denote the minimal bid that is ever submitted and then revealed in round $K - 1$. A contradiction will be reached if, regardless of their values, bidders strictly prefer to wait until round K rather than bid $\underline{b}$ in round $K - 1$.

Observe first that, in any symmetric equilibrium, any bidder who has not bid prior to round K will enter in round K if and only if his value exceeds a symmetric threshold that depends on the history of previously-revealed bids. However, since the seller will

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always choose to reveal whatever bids encourage the most round-K entry, anyone who submits a bid in round $K - 1$ is always at least weakly worse off than if he waited until round K to submit the same bid.

Next, note that waiting until round K gives bidders an option not to participate in the auction. In particular, since $\underline{b}$ is the minimal bid ever revealed, waiting until round K is strictly better than bidding $\underline{b}$ (say, for bidder i) if there is any chance that the seller will reveal any bid, were bidder i to wait. This leads to a contradiction, unless the seller *never* reveals any bid when bidder i chooses to wait. By bidder symmetry, then, we get a contradiction unless the seller never reveals any bids after round $K - 1$ unless all bidders have already bid! Of course, if the seller were to follow such a bid-revelation rule, the outcome of the game would be the same as if the seller never revealed any round-$(K - 1)$ bids at all.

We conclude that, if a symmetric equilibrium exists in which bids are sometimes made and then revealed in round $K - 1$, another symmetric equilibrium exists in which all bids made in round $K - 1$ are instead made to round K. In this new equilibrium, some bids that were made in previous rounds might still be revealed in round $K - 1$. However, since no bids are made in round $K - 1$, we can construct yet another outcome-equivalent equilibrium in which these previous bids are revealed in round $K - 2$ instead of in round $K - 1$. With no activity at all in round $K - 1$, the previous argument can now be applied again to round $K - 2$, to show that equilibrium outcomes starting from round $K - 2$ are the same as if all bidders wait until round K, and so on all the way back to round 1. All together, then, we conclude that every symmetric equilibrium is outcome-equivalent to a silent symmetric equilibrium. $\qquad\square$

5 Concluding Remarks

A standard argument in favor of conducting a sealed-bid auction is that sealed bids can make it more difficult for bidder-cartels to monitor and enforce a collusive agreement (Marshall and Marx (2012)). This paper emphasizes a countervailing upside associated with making bidding activity observable during the auction, that such "dynamic bidding" facilitates bidder coordination and hence reduces excess entry. This improved coordination increases bidder interim expected surplus and total expected welfare for any given reserve price (Propositions 2-3) and increases expected revenue when the seller sets an optimal reserve (Proposition 1). That said, equilibrium entry remains excessive, even in the limit as the number of bidding rounds goes to infinity, with a positive measure of high-value bidders entering immediately in the first round (Theorem 2).[4]

If bidders can costlessly communicate with the seller, this remaining entry inefficiency can be easily addressed.[5] Consider a class of mechanisms that specify who should pay the entry cost as well as who wins and what price they pay. The optimal mechanism in this context can be implemented by conducting a "virtual auction" based on bidders' costless reports, inducing only the winner of this virtual auction to pay the entry cost, and then charging this winner the final price in the virtual auction. Working to identify optimal mechanisms when communication is costly is a worthwhile goal for future research. For recent progress on this problem, see e.g. Mookherjee and Tsumagari (2012).

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[4]This pooling feature of the equilibrium differentiates this paper's signaling mechanism from those in other auction models, such as Daley, Schwarz, and Sonin (2012), in which bidders can perfectly signal their value by burning money.

[5]Others who have explored how costless communication can help coordinate entry when participation is costly include Campbell (1998) and Miralles (2010), who allow for cheap talk among the bidders before the auction, and Quint and Hendricks (2013), who allow for indicative bidding.

Appendix

Proof of Theorem 1

Part One: (2,3) are sufficient for STE. Consider bidder i's best response, if all bidders $j \neq i$ adopt $(e^{1|K}, ..., e^{K|K})$-threshold strategies where $(e^{1|K}, ..., e^{K|K})$ satisfy (2, 3). Suppose first that $v_{-i} < e^{K-1}$, so that round K can be reached with no prior bids (if bidder i does not bid prior to round K). By entering in round K, bidder i wins at the reserve price when $v_{-i} < e^K$ and wins at price v_{-i} when $v_{-i} \in (e^K, \min\{v_i, e^{K-1}\})$, yielding expected payoff (expressed for convenience in ex ante terms)

$$X^K(v_i) = (v_i - r)G(e^K) - cG(e^{K-1}) \text{ if } v_i \leq e^K \tag{8}$$
$$= (v_i - r)G(e^K) + \int_{e^K}^{\min\{v_i, e^{K-1}\}} (v_i - v_{-i})dG(v_{-i}) - cG(e^{K-1}) \text{ if } v_i \geq e^K$$

Note that $X^K(v_i)$ is strictly increasing in v_i and, by (2), $X^K(e^K) = 0$. So, entering in round K is bidder i's best response if and only $v_i \geq e^K$.

Next, suppose that $v_{-i} < e^{k-1}$, so that round $k = 1, ..., K - 1$ can be reached with no prior bids. Relative to waiting and entering in round $k + 1$, entering in round k has three effects, depending on others' values.

Case #1: $v_{-i} < e^{k+1}$. No one else enters in round k or would enter in round $k + 1$, so bidder i wins at the reserve price (for ex post payoff $v_i - r - c$) whether he enters in round k or waits to enter in round $k + 1$.

Case #2: $v_{-i} \in (e^{k+1}, e^k)$. Bidder i is better off entering in round k, since doing so deters others from entering in round $k + 1$. Such entry deterrence allows bidder i to win at the reserve price rather than at price v_{-i} when $v_i \geq v_{-i}$ (for ex post gain $v_{-i} - r$), or to avoid losing the auction when $v_i < v_{-i}$ (for ex post gain $v_i - r$). Overall, then, bidder i's (ex

ante) expected gain due to entering in round k when $v_{-i} \in (e^{k+1}, e^k)$ is

$$Y^k(v_i) = \int_{e^{k+1}}^{e^k} (v_{-i} - r)dG(v_{-i}) \text{ if } v_i \geq e^k \tag{9}$$

$$= (v_i - r)(G(e^k) - G(\max\{v_i, e^{k+1}\})) + \int_{e^{k+1}}^{\max\{v_i, e^{k+1}\}} (v_{-i} - r)dG(v_{-i}) \text{ if } v_i \leq e^k$$

Case #3: $v_{-i} \in (e^k, e^{k-1})$. Bidder i is at least weakly worse off entering in round k, since there is an option value to waiting and observing what others' round-K bids before deciding whether to enter the auction. In particular, waiting until round $k+1$ allows bidder i to avoid incurring a loss of $c - \max\{0, v_i - v_{-i}\}$ when $v_{-i} > e^k$ and $v_{-i} > v_i - c$.[6] Overall, bidder i's (ex ante) expected loss due entering in round k when $v_{-i} \in (e^k, e^{k-1})$ is

$$Z^k(v_i) = c(G(e^{k-1}) - G(e^k)) \text{ if } v_i \leq e^k \tag{10}$$

$$= \int_{\max\{e^k, v_i - c\}}^{\min\{e^{k-1}, v_i\}} (c - v_i + v_{-i})dG(v_{-i}) + c(G(e^{k-1}) - G(\min\{e^{k-1}, v_i\})) \text{ if } e^k \leq v_i \leq e^{k-1} + c$$

$$= 0 \text{ if } v_i \geq e^{k-1} + c$$

(To parse (10) in the most complex case when $v_i \in (e^k, e^{k-1} + c)$, note that (i) round-$k$ entry leads to a loss of c when others also enter and bidder i loses, i.e. when $v_{-i} \in (e^k, e^{k-1})$ and $v_{-i} > v_i$, and (ii) round-k entry leads to a loss of $c - v_i + v_{-i}$ when others also enter and bidder i wins at a price greater than $v_i - c$, i.e. when $v_{-i} \in (e^k, e^{k-1})$, $v_{-i} < v_i$, and $v_{-i} > v_i - c$.)

By (3), $X^k(e^k) = Y^k(e^k)$. Note further by inspection of (9,10) that, for any $v'' > e^k > v'$, $Y^k(v'') = Y^k(e^k) > Y^k(v')$ and $Z^k(v'') < Z^k(e^K) = Z^k(v')$. So, $Y^k(v_i) \geq Z^k(v_i)$ for

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[6]If bidder i waits until round $k+1$ and $v_{-i} > e^k$ but $v_{-i} < v_i - c$, bidder i will jump in and outbid the highest round-k bidder once round $k+1$ is reached. So, in this case, he wins at price v_{-i} whether he bids in round k or waits until round $k+1$.

all $v_i \geq e^k$ while $Y^k(v_i) < Z^k(v_i)$ for all $v_i < e^k$. So, bidder i's best response is to enter in round k when $v_i \geq e^k$ but not enter in round k when $v_i < e^k$.

So far, I have shown that bidder i's best response if any round k is reached with no prior bids is to enter (and bid truthfully[7]) if and only if $v_i \geq e^k$. To complete the proof, observe that if bidder i follows this rule in all subgames with no prior bids, bidder i's best response is never to bid in subgames with prior bids. Why? Suppose that the first bids received were in round $k' < k$ and that bidder i has not bid prior to round k. All bids submitted in round k' are at least $e^{k'}$ but, since bidder i did not bid in rounds $1, ..., k'$, bidder i's own value must be less than $e^{k'}$. Clearly, then bidder i prefers not to bid. All together, then, bidder i's best response when all others adopt $(e^1, ..., e^K)$-threshold strategies is to do so as well. This completes the proof that (2,3) are necessary and sufficient for existence of a STE with thresholds $(e^1, ..., e^K)$.

Part Two: Uniqueness of STE. To establish uniqueness, I need to show that (2,3) has a unique solution. Note that (2,3) can be re-written as

$$G(e^{K-1}) = G(e^K) \left(\frac{e^K - r}{c} \right) \text{ and} \tag{11}$$

$$G(e^{k-1}) - G(e^k) = (G(e^k) - G(e^{k+1})) \frac{E[v_{-i} - r | v_{-i} \in [e^{k+1}, e^k]]}{c} \tag{12}$$

for all $k = 1, ..., K - 1$. (Recall that $e^0 = \overline{V}$, so this is a system of K equations with K unknowns.)

Suppose for a moment that $e^K = r + c$. If so, (2) implies that $e^{K-1} = r + c$ while (3) implies that $e^{K-2} = ... = e^1 = e^0 = r + c$. This is a contradiction, clearly, since

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[7]Bidding more than one's true value could be optimal, if such overbidding deters others from bidding later. However, since any round-k bid greater than e^k is sufficient to deter all future entry, and bidder i's value $v_i \geq e^k$ whenever he enters in round k, truthful bidding is sufficient to deter all future entry. Moreover, as usual, truthful bidding ensures that bidder i only wins in round k when his value exceeds the price that he will pay to win.

$e^0 = \overline{V} > r + c$. Similarly, for $e^K > r + c$, (2) determines e^{K-1} as a function of e^K (call it $e^{K-1}(e^K)$) while (3) inductively determines the other thresholds $(e^{K-2}, ..., e^1, e^0)$ as functions $(e^{K-2}(e^K), ..., e^1(e^K), e^0(e^K))$ of e^K. ((12) determines e^{K-2} as a function of (e^{K-1}, e^K). Since e^{K-1} is determined by e^K, so is e^{K-2}. Repeating this logic inductively determines $e^{K-3}, ..., e^1, e^0$ as functions of e^K.)

$(e^0 = \overline{V}, e^{1*}, ..., e^{K*})$ solves (2,3) if and only if (i) $e^{1*} = e^1(e^{K*}), ..., e^{K-1*} = e^{K-1}(e^{K*})$ and (ii) $e^0(e^{K*}) = \overline{V}$ or, equivalently, $G(e^0(e^{K*})) = 1$. Next, note that (11) implies that $G(e^{K-1}(e^K)) - G(e^K) > 0$ is continuous and strictly increasing in e^K, while (12) implies by induction that $G(e^{k-1}(e^K)) - G(e^k(e^K))$ is continuous and strictly increasing in e^K, for all $k = 1, ..., K - 1$. In particular, $G(e^0(e^K))$ is continuous and strictly increasing in e^K. So, there is a unique solution e^{K*} to $G(e^0(e^{K*})) = 1$ and hence a unique solution $(e^{1*}, ..., e^{K*})$ to (2,3). $\qquad\square$

Proof of Theorem 2

It is convenient to rewrite conditions (2,3) as

$$\frac{e^{K|K} - r}{c} = \frac{G(e^{K-1|K})}{G(e^{K|K})} \tag{13}$$

$$\frac{E[v_{-i} - r | v_{-i} \in (e^{k+1|K}, e^{k|K})]}{c} = \frac{G(e^{k-1|K}) - G(e^{k|K})}{G(e^{k|K}) - G(e^{k+1|K})} \text{ for all } k = 1, ..., K - 1 \tag{14}$$

Step One: $e^{K|K}$ is decreasing in K. Suppose for the sake of contradiction that $e^{K'|K'} \geq e^{K|K}$ for some $K' > K$. This implies $G(e^{K'|K'}) \geq G(e^{K|K})$ and, by (13), $\frac{G(e^{K'-1|K'})}{G(e^{K'|K'})} \geq \frac{G(e^{K-1|K})}{G(e^{K|K})}$. So, $e^{K'-1|K'} \geq e^{K-1|K}$ and $G(e^{K'-1|K'}) - G(e^{K'|K'}) \geq G(e^{K-1|K}) - G(e^{K|K})$. By (14), this then recursively implies both $e^{K'-t|K'} \geq e^{K-t|K}$ and $G(e^{K'-t|K'}) - G(e^{K'-t+1|K'}) \geq G(e^{K-t|K}) - G(e^{K-t+1|K})$ for all $t = 2, ..., K$. Why? Consider $t = 2$. Since $e^{K'|K'} \geq e^{K|K}$ and $e^{K'-1|K'} \geq e^{K-1|K}$, $E[v_{-i} - r | v_{-i} \in (e^{K'|K'}, e^{K'-1|K'})] \geq E[v_{-i} - r | v_{-i} \in (e^{K|K}, e^{K-1|K})]$.

Since $G(e^{K'-1|K'}) - G(e^{K'|K'}) \geq G(e^{K-1|K}) - G(e^{K|K})$, condition (14) for $k = K - 1$ then implies that $G(e^{K'-2|K'}) - G(e^{K'-1|K'}) \geq G(e^{K-2|K}) - G(e^{K-1|K})$. $e^{K'-2} \geq e^{K-2}$ then follows immediately from $e^{K'-1} \geq e^{K-1}$. Repeating this argument recursively, using condition (14) for $k = K - 2, ..., 1$, establishes in the end that $e^{K'-K|K'} \geq e^{0|K} = \overline{V}$. But this is a contradiction since, by Theorem 1, $e^{K'-K|K'} < \overline{V}$. $\square$

Step Two: $\lim_{K \to \infty} e^{K|K} = r + c$. Let $\underline{v} = \lim_{K \to \infty} e^{K|K}$. Suppose for the sake of contradiction that $\underline{v} > r + c$, and fix $\hat{v} \in (r + c, \underline{v})$. Since $e^{K|K}$ is decreasing in K, a bidder with value $\hat{v}$ must find it unprofitable to enter in every round k of every K-round bidding game. Such entry allows a bidder with value $\hat{v}$ to win at the reserve price with (ex ante) probability $G(e^{k|K})$ while incurring cost c with probability $G(e^{k-1|K})$. For this to be unprofitable, the conditional probability that someone else enters in round k must be sufficiently high, namely, $\frac{G(e^{k-1|K}) - G(e^{k|K})}{G(e^{k-1|K})} \geq \frac{\hat{v}-r-c}{\hat{v}-r}$. Now, consider any $K > \frac{\hat{v}-r}{G(r+c)(\hat{v}-r-c)}$ and define $\hat{k} \in \arg\min_k (G(e^{k-1|K}) - G(e^{k|K}))$. Since $e^{1|K} > ... > e^{K|K}$, $\min_k (G(e^{k-1|K}) - G(e^{k|K})) \leq \frac{1}{K} < \frac{G(r+c)(\hat{v}-r-c)}{\hat{v}-r}$. So, $\frac{\hat{v}-r-c}{\hat{v}-r} > \frac{G(e^{\hat{k}-1|K}) - G(e^{\hat{k}|K})}{G(r+c)} > \frac{G(e^{\hat{k}-1|K}) - G(e^{\hat{k}|K})}{G(e^{\hat{k}-1|K})}$ and entering in round $\hat{k}$ is profitable for a bidder with value $\hat{v}$, a contradiction. $\square$

Step Three: Characterizing $e^{k|K}$ as a function of $(e^{k-1|K}, e^{K|K})$. In the K-round game, a bidder with value $v_i \in (e^{k|K}, e^{k-1|K})$ finds it optimal to bid his true value if round k is reached with no prior bids. In particular, $v_i \in \arg\max_{b_i} \Pi^K(b_i; v_i)$, where $\Pi^K(b_i; v_i) = (v_i - r)G(e^{k|K}) + \int_{e^{k|K}}^{b_i} (v_i - v_{-i})dG(v_{-i}) - cG(e^{k-1|K})$ and $\Pi^K(v_i) = \Pi^K(v_i; v_i)$ is type v_i's expected equilibrium payoff. By the Envelope Theorem, $d[\Pi^K(v_i)]/dv_i = d[\Pi^K(b_i; v_i)]/dv_i = G(v_i)$ for all $v_i \in (e^{k|K}, e^{k-1|K})$, for all $k = 1, ..., K$. Moreover, by the indifference conditions (2,3), $\Pi^K(v_i)$ is continuous at $v_i \in \{e^{1|K}, ..., e^{K|K}\}$. So,

$$\Pi^K(v_i) = \int_{e^{K|K}}^{v_i} G(x)dx \text{ for all } v_i \geq e^{K|K}. \tag{15}$$

At each threshold $e^{k|K}$, we can also express $\Pi^K(e^{k|K}) = (e^{k|K} - r)G(e^{k|K}) - cG(e^{k-1|K})$.

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(The threshold type $e^{k|K}$ only wins if no one else enters in round k.) In particular, for every K and $k \le K$, $e^{k|K}$ must be the (unique) solution to

$$(v_i - r)G(v_i) - \int_{e^{K|K}}^{v_i} G(x)dx = cG(e^{k-1|K}), \tag{16}$$

where $(e^{k-1|K}, e^{K|K})$ are parameters that vary with K.

The desired result that $e^{k|K}$ is increasing in K emerges as a simple comparative static of this solution. Consider first $k = 1$. Since $e^{0|K} = \overline{V}$ for all K, the right-hand side of (16) does not depend on K. On the other hand, since $e^{K|K}$ is decreasing in K (shown earlier), the left-hand side of (16) is decreasing in K. Since $\frac{d[(v_i - r)G(v_i)]}{dv_i} - \frac{d[\int_{e^{K|K}}^{v_i} G(x)dx]}{dv_i} = (v_i - r)g(v_i) > 0$, the solution $e^{1|K}$ of (16) must therefore be increasing in K. The rest of the proof is by induction on k. As long as $e^{k-1|K}$ is increasing in K, the right-hand side of (16) is increasing in K while, as in the $k = 1$ case, the left-hand side of (16) is decreasing in K. So, the solution $e^{k|K}$ of (16) must be increasing in K, and the limit $e^{k|\infty}$ exists for all k. (13) and (14) now follow from (16) for $k = 1$ and $k > 1$, respectively, by continuity in the limit as $e^{K|K} \to r + c$. $\qquad\square$

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