Aggregating Tastes, Beliefs, and Attitudes under Uncertainty*

Eric Danan† Thibault Gajdos‡ Brian Hill§ Jean-Marc Tallon¶

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Abstract

We provide results on the aggregation of beliefs and tastes for Monotone, Bernoullian and Archimedian preferences of Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2011). We propose a new axiom, Unambiguous Pareto Dominance, which requires that if the unambiguous part of individuals' preferences agree, then society should follow it. We characterize the resulting social preferences and show that it is enough that individuals share a prior to allow non dictatorial aggregation. A further weakening of this axiom on common taste acts, where cardinal preferences are identical, is also characterized. It gives rise to beliefs at the social level that can be any subset of the convex hull of individuals'. We then apply this general results to the Maxmin Expected Utility model, the Choquet Expected Utility model and the Smooth Ambiguity model. We end with a characterization of the aggregation of ambiguity attitudes.

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1 Introduction

Many social decisions have to be made in uncertain environments where individual agents have both different tastes over the possible outcomes and different beliefs over the possible states of the world. The separation of beliefs and tastes is certainly conceptually important, but rests mostly on the interpretation of representation theorems (as recalled, e.g., by Gayer, Gilboa, Samuelson, and Schmeidler, 2014 and Blume, Cogley, Easley, Sargent, and Tsyrennikov, 2014). A basis for this separation is, as the old saying goes "De Gustibus Non est Disputandum", that tastes are stable and not subject to dispute while beliefs can change more rapidly upon, say, persuasion. Gayer, Gilboa, Samuelson, and Schmeidler (2014) provide examples and a discussion

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†THEMA, Université de Cergy-Pontoise, CNRS, 33 boulevard du Port, 95011 Cergy-Pontoise cedex, France (eric.danan@u-cergy.fr).
‡GREQAM, CNRS, Aix-Marseille University (thibault.gajdos@univ-amu.fr).
§CNRS, HEC Paris (hill@hec.fr).
¶Paris School of Economics, Université Paris I Panthéon-Sorbonne, CNRS (jean-marc.tallon@univ-paris1.fr).

of the reasons why the distinction is particularly relevant when dealing with social decisions, as well as references to previous literature on this subject. We endorse this view and explore in this paper the separate aggregation of tastes and beliefs in a general uncertain environment.

Subjective Expected Utility (henceforth SEU; Savage, 1954) captures beliefs through a probability distribution. Hylland and Zeckhauser (1979) and Mongin (1995, 1998) showed that for SEU preferences, simultaneous heterogeneity of tastes and beliefs leads to the impossibility to aggregate individual preferences into a social preference. Spurious unanimity, i.e., differences in beliefs and tastes that "cancel" each other, can be identified as the source of the problem. The possibility of Paretian preference aggregation can be restored by relaxing the Pareto principle for acts (choice alternatives) that depend on events over which individuals have different beliefs and retaining it only for common-belief acts (Gilboa, Samet, and Schmeidler, 2004).

When SEU fails to apply, beliefs are not captured by a single probability distribution but a separation of beliefs and tastes (that we assimilate to preferences over outcomes) is still desirable. Such a separation has been proposed in great generality by Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2011) in an Ansombe-Aumann setting, building on the notion of "unambiguous preference" introduced by Ghirardato, Maccheroni, and Marinacci (2004) and Nehring (2007). A third component, beyond beliefs and tastes, appears: ambiguity attitudes. These are distinct from tastes (preferences over outcomes) and beliefs although some models, such as the Choquet Expected Utility model, provide representations where beliefs and ambiguity attitudes are mingled. This paper seeks to identify the taste, belief, and attitude components of preference aggregation. The following examples motivate this goal.

Example 1. Countries have to take actions to prepare for changes due to global warming. A country has both an important freshwater fish industry and an agriculture sector. If the temperature increases by 4 degrees or more, agriculture will need some new water infrastructure (dam) to survive. This infrastructure is also beneficial for the fish industry but only if the temperature increases by 2 degrees or less as it increases the lakes' surface in the region and the moderate increase in temperature has no impact on fish. The fish lobby thinks that temperature rise will be around 2 degrees while the agriculture lobby believes that it will be 4 or more. It is thus decided that the infrastructure will be developped. Although this decision is made on the basis of the Pareto principle, eventually, either the fish or the agriculture sector will regret this choice and think that, had they had the right beliefs, they would have disagreed with the infrastructure to begin with. This is a case of spurious unanimity as described by Mongin (1998) and Gilboa, Samet, and Schmeidler (2004).

Both lobbies now read in the IPCC report that "Changes in the global water cycle in response to the warming over the 21st century will not be uniform. The contrast in precipitation between wet and dry regions and between wet and dry seasons will increase, although there may be regional exceptions" and that "based on projected decreases in regional crop yields and water availability, risks of unevenly distributed impacts are high for additional warming above 2°C (medium confidence)". Understanding that the chances with which various scenarii will happen are not so well established, they understand that the best they can come up with are ambiguous beliefs in the form of sets of probability distributions. Is it now possible to aggregate beliefs and tastes? If fishermen are uncertainty seeking while farmers are uncertainty averse, would this make a difference to the aggregation of tastes and beliefs? Can these attitudes toward uncertainty be aggregated as well?

Example 2. Speculation (trade based exclusively on different beliefs – same tastes) can also be qualified as spurious unanimity. Consider an asset that pays off $+1$ in state s and -1 in state t. "Bulls" (putting a high probability on t) will then be buying that asset that "Bears" (putting a high probability on s) will be willing to sell. Given this difference in beliefs, the Pareto axiom is not beyond criticism (Gilboa, Samet, and Schmeidler, 2004) and cannot be invoked to allow for this kind of trade. What if now the traders' beliefs are ambiguous? Should the ambiguity attitudes of the traders be considered for defining what trades are deemed "acceptable"?

There are few results on preference aggregation with ambiguous beliefs in the literature. Gajdos, Tallon, and Vergnaud (2008) showed that within a wide class of "Rank-Dependent Additive" preferences, the standard Pareto principle yields an impossibility result that goes beyond the one identified under probabilistic beliefs in that it holds even if all individuals have the same beliefs (or if the Pareto principle is only imposed on common-belief acts). Recently, Herzberg (2013) extended this result to the very same class of preferences we consider in this paper. We call "spurious hedging" the mechanism underlying this impossibility result, whereby hedging (i.e. a particular mixture of two ambiguous acts) that is valued at the individual level might be irrelevant at the social level.[1] Crès, Gilboa, and Vieille (2011) showed that within the class of Maxmin Expected Utility (MEU) preferences, aggregation under the standard Pareto principle is possible if all individuals have the same tastes. Recently, Qu (2014) extended this result to the class of Choquet Expected Utility (CEU) preferences, with possibly heterogeneous tastes, under a weakening of the standard Pareto principle to "common-taste" acts.

In this paper, we propose a new weakening of the standard Pareto principle, called Unambiguous Pareto Dominance, according to which society must prefer an act f to an act g whenever all individuals unambiguously prefer f to g, in the sense of Ghirardato, Maccheroni, and Marinacci (2004) and Nehring (2007). In the Anscombe-Aumann framework, f is unambiguously preferred to g if any mixing of f with some other act h is also preferred to the same mixing of g with h. This notion induces an incomplete preference ordering. For a wide set of preference relations, dubbed "MBA preferences" by Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2011)[2], this incomplete preference relation can be represented via a utility function and a set of priors, reflecting tastes and beliefs or "revealed uncertainty". Ambiguity attitude is then reflected by a parameter in the representation of the complete preference relation.[3]

Unambiguous Pareto Dominance has the distinctive feature of placing restrictions only on the "linear" part of the preferences, which fits well with the aggregation literature, where we know that essentially only linear preferences can be aggregated. Said differently, the axiom deals only with the "rational" (if one sees Independence as a rationality requirement) part of the preferences, that is, the part that satisfies Transitivity, Monotonicity and Independence.

[1] See example 6 for an illustration.

[2] MBA stands for Monotonic, Bernouillian, Archimedian.

[3] Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) also characterize the "unambiguous" part of a preference relation through a utility function and a set of priors. They interpret this incomplete preference as reflecting "objective rationality", i.e. preferences whose beliefs are based on facts that could be used to persuade other decision makers.

We show, in the general class of MBA preferences, that Unambiguous Pareto Dominance yields separate aggregation of tastes and beliefs, independently of ambiguity attitudes. We further characterize the form of these aggregations. Utility functions are aggregated linearly (as Unambiguous Pareto Dominance implies Pareto Dominance on constant acts which then yields linear aggregation à la Harsanyi). The set of probability distributions for the social preference is a subset of the intersection of the individual sets. This is in line with the view defended, in the MEU model by Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) that these sets represent "objective rationality". It implies, if we want social preferences not to be dictatorial, that individuals have compatible beliefs, in the sense that they share at least a prior.

We also characterize a weakening of Unambiguous Pareto Dominance to common-taste acts[4] and show that social beliefs can now also be a superset of the individual beliefs, therefore allowing aggregation even in the presence of conflicting probabilistic beliefs as in the speculation example above. This aggregation result does not require that individuals have compatible beliefs. Finally, we introduce a new axiom allowing aggregation of ambiguity attitudes, independently of beliefs and tastes.

Compared to the papers mentioned above, our approach is not restricted to specific models of ambiguity. The MEU and CEU models, two pioneering models of decision-making under ambiguity, are quite specific regarding ambiguity attitudes. In the MEU model only ambiguity aversion is possible, not ambiguity seeking. In the CEU model both ambiguity aversion and ambiguity seeking are possible but beliefs and ambiguity attitudes are jointly captured by a "capacity" (non-additive probability); only these capacities are aggregated in Qu's result, not beliefs *per se*. In contrast, our results apply to a wide range of recent models allowing for more general forms of ambiguity attitudes, such as the Smooth Ambiguity model (Klibanoff, Marinacci, and Mukerji, 2005), and yield separate aggregation of beliefs and ambiguity attitudes in all these models.

Related to the present analysis, the question of how to rank allocations of contingent claims when individuals have heterogeneous beliefs has recently received a renewed attention. While using a different conceptual framework and focusing exclusively on SEU maximizers with heterogeneous beliefs, Gilboa, Samuelson, and Schmeidler (2014), Gayer, Gilboa, Samuelson, and Schmeidler (2014), Blume, Cogley, Easley, Sargent, and Tsyrennikov (2014), and Brunnermeier, Simsek, and Xiong (2014) are particularly relevant. These papers propose various restrictions of the Pareto principle that aim at excluding Pareto improvements that might be based on spurious unanimity.

Gilboa, Samuelson, and Schmeidler (2014) only retain those Pareto improvements that could be *rationalized* in the following sense. Assume that an allocation f dominates allocation g in the usual (strong) Pareto sense, i.e. each individual strictly prefers f to g. In this comparison, assuming individuals are SEU maximizers, each individual uses his/her own beliefs, p_i. Now, f qualifies as a *no-betting Pareto improvement* over g if there exists a belief p_0 such that all

[4] Acts are *common-taste acts* if all individuals have the same cardinal preferences on the (convex hull of the) lotteries involved in these acts. In view of the discussion on spurious unanimity, it would also have made sense to restrict Pareto Dominance to common-belief acts. Interestingly, these two concepts are very closely related; they are essentially equivalent in the Savage framework and Pareto Dominance on common-taste acts implies Pareto Dominance on comme-belief acts in the Anscombe-Aumann setup.

individuals would also prefer f to g, if they had p_0 as a common belief. In other words, a Pareto improvement is a no-betting Pareto improvement if it can be rationalized by a common belief. Importantly, the "rationalizing" belief need not be in the range of individuals' actual beliefs. Note furthermore that the "rationalizing" common belief depends on the allocations one compares, and therefore the no-betting Pareto relation is not transitive. On the other hand, one can argue that it is a minimal restriction of the usual classical Pareto relation.

Gayer, Gilboa, Samuelson, and Schmeidler (2014) and Brunnermeier, Simsek, and Xiong (2014) take a different route: they restrict Pareto dominance by asking for some *robustness* in the comparison of allocations. Consider again two allocations f and g, such that f Pareto dominates g in the usual (strong) sense. Gayer, Gilboa, Samuelson, and Schmeidler (2014) would say that f is a *unanimity Pareto improvement* if, moreover, each individual still prefers f to g, no matter whose beliefs she uses. On the other hand, Brunnermeier, Simsek, and Xiong (2014) use a slightly different notion. They say that an allocation f is *belief-neutral Pareto efficient* if there is no alternative g , and no belief p in the convex hull of individuals' beliefs, such that all individuals prefer g to f, given the shared belief p. Note that the belief p may depend on the alternative under consideration.

These contributions do not aim at deriving a social welfare function from the various Pareto principles they propose, and are therefore somewhat different from ours. However, Brunnermeier, Simsek, and Xiong (2014) propose a closely related social welfare function in which the social belief is the convex hull of individual beliefs. Clearly, this is closely related in spirit to their belief-neutral Pareto efficiency concept (while not formally derived). This result can be compared to the one we obtained in Theorem 2. Essentially, the social unambiguous preference we derive from Common-Taste Unambiguous Pareto Dominance is a generalization of the social welfare functional proposed by Brunnermeier, Simsek, and Xiong (2014). More precisely, we obtain exactly the same result when the set of social beliefs is the largest one identified. Therefore, Common-Taste Unambiguous Pareto Dominance can be view as an axiomatic basis for, and a generalization of, Brunnermeier, Simsek, and Xiong (2014)'s proposal.

Finally, Blume, Cogley, Easley, Sargent, and Tsyrennikov (2014) also propose a social welfare functional based on a robustness approach. They propose to evaluate allocations by the expected utility of the worst-off individuals, when one minimizes individuals' utilities over individual and social beliefs. The obtained social welfare function is always complete. They do not provide any axiomatic basis for it.

The paper is constructed as follows. The setup is exposed in Section 2. Section 3 provides the relevant material on MBA preferences from Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2011). Section 4 states Harsanyi's result in our framework and provides examples of the sources of the impossibility results mentioned above (spurious unanimity and spurious hedging). The main axioms (Unambiguous Pareto Dominance and its restrictions to common-taste acts) and results (aggregation results) are contained in section 5. We then apply these general results to three classes of preferences (MEU, CEU and Smooth ambiguity) in Section 6. Section 7 deals with the aggregation of ambiguity attitudes. Proofs are gathered in the Appendix.

2 Setup

We adopt a simple Anscombe-Aumann setting with a finite set $\mathcal{S}$ of *states of the world* and a finite set of $\mathcal{X}$ of *prizes*. Let $\mathcal{P} = \Delta(\mathcal{X})$ denote the set of all (roulette) *lotteries* and $\mathcal{F} = \mathcal{P}^{\mathcal{S}}$ denote the set of all acts (horse-roulette lotteries). Given two acts $f, g \in \mathcal{F}$ and $\lambda \in [0, 1]$, let $\lambda f + (1 - \lambda)g \in \mathcal{F}$ denote the act yielding lottery $\lambda f(s) + (1 - \lambda)g(s)$ in each state $s \in \mathcal{S}$.

Given a lottery $p \in \mathcal{P}$, we abuse notation by also letting $p \in \mathcal{F}$ denote the corresponding constant act. Given a utility function $u : \mathcal{X} \to \mathbb{R}$ and a lottery $p \in \mathcal{P}$, let $u \circ p = E_p(u(x)) \in \mathbb{R}$, where E_p denotes the expectation operator with respect to p. Given a utility function $u : \mathcal{X} \to \mathbb{R}$ and an act $f \in \mathcal{F}$, let $u \circ f = (u \circ (f(s)))_{s \in \mathcal{S}} \in \mathbb{R}^{\mathcal{S}}$.

Given a binary relation $\succsim$ on $\mathcal{F}$, let $\succ$ and $\sim$ denote its asymmetric and symmetric components, respectively. As a benchmark we recall the classical axioms and representation of the Subjective Expected Utility (henceforth SEU) model in our Anscombe-Aumann setting setting.

Axiom 1 (Completeness)**.** For all $f, g \in \mathcal{F}$, $f \succsim g$ or $g \succsim f$.

Axiom 2 (Transitivity)**.** For all $f, g, h \in \mathcal{F}$, if $f \succsim g$ and $g \succsim h$ then $f \succsim h$.

Axiom 3 (Non-Triviality)**.** There exist $f, g \in \mathcal{F}$ such that $f \succ g$.

Axiom 4 (Monotonicity)**.** For all $f, g \in \mathcal{F}$, if $f(s) \succsim g(s)$ for all $s \in \mathcal{S}$ then $f \succsim g$.

Axiom 5 (Mixture Continuity)**.** For all $f, g, h \in \mathcal{F}$, the sets $\{\lambda \in [0, 1] : \lambda f + (1 - \lambda)g \succsim h\}$ and $\{\lambda \in [0, 1] : h \succsim \lambda f + (1 - \lambda g)\}$ are closed.

Axiom 6 (Independence)**.** For all $f, g, h \in \mathcal{F}$ and $\lambda \in (0, 1)$, if $f \succsim g$ then $\lambda f + (1 - \lambda)h \succsim \lambda g + (1 - \lambda)h$.

Anscombe and Aumann (1963) showed that a binary relation $\succsim$ on $\mathcal{F}$ satisfies these axioms if and only if, for all $f, g \in \mathcal{F}$,

$$f \succsim g \Leftrightarrow E_m(u \circ f) \geq E_m(u \circ g), \tag{1}$$

where $u : X \to \mathbb{R}$ in non-constant and $m \in \Delta(\mathcal{S})$ (u is unique up to a positive affine transformation and m). We then say that $\succsim$ is an *SEU preference relation* and call *SEU representation* of $\succsim$ any couple (u, m) satisfying (1).

3 Preference and unambiguous preference

We consider agents (individuals and society) described by the following decison model. First, each agent is endowed with a *preference* relation $\succsim$ on $\mathcal{F}$. We assume that $\succsim$ is an *MBA preference relation* (for Monotone, Bernoullian, Archimedean), i.e. that it satisfies the axioms of the SEU model except for Independence, which is weakened as follows.

Axiom 7 (Risk Independence)**.** For all $p, q, r \in \mathcal{P}$ and $\lambda \in (0, 1)$, if $p \succsim q$ then $\lambda p + (1 - \lambda)r \succsim \lambda q + (1 - \lambda)r$.

The MBA class thus constrains preferences over lotteries to conform to the Expected Utility model but essentially leaves preferences over uncertain acts unconstrained, except through Monotonicity. Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2011) showed that a binary relation $\succsim$ on $\mathcal{F}$ satisfies these axioms if and only if, for all $f, g \in \mathcal{F}$,

$$f \succsim g \Leftrightarrow J(u \circ f) \geq J(u \circ g), \tag{2}$$

where $u : \mathcal{X} \to \mathbb{R}$ is non-constant and $J : \text{conv}(u(\mathcal{X}))^{\mathcal{S}} \to \mathbb{R}$ is monotonic, continuous, and normalized (u is unique up to a positive affine transformation, and J is unique given u).[5] We call *MBA representation* of $\succsim$ any couple (u, J) satisfying (2). Besides the SEU model, many popular ambiguity models fall within the MBA class and correspond to specifications of the MBA functional J, such as the Maxmin Expected Utility (henceforth MEU; Gilboa and Schmeidler, 1989), Choquet Expected Utility (henceforth CEU; Schmeidler, 1989), and Smooth Ambiguity (Klibanoff, Marinacci, and Mukerji, 2005) models.

Second, each agent is also endowed with an *unambiguous preference* relation $\succsim^*$ on $\mathcal{F}$ identifying the preference comparisons that are unaffected by ambiguity. For instance, in the classical Ellsberg (1961) 3-color urn experiment with 30 red balls and 60 black or yellow balls in unknown proportions, and $f_{s,x} \in \mathcal{F}$ denoting a bet yielding payoff $x \in \mathbb{R}$ if a ball of color $s \in \{R, B, Y\}$ is drawn, an individual who perceives (and dislikes) ambiguity would display $f_{B,100} \succ f_{B,50}$ and $f_{R,100} \succ f_{B,100}$. At the same time, since only the former comparison is affected by perceived ambiguity, (s)he would display $f_{B,100} \succ^* f_{B,50}$ but not $f_{R,100} \succ^* f_{B.100}$.

We assume that $\succsim^*$ is a *Bewley preference relation*, i.e. that is satisfies the axioms of the SEU model except for Completeness, which is weakened as follows.[6]

Axiom 8 (Risk Completeness)**.** For all $p, q \in \mathcal{P}$, $p \succsim q$ or $q \succsim p$.

Thus, contrary to $\succsim$, $\succsim^*$ can violate Completeness but must satisfy Independence. The former point allows for perceived ambiguity, which is only prevented by Risk Completeness from affecting preferences between lotteries. The latter one prevents an unambiguous preference from being reversed by mixing, in line with the usual interpretation of such reversals in terms of *hedging*. In the above example, the individual would display $\frac{1}{2}f_{B,100} + \frac{1}{2}f_{Y,100} \succ \frac{1}{2}f_{R,100} + \frac{1}{2}f_{Y,100}$ since mixing with $f_{Y,100}$ hedges the ambiguity of $f_{B,100}$ but not of $f_{R,100}$, thereby reversing the (ambiguous) preference $f_{R,100} \succ f_{B,100}$. Bewley (2002) showed that a binary relation $\succsim^*$ on $\mathcal{F}$ satisfies these axioms if and only if, for all $f, g \in \mathcal{F}$,

$$f \succsim^* g \Leftrightarrow \left[E_m(u \circ f) \geq E_m(u \circ g) \text{ for all } m \in M\right], \tag{3}$$

where $u : \mathcal{X} \to \mathbb{R}$ is non-constant and $M \subseteq \Delta(\mathcal{S}$ is non-empty, compact, and convex (u is unique up to a positive affine transformation and M is unique). (3) states that f is unambiguously preferred to g if and only if f has a higher subjective expected utility than g for all probability distributions in some set M, which is then interpreted as capturing the uncertainty perceived by the agent (we also say that M is the set of *relevant priors* for the agent). Clearly, M is a

[5] conv denotes convex hull. That J is monotonic means that for all $f, g \in \mathcal{F}$, if $u \circ f(s) \geq u \circ g(s)$ for all $s \in \mathcal{S}$ then $J(u \circ f) \geq J(u \circ g)$. That J is normalized means that $J(u \circ p) = u \circ p$ for all $p \in \mathcal{P}$.

[6] This weakening is usually named "Certainty Completeness". We rename it here for clarity.

singleton if and only if $\succsim^*$ is an SEU preference relation (i.e. if and only if the agent perceives no ambiguity). We call *Bewley representation* of $\succsim$ any couple (u, M) satisfying (3).

We finally assume that the two relations $\succsim$ and $\succsim^*$ are consistent in the following sense.

Axiom 9 (Consistency)**.** For all $f, g \in \mathcal{F}$, if $f \succsim^* g$ then $f \succsim g$.

We say that $(\succsim, \succsim^*)$ is a *consistent MBA-Bewley pair* if $\succsim$ is an MBA preference relation, $\succsim^*$ is a Bewley preference relation, and $(\succsim, \succsim^*)$ satisfies Consistency. Consistent MBA-Bewley pairs admit a joint representation. To state it, we say that an act $f \in \mathcal{F}$ is $\succsim^*$*-crisp* if $f \sim^* p$ for some $p \in \mathcal{P}$.

Proposition 1 (Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi, 2011)**.** A couple $(\succsim, \succsim^*)$ of binary relations on $\mathcal{F}$ is a consistent MBA-Bewley pair if and only if there exist a non-constant function $u : \mathcal{X} \to \mathbb{R}$, a non-empty, compact, convex set $M \subseteq \Delta(\mathcal{S})$ and a function $\alpha : \mathcal{F} \to [0, 1]$ such that:

- The functional $J : \mathrm{conv}(u(\mathcal{X}))^{\mathcal{S}} \to \mathbb{R}$ defined by, for all $f \in \mathcal{F}$,

$$J(u \circ f) = \alpha(f) \min_{m \in M} E_m(u \circ f) + (1 - \alpha(f)) \max_{m \in M} E_m(u \circ f) \tag{4}$$

  is monotonic, continuous, and normalized,

- (u, J) is an MBA representation of $\succsim$,

- (u, M) is a Bewley representation of $\succsim^*$.

Moreover, u is unique up to a positive affine transformation, M is unique, and α is unique on $\succsim^*$-non-crisp acts.[7]

(4) states that $\succsim$ admits a representation in the spirit of Hurwicz (1951): the utility of f is a convex combination of the minimum and maximum subjective expected utilities of f (over the same set M of relevant priors and with the same utilty function u that represent $\succsim^*$), the weight $\alpha(f)$ being interpreted as an index of *ambiguity aversion* for act f. We call *generalized Hurwicz representation* of $(\succsim, \succsim^*)$ any triple (u, M, α) satisfying the three properties of Proposition 1.

Introducing $\succsim^*$ as an additional primitive relation (related to $\succsim$ through Consistency) enables it to embody additional data with respect to $\succsim$, pertaining for instance to objective information (Gajdos, Hayashi, Tallon, and Vergnaud, 2008) or objective rationality (Gilboa, Maccheroni, Marinacci, and Schmeidler, 2010). Alternatively, it is possible to take $\succsim$ as only primitive and derive $\succsim^*$ from it (provided this derivation yields a Bewley preference relation and respects Consistency), as we now illustrate.

Example 3. Ghirardato, Maccheroni, and Marinacci (2004) and Nehring (2007) define the unambiguous preference relation $\succsim^{*\mathrm{GMMN}}$ by, for all $f, g \in \mathcal{F}$,

$$f \succsim^{*\mathrm{GMMN}} g \Leftrightarrow [\lambda f + (1 - \lambda) h \succsim \lambda g + (1 - \lambda) h \text{ for all } h \in \mathcal{F} \text{ and } \lambda \in (0, 1]] \,. \tag{5}$$

[7] An act f is crisp if and only if the minimum and the maximum agree in (4), in which case the coefficient $\alpha(f)$ is irrelevant. This is a straightforward extension of Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2011)'s result, which focuses on the "only if" part and on the specific unambiguous preference relation $\succsim^{*\mathrm{GMMN}}$ defined below.

$\succsim^{*\mathrm{GMMN}}$ deems unambiguous all preferences that cannot be reversed by mixing. It is thus the most complete unambiguous preference that can be derived from $\succsim$: if $\succsim^*$ is a Bewley preference relation and $(\succsim, \succsim^*)$ satisfies Consistency then $f \succsim^* g$ implies $f \succsim^{*\mathrm{GMMN}} g$ for all $f, g \in \mathcal{F}$. Equivalently, in terms of generalized Hurwicz representation, $\succsim^{*\mathrm{GMMN}}$ yields the smallest possible set of relevant priors.

Example 4. Klibanoff, Mukerji, and Seo (2014) consider an MBA preference relation $\hat{\succsim}$ on the set $\hat{\mathcal{F}}$ of simple acts $\hat{f} : \mathcal{S}^\infty \to \mathcal{P}$, satisfying an "Event Symmetry" axiom stating that the agent views all ordinates of Ω as identical as well as a "Monotone Continuity" axiom. They derive from $\succsim$ a non-constant function $u : \mathcal{X} \to \mathbb{R}$ as well as a non-empty, closed set $R \subseteq \Delta(\mathcal{S})$ of "relevant mesaures" as follows: a measure $m \in \Delta(\mathcal{S})$ is deemed relevant if for all neighborhood M of m, the event that the limiting frequency over $\mathcal{S}$ lies in M is non-null. We may then consider the consistent MBA-Bewley pair $(\succsim, \succsim^{*\mathrm{KMS}})$ where $\succsim^{*\mathrm{KMS}}$ admits Bewley representation $(u, \mathrm{conv}(R))$ and for all $f, g \in \mathcal{F}$, $f \succsim g$ if and only if $\hat{f} \hat{\succsim} \hat{g}$ for some $\hat{f}, \hat{g} \in \hat{\mathcal{F}}$ contingent on a single ordinate of $\mathcal{S}^\infty$ and agreeing with f and g, respectively, on this ordinate. In generaal, $\succsim^{*\mathrm{KMS}}$ is less complete (or equivalently, yields a larger set of relevant priors) than $\succsim^{*\mathrm{GMMN}}$.

4 Paretian Aggregation

We consider a finite set $\mathcal{I} = \{1, \ldots, |\mathcal{I}|\}$ of at least two individuals and let $\mathcal{I}' = \{0\} \cup \mathcal{I}$, where 0 stands for society itself. Each $i \in \mathcal{I}$ is endowed with an MBA preference relation $\succsim_i$ on $\mathcal{F}$. We say that an individual $i \in \mathcal{I}$ is *null* if there exist no $p, q \in \mathcal{P}$ such that $p \succ_0 q$ and $p \sim_j q$ for all $j \in \mathcal{I} \setminus \{i\}$. We assume that the profile $(\succsim_i)_{i \in \mathcal{I}}$ of individual preference relations satisfies the following axiom.

Axiom 10 (Risk Diversity)**.** For all $i \in \mathcal{I}$, there exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ and $p \sim_j q$ for all $j \in \mathcal{I} \setminus \{i\}$.

This is a standard axiom in the preference aggregation literature.[8] For some results we only require the following, weaker axiom.[9]

Axiom 11 (Risk Minimal Agreement)**.** There exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ for all $i \in \mathcal{I}$.

We shall restrict attention throughout to Pareto criteria involving weak preferences only, but the usual variants involving strict preferences could be straightforwardly accomodated and we omit the details. We start with the standard Pareto Dominance axiom, as well as a restriction of this axiom to the subdomain of lotteries.

Axiom 12 (Pareto Dominance)**.** For all $f, g \in \mathcal{F}$, if $f \succsim_i g$ for all $i \in \mathcal{I}$ then $f \succsim_0 g$.

Axiom 13 (Risk Pareto Dominance)**.** For all $p, q \in \mathcal{P}$, if $p \succsim_i q$ for all $i \in \mathcal{I}$ then $p \succsim_0 q$.

Since MBA preferences satisfy Risk Independence, it is a direct consequence of Harsanyi's aggregation theorem that Risk Pareto Dominance is necessary and sufficient for linear aggregation of tastes.

[8] It is usually named "Independent Prospects". We rename it here for clarity.

[9] Note that this axiom implies in particular that $\succsim_i$ is non-trivial for all $i \in \mathcal{I}$. To check that it is weaker than Risk Diversity, simply take two lotteries $p_i, q_i \in \mathcal{P}$ as in Risk Diversity for all $i \in \mathcal{I}$ and mix them with strictly positive weights.

Proposition 2 (Harsanyi, 1955)**.** Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Risk Pareto Dominance if and only if, for all MBA representations $(u_i, J_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}^{\mathcal{I}}_+$ and $\gamma \in \mathbb{R}$ such that

$$u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma. \tag{6}$$

Moreover, if $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity then θ and γ are unique given $(u_i)_{i\in\mathcal{I}'}$ and an individual $i \in \mathcal{I}$ is null if and only if $\theta_i = 0$.

On the other hand, if $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity then $(\succsim_i)_{i\in\mathcal{I}'}$ cannot satisfy Pareto Dominance (that is, on the whole domain of acts) in general. This was shown by Mongin (1998) within the class of SEU preferences. For such preferences, Pareto Dominance can only be satisfied if $m_0 = m_i$ for all non-null individuals $i \in \mathcal{I}$.[10] The following example illustrates this impossibility of simultaneously aggregating heterogeneous tastes and beliefs.

Example 5. Let $\mathcal{S} = \{s_1, s_2\}$, $\mathcal{X} = \{x, y, z\}$, and $\mathcal{I} = \{1, 2\}$. Assume that $\succsim_i$ is an SEU preference relation with SEU representation (u_i, m_i) for all $i \in \{0, 1, 2\}$, where

$$\begin{aligned} &u_1(x) = 1, u_1(y) = 0, u_1(z) = 0, && m_1(s_1) = \tfrac{1}{4}, \\ &u_2(x) = 0, u_2(y) = 1, u_2(z) = 0, && m_2(s_1) = \tfrac{3}{4}. \end{aligned}$$

Note that $(\succsim_i)_{i=1,2}$ satisfies Risk Diversity. Then $(\succsim_i)_{i=0,1,2}$ can only satisfy Pareto Dominance if either 1 or 2 is null. Indeed, suppose that $\theta_1\theta_2 > 0$ in (6) and, assuming without loss of generality that $\theta_1 + \theta_2 = 1$, define the acts $f, g \in \mathcal{F}$ by

$$\begin{aligned} &f(s_1) = p, f(s_2) = q, && \text{where} && p = \theta_2 x + \theta_1 z, \\ &g(s_1) = q, g(s_2) = p, && && q = \theta_1 y + \theta_2 z. \end{aligned}$$

Note that $p \succ_1 q$ and, hence, $g \succ_1 f$ since $m_1(s_2) > m_1(s_1)$. Similarly, $q \succ_2 p$ and, hence, $g \succ_2 f$ since $m_2(s_1) > m_2(s_2)$. On the other hand, $p \sim_0 q$ and, hence, $f \sim_0 g$ independently of m_0, so Pareto Dominance is violated.[11]

Mongin uses the term *spurious unanimity* to describe situations as in this example: individuals unanimously rank g above f but for "different reasons" (opposite differences in tastes and beliefs). Under Risk Diversity, there are always acts inducing spurious unanimity while being socially indifferent, leading to violations of Pareto Dominance.

Gilboa, Samet, and Schmeidler (2004) argue that Pareto Dominance, besides yielding an impossibility, is also less compelling when unanimity results from disagreeing beliefs, and proceed to restrict it to "common-belief" acts. In a Savage setting, they show that this restriction allows to aggregate heterogeneous beliefs and is, more precisely, characterized by linear aggregation of beliefs.

[10] In this common-belief case, any act $f \in \mathcal{F}$ can be identified with the lottery $\sum_{s\in\mathcal{S}} m(s)f(s)$, where $m \in \Delta(\mathcal{S})$ is the common belief, so Pareto Dominance reduces to Risk Pareto Dominance.

[11] Strictly speaking, this only violates the "strict" version of Pareto Dominance. However, it is easy to obtain a violation of the "weak" version by replacing g with $(1-\varepsilon)g + \varepsilon z$ for a small enough $\varepsilon \in (0, 1)$. We omit the details here and in subsequent examples.

Moving outside the SEU class to consider ambiguity preferences gives rise to another type of impossibility, extending even to the common-belief case. This was shown by Gajdos, Tallon, and Vergnaud (2008, henceforth GTV) within the classes of MEU and CEU preferences. For such preferences, Pareto Dominance can only be satisfied if $\succsim_i$ is an SEU preference relation for all non-null individuals $i \in \mathcal{I}$, provided there are least two such individuals.[12] GTV also provide an example of Smooth Ambiguity preferences in which in which the same impossibility arises. The following example illustrates this general impossibility within the class of MEU preferences.

Example 6. Consider again Example 5 but assume now that $\succsim_i$ is an MEU preference relation with MEU representation (u_i, M_i) for all $i \in \{0, 1, 2\}$, where u_1 and u_2 are as above and

$$M_1 = M_2 = \left\{m \in \Delta(\mathcal{S}) : \tfrac{1}{4} \leq m(s_1) \leq \tfrac{3}{4}\right\}.$$

Then even though individuals now have the same beliefs, $(\succsim_i)_{i=0,1,2}$ can only satisfy Pareto Dominance if either 1 or 2 is null. Indeed, suppose that $\theta_1\theta_2 > 0$ in (6) and consider the act f as above and the lottery $r = \frac{1}{2}p + \frac{1}{2}q \in \mathcal{P}$. Then $r \succ_1 f$ since $p \succ_1 q$ and $\min_{m \in M_1} m(s_1) < \frac{1}{2}$. Similarly, $r \succ_2 f$ since $q \succ_2 p$ and $\min_{m \in M_2} m(s_2) < \frac{1}{2}$. On the other hand, $r \sim_0 f$ independently of M_0 since $p \sim_0 q$, so Pareto Dominance is violated.

In line with Mongin's terminology, we refer to situations as in this example as *spurious hedging*: $r = \frac{1}{2}f + \frac{1}{2}g$ hedges the ambiguity of f for all individuals but for "different reasons" (1 ranks p above q and is therefore concerned with $m(s_1)$ being low when evaluating f, and the opposite for 2). Under Risk Diversity, there are always acts inducing spurious hedging while being socially unambiguous, leading to violations of Pareto Dominance.

Another interpretation of this example is that ambiguity by itself induces some form of disagreement in beliefs: even though all individuals have the same set of priors, they use different priors to evaluate f ($m(s_1) = \frac{1}{4}$ for 1 and $m(s_1) = \frac{3}{4}$ for 2). We may therefore argue, along the lines of Gilboa, Samet, and Schmeidler (2004), that Pareto Dominance is also less compelling when unanimity results from spurious hedging.

5 Unambiguous Pareto Dominance

We now introduce our main axiom.

Axiom 14 (Unambiguous Pareto Dominance)**.** For all $f, g \in \mathcal{F}$, if $f \succsim_i^* g$ for all $i \in \mathcal{I}$ then $f \succsim_0^* g$.

The axiom states that if all individuals *unambiguously* prefer f to g then so does society. Since preferences over lotteries are always unambiguous, Unambiguous Pareto Dominance implies Risk Pareto Dominance. On the other hand, Unambiguous Pareto Dominance weakens Pareto Dominance by only constraining social preferences when individual preferences are both

[12] By Mongin's result, these individuals must in addition share the same beliefs. GTV show more generally that within the wider class of "Rank-Dependent Additive" preferences, these individuals must have " uncertainty-neutral betting preferences".

unanimous and unambiguous.[13] Intuitively, this weakening makes the axiom robust to the situations of spurious hedging that arise when individuals are not uncertainty-neutral, since an unambiguous preference is by definition robust to hedging (thus in Example 6, we have $r \succ_i f$ but not $r \succ_i^* f$, for $i = 1, 2$). Formally, we obtain the following aggregation result for MBA preferences.

Theorem 1. Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Unambiguous Pareto Dominance if and only if, for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}_+^{\mathcal{I}}$ and $\gamma \in \mathbb{R}$ such that (6) holds and

$$M_0 \subseteq \bigcap_{i\in\mathcal{I},\theta_i>0} M_i. \tag{7}$$

(7) states that each relevant prior for society must also be relevant for each non-null individual. Thus Unambiguous Pareto Dominance allows aggregation of amgibuity preferences, even with different sets of relevant priors, as long as they share at least one relevant prior. When this is the case we say that individual beliefs are *compatible*. If there are several common priors then society can adopt any subset of common priors. In particular, society can have SEU preferences even if all individuals have ambiguity preferences. If there is exactly one common prior then society must have SEU preferences with this prior. In particular, any non-null individual with SEU preferences forces society to have SEU preferences with his/her prior.

Theorem 1 shows that Unambiguous Pareto Dominance is characterized, besides linear aggregation of tastes, by a relationship between individual and social beliefs, independently of ambiguity attitudes. This is intuitive since unambiguous preferences are by definition unaffected by ambiguity attitudes. In the particular case where all individuals have SEU preferences and, hence, perceive no ambiguity, Unambiguous Pareto Dominance reduces to Pareto Dominance and we obtain the following generalization of Mongin's result, in which society is shown rather than assumed to have SEU preferences.

Corollary 1. Assume that $\succsim_0$ is an MBA preference relation on $\mathcal{F}$ and $\succsim_i$ is an SEU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}$, and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Pareto Dominance if and only if $\succsim_0$ is an SEU preference relation and, for all SEU representations $(u_i, m_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}_+^{\mathcal{I}}$ and $\gamma \in \mathbb{R}$ such that (6) holds and $m_0 = m_i$ for all non-null individuals $i \in \mathcal{I}$.

Except in this SEU case, (7) does not force non-null individuals with ambiguity preferences to have identical sets of relevant priors. In other words, Unambiguous Pareto Dominance is compatible not only with situations of spurious hedging but also with some situations of spurious unanimity, namely those in which some individual preferences are ambiguous. Yet the allowed heterogeneity in beliefs is limited as aggregation is impossible when non-null individuals have incompatible beliefs.

[13]On the other hand, the social preference for f over g must then be unambiguous. However, it would be equivalent to require a mere social preference for f over g by Ghirardato, Maccheroni, and Marinacci (2004, Proposition 4), so the axiom is indeed weaker than Pareto Dominance.

In order to allow aggregation of incompatible beliefs, a further weakening of Unambiguous Pareto Dominance is needed, which makes it robust to unambiguous spurious unanimity. To state it, we say that $f, g \in \mathcal{F}$ are *common-taste acts* if $p \succsim_i q \Leftrightarrow p \succsim_j q$ for all $i, j \in \mathcal{I}$ and $p, q \in \text{conv}(f(\mathcal{S}) \cup g(\mathcal{S}))$. Intuitively, common-taste acts are acts involving only lotteries over which all individuals have the same cardinal preferences.

Axiom 15 (Common-Taste Unambiguous Pareto Dominance)**.** *For all common-taste acts $f, g \in \mathcal{F}$, if $f \succsim_i^* g$ for all $i \in \mathcal{I}$ then $f \succsim_0^* g$.*

The axiom states that if all individuals *unambiguously* prefer f to g *and f and g are common-taste acts* then so does society. This weakening to common-taste acts is compatible with situations of spurious unanimity, since such situations arise from simultaneous differences in tastes and beliefs. Note that Common-Taste Unambiguous Pareto Dominance can alternatively be viewed as the unambiguous analogue of the following axiom.

Axiom 16 (Common-Taste Pareto Dominance)**.** *For all common-taste acts $f, g \in \mathcal{F}$, if $f \succsim_i g$ for all $i \in \mathcal{I}$ then $f \succsim_0 g$.*

This latter axiom was recently introduced by Qu (2014), who showed that it is characterized by linear aggregation of beliefs.[14] For general MBA preferences we obtain the following characterization of Common-Taste Unambiguous Pareto Dominance.

Theorem 2. *Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Unambiguous Pareto Dominance if and only if, for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$,*

$$M_0 \subseteq \text{conv}\left(\bigcup_{i\in\mathcal{I}} M_i\right). \tag{8}$$

(8) states that any socially relevant prior must be a convex combination of some individually relevant priors. The weights in these combinations are not unique in general.[15] Note that Risk Pareto Dominance is not implied by Common-Taste Unambiguous Pareto Dominance and should therefore be explicitly assumed in order to obtain linear aggregation of tastes in addition to (8).

Thus Common-Taste Unambiguous Pareto Dominance allows aggregation of ambiguity preferences even with incompatible beliefs. As in Theorem 1, society can have SEU preferences even if all individuals have ambiguity preferences. The converse is now also possible: society can have ambiguity preferences even if all individuals have SEU preferences, in which case social ambiguity results from individual heterogeneity in beliefs. We formally state this corollary, which generalizes Qu's result.

[14] Qu's axiom is stated in a slightly weaker form, by restricting Pareto Dominance to acts involving only mixtures of two exogenously given lotteries over which individuals have a unanimous strict preference. His characterization is the same that was obtained by Gilboa, Samet, and Schmeidler (2004) in a Savage setting, by restricting Pareto Dominance to "common-belief" rather than common-taste acts. In our Anscombe-Aumann setting, such a restriction to common-belief acts would be weaker than the restriction to common-taste acts.

[15] A sufficient but very demanding condition for uniqueness of the weights is that any collections $(m_i, m_i')_{i\in\mathcal{I}} \in \bigcup_{i\in\mathcal{I}} M_i^2$ with $m_i \neq m_i'$ for some $i \in \mathcal{I}$ is linearly independent.

Corollary 2. Assume that $\succsim_0$ is an MBA preference relation on $\mathcal{F}$ and $\succsim_i$ is an SEU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}$, and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Unambiguous Pareto Dominance if and only if, for all generalized Hurwicz representation (u_0, M_0, α_0) of $\succsim_0$ and SEU representations $(u_i, m_i)_{i\in\mathcal{I}}$ of $(\succsim_i)_{i\in\mathcal{I}}$,

$$M_0 \subseteq \operatorname{conv}\left(\{m_i : i \in \mathcal{I}\}\right). \tag{9}$$

6 Applications

Ghirardato, Maccheroni, and Marinacci (2004) and Ghirardato and Siniscalchi (2012) provide explicit computations of the set of relevant priors for various subclasses of MBA preferences. We use these computations to translate Theorems 1 and 2 in terms of the corresponding representations.

6.1 MEU preferences

MEU preferences correspond to MBA functionals of the form

$$J(u \circ f) = \min_{m\in M} E_m(u \circ f), \tag{10}$$

where $M \subseteq \Delta(\mathcal{S})$ is non-empty, compact, convex. M is unique and is in fact the set of relevant priors (Ghirardato, Maccheroni, and Marinacci, 2004; Nehring, 2007). Hence Theorems 1 and 2 translate verbatim in terms of the MEU representations $(u_i, M_i)_{i\in\mathcal{I}}$ of $(\succsim_i)_{i\in\mathcal{I}}$.

Theorem 1 then stands in contrast with the GTV's impossibility result: weakening Pareto Dominance to Unambiguous Pareto Dominance allows aggregation of MEU preferences, provided the set of priors are compatible. Theorem 2, on the other hand, can be compared with an analogous result recently obtained by Qu (2014). Building on results from Crès, Gilboa, and Vieille (2011), Qu introduces a strengthening of Common-Taste Pareto Dominance and shows that it is characterized by linear aggregation of the sets of priors, i.e.

$$M_0 = \left\{ \sum_{i\in\mathcal{I}} \lambda_i m_i : (m_i)_{i\in\mathcal{I}} \in \prod_{i\in\mathcal{I}} M_i \right\} \tag{11}$$

for some $\lambda \in \Delta(\mathcal{I})$. No exact characterization of Common-Taste Pareto Dominance is known. We first note that for MEU preferences, Common-Taste Pareto Dominance (and, hence, Qu's axiom) is stronger than Common-Taste Unambiguous Pareto Dominance (see Proposition 6 in the Appendix). The following proposition helps assess the additional content of the former with respect to the latter.

Proposition 3. Assume that $\succsim_i$ is an MEU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. If $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance then, for all MEU representations $(u_i, M_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, (8) holds and for all $(m_i)_{i\in\mathcal{I}} \in \prod_{i\in\mathcal{I}} M_i$,

$$M_0 \cap \operatorname{conv}(\{m_i : i \in \mathcal{I}\}) \neq \emptyset. \tag{12}$$

Thus, Common-Taste Pareto Dominance implies not only that social beliefs must be linear combinations of individual beliefs, but also that any possible selection of individual priors must be aggregated into a social prior (and Qu's axiom further implies that all these selections are aggregated through the same vector of individual weights). This requires the social set of priors to be sufficiently large, and in particular we must have $\bigcap_{i\in\mathcal{I}} M_i \subseteq M_0$. Hence, for instance, if individuals share more than one prior then society can have SEU preferences under Common-Taste Unambiguous Pareto Dominance but not under Common-Taste Pareto Dominance.

6.2 CEU preferences

CEU preferences correspond to MBA functionals of the form

$$J(u \circ f) = \int_{\mathcal{S}} u \circ f d\nu, \tag{13}$$

where $\nu : 2^{\mathcal{S}} \to [0,1]$ is a capacity and the integral is taken in the sense of Choquet.[16] Equivalently, if we write $\mathcal{S} = \{s_1, \dots, s_N\}$ and define, for all $\sigma \in \mathrm{perm}(N)$, the probability distribution $m_{\nu,\sigma} \in \Delta(\mathcal{S})$ by

$$m_{\nu,\sigma}(s_n) = \nu(\{s_{\sigma(1)}, \dots, s_{\sigma(n)}\}) - \nu(\{s_{\sigma(1)}, \dots, s_{\sigma(n-1)}\})$$

for all $n = 1, \dots, N$, then we have

$$J(u \circ f) = E_{m_{\nu,\sigma}}(u \circ f) \tag{14}$$

for any $\sigma \in \mathrm{perm}(N)$ such that $u \circ f(s_{\sigma(1)}) \geq \dots \geq u \circ f(s_{\sigma(N)})$.[17] The capacity ν is unique and

$$M = \mathrm{conv}\left(\{m_{\nu,\sigma} : \sigma \in \mathrm{perm}(N)\}\right) \tag{15}$$

(Ghirardato, Maccheroni, and Marinacci, 2004). Thus a decision-maker with CEU preferences reveals a set of relevant priors and evaluates an act by selecting an element of this set depending on the ranking of states induced by his preferences over the lotteries this acts yields. The capacity ν incorporates both perceived ambiguity and ambiguity attitude, in the sense that

$$\nu(S) = \alpha(f) \min_{m\in M} m(S) + (1 - \alpha(f)) \max_{m\in M} m(S) \tag{16}$$

for any event $S \subseteq \mathcal{S}$ and any act $f \in \mathcal{F}$ such that $f(s) = p$ for all $s \in S$ and $f(s) = q$ for all $s \in \mathcal{S} \setminus S$, where $p, q \in \mathcal{P}$ are such that $p \succ q$. We now translate Theorems 1 and 2 in terms of the CEU representations $(u_i, \nu_i)_{i\in\mathcal{I}}$ of $(\succsim_i)_{i\in\mathcal{I}}$.

Corollary 3. Assume that $\succsim_i$ is a CEU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Unambiguous Pareto Dominance if and only if, for all CEU representations $(u_i, \nu_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}^{\mathcal{I}}_+$ and $\gamma \in \mathbb{R}$ such that (6)

[16]That ν is a capacity means that $\nu(\emptyset) = 0$, $\nu(\mathcal{S}) = 1$, and $\nu(S) \leq \nu(T)$ whenever $S \subseteq T$.

[17]$\mathrm{perm}(N)$ denotes the set of all permutations of $\{1, \dots, N\}$.

holds and for all $\sigma \in \mathrm{perm}(N)$,

$$m_{\nu_0,\sigma} \in \bigcap_{i\in\mathcal{I},\theta_i>0} \mathrm{conv}(\{m_{\nu_i,\tau} : \tau \in \mathrm{perm}(N)\}). \tag{17}$$

Corollary 4. Assume that $\succsim_i$ is a CEU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Unambiguous Pareto Dominance if and only if, for all CEU representations $(u_i, \nu_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$ and all $\sigma \in \mathrm{perm}(N)$,

$$m_{\nu_0,\sigma} \in \mathrm{conv}(\{m_{\nu_i,\tau} : \tau \in \mathrm{perm}(N), i \in \mathcal{I}\}). \tag{18}$$

Again, Corollary 3 stands in contrast with the GTV's impossibility result, and Corollary 4 can be compared with an analogous result recently obtained by Qu (2014). Indeed, Qu shows that for CEU preferences, his strengthening of Common-Taste Pareto Dominance is characterized by linear aggregation of the capacities, i.e.

$$\nu_0 = \sum_{i\in\mathcal{I}} \lambda_i \nu_i \tag{19}$$

for some $\lambda \in \Delta(\mathcal{I})$. No exact characterization of Common-Taste Pareto Dominance is known. For CEU preferences, Common-Taste Pareto Dominance (and, hence, Qu's axiom) is again stronger than Common-Taste Unambiguous Pareto Dominance (see Proposition 6 in the Appendix). The following proposition helps assessing the additional content of the former with respect to the latter.

Proposition 4. Assume that $\succsim_i$ is a CEU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. If $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance then, for all CEU representations $(u_i, \nu_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$ and all $\sigma \in \mathrm{perm}(N)$,

$$m_{\nu_0,\sigma} \in \mathrm{conv}(\{m_{\nu_i,\sigma} : i \in \mathcal{I}\}). \tag{20}$$

(20) strengthens (18) by requiring that the prior used by society to evaluate an act be a linear combination of individual priors *corresponding to the same ordering of states* (note that these priors are generally different from the priors individuals use to evaluate the same act). This reflects the fact that Common-Taste Pareto Dominance (and Qu's axiom) jointly relates social beliefs and ambiguity attitudes to individual beliefs and ambiguity attitudes. Common-Taste Unambiguous Pareto Dominance, on the other hand, only relates social beliefs to individual beliefs, independently of ambiguity attitudes.

6.3 Smooth Ambiguity preferences

Smooth Ambiguity preferences correspond to MBA functionals of the form

$$J(u \circ f) = \phi^{-1}(E_\mu(\phi(E_m(u \circ f)))), \tag{21}$$

where $\phi : \mathrm{conv}(u(\mathcal{X})) \to \mathbb{R}$ is continuous and strictly increasing and μ is a countably additive probability measure over $\Delta(\mathcal{S})$. ϕ is unique up to a positive affine transformation given u, and

μ is unique. Moreover, if ϕ is continuously differentiable then

$$M = \mathrm{cl}\left(\mathrm{conv}\left(\left\{\left(\frac{E_\mu(m(s)\phi'(E_m(u\circ f)))}{E_\mu(\phi'(E_m(u\circ f)))}\right)_{s\in\mathcal{S}} : f\in\mathrm{int}(\mathcal{F})\right\}\right)\right). \tag{22}$$

(Ghirardato and Siniscalchi, 2012).[18] Again, this expression can be used to translate Theorems 1 and 2 in terms of the Smooth Ambiguity representations $(u_i, \phi_i, \mu_i)_{i\in\mathcal{I}}$ of $(\succsim_i)_{i\in\mathcal{I}}$.

A natural interpretation of the Smooth Ambiguity representation is that the second-order prior μ captures perceived ambiguity whereas the second-order utility function ϕ captures ambiguity attitude. This interpretation leads to consider the support of μ as reflecting the ambiguity perceived by the decision-maker. (22) then shows that the separation between ambiguity and ambiguity attitude provided by the unambiguous preference relation does not agree with the one provided by the Smooth Ambiguity representation. More precisely, it is always the case that

$$M \subseteq \mathrm{cl}(\mathrm{conv}(\mathrm{supp}(\mu))), \tag{23}$$

but the converse does not hold in general.[19]

7 Aggregating ambiguity attitudes

Besides aggregating tastes and beliefs, it may also be of interest to aggregate ambiguity attitudes. That is to say, society may want to react to social ambiguity depending on how individuals react to individual ambiguity. In this section we simply show that such an aggregation is possible, by stating and characterizing an elementary axiom relating individual and social ambiguity attitudes.

Given a preference relation $\succsim$ on $\mathcal{F}$ and an act $f\in\mathcal{F}$, we say that a lottery $\underline{p}\in\mathcal{P}$ is a *lower certainty-equivalent* of f if

$$f \succsim^* p \Leftrightarrow \underline{p} \succsim p$$

for all $p\in\mathcal{P}$, and that a lottery $\overline{p}\in\mathcal{P}$ is an *upper certainty-equivalent* of f if

$$p \succsim^* f \Leftrightarrow p \succsim \overline{p}$$

for all $p\in\mathcal{P}$. That is to say, a lower certainty-equivalent of f is a best lottery that is unambiguously worse than f, wheres an upper certainty-equivalent of f is a worst lottery that is better than f. In terms of generalized Hurwicz representation, we have

$$u\circ\underline{p} = \min_{m\in M} E_m(u\circ f), \qquad u\circ\overline{p} = \max_{m\in M} E_m(u\circ f).$$

[18] cl and int denote closure and interior, respectively. This result (and a more general one weakening the continuous differentiability assumption) appears in an unpublished version of the paper.

[19] $\mathrm{supp}(\mu)$ denotes the support of μ. To check (23), simply note that the binary relation represented by $\mathrm{cl}(\mathrm{conv}(\mathrm{supp}(\mu)))$ in the sense of (**??**) is a sub-relation of $\succsim$ satisfying Independence and, hence, is a sub-relation of $\succsim^*$. Ghirardato and Siniscalchi (2012) show that if $u(\mathcal{X})$ is unbounded (which is impossible in our setting) then (23) holds with equality when ϕ is a CARA utility function.

We therefore have $\overline{p} \succsim f \succsim \underline{p}$, with $\overline{p} \sim f \sim \underline{p}$ if and only if f is $\succsim$-crisp. Thus for an SEU preference relation, lower and upper certainty-equivalents coincide for all acts, whereas for an ambiguity preferencd there are always acts for which they do not. For such acts, we then have

$$f \sim \alpha(f)\underline{p} + (1-\alpha(f))\overline{p}. \tag{24}$$

Let us consider the following axiom.

Axiom 17 (Uncertainy-Adjusted Pareto Dominance)**.** For all $f \in \mathcal{F}$, all lower and upper certainty equivalents $(\underline{p}_i, \overline{p}_i)_{i\in\mathcal{I}} \in (\mathcal{P}^2)^{\mathcal{I}'}$, and all $\lambda \in [0,1]$,

- if $f \succsim_i \lambda\underline{p}_i + (1-\lambda)\overline{p}_i$ for all $i \in \mathcal{I}$ then $f \succsim_0 \lambda\underline{p}_0 + (1-\lambda)\overline{p}_0$,
- if $\lambda\underline{p}_i + (1-\lambda)\overline{p}_i \succsim_i f$ for all $i \in \mathcal{I}$ then $\lambda\underline{p}_0 + (1-\lambda)\overline{p}_0 \succsim_0 f$.

The axiom states that if a λ-mixture of lower and uppoer certainty-equivalents of f is ranked below (resp. above) f for all individuals then so is it for society. Note that the mixing coefficient λ is fixed whereas the lower and upper certainty-equivalents are specific to each individual and society. Thus the axiom is a form of Pareto criterion involving an act and a lottery, but the lottery is adjusted for each individual and society in order to compensate for differences in percceived ambiguity. In view of (24), we immediately obtain the following characterization.

Theorem 3. Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Uncertainty-Adjusted Pareto Dominance if and only if, for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$ and all $f \in \mathcal{F}$ that is not crisp for 0,

$$\alpha_0(f) \in \operatorname{conv}(\{\alpha_i(f) : i \in \mathcal{I}, f \text{ is not crisp for } i\}). \tag{25}$$

(25) states that the social ambiguity aversion index for a socially non-crisp act is a convex combination of the ambiguity aversion indices for this act of all individuals for which the act is not crisp. In other words, society must be less ambigtuity averse than the most ambiguity averse invidual and more ambiguity averse than the least ambiguity averse individual.

The aggregation of ambiguity attitudes is independent of the aggregation of beliefs (and tastes). Thus, for instance, society could disregard individual beliefs while taking into account individual ambiguity attitudes. On the other hand, the aggregation results for tastes, beliefs, and attitudes can be combined. As an illustration, the most general combination of our axioms yields the following corollary.

Corollary 5. Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Risk Pareto Dominance, Common-Taste Unambiguous Pareto Dominance, and Uncertainty-Adjusted Pareto Dominance if and only if, for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}^{\mathcal{I}}_+$ and $\gamma \in \mathbb{R}$ such that (6), (8), and (25) hold.

A Proofs

A.1 Preliminaries

We briefly recall two representation results for incomplete preferences, which will be used to prove Theorems 1 and 2. Second, define a *State-Dependent Expected Multi-Utility preference relation* (henceforth SDEMU) on $\mathcal{F}$ as one satisfying the axiom s of the SEU model except Monotonicity and Completeness, the latter being weakened as follows.

Axiom 18 (Reflexivity)**.** For all $f \in \mathcal{F}$, $f \succsim f$.

Note that a Bewley preference relation is an SDEMU preference relation since Risk Completeness and Monotonicity together imply Reflexivity.

Proposition 5 (Nau, 2006)**.** A binary relation $\succsim$ on $\mathcal{F}$ satisfies Reflexivity, Transitivity, Mixture Continuity, Independence if and only if there exists a non-empty, closed, convex set $W \subseteq \mathbb{R}^{\mathcal{X}\times\mathcal{S}}$ such that for all $f, g \in \mathcal{F}$,

$$f \succsim g \Leftrightarrow \left[\sum_{s\in\mathcal{S}}(w(\cdot,s)\circ f(s)) \geq \sum_{s\in\mathcal{S}}(w(\cdot,s)\circ g(s)) \text{ for all } w \in W\right]. \tag{26}$$

Moreover, $\mathrm{cl}(\mathrm{cone}(W)+\mathbb{R}^{\mathcal{S}})$ is unique.[20]

We call *SDEMU representation* of $\succsim$ any set W satisfying (26).

A.2 Proof of Theorems 1 and 2

Since the unambiguous preference relation associated with an MBA prerference relation is a Bewley preference relation, Theorems 1 and 2 are direct consequences of the following results on aggregation of Bewley preferences, respectively.

Theorem 4. Assume that $\succsim_i$ is a Bewley preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Pareto Dominance if and only if, for all Bewley representations $(u_i, M_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}^{\mathcal{I}}_+$ and $\gamma \in \mathbb{R}$ such that (6) and (7) hold.

Theorem 5. Assume that $\succsim_i$ is a Bewley preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance if and only if, for all Bewley representations $(u_i, M_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, (8) holds.

We note that these results are also of separate interest since Bewley incomplete preferences allow to model either individual indecisiveness or social inability to fully observe individual preferences (Danan, Gajdos, and Tallon, 2013, 2014). In order to prove these results, we first establish a more general result on aggregation of SDEMU preferences.

[20] cone denotes conical hull. We abuse notation by identifying $\mathbb{R}^{\mathcal{S}}$ with the set of functions $w \in R^{\mathcal{X}\times\mathcal{S}}$ such that $w(\cdot,s)$ is constant for all $s \in \mathcal{S}$.

Lemma 1. Assume that $\succsim_i$ is an SDEMU preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$. Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Pareto Dominance if and only if, for all SDEMU representations $(W_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$,

$$W_0 \subseteq \mathrm{cl}\left(\sum_{i\in\mathcal{I}} \mathrm{cone}(W_i) + \mathbb{R}^{\mathcal{S}}\right). \tag{27}$$

Proof. The proof is a straightforward generalization of the proof of the aggregation theorem in Danan, Gajdos, and Tallon (2014). For all $i \in \mathcal{I}'$, the set

$$K_i = \{a(f-g) : a \in \mathbb{R}_+, f, g \in \mathcal{F}, f \succsim_i g\}$$

is a non-empty, closed, convex cone in $\mathbb{R}^{\mathcal{X}\times\mathcal{S}}$, orthogonal to $\mathbb{R}^{\mathcal{S}}$, and a non-empty, closed, convex set $W_i \subseteq \mathbb{R}^{\mathcal{X}\times\mathcal{S}}$ is a SDEMU representation of $\succsim_i$ if and only if $\mathrm{cl}(\mathrm{cone}(W_i) + \mathbb{R}^{\mathcal{S}}) = K_i^*$.[21] Moreover, $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Pareto Dominance if and only if $\bigcap_{i\in\mathcal{I}} K_i \subseteq K_0$, which is equivalent to $K_0^* \subseteq \mathrm{cl}(\sum_{i\in\mathcal{I}} K_i^*)$ (Rockafellar, 1970, Corollary 16.4.2) and, hence, to (27) for all SDEMU representations $(W_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}''}$. □

Next, we now show that the closure operator in (27) is redundant if $\succsim_i$ is a Bewley preference relation for all $i \in \mathcal{I}$ and $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Given $u : \mathcal{X} \to \mathbb{R}$ and $m \in \Delta(\mathcal{S})$, define $w_{u,m} : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$ by $w_{u,m}(x,s) = m(s)u(x)$. Given $u : \mathcal{X} \to \mathbb{R}$ and $M \subseteq \Delta(\mathcal{S})$, let $W_{u,M} = \{w_{u,m} : m \in M\}$. Note that if $\succsim$ is a Bewley prefrence relation and (u, M) is a Bewley representation of $\succsim$, then $W_{u,M}$ is a SDEMU representation of $\succsim$.

Lemma 2. Assume that $\succsim_i$ is a Bewley preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement. Then for all Bewley representations $(u_i, M_i)_{i\in\mathcal{I}}$ of $(\succsim_i)_{i\in\mathcal{I}}$, $\sum_{i\in\mathcal{I}} \mathrm{cone}(W_{u_i,M_i}) + \mathbb{R}^{\mathcal{S}}$ is closed.

Proof. Assume that there exist $p, q \in \mathcal{P}$ such taht $p \succ_i q$ for all $i \in \mathcal{I}$. Fix Bewley representations $(u_i, M_i)_{i\in\mathcal{I}}$ of $(\succsim_i)_{i\in\mathcal{I}}$. We first show that $\mathrm{cone}(W_{u_i,M_i}) + \mathbb{R}^{\mathcal{S}}$ is a closed, convex cone for all $i \in \mathcal{I}$. Convexity is obvious. For closedness, fix $i \in \mathcal{I}$. Since $p \succ_i q$, u_i is non-constant. Hence $0 \notin W_{u_i,M_i}$ and, hence, $\mathrm{cone}(W_{u_i,M_i})$ is closed since W_{u_i,M_i} is convex and compact (Rockafellar, 1970, Corollary 9.6.1). Moreover, $\mathrm{cone}(W_{u_i,M_i}) \cap \mathbb{R}^{\mathcal{S}} = \{0\}$, so $\mathrm{cone}(W_{u_i,M_i}) + \mathbb{R}^{\mathcal{S}}$ is closed (Rockafellar, 1970, Corollary 9.1.3).

It remains to show that the sum over $\mathcal{I}$ of these closed, convex cones is itself closed. To this end it is sufficient to show that the cones $(K_i)_{i\in\mathcal{I}}$ defined in the proof of Lemma 1 have a common point in their relative interiors (Rockafellar, 1970, Corollary 16.4.2) or, equivalently, that $(\succsim_i)_{i\in\mathcal{I}}$ satisfy the following property: there exist $f, g \in \mathcal{F}$ such that, for all $i \in \mathcal{I}$ and $g_i \in \mathcal{F}$ such that $f \succsim_i g_i$, there exist $g_i' \in \mathcal{F}$ and $\lambda_i \in (0,1)$ such that $f \succsim_i g_i'$ and $g = \lambda_i g_i + (1-\lambda_i) g_i'$ (Danan, Gajdos, and Tallon, 2014).

To establish this property, recall that $p \succ_i q$ for all $i \in \mathcal{I}$. By Mixture Continuity and since $\mathcal{X}$ is finite, for all $i \in \mathcal{I}$, there exists an open neighborhood P_i of q in $\mathcal{P}$ such that $p \succ_i q'$ for all $q' \in P_i$. Let $P = \bigcap_{i\in\mathcal{I}} P_i$, so that $p \succ_i q'$ for all $q' \in P$ and $i \in \mathcal{I}$. Since $\mathcal{I}$ is finite, there exists a lottery $r \in P \cap \mathrm{int}(\mathcal{P})$. Now, fix $i \in \mathcal{I}$ and $g_i \in \mathcal{F}$ such that $p \succsim_i g_i$. Given $\lambda \in (0,1)$, let $g_i' = \frac{1}{1-\lambda} r - \frac{\lambda}{1-\lambda} g$, so that $r = \lambda g_i + (1-\lambda) g_i'$. Since $\mathcal{S}$ is finite, there exists $\lambda \in (0,1)$ small

[21] K_i^* denotes the dual cone of K_i, i.e. $K_i^* = \{w \in \mathbb{R}^{\mathcal{X}\times\mathcal{S}} : \sum_{(x,s)\in\mathcal{X}\times\mathcal{S}} w(x,s)k(x,s) \geq 0 \text{ for all } k \in K_i\}$.

enough so that $g_i' \in P^{\mathcal{S}} \subset \mathcal{F}$ and, hence, $p \succ_i g_i'(s)$ for all $s \in \mathcal{S}$. By Monotonicity, it follows that $p \succsim_i g'$. $\square$

We are now ready to prove Theorems 4 and 5.

Proof of Theorem 4. Obviously, (6) and (7) imply that $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Pareto Dominance. Conversely, assume $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Pareto Dominance. Fix Bewley representations $(u_i, M_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$. First, restricting attentino to lotteries and using Pareto Dominance and Risk Diversity, we apply Harsanyi (1955)'s Aggregation Theorem to obtain unique $\theta \in \mathbb{R}_+^{\mathcal{I}}$ and $\gamma \in \mathbb{R}$ such that (6) holds. Second, fix $m_0 \in M_0$. In order to establish (7), it is sufficient to show that $m_0 \in M_i$ for all $i \in \mathcal{I}$ such that $\theta_i > 0$. To this end, note that by Lemmas 1 and 2, there exist $(m_i)_{i\in\mathcal{I}} \in \prod_{i\in\mathcal{I}} M_i$, $\theta' \in \mathbb{R}_+^{\mathcal{I}}$, and $c \in \mathbb{R}^{\mathcal{S}}$ such that

$$w_{u_0,m_0} = \sum_{i\in\mathcal{I}} \theta_i' w_{u_i,m_i} + c$$

and, hence,

$$m_0(s)u_0(x) = \sum_{i\in I} \theta_i' m_i(s)u_i(x) + c(s) \tag{28}$$

for all $s \in \mathcal{S}$ and $x \in \mathcal{X}$. Summing over $\mathcal{S}$ yields

$$u_0(x) = \sum_{i\in I} \theta_i' u_i(x) + \sum_{s\in\mathcal{S}} c(s)$$

for all $x \in \mathcal{X}$, so that $\theta = \theta'$ and $\gamma = \sum_{s\in\mathcal{S}} c(s)$. Hence (28) implies that

$$m_0(s)(u_0 \circ p - u_0 \circ q) = \sum_{i\in\mathcal{I}} \theta_i m_i(s)(u_i \circ p - u_i \circ q)$$

and, hence, using (6), that

$$\sum_{i\in\mathcal{I}} \theta_i(m_0(s) - m_i(s))(u_i \circ p - u_i \circ q) = 0 \tag{29}$$

for all $s \in \mathcal{S}$ and $p, q \in \mathcal{P}$. Now, fix $i \in \mathcal{I}$ such that $\theta_i > 0$. By Risk Diversity, there exist $p, q \in \mathcal{P}$ such that $u_i \circ p > u_i \circ q$ and $u_j \circ p = u_j \circ q$ for all $j \in \mathcal{I} \setminus \{i\}$. By (29), it follows that $m_0(s) = m_i(s)$ for all $s \in \mathcal{S}$, so that $m_0 = m_i \in M_i$. $\square$

Proof of Theorem 5. Obviously, (8) implies that $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance. Conversely, assume $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance. Fix Bewley representations $(u_i, M_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$. By Risk Minimal Agreement, there exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ for all $i \in \mathcal{I}$. Hence all acts in $\operatorname{conv}(\{p, q\})^{\mathcal{S}}$ are common-taste acts. It follows that $p \succ_0 q$ by Common-Taste Pareto Dominance, so that individual and social preferences all agree on $\operatorname{conv}(\{p, q\})$. Hence for all $i \in \mathcal{I}$, there exist $a_i \in \mathbb{R}_{++}$ and $b_i \in \mathbb{R}$ such that

$$u_i \circ r = (a_i u_0 + b_i) \circ r \tag{30}$$

for all $r \in \text{conv}(\{p, q\})$. We can therefore use Common-Taste Pareto Dominance to show, as in the proof of Theorem 4, that for all $m_0 \in M_0$, there exist $(m_i)_{i \in \mathcal{I}} \in \prod_{i \in \mathcal{I}} M_i$, $\theta' \in \mathbb{R}_+^{\mathcal{I}}$, and $c' \in \mathbb{R}^{\mathcal{S}}$ such that

$$m_0(s) u_0 \circ r = \sum_{i \in \mathcal{I}} \theta'_i m_i(s) u_i \circ r + c'(s) \tag{31}$$

for all $s \in \mathcal{S}$ and $r \in \text{conv}(\{p, q\})$. Summing over $\mathcal{S}$ and using (30) yields

$$u_0 \circ r = \sum_{i \in \mathcal{I}} \theta'_i u_i \circ r + \sum_{s \in \mathcal{S}} c'(s) = \sum_{i \in \mathcal{I}} \theta'_i a_i u_0 \circ r + \sum_{i \in \mathcal{I}} \theta'_i b_i + \sum_{s \in \mathcal{S}} c'(s)$$

for all $r \in \text{conv}(\{p, q\})$, so that $\sum_{i \in \mathcal{I}} \theta'_i a_i = 1$ and $\sum_{i \in \mathcal{I}} \theta'_i b_i = -\sum_{s \in \mathcal{S}} c'(s)$ since u_0 is non-constant on $\text{conv}(\{p, q\})$. Hence (31) implies that

$$m_0(s)(u_0 \circ p - u_0 \circ q) = \sum_{i \in \mathcal{I}} \theta'_i m_i(s)(u_i \circ p - u_i \circ q) = \sum_{i \in \mathcal{I}} \theta'_i m_i(s) a_i (u_0 \circ p - u_0 \circ q)$$

and, hence, that

$$m_0(s) = \sum_{i \in \mathcal{I}} \theta'_i a_i m_i(s)$$

for all $s \in \mathcal{S}$, so that $m_0 = \sum_{i \in \mathcal{I}} \theta'_i a_i m_i$. Let $\lambda = (\theta'_i a_i)_{i \in \mathcal{I}} \in \mathbb{R}^{\mathcal{I}}$. Since $\theta'_i \geq 0$ and $a_i > 0$ for all $i \in \mathcal{I}$ and $\sum_{i \in \mathcal{I}} \theta'_i a_i = 1$, we have $\lambda \in \Delta(\mathcal{I})$ and, hence, $m_0 \in \text{conv}(\{m_i : i \in \mathcal{I}\})$, establishing (8). $\square$

A.3 Proof of Propositions 3 and 4

We first show that Common-Taste Pareto Dominance implies Common-Taste Unambiguous Pareto Dominance within the classes of MEU and CEU preferences. Ghirardato, Maccheroni, and Marinacci (2004) define an *Invariant Biseparable preference relation* on $\mathcal{F}$ as one satisfying the axioms of the SEU model except Independence, which is weakened to the "Certainty Independence" of Gilboa and Schmeidler (1989). The Invariant Biseparable class is contained in the MBA class and contains the MEU and CEU classes.

Proposition 6. Assume that $\succsim_i$ is an Invariant Biseparable preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$. If $(\succsim_i)_{i \in \mathcal{I}'}$ satisfies Common-Taste Paredo Dominance then $(\succsim_i)_{i \in \mathcal{I}'}$ satisfies Unambiguous Common-Taste Paredo Dominance.

Proof. Assume $(\succsim_i)_{i \in \mathcal{I}'}$ satisfies Common-Taste Paredo Dominance. Fix two common-taste acts $f, g \in \mathcal{F}$ and let $P = \text{conv}(f(\mathcal{S}) \cup g(\mathcal{S}))$. Given a binary relation $\succsim$ on $\mathcal{F}$, let $\succsim|_P$ denote its restriction to $P^{\mathcal{S}}$. Clearly, $(\succsim_i|_P)_{i \in \mathcal{I}'}$ satisfies Common-Taste Paredo Dominance and, hence, Common-Taste Unambiguous Pareto Dominance, i.e. $(\succsim_i|_P^*)_{i \in \mathcal{I}'}$ satisfies Common-Taste Paredo Dominance. We want to show that $(\succsim_i|_P)_{i \in \mathcal{I}'}$ satisfies Common-Taste Unambiguous Paredo Dominance, i.e. that $(\succsim_i^*|_P)_{i \in \mathcal{I}'}$ satisfies Common-Taste Paredo Dominance. To this we show that $\succsim|_P^* = \succsim^*|_P$ for any Invariant Biseparable preference relation $\succsim$ on $\mathcal{F}$.

If $\succsim|_P$ then the result trivially follows from Monotonicity, so suppose it is not. By Ghirardato, Maccheroni, and Marinacci (2004)'s Lemma 1, there then exists a non-constant function $u : \mathcal{X} \to \mathbb{R}$ and a monotonic, constant-linear functional $J' : \mathbb{R}^{\mathcal{S}} \to \mathbb{R}$ such that

$$f' \succsim g' \Leftrightarrow J'(u \circ f') \geq J'(u \circ g')$$

for all $f', g' \in \mathcal{F}$. Moreover, J' is unique (independently of u). Hence J' is also the unique such functional such that

$$f' \succsim|_P g' \Leftrightarrow J'(u \circ f') \geq J'(u \circ g')$$

for all $f', g' \in P^{\mathcal{S}}$. By Ghirardato, Maccheroni, and Marinacci (2004)'s Theorem 14, it follows that $\succsim$ and $\succsim|_P$ induce the same set of relevant priors (namely the Clarke differential of J' at 0). Hence $\succsim|_P^* = \succsim^*|_P$ by (**??**). $\square$

Proof of Proposition 3. Assume $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance. Fix MEU representations $(u_i, M_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$. Then (8) holds by Proposition 6 and Theorem 2. We prove (12) by contradiction.

Suppose there exists $(m_i)_{i\in\mathcal{I}} \in \prod_{i\in\mathcal{I}} M_i$ such that $M_0 \cap \mathrm{conv}(\{m_i : i \in \mathcal{I}\}) = \emptyset$. There then exist $c \in \mathbb{R}^{\mathcal{S}}$ and $d \in \mathbb{R}$ such that $\sum_{s\in\mathcal{S}} c(s)m(s) > d$ for all $m \in M_0$ and $\sum_{s\in\mathcal{S}} c(s)m_i(s) \leq d$ for all $i \in \mathcal{I}$. We can assume without loss of generality that $|c(s)| \leq \frac{1}{2}$ for all $s \in \mathcal{S}$ and $|d| \leq \frac{1}{2}$. Moreover, by Risk Minimal Agreement, there exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ for all $i \in \mathcal{I}$ and, hence, $p \succ_0 q$ by Common-Taste Pareto Dominance. Define $r \in \mathcal{P}$ and $f \in \mathcal{F}$ by

$$r = \left(\tfrac{1}{2} + d\right) p + \left(\tfrac{1}{2} - d\right) q, \qquad\qquad f(s) = \left(\tfrac{1}{2} + c(s)\right) p + \left(\tfrac{1}{2} - c(s)\right) q$$

for all $s \in \mathcal{S}$. Note that f and r are common-taste acts. Moreover, for all $m \in \Delta(\mathcal{S})$ and $u \in \mathbb{R}^{\mathcal{X}}$,

$$E_m(u \circ f) - u \circ r = \left(\sum_{s\in\mathcal{S}} c(s)m(s) - d\right)(u \circ p - u \circ q).$$

Hence

$$\min_{m\in M_i} E_m(u_i \circ f) - \min_{m\in M_i} E_m(u_i \circ r) = \min_{m\in M_i} E_m(u_i \circ f) - u_i \circ r \leq E_{m_i}(u_i \circ f) - u_i \circ r \leq 0$$

for all $i \in \mathcal{I}$, whereas

$$\min_{m\in M_0} E_m(u_0 \circ f) - \min_{m\in M_0} E_m(u_0 \circ r) = \min_{m\in M_0} E_m(u_0 \circ f) - u_0 \circ r > 0$$

since M_0 is compact. Hence $r \succsim_i f$ for all $i \in \mathcal{I}$ and $f \succ_0 r$, contradicting Common-Taste Pareto Dominance. $\square$

Proof of Proposition 4. Assume $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Pareto Dominance. Fix CEU representations $(u_i, \nu_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$. We prove (20) by contradiction.

Suppose there exists $\sigma \in \mathrm{perm}(N)$ such that $m_{\nu_0,\sigma} \notin \mathrm{conv}(\{m_{\nu_i,\sigma} : i \in \mathcal{I}\})$. There then exist $c \in \mathbb{R}^{\mathcal{S}}$ and $d \in \mathbb{R}$ such that $\sum_{s\in\mathcal{S}} c(s)m_{\nu_0,\sigma}(s) > d$ and $\sum_{s\in\mathcal{S}} c(s)m_{\nu_i,\sigma}(s) \leq d$ for all $i \in \mathcal{I}$. We can assume without loss of generality that $|c(s)| \leq \frac{1}{N+1}$ for all $s \in \mathcal{S}$ and $|d| \leq \frac{1}{N+1}$. By Risk Minimal Agreement, there exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ for all $i \in \mathcal{I}$ and, hence, $p \succ_0 q$ by Common-Taste Pareto Dominance. Define $f, g \in \mathcal{F}$ by

$$f(s_{\sigma(n)}) = \left(\tfrac{N+1-n}{N+1} + d\right) p + \left(\tfrac{n}{N+1} - d\right) q,$$
$$g(s_{\sigma(n)}) = \left(\tfrac{N+1-n}{N+1} + c(s)\right) p + \left(\tfrac{n}{N+1} - c(s)\right) q,$$

for all $n = 1 \ldots N$. Note that f and g are common-taste acts and that $u_i \circ f(s_{\sigma(1)}) \geq \ldots \geq u_i \circ f(s_{\sigma(N)})$ and $u_i \circ g(s_{\sigma(1)}) \geq \ldots \geq u_i \circ g(s_{\sigma(N)})$ for all $i \in \mathcal{I}'$. Hence

$$\begin{aligned}\int_{\mathcal{S}} u_i \circ g d\nu - \int_{\mathcal{S}} u_i \circ f d\nu &= E_{m_{\nu_i,\sigma}}(u \circ g) - E_{m_{\nu_i,\sigma}}(u \circ f)\\ &= \left(\sum_{s\in\mathcal{S}} c(s)m(s) - d\right)(u_i \circ p - u_i \circ q)\end{aligned}$$

for all $i \in \mathcal{I}'$. Hence $f \succsim_i g$ for all $i \in \mathcal{I}$ and $g \succ_0 f$, contradicting Common-Taste Pareto Dominance. □

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