Dynamic Coalitions*

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Abstract

We present a theory of dynamic coalitions for a legislative bargaining game in which policies can be changed in every period but continue in effect in the absence of new legislation. We characterize Markov perfect equilibria with dynamic coalitions, which are decisive sets of legislators whose members prefer to continue the current policy rather than change to another policy. The policies supported by dynamic coalitions satisfy necessary and sufficient existence conditions of *internal stability* and *exclusion risk*. Dynamic coalitions can be minimal winning or surplus and can award positive allocations to non-coalition members. The range of attainable dynamic payoffs is characterized, and the policies supported can be efficient or inefficient. Vested interests can support policies that no legislator would propose if forming a new coalition. If uncertainty is associated with policy implementation, a continuum of policies are supported. These equilibria have the same allocation in every period when the coalition persists. Dynamic coalitions also exist in which members tolerate a degree of implementation uncertainty, resulting in policies that can change without the coalition changing.

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1 Introduction

Many economic decisions involve dynamic considerations. Most laws and government programs are continuing and remain in effect in the absence of new legislation. Rules promulgated by regulatory commissions also remain in effect until modified or rescinded. Social security, welfare, and other redistributive programs are typically continuing, and distributions are governed by formulas that are changed only infrequently. Tax rates also continue in the absence of new legislation. At the state level, legislatures establish continuing policies including Medicaid eligibility and benefits, and regulatory commissions establish prices, rules governing lifeline and other cross-subsidization programs, and environmental and energy-efficiency policies. These programs have the property that the policy adopted or in place in the current period becomes the status quo for the next period.

Policy choice can be viewed as a dynamic bargaining game with an endogenous status quo in which a legislature has the opportunity to choose the policy in every period and agenda-setting control can change over time. Despite their dynamic nature and the opportunities for change, many policies are durable and are supported by coalitions that persist from one period to the next. Government formation in parliamentary systems also involves dynamic considerations and policies, and often government coalitions and their policies persist in both interelection and election periods. In the post-World War II period countries such as Germany, India, and Japan have experienced extended periods of stable governments, and despite Italy having 44 governments between 1948 and 1991 all government coalitions were led by the Christian Democratic Party and all excluded the Communist Party (Merlo, 1998; Nishikawa, 2012).

We study endogenous dynamic coalitions and the policies they support and provide an explanation for and characterization of durable policies and coalitions. We consider *basic strategies* with the property that legislators propose the status quo if it is in a particular set of policies that we call the *supported set*. If the status quo is not in the supported set, legislators propose a policy they most prefer in the supported set and randomize among coalition partners. For status quo policies not in the supported set there is uncertainty over which coalition will form in the future, and this uncertainty provides an incentive for current coalition members to maintain the status quo – to avoid the risk of being excluded from a future coalition. The coalition is dynamic because in equilibrium its members strictly prefer that the coalition and its policy continue, taking into account the future consequences of having the coalition end.

Despite the dynamic nature of the game, there are two straightforward conditions that are necessary and sufficient for a set of policies to be supported by a stationary Markov perfect equilibrium with dynamic coalitions using basic strategies: (i) *internal stability* of the supported set of policies and (ii) *exclusion risk*, which formalizes the preferences of the members of the dynamic coalition to preserve the current policy and coalition rather than risk exclusion when a new policy and coalition are chosen.[1] The exclusion risk is not player-specific but rather is collective in that all coalition members bear the risk when the policy changes and a new coalition formation round commences in the next period. The punishment for defection thus is collective. The risk of exclusion can be present in any dynamic voting game with a less than unanimity collective choice rule. The approach taken here is applicable to a broad class of dynamic political economy problems with a collective choice rule and players who face a risk of exclusion from future coalitions.

Studies of distributive politics typically find that benefits are allocated to more than a minimal majority of legislative districts even though a minimal winning coalition could do strictly better.[2] In parliamentary systems governments often include more parties than required for a majority in parliament, and minority governments are observed in a number of political systems.[3] The theory presented here shows that dynamic coalitions may be minimal winning or surplus, may allocate benefits to legislators not in the coalition, and can be interpreted as minority governments.

Distributive programs, such as those providing pork, are often viewed as inefficient, and dynamic coalitions can support inefficient policies. One source of inefficiency is due to a class of dynamic coalitions we call *vested interests*. These vested interests support a policy that no legislator would propose when forming a new coalition. That policy is supported because a decisive set of legislators fear the risk of exclusion and hence are willing to support it when it is the status quo even if it is Pareto inefficient. When inefficient policies are supported, dynamic coalitions result in political failure in the sense of Besley and Coate (1998).

Many policies have a degree of uncertainty associated with their implementation. The uncertainty could be due to exogenous factors or to endogenous factors associated with delegation to an administrative agency or regulatory commission or to choices made by those affected by the policy. The basic model is extended to include implementation uncertainty, and a class of specific-policy coalition equilibria is characterized in which a dynamic coalition persists as long as uncertainty is not realized and ends when it is realized. The dynamic coalition, while it persists, implements the same policy in each period, and the originator of the dynamic coalition shares the gains from proposal power with the coalition partner but not necessarily equally.

A specific-policy coalition ends when implementation uncertainty changes the policy, but a coalition could tolerate a degree of change due to uncertainty. Coalition equilibria exist that support a set of tolerated policies where the coalition persists if the policy remains in the set and ends if it is outside

[1] Diermeier, Swaab, Medvec and Kern (2008) refer to the second condition as the "fear of exclusion."

[2] See Primo and Snyder (2008), Stein and Bickers (1994), and Weingast (1994).

[3] See Ansolabehere, Snyder, Strauss and Ting (2005), Laver and Shepsle (1996), and Strom (1990) for theory and evidence on government formation.

the set. Because of the realized implementation uncertainty, the coalition partner could in some periods have a larger allocation than the originator of the coalition. Tolerant coalitions provide an explanation for coalition governments that survive small shocks but fail in crises.

The dynamic legislative bargaining game considered is an extension of the sequential legislative bargaining game introduced by Baron and Ferejohn (1989). In that game the stationary equilibrium outcome is reached with the first proposal, and the decisive set supporting the bargain is minimal winning. The proposer captures what otherwise would be the allocation of those legislators excluded from the decisive set and does not share the gains with other members of the decisive set. We show that with an endogenous status quo, a dynamic coalition forms in one-step and in the absence of implementation uncertainty persists thereafter. For simple majority rule and an odd number of legislators, the policy in the Baron and Ferejohn (1989) model cannot be supported in a coalition equilibrium because the internal stability condition is not satisfied; i.e., a legislator not in the coalition can propose a replacement policy that is attractive to a decisive set of legislators, so proposal power is mitigated within the dynamic coalition. With supermajority rule or with an even number of legislators, however, the Baron and Ferejohn (1989) equilibrium payoffs can be supported arbitrarily closely when legislators are sufficiently patient.

Kalandrakis (2004, 2010) first characterized Markov perfect equilibria for a dynamic legislative bargaining model, and in the equilibrium a rotating dictator captures all the surplus in a period. The equilibrium policies rotate because the indifference rule used specifies that legislators accept a proposal when indifferent, which allows a proposer to capture the entire surplus after the first two periods. For the same legislative bargaining model Battaglini and Palfrey (2012) assume that legislators accept a proposal with probability one-half when indifferent and for a finite policy space characterize quantal response equilibria that converge to rotating equilibria that approximate those of Kalandrakis (2004) when utilities are linear. In our equilibria policies are stable because legislators vote for the status quo when indifferent.

In our model stable policies are reached in no more than one step. Stability in one step is also present in Anesi (2010), Acemoglu, Egorov and Sonin (forthcoming), and Diermeier, Egorov and Sonin (2013), who characterize Markov perfect equilibria and relate them to the solution concept of stable sets. To do this, they use a finite policy space and a discount factor approximately equal to one. Diermeier, Egorov and Sonin (2013), for example, use von Neumann and Morgenstern (1944) internal and external stability conditions to characterize the set of supported policies. Anesi (2010), however, shows that external stability is not a necessary condition for a policy to be supported as a stationary Markov perfect equilibrium (MPE). Our exclusion risk condition is the necessary condition for existence of Markov perfect equilibria in basic strategies that is the analogue of external stability.

The relation between their approach and ours is discussed in more detail in Section 5.3.

Anesi and Seidmann (forthcoming) consider a legislative bargaining game and characterize stationary MPE, the outcomes of which are referred to as simple solutions. In equilibrium each legislator proposes a particular policy and collectively the policies are such that each legislator receives a low payoff in one of those policies. The low payoff poses a threat that plays the same role as our exclusion risk. They provide sufficient conditions under which simple solutions exist, and those solutions support almost all policies. We provide conditions that are necessary and sufficient for the existence of a coalition equilibrium and characterize the range of payoffs for all coalition equilibria. The equilibria in Anesi and Seidmann (forthcoming) need not have equilibrium coalitions in which each member includes the other in his coalition with positive probability. We characterize equilibria that support sets of policies in which all legislators have the same opportunities for payoffs and coalition members make offers to each other with positive probability. Despite the same opportunities, equilibrium payoffs can differ among coalition members, and coalitions ranging from minimal winning to universal can be supported in stationary MPE with basic strategies. The size-principle of Riker (1962) thus is not supported, since although minimal winning coalitions are supported in coalition equilibria, surplus coalitions are also be supported.

Bowen and Zahran (2012) and Richter (2014) identify equilibria that exhibit compromise where a surplus majority receives an allocation. Bowen and Zahran (2012) find compromise in a Markov perfect equilibrium with risk-averse legislators. When inefficient policies can be used as threats, Richter (2014) identifies Markov perfect equilibria in which all legislators share the benefits equally.

The challenge in this line of research is to identify equilibria with simple strategies and intuitive properties that can be applied in a variety of settings. We provide such a characterization and use it to study the dynamics of distributive policy and the stability and efficiency of those policies. The equilibria constructed are supported by the threat of collective punishment for all coalition members for any deviation from the equilibrium path, which allows a straightforward existence construction with a natural and focal interpretation.

The models discussed above are pure distribution, but continuing policies (or an endogenous status quo) have been considered with other policy spaces suggesting that the approaches used here are more broadly applicable. Baron (1996) considers a unidimensional policy space and provides a dynamic median voter theorem. Zápal (2012), Piguillem and Riboni (2012), and Bowen, Chen, Eraslan and Zápal (2015) also study equilibria in related models for a unidimensional policy space. Baron, Diermeier and Fong (2012) consider a multidimensional spatial dynamic government formation model with a proportional representation system in which Pareto-dominated policies can result. In these models parties act opportunistically and replace governments when indifferent, whereas in coalition equilibria parties maintain the status quo when indifferent. Baron (2014) considers a multidimensional spatial model of government formation and policy choice in a parliamentary system with proportional representation elections where parties maintain their governments when indifferent.[4]

Policy moderation in a dynamic policy-making environment has been studied in Dixit, Grossman and Gul (2000), Lagunoff (2001), and Acemoglu, Golosov and Tsyvinski (2011). Besley and Coate (1998), Battaglini and Coate (2007), Battaglini and Coate (2008), and Acemoglu, Egorov and Sonin (2012) show that dynamic incentives can lead to inefficiency. In our distributive policy setting we show that dynamic coalitions with basic strategies can support an equal distribution of benefits because the members of the coalition prefer that the current policy continue rather than risk exclusion from a new coalition with a Pareto dominated policy.

Duggan and Kalandrakis (2012) provide a general existence result for dynamic legislative bargaining games with an endogenous status quo when there is uncertainty over legislators' preferences.[5] In the environment considered here, legislators' preferences are fixed so the results of Duggan and Kalandrakis (2012) do not apply. We establish existence of equilibria by construction. Ray and Vohra (2014) embed a bargaining model using deterministic agenda setting and voting protocols that accommodate cooperative game solution concepts. Acemoglu, Egorov and Sonin (2012) and Diermeier, Egorov and Sonin (2013) use these protocols in their models. The random recognition rule used for selection of a proposer in our model is not in the class of protocols considered by Ray and Vohra (2014).

The basic model is introduced in the next section, and Section 3 presents the basic strategies used in the coalition equilibria. Section 4 presents necessary and sufficient conditions for the existence of coalition equilibria for the basic model and characterizes the set of attainable payoffs, and Section 5 presents some important special cases of dynamic coalitions. Section 6 introduces implementation uncertainty, and specific-policy coalition equilibria are characterized in Section 7. Section 8 characterizes tolerant coalition equilibria, and conclusions are provided in the final section.

2 The Basic Model

The model represents a political process with an endogenous status quo where in each period legislators can adopt a new policy or leave the status quo in place. Legislators in this model could be thought of as members of a unicameral legislature, party leaders in a parliamentary system forming

[4] Baron and Herron (2003), Bernheim, Rangel and Rayo (2006), Anesi (2010), Cho (2011, 2014), Diermeier and Fong (2011), Zápal (2012), Nunnari (2014), Nunnari and Zápal (2013), Bowen, Eraslan and Chen (2014), Dziuda and Loeper (forthcoming), Piguillem and Riboni (forthcoming), and Bowen (forthcoming) also consider bargaining games with an endogenous status quo.

[5] Duggan (2012) also proves a general existence result for MPE in noisy stochastic games that requires norm-continuity of state transition probabilities. Norm-continuity is violated with voting, however.

a government or governing once in office, factions or blocks of legislators with aligned preferences, or members of a commission bargaining each period over the allocation of a budget or a regulatory policy. In each period $t = 1, 2, \ldots$, legislator $i \in \{1, \ldots, n\}$, $n \geq 3$, is selected with probability $p = \frac{1}{n}$ to propose a policy, which is then voted against the status quo policy from the previous period according to an m-majority rule, where $\lceil \frac{n+1}{2} \rceil \leq m \leq n-1$. The winner becomes the policy in place in the current period and the status quo for the next period. In each period legislators allocate a dollar, possibly with waste, so the feasible set of policies in each period is $X = \{x \in \mathbb{R}^n_+ | \sum_{i=1}^n x_i \leq 1\}$. A proposal by a legislator in period t is a policy $y^t \in X$, and the status quo policy at the beginning of period t is denoted $q^{t-1} \in X$. The agenda on which legislators vote is $\{q^{t-1}, y^t\}$, and the implemented policy in period t is denoted by x^t and $q^t = x^t$. Legislator i derives utility $u(x_i^t)$ from the allocation he receives in period t, where u is increasing. Legislators maximize the expectation of the discounted, infinite stream of utilities $\sum_{t=1}^{\infty} \delta^{t-1} u(x_i^t)$, where $\delta \in [0, 1)$ is the discount factor. An extension in which discount factors and selection probabilities differ among legislators is presented in Appendix A, along with a generalization of the results presented in Section 4 for the basic model.

A history of the game includes all proposals made, the identity of the proposers, votes cast and policies implemented. A stationary Markov perfect equilibrium is a subgame perfect equilibrium in which strategies depend only on the payoff-relevant history, which at the proposal stage is the status quo q^{t-1}, and do not depend on calendar time. A stationary Markov strategy for legislator i is a pair of functions (σ_i, ω_i), where $\sigma_i : X \to X$ is a proposal strategy and $\omega_i : X \times X \to \{0, 1\}$ is a voting strategy.[6] Legislator i's proposal strategy $\sigma_i(q^{t-1}) = y^t$ selects a proposal y^t conditional on the status quo. Legislator i's voting strategy $\omega_i(q^{t-1}, y^t)$ assigns a vote conditional on the proposal and the status quo, where $\omega_i(q^{t-1}, y^t) = 1$ denotes a vote for the proposal. We denote a profile of strategies by (σ, ω).

A proposal is approved if and only if $\sum_{i=1}^n \omega_i(q^{t-1}, y^t) \geq m$. The status quo $q^t = (q_1^t, \ldots, q_n^t)$ in period $t+1$ is then

$$q^t = \begin{cases} q^{t-1} & \text{if } \sum_{i=1}^n \omega_i(q^{t-1}, y^t) < m \\ y^t & \text{if } \sum_{i=1}^n \omega_i(q^{t-1}, y^t) \geq m. \end{cases}$$

The status quo thus evolves as proposals are made and votes are cast. The next period's status quo policy q^t is therefore a function of strategies (σ, ω) and the current status quo q^{t-1}.

The continuation value $v_i(\sigma, \omega | q^{t-1})$ for i depends on t only through the state and is defined by

$$v_i(\sigma, \omega | q^{t-1}) = E^t[u(q_i^t) + \delta v_i(\sigma, \omega | q^t)],$$

where E^t denotes expectation with respect to the selection of the proposer, the proposal, and any

[6] We abuse notation slightly by writing proposal strategies as pure strategies. The coalition equilibria we characterize involve mixing, but the mixing among pure strategies is simple, so for clarity we use pure strategy notation.

uncertainty affecting payoffs and transitions.

A perfect equilibrium requires that in every subgame every legislator's dynamic payoff is optimal given the equilibrium strategies of the other legislators. To make this precise, we write $q^t(\sigma, \omega; q^{t-1})$ so that the conditioning of next period's status quo on strategies and the current status quo is explicit. A stationary Markov strategy profile (σ^*, ω^*) is a perfect equilibrium if and only if

1. Proposal strategies σ^* satisfy

$$\begin{aligned} &u(q_i^t(\sigma^*, \omega^*; q^{t-1})) + \delta v_i(\sigma^*, \omega^* | q^t(\sigma^*, \omega^*; q^{t-1})) \\ &\geq u(q_i^t(\hat{\sigma}_i, \sigma_{-i}^*, \omega^*; q^{t-1})) + \delta v_i(\sigma^*, \omega^* | q^t(\hat{\sigma}_i, \sigma_{-i}^*, \omega^*; q^{t-1})) \end{aligned}$$

   for all $i = 1, \ldots, n$ and all $q^{t-1} \in X$, and all $\hat{\sigma}_i$ where $\hat{\sigma}_i$ may depend on any history of actions and the status quo.

2. Voting strategies ω^* satisfy

$$\begin{aligned} &u(q_i^t(\sigma^*, \omega^*; q^{t-1})) + \delta v_i(\sigma^*, \omega^* | q^t(\sigma^*, \omega^*; q^{t-1})) \\ &\geq u(q_i^t(\sigma^*, \hat{\omega}_i, \omega_{-i}^*; q^{t-1})) + \delta v_i(\sigma^*, \omega^* | q^t(\sigma^*, \hat{\omega}_i, \omega_{-i}^*; q^{t-1})) \end{aligned}$$

   for all $i = 1, \ldots, n$ and all $q^{t-1} \in X$, and all $\hat{\omega}_i$, where $\hat{\omega}_i$ may depend on any history of actions and the status quo.

We restrict attention to equilibria in which proposals are accepted on the equilibrium path when the proposal is not the status quo. That is, $\sum_{j=1}^{n} \omega_j^*(\sigma_i^*; q^{t-1}) \geq m$ for all i and for all $q^{t-1} \in X$ when $\sigma_i^* \neq q^{t-1}$.[7] Henceforth we refer to a such a stationary Markov perfect equilibrium as an equilibrium.

In the next section we introduce a class of *basic strategies*, and stationary Markov perfect equilibria using basic strategies are referred to as *coalition equilibria*. The focus is on dynamic coalitions of legislators and the policies they support. In Section 4 conditions that are necessary and sufficient for the existence of a coalition equilibrium are presented, and the policies and dynamic coalitions that arise in coalition equilibria are characterized.

3 Basic Strategies and Coalition Equilibria

This section introduces a class of strategies that are simple, result in one-step equilibria, and have proposals in a subset of X. The strategies are symmetric and hence do not favor any particular legislator in coalition formation. Equilibria employing basic strategies are coalition equilibria.

We focus on supporting in equilibrium a symmetric set $Z \subset X$ of policies, and we call Z the *supported set*. Let $S \subset X$ be a set of policies, and let $\mathcal{Z}(\cdot)$ be the operator that returns the set

[7] The restriction that all equilibrium proposals are accepted does not affect the set of attainable payoffs. Any equilibrium in which a proposal is rejected for some status quo $\hat{q}^{t-1}$ is payoff equivalent to one in which the status quo $\hat{q}^{t-1}$ is proposed.

of all permutations of the policies in S. The set $Z = \mathcal{Z}(S)$ thus is symmetric and provides the same opportunities to each legislator, so no legislator is advantaged. As an example, let $n = 3$ and $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$. Then $\mathcal{Z}(S) = \{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2})\}$, and $Z = \mathcal{Z}(S)$ is symmetric. As a convention, we will order each element $z \in S$ so that $u(z_i) \geq u(z_{i+1})$, for $i = 1, \ldots, n-1$.[8]

To ensure basic strategies are well-defined, we assume Z is compact and denote the maximum allocation in Z as $z_{\max}$. With basic strategies the sets Z_i of policies proposed by i are those for which he receives the largest allocation; i.e., $Z_i = \{z \in Z : z_i = z_{\max}\}$. In the example $Z_1 = \{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2})\}$. In equilibrium, proposing policies in Z_i is equivalent to choosing coalition partners among whom the proposer is indifferent. To facilitate the characterization of equilibria, the proposal sets Z_i are assumed to be finite.

Definition 1. A *basic strategy* profile is a (σ, ω) such that for all $i = 1, \ldots, n$ and for some Z:

(i) Legislators propose the status quo if it is in the supported set Z and otherwise randomize over favorable policies in Z to form a new coalition

$$\sigma_i(q^{t-1}) = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in Z \\ z \in Z_i \text{ with probability } \frac{1}{|Z_i|} & \text{if } q^{t-1} \notin Z, \end{cases}$$

(ii) voting strategies are stage undominated and legislator i votes for the status quo when indifferent between the status quo and the proposal y^t

$$\omega_i(q^{t-1}, y^t) = \begin{cases} 1 & \text{if } u(y_i^t) + \delta v_i(\sigma, \omega | y^t) > u(q_i^{t-1}) + \delta v_i(\sigma, \omega | q^{t-1})] \\ 0 & \text{if } u(y_i^t) + \delta v_i(\sigma, \omega | y^t) \leq u(q_i^{t-1}) + \delta v_i(\sigma, \omega | q^{t-1}). \end{cases}$$

A basic strategy incorporates an *indifference rule* in (ii) under which a legislator votes for the status quo when indifferent between it and a proposal.

With basic strategies once the status quo is in the supported set Z, legislators continue to propose the status quo even if they are not members of the dynamic coalition because no other policy can receive a majority of votes. If the status quo is not in the supported set Z, the proposer j randomizes over the policies in Z_j, so a policy in the supported set is reached in one step and is stable thereafter. The proposer's indifference among all coalition partners is a conjecture that is verified for a coalition equilibrium. If there is a deviation from the equilibrium path to a policy not in Z, a new coalition formation round commences in the next period. Members of the dynamic coalition then face a risk of exclusion from the next coalition, which serves as a collective punishment for all members of the dynamic coalition. Legislators vote as if they are pivotal when their preferences are strict; i.e., as if

[8] This ordering is without loss of generality. If $z \in S$ is supported in a coalition equilibrium, then so are all permutations of z.

their vote determines the outcome. The indifference rule requires that legislators vote for the status quo when indifferent between it and a proposal, which assures the stability of the coalition.

We define a coalition equilibrium as a stationary Markov perfect equilibrium in which basic strategies are employed.

Definition 2. The strategy profile (σ, ω) is a *coalition equilibrium* if it is a stationary Markov perfect equilibrium using basic strategies.

We formally define a dynamic coalition in a coalition equilibrium as follows.

Definition 3. For a given (n, m, u) a *dynamic coalition in a coalition equilibrium* is a decisive set of legislators who support the continuation of the status quo policy from one period to the next rather than change to another policy that results in a new coalition formation round for some δ. Specifically, given a coalition equilibrium (σ, ω) supporting a set Z, a dynamic coalition $C(q^{t-1}; Z)$ corresponding to $q^{t-1} \in Z$ is a decisive set $C(q^{t-1}; Z) \equiv \{i | u(q_i^{t-1}) + \delta v_i(\sigma, \omega | q^{t-1}) > u(x_i) + \delta v_i(\sigma, \omega | x), \text{ for all } x \in X \setminus Z \text{ and some } \delta \in [0, 1)\}$.

In Section 4.3 we illustrate the identification of the members of a dynamic coalition.

To analyze dynamic coalitions and the policies they support, we ask which sets Z are supported in coalition equilibria. We provide an example in the next section and characterize coalition equilibria more generally in the following sections.

3.1 A Coalition Equilibrium Example

This example illustrates a coalition equilibrium, identifies sufficient conditions for existence, and identifies the members of the dynamic coalition. Let $n = 3$, $m = 2$, $u(x_i) = x_i$, and consider the set $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$ from which the supported set Z is generated. In a coalition equilibrium the status quo $q^{t-1} \in Z$ continues in place, so the continuation value for legislators i and j receiving $\frac{1}{2}$ in each period under the status quo is $\frac{1}{2(1-\delta)}$ and for the other legislator k it is 0.

Given $q^{t-1} \notin Z$, with probability $\frac{1}{3}$ the proposer i selected proposes a policy $y \in Z_i$, which in the conjectured equilibrium is approved. In expectation a legislator i obtains $\frac{2}{3}\left(\frac{1}{2(1-\delta)}\right) = \frac{1}{3(1-\delta)}$. Note that this is the continuation value for all policies not in Z. We show next that basic strategies constitute a Markov perfect equilibrium.

Suppose that $q^{t-1} = (\frac{1}{2}, \frac{1}{2}, 0)$ and legislator 3 is selected as the proposer and proposes to legislator 2 that they form a replacement coalition with policy $y = (0, \frac{1}{2}, \frac{1}{2})$ with continuation value $\frac{1}{2(1-\delta)}$. That is, a deviation to another policy in Z is proposed. Legislator 2 is pivotal and indifferent between voting for the status quo and the proposal, and according to the indifference rule in basic strategies, along with legislator 1 votes for the status quo sustaining the present coalition $\{1, 2\}$ and policy q^{t-1}. Since legislators 1 and 2 vote against any other proposal in Z, we say Z satisfies *internal stability*.[9]

Next suppose legislator 3 offers more than $\frac{1}{2}$ to legislator 2. That is, a policy outside the set Z is proposed. If 2 votes for the proposal, the dynamic coalition of 1 and 2 then ends and a new coalition formation round commences in the next period. Since the continuation value $\frac{1}{3(1-\delta)}$ is the same for all policies not in Z, to induce 2 to vote for the proposal, 3 offers the most possible, i.e., (0,1,0). The dynamic payoff to 2 if he votes for the proposal is $1 + \delta\frac{1}{3(1-\delta)}$. Legislator 2 rejects the proposal if and only if

$$\frac{1}{2} + \delta\frac{1}{2(1-\delta)} \geq 1 + \delta\frac{1}{3(1-\delta)}, \tag{1}$$

which is satisfied if and only if $\delta \geq \frac{3}{4}$. Since $(0, 1, 0)$ is the most attractive proposal, there is no proposal by 3 (nor by 1 or 2) that can break the coalition for $\delta \geq \frac{3}{4}$. As long as the continuation payoff for a proposal outside Z is lower than the continuation payoff to coalition members from continuing a policy in Z, the coalition is sustained. The continuation payoff is lower because of the possibility in a new coalition formation round of being excluded from the coalition formed in the next period, i.e., *exclusion risk*.

If $q^{t-1} \notin Z$, the selected legislator i is to propose a policy in Z_i. To show that the proposer has no incentive to propose a policy not in Z_i, consider the most attractive status quo $q^{t-1} = (1, 0, 0)$ for legislator 1. If legislator 1 proposes the status quo, his dynamic payoff is $1 + \delta\frac{1}{3(1-\delta)}$ as in (1). If legislator 1 proposes $y \in Z_1$, the policy is voted against q^{t-1}. Suppose $y = (\frac{1}{2}, \frac{1}{2}, 0)$. Legislator 3 strictly prefers q^{t-1} to y, and hence votes for the status quo for all δ. Legislator 2 votes for the proposal for all δ. If $\delta = \frac{3}{4}$, legislator 1 is indifferent between y and q^{t-1}, and under the indifference rule votes for the status quo. The discount factor thus must be strictly greater than $\frac{3}{4}$, in which case legislator 1 proposes $y \in Z_1$.

The set Z thus is supported in a coalition equilibrium if $\delta > \frac{3}{4}$. The coalition equilibrium is attained in one step because legislators propose in $Z_i \subset Z$, and the dynamic coalition sustains that policy thereafter.[10] Note that in the proof of existence checking all possible deviations from the conjectured equilibrium strategies is simple because the continuation value is the same for all $x \notin Z$. Naturally, legislators receiving $\frac{1}{2}$ under the status quo constitute the dynamic coalition for all $\delta > \frac{3}{4}$ because these legislators constitute a decisive set of legislators who prefer that the coalition continue to having any other policy $x \notin Z$ implemented.

[9] Note that the set Z admits multiple policies, and the internal stability requirement places restrictions on the policies admissible in Z. This is shown formally in Section 4.3. For $n = 3$ the internal stability requirement implies that members of the dynamic coalition have equal allocations in any coalition equilibrium. This is formalized in Section 5.3.

[10] The indifference rule used here is thus fundamentally different from the indifference rule used in sequential legislative bargaining. In the latter a legislator is assumed to vote for the proposal when indifferent between it and the alternative so as to avoid an open set of accepted proposals, whereas here the indifference rule provides stability within the set of supported policies.

4 Coalition Equilibria and Dynamic Coalitions in the Basic Model

4.1 Continuation Values

For any set Z the continuation values in a coalition equilibrium are straightforward to characterize as illustrated by the example. Basic strategies call for all legislators to propose the status quo if it is in Z, so any policy in Z is an absorbing state. For any status quo not in Z, legislator i, if selected, proposes with equal probability all elements of the set Z_i, each of which is approved in the conjectured coalition equilibrium. Symmetry implies that $|Z_i| = |Z_j|$, so the probabilities of receiving particular allocations are identical across legislators. This gives Lemma 1, the proof of which is straightforward and hence is omitted. To simplify the notation, let $v_i(q^{t-1})$ denote $v_i(\sigma, \omega | q^{t-1})$.

Lemma 1. *In the basic model if (σ, ω) is a coalition equilibrium with supported set Z:*

(i) *The continuation value for player i for $q^{t-1} \in Z$ is $v_i(q^{t-1}) = \frac{u(q_i^{t-1})}{1-\delta}$.*

(ii) *The continuation value for player i for $q^{t-1} \notin Z$ is $v_i(q^{t-1}) = v^* \equiv \frac{\overline{u}}{1-\delta}$, where*

$$\overline{u} \equiv \frac{1}{n|Z_j|} \sum_{z \in Z_j} \sum_{i=1}^{n} u(z_i).$$

Because of the symmetry of Z the continuation value is v^* for any legislator i for all $q^{t-1} \notin Z$, where v^* is the discounted average utility $\bar{u}$ from the proposals made in the sets Z_i. This feature of basic strategies simplifies the characterization of coalition equilibria, because every possible deviation from the set Z is met by the same response (random formation of a new coalition in the next period).

4.2 A Class of Simple Coalition Equilibria

The following proposition identifies a class of coalition equilibria that includes the example in Section 3.1. The proof is presented as a special case of an extended model with heterogeneous selection probabilities and discount factors to demonstrate the robustness of coalition equilibria. All proofs for Section 4 are presented in Appendix A.

Proposition 1. *In the basic model a symmetric set of policies Z is supported by a coalition equilibrium if $\delta > \underline{\delta} \equiv \frac{u(1) - u(z_{\max})}{u(1) - \overline{u}}$, and*

(a) *$z_i = z_{\max}$ for at least m legislators, for all $z \in Z$.*

(b) *$u(z_j) < u(z_{\max})$ for some j and some $z \in Z$.*

Proposition 1 requires that at least a minimal majority of legislators receives an allocation equal to the maximum allocation in Z. This condition ensures that a policy in Z cannot be replaced by another policy in Z, since no group of m or more legislators can be made strictly better off with another policy in Z. That is, Condition (a) ensures internal stability. Condition (b) requires that at least one policy in Z give strictly lower utility than $z_{\max}$. This condition assures that there is exclusion risk for the dynamic coalition supporting a policy $z \in Z$. That is, each legislator i in the dynamic coalition fears being excluded from the next coalition formed and receiving a payoff strictly lower than $u(z_{\max})$. The bound on the discount factor is determined by the allocation to a pivotal legislator, and in the equilibria in Proposition 1 some legislator receiving z_{max} is always pivotal. This is also the bound determined from the proposer's incentive constraint.

To illustrate the argument underlying the proof of Proposition 1, note that with basic strategies any $q^{t-1} \in Z$ is an absorbing state, so any deviation to a $z' \in Z$ from the coalition equilibrium supporting Z involves for legislator i a comparison between q_i^{t-1} and z_i'. Since at least a majority have $q_i^{t-1} = z_{\max}$, under the indifference rule no deviation in Z receives a majority vote. A legislator i not in the dynamic coalition could also propose a policy $y^t \notin Z$ to attract a majority of votes, in which case the next period would be a new coalition formation round. Since at least m legislators receive $q_i^{t-1} = z_{\max}$ and the most that can be offered to any one of those legislators is $y_j^t = 1$, legislator j votes against y^t for $\delta > \underline{\delta}$. Then, legislator i has no incentive to propose $y^t \neq q^{t-1} \in Z$.

For $q^{t-1} \notin Z$ legislator i proposes at random a policy in Z_i, and each of those proposals must in equilibrium be approved by a majority over all $q^{t-1} \notin Z$. The most difficult status quo to defeat has $q_k^{t-1} = 1$ for some legislator k. Legislator k can obtain a dynamic payoff $u(1) + \delta v^*$ by voting for q^{t-1}, whereas on the equilibrium path k receives a dynamic payoff $\frac{u(z_{\max})}{1-\delta}$, which is strictly greater than the dynamic payoff with q^{t-1} for a discount factor $\delta > \underline{\delta}$. Consequently, legislator k and the other legislators receiving $z_{\max}$ in y^t vote for $y^t \in Z_i$ over q^{t-1}. Similarly, a majority receiving $y_k^t = z_{\max}$ in $y^t \in Z$ votes for y^t over any $q^{t-1} \notin Z$ if $\delta > \underline{\delta}$. The bound $\underline{\delta} < 1$ because $u(z_{\max}) > \overline{u}$, which is implied by $u(z_j) < u(z_{\max})$. Consequently, the proposer i has no incentive to deviate from proposing $y^t \in Z_i$ to any $q^{t-1} \notin Z$ if $\delta > \underline{\delta}$. Note that $\underline{\delta}$ depends on the elements of Z, but we omit this conditioning for ease of exposition.

For $q^{t-1} \in Z$ any legislator i with $q_i^{t-1} = z_{\max}$ is in the dynamic coalition. It is also possible that legislators with other allocations are in the dynamic coalition depending on the elements of Z and the discount factor. The following corollary identifies properties that a dynamic coalition formed in the coalition equilibria identified in Proposition 1 can have.

Corollary 1. *Dynamic coalitions in Proposition 1 can support policies with the following properties:*

(a) *Dynamic coalitions can support both efficient and inefficient policies.*

(b) *Dynamic coalitions can support positive allocations to legislators outside the dynamic coalition.*

(c) *Dynamic coalitions can have surplus members; that is, the size of the dynamic coalition can be strictly larger than minimal winning.*

The properties identified in Corollary 1 result from all players believing that a proposal that is not in Z results in a new coalition round commencing in the next period. This results in exclusion risk, and the risk is the same for all supported policies. Part (a) states that dynamic coalitions can support inefficient policies. That is, a policy with waste can be sustained in a coalition equilibrium, representing a political failure in the sense of Besley and Coate (1998). An inefficient policy is sustained because the coalition members fear being excluded from a future coalition and receiving a lower continuation payoff. The dynamics sustaining the coalition are similar to the dynamics described in Acemoglu, Egorov and Sonin (2012) and Anesi and Seidmann (forthcoming), who also find that equilibria can support inefficient policies. Part (b) states that dynamic coalitions can support policies in which non-coalition members receive positive allocations. This is observed in practice as noted in the Introduction, where benefits are distributed to more than a minimal majority. Part (c) states that a dynamic coalition can be strictly larger than minimal winning, as in parliamentary governments that have a surplus of parties. This again results because the members of the dynamic coalition fear the risk of exclusion in a new coalition formation round.

Coalition equilibria have stable policies and hence provide perfect risk smoothing over time. For risk averse legislators this allows coalition equilibria to exist for lower discount factors compared to risk neutral preferences. Proposition 1 identifies the bound on the discount factor corresponding to a set Z supported by a coalition equilibrium, and the following Corollary presents the bound for a class of particularly simple coalition equilibria in which non-coalition members receive no allocation.

Corollary 2. *For all $\delta > \underline{\delta}$ there exists a coalition equilibrium in which $\kappa \in [m, n-1]$ legislators receive $z_{\max} \leq \frac{1}{\kappa}$, and $n-\kappa$ legislators receive 0. The bound $\underline{\delta}$ is*

$$\underline{\delta} = \frac{u(1) - u(z_{\max})}{u(1) - \left[\frac{\kappa}{n} u(z_{\max}) + \frac{n-\kappa}{n} u(0)\right]},$$

which is strictly increasing in κ.

For u concave enough, $\underline{\delta}$ can be arbitrarily small. That is, normalize u so that $u(1) = 1$ and $u(0) = 0$, and note that as u becomes more concave, $u(z_{\max})$ approaches 1 and $\underline{\delta}$ approaches 0. The theory of coalition equilibria thus can be applied to a large class of political settings, even where risk-averse political actors place a low value on the future. The discount factor in a particular application depends on the length of a period. In a legislative application the time between proposals can be relatively short in which case discount factors may be high, whereas if a period corresponds to an interelection period in a parliamentary system, the discount factor could be relatively low. As Corollary 2 indicates, a coalition equilibrium exists for a relatively low discount factor if parties or their leaders are risk averse.[11]

4.3 Existence and Characterization of Coalition Equilibria

Proposition 1 identifies a class of coalition equilibria, and the following proposition provides conditions that are necessary and sufficient for the existence of a coalition equilibrium supporting a set Z. Let M denote a set of m legislators, and let $\mathcal{M}$ denote the collection of all such M. Let $W(z)$ denote the set of policies in Z that are strictly preferred by a majority M of legislators; i.e., $W(z) = \{z' \in Z | \exists M \in \mathcal{M} \ni u(z'_i) > u(z_i), i \in M\}$.[12] Then, if $W(z) = \emptyset$, there is no policy $z' \in Z$ that defeats z. To form a dynamic coalition, a proposal must attract the vote of the legislator with the m^{th} largest allocation for any $z \in Z$, and let the smallest of these be denoted by $z_m^{\min}$.

Proposition 2. *In the basic model there exists a* $\underline{\underline{\delta}}$ *and a* δ^* *with* $\max\{\underline{\underline{\delta}}, \delta^*\} \leq \frac{u(1)-u(z_m^{\min})}{u(1)-\bar{u}} < 1$ *such that a coalition equilibrium supporting the set* Z *exists if and only if* $\delta \geq \underline{\underline{\delta}}$, $\delta > \delta^*$, *and:*

- (a) *No policy in* Z *can be defeated by another policy in* Z *in a pairwise comparison; i.e.,* $W(z) = \emptyset$ *for all* $z \in Z$. *Equivalently, for any* $x, y \in Z$ *such that* $x_i \geq x_{i+1}$ *and* $y_i \geq y_{i+1}$ *for all* $i \in \{1, \ldots, n-1\}$, *we have* $u(x_i) \leq u(y_{i+n-m})$ *for some* $i \in \{1, \ldots, m\}$.
- (b) *A majority* M *of legislators strictly prefers to continue the policy* $z \in Z$ *and the corresponding coalition than change the policy and risk being excluded from a future coalition; i.e.,* $u(z_m^{\min}) > \bar{u}$.

Proposition 2 provides two intuitive conditions for the existence of a coalition equilibrium supporting a set Z. Condition (a) is the internal stability condition, and Condition (b) is the exclusion risk condition. Note that Condition (b) implies $v_m(z) > v^*$, thus it is explicitly dynamic and hence is different from von Neumann Morgenstern external stability, which requires that no policy not in Z can defeat a policy in Z.[13]

There is a weak lower bound on the discount factor and a strict lower bound. Two incentive constraints determine the weak lower bound. The first ensures that if $q^{t-1} \notin Z$ a proposer in a new coalition formation round prefers a proposal in Z_i to continuing with a status quo that gives him an allocation of 1. This condition requires the discount factor to be greater than $\underline{\delta}$. The second condition ensures that a majority prefers to maintain the status quo $q^{t-1} \in Z$ rather than accept a policy $q^{t-1} \notin Z$. We denote this lower bound as $\hat{\delta}^*$ and thus $\underline{\underline{\delta}} = \max\{\underline{\delta}, \hat{\delta}^*\}$. The strict lower bound ensures that a majority strictly prefers (because of the indifference rule) to accept the proposal in Z

[11] We characterize the lowest discount factor such that a coalition equilibrium exists in Proposition 3 in Section 4.4.

[12] Strict preference is used because of the indifference rule.

[13] Condition (b) is thus different from the external stability conditions used by Anesi (2010) and Diermeier, Egorov and Sonin (2013). This difference is considered in more detail in Section 5.3.

from any status quo $q^{t-1} \notin Z$. The pivotal legislators for a given status quo will in general depend on the elements of Z, and the status quo. In Sections 4.4 and 5 we illustrate the determination of the lower bound on the discount factor. The exclusion risk condition ensures $\max\{\underline{\delta}, \delta^*\} < 1$.

Proposition 2 can be extended to a model in which legislators have different discount factors δ_i and different selection probabilities p_i.[14] The continuation values of the legislators are irrelevant to the selection of coalition partners even though when the discount factors are unequal the continuation values are ordered by $p_i > p_j$, i.e., $v_i(q^{t-1}) > v_j(q^{t-1})$ for $q^{t-1} \notin Z$. All that matters to proposers when $q^{t-1} \notin Z$ is that they receive the largest payoff $z_{\max}$ in the next period, since a proposal in Z_i persists thereafter.

If Z is supported by a coalition equilibrium, for each $q^{t-1} \in Z$ and δ there is a unique dynamic coalition whose members support the continuation of the status quo rather than change the policy and face a new coalition formation round. The following two corollaries provide examples of policies that satisfy Condition (a) but fail Condition (b) in Proposition 2, and hence are not supported by dynamic coalitions.

Corollary 3. *Dynamic coalitions cannot support a dictator outcome* $(1, 0, \ldots, 0)$ *or any policy such that fewer than* m *legislators receive positive allocations.*

It is straightforward to see that Condition (b) is violated in the case of the dictator outcome. It is natural to expect dynamic coalitions not to support the dictator outcome, since fewer than a minimal majority receive a positive allocation.

Corollary 4. *If* $u(x_i)$ *is convex and all policies in* S *are efficient, no coalition equilibrium supports the universal policy (with all legislators receiving equal allocations). If* S *is a singleton, no coalition equilibrium can support the universal policy for any* u.

Corollary 4 implies that the set $S = \{(\frac{1}{n}, \ldots, \frac{1}{n})\}$ is not supported by a coalition equilibrium, since $\bar{u} = u(\frac{1}{n})$ and hence there is no exclusion risk. If an inefficient policy is included in S, for example $S' = \{(\frac{1}{n}, \ldots, \frac{1}{n}), (\frac{1}{n}, \ldots, \frac{1}{n}, 0, \ldots, 0)\}$ with m legislators receiving $\frac{1}{n}$ in the second policy, then $\mathcal{Z}(S')$ is supported by a coalition equilibrium. The inclusion of the policy $(\frac{1}{n}, \ldots, \frac{1}{n}, 0, \ldots, 0)$ in S' provides the exclusion risk, i.e., $\bar{u} < u(\frac{1}{n})$.[15] Universal coalitions thus can form when a threat is present.

We provide an illustration of the identification of the members of a dynamic coalition with less simple policies than the ones discussed so far. Consider $n = 5$, $m = 3$, $u(x_i) = x_i$, and $S = \{(c, c, c, a, 0)\}$ with $c > a$ and $\bar{u} < \frac{1}{5}$. For all $\delta > \underline{\delta} = \delta^* = \frac{1-c}{1-\bar{u}}$ the set $Z = \mathcal{Z}(S)$ is supported in a coalition equilibrium. If $z = (c, c, c, a, 0)$ legislators 1, 2, and 3 prefer to continue the coalition rather

[14] See Section A.4 for the extended model.

[15] This is similar to Richter (2014), who shows that equal division can be supported by allowing some of the dollar to be wasted.

than change to any policy $x \notin Z$ for all $\delta > \underline{\delta}$, thus $i \in C(z;Z)$ for all $i \in \{1,2,3\}$ and for all $\delta > \underline{\underline{\delta}}$. Suppose $a < \bar{u}$, then there is no discount factor such that legislator 4 prefers $z = (c,c,c,a,0)$ to any policy $x \notin Z$, so legislator 4 is not in the dynamic coalition. Thus, $C(z;Z) = \{1,2,3\}$ for all $\delta > \underline{\underline{\delta}}$ when $a < \overline{u}$. If $a = a' > \bar{u}'$, $c > a'$, $S' = \{(c,c,c,a',0)\}$, and $Z' = \mathcal{Z}(S')$, then, for $z' = (c,c,c,a',0)$, legislators $\{1,2,3,4\}$ accept all policies in Z for $\delta > \frac{1-a'}{1-\bar{u}'}$, and thus $C(z';Z') = \{1,2,3,4\}$. In general if $u(z_i) > \overline{u}$ for some $z \in Z_i$, then legislator i is in the dynamic coalition.

As the example illustrates, a parliamentary system can have a minimal winning governing coalition, all of whose members vote against every proposal by an out party, and it can have a surplus coalition for sufficient political patience. A party in government can be a "junior" member in the sense that it has a lower allocation than the other members. Minimal winning and surplus governments can also allocate benefits to an out party.

Coalition equilibria also exist with one member of the dynamic coalition having a strictly greater allocation than any other coalition member, which in the context of parliamentary systems can be thought of as the head of government receiving more of the benefits than other coalition parties. In terms of government formation this could represent a single-party minority government supported by enough other parties to yield a majority.

The requirement that $z_j \geq \bar{u}$ for all coalition members, as shown in the previous example, and the necessity of Condition (a) of Proposition 2 provides a stark prediction for dynamic coalitions in the case of n odd, simple majority rule, and minimal winning dynamic coalitions. Suppose that at least two coalition members have different allocations in a policy $z \in Z$. Then, a proposer can propose a permutation of z such that each of the non-coalition members is strictly better off and $z_{\max}$ can be offered to the dynamic coalition member with the lowest allocation. Internal stability is then violated, so all members of a minimal winning dynamic coalition must have the same allocation. Non-members may have different payoffs as long as their payoffs are below the average utility $\overline{u}$. This is summarized in the next corollary.

Corollary 5. *For n odd, simple majority rule, and minimal winning dynamic coalitions, all coalition members must have the same allocation.*

4.4 Bounds on Attainable Allocations

This section characterizes for a given n, m, u and δ the lower and upper bounds on allocations in any coalition equilibrium.[16] For this section we assume u is strictly increasing.

By Proposition 1 for $\delta > \underline{\delta}$ there is an equilibrium in which some legislators not in the dynamic

[16] Although not all payoffs can be supported in a coalition equilibrium with basic strategies, coalition equilibria can be used to support a larger set of payoffs using player-specific punishments provided by a coalition equilibrium in which a deviator receives zero thereafter. We illustrate this with an example in the Appendix Section A.9.

coalition receive an allocation of zero. Zero is thus the lower bound on any allocation in a coalition equilibrium. From Condition (b) of Proposition 2 the upper bound $\bar{z}$ on the allocation for a non-coalition member is given by $u(\bar{z}) = \bar{u}$. Since Proposition 2 holds for arbitrarily inefficient policies, the infimum of the allocations for a coalition member is also zero. The least upper bound on allocations for coalition members is characterized next.

To characterize the least upper bound, it suffices to consider only efficient policies and policies of two forms.[17] One is $z^m = (\frac{1}{m}, \ldots, \frac{1}{m}, 0 \ldots, 0)$, which allocates $\frac{1}{m}$ to m legislators, and denote $Z^m \equiv \mathcal{Z}(\{z^m\})$. The other is $z^a = (a, b, \ldots, b, 0, \ldots, 0)$ in which $\kappa - 1$, $\kappa \in \{m, ..., n-1\}$, coalition members receive b, with $a > b > 0$.[18] Denote $Z^a \equiv \mathcal{Z}(\{z^a\})$. The policy z^a will be efficient to achieve the maximum allocation, but for now we do not assume that z^a is efficient. Note that for n odd and simple majority rule ($m = \frac{n+1}{2}$) the set Z^a is supported by a coalition equilibrium only if $\kappa \geq m+1$ (a surplus coalition), since for $\kappa = m$ the internal stability condition is not satisfied and Corollary 5 holds. Because $\kappa \geq m$ and $\kappa \leq n-1$, $n \geq 4$ is required for Z^a to be supported, so $m \geq 3$.

We discuss the bounds on the discount factors for Z^m and Z^a, making the conditioning of $\underline{\delta}$ on Z explicit. For a policy in Z^m all coalition members have the same allocation, hence the bound on the discount factor for the proposer is the same as the bound on the discount factor for the coalition members as given in Proposition 1. That is $\underline{\delta}(Z^m) = \frac{u(1)-u(\frac{1}{m})}{u(1)-\overline{u}^m}$, where $\overline{u}^m \equiv \frac{1}{n}[mu(\frac{1}{m}) + (n-m)u(0)]$. For a policy in Z^a the binding constraint for the proposer occurs when the proposer's status quo allocation is 1, yielding the lower bound on the discount factor $\underline{\delta}(Z^a) = \frac{u(1)-u(a)}{u(1)-\overline{u}^a}$, where $\overline{u}^a \equiv \frac{1}{n}[u(a) + (\kappa-1)u(b) + (n-\kappa)u(0)]$. We must consider that the equilibrium policy is accepted beginning from any initial status quo $q^0 \notin Z$. If a legislator with a status quo allocation of 1 is offered b by the proposer, there are $\kappa - 2$ other legislators with a status quo allocation of 0 that are offered b and will approve the proposal along with the proposer. The proposal defeats q^0 if the $\kappa - 1$ legislators constitute a majority. The most difficult status quo to beat has $\kappa - m + 1$ legislators, each with a status quo allocation of $\frac{1}{\kappa - m+1}$. The lower bound on the discount factor such that these coalition members accept the proposal is denoted $\delta^{*a}(\kappa, b)$ and this is greater than $\hat{\delta}^*$.[19] This is summarized in the following lemma.

Lemma 2. *(i) A coalition equilibrium supporting $Z = Z^m$ exists if and only if $\delta > \underline{\delta}(Z^m)$. (ii) A coalition equilibrium supporting $Z = Z^a$ exists if and only if $n \geq 4$, $\kappa \in \{\underline{\kappa}, ..., n-1\}$, $u(b) > \overline{u}^a$,*

[17] This is shown formally in the proof of Proposition 3.

[18] Note that if $S = \{z^m, z^a\}$ the set $Z = \mathcal{Z}(S)$ is not supported by a coalition equilibrium, since the internal stability Condition (a) of Proposition 2 is not satisfied.

[19] This is shown formally in the proof of Lemma 2 in the Appendix.

$\delta \geq \underline{\delta}(Z^a)$, and $\delta > \delta^{*a}(\kappa, b)$ where

$$\delta^{*a}(\kappa, b) \quad \equiv \quad \frac{u\left(\frac{1}{\kappa-m+1}\right) - u(b)}{u\left(\frac{1}{\kappa-m+1}\right) - \bar{u}^a}.$$

Furthermore $\underline{\kappa} = m+1$ for n odd and simple majority rule (or $m = \frac{n+1}{2}$), and $\underline{\kappa} = m$ otherwise.

Note from Lemma 2, if $m = \frac{n+1}{2}$, the minimum coalition size is $\kappa = m+1$, thus a surplus coalition is required if the supremum is attained for a policy in Z^a. If $\kappa = m > \frac{n+1}{2}$, then $\delta^{*a}(\kappa, b) > \underline{\delta}(Z^a)$ since $a > b$.

For a fixed (n, m, u, δ) the maximum allocation is either $\frac{1}{m}$ or a for some policy in Z^a. Define the function $\underline{b}(\kappa, \delta)$ as the solution of b to $u(b) = (1-\delta)u\left(\frac{1}{\kappa-m+1}\right) + \delta\bar{u}^a$ with $a = 1-(\kappa-1)b$. The allocation $\underline{b}(\kappa, \delta)$ is the infimum allocation for a coalition member, given δ such that the coalition member has an incentive to accept the equilibrium proposal in Z^a from any $q^{t-1} \notin Z$.[20] The supremum $\bar{a}(\kappa, \delta)$ of the proposer's allocation in Z^a is thus

$$\bar{a}(\kappa, \delta) = 1 - (\kappa - 1)\underline{b}(\kappa, \delta), \tag{2}$$

which is strictly increasing in δ.[21] Satisfying the proposer's incentive constraint requires $u(\bar{a}(\kappa, \delta)) > (1-\delta)u(1) + \delta\bar{u}^a$. The right side of this expression is decreasing in δ, since $u(1) > \bar{u}^a$ for any strictly increasing u. For some $\delta^a(\kappa) \in (0,1)$ we have $u(\bar{a}(\kappa, \delta)) > (1-\delta)u(1) + \delta\bar{u}^a$ for all $\delta > \delta^a(\kappa)$. The bound $\delta^a(\kappa) = \underline{\delta}(Z^a)$ when $a = \bar{a}(\kappa, \delta)$ and $b = \underline{b}(\kappa, \delta)$. Thus $\bar{a}(\kappa, \delta)$ is a feasible allocation for all $\delta > \delta^a(\kappa)$.

Denote by $\hat{\kappa}^*$ the maximizer of $\bar{a}(\kappa, \delta)$ with respect to the coalition size κ. The supremum allocation $\bar{a}(\kappa, \delta)$ is not necessarily decreasing in κ, and the maximizer $\hat{\kappa}^*$ may be greater than $\underline{\kappa}$, which, for n odd and simple majority rule, is $m+1$, and is m otherwise by Lemma 2. The supremum allocation when the discount factor is high enough is obtained for $\kappa^* \equiv \max\{\hat{\kappa}^*, \underline{\kappa}\}$. The discount factor such that $\bar{a}(\kappa^*, \delta) = \frac{1}{m}$ is $\delta^{*m} \equiv \delta^{*a}(\kappa^*, \frac{m-1}{(\kappa^*-1)m})$, since the coalition is size κ^* and each coalition member receives $b = \frac{1-\frac{1}{m}}{\kappa^*-1}$. Thus $\bar{a}(\kappa^*, \delta) \geq \frac{1}{m}$ for all $\delta \geq \delta^{*m}$, since $\bar{a}(\kappa^*, \delta)$ is strictly increasing in δ. Note also that $\delta^a(\kappa^*) \leq \delta^{*m}$ when a coalition equilibrium exists supporting Z^a, thus δ^{*m} is a bound on the discount factor above which the supremum allocation is for a policy in Z^a. The minimum discount factor such that a coalition equilibrium exists is $\delta^{\min} \equiv \min\{\underline{\delta}(Z^m), \delta^a(\kappa^*)\}$. This is summarized in the next proposition.

Proposition 3. *If $\delta^{*m} \geq \delta^a(\kappa^*)$, then the maximum allocation is $\frac{1}{m}$ for $\delta \in (\delta^{\min}, \delta^{*m}]$, and the supremum allocation is $\bar{a}(\kappa^*, \delta)$ for $\delta \in (\delta^{*m}, 1)$. If $\delta^{*m} < \delta^a(\kappa^*)$ and $\delta^a(\kappa^*) \geq \underline{\delta}(Z^m)$, then the*

[20] The necessity of this bound on b is discussed in the proof of Lemma 2 in Appendix A.

[21] Note that $\underline{b}(\kappa, \delta)$ is strictly decreasing in δ for any feasible value of $\underline{b}(\kappa, \delta)$. To see this, note that $u\left(\frac{1}{\kappa-m+1}\right) > \bar{u}^a$ in any coalition equilibrium, otherwise $\underline{b}(\kappa, \delta) \leq \bar{u}^a$ violating Proposition 2 part (b). The comparative static with respect to δ is then straightforward since u is strictly increasing.

maximum allocation is $\frac{1}{m}$ *for* $\delta \in (\delta^{\min}, \delta^a(\kappa^*)]$ *and the supremum allocation is* $\overline{a}(\kappa^*, \delta)$ *for all* $\delta \in (\delta^a(\kappa^*), 1)$*. If* $\delta^{*m} < \delta^a(\kappa^*) < \underline{\delta}(Z^m)$*, then the supremum allocation is* $\overline{a}(\kappa^*, \delta)$ *for all* $\delta \in (\delta^{\min}, 1)$*. Furthermore, the supremum allocation is increasing in* δ*. There is no coalition equilibrium for any* Z *if* $\delta \leq \delta^{\min}$*.*

Proposition 3 states that the supremum allocation over all coalition equilibria is weakly increasing in δ since $\overline{a}(\kappa^*, \delta)$ is strictly increasing in δ. The proposition implies that for all $\delta \geq \max\{\delta^{*m}, \delta^{*a}\}$ the supremum allocation is greater than $\frac{1}{m}$ and is attained with a surplus coalition for $m = \frac{n+1}{2}$, and may be attained for a surplus coalition for $m > \frac{n+1}{2}$.

We can say more about the bounds on the supremum allocation with linear utility. In this case $\hat{\kappa}^* = m - 1 + \frac{\sqrt{n\delta(m-2)(1-\delta)}}{\delta}$ for $\delta > \delta^a(\kappa^*)$.[22] This is strictly increasing in m and for $m > 2 + \frac{\delta}{n(1-\delta)}$, the maximal coalition size is strictly greater than m. The maximal coalition size κ^* is decreasing in δ. As an example, suppose $n = 5$. When $m = 3$, the only possible coalition size for Z^a is $\kappa = 4 > m$. We have $\delta^{\min} = \underline{\delta}(Z^m) = \frac{5}{6}$ and $\delta^a(\kappa^*) = \frac{15}{17} < \frac{25}{27} = \delta^{*m}$. When $m = 4$ the only possible coalition size is $m = 4$, and we have $\delta^{\min} = \underline{\delta}(Z^m) = \delta^a(\kappa^*) = \delta^{*m} = \frac{15}{16}$. The maximum and supremum payoffs are illustrated in Figure 1 below.

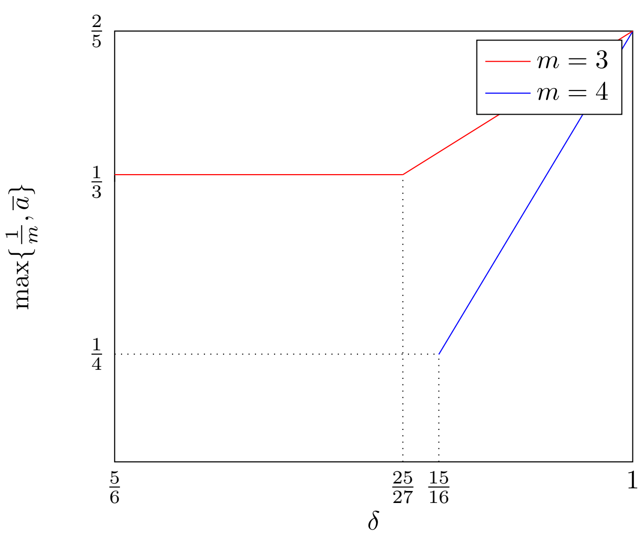


Figure 1: Supremum allocations for $u(x_i) = x_i$, $n = 5$

Note that for the case of linear utility, $n = 5$ and $m = 3$ we have $\delta^{*m} \leq \delta^a(\kappa^*)$, but we provide an example where this is not the case. Consider $u(x_i) = x_i$, $n = 14$, $m = 8$. In this case $\kappa^* = 9 > m$ for δ sufficiently small, and $\frac{175}{192} = \delta^{*m} < \delta^a(\kappa^*) = \frac{56}{61}$. Furthermore $\delta^{\min} = \delta^a(\kappa^*) < \underline{\delta}(Z^m) = \frac{49}{52}$. Thus for $u(x_i) = x_i$, $n = 14$ and $m = 8$, the supremum allocation in a coalition equilibrium is greater than

[22] More precisely, since $\hat{\kappa}^*$ must be an integer, $\hat{\kappa}^* \in \left\{ \left\lfloor m - 1 + \frac{\sqrt{n\delta(m-2)(1-\delta)}}{\delta} \right\rfloor, \left\lceil m - 1 + \frac{\sqrt{n\delta(m-2)(1-\delta)}}{\delta} \right\rceil \right\}$.

$\frac{1}{m}$ for all feasible discount factors and is obtained with a surplus coalition for some discount factors.

The following proposition identifies bounds on the supremum $\bar{a}(\kappa, \delta)$ of the largest allocation for risk neutral, risk averse, and risk loving preferences.

Proposition 4. *For (n, m, δ), $\kappa \in \{m, n-1\}$, $n \geq 4$, efficient policies, and u strictly concave (linear) (convex), and strictly increasing the supremum $\bar{a}(\kappa, \delta)$ satisfies*

$$\bar{a}(\kappa, \delta) > (=)(<)\ 1 - (\kappa - 1)\left(\frac{1-\delta}{\kappa - m + 1} + \frac{\delta}{n}\right). \tag{3}$$

Risk aversion makes a new coalition round less attractive than with risk neutrality, so proposers can attain a greater allocation when u is strictly concave than when it is linear. Since $\bar{a}(\kappa, \delta)$ is the supremum of the largest allocation for a sufficiently high discount factor, the largest allocation is greater with u strictly concave than for u linear, which is strictly greater than the largest allocation attainable with u strictly convex.

5 Special Cases of Dynamic Coalitions

5.1 Vested Interests

Proposition 2 does not require the maximum allocation in each policy in S to be the same. This suggests that coalition equilibria can support a policy that is not proposed from status quos other than itself. That is, the set Z supported by a coalition equilibrium includes the policies in $\bigcup_i Z_i$ that are proposed by legislators when the status quo is not in Z, and it can also include policies that are supported in a coalition equilibrium but are never proposed by any legislator when forming a new coalition. These policies can be thought of as supported by interests vested in the initial status quo, and let that set of policies be denoted by Q^0. That is, $Q^0 = Z \setminus (\bigcup_i Z_i)$. The set Q^0 is empty for $|S| = 1$, but for $|S| > 1$ it can be nonempty. As an example, consider $u(x_i) = x_i$, $n = 7$, $m = 5$, and two policies $z' = (a, b, b, b, b, c, c)$ and $z'' = (b, b, b, b, b, d, d)$, with $a > b > \overline{u} > c > d$. Legislators only propose a permutation of z' and never propose a permutation of z'' when a new coalition is being formed. If the initial status quo is z'', however, it persists thereafter because there is no permutation of z' that is strictly preferred by a majority to z''. Hence $Q^0 = \mathcal{Z}(\{z''\})$. For this example, $\underline{\delta} = \frac{1-a}{1-\overline{u}}$, and the lower bound for coalition members to accept z' from $q^{t-1} \notin Z$ is $\delta^* = \frac{1-b}{1-\overline{u}}$. If the status quo is z'' we must ensure legislator 7 with a status quo allocation of d cannot form a coalition to accept a policy $x \notin Z$. Legislator 7's coalition will consist of legislators $\{3, 4, 5, 6, 7\}$. Legislator 7 can offer this coalition a policy $x = (0, 0, x_3, x_4, x_5, x_6, x_7)$ such that $\sum_{j=3}^{7} x_j = 1$. Legislator j rejects the proposal if $(1-\delta)x_j + \delta\overline{u} \leq z''_j \Leftrightarrow \delta \geq \frac{x_j - z''_j}{x_j - \overline{u}} \equiv \hat{\delta}^*_j$. If $z''_j < \delta\overline{u}$, legislator j prefers any proposal $x \notin Z$ to z'' for any $\delta \in [0, 1)$ and accepts a proposal x with $x_j = 0$. Assume $d < \delta\overline{u}$ so $x_6 = x_7 = 0$ is accepted by legislators 6 and 7. Then $\hat{\delta}^*_j = \frac{\frac{1}{3} - b}{\frac{1}{3} - \overline{u}} = \hat{\delta}^*$ for all $j \in \{3, 4, 5\}$. The weak lower bound on the discount factor is thus $\underline{\underline{\delta}} = \max\{\underline{\delta}, \hat{\delta}^*\}$ and the strict lower bound is $\delta^* = \frac{1-b}{1-\bar{u}} > \underline{\underline{\delta}}$. Consequently, a coalition equilibrium exists supporting $\mathcal{Z}(\{z', z''\})$ if and only if $\delta > \delta^*$, but policies $\mathcal{Z}(\{z''\}) \in Q^0$ are never proposed from any $q^{t-1} \notin Z$.

A variety of policies can be sustained by equilibria when there are vested interests, including the universal policy $\hat{z}$ in which each legislator receives $\frac{1}{n}$ and Pareto dominated policies. Vested interests can support Pareto dominated policies, since a majority can prefer such a policy rather than face exclusion risk. In the example above, $c \geq d$ so the policy z' Pareto dominates z'', but there is a coalition equilibrium for a sufficiently high discount factor such that if $q^0 = z''$, it persists thereafter. Vested interests thus can result in a political failure in the form of a Pareto dominated policy.

5.2 The Sequential Legislative Bargaining Outcome

The sequential legislative bargaining model of Baron and Ferejohn (1989) involves the division of a dollar with a bargaining protocol in which a legislator is selected at random to make a proposal and legislators vote for the proposal or the status quo in which everyone receives 0. If the proposal is accepted, the game ends. If it is rejected, another proposer is selected at random to make a proposal, and it is voted against the same status quo in which everyone receives 0. The ex ante values of the game are the same for all legislators, the first proposal is accepted, all legislators have the same probability of being in the winning coalition, and the set of equilibrium allocations is symmetric, which are also the predictions for coalition equilibria in the dynamic legislative bargaining game. In sequential bargaining, however, the coalition is always minimal winning and the proposer captures all the benefits of proposal power; that is, the equilibrium policies with $\delta = 1$ are of the form $(1 - \frac{m-1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}, 0 \ldots, 0)$, with $m-1$ legislators receiving $\frac{1}{n}$ and $n-m$ legislators receiving 0. If $u(x_i) = x_i$, the sequential bargaining policy is the limit as $\delta \to 1$ of a coalition equilibrium except for the case of n odd, simple majority rule, and minimal winning coalitions, since the only coalition equilibria with efficient policies in that case give $\frac{1}{m}$ to m legislators as in Corollary 5. Otherwise, policies of the form $(a, b, \ldots, b, 0, \ldots, 0)$ with $m-1$ legislators receiving b approximate the sequential bargaining policy arbitrarily closely as $\delta \to 1$, since, as shown in Lemma 2, Z^a with $\kappa = m$ is supported for sufficiently high discount factors and $m > \frac{n+1}{2}$. This yields the following corollary to Proposition 3.

Corollary 6. *(i). For $m = \frac{n+1}{2}$, that is, for n odd and simple majority rule, the sequential equilibrium policies cannot be attained in a coalition equilibrium. (ii). For $m > \frac{n+1}{2}$ or a surplus coalition and u linear the sequential equilibrium policies are approximated arbitrarily closely in a coalition equilibrium in the limit as $\delta \to 1$.*

5.3 A Three-Member Legislature

For $n = 3$ Proposition 1 indicates that any policy of the form (c, c, w) with $c > w$ is supported in a coalition equilibrium. The conditions in Proposition 1 are also necessary when u is strictly increasing.

Proposition 5. *Suppose $n = 3$, $m = 2$. A coalition equilibrium supports the set $Z = \mathcal{Z}(\mathcal{S})$ if and only if for any $z, z' \in S$ we have $z = (c, c, d)$ and $z' = (c, c, d')$ $c > d$ or $c > d'$ and $\delta > \underline{\delta} = \frac{u(1)-u(c)}{u(1)-\overline{u}}$.*

The set of feasible allocations follows from the internal stability condition, and more generally for n odd and $m = \frac{n+1}{2}$, all members of a minimal winning dynamic coalition have the same allocation. If not, a proposal exists that is supported by the $\frac{n-1}{2}$ legislators outside the dynamic coalition and a coalition member with the lowest allocation. Exclusion risk requires either $c > d$ or $c > d'$. Thus these policies all have equal sharing within the dynamic coalition. The universal policy in which all legislators receive c is also supported, Since Proposition 5 covers $d = c$ and $d' < c$. In a three-member legislature the proposer in a dynamic coalition has no advantage over his coalition partner.

The efficient policies supported by a coalition equilibrium for the case of $n = 3$ are illustrated in Figure 2.

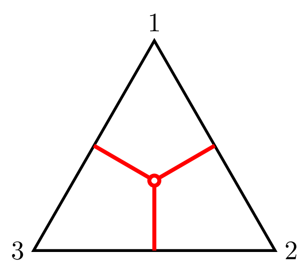


Figure 2: Efficient supported policies

Restricting attention to the case of efficient policies and linear utility yields the following corollary.

Corollary 7. *If $n = 3$, $m = 2$, $u(x_i) = x_i$ and $S = \{(c, c, 1 - 2c)\}$ for some $c \in (\frac{1}{3}, \frac{1}{2}]$, for any $\delta \in \left(\frac{3}{4}, 1\right)$ policies supported by a dynamic coalition satisfy $c > 1 - \frac{2}{3}\delta$.*

Figure 3 illustrates the efficient supported policies in Corollary 7 as a function of the discount factor. The lower diagonal border is open as is the vertical border at $\delta = 1$. The upper bound represents the example presented in Section 3.1. The higher the discount factor the larger is the set of policies that are supported by a coalition equilibrium because the more patient are legislators the greater is the set of short-run temptations a legislator can resist.

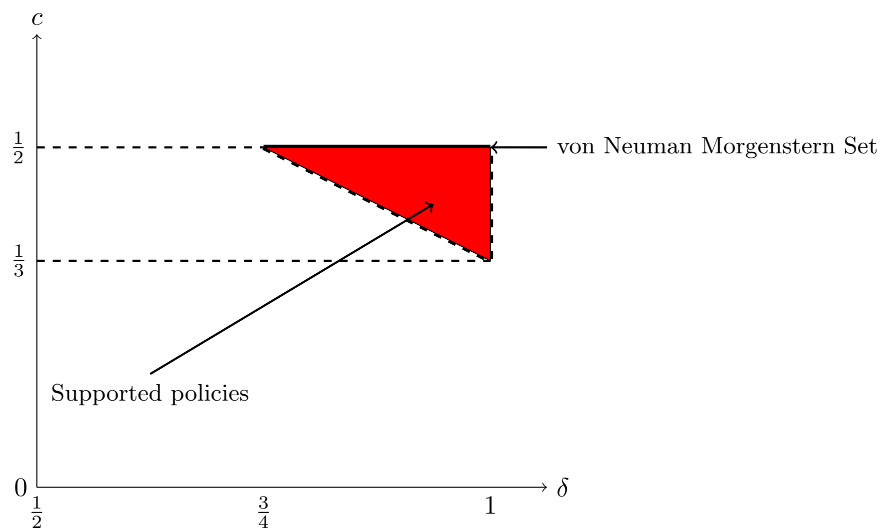


Figure 3: Supported policies as a function of δ

5.3.1 Relation to the Literature

The special case of a three-member legislature provides a simple comparison of policies supported by dynamic coalitions and other notions of stability. The von Neumann Morgenstern stable set, which for $n = 3$ is the set of policies $\{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2})\}$, is supported in a coalition equilibrium and is the set of policies that deliver the highest payoffs to coalition members. The internal stability and exclusion risk conditions in Proposition 2 are necessary and sufficient for a coalition equilibrium to exist and allow a set of policies larger than the stable set to be supported and for discount factors as low as $\frac{3}{4}$ for a linear utility function. Also, the limit as $\delta \to 1$ of the supported set is not the stable set but a larger set. Moreover, if the stable set is not in the supported set Z, it is defeated by all policies in Z for δ sufficiently high. That is, the incentives in a dynamic game are fundamentally different from those in the static game.

Anesi (2010), Acemoglu, Egorov and Sonin (2012), and Diermeier, Egorov and Sonin (2013) consider models for the case in which $\delta \approx 1$, which allows them to compare the Markov perfect equilibria in a bargaining game to von Neumann Morgenstern stable sets.[23] Anesi (2010) and Diermeier, Egorov and Sonin (2013) focus on policies that satisfy internal and external stability conditions consistent with the von Neumann Morgenstern stable set, whereas Acemoglu, Egorov and Sonin (2012) develop a related dynamic stability concept.[24]

[23] The assumption that $\delta \approx 1$ corresponds to choices that can be changed immediately as Acemoglu, Egorov and Sonin (2012) note.

[24] These models rely on a finite policy space, and Diermeier, Egorov and Sonin (2013) find that even though some players have vetoes and those players also have monopoly proposal rights, the non-veto players effectively form a blocking group that prevents the veto players from fully exploiting them. Nunnari (2014) considers a dynamic legislative bargaining model with a veto player and shows that the veto player fully exploits the non-veto players. Similarly,

For a model with veto players who are also monopoly proposers Diermeier, Egorov and Sonin (2013) show that a policy is in the von Neuman Morgenstern stable set if and only if it is supported by a Markov perfect equilibrium in which veto players propose only policies they strictly prefer.[25] For a dynamic legislative bargaining model as in the present paper Anesi (2010) shows that every policy in the von Neuman Morgenstern stable set can be supported by a Markov perfect equilibrium but the converse is not true. The difference between the two results is that, as in this paper, Anesi (2010) uses the indifference rule of voting for the status quo when indifferent and Diermeier, Egorov and Sonin (2013) use the indifference rule of voting for the proposal when indifferent. Thus, Diermeier, Egorov and Sonin (2013) find that internal and external stability conditions are both sufficient and necessary for a policy to be supported by a MPE, whereas Anesi (2010) finds that external stability is not necessary, so a set larger than the stable set is supported for $\delta \approx 1$. In the theory presented here for $\delta \approx 1$ all policies of the form in Corollary 7 with $c \in \left(\frac{1}{3}, \frac{1}{2}\right]$ are supported. The exclusion risk condition in Proposition 2 can be thought of as the replacement for the external stability condition for a dynamic legislative bargaining model with basic strategies.

6 Implementation Uncertainty

Uncertainty can be associated with the implementation of a policy, and that uncertainty can affect not only the payoff in the current period but also the status quo for the following period. Since the uncertainty affects the policy that is implemented, the status quo can move away from the coalition equilibrium policy in which case the coalition dissolves and a new coalition form as in Section 7 or the coalition members could tolerate the new policy as in Section 8.

Proposition 5 provides a benchmark for the case of implementation uncertainty. If efficient policies with non-coalition members receiving a zero allocation are focal, Proposition 5 provides a unique benchmark $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$ for comparison. Furthermore, $(\frac{1}{2}, \frac{1}{2}, 0)$ is supported by a coalition equilibrium only when $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$. That is, if $(\frac{1}{2}, \frac{1}{2}, 0) \in S$, then to satisfy internal stability requires that no other policy can be included in S. In contrast to these equilibria in which the gains from proposal power are shared equally within the coalition, with implementation uncertainty dynamic coalitions with unbalanced allocations are possible, as considered in Sections 6-8. That is, the internal stability condition can be weakened when there is implementation uncertainty.

Kalandrakis (2010) shows that a player with monopoly proposal rights can fully exploit the other players using a mixed proposal strategy. Nunnari (2014) and Kalandrakis (2010) assume a continuous policy space. Kalandrakis (2010) also argues that for any fixed $\delta < 1$ the pure strategy equilibrium in Diermeier and Fong (2011), which is similar to that in Diermeier, Egorov and Sonin (2013), does not exist as the finite grid on the policy space becomes finer. Kalandrakis (2010) notes that whether these equilibria exist with a continuous policy space is an open question.

[25]The restriction of equilibria to proposals that strictly improve a veto player's payoff is substantive as it rules at equilibria with cycles. That is, equilibria in which policies cycle according to the identity of the proposer, for example, are ruled out.

The analysis is presented for $n = 3$, $m = 2$, $u(x_i) = x_i$ and efficient policies, which is the case considered by Kalandrakis (2004) and Battaglini and Palfrey (2012).[26] Implementation uncertainty could take many forms and the model used simplifies the analysis of the incentive constraints and facilitates comparative statics analysis of the set of policies supported by dynamic coalitions.

Uncertainty resulting from implementation represents the observation that policy does not always work as intended. The implementation of legislation is typically delegated to administrative agencies or regulatory commissions that develop the details for the application of the policy. A degree of uncertainty can be associated with that delegation, and legislators take that uncertainty into account in choosing a policy. The uncertainty could also be associated with the response to the enacted policy by those affected, and the realization of that uncertainty can affect the status quo and the strategies of legislators in the future. As an extreme example, with the support of the American Association of Retired Persons Congress overwhelmingly enacted the Medicare Catastrophic Coverage Act of 1988, which provided generous benefits for catastrophic care under Medicare and financed the benefits through increases in Medicare premiums. Before the change could be fully implemented, Medicare recipients began protesting the forthcoming premium increases, and facing uncertainty about the impact of the Act, Congress quickly repealed it.

Implementation uncertainty could depend on whether the legislature retains the current policy or chooses a new policy. When the legislature retains the current policy, implementation uncertainty is assumed to be present with probability η, and when the legislature chooses a new policy, the corresponding probability is $\gamma < 1$. With the complementary probabilities there is no uncertainty and hence the policy implemented equals that adopted by the legislature. When implementation uncertainty is realized, its magnitude is represented by a continuous, mean zero, random shock that is publicly observable. The specification for the occurrence of implementation uncertainty allows comparative statics analysis in terms of parameters η and γ, and the specification of a continuous random shock means that the probability is zero that the shocked policy equals the policy adopted by the legislature. A legislator cannot receive more that 1 or less than 0, so the shocked allocation may be truncated, in which case the truncated amount is assumed to be reallocated among the legislators to ensure the policy remains in the feasible set. Details of the specification of implementation uncertainty are given in the online Appendix B, and we present the substance here.

Let $Z(c) \equiv \mathcal{Z}(S(c))$, where $S(c) = \{(1-c, c, 0)\}$ and $c \leq \frac{1}{2}$, so $z_{\max} = 1 - c$.

[26] In their MLQRE Battaglini and Palfrey (2012) assume that players use behavioral strategies that place positive probability on every available action (on a grid). That probability is proportional to the continuation value, and as that proportion increases the limit points correspond to MPE. This uncertainty affects strategies and hence payoffs but does not affect state transitions other than through the strategies. Duggan and Kalandrakis (2012) show the existence of stationary MPE in pure strategies for a class of dynamic games that accommodate uncertainty in the current period payoffs and in the transitions from one state to another. Our assumed uncertainty is different from theirs, so their results do not apply to the equilibria we characterize.

Assumption 1 (Substance)**.** *Suppose* $q^{t-1} \in Z(c)$ *and* $y^t \in Z(c)$. *If the proposal* $y^t = q^{t-1}$, *with probability* $1-\eta$, *the status quo* $q^t = q^{t-1}$ *and with probability* η, *a shock* $\tilde{\theta}^t$ *distorts the policy as follows:*

$$q_i^t = \begin{cases} 1-c+\theta^t & \text{if } y_i^t = 1-c \\ c-\theta^t & \text{if } y_i^t = c, \\ 0 & \text{if } y_i^t = 0. \end{cases}$$

where $\tilde{\theta}^t$ *is uniformly distributed on* $[-\underline{\theta}, \underline{\theta}]$ *and* θ^t *is the realization of* $\tilde{\theta}^t$. *If* $y^t \neq q^{t-1}$ *is approved, with probability* $1-\gamma$, $q^t = y^t$, *and with probability* γ *a shock* $\tilde{\varepsilon}^t$ *distorts the policy as above but with the realization* ε^t *replacing* θ^t, *where* $\tilde{\varepsilon}^t$ *is distributed uniformly on* $[-\underline{\varepsilon}, \underline{\varepsilon}]$. *If* $q^{t-1} \notin Z(c)$ *or* $y^t \notin Z(c)$ *is approved, allocations are similarly affected.*[27]

Implementation uncertainty could be greater when a new policy is adopted than when the current policy is continued.

Assumption 2. *Implementation of a new policy* $y^t \neq q^{t-1}$ *has a higher probability of a shock than implementation of the current (status quo) policy, i.e.,* $1 > \gamma \geq \eta$, *and has a stochastically larger shock, i.e.,* $\underline{\varepsilon} \geq \underline{\theta}$.

7 Specific-Policy Equilibria with Implementation Uncertainty

This section shows by construction the existence of a class of coalition equilibria that support policies with unbalanced allocations to the members of a dynamic coalition; i.e., in $Z(c)$. In these equilibria $S(c)$ consists of a single policy, so the policy is specific to that c. The originator of a specific-policy coalition has proposal power and may not share the gains equally with the coalition partner. These coalitions persist with probability $1-\eta$ and dissolve with probability η when implementation uncertainty is realized. A new coalition then forms in the next period. The following bound on the shocks to the policy facilitates the exposition by simplifying the expressions for the continuation values.

Assumption 3. $\underline{\varepsilon} \leq \frac{1}{3}$.[28]

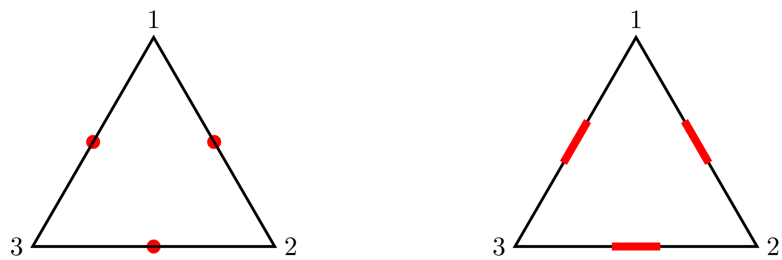
Basic strategies are used in establishing the existence of a *specific-policy equilibrium* supporting $Z(c)$.

[27] See Assumption 1 in Appendix B for details.

[28] The requirement that $\underline{\varepsilon} \leq \frac{1}{3}$ assures that on the equilibrium path the payoffs to coalition members are in $[0,1]$ with probability 1 if $c > \frac{1}{3}$. We show in Lemma 8 in Appendix B that $c > \frac{1}{3}$ in any equilibrium characterized in Proposition 6.

Proposition 6. *With implementation uncertainty given in Assumptions 1-3, there exists a* $c^+ \leq \frac{1}{2}$*, such that for all* $\delta > \delta^* \equiv \frac{3-\frac{3}{2}\eta\underline{\theta}}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)\eta\underline{\theta}}$*, all* $c \in [c^+, \frac{1}{2}]$ *and not too much uncertainty; i.e.,* $(\gamma, \eta) \in R(\underline{\theta}) = \{(\gamma, \eta) | 1 - \gamma - 3\eta(1 - \frac{\eta\underline{\theta}}{2}) > 0\}$*, a coalition equilibrium exists supporting* $Z(c)$*.*

Proposition 6 identifies a class of specific-policy equilibria that are indexed by the allocation c to the coalition partner, where the originator of the coalition receives the larger allocation $1 - c$. By Proposition 5 the only c supportable in the absence of uncertainty, i.e., when $\gamma = \eta = 0$, is $c = \frac{1}{2}$, but with implementation uncertainty strictly unbalanced allocations are supported, as illustrated in Figure 4.



(a) no uncertainty (b) with uncertainty

Figure 4: The supported sets $Z(c), c \in [c^+, \frac{1}{2}]$

Proposition 6 is proven in four steps. First, the continuation values corresponding to basic strategies are derived. Second, bounds on c are identified such that legislators have no incentive to deviate from the basic strategies. Third, we derive a bound δ^o such that for all $\delta > \delta^o$ there is a non-empty set of c satisfying these bounds. Finally, we establish restrictions on the implementation uncertainty such that δ^o is strictly less than one, completing the proof. These results are stated and proven in the online Appendix B, and the intuition is developed in this section.

The originator of the coalition receives an allocation of $1 - c \geq \frac{1}{2}$ every period in which the coalition continues, and the coalition partner receives $c > \frac{1}{3}$. The continuation value of the originator is thus greater than that of the partner, and a higher probability η of implementation uncertainty decreases the continuation value to the coalition members and increases the continuation value to the legislator not in the coalition, so greater uncertainty means that sustaining a coalition is more difficult. The continuation values are increasing in δ and independent of γ. For a status quo not in the supported set $Z(c)$ the continuation value is $\hat{v} = \frac{1}{3(1-\delta)}$ because of the random selection rule and the proposer's randomization among potential partners. The difference between the dynamic payoffs when the coalition persists and $\hat{v}$ when the coalition dissolves is due to exclusion risk, which provides the incentive for the partner to accept the lower payoff.

When c is not too small, the coalition partner has an incentive both to accept the coalition proposal and maintain it once the dynamic coalition has formed. We demonstrate that all supportable c are strictly greater than $\frac{1}{3}$.

The allocation c must be such that the coalition partner receiving c in the status quo has no incentive to propose (or accept) an allocation in which he receives $1-c$, and this requires that $\eta < \gamma$ for $c < \frac{1}{2}$. Greater uncertainty from changing the policy allows the allocations to the dynamic coalition members to differ.

The allocation c also must be such that the coalition partner being offered c accepts the proposal for status quos not in $Z(c)$. Remaining at a status quo outside $Z(c)$ has a continuation value of $\hat{v}$, and for c sufficiently high the coalition partner is willing to accept the greater uncertainty associated with a new policy.

We show that for a sufficiently high discount factor an equilibrium with $c \in (\frac{1}{3}, \frac{1}{2}]$ exists and that the lower bound on the discount factor is strictly less than one. This requires that implementation uncertainty is not too great. For example, consider a status quo policy $(0, 1, 0)$ for which legislator 1 makes the proposal $(1-c, c, 0)$. The status quo is very attractive for legislator 2, and with γ high it is very likely that the coalition allocation c is never realized. Furthermore with η sufficiently high once the coalition has formed it dissolves with high probability, so accepting the proposal is not attractive. So for γ and η sufficiently high legislator 2 could reject the proposal regardless of the discount factor. We establish that $\delta^o < 1$ if $(\gamma, \eta) \in R(\underline{\theta})$, which completes the proof of Proposition 6.

Corollary 8. *The set of policies supported by specific-policy coalition equilibria is strictly increasing in δ for $\delta \in (\delta^o, 1)$.*

The set of policies supported by a specific-policy equilibrium is strictly increasing in the discount factor because the coalition partner has a stronger incentive to maintain the policy and coalition the more important is the future. In a specific-policy coalition equilibrium with $c < \frac{1}{2}$ the originator of the coalition does not share the gain from proposal power equally with the coalition partner. The lower bound c^+ is greater than $\frac{1}{3}$, however, so the coalition partner receives more in each period than in sequential legislative bargaining because of his opportunity in a new coalition formation round to propose the coalition originator's policy.

If the probability of implementation uncertainty for both the current and a new policy are equal $(\gamma = \eta)$ and η is not too large, equal division within the coalition is the only policy supported by a specific-policy coalition equilibrium.

Corollary 9. *For $\gamma = \eta$, the set of allocations c that can be supported as a coalition equilibrium is the singleton $\{\frac{1}{2}\}$ for $\eta < \frac{1}{3\underline{\theta}}\left[4 - 2(4 - \frac{3}{2}\underline{\theta})^{\frac{1}{2}}\right]$.*

The equal division property in Proposition 5 is thus robust to implementation uncertainty if the probability that uncertainty is realized is the same when a new policy is adopted as when the policy remains the same and that probability is not too high.

8 Tolerant Coalitions

Specific-policy dynamic coalitions dissolve if implementation uncertainty is realized because the shock moves the implemented policy away from the coalition policy. A dynamic coalition could, however, tolerate some change in policy due to implementation uncertainty. This section identifies tolerant coalition equilibria in which coalition members accept a degree of variation in the coalition policy, i.e., the coalition persists if the policy remains in a tolerated set of policies and dissolves if it is outside the set. The coalition thus withstands small but not large shocks.

We show that basic strategies support policies in a set $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for some $\underline{c} \leq \frac{1}{2}$. Letting $\zeta \equiv [\underline{c}, 1-\underline{c}]$, the coalition persists when the realization θ^t of the implementation uncertainty satisfies $c - \theta^t \in \zeta$. If a status quo policy is not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, the proposer randomizes with basic strategies over policies that give him $1-\underline{c}$. If the status quo is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for any $c \in \zeta$, the status quo is proposed. The following assumption assures that the implementation uncertainty is sufficiently great that the coalition can dissolve for all $c \in \zeta$ from either a very high or a very low realization of $\tilde{\theta}^t$. This simplifies the expressions for the continuation values and facilitates the comparison between tolerant coalition and specific-policy coalition equilibria.

Assumption 4. $1 - 2\underline{c} \leq \underline{\theta} \leq \underline{\varepsilon} \leq \underline{c}$.

The bound $\underline{c}$ of the set ζ is obtained from the incentive constraints corresponding to deviations at the boundaries of ζ and X, and an additional deviation must be considered with tolerant equilibria. The status quo could be a policy $(1-a, a, 0)$ where $a \in (\underline{c}, 1)$, which has the property that the current period allocation $1 - a + \theta^t$ is less likely to be truncated at 1 than $1 - c + \theta^t$ and also has a higher probability that the allocation will be in ζ than a status quo $(1, 0, 0)$. A characterization of the a that maximizes the dynamic payoff and the corresponding bound is provided in Appendix C for the case of $\eta = 0$. In addition, starting from a status quo not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, the proposing legislator may propose $(1-a, a, 0)$ rather than a proposal in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. Ensuring that no such deviation is attractive requires an upper bound.

An equilibrium supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c), \underline{c} \in [\underline{c}^{+}, \frac{1}{2}]$, is referred to as a *tolerant coalition equilibrium.* Coalition members in period $t+1$ propose the status quo when the allocation in period t is in ζ. If the realized implementation uncertainty θ^t is such that $c - \theta^t$ is not in ζ, the coalition dissolves and the legislator i selected in the next period proposes a policy in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ that yields $1 - \underline{c}$ to i. Tolerant coalitions thus form immediately with the composition of the coalition determined by the selection of a proposer and the random selection of a coalition partner. When a tolerant coalition forms, the originator of the coalition receives a greater allocation than the coalition partner, but following a tolerated realization of the implementation uncertainty, the allocation to the partner and the corresponding continuation value can be larger. When a tolerant coalition forms with $c = \underline{c}$, the probability that it dissolves due to implementation uncertainty is $\frac{1}{2}\eta$, since the distribution of $\tilde{\theta}^t$ is symmetric about 0. If the coalition persists beyond one period, the probability that it dissolves in the next period is smaller, since $c \geq \underline{c}$.

As with specific-policy equilibria, restrictions are required so that the implementation uncertainty is not so great that the set of tolerated allocations is empty. Identifying the set $R^T(\underline{\varepsilon}, \underline{\theta})$ of allowable implementation uncertainty and a bound on the discount factor is complex, so existence of tolerant coalition equilibria is established in Section 8.1 for the case in which $\gamma > \eta = 0$. By continuity tolerant coalition equilibria exist for at least small η.

The continuation values corresponding to a tolerant coalition equilibrium are given in Appendix C and the continuation values for a specific-policy coalition corresponding to c^+ can be compared to the continuation values for a tolerant coalition with $\underline{c} = c^+$.

Proposition 7. *Consider a* $\underline{c} = c^+ < \frac{1}{2}$ *such that both a specific-policy coalition equilibrium and a tolerant coalition equilibrium exist. If* $\eta > (=)\ 0$*, the continuation values for specific-policy coalition members are strictly less than (equal to) the continuation values for tolerant coalition members.*

With $\eta = 0$ in a specific-policy coalition equilibrium, once formed the coalition continues with probability one, as does a tolerant coalition. The continuation values on the equilibrium path thus are the same in the two equilibria. For $\eta > 0$, implementation uncertainty can be realized on the equilibrium path in which case a specific-policy coalition dissolves, whereas the probability is positive that the allocation remains in ζ and the tolerant coalition continues. A tolerant coalition thus is more valuable to its members than is the corresponding specific-policy coalition when $\eta > 0$.

8.1 Tolerant Coalitions with Uncertainty Only When Policy Changes

To provide a further characterization of tolerant coalition equilibria, consider the case in which there is implementation uncertainty ($\gamma > 0$) associated with a change in policy but no implementation uncertainty ($\eta = 0$) when the policy is not changed. This allows a complete characterization of the set of policies supported by a tolerant coalition equilibrium. When $\eta = 0$ once the coalition policy is on the equilibrium path it remains there, so a tolerant coalition is no more valuable than a specific-policy coalition, yet the equilibria are not the same.

The following proposition states that for sufficiently high discount factors a tolerant coalition equilibrium exists with the equilibrium policy of the most tolerant coalition identified by the partner's incentive to propose or accept the larger coalition allocation $1-c$. The proof of Proposition 8 is presented in Appendix C.

Proposition 8. *With implementation uncertainty given in Assumptions 1 and 4 and for $\eta = 0$, there exists a $\delta^{\zeta} < 1$ such that for all $\delta > \delta^{\zeta}$, a coalition equilibrium exists supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for all $\underline{c} \in [c^{**}, \frac{1}{2}]$.*

The intuition underlying Proposition 8 is as follows. When $\eta = 0$ the value of a specific-policy coalition corresponding to $\underline{c}$ equals the value of a tolerant coalition corresponding to $\underline{c}$, since once on the equilibrium path the policy does not change provided no coalition member deviates from the equilibrium strategies.[29] A deviation from the tolerant coalition equilibrium strategies, however, is not as costly to a coalition member as is a deviation from the specific-policy equilibrium strategies because the former deviation could still result in an allocation in the set ζ as a result of the realization of $\tilde{\varepsilon}^t$, whereas with a specific-policy coalition the coalition dissolves with probability one whenever the shock is realized. The incentive constraint is thus tighter for a tolerant coalition equilibrium than for a specific-policy coalition equilibrium, and $c^{**} > c^*$.

When $\eta > 0$, the probability that a tolerant coalition persists is higher than the probability that a specific-policy coalition persists, since the shocked allocation can remain in the set ζ. The higher probability means that the continuation value for a tolerant coalition is higher than that in Proposition 7. This effect is in the opposite direction of the effect characterized in Proposition 8, and the bound c^{**} can be lower than c^* for a specific-policy coalition if $\frac{\eta(1-\delta(1-\gamma))}{\gamma(1-\delta(1-\eta))} \geq \frac{\underline{\theta}}{\underline{\varepsilon}}$, which requires $\underline{\theta} < \underline{\varepsilon}$ when $\eta < \gamma$. A tolerant coalition equilibrium thus could have a greater divergence between the allocations of the coalition members than in a specific-policy coalition.

9 Conclusions

Public policymaking is a dynamic process in which the opportunity to set the agenda gives legislators temporary power that can be used to their advantage. Distributive policy in particular could be prone to opportunistic behavior, and shifting agenda-setters could lead to policy instability. Yet most policies exhibit a measure of stability. This paper shows that dynamic coalitions can form in one step beginning from any status quo and once formed support policies that are stable.

The originator of a dynamic coalition has agenda-setting or proposal power as in sequential bargaining theory, but in contrast to that theory the originator of a dynamic coalition shares proposal power more equally with the other members of the coalition. Sharing is required to satisfy dynamic incentive constraints resulting from the opportunity of coalition members to propose or accept alternative policies and to vote against the coalition policy when the status quo is favorable. In the basic

[29] For $\eta = 0$ the continuation values are the same for specific-policy and tolerant coalition equilibria.

model for a three member legislature the dynamic incentives require the originator to share proposal power equally with the coalition partner.

Uncertainty can be associated with the implementation of policies, and that uncertainty can be greater when the policy changes than when it remains unchanged. Specific-policy coalition equilibria exist in the presence of implementation uncertainty, and in the supported policies the originator of the coalition can receive more than the coalition partner. The coalition partner accepts the smaller allocation rather than break the coalition and risk exclusion when a new coalition is formed.

Coalitions in specific-policy equilibria dissolve when implementation uncertainty is realized, but a coalition could tolerate a degree of uncertainty and continue to the next period. A tolerant coalition continues when the implemented policy remains in a set of tolerated policies, but if the shock is large as in a crisis, the coalition dissolves. A tolerant coalition thus can persist over time, and policies have a degree of stability, provided that the realized implementation uncertainty is not too large.

Coalition equilibria are based on particularly simple strategies with legislators proposing the status quo if it is in the supported set of policies and otherwise proposing a new coalition with coalition partners selected randomly. When legislators are risk averse, coalition equilibria seem natural, since they provide perfect risk smoothing over time. These equilibria could arise in the laboratory with the strategies of legislators coordinated through straightforward communication between the coalition originator and potential coalition partners.

Several experiments implementing the sequential legislative bargaining game find behavior that only weakly supports the theory.[30] These experiments do not allow communication among participants, and Agranov and Tergiman (2012) show that allowing participants in a legislative bargaining experiment to communicate results in behavior that more strongly supports the theory. The cheap talk communication allowed in the experiment takes place after a proposer has been selected but before a proposal is made, and any player can send a message to any subset of other players. Participants in the experiment used the communication opportunity to learn about the reservation values of other participants and to induce the proposer to include them in the majority. The theory developed here could be taken to the laboratory to study the effect of communication on strategy choice and policy outcomes.

The theory of dynamic coalitions can be applied to understand government formation in a multiparty parliamentary democracy. The theory predicts that governments can be formed even though party leaders are politically impatient. The theory explains surplus and minority governments as well as minimal-winning government coalitions, and a consensus government can form if there is exclusion risk. If there is uncertainty in implementing policies, governments can survive small shocks. However,

[30]See Frechette, Kagel and Lehrer (2003), Frechette, Kagel and Morelli (2005*a*), and Frechette, Kagel and Morelli (2005*b*).

large shocks, which can be thought of as crises, can lead to the dissolution of a government. The theory thus provides an explanation for why and when governments fail. The theory also provides an explanation for failed government formation attempts when there are interests vested in the initial status quo that prevent a Pareto improving government from forming.

The theory of dynamic coalitions can be extended in a number of directions. In the pure distribution game considered here, the preferences of legislators are directly opposing yet stable coalitions can form. With a policy space in which preferences are partially aligned, dynamic coalitions should also be present although their characteristics could differ. In particular, the extent to which proposal power is shared depends on the preference alignment, as do coalition size and the set of policies supported. For example, in every period the legislature could allocate a budget between a public good and a distributive policy with legislators having quasi-linear preferences. The model could be extended to incorporate a richer model of government in which investment, debt and tax policies are chosen. In this setting, the focus of the analysis could be the role of dynamic coalitions in limiting the inefficiency resulting from the inability of government to commit to long-term policies. Another extension would be to enrich the model of politics by incorporating features of political institutions such as bicameralism and committees with agenda control. This could also include incorporating periodic elections with the legislative bargaining model representing government formation and policy choice. Baron (2014) presents a theory of government formation and policy choice in a spatial model for a parliamentary system with a proportional representation electoral system. The theory of dynamic coalitions could provide a foundation for a theory of endogenous political parties that arise through representation of voters' interests in legislative bargaining. A number of these extensions could be taken to the laboratory.

Appendix A

A Dynamic Coalitions in the Basic Model

We first present a more general *extended model* and prove Proposition 1e, the analogue to Proposition 1 in the basic model. The basic model is a special case of the extended model, so the proof of Proposition 1 is immediate.

A.1 The extended model

The extended model is the same as the basic model, but we allow heterogeneous discount factors δ_i and heterogenous proposal probabilities $p_i < 1$ for legislator i. Continuation values in the extended model are accordingly $v_i(\sigma, \omega | q^{t-1}) = E^t[u(q_i^t) + \delta_i v_i(\sigma, \omega | q^t)]$. All other definitions including the definition of basic strategies in the extended model are analogous to those in the basic model.

Lemma 1e gives continuation values for the extended model.

Lemma 1e. *In the extended model, if (σ, ω) is a coalition equilibrium supporting a set Z:*

(*a*) *The continuation value for legislator i for $q^{t-1} \in Z$ is $v_i(q^{t-1}) = \frac{u(q_i^{t-1})}{1-\delta_i}$.*

(*b*) *The continuation value for legislator i for $q^{t-1} \notin Z$ is*

$$v_i(q^{t-1}) = v_i^* \equiv \frac{p_i u(z_{\max}) + (1 - p_i)\overline{\overline{u}}}{1 - \delta_i}. \tag{A.1}$$

where

$$\overline{\overline{u}} \equiv \frac{1}{|Z_j|} \sum_{z \in Z_j} u(z_i), \text{ for } j \neq i.$$

Let $p_{\max} \equiv \max\{p_1, \ldots, p_n\}$ and $\delta_{\min} \equiv \min\{\delta_1, \ldots, \delta_n\}$.

Proposition 1e. *In the extended model a set Z is supported by a coalition equilibrium if $\delta_{\min} > \underline{\underline{\delta}} \equiv \frac{u(1)-u(z_{\max})}{u(1)-p_{\max}u(z_{\max})-(1-p_{\max})\overline{\overline{u}}}$, and*

(*a*) *$z_i = z_{\max}$ for at least m legislators, for all $z \in Z$.*

(*b*) *$u(z_j) < u(z_{\max})$ for some j and some $z \in Z$.*

Proof. The proof proceeds by checking incentives to deviate from basic strategies, assuming Conditions (*a*) and (*b*) in Proposition 1e are true and $\delta_{\min} > \underline{\underline{\delta}}$.

First, consider $\mathbf{q^{t-1} \in Z}$. With basic strategies the proposal is the same as the status quo, so on the equilibrium path legislators vote for the status quo under the indifference rule, and $q^t = q^{t-1}$. Suppose that i is the proposer. If $q^{t-1} \in Z_i$, i receives a dynamic payoff $\frac{u(z_{\max})}{1-\delta_i}$ and has no incentive to deviate from proposing q^{t-1} to proposing $y^t \in Z \setminus Z_i$.

Consider proposer i's incentives to deviate from proposing $q^{t-1} \in Z \setminus Z_i$. The best potential deviation is $y^t \in Z_i$, which gives i a dynamic payoff of $\frac{u(z_{\max})}{1-\delta_i}$ if it defeats q^{t-1}. Since $q^{t-1} \in Z$, at least m legislators receive $z_{\max}$, so a proposal $y^t \in Z_i$ is rejected, since at least one legislator receiving $z_{\max}$ is indifferent between y^t and q^{t-1} and hence under the indifference rule votes for q^{t-1}. Proposer i thus has no incentive to deviate from proposing $y^t = q^{t-1} \in Z \setminus Z_i$.

Consider a proposer's incentive to deviate to a policy $y^t \notin Z$. The proposal y^t must attract the votes of at least $m$legislaotrs, and at least one of them, say i, receives $z_{\max}$ in q^{t-1}. The most that can be offered to i is an allocation of 1 in y^t, and i prefers q^{t-1} if $u(1) + \delta_i v_i^* \leq \frac{u(z_{\max})}{1-\delta_i}$, or

$$\begin{aligned} u(1) + \delta_i v_i^* &\leq \frac{u(z_{\max})}{1-\delta_i} \\ \Leftrightarrow u(1) + \delta_i \frac{p_i u(z_{\max}) + (1-p_i)\overline{\overline{u}}}{1-\delta_i} &\leq \frac{u(z_{\max})}{1-\delta_i} \\ \Leftrightarrow \frac{u(1) - u(z_{\max})}{u(1) - p_i u(z_{\max}) - (1-p_i)\overline{\overline{u}}} \equiv \underline{\delta}_i &\leq \delta_i, \end{aligned} \tag{A.2}$$

where $\underline{\delta}_i < 1$ because $u(z_{\max}) > \overline{\overline{u}}$. The left hand side of (A.2) is increasing in p_i, so $\delta_i \geq \underline{\underline{\delta}}$ implies that proposer i prefers q^{t-1} to y^t. Consequently, if $\delta_{min} \geq \underline{\underline{\delta}}$, any legislator receiving $z_{\max}$ in q^{t-1} votes for q^{t-1} over a $y^t \notin Z$ that offers him 1. All other legislators receiving $z_{\max}$ also vote for q^{t-1} over y^t, so y^t is rejected. Proposer i thus has no incentive to deviate from proposing $q^{t-1} \in Z$ for $\delta_i \geq \underline{\underline{\delta}}_i$. Thus, no proposer has an incentive to deviate for $\delta_{min} \geq \underline{\underline{\delta}}$.

Second, consider $\mathbf{q^{t-1} \notin Z}$. Legislators are to propose $y^t \in Z_i$, and the proposal has to be accepted over all $q^{t-1} \notin Z$. Consider a status quo such that $q_i^{t-1} = 1$. If i proposes $y^t \in Z_i$, at least $m-1$ other legislators who receive $z_{\max}$ in y^t vote for it over q^{t-1} in which they receive 0 in the current period followed by a new coalition formation round in the next period. The proposer i prefers y^t if $u(1) + \delta_i v_i^* \leq \frac{u(z_{\max})}{1-\delta_i}$, so, as above in (A.2), $\delta_i \geq \underline{\underline{\delta}}$ implies that proposer i prefers y^t to q^{t-1}. Because of the indifference rule, for i to vote for y^t i must strictly prefer y^t to q^{t-1}, which requires $\delta_i > \underline{\underline{\delta}}$. Then, y^t defeats q^{t-1}, and hence i has no incentive to deviate from proposing $y^t \in Z_i$ to proposing q^{t-1}. For any other $q^{t-1} \in Z$, since at least m legislators receive $z_{\max}$ in $y^t \in Z_i$ and the most that any one of them can receive in q^{t-1} is 1, each legislator i receiving $z_{\max}$ strictly prefers y^t to q^{t-1} if $\delta_i > \underline{\underline{\delta}}$, so proposer i has no incentive to deviate from proposing $y^t \in Z_i$. Thus, for $\delta_{min} > \underline{\underline{\delta}}$ no legislator has an incentive to deviate from proposing $y^t \in Z_i$ when $q^{t-1} \in Z$. ■

A.2 Proof of Proposition 1

The proof of Proposition 1 follows from Proposition 1e. It is straightforward to verify that for $p_i = \frac{1}{n}$ and $\delta_i = \delta$ for all i the lower bound $\underline{\underline{\delta}}$ on the discount factor simplifies to $\underline{\underline{\delta}} \equiv \frac{u(1)-u(z_{\max})}{u(1)-\overline{u}}$.

A.3 Proof of Proposition 2

We use the following notation in proving the equivalence in Condition (a) of Proposition 2. For $x, y \in X$ we say $x >_m y$ if there is a coalition of at least m legislators that strictly prefer x to y. That is, $x >_m y$ if and only if $|\{i \in \{1, \ldots, n\} : x_i > y_i\}| \geq m$. We say $x \not>_m y$ if there is no coalition of m legislators that prefer x to y. That is, $x \not>_m y$ if and only if $|\{i \in \{1, \ldots, n\} : x_i > y_i\}| < m$.

Lemma 3. *$x >_m \hat{y}$ for some $\hat{y} \in \mathcal{Z}(y)$ if and only if $u(x_i) > u(y_{i+n-m})$, for all $i = 1, \ldots, m$.*

Conversely, $x \not>_m \hat{y}$ for any $\hat{y} \in \mathcal{Z}(y)$ if and only if $u(x_i) \leq u(y_{i+n-m})$ for some $i \in \{1, \ldots, m\}$.

Proof. To show the "if," it is straightforward to see that x is strictly preferred by a coalition of at least m legislators to any permutation of y in which legislator i is allocated y_{i+n-m} for legislators $i = 1, \ldots, m$. That is, the payoffs to $x_i, i = 1, \ldots, m$, are strictly greater than the payoffs to m allocations in y, so $x >_m y$ and similarly for any permutation of y.

To show the "only if," suppose by way of contradiction that $u(x_1) \leq u(y_{1+n-m})$. Then $u(x_n) \leq \ldots \leq u(x_1) \leq u(y_{1+n-m}) \leq \ldots \leq u(y_1)$. There are at most $n - (1 + n - m) = m - 1$ remaining allocations $y_j, j \in [2 + n - m, n]$, so there at most $m - 1$ legislators that strictly prefer x to y. The same is true if $u(x_2) \leq u(y_{2+n-m})$. That is $u(x_n) \leq \ldots \leq u(x_2) \leq u(y_{2+n-m}) \leq \ldots \leq u(y_1)$. Suppose $u(y_1) < u(x_1)$, then there are at most $n-(2+n-m) = m-2$ remaining legislators with allocations in y that can be made strictly better off with an allocation in x. Thus, no more than $m-1$ legislators can be made strictly better off with an allocation in x. By induction the same is true for any $\ell = 1, \ldots, m$.

The converse is immediate. ■

Lemma 4. *The following statements are equivalent:*

1. *$W(z) = \emptyset$ for all $z \in Z$.*
2. *For any $x, y \in Z$ such that $x_i \geq x_{i+1}$ and $y_i \geq y_{i+1}$ for all $i \in \{1, \ldots, n-1\}$, we have $u(x_i) \leq u(y_{i+n-m})$ for some $i \in \{1, \ldots, m\}$.*

Proof. Pick $x, y \in S \subset Z$ such that $u(x_i) \leq u(y_{i+n-m})$ for some $i = 1, \ldots, m$. Note $x, y \in S$ are ordered appropriately by assumption. Then by Lemma 3 $x \not>_m \hat{y}$ for any $\hat{y} \in \mathcal{Z}(y)$. Since this must be true for any $x, y \in S$, it is also true that $y \not>_m \hat{x}$ for any $\hat{x} \in \mathcal{Z}(x)$. Then for any x, y or permutations of x, y there is no policy in Z that is strictly preferred by a coalition of m legislators. This is true if and only if $W(z) = \emptyset$ for all $z \in Z$. ■

Sufficiency: The proof of sufficiency proceeds by checking incentives to deviate from basic strategies assuming (a) and (b), $\delta \geq \underline{\delta}$ and $\delta > \delta^*$.

First consider $\mathbf{q^{t-1} \in Z}$. If i is the proposer, consider the votes of other legislators. With a proposal $y^t = q^{t-1}$, legislators are indifferent and vote for q^{t-1} under the indifference rule. Consider legislator i's incentives to make a deviation proposal. The best potential deviation inside Z gives legislator i a dynamic payoff of $\frac{u(z_{\max})}{1-\delta}$, and with the status quo legislator i's dynamic payoff is $\frac{u(q_i^{t-1})}{1-\delta}$. If $u(q_i^{t-1}) < u(z_{\max})$, legislator i has an incentive to deviate to an allocation in Z unless a minimal-winning coalition will reject the proposal. If $W(z) = \emptyset$ for all $z \in Z$, there is no deviation in Z that can defeat q^{t-1}. Legislator i therefore has no incentive to propose this deviation.

To deviate to a policy $z' \notin Z$, legislator i must find an m-member coalition to accept z'. It is sufficient to ensure the legislator with the m^{th} largest allocation for any $z \in Z$ has no incentive to accept a deviation. The smallest of these is $z_m^{\min}$. This deviation is rejected by legislator m if

$$\begin{aligned} u(z_i') + \delta v^* &\leq \frac{u(z_m^{\min})}{1-\delta} \\ \Leftrightarrow (1-\delta)u(z_i') + \delta \overline{u} &\leq u(z_m^{\min}) \\ \Leftrightarrow \delta &\geq \hat{\delta}_m^* \equiv \frac{u(z_i') - u(z_m^{\min})}{u(z_i') - \overline{u}}. \end{aligned}$$

Since $\hat{\delta}^*$ is increasing in z_i', then $\hat{\delta}_m^*$ is less than some $\hat{\delta}^* \leq \frac{u(1) - u(z_m^{\min})}{u(1) - \overline{u}}$.

Second, consider $\mathbf{q^{t-1} \notin Z}$. Repeating the argument above, some majority prefers $z \in Z$ to any $z' \notin Z$ if δ is strictly greater than some $\delta^* < \frac{u(1) - u(z_m^{\min})}{u(1) - \overline{u}}$, so the equilibrium proposal $z \in Z_i$ is accepted. The proposer also prefers any $z \in Z_i$ to any $z' \notin Z$. Legislator i does not have an incentive to propose any policy in $Z \setminus Z_i$, since the dynamic payoff is no greater than the dynamic payoff $\frac{u(z_{\max})}{1-\delta}$ from the equilibrium proposal if $\delta \geq \underline{\delta}$. Since $\delta^* \leq \frac{u(1) - u(z_m^{\min})}{u(1) - \overline{u}}$, for $\delta > \delta^*$ every $z \in Z_i$ is accepted when it is proposed and hence the proposer is indifferent among coalition partners and hence randomization is incentive compatible.

Necessity: We show necessity by way of contradiction. Suppose Z can be supported by a coalition equilibrium, and suppose $W(z') \neq \emptyset$ for some $z' \in Z$. Then there exists a $z'' \in Z$ such that m legislators strictly prefer z'' to z'. Consequently, a proposer may propose z'' that makes him better off and a majority prefers to vote for a deviation to z'', so z' cannot be supported as a coalition equilibrium. Condition (a) thus is necessary.

We show the necessity of the lower bounds on the discount factor. Suppose the status quo is $q^{t-1} = z' \notin Z$ with $q_i^{t-1} = 1$ and legislator i proposes $z \in Z$ with $z_1 \geq \ldots \geq z_n$. For legislator i to have an incentive to propose such a deviation requires $\frac{u(z_{\max})}{1-\delta} \geq u(1) + \delta \frac{\overline{u}}{1-\delta}$. For $\delta < \underline{\delta}$ this condition is not satisfied hence this lower bound on the discount factor is necessary. Suppose the status quo is $z \in Z$ (as before), then legislator n has an incentive to make a proposal $x \notin Z$ to a minimum winning coalition, such that $\sum_{j \in M} x_j = 1$. Some member k of the minimum winning coalition rejects the proposal only if

$\delta \geq \frac{u(x_k)-u(z_k)}{u(x_k)-\overline{u}} \equiv \hat{\delta}^*_k$. To prevent such a deviation it is necessary that the minimum winning coalition has no incentive to accept and we denote by $\hat{\delta}^*$, the discount factor such that for $\delta \geq \hat{\delta}^*$ legislator n cannot form a minimum winning coalition to accept a deviation to $x \notin Z$ starting from any $z \in Z$. Thus $\delta \geq \hat{\delta}^*$ is necessary for a coalition equilibrium to exist, and by the same arguments as before $\hat{\delta}^* \leq \frac{u(1)-u(z_m^{\min})}{u(1)-\overline{u}}$. An m-member coalition, $M \in \mathcal{M}$, accepts the equilibrium proposal $z \in Z_i$ starting from status quo $q^{t-1} \notin Z$ only if $\frac{u(z_j)}{1-\delta} > u(q_j^{t-1}) + \delta \frac{\overline{u}}{1-\delta}$ for all $j \in M$. This implies that $\delta > \frac{u(q_j^{t-1})-u(z_j)}{u(q^{t-1})-\overline{u}}$ for all $j \in M$. Consider the maximum bound for any legislator $j \in M$, for all status quos $q^{t-1} \notin Z$ and all proposals $z \in Z_i$. This bound can be written as $\max_{j \in M, q^{t-1} \notin Z, z \in Z_i} \left\{ \frac{u(q_j^{t-1})-u(z_j)}{u(q_j^{t-1})-\overline{u}} \right\}$. For the coalition M to accept requires $\delta > \max_{j \in M, q^{t-1} \notin Z, z \in Z_i} \left\{ \frac{u(q_j^{t-1})-u(z_j)}{u(q_j^{t-1})-\overline{u}} \right\}$. We find the necessary bound such that a legislator i as proposer can satisfy this constraint for some M. This necessary bound is the minimum such bound for all $M \in \mathcal{M}$. Thus $\delta^* = \min_{M \in \mathcal{M}} \left\{ \max_{j \in M, q^{t-1} \notin Z, z \in Z_i} \left\{ \frac{u(q_j^{t-1})-u(z_j)}{u(q_j^{t-1})-\overline{u}} \right\} \right\}$. As shown above this bound is no greater than $\frac{u(1)-u(z_m^{\min})}{u(1)-\overline{u}}$.

Finally, we show that Condition (b) is necessary. Suppose that $Z \subset X$ is supported by a coalition equilibrium and Condition (b) is not satisfied. That is $u(z_m^{\min}) < \overline{u}$. Note that $z_m^{\min} \leq \frac{1}{m}$, because if not, then $\sum_{i=1}^m z_i > 1$ and hence $z \notin X$. If $z_m^{\min} = \frac{1}{m}$, then Condition (b) is not violated since it must be that z awards $\frac{1}{m}$ to m legislators. Suppose therefore $z_m^{\min} < \frac{1}{m}$ and $u(z_m^{\min}) \leq \overline{u}$, so Condition ($b$) is violated. Further, suppose the status quo is $z' = (0, \ldots, 0, \frac{1}{m}, \ldots, \frac{1}{m}) \notin Z$ with $u(z'_i) = u(0)$ for $i \in \{1, \ldots, m-1\}$, and $u(z'_k) = u(\frac{1}{m})$ for legislators $k \in \{m+1, \ldots, n\}$. Consider when legislator 1 proposes $z \in Z_1$ such that $z_1 \geq \ldots \geq z_n$ and legislator m is offered $z_m^{\min}$ such that $u(z_m^{\min}) < \overline{u}$. Legislators $i \in \{1, \ldots, m-1\}$ accept the proposal if δ is sufficiently large and $u(z_i) > \overline{u}$. To obtain legislator m's vote requires $\frac{u(z_m^{\min})}{1-\delta} > u(\frac{1}{m}) + \delta \frac{\overline{u}}{1-\delta}$. Since $z_m^{\min} < \frac{1}{m}$, then u increasing implies $u(\frac{1}{m}) \geq u(z_m^{\min})$ and Condition (b) is violated, then there is no $\delta \in [0, 1)$ such that legislator m votes for the proposal. To obtain the vote of legislators $k \in \{m+1, \ldots, n\}$ requires $\frac{u(z_k)}{1-\delta} > u(\frac{1}{m}) + \delta \frac{\overline{u}}{1-\delta}$, but $u(z_k) \leq u(z_m^{\min}) \leq u(\frac{1}{m})$ and these legislators also reject the proposal for all $\delta \in [0, 1)$. The proposal therefore does not obtain m votes for any $\delta \in [0, 1)$ and hence Z is not supported in a coalition equilibrium. ■

A.4 Dynamic Coalitions in the Extended Model

Proposition 2e. *In the extended model, for each i, there exists a $\underline{\underline{\delta}}_i$ and a δ_i^* with $\max\{\underline{\underline{\delta}}_i, \delta_i^*\} \leq \frac{u(1)-u(z_m^{\min})}{u(1)-p_i u(z_{\max})-(1-p_i)\overline{u}} < 1$ for all i, and such that a coalition equilibrium supporting the set Z exists if and only if $\delta_i \geq \underline{\underline{\delta}}_i$, $\delta_i > \delta_i^*$ for all i and:*

(a) *No policy in Z can be defeated by another policy in Z in a pairwise comparison; i.e., $W(z) = \emptyset$ for all $z \in Z$, or equivalently, for any $x, y \in Z$ such that $x_i \geq x_{i+1}$ and $y_i \geq y_{i+1}$ for all*

$i \in \{1,\ldots,n-1\}$, we have $u(x_i) \leq u(y_{i+n-m})$ for some $i \in \{1,\ldots,m\}$.

(*b*) *A majority M of legislators strictly prefers to continue the policy $z \in Z$ and the corresponding coalition than change the policy and risk being excluded from a future coalition; i.e., $u(z_m^{\min}) > \bar{u}$.*

The proof is analogous to the proof of Proposition 2.

A.5 Proof of Lemma 2

We prove part (i). By Proposition 1 a coalition equilibrium exists supporting $Z = Z^m$ if $\delta > \underline{\delta}(Z^m)$. Furthermore $\delta > \underline{\delta}(Z^m)$ is necessary because if $\delta \leq \underline{\delta}(Z^m)$, then any coalition member with a status quo allocation of 1 $q_i^{t-1} = 1$ rejects the proposal $z \in Z^m$, thus no more than $m-1$ votes are obtained and the proposal does not pass.

We prove part (ii). We substitute $\underline{\kappa}$ with $\max\{n-m+2, m\}$ in what follows and show the equivalence at the end of the proof. To show sufficiency we check incentives to deviate. Suppose $Z = Z^a$ with $n \geq 4$, $\kappa \in \{\max\{n-m+2,m\},\ldots,n-1\}$, $u(b) > \overline{u}^a$ and $\delta > \max\{\underline{\delta}(Z^a), \delta^{*a}(\kappa, b)\}$. Suppose $q^{t-1} \in Z^a$. Note $\kappa \geq n-m+2$, so $u(z_2) = u(z_{n-m+2})$ whenever $\kappa \in \{\max\{n-m+2,m\},\ldots,n-1\}$. Thus Proposition 2 Condition (a) is satisfied and there is no incentive to deviate to another policy in Z if $q^{t-1} \in Z$. Now suppose $z = z^a = (a, b, \ldots, b, 0, \ldots, 0)$ and consider a deviation to a policy not in Z. The legislator with the greatest incentive to propose such a policy is legislator n, since all legislators have the same continuation payoff if $q^{t-1} \notin Z$ and the continuation payoff for $z \in Z$ is $\frac{u(z_i)}{1-\delta}$ for legislator i. Legislator n prefers a policy $x \notin Z$ with $u(x_n) = u(0)$ to z^a, since $u(0) + \delta \frac{\overline{u}^a}{1-\delta} > u(0)$ for all $\delta \in [0, 1)$. Legislator n will form a minimum winning coalition consisting of $n-1-\kappa$ legislators with a status quo allocation of 0 and $m+\kappa-n$ legislators with a status quo allocation of b. Legislators with a zero status quo allocation accept the proposal, and the most legislator n can offer each legislator with a status quo allocation of b is $\frac{1}{m-n+\kappa}$. This is rejected by legislators with a status quo allocation of b if $\frac{u(b)}{1-\delta} \geq u(\frac{1}{m-n+\kappa}) + \delta \frac{\overline{u}^a}{1-\delta} \Leftrightarrow \delta \geq \frac{u\left(\frac{1}{m-n+\kappa}\right) - u(b)}{u\left(\frac{1}{m-n+\kappa}\right) - \overline{u}^a} = \hat{\delta}^*$. If $\delta > \delta^{*a}(\kappa, b) = \frac{u\left(\frac{1}{\kappa-m+1}\right) - u(b)}{u\left(\frac{1}{\kappa-m+1}\right) - \overline{u}^a}$, then $\delta > \hat{\delta}^*$. To see this note that $\frac{u\left(\frac{1}{\kappa-m+1}\right) - u(b)}{u\left(\frac{1}{\kappa-m+1}\right) - \overline{u}^a} \geq \frac{u\left(\frac{1}{m-n+\kappa}\right) - u(b)}{u\left(\frac{1}{m-n+\kappa}\right) - \overline{u}^a} \Leftrightarrow u\left(\frac{1}{\kappa-m+1}\right) \geq u\left(\frac{1}{m-n+\kappa}\right)$. The latter inequality holds since $m \geq \frac{n+1}{2}$. Thus a minimum winning coalition will not accept legislator n's deviation proposal.

We check incentives to propose and accept the equilibrium proposal z^a if $q^{t-1} \notin Z$. If $q_i^{t-1} = 1$ for some i, then legislator i has no incentive to deviate from proposing $z \in Z^a$ that gives i an allocation of a if $\delta \geq \underline{\delta}(Z^a) = \frac{u(1)-u(a)}{u(1)-\overline{u}^a}$. Since $v_i(q^{t-1}) = \frac{u(q_i^{t-1})}{1-\delta}$ and $\kappa \geq m$ for all $q^{t-1} \in Z$, we require only m legislators with positive allocations under z^a to accept the equilibrium proposal. Suppose legislator j is offered $z_j^a = b$. If legislator j's status quo allocation is 0, then legislator j accepts the equilibrium proposal with $z_j^a = b$ as shown before. Denote by $c^{0,b}$ the number of legislators with a status quo allocation of 0 who are offered a positive allocation (a or b). If $c^{0,b} \geq m$ then the proposal passes for all $\delta \geq \underline{\delta}(Z^a)$. If $c^{0,b} < m$, then $m - c^{0,b}$ votes are required from legislators with a strictly positive status quo allocation. There are $\kappa - c^{0,b}$ legislators with a strictly positive allocation in the status quo that are offered a positive allocation under the equilibrium proposal z. Each of these legislators accepts the proposal if $\frac{u(z_j)}{1-\delta} > u(x_j) + \delta \frac{\overline{u}^a}{1-\delta}$. This constraint is tightest when x_j is maximized for all of the $\kappa - c^{0,b}$ legislators with positive allocations. The tightest constraint thus occurs when $\kappa - c^{0,b}$ legislators divide the dollar equally, each receiving $\frac{1}{\kappa - c^{0,b}}$ under the status quo, and they are offered b. This status quo allocation is strictly increasing in $c^{0,b}$ hence the status quo policy q^{t-1} for which the allocation is largest is when $c^{0,b} = m-1$ (recalling $c^{0,b} < m$ and $c^{0,b}$ must be an integer). The $\kappa - (m-1)$ legislators vote for the equilibrium proposal z^a if $\frac{u(b)}{1-\delta} > u(\frac{1}{\kappa-m+1}) + \delta \frac{\overline{u}^a}{1-\delta} \Leftrightarrow \delta > \frac{u\left(\frac{1}{\kappa-m+1}\right) - u(b)}{u\left(\frac{1}{\kappa-m+1}\right) - \overline{u}^a} = \delta^{*a}(\kappa, b)$. (Note if $\delta > \delta^{*a}(\kappa, b)$, then any status quo with $q_j^{t-1} > 0$ where j is offered a in the proposal is defeated by the proposal.)

We now show necessity of $\kappa \in \{\max\{n-m+2, m\}, \ldots, n-1\}$. We first show that $\kappa \geq m$. Suppose by way of contradiction that $\kappa < m$. Then $u(z_m^{\min}) \leq u(0)$ contradicting Proposition 2 Condition (b). We next show that $\kappa \geq n - m + 2$. Suppose by way of contradiction that $\kappa < n - m + 2$. Then for all $i \in \{1, \ldots, m\}$ we have $u(z_i) > u(z_{i+n-m})$ contradicting Proposition 2 Condition (a). Suppose $\kappa = n > n-1$. Then $u(b) < \overline{u}$, contradicting Proposition 2 Condition (b). Note that the range of κ is feasible if and only if $n \geq 4$.

The necessity of $u(b) > \overline{u}^a$ is given by Proposition 2 Condition (b).

We show necessity of $\delta \geq \underline{\delta}(Z^a)$. Suppose $\delta < \underline{\delta}(Z^a)$, then this contradicts Proposition 2 and thus Z^a is not supported in a coalition equilibrium.

We show necessity of $\delta > \delta^{*a}(\kappa, b)$. Suppose $\delta \leq \delta^{*a}(\kappa, b)$, by way of contradiction, and $Z = Z^a$ is supported in a coalition equilibrium. Consider $q^{t-1} \notin Z$, with $q_j^{t-1} = \frac{1}{\kappa-m+1}$ for legislators $j \in \{m, \ldots, \kappa\}$, $q_k^{t-1} = 0$ for all other legislators, and $z^a = (a, b, \ldots, b, 0, \ldots, 0)$ is proposed by legislator 1. Legislators $k \in \{\kappa+1, \ldots, n\}$ reject the proposal because $\frac{u(0)}{1-\delta} < u(0) + \delta \frac{\overline{u}^a}{1-\delta}$ for all $\delta \in (0,1)$. Legislators $i \in \{1, \ldots, m-1\}$ accept the proposal because $u(b) > \overline{u}^a$. Legislators $j \in \{m, \ldots, \kappa\}$ reject the proposal because $\delta \leq \delta^{*a}(\kappa, \delta)$, and thus only $m-1$ votes are obtained and the proposal is rejected. Then Z^a is not supported in a coalition equilibrium.

Finally we show

$$\max\{n-m+2, m\} \quad = \quad \begin{cases} m+1 & \text{if } m \text{ is odd and } m = \frac{n+1}{2} \\ m & \text{otherwise.} \end{cases}$$

Note $n - m + 2 > m \Leftrightarrow \frac{n}{2} + 1 > m$. For n even, the minimum value of m is $\frac{n}{2} + 1$ violating the previous inequality. For n odd, the minimum value of $m = \frac{n+1}{2} < \frac{n}{2} + 1$. Thus the inequality is satisfied for n

odd and $m = \frac{n+1}{2}$. Since m must be an integer, the next greatest value for n odd is $m = \frac{n+3}{2} > \frac{n}{2} + 1$, which violates the inequality. Thus the only case for which $\max\{n - m + 2, m\} = n - m + 2$ is the case of n odd and $m = \frac{n+1}{2}$. Substituting $n = 2m - 1$ then gives $n - m + 2 = m + 1$.■

A.6 Proof of Proposition 3

For a given (n, m, u, δ), Proposition 3 characterizes the highest allocation in Z. We characterize, for a given (n, m, u), the lowest discount factor such that a coalition equilibrium exists in Lemma 6. To do this, we first provide a useful comparative static result in Lemma 5. The remainder of the proof characterizes the highest allocation for any discount factors such that a coalition equilibrium exists.

Making the condition of $\overline{u}$ on elements of Z_i explicit, define the function $\tilde{\delta}(z_j, Z_i; x_j) = \frac{u(x_j)-u(z_j)}{u(x_j)-\overline{u}(Z_i)}$, where z_j is the j^{th} element of a policy $z \in Z_i$, and x_j is the j^{th} element of $x \in X$.

Lemma 5. *If $u(x_j) > \frac{\sum_{k\neq j} u(z_k)+\sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)}{n|Z_i|-1}$, then $\tilde{\delta}(z_j, Z_i; x_j)$ is (i) decreasing in z_j, and (ii) increasing in $z'_k \neq z_j$ for all $k \in \{1, \ldots, n\}$ and $z' \in Z_i$.*

Proof. To show part (i), consider a set Z' identical to Z, but with $z_j < 1$ for some $z \in Z$ replaced with $z_j + \varepsilon < 1$ with $\varepsilon > 0$. Note policies in Z and Z' may be inefficient. We have

$$\tilde{\delta}(z_j + \varepsilon, Z'_i; x_j) = \frac{u(x_j) - u(z_j + \varepsilon)}{u(x_j) - \frac{1}{n|Z_i|}\left[u(z_j + \varepsilon) + \sum_{k\neq j} u(z_k) + \sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)\right]}.$$

Then

$$\tilde{\delta}(z_j + \varepsilon, Z'_i; x_j) < \tilde{\delta}(z_j, Z_i; x_j) \quad \Leftrightarrow \quad \frac{u(x_j) - u(z_j + \varepsilon)}{u(x_j) - \frac{1}{n|Z_i|}\left[u(z_j + \varepsilon) + \sum_{k\neq j} u(z_k) + \sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)\right]}$$
$$< \frac{u(x_j) - u(z_j)}{u(x_j) - \frac{1}{n|Z_i|}\left[u(z_j) + \sum_{k\neq j} u(z_k) + \sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)\right]}. \tag{A.3}$$

Simplifying and rearranging (A.3) gives

$$0 < [u(z_j + \varepsilon) - u(z_j)] \left[u(x_j)(n|Z_i| - 1) - \sum_{k\neq j} u(z_k) - \sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)\right].$$

This expression holds since $u(x_j) > \frac{\sum_{k\neq j} u(z_k)+\sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)}{n|Z_i|-1}$, and $u(z_j + \varepsilon) - u(z_j) > 0$ because u is strictly increasing .

To show part (ii), note that an increase in z'_k such that $z'_k \neq z_j$ and $z' \in Z'_i$ implies an increase in $\overline{u}(Z_i)$ holding everything else fixed. Thus $\hat{\delta}(z_j, Z_i)$ increases with z'_k. ■

The condition $u(x_j) > \frac{\sum_{k\neq j} u(z_k)+\sum_{z'\in Z_i\setminus\{z\}} \sum_{k=1}^{n} u(z'_k)}{n|Z_i|-1}$ implies that the average policy in Z_i, excluding z_j gives lower utility than x_j. The allocation x_j may come from a possible deviation. Next we give the lower bound on discount factors such that a coalition equilibrium exists.

Lemma 6. *For a fixed (n, m, u) with u strictly increasing, a coalition equilibrium exists for some $Z \subset X$ if only if $\delta > \min\{\underline{\delta}(Z^m), \delta^{*a}(\kappa^*, b^*)\}$ where b^* satisfies $\underline{\delta}(Z^a) = \delta^{*a}(\kappa^*, b^*)$, with $Z^a = \mathcal{Z}(\{(1-(\kappa^*-1)b^*, b^*, \ldots, b^*, 0, \ldots, 0)\})$.*

Proof. To show the "if", by Lemma 2 a coalition equilibrium exists supporting Z^m if $\delta > \underline{\delta}(Z^m)$, and a coalition equilibrium exists supporting $Z^a = \mathcal{Z}(\{(1-(\kappa^*-1)b^*, b^*, \ldots, b^*, 0, \ldots, 0)\})$ if $\delta > \underline{\delta}(Z^a) = \delta^{*a}(\kappa^*, b^*)$.

We prove the "only if" by contradiction. Suppose that $\delta \in [0, \min\{\underline{\delta}(Z^m), \delta^{*a}(\kappa^*, b^*)\}]$ and a coalition equilibrium exists for some Z.

First, suppose $Z = Z^m$. This contradicts Lemma 2 that says $\delta > \underline{\delta}(Z^m)$ is necessary to support Z^m.

Next, suppose $Z \neq Z^m$. Consider $z' \in Z_i$ with $z'_i \geq z'_{i+1}$ for all $i \in \{1, \ldots, n-1\}$. Such a z' satisfies either (i) $0 < z'_j < \frac{1}{m}$ for some $j \in \{1, \ldots, m\}$ and $z'_j = 0$ for all $j \in \{m+1, \ldots, n\}$ and the policy is inefficient, or (ii) $0 < z'_j < \frac{1}{m}$ for some $j \in \{1, \ldots, m\}$ and $z'_j > 0$ for some $j \in \{m+1, \ldots, n\}$, and the policy may or may not be inefficient.

In the first case, consider legislator m. This is the legislator with the lowest positive allocation in z', and is the pivotal coalition member for the policy z'. Consider q^{t-1} such that $q^{t-1}_m = 1$. Legislator m accepts the proposal if and only if $\delta > \tilde{\delta}(z'_m, Z_i; q^{t-1})$. The payoff $u(1)$ is greater than any other payoff in Z_i (satisfying the condition for $u(x_j)$ in Lemma 5), thus $\tilde{\delta}(z'_m, Z_i; q^{t-1})$ is increasing in $z_k \neq z'_m$ by Lemma 5. To obtain the Z_i that requires the lowest discount factor such that z' is sustainable, set $z_k = z'_m$ for all $k = 1, \ldots, m$, and set $z_l = 0$ for all $l = m+1, \ldots, n$, for all the $z \in Z_i$ which are ordered, i.e. all the $z \in Z_i$ with $z_i \geq z_{i+1}$ for all $i \in \{1, \ldots, n-1\}$. Then

$$\overline{u}(Z_i) = \tfrac{mu(z'_m)+(n-m)u(0)}{n} \equiv \overline{u}^*(z'_m), \tag{A.4}$$

and

$$\tilde{\delta}(z'_m, Z_i; q^{t-1}) = \tfrac{u(1)-u(z'_m)}{u(1)-\overline{u}^*(z'_m)} \equiv \tilde{\delta}^*(z'_m).$$

It is straightforward to show $\tilde{\delta}^*(z'_m)$ is a decreasing function, thus legislator m rejects the proposal since $\delta < \underline{\delta}(Z^m) = \tilde{\delta}^*(\frac{1}{m}) \leq \tilde{\delta}^*(z'_m)$.[31] Then no more than $m-1$ legislators accept the proposal, contradicting that Z is supported by a coalition equilibrium.

Consider the second case. For any inefficient policy, the previous analysis shows such a Z cannot be supported with a discount factor less than $\underline{\delta}(Z^m)$. Consider z' efficient with $z'_m = \frac{1}{m} - \varepsilon$ and

[31] To see $\tilde{\delta}^*(z'_m)$ decreasing, consider $z'_m < z''_m$. Then $\tilde{\delta}^*(z'_m) - \tilde{\delta}^*(z''_m) = \frac{u(1)-u(z'_m)}{u(1)-\overline{u}^*(z'_m)} - \frac{u(1)-u(z''_m)}{u(1)-\overline{u}^*(z''_m)} = \frac{[u(1)-u(z'_m)][u(1)-\overline{u}^*(z''_m)]-[u(1)-u(z''_m)][u(1)-\overline{u}^*(z'_m)]}{[u(1)-\overline{u}^*(z'_m)][u(1)-\overline{u}^*(z''_m)]}$. Using the definition of $\overline{u}^*$ in (A.4), this simplifies to $\frac{\frac{n-m}{n}[u(z''_m)-u(z'_m)][u(1)-u(0)]}{[u(1)-\overline{u}^*(z'_m)][u(1)-\overline{u}^*(z''_m)]} > 0$.

$z'_{m+1} = \varepsilon$. For ε sufficiently small, the binding constraint is for the proposing legislator to propose $z'_1 \approx \frac{1}{m}$, when the status quo awards 1 to the proposer. This constraint is violated for $\delta \leq \underline{\delta}(Z^m)$ from before.

To relax the constraint on the discount factor (i.e., to find a set Z such that the necessary bound on the discount factor is lower) requires $z'_1 > \frac{1}{m}$ and such that the conditions of Proposition 2 are satisfied. By Lemma 5, the proposal with the lowest bound on the discount factor satisfying these is z^a with $a = 1 - (\kappa - 1)b$, and z^a is supported if and only if $\delta \geq \underline{\delta}(Z^a)$ and $\delta > \delta^{*a}(\kappa, b)$ by Lemma 2. The coalition size that minimizes $\delta^{*a}(\kappa, b)$ is κ^*. The bound $\underline{\delta}(Z^a)$ is strictly decreasing in b and $\delta^{*a}(\kappa^*, b)$ is strictly increasing in b, thus the minimum discount factor such that Z^a is supported is attained for b^* such that $\underline{\delta}(Z^a) = \delta^{*a}(\kappa^*, b^*)$. Thus for $\delta \leq \delta^{*a}(\kappa^*, b^*)$, $Z = Z^a$ is not supportable as a coalition equilibrium for any b and κ. ■

For any feasible discount factor we now characterize the highest allocation. First consider $Z = Z^m$. By Lemma 2 this set is supportable in a coalition equilibrium for $\delta \geq \underline{\delta}(Z^m)$. The maximum allocation a coalition member can receive is $\frac{1}{m}$ under this policy.

Now consider Z^a. By Lemma 2 Z^a is supported only for $n \geq 4$. The supremum allocation a coalition member can receive is $a = 1 - (\kappa - 1)b$. The greatest upper bound for a minimizes b. The proof of Lemma 2 argues that the tightest constraint for coalition member j with $z^a_j = b$ is when $q^{t-1}_j = \frac{1}{\kappa - m + 1}$. Thus the infimum of b satisfies $u(b) = (1-\delta)u(\frac{1}{\kappa-m+1}) + \overline{u}^a$. Denote this infimum as $\underline{b}(\kappa, \delta)$. The supremum of a is thus $\overline{a}(\kappa, \delta) = 1 - (\kappa - 1)\underline{b}(\kappa, \delta)$. The maximizer of this is $\hat{\kappa}^*$, so $\kappa^* = \max\{\hat{k}^*, n-m+1, m-1\}$ by Lemma 2. For $\overline{a}(\kappa^*, \delta)$ to be feasible, we require $\overline{a}(\kappa^*, \delta) > (1-\delta)u(1) + \delta\overline{u}^a$. The left side is increasing in δ and the right side is decreasing with δ. Define $\delta^a(\kappa^*)$ as the threshold such that for all $\delta > \delta^a(\kappa^*)$ we have $\overline{a}(\kappa^*, \delta) > (1-\delta)u(1) + \delta\overline{u}^a$. Thus the supremum of a is $\overline{a}(\kappa^*, \delta)$ if $\delta > \delta^{*a}(\kappa^*, \frac{1-\frac{1}{m}}{\kappa^*-1}) = \delta^{*m} \geq \delta^a(\kappa^*)$. In this case for $\delta \in (\min\{\underline{\delta}(Z^m), \delta^{*a}(\kappa^*)\}, \delta^{*m})$ the maximum is $\frac{1}{m}$ because either a proposal in Z^a is not feasible, or $\frac{1}{m}$ is greater than the highest payoff in Z^a. If $\delta^{*m} < \delta^a(\kappa^*)$, then there are two cases. First suppose $\delta^a(\kappa^*) < \underline{\delta}(Z^m)$, the supremum allocation is $\overline{a}(\kappa^*, \delta)$ for all feasible discount factors by Lemma 6 (we show below that $\delta^a(\kappa^*) = \delta^{*a}(\kappa^*, b^*)$. Next suppose $\underline{\delta}(Z^m) \leq \delta^a(\kappa^*)$, then for $\delta \in (\underline{\delta}(Z^m), \delta^a(\kappa^*)]$ the maximum allocation is $\frac{1}{m}$, and for $\delta \in (\delta^a(\kappa^*), 1)$ the supremum is $\overline{a}(\kappa^*, \delta)$ otherwise.

We show that no set of policies can give higher allocations than the maximum available in Z^a or Z^m. The greatest upper bound for any allocation minimizes $\overline{u}$ since any coalition member's allocation is increasing in $\overline{u}$. Recall $\overline{u} = \frac{1}{n|Z_j|}\sum_{z \in Z_j}\sum_{i=1}^n u(z_i)$. Denote $\tilde{z} = (\bar{a}, \underline{b}, \ldots, \underline{b}, 0, \ldots, 0)$, with $\bar{a} > \underline{b}$. First, suppose $\tilde{Z} = \mathcal{Z}(\{\tilde{z}\})$, then $\overline{u} = \overline{u}^a$ with $a = \bar{a}$ and $b = \underline{b}$. We show by contradiction that there does not exist a $\hat{Z}$ supported by a coalition equilibrium such that the infimum of $\overline{u}$ is lower than $\overline{u}^a$, and such that $z_{\max} \geq \bar{a}$, where $z_{\max}$ is the maximum allocation in $\hat{Z}$. Suppose

$\inf \overline{u} < \frac{u(\overline{a})+(\kappa-1)u(\underline{b})+(n-\kappa)u(0)}{n}$, and $\hat{Z}$ is such that $z_{\max} \geq \overline{a}$. Then there exists $\hat{z} \in \hat{Z}_i \subset \hat{Z}$ with $\hat{z}_i \geq \hat{z}_{i+1}$ for all $i \in \{1, \ldots, n-1\}$ and such that $u(\hat{z}_i) < u(\tilde{z}_i)$. Since $\hat{z} \in \hat{Z}_i$, we know $\hat{z}_1 \geq \overline{a}$. Further, since $\tilde{z}_i = 0$ for $i = \kappa+1, \ldots, n$, then $\hat{z}_i \geq \overline{z}_i$ for $i = \kappa+1, \ldots, n$. Suppose $u(\hat{z}_2) < u(\underline{b})$. Since the elements of $\hat{z}$ are ordered, this implies $u(\hat{z}_i) < u(\underline{b})$ for all $i \in \{2, \ldots, \kappa\}$. Then $u(\hat{z}_m) < u(\underline{b}) = \frac{1}{n-k}u(\overline{a}) + \frac{n-k-1}{n-k}u(0) \leq \frac{1}{n-k}u(\hat{z}_{\max}) + \frac{n-k-1}{n-k}u(0) \leq \overline{u}$, which contradicts Proposition 2, Condition (b), thus $\hat{Z}$ cannot be supported in a coalition equilibrium, which is a contradiction. A similar argument shows that if $u(\hat{z}_i) < u(\underline{b})$ for any $i \in \{2, \ldots, \kappa\}$, then we have $u(\hat{z}_m) \leq \overline{u}$ violating Proposition 2, Condition (b) so a coalition equilibrium cannot exist supporting such a $\hat{Z}$.

The value $\overline{a}(\kappa^*, \delta)$ is increasing in δ because $\underline{b}(\kappa^*, \delta)$ is decreasing in δ.

There does not exist a coalition equilibrium for $\delta \leq \min\{\underline{\delta}(Z^m), \delta^{*a}(\kappa^*, b^*)\}$ by Lemma 6. It is straightforward to show that $\delta^a(\kappa^*) = \delta^{*a}(\kappa^*, b^*)$. This is true because for $\delta = \delta^a(\kappa^*)$ we have $\underline{\delta}(Z^a) = \delta^{*a}(\kappa^*, \underline{b})$, and thus it must be that $\underline{b} = b^*$ by the definition of b^*. ■

A.7 Proof of Proposition 4

For u linear

$$\bar{a}(\kappa, \delta) = 1 - (\kappa - 1)\left(\tfrac{1-\delta}{\kappa-m+1} + \tfrac{\delta}{n}\right).$$

For u strictly concave (convex) by Jensen's inequality

$$\bar{u} = \frac{u(1-(\kappa-1)b) + (\kappa-1)u(b) + (n-\kappa)u(0)}{n} < (>) u(\tfrac{1}{n}).$$

Then, for $u(\cdot)$ strictly concave by Jensen's inequality

$$(1-\delta)u(\tfrac{1}{\kappa-m+1}) + \delta u(\tfrac{1}{n}) < u(\tfrac{1-\delta}{\kappa-m+1} + \tfrac{\delta}{n}),$$

and

$$\begin{aligned}
\bar{a}(\kappa, \delta) &= 1 - (\kappa-1)u^{-1}\left((1-\delta)u(\tfrac{1}{\kappa-m+1}) + \delta\bar{u}\right) \\
&> 1 - (\kappa-1)u^{-1}\left((1-\delta)u(\tfrac{1}{\kappa-m+1}) + \delta u(\tfrac{1}{n})\right) \\
&> 1 - (\kappa-1)u^{-1}\left(u(\tfrac{1-\delta}{\kappa-m+1} + \tfrac{\delta}{n})\right) \\
&> 1 - (\kappa-1)\left(\tfrac{\delta}{\kappa-m+1} + \tfrac{\delta}{n}\right).
\end{aligned}$$

Repeating the argument for u strictly convex yields $\bar{a}(\kappa, \delta) < 1 - (\kappa-1)\left(\frac{\delta}{\kappa-m+1} + \frac{\delta}{n}\right)$.

A.8 Proof of Proposition 5

Proposition 1 shows that Conditions (a) and (b) are sufficient for an equilibrium to exist for $\delta > \underline{\delta}$. We show that Conditions (a) and (b) are necessary and sufficient for $n = 3$.

Proof. First, the necessary form of equilibrium policies is established. For $|S| = 1$ assume that there is a coalition equilibrium in which $z = (c, b, a) \in S$ with $c > b > a$ is supported. Legislator 2 can propose $z' = (a, c, b) \in Z$, which defeats z. Consequently, $c = b$ or $b = a$. Suppose that it is the latter. Then, $\bar{u} = \frac{2u(b)+u(a)}{3} > u(b)$, which violates Condition (b) of Proposition 2. Consequently, $c = b$ is necessary. Next, consider $c = b = a$, in which case $\bar{u} = u(c)$; i.e., there is no collective punishment, so there is no coalition equilibrium supporting $z = (c, c, c)$. Consequently, it is necessary that the supported policies are of the form $z = (c, c, a), c > a$.

Next consider $|S| > 1$. Suppose first that $z_{\max}$ is an element of each $s \in S$. Then, the argument above implies that all $s \in S$ must be of the form $s = (c, c, a^s), c > a^s$, where $c = z_{\max}$ and a^s can differ among the elements of S. Suppose that some $s' \in S$ has a maximal element less than $z_{\max} = c$. Then, $z = (c, c, a^s)$ defeats $z' = s'$, contradicting Condition (a) of Proposition 2. Consequently, all $s \in S$ are of the form $s = (c, c, a^s), c > a^s$.

Consider $S = \{(c, c, a)\}$, with $a < c$. Such a set is sustained if and only if $\delta \geq \underline{\delta}$ by the same arguments used to prove Lemma 2 part (i). Next, consider $|S| > 1$. Then $c > a^s$ for all $s \in S$ and $\delta > \underline{\delta}$ are sufficient from Proposition 1. Necessity of $\delta > \underline{\delta}$ is established as for the case of $|S| = 1$ by considering the incentive for legislator i to deviate from proposing $z \in S$ when $q^{t-1} \notin Z$. ■

A.9 Universal Efficient Allocations and Player-Specific Punishments

The universal allocation can be obtained with strategies that are conditioned on information in addition to the status quo. To sketch the argument, denote $\bar{x} = (\frac{1}{n}, \ldots, \frac{1}{n})$ as the universal allocation, and consider proposal strategies where a legislator proposes $\bar{x}$ for any status quo as long as no deviation has occurred and assume that the proposals $\bar{x}$ are approved under stage-undominated voting strategies. If legislator i deviates and proposes $y^t \neq \bar{x}$, a punishment phase commences with all legislators $j \neq i$ punishing i by playing the coalition equilibrium in which they form a dynamic coalition and each receives $\frac{1}{n-1}$ leaving 0 for the deviator. If legislator i becomes the proposer after the deviation, legislator i proposes the status quo. Voting strategies in the punishment phase are stage undominated with legislators voting for the proposal if indifferent between the proposal and the status quo and the game is not in the coalition equilibrium punishment phase. This punishment is player-specific rather than collective, and uses the history of the game beyond what is used in the equilibrium characterized in Proposition 2.[32] Note that legislators $j \neq i$ strictly prefer to punish legislator i, because their dynamic payoff is $\frac{u(\frac{1}{n-1})}{1-\delta}$ rather than $\frac{u(\frac{1}{n})}{1-\delta}$ if they return to the equilibrium path.

If $q_i^{t-1} = 1$, the best deviation for i is to propose $y^t = q^{t-1}$. The continuation value v_i^o for i then

[32] That is, Markov strategies are conditioned on only the status quo, and if $q^{t-1} \notin Z$ after $t > 1$, it is known that some player has deviated from the equilibrium path, but which player deviated is not identified. A player-specific punishment is conditioned on more of the history of play than just the status quo policy. In the particular player-specific punishment discussed here, the status quo policy and the identity of the deviator are needed to specify the strategies.

satisfies

$$v_i^o = \tfrac{1}{n}\left[u(1) + \delta v_i^o\right] + \tfrac{n-1}{n}\left[u(0) + \delta \tfrac{u(0)}{1-\delta}\right],$$

which yields

$$v_i^o = \tfrac{1}{n-\delta}\left[u(1) + (n-1)\tfrac{u(0)}{1-\delta}\right].$$

Legislator i has no incentive to deviate if

$$\begin{aligned} \tfrac{u(\frac{1}{n})}{1-\delta} &> u(1) + \delta v_i^o \\ \Leftrightarrow \delta &> \bar{\delta} \equiv \tfrac{n(u(1)-u(\frac{1}{n}))}{nu(1)-(n-1)u(0)-u(\frac{1}{n})}. \end{aligned}$$

Player-specific punishments using a coalition equilibrium in which the deviator receives 0 thus can be used to generate a larger class of equilibria than can be attained with Markov strategies.

Appendix B - For online publication

B Implementation uncertainty

The maintained assumption on implementation uncertainty ensures that when uncertainty is realized the policy remains in the feasible set X. Assumption 1 gives details of the specification of implementation uncertainty.

Assumption 1. *If a proposal $y^t = q^{t-1}$ is adopted, with probability $1-\eta$ the policy implemented equals the proposal, and with probability $1 > \eta \geq 0$ the policy is distorted by a uniformly distributed shock $\tilde{\theta}^t$ with mean zero and support $[-\underline{\theta}, \underline{\theta}]$. (i) For $y^t \in Z(c)$, if the realization θ^t is such that $c - \theta^t \geq 0$, the legislators in the coalition receive $1 - c + \theta^t$ and $c - \theta^t$. If $c - \theta^t < 0$ for legislator ℓ, ℓ receives 0 and the other coalition member ℓ' receives 1. (ii) For a proposal $y^t = q^{t-1} \notin Z(c)$, if a legislator ℓ receives 1 in y^t and $1 + \theta^t \geq 1$, ℓ receives 1. If $1 + \theta^t < 1$, ℓ receives $1 + \theta^t$ and one other legislator selected at random receives $-\theta^t$. If only two legislators receive positive allocations in $y^t = (1 - x_\ell, x_\ell, 0)$, where $0 < x_\ell \leq \frac{1}{2}$, they receive $1 - x_\ell + \theta^t$ and $x_\ell - \theta^t$, respectively, if $x_\ell - \theta^t \geq 0$. If $x_\ell - \theta^t \leq 0$, ℓ receives 1 and ℓ' receives 0. If all three legislators receive positive allocations in y^t, the allocations with the shock are $x_\ell + \alpha_\ell \theta^t, \ell = i, j, k$, where $|\alpha_\ell| \leq 1, \ell = i, j, k$, and $\alpha_i + \alpha_j + \alpha_k = 0$. If $x_{\ell'} + \alpha_{\ell'} \theta^t \leq 0$ for some ℓ', ℓ' receives 0 and $-\alpha_{\ell'} \theta^t$ is allocated randomly among the other legislators.*

If a proposal $y^t \neq q^{t-1}$ is adopted, with probability $1-\gamma$ the policy implemented equals the proposal, and with probability $1 > \gamma \geq 0$ the policy is distorted from the proposal by a uniformly distributed shock $\tilde{\varepsilon}^t$ with support $[-\underline{\varepsilon}, \underline{\varepsilon}]$. (iii) For $y^t \in Z(c)$ the allocations are as in (i) with the realization ε^t replacing θ^t. (iv) For $y^t \notin Z(c)$ the allocations are as in (ii) with the realization ε^t replacing θ^t.

B.1 Specific-policy equilibrium policies

For notational convenience define $z_{ij}(c) \in Z(c)$ as the policy that allocates $1 - c$ to legislator i, c to legislator j, and 0 to legislator k, so legislator i receives the highest allocation and legislator j is the coalition partner.

Lemma 7. *With implementation uncertainty given in Assumptions 1-3, if (σ, ω) is a coalition equilibrium for a set $Z(c)$ with $c > \frac{1}{3}$:*

(*i*) *The continuation value for legislator i for $q^{t-1} \in Z(c)$ is*

$$v_i(q^{t-1}) = \frac{3(1-\delta)q_i^{t-1} + \eta\delta}{3(1-\delta)(1-\delta(1-\eta))}, \; q_i^{t-1} \in \{1-c, c, 0\}. \tag{B.1}$$

(*ii*) *The continuation value for legislator i for $q^{t-1} \notin Z(c)$ is*

$$v_i(q^{t-1}) = \hat{v} \equiv \frac{1}{3(1-\delta)}.$$

Proof. Let $v_i(y^t), i = 1,2,3$, denote the continuation value when the proposal is y^t, conditional on no shock being realized. Let $v_i(y^{\varepsilon^t})$ denote the continuation value when y^t is implemented conditional on the shock ε^t being realized, and $v_i(y^{\theta^t})$ denote the continuation value when y^t is implemented conditional on the shock θ^t being realized.

The continuation value $\hat{v}_1$ when $q^{t-1} \notin Z(c)$ is given by

$$\begin{aligned}
\hat{v}_1 \quad = \quad & (1-\gamma)\left[\tfrac{1}{3}\left[\tfrac{1}{2}(1-c+\delta v_1(z_{12}(c)))\right] + \left[\tfrac{1}{2}(1-c+\delta v_1(z_{13}(c)))\right]\right] \\
& + \tfrac{1}{3}\left[\tfrac{1}{2}\left(c+\delta v_1(z_{21}(c))\right) + \tfrac{1}{2}\delta v_1(z_{23}(c))\right] \\
& + \tfrac{1}{3}\left[\tfrac{1}{2}\left(c+\delta v_1(z_{31}(c))\right) + \tfrac{1}{2}\delta v_1(z_{32}(c))\right] \\
& + \gamma\left[\tfrac{1}{3}\left[\tfrac{1}{2}(1-c+E^t\tilde{\varepsilon}^t+\delta v_1(z_{12}^{\varepsilon^t})) + \tfrac{1}{2}(1-c+E^t\tilde{\varepsilon}^t+\delta v_1(z_{13}^{\varepsilon^t}))\right]\right] \\
& + \tfrac{1}{3}\left[\tfrac{1}{2}(c-E^t\tilde{\varepsilon}^t+\delta v_1(z_{21}^{\varepsilon^t})) + \tfrac{1}{2}\delta v_1(z_{23}^{\varepsilon^t})\right] \\
& + \tfrac{1}{3}\left[\tfrac{1}{2}(c-E^t\tilde{\varepsilon}^t+\delta v_1(z_{31}^{\varepsilon^t})) + \tfrac{1}{2}\delta v_1(z_{32}^{\varepsilon^t})\right], \qquad \text{(B.2)}
\end{aligned}$$

and the continuation values $\hat{v}_2$ and $\hat{v}_3$ for the other legislators are given analogously. By symmetry $\hat{v}_i = \hat{v}, i = 1,2,3$.

Given $q^{t-1} \notin Z(c)$, the policy $z_{ij}^{\varepsilon^t}$ resulting from a proposal $y^t \neq q^{t-1}$ is not in $Z(c)$ with probability one, so the continuation values $v_i(z_{ij}^{\varepsilon^t}) = \hat{v}, i, = 1,2,3, i \neq j$. For $q^{t-1} = z_{ij}(c)$ the allocation $z_{ij}^{\theta^t} \in Z(c)$ with probability zero, so the continuation value $v_i(z_{ij}^{\theta^t}) = \hat{v}, i = 1,2,3,\ i \neq j$. For $q^{t-1} \in Z(c)$ the dynamic payoffs $v_i(z_{12}(c)), i = 1,2,3$, are given by

$$v_1(z_{12}(c)) \quad = \quad (1-\eta)[1-c+\delta v_1(z_{12}(c))] + \eta[1-c+E^t\tilde{\theta}^t+\delta\hat{v}] \qquad \text{(B.3)}$$

$$v_2(z_{12}(c)) \quad = \quad (1-\eta)[c+\delta v_2(z_{12}(c))] + \eta[c+E^t\tilde{\theta}^t+\delta\hat{v}] \qquad \text{(B.4)}$$

$$v_3(z_{12}(c)) \quad = \quad (1-\eta)\delta v_3(z_{12}(c)) + \eta\delta\hat{v}. \qquad \text{(B.5)}$$

Continuation values for the other policies in $Z(c)$ are defined analogously. Solving (B.2)-(B.5) and the analogous conditions simultaneously yields the continuation values in Lemma 7. ■

Lemma 8. *(i) No legislator has an incentive to deviate from the basic strategies if and only if*[33]

$$\begin{aligned} c^* \leq \quad & c \quad \leq \tfrac{1}{2}, \ \textit{and} \\ c^o < \quad & c \quad < 1-c^o, \end{aligned}$$

where

$$\begin{aligned} c^* &= \frac{3-2\delta(\gamma-\eta)}{3(2-\delta(\gamma-\eta))} \\ c^o &= \frac{3-\delta(2+\gamma-3\eta)-\frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}, \end{aligned} \qquad \text{(B.6)}$$

and $(\gamma,\eta) \in R(\underline{\theta}) = \{(\gamma,\eta) | 1-\gamma-3\eta(1-\frac{\eta\underline{\theta}}{2}) > 0\}$*; (ii)* $c^* > \frac{1}{3}$.

[33]Lemma 8 indicates that Assumption 3 can be weakened to $\underline{\varepsilon} \leq \min\{c^*, c^o\}$ for $\delta > \delta^o$.

Proof. The proof of part (i) proceeds by checking incentives to deviate from basic strategies given the continuation values in Lemma 7.

Suppose $\mathbf{q^{t-1}} \in \mathbf{Z(c)}$, so $q^{t-1} = z_{ij}(c)$ for some i and j. Consider the incentives of legislators to vote for the equilibrium proposal. With basic strategies the proposal $z_{ij}(c) \in Z$ is the same as the status quo, so legislators vote for the status quo.

Consider i's incentive to propose a deviation. Proposing $z_{ji}(c)$ or $z_{ki}(c)$ with j and k voting for the proposal changes the status quo if approved, and since $c \leq \frac{1}{2}$ and $\eta \leq \gamma$, $z_{ij}(c)$ is preferred by i. Formally, the incentive constraint is

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)[c+\delta v_i(z_{ji}(c))] + \gamma[c - E^t\tilde{\varepsilon}^t + \delta\hat{v}],$$

which is satisfied for $\eta \leq \gamma$.

Proposing $z_{ik}(c)$ results in a change in the status quo if approved, so i prefers to propose $z_{ij}(c)$, since by stochastic dominance

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)[1-c+\delta v_i(z_{ik}(c))] + \gamma[1-c+E^t\tilde{\varepsilon}^t + \delta\hat{v}].$$

Proposing $z_{jk}(c)$ or $z_{kj}(c)$ changes the status quo if approved, and i prefers to propose $z_{ij}(c)$, since

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)\delta v_i(z_{jk}(c)) + \gamma\delta\hat{v}.$$

The best proposal deviation for i outside the set $Z(c)$ gives 1 to i with i and k voting for the proposal. Proposer i prefers not to deviate if and only if

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq 1 - \gamma\tfrac{\underline{\varepsilon}}{4} + \delta\tfrac{1}{3(1-\delta)},$$

where $1-\gamma\frac{\underline{\varepsilon}}{4} = 1-\gamma+\gamma\left[\int_{-\underline{\varepsilon}}^{0}(1+\varepsilon^t)\frac{d\varepsilon^t}{2\underline{\varepsilon}} + \int_0^{\underline{\varepsilon}}\frac{d\varepsilon^t}{2\underline{\varepsilon}}\right]$ is i's expected truncated payoff in period t. Then legislator i does not deviate if

$$c \leq c^u \equiv \tfrac{\underline{\varepsilon}\gamma(1-\delta(1-\eta))}{4} + \tfrac{2\delta(1-\eta)}{3}.$$

The upper bound c^u is at least $\frac{1}{2}$ for

$$\delta \geq \delta^u \equiv \frac{6(1-\frac{1}{2}\underline{\varepsilon}\gamma)}{(1-\eta)(8-3\underline{\varepsilon}\gamma)},$$

and $\delta^u < 1$ if and only if $(\gamma,\eta) \in R^u(\underline{\varepsilon}) \equiv \{(\gamma,\eta)|2-\eta(8-3\varepsilon\gamma) > 0\}$.

Consider j's incentives to propose a deviation. Proposals $z_{ki}(c)$ or $z_{ik}(c)$ give j an allocation of 0, and j has no incentive to propose these over $z_{ij}(c)$, since

$$(1-\eta)[c+\delta v_j(z_{ij}(c))] + \eta[c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)[\delta v_j(z_{ki}(c))] + \gamma[E^t\tilde{\varepsilon}^t + \delta\hat{v}].$$

If j proposes $z_{kj}(c)$, j and i vote against it as above.

If j proposes $z_{ji}(c)$ or $z_{jk}(c)$, j receives $1-c$ in expectation in the current period, and with probability $1-\gamma$ the continuation value is $v_j(z_{ji}(c)) = v_i(z_{ij}(c))$ and with probability γ the continuation value is $\hat{v}$. Legislator j has no incentive to deviate if and only if

$$\begin{aligned} (1-\eta)[c+\delta v_j(z_{ij}(c))] + \eta[c - E^t\tilde{\theta}^t + \delta\hat{v}] &\geq (1-\gamma)[1-c+\delta v_j(z_{jk}(c))] + \gamma[1-c+E^t\tilde{\varepsilon}^t + \delta\hat{v}] \\ \Leftrightarrow c &\geq c^* \equiv \tfrac{3-2\delta(\gamma-\eta)}{3(2-\delta(\gamma-\eta))}, \qquad \text{(B.7)} \end{aligned}$$

and $c^* - \frac{1}{2} = -\frac{\delta(\gamma-\eta)}{6(2-\delta(\gamma-\eta))} \leq 0$. So, $c^* = \frac{1}{2}$ if $\gamma = \eta$, and $c^* < \frac{1}{2}$ if $\gamma > \eta$. Since $c \leq \frac{1}{2}$ by assumption and $c \geq c^*$, $c^* \leq c \leq \frac{1}{2}$ is necessary.

If j proposes $y^t \notin Z(c)$, the best proposal gives 1 to j (j and k vote for it), and the expected truncated payoff is $1-\gamma\frac{\varepsilon}{4}+\delta\frac{1}{3(1-\delta)}$. Legislator j prefers the equilibrium proposal if and only if

$$\begin{aligned} (1-\eta)[c+\delta v_j(z_{ij}(c))] + \eta[c - E^t\tilde{\theta}^t + \delta\hat{v}] &\geq 1-\gamma\tfrac{\varepsilon}{4}+\delta\tfrac{1}{3(1-\delta)} \qquad \text{(B.8)} \\ \Leftrightarrow c &\geq c^\ell \equiv \tfrac{3-2\delta(1-\eta)}{3} - \tfrac{\varepsilon\gamma(1-\delta(1-\eta))}{4}. \qquad \text{(B.9)} \end{aligned}$$

Note that $c^\ell + c^u = 1$, so $c^\ell \leq \frac{1}{2}$ for $\delta \geq \delta^u$ or $(\gamma,\eta) \in R^u(\underline{\varepsilon})$.

Consider k's incentive to propose a deviation. As shown above if $c \in [\max\{c^*, c^\ell\}, c^u]$, i and j prefer $z_{ij}(c)$ to any other allocation. Hence, any proposal by k different from $z_{ij}(c)$ will be rejected. Legislator k's payoff is the same if he proposes $z_{ij}(c)$ and it is accepted, or proposes another allocation that is rejected, hence legislator k has no incentive to deviate from the equilibrium strategies.

Suppose $\mathbf{q^{t-1}} \notin \mathbf{Z(c)}$. Consider j's incentive to vote for the equilibrium proposal $z_{ij}(c)$. The best status quo for j, gives 1 to j. Legislator j along with i vote for $z_{ij}(c)$ rather than the status quo if and only if

$$(1-\gamma)[c+\delta v_j(z_{ij}(c))] + \gamma[c - E^t\tilde{\varepsilon}^t + \delta\hat{v}] > 1-\eta\tfrac{\underline{\theta}}{4}+\delta\tfrac{1}{3(1-\delta)}, \qquad \text{(B.10)}$$

where $1-\eta\frac{\underline{\theta}}{4} = 1-\eta+\eta\left[\int_{-\underline{\theta}}^{0}(1+\theta^t)\frac{d\theta^t}{2\underline{\theta}} + \int_0^{\underline{\theta}}\frac{d\theta^t}{2\underline{\theta}}\right]$ is the expected truncated payoff in period t. Legislator j accepts the proposal if

$$c > c^o = \tfrac{3-\delta(2+\gamma-3\eta)-\frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}.$$

The lower bound $c^o < \frac{1}{2}$ if and only if

$$\delta > \delta^o = \frac{3-\frac{3}{2}\eta\underline{\theta}}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)\eta\underline{\theta}},$$

and $\delta^o < 1$ for $(\gamma,\eta) \in R(\underline{\theta}) \equiv \{(\gamma,\eta) | 1-\gamma-3\eta(1-\frac{\eta\underline{\theta}}{2}) > 0\}$.

Lemma 9. *$(\gamma,\eta) \in R(\underline{\theta}) \Rightarrow c^u \geq \frac{1}{2}$, and hence $c^\ell \leq \frac{1}{2}$.*

Proof. $(\gamma,\eta)\in R(\underline{\theta})$ if and only if

$$\begin{aligned} 0<1-\gamma+3\eta+\tfrac{3}{2}\eta^2\underline{\theta} &< 1-4\eta+\tfrac{3}{2}\eta^2\underline{\theta} \\ &< 1-4\eta+\tfrac{3}{2}\eta\underline{\varepsilon} \\ &< 2\left(1-4\eta+\tfrac{3}{2}\eta\underline{\varepsilon}\right)=2-\eta(8-3\gamma\underline{\varepsilon}). \end{aligned}$$

The last inequality implies $(\gamma,\eta)\in R^u(\underline{\varepsilon})$ and hence $c^u\le\frac{1}{2}$ and $c^\ell\le\frac{1}{2}$. ■

Consider i's incentive to make a proposal other than $z_{ij}(c)$. Using the continuation values in (B.1), $z_{ij}(c)$ gives i the highest payoff among proposals in $Z(c)$, so there is no incentive to make any other proposal in $Z(c)$.

If q^{t-1} gives 1 to i, i strictly prefers $z_{ij}(c)$ to the status quo if and only if

$$\begin{aligned} (1-\gamma)[1-c+\delta v_i(z_{ij}(c))]+\gamma[1-c+E^t\tilde{\varepsilon}^t+\delta\hat{v}] &> 1-\eta\tfrac{\underline{\theta}}{4}+\delta\tfrac{1}{3(1-\delta)} \\ \Leftrightarrow \hat{c}_1\equiv\tfrac{2\delta(1-\gamma)+\frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))} &> c. \end{aligned}$$

Since $\hat{c}_1=1-c^o$, $\hat{c}_1\ge\frac{1}{2}$ if and only if $\delta>\delta^o$ for $(\gamma,\eta)\in R(\underline{\theta})$. The difference in the two upper bounds is

$$c^u-\hat{c}_1=\tfrac{1-\delta(1-\eta)}{12(1-\delta(\gamma-\eta))}\left(3(\underline{\varepsilon}\gamma-\underline{\theta}\eta)+\delta(\gamma-\eta)(8-3\underline{\varepsilon}\gamma)\right),$$

which is nonnegative for all $\delta\in[0,1)$ and for all (γ,η). Consequently, the incentive constraint generating c^u is not binding.

Since $c^u=1-c^\ell\ge\hat{c}_1=1-c^o$, $c^o\ge c^\ell$. The incentive constraint (B.8) thus is not binding.

If i does not receive 1 in q^{t-1}, i prefers a proposal $z_{ij}(c)$ to a proposal that gives 1 to i if and only if

$$\begin{aligned} 1-\gamma\tfrac{\underline{\varepsilon}}{4}+\delta\tfrac{1}{3(1-\delta)} &\le (1-\gamma)[1-c+\delta v_i(z_{ij}(c))]+\gamma[1-c+E^t\tilde{\varepsilon}^t+\delta\hat{v}], \\ \Leftrightarrow c &\le \hat{c}_2\equiv\tfrac{2\delta(1-\gamma)+\frac{3}{4}\gamma\underline{\varepsilon}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}. \end{aligned}$$

Note that $\hat{c}_1\le\hat{c}_2$, since $\eta\underline{\theta}\le\gamma\underline{\varepsilon}$, so $\hat{c}_2$ is not binding.

To prove part (*ii*), note that $c^*-\frac{1}{3}=\frac{1}{3}\left(\frac{1-\delta(\gamma-\eta)}{2-\delta(\gamma-\eta)}\right)>0$, for all $\gamma\in[0,1)$ and $\eta\in[0,1)$. ■

Consequently, if $(1-c,c,0)$ is a coalition equilibrium proposal, $c-\underline{\varepsilon}>0$, and hence $c-\underline{\theta}>0$. Also, $1-c+\underline{\varepsilon}<1$, so $1-c+\underline{\theta}<1$. Then $\underline{\varepsilon}\le\frac{1}{3}$ is sufficient for the coalition allocations c and $1-c$ to be in $[0,1]$.

Corollary 10. *For $\gamma \leq \frac{2}{3}$, $c^+ = c^*$ for $\delta \geq \delta^+$ and $c^+ = c^o$ for $\delta^o < \delta < \delta^+$, where*

$$\delta^+ \equiv \frac{(4(1-\eta) + \frac{3}{4}\eta\underline{\theta}(2+\gamma-3\eta))}{2(\gamma-\eta)(2-\gamma-\eta-\frac{3}{4}\eta\underline{\theta}(1-\eta))} - \frac{\sqrt{(4(1-\eta)+\frac{3}{4}\eta\underline{\theta}(2+\gamma-3\eta))^2 - 4(3-\frac{3}{2}\eta\underline{\theta})(\gamma-\eta)(2-\gamma-\eta-\frac{3}{4}\eta\underline{\theta}(1-\eta))}}{2(\gamma-\eta)(2-\gamma-\eta-\frac{3}{4}\eta\underline{\theta}(1-\eta))}. \tag{B.11}$$

Proof. The difference $c^* - c^o$ is increasing in δ for $\gamma \leq \frac{2}{3}$. To show this, differentiation yields

$$\begin{aligned} \frac{\partial(c^*-c^o)}{\partial\delta} &= -\frac{\gamma-\eta}{3[2-\delta(\gamma-\eta)]^2} + \frac{(2-\frac{3}{4}\eta\underline{\theta})(1-\gamma)}{3[1-\delta(\gamma-\eta)]^2} \\ &> -\frac{\gamma-\eta}{3[2-\delta(\gamma-\eta)]^2} + \frac{(2-\frac{3}{4}\eta\underline{\theta})(1-\gamma)}{3[2-\delta(\gamma-\eta)]^2}. \end{aligned} \tag{B.12}$$

If $\gamma = \eta$, the first line of (B.12) is positive. If $\gamma > \eta$, the second line is positive if $2 - 3\gamma + \eta[1 - \frac{3}{4}\underline{\theta}(1-\gamma)] > 0$, which is the case for $\gamma \leq \frac{2}{3}$. The greater lower bound is then c^* if and only if $\delta \geq \delta^+$, where δ^+ in (B.11) is obtained by equating c^o and c^* in (B.6). ■

Discussion: When the discount factor is high ($\delta \geq \delta^+$), the binding incentive constraint is for the coalition partner to stay on the equilibrium path; i.e., to accept the allocation c and not propose a policy in $Z(c)$ that would yield $1-c$. When $\delta \in (\delta^o, \delta^+)$, the binding incentive constraint is for the potential coalition partner to accept the coalition originator's proposal for any status quo. The binding incentive constraints are associated with the coalition member who receives the lower allocation.

The following lemma characterizes c^+ in Proposition 6 in terms of the probability γ of implementation uncertainty with $c^+ = c^*$ for low γ and $c^+ = c^o$ for higher γ.

Lemma 10. *$c^+ = c^*$ for $\gamma \leq \gamma^e$ and $c^+ = c^o$ for $\gamma > \gamma^e$, where*

$$\gamma^e \equiv 1 + \frac{1}{8\delta}\left[3\eta\underline{\theta}(1-\delta(1-\eta)) - [(8-3\eta\underline{\theta})(1-\delta(1-\eta))(16+(8-3\eta\underline{\theta})[1-\delta(1-\eta)])]^{\frac{1}{2}}\right]. \tag{B.13}$$

Proof. The bound c^* is decreasing in γ, and c^o is increasing in γ, so the difference $c^* - c^o$ is decreasing in γ. The greater lower bound is then c^* if and only if $\gamma \geq \gamma^e$, where γ^e in (B.13) is obtained by equating c^o and c^* in (B.6). ■

Proof of Corollary 9

From the proof of Lemma 8 $\gamma = \eta$ implies $c^* = \frac{1}{2}$. The condition $\eta < \frac{1}{3\underline{\theta}}[4 - 2(4-\frac{3}{2}\underline{\theta})^{\frac{1}{2}}]$ implies $(\eta, \eta) \in R(\underline{\theta})$, so $\delta^o < 1$ and hence $c^o < \frac{1}{2}$. ■

Appendix C - For online publication

C Tolerant coalition equilibrium

Lemma 11. *With implementation uncertainty given in Assumptions 1, 2, and 4, if* (σ, ω) *is a coalition equilibrium for some set* $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$*:*

(i) The continuation value $\overline{v}_i(q^{t-1})$ *for legislator* i *for* $q^{t-1} \in \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ *is*

$$\overline{v}_i(q^{t-1}) = \frac{3(1-\delta)q_i^{t-1}+\eta\delta+3(1-\delta)\eta\delta\nu(\underline{c})}{3(1-\delta)(1-\delta(1-\eta))} \ , \ q_i^{t-1} \in \{1-c, c, 0\}, \tag{C.1}$$

where

$$\nu(c) \equiv \frac{1-2c}{6[2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2c)]}. \tag{C.2}$$

(ii) the continuation value $\overline{v}_i(q^{t-1})$ *for legislator* i *for* $q^{t-1} \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ *is*

$$\overline{v}_i(q^{t-1}) = \hat{v} \equiv \frac{1}{3(1-\delta)},$$

(iii) $\nu(\underline{c}) > 0$ *if* $\underline{c} < \frac{1}{2}$.

Proof. For $c \in \zeta = [\underline{c}, 1-\underline{c}]$ the dynamic payoffs are given by

$$\begin{aligned} \bar{v}_i(z_{ij}(c)) &= (1-\eta)\left[1-c+\delta\bar{v}_i(z_{ij}(c))\right] + \eta[1-c+E^t\tilde{\theta}^t + \delta\bar{v}_i(z_{ij}^{\theta^t})] &\text{(C.3)}\\ \bar{v}_j(z_{ij}(c)) &= (1-\eta)\left[c+\delta\bar{v}_j(z_{ij}(c))\right] + \eta[c-E^t\tilde{\theta}^t + \delta\bar{v}_j(z_{ij}^{\theta^t})] &\text{(C.4)}\\ \bar{v}_k(z_{ij}(c)) &= (1-\eta)\delta\bar{v}_k(z_{ij}(c)) + \eta\delta\bar{v}_k(z_{ij}^{\theta^t}), &\text{(C.5)}\end{aligned}$$

where

$$\begin{aligned} \bar{v}_\ell(z_{ij}^{\theta^t}) &= \int_{-\underline{\theta}}^{c+\underline{c}-1} \bar{v}_\ell(z_{ij}(c-\theta^t))\frac{1}{2\underline{\theta}}d\theta^t + \int_{c+\underline{c}-1}^{c-\underline{c}} \bar{v}_\ell(z_{ij}(c-\theta^t))\frac{1}{2\underline{\theta}}d\theta^t \\ &\quad + \int_{c-\underline{c}}^{\underline{\theta}} \bar{v}_\ell(z_{ij}(c-\theta^t))\frac{1}{2\underline{\theta}}d\theta^t. &\text{(C.6)}\end{aligned}$$

In the first and third integrals in (C.6), $c-\theta^t \notin \zeta$, so $\bar{v}_\ell(z_{ij}(c-\theta^t)) = \bar{v}_\ell(z_{ij}(c'))$ for some $c' \notin \zeta$, where $\bar{v}_\ell(z_{ij}(c'))$ is not a function of c or θ according to the equilibrium strategies.

In the second integral in (C.6) $c-\theta^t \in \zeta$. Conjecture that for $c \in \zeta$, $\bar{v}_i(z_{ij}(c))$ is linear in $1-c$, $\bar{v}_j(z_{ij}(c))$ is linear in c and that these are given by $\bar{v}_i(z_{ij}(c)) = a_i + b_i(1-c)$ and $\bar{v}_j(z_{ij}(c)) = a_j + b_j c$. Then $\bar{v}_i(z_{ij}(c-\theta^t)) = a_i + b_i(1-c+\theta^t)$, and $\bar{v}_j(z_{ij}(c-\theta^t)) = a_j + b_j(c-\theta^t)$. Conjecture that $\bar{v}_k(z_{ij}(c))$ is constant in c. Then for $\ell = i, j$

$$\begin{aligned} \bar{v}_\ell(z_{ij}^{\theta^t}) &= \frac{\bar{v}_\ell(z_{ij}(c'))(2\underline{c}-1+2\underline{\theta})}{2\underline{\theta}} + \frac{(1-2\underline{c})(2a_\ell+b_\ell)}{4\underline{\theta}} &\text{(C.7)}\\ \bar{v}_k(z_{ij}^{\theta^t}) &= \frac{\bar{v}_\ell(z_{ij}(c'))(2\underline{c}-1+2\underline{\theta})}{2\underline{\theta}} + \frac{(1-2\underline{c})a_k}{2\underline{\theta}}. &\text{(C.8)}\end{aligned}$$

Substituting into (C.3)-(C.5) and matching coefficients gives

$$\begin{aligned} a_i = a_j &= \tfrac{\delta\eta(1-2\underline{c})}{2[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]} + a_k \\ a_k &= \tfrac{\delta\eta v_k(z_{ij}(c'))(2\underline{c}-1+2\underline{\theta})}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} \\ b_i = b_j &= \tfrac{1}{1-\delta(1-\eta)}. \end{aligned}$$

Substituting the coefficients and simplifying (C.7)-(C.8) gives

$$\bar{v}_\ell(z_{ij}^{\theta^t}) = \tfrac{(1-2\underline{c})[1-2\bar{v}_\ell(z_{ij}(c'))(1-\delta)]}{2[2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]} + \bar{v}_\ell(z_{ij}(c')) \tag{C.9}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \tfrac{(1-2\underline{c})\bar{v}_k(z_{ij}(c'))(1-\delta)}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_k(z_{ij}(c')). \tag{C.10}$$

Simplifying (C.3)-(C.5) gives

$$\bar{v}_i(z_{ij}(c)) = \tfrac{1-c}{1-\delta(1-\eta)} + \beta_i \tag{C.11}$$

$$\bar{v}_j(z_{ij}(c)) = \tfrac{c}{1-\delta(1-\eta)} + \beta_j \tag{C.12}$$

$$\bar{v}_k(z_{ij}(c)) = \beta_k, \tag{C.13}$$

where for $\ell = i, j$,

$$\begin{aligned} \beta_\ell &= \tfrac{\eta\delta\bar{v}_\ell(z_{ij}(c'))}{1-\delta(1-\eta)} + \tfrac{\eta\delta(1-2\underline{c})[1-2(1-\delta)\bar{v}_\ell(z_{ij}(c'))]}{2[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]} \text{ and} \\ \beta_k &= \tfrac{\eta\delta\bar{v}_k(z_{ij}(c'))}{1-\delta(1-\eta)} + \tfrac{\eta\delta(1-2\underline{c})(1-\delta)\bar{v}_j(z_{ij}(c'))}{[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]}. \end{aligned}$$

For $q^{t-1} \neq z_{ij}(c)$ for all $c \in \zeta$, the continuation value $\bar{v}_\ell(q^{t-1})$ is, using the equilibrium strategies,

$$\begin{aligned} \bar{v}_\ell(q^{t-1}) = {} & (1-\gamma)\left[\tfrac{1}{3}\left[\tfrac{1}{2}(1-c+\delta\bar{v}_\ell(z_{ij}(c))) + \tfrac{1}{2}(1-c+\delta\bar{v}_\ell(z_{ik}(c)))\right]\right. \\ & +\tfrac{1}{3}\left[\tfrac{1}{2}\left(c+\delta\bar{v}_\ell(z_{ji}(c))\right) + \tfrac{1}{2}\delta\bar{v}_\ell(z_{jk}(c))\right] \\ & \left.+\tfrac{1}{3}\left[\tfrac{1}{2}\left(c+\delta\bar{v}_\ell(z_{ki}(c))\right) + \tfrac{1}{2}\delta\bar{v}_\ell(z_{kj}(c))\right]\right] \\ & +\gamma\left[\tfrac{1}{3}\left[\tfrac{1}{2}\left(1-c+E^t\tilde{\varepsilon}^t + \delta\bar{v}_l(z_{ij}^{\varepsilon^t})\right) + \tfrac{1}{2}\left(1-c+E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(c_{ik}^{\varepsilon^t})\right)\right]\right. \\ & +\tfrac{1}{3}\left[\tfrac{1}{2}\left(c-E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(c_{ji}^{\varepsilon^t})\right) + \tfrac{1}{2}\delta\bar{v}_\ell(c_{jk}^{\varepsilon^t})\right] \\ & \left.+\tfrac{1}{3}\left[\tfrac{1}{2}(c-E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(z_{ki}^{\varepsilon^t})) + \tfrac{1}{2}\delta\bar{v}_\ell(c_{kj}^{\varepsilon^t})\right]\right], \end{aligned} \tag{C.14}$$

where $\bar{v}_\ell(z_{ij}(c)), \ell = i, j, k$, are given by (C.11)-(C.13) and

$$\begin{aligned} \bar{v}_\ell(z_{ij}^{\varepsilon^t}) = {} & \int_{-\underline{\varepsilon}}^{c+\underline{c}-1} \bar{v}_\ell(z_{ij}(c-\varepsilon^t))\tfrac{1}{2\underline{\varepsilon}}d\varepsilon^t + \int_{c+\underline{c}-1}^{c-\underline{c}} \bar{v}_\ell(z_{ij}(c-\varepsilon^t))\tfrac{1}{2\underline{\varepsilon}}d\varepsilon^t \\ & + \int_{c-\underline{c}}^{\underline{\varepsilon}} \bar{v}_\ell(z_{ij}(c-\varepsilon^t))\tfrac{1}{2\underline{\varepsilon}}d\varepsilon^t. \end{aligned} \tag{C.15}$$

In the first and third integrals in (C.15), $c-\varepsilon^t \notin \zeta$, so $\bar{v}_\ell(z_{ij}(c-\varepsilon^t)) = \bar{v}_\ell(q^{t-1})$. In the second integral in (C.15) $c-\varepsilon^t \in \zeta$, so $\bar{v}_\ell(z_{ij}(c-\varepsilon^t))$ is given by (C.11)-(C.13). Then substituting from (C.11)-(C.13) and simplifying gives

$$\bar{v}_i(z_{ij}^{\varepsilon^t}) = \left(\frac{1-2\underline{c}}{2\underline{\varepsilon}}\right)\frac{\underline{\theta}[1-2(1-\delta)\bar{v}_i(q^{t-1})]}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_i(q^{t-1}) \tag{C.16}$$

$$\bar{v}_j(z_{ij}^{\varepsilon^t}) = \left(\frac{1-2\underline{c}}{2\underline{\varepsilon}}\right)\frac{\underline{\theta}[1-2(1-\delta)\bar{v}_j(q^{t-1})]}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_j(q^{t-1}) \tag{C.17}$$

$$\bar{v}_k(z_{ij}^{\varepsilon^t}) = -\left(\frac{1-2\underline{c}}{3\underline{\varepsilon}}\right)\frac{\underline{\theta}(1-\delta)\bar{v}_k(q^{t-1})}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_k(q^{t-1}). \tag{C.18}$$

By symmetry $\bar{v}_i(q^{t-1}) = \bar{v}_j(q^{t-1}) = \bar{v}_k(q^{t-1}) = \bar{v}_\ell(q^{t-1})$. Substituting (C.16)-(C.18) into (C.14) and solving gives

$$\bar{v}_\ell(q^{t-1}) = \hat{v} = \frac{1}{3(1-\delta)}, \ell = i, j, k. \tag{C.19}$$

This proves part (ii) of the lemma.

To prove part (i), by part (ii) $\hat{v} = \frac{1}{3(1-\delta)}$ is the continuation payoff for any allocation such that $c \notin \zeta$, hence $\bar{v}_\ell(z_{ij}(c')) = \hat{v} = \frac{1}{3(1-\delta)}$, for $c' \notin \zeta$. Substituting $\bar{v}_\ell(z_{ij}(c')) = \frac{1}{3(1-\delta)}$ into (C.11)-(C.13) yields (C.1).

To prove part (iii), first note that the numerator of (C.2) is nonnegative, since $\underline{c} \leq \frac{1}{2}$. Using $\underline{\theta} \geq \frac{1}{2} - \underline{c}$ from Assumption 4 the denominator of (C.2) yields

$$2\underline{\theta}(1-\delta(1-\eta)) - 2\delta\eta\left(\tfrac{1}{2}-\underline{c}\right) \geq 2\underline{\theta}(1-\delta) > 0,$$

so $\nu(\underline{c}) \geq 0$ ■

C.1 Proof of Proposition 7

The difference between the continuation values in (C.1) and (B.1) for the coalition originator i receiving $1-c$ under the status quo q^{t-1} is

$$\bar{v}_i(q^{t-1}) - v_i(q^{t-1}) = \frac{\delta\eta\nu(\underline{c})}{1-\delta(1-\eta)},$$

which is positive for $\eta > 0$ and $\underline{c} < \frac{1}{2}$. If $\eta = 0$, the continuation values are the same. The same argument establishes the result for the coalition partner receiving c. ■

C.2 Proof of Proposition 8

First note from the proof of Proposition 7 in Section C.1 that when $\eta = 0$ continuation values on the equilibrium path are the same as for the specific policy coalition. That is for $q^{t-1} \in \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ we have $\bar{v}_i(q^{t-1}) = \frac{q_i^{t-1}}{1-\delta}$ for all i, and for $q^{t-1} \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ we have $\bar{v}_i(q^{t-1}) = \hat{v}$ for all i. The following lemma identifies the continuation values off the equilibrium path, when a coalition dissolves. This is required to check incentives to deviate.

Lemma 12. *If $q^{t-1} = z_{ij}(c)$ for $c \in \zeta$, the continuation value $\bar{v}_\ell(z_{ij}^{\varepsilon^t})$ for $c - \underline{\varepsilon} < \underline{c}$ when $y^t \neq q^{t-1}$ is* *proposed is given by*

$$\bar{v}_i(z_{ij}^{\varepsilon^t}) = \bar{v}_j(z_{ij}^{\varepsilon^t}) = \tfrac{1}{3(1-\delta)} + \tfrac{\theta}{\underline{\varepsilon}}\nu(\underline{c}) \tag{C.20}$$

$$\bar{v}_k(z_{ij}^{\varepsilon^t}) = \tfrac{1}{3(1-\delta)} - 2\tfrac{\theta}{\underline{\varepsilon}}\nu(\underline{c}). \tag{C.21}$$

If $q^{t-1} = z_{ij}(c)$ for $c \in \zeta$, the continuation value $\bar{v}_\ell(z_{ij}^{\theta^t})$ for $c - \underline{\theta} < \underline{c}$ when $y^t = z_{ij}(c)$ is proposed is given by

$$\bar{v}_i(z_{ij}^{\theta^t}) = \bar{v}_j(z_{ij}^{\theta^t}) = \tfrac{1}{3(1-\delta)} + \nu(\underline{c}) \tag{C.22}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \tfrac{1}{3(1-\delta)} - 2\nu(\underline{c}). \tag{C.23}$$

Proof. The first part follows from substituting $\bar{v}_\ell(q^{t-1}) = \frac{1}{3(1-\delta)}$ into (C.16)–(C.18). The second part follows from substituting $\bar{v}_\ell(z_{ij}(c')) = \frac{1}{3(1-\delta)}$ into (C.9) and (C.10). ∎

We show in the next lemma that the optimal proposal for the originator of a coalition gives the proposer $1 - \underline{c}$.

Lemma 13. *For $y^t \neq q^{t-1} \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, the optimal proposal by the originator i of a tolerant coalition is $y^t = z_{i\ell}(\underline{c}), \ell = j, k$.*

Proof. Legislator i proposes $z_{ij}(c) \in \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, which yields an expected dynamic payoff $EU_i(c)$ given by

$$EU_i(c) = (1-\gamma)[1 - c + \delta\bar{v}_i(z_{ij}(c))] + \gamma[1 - c + E^t\tilde{\varepsilon}^t + \delta\bar{v}_i(z_{ij}^{\varepsilon^t})], \tag{C.24}$$

where $\bar{v}_i(z_{ij}(c))$ is given in (C.1) and $\bar{v}_i(z_{ij}^{\varepsilon^t})$ is given in (C.20). From Lemma 12 $\bar{v}_i(z_{ij}^{\varepsilon^t})$ does not depend on c, so differentiating (C.24) yields

$$\tfrac{dEU_i(c)}{dc} = -1 - \tfrac{\delta(1-\gamma)}{1-\delta} < 0.$$

Consequently, i prefers the lowest $c \in \zeta$, so $c = \underline{c}$ is optimal. ∎

The following lemma identifies policies that can be supported by tolerant dynamic coalitions. When $\eta = 0$, the analogues c^{**} of c^* and c^{oo} of c^o in (B.6) are

$$c^{**} = c^* + \frac{\delta(1-\delta)\gamma\frac{\theta}{\underline{\varepsilon}}\nu(c^{**})}{2-\delta\gamma} = \frac{3-\delta\gamma(2-\frac{1}{4\underline{\varepsilon}})}{3(2-\delta\gamma(1-\frac{1}{6\underline{\varepsilon}}))} \tag{C.25}$$

$$c^{oo} = c^o - \frac{\delta(1-\delta)\gamma\frac{\theta}{\underline{\varepsilon}}\nu(c^{oo})}{1-\delta\gamma} = \frac{3-2\delta-\delta\gamma(1+\frac{1}{4\underline{\varepsilon}})}{3(1-\delta\gamma(1+\frac{1}{6\underline{\varepsilon}}))}. \tag{C.26}$$

Lemma 14. *With implementation uncertainty given in Assumptions 1, 2, and 4 basic strategies are a coalition equilibrium supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for all $\underline{c} \in [\underline{c}^+, \frac{1}{2}]$, if $\delta > \max\{\hat{\hat{\delta}} \equiv \frac{3\underline{\varepsilon}[8+\gamma(4-\gamma)]}{2\gamma(8-3\gamma\underline{\varepsilon})}, \delta^o = \frac{3}{4-\gamma}\}$, and*

$$\underline{c}^+ \equiv \max\{c^{**}, c^{oo}\}. \tag{C.27}$$

Proof. **Suppose $\mathbf{q^{t-1}} \in \bigcup_{\mathbf{c}=\underline{\mathbf{c}}}^{\frac{1}{2}} \mathbf{Z(c)}$.** Then $q^{t-1} = z_{ij}(c)$ for some i and j and some $c \in \zeta$. Consider the incentives of legislators to accept the equilibrium proposal. With basic strategies the proposal is the same as the status quo, so legislators vote for the status quo.

Consider i's and j's incentives to propose a deviation. The lowest allocation for i or j when the status quo is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ is $\underline{c}$, so consider $q^{t-1} = z_{ij}(\underline{c})$ and j's incentives, since j receives the allocation c. Since i's payoff is higher, i has no incentive to deviate if j does not have an incentive to deviate.

From Lemma 13 the best deviation proposal for j in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ gives j the allocation $1-\underline{c}$. Legislator j will not propose this if

$$\begin{aligned}\underline{c} + \delta\bar{v}_j(z_{ij}(\underline{c})) = \frac{\underline{c}}{1-\delta} &\geq (1-\gamma)[1-\underline{c}+\delta\bar{v}_j(z_{ji}(\underline{c}))] + \gamma[1-\underline{c}+E^t\tilde{\varepsilon}^t + \delta\bar{v}_j(z_{ji}^{\varepsilon^t})] \\ \Leftrightarrow \underline{c} &\geq c^{**}. \end{aligned} \tag{C.28}$$

The bound c^{**} is strictly less than $\frac{1}{2}$ for all $\delta \in [0,1)$.

If j proposes $y^t \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, the best proposal such that the realized policy is not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ gives 1 to j. Legislator j has no incentive to deviate if

$$\begin{aligned}\underline{c} + \delta\bar{v}_j(z_{ij}(\underline{c})) &\geq (1-\gamma)(1+\delta\hat{v}) + \gamma E^t(1+\tilde{\varepsilon}^t + \delta\bar{v}_j(y^{\varepsilon^t})) \\ &\geq (1-\gamma)(1+\delta\hat{v}) + \gamma\left[\int_0^{\bar{\varepsilon}}(1+\delta\hat{v})\frac{d\varepsilon^t}{2\underline{\varepsilon}} + \int_{-\underline{\varepsilon}}^0 (1+\varepsilon^t+\delta\hat{v})\frac{d\varepsilon^t}{2\underline{\varepsilon}}\right] \\ &= 1 - \gamma\tfrac{\underline{\varepsilon}}{4} + \delta\tfrac{1}{3(1-\delta)} \qquad \text{(C.29)} \\ \Leftrightarrow \underline{c} &\geq c^{\ell}. \end{aligned}$$

The lower bound $c^{\ell} \leq \frac{1}{2}$ for $(\gamma, 0) \in R(\underline{\theta})$, which is the case for all $\lambda < 1$.

Legislator j may propose $y^t \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ such that the realized policy has some probability of being in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. Consider a policy that awards $1-a^1$ to legislator j, where $0 \leq a^1 < \underline{c}$. Legislator j has no incentive to deviate if

$$\underline{c} + \delta\bar{v}_j(z_{ij}(\underline{c})) \geq (1-\gamma)[1-a^1+\delta\hat{v}] + \gamma E^t[1-a^1+\tilde{\varepsilon}^t + \delta\bar{v}_j(y^{\varepsilon^t})], \tag{C.30}$$

where $\bar{v}_j(y^{\varepsilon^t})$ is the continuation payoff from the realized policy $y^{\varepsilon^t} = 1-a^1+\varepsilon^t$. This realized policy may be in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ or not. If the realized allocation can be less than $\underline{c}$, then by Assumption 1

$$\begin{aligned} & (1-\gamma)[1-a^1+\delta\hat{v}] + \gamma E^t[1-a^1+\tilde{\varepsilon}^t+\delta\bar{v}_j(y^{\varepsilon^t})] \\ = \; & (1-\gamma)(1-a^1) + \gamma E^t(1-a^1+\varepsilon^t) \\ & +\delta\left[(1-\gamma)\hat{v} + \gamma\int_{-\underline{\varepsilon}}^{a^1+\underline{c}-1} \hat{v}\tfrac{d\varepsilon^t}{2\underline{\varepsilon}} + \gamma\int_{a^1+\underline{c}-1}^{a^1-\underline{c}} \bar{v}_j(z_{ji}(a^1-\varepsilon^t))\tfrac{d\varepsilon^t}{2\underline{\varepsilon}^t} + \gamma\int_{a^1-\underline{c}}^{\underline{\varepsilon}} \hat{v}\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}\right] \\ = \; & 1-a^1 - \gamma\tfrac{(\underline{\varepsilon}-a^1)^2}{4\underline{\varepsilon}} + \delta\left[\tfrac{\hat{v}(2\underline{\varepsilon}+\gamma(2\underline{c}-1))}{2\underline{\varepsilon}} + \tfrac{\gamma(1-2\underline{c})}{4\underline{\varepsilon}(1-\delta)}\right]. \end{aligned} \tag{C.31}$$

The most attractive deviation maximizes this expected payoff with respect to a^1. This expected dynamic payoff is decreasing in a^1, so the maximum $a^1 = 0$. The right side of (C.30) then is the same as the right side of (C.29), so (C.30) is satisfied for all $\underline{c} \geq c^\ell$.

If the realized allocation cannot be less than $\underline{c}$, i.e.,$1-\underline{c}-\underline{\varepsilon} > a^1$, then

$$\begin{aligned} & E^t[1-a^1+\tilde{\varepsilon}^t+\delta\bar{v}_j(y^{\varepsilon^t})] \\ = \;& 1-a^1+\int_{-\underline{\varepsilon}}^{a^1-\underline{c}}\left(\varepsilon^t+\delta\bar{v}_j(z_{ji}(a^1-\varepsilon^t))\right)\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}+\int_{a^1-\underline{c}}^{a^1}\left(\varepsilon^t+\delta\hat{v}\right)\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}+\int_{a^1}^{\underline{\varepsilon}}\delta\hat{v}\tfrac{d\varepsilon^t}{2\underline{\varepsilon}} \end{aligned} \tag{C.32}$$

Using (C.32), the right side of (C.30) is quadratic and strictly concave in a^1 with a maximum at $\hat{a}^1$ given by

$$\hat{a}^1 = \tfrac{6\underline{\varepsilon}(1-\delta)-\delta\gamma(2-3\underline{\varepsilon})}{3\gamma(1-2\delta)}, \tag{C.33}$$

for $\delta > \frac{1}{2}$ and which must satisfy the constraint $1-\underline{c}-\underline{\varepsilon} > \hat{a}^1$.[34] This constraint implies $\underline{c} < 1-\hat{a}^1-\underline{\varepsilon} \equiv \hat{c}^\ell$. If $\delta > \hat{\hat{\delta}} \equiv \frac{3\underline{\varepsilon}[8+\gamma(4-\gamma)]}{2\gamma(8-3\gamma\underline{\varepsilon})}$, then $\hat{c}^\ell < c^\ell$. Then, (C.30) is

$$\underline{c} \geq c^\ell + (1-\delta)\left[-\hat{a}^1+\frac{\gamma(\hat{a}^1)^2}{4\underline{\varepsilon}}+\frac{\delta\gamma}{1-\delta}\int_{-\underline{\varepsilon}}^{a^1-\underline{c}}\left(\frac{2}{3}-\hat{a}^1+\varepsilon^t\right)\frac{d\varepsilon^t}{2\underline{\varepsilon}}\right]. \tag{C.34}$$

The right side of (C.34) is at least c^ℓ and is strictly decreasing in $\underline{c}$, so (C.30) is satisfied for all $\underline{c} \geq \hat{c}^\ell$.

Consider k's incentive to propose a deviation. If $\underline{c} \in [\max\{c^{**}, \hat{c}^\ell\}, \frac{1}{2}]$, i and j prefer $z_{ij}(c)$ for all $c \in [\underline{c}, 1-\underline{c}]$ to any new allocation. Hence, any proposal by k other than the status quo will be defeated. Hence legislator k has no incentive to deviate from the equilibrium strategies.

Suppose $\mathbf{q^{t-1}} \notin \bigcup_{\mathbf{c}=\underline{\mathbf{c}}}^{\frac{1}{2}} \mathbf{Z(c)}$. Consider j's incentive to vote for the equilibrium proposal $z_{ij}(\underline{c})$. The best status quo for j such that the realized policy from the status quo is not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ gives 1 to j. Legislator j votes for $z_{ij}(\underline{c})$ rather than the status quo, and i also votes for $z_{ij}(c)$ if and only if

$$\begin{aligned} (1-\gamma)[\underline{c}+\delta\bar{v}_j(z_{ij}(\underline{c}))]+\gamma[\underline{c}-E^t\tilde{\varepsilon}^t+\delta\bar{v}_j(z_{ij}^{\varepsilon^t})] \quad &> \quad 1+\delta\tfrac{1}{3(1-\delta)} \\ \Leftrightarrow \underline{c} \quad &> \quad c^{oo}. \end{aligned}$$

Compare this constraint to (C.29). The left side of (C.29) is larger than the left side of the above because of uncertainty. Furthermore the right side of (C.29) is smaller than the right side above. Thus the constraint giving the bound c^{oo} is tighter than for $\hat{c}^\ell$, so $\hat{c}^\ell$ is not binding. The bound $c^{oo} < \frac{1}{2}$ for $\delta > \delta^o$.

Consider i's incentive to propose $y^t \neq z_{ij}(\underline{c})$. By Lemma 11, $z_{ij}(\underline{c})$ gives i the highest dynamic payoff among proposals in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, so there is no incentive to make any other proposal in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. Since legislator j has no incentive to deviate when receiving 1 then neither does legislator i, thus legislator i has no incentive to deviate to a policy not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ when $\underline{c} > c^{oo}$.

[34] Also, $\hat{a}^1$ must be non-negative, which is the case for $\delta \geq \frac{6\underline{\varepsilon}}{\gamma(2-3\underline{\varepsilon})+6\underline{\varepsilon}}$.

Consider the case in which i does not receive 1 in q^{t-1}. Legislator i prefers a proposal $z_{ij}(\underline{c})$ to a proposal that gives 1 to i if and only if

$$(1-\gamma)[1-\underline{c}+\delta\bar{v}_i(z_{ij}(\underline{c}))]+\gamma[1-\underline{c}+E^t\tilde{\varepsilon}^t+\delta\bar{v}_i(z_{ij}^{\varepsilon^t})] \quad \geq \quad 1-\gamma\tfrac{\underline{\varepsilon}}{4}+\delta\tfrac{1}{3(1-\delta)}.$$

Note that the right side of the inequality is less than $1+\delta\frac{1}{3(1-\delta)}$, so $\underline{c} > c^{oo}$ is sufficient for this to be satisfied.

Legislator i may also propose $y^t \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ such that the realized proposal is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ with positive probability. From the previous analysis, if $\delta > \hat{\hat{\delta}}$ the optimal proposal gives 1 to the proposer, and thus this constraint is satisfied if $\underline{c} > c^{oo}$ and $\delta > \hat{\hat{\delta}}$. ■

The next lemma shows for δ large enough $c^{**} \geq c^{oo}$.

Lemma 15. *For $\eta = 0$ there exists a δ^{**} such that for all $\delta \geq \delta^{**}$ we have $c^{**} \geq c^{oo}$.*

Proof. The difference between c^{**} and c^{oo} is

$$c^{**}-c^{oo} = \frac{3-2\delta\gamma+\frac{\delta\gamma}{4\underline{\varepsilon}}}{3(2-\delta\gamma(1-\frac{1}{6\underline{\varepsilon}}))} - \frac{3-\delta(2+\gamma)-\frac{\delta\gamma}{4\underline{\varepsilon}}}{3(1-\delta\gamma(1+\frac{\delta\gamma}{6\underline{\varepsilon}}))}. \tag{C.35}$$

Evaluating (C.35) at $\delta = 0$ yields $(c^{**}-c^{oo})_{|\delta=0} = -\frac{1}{2}$. Taking the limit as $\delta \to 1$ yields

$$\sup_{\delta\to 1}(c^{**}-c^{oo}) = \frac{(1-\gamma)^2+\frac{\gamma}{12\underline{\varepsilon}}}{9(2-\gamma(1-\frac{1}{6\underline{\varepsilon}}))(1-\gamma(1+\frac{1}{6\underline{\varepsilon}}))} > 0.$$

By the mean value theorem there exists one or more solutions to $c^{**}-c^{oo} = 0$ in $(0,1)$. Let the largest of these be denoted by δ^{**}.■

By Lemma 15 for $\delta > \delta^{**}$, c^{**} is the greatest lower bound. Thus, by Lemma 14 with implementation uncertainty given in Assumptions 1, 2, and 4 basic strategies are a coalition equilibrium supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for all $\underline{c} \in [c^{**}, \frac{1}{2}]$, if $\delta > \delta^{\zeta} \equiv \max\{\hat{\hat{\delta}}, \delta^{**}, \delta^{o}\}$. This completes the proof of Proposition 8.

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