Language Games*

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Abstract

A language game is a communication game in which players are uncertain about each other's ability to produce and interpret messages. We consider two types of language games, public-announcement games and pre-play communication games. A public-announcement game is a finite static complete-information game, the base game, preceded by the observation of a public signal, interpreted as an announcement. In a pre-play communication game the base game is preceded by one round of simultaneous public pre-play communication. We show that for any base game all rational correlated equilibrium (CE) distributions of that game can be realized as Bayesian Nash equilibrium distributions of a language game with that base game. If we define a language equilibrium (LE) of a game as a Bayesian Nash equilibrium of a language game with that base game, our result can be succinctly stated as "The sets of rational CE and LE distributions coincide." Thus, uncertainty about the meaning of messages can fully substitute for correlation.

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...I can direct a man who has learned only German, only by using the German language.

Describe the aroma of coffee. — Why can't it be done? Do we lack the words?

Ludwig Wittgenstein, *Philosophical Investigations*

1 Introduction

A key puzzle in a game-theoretic approach to meaning is that in equilibria of communication games meaning arises endogenously while "real world" communication evidently would be impossible without access to a shared language. On one hand we have "strategic meaning" of messages and on the other "semantic meaning." By themselves neither of these accounts of meaning is satisfactory. Semantic meaning alone is strategically naive and strategic meaning alone fails to differentiate between the details of a given strategic encounter and the linguistic context of that encounter.

The literature has explored different avenues through which an exogenously given language can shape equilibrium meaning. Farrell [17] uses a pre-existing language to endow out-of-equilibrium messages with meaning. Crawford [13] considers players who are drawn from a population that includes behavioral types with various degrees of naiveté in their use of a pre-existing language, including truthful senders and gullible receivers. Kartik [27] and Kartik, Ottaviani and Squintani [28] let senders incur a cost that is increasing in the magnitude of the departure from truth. Franke [20] uses semantic meaning to pin down the behavior of level-0 players in a model of iterative reasoning that he applies to pragmatic inference. Mialon and Mialon [32] induce message meaning both by limiting message availability according to the sender's payoff type and by constraining message interpretations of naive receivers.[1] Halpern and Kets [25] endow different agents

[1]There is a long tradition in the disclosure literature beginning with Grossman [24]

with different semantics (mappings that assign truth values to pairs of states and propositions). Blume and Board [10] are also interested in understanding the interaction of agents with different semantics, but shift the balance from semantic to strategic meaning. In their setup players face language constraints that limit their ability to send and interpret messages, without any prior assignment of meaning to messages.[2] Blume and Board rely on negative rather than positive semantic restrictions. Instead of endowing messages with meaning or nudging meaning toward a given semantics the constraints restrict and individualize the possible equilibrium meanings of messages. Different semantics can arise in equilibrium as a consequence of uncertainty about the language constraints faced by others; as a result agents may disagree about the *equilibrium* meaning of messages.

Here we adopt the apparatus of Blume and Board to investigate the impact of disagreement about message meaning on communication prior to the play of finite static complete-information games. We depart from Blume and Board by dropping the common-interest assumption with its attendant focus on efficiency losses from language constraints. Instead we investigate the possibility of expanding the set of equilibrium outcomes by leveraging the disagreement about meaning that results from uncertainty about language constraints.

We investigate uncertainty about language in a simple class of *communication games* in which players receive or exchange public messages prior to taking part in a static complete-information (base) game. We consider two types of such games, public-announcement games and pre-play communica-

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and Milgrom [33] to fix message meaning via truth-telling constraints. In the mechanism design literature Green and Laffont [23] have explored the consequences of payoff type contingent message spaces.

[2]The literature has considered various versions of constraints on players' language, including finite message spaces, Crémer, Garicano, and Prat [14] and Jäger, Metzger, and Riedel [26]; symmetry constraints on strategies, Crawford and Haller [12] and Blume [8]; limited sets of relations on partially nameless objects, Rubinstein [36]; constraints on individual decision making, Rubinstein [37]; and, clarification and comprehension costs, Dewatripont and Tirole [16]. None of these address the consequences of uncertainty about language constraints for pre-play communication.

tion games. In a public-announcement game players observe a public signal, interpreted as an announcement, prior to the base game. In a pre-play communication game the base game is preceded by one round of simultaneous public pre-play communication.

As is well known, embedding a base game into a communication game permits players to correlate their actions in the base game. In the Battle of the Sexes game (BSG), for example, the public message could be the outcome of a coin flip and players could play one of the pure Nash equilibria after "Heads" and the other after "Tails." With asymmetric three-sided "coins" all convex combinations of Nash equilibrium payoffs of the BSG can be realized in this manner. As first noted by Aumann, Maschler and Stearns [3] the same can effect be achieved by a simultaneous exchange of messages before the base game: To implement a distribution that puts equal weight on the two pure-strategy Nash equilibria of the BSG, one can have both players randomize uniformly over two messages, say "H" and "T" and prescribe that they play one equilibrium if the messages differ and the other if they are the same; importantly, in this "jointly controlled lottery" neither player can affect the probability of which BSG equilibrium will be played by a unilateral deviation at the communication stage.

Adding public announcements to a game makes it possible to induce all distributions over outcomes that are in the convex hull of the set of Nash equilibrium distributions of the base game as Nash equilibrium distributions of the extended gams. Sometimes one can go further, and support distributions outside of the convex hull of Nash equilibrium distributions, by letting players receive *private* messages from an impartial mediator. The set of Bayesian Nash equilibria of a base game that has been augmented by adding a device that sends private messages to players is the set of *correlated equilibria* of that base game. Correlated equilibria were introduced by Aumann [1] [2] and characterize what can be achieved with (possibly mediated) communication prior to a static complete information game; for incomplete information games, Myerson [34] and Forges [18] show that the corresponding characterization is in terms of the set of communication equilibria.

A simultaneous exchange of messages prior to the base game is an instance of a cheap-talk extensions of that base game. In a cheap-talk extension players communicate directly, either privately or publicly according to some protocol, without reliance on an impartial mediator. Evidently, every Bayesian Nash equilibrium (BNE) distribution of a cheap-talk extension of a base game is a correlated equilibrium distribution of the base game. A natural question is whether there are conditions under which the converse holds as well. When is it the case that for every correlated equilibrium distribution of the base game there is an equivalent BNE distribution of some cheap-talk extension of the base game? With only two players, regardless of the communication protocol, once players choose their actions in the base game they have the same information and hence only distributions in the convex hull of the set of BNE distributions of the base game can be supported. In contrast, with four or more players, Bárány [4] shows that for finite static complete information games every correlated equilibrium distribution is a BNE distribution of an appropriate cheap-talk extension. Analogously, for incomplete information games Forges [19] shows that with four or more players every communication equilibrium distribution is a BNE distribution of an appropriate cheap-talk extension. Bárány's [4] construction requires that players can sometimes check past messages and is not sequentially rational. Ben-Porath [5] obtains a similar result as Bárány, with message checking, for the case of three players while maintaining sequential rationality. Ben-Porath [6] shows that for incomplete information games with three or more players communication equilibrium distributions with payoffs for every type above the payoffs from a BNE of the base games can be attained via sequential equilibria of a cheap-talk extension of the base game. Gerardi [21] shows that with five or more players every rational correlated equilibrium distribution can be achieved as a sequential equilibrium distribution for some cheap-talk extension of the base, without verification of messages. Lehrer and Sorin [31], building on Lehrer [29] and [30], demonstrate that regardless of the number of players the entire set of rational correlated equilibrium distributions can be achieved with a mediator that receives one round of private messages from the players and sends public signals as a deterministic function of the players' messages.

In the present paper we will show that uncertainty about language in simple communication games, with either a single public pre-play announcement or one round of simultaneous message exchange, can replace mediation and elaborate cheap-talk extensions. For this purpose, we introduce the notion of a *language game.* A language game is a communication game in which players are language constrained and uncertain about the language constraints faced by others. Specifically, following Blume and Board [10], we assume that players are constrained in their ability to distinguish different messages and uncertain about which messages can be distinguished by other players.

In a language game players will have private information about their ability to distinguish messages, their *language types.* Private information might permit players to correlate their actions in the base game, even without communication. We close down this avenue for correlation by considering, in the spirit of Crawford and Haller [12], games in which players can make no *a priori* distinctions among messages (they face an *a priori* lack of a common language for messages) and by only admitting language types that are identical up to permutations of messages. As a result, the only distinctions players can make at the interim stage are the ones dictated by their language types and since language types are the same up to permutations of indistinguishable messages, by themselves they are useless for correlating behavior in the base game. Thus our language games are designed so that language is irrelevant without communication.

We show that for any finite static complete-information base game (including games with only two players) all rational correlated equilibrium (CE) distributions of that game can be realized as Bayesian Nash equilibrium distributions of a language game with that base game.[3] If we define a language

[3]de Jaegher [15] is similarly motivated as we are in connecting correlated equilibria

equilibrium (LE) of a game as a Bayesian Nash equilibrium of a language game with that base game, our result can be succinctly stated as "The sets of rational CE and LE distributions coincide." Thus, uncertainty and disagreement about meaning can fully substitute for correlation.[4]

We informally introduce our setup and motivate our results in Section 2. Our model and results are in Section 3. We conclude with an example showing that there are instances in which one can use language games to enlarge the set of equilibrium distributions beyond the convex hull of the set of BNE distributions even when language type distributions are constrained to be independent.

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with uncertainty about meaning, but has no equivalent to language games or interlocutors being privately informed about their language types. He gives two examples in which he relates correlated equilibrium to vagueness in language. In the first players differ in how they partition degrees of darkness into "night" and "day," in the second players arrive at different partitions of the set of states as a result of communication through a noisy channel.

The latter example is similar in spirit to Myerson's [35] messenger pigeon example, which illustrates the benefits of transmitting private information through noisy channels. General results for the role of noisy channels and mediated communication in variants of Crawford-Sobel [11] games have been obtained by Blume, Board and Kawamura [9] and Goltsman, Hörner, Pavlov and Squintani [22]. In the present paper we focus exclusively on the role of private information about language types in the context of pre-play communication prior to static games of complete information.

[4] In the case of public announcements there is a simple way to see how language constraints, even without private information about those constraints, can enable the implementation of the entire set of correlated equilibria. Let each player in the game understand a natural language that no other player understands, so that for example one is the sole Hungarian speaker, one the sole Quechua speaker and so on. A mediator who speaks all of those languages can then make public announcements that address each player in the language only that player understands. Since any correlated equilibrium can be achieved as a Nash equilibrium of a game in which an impartial mediator sends private messages, and since in the environment just described equivalents of private messages can be conveyed through a public channel, it is clear that all correlated equilibrium distributions can be achieved with public announcements. While useful intuition can be gleaned from this setup, we will pursue a different direction, which does not impose onerous requirements on the public announcer's language ability, is privacy preserving in the sense that the announcer cannot predict players' actions as a function of his announcements and, most importantly, generalizes to the case of pre-play communication among the players.

2 Laboratory Thought Experiments

In this section we set up a series of imaginary laboratory experiments. The purpose is threefold. First, and foremost, the experimental allegory permits us to highlight details of the strategic encounter that are ordinarily treated as incidental and not included in the formal description of the game. Second, by varying the experimental conditions we can introduce scenarios that closely match our key concepts *absence of a common language* and *uncertainty about language* and scenarios which naturally give rise to considerations of correlated equilibria. Third, we give a self-contained preview of our results in the context of a simple example.

Imagine having individuals in an experimental laboratory play the game of Chicken in Figure 1. Everyone is anonymous and seated in a separate cubicle. Half of the participants are designated to be row players and the others column players; each row player is matched with one column player; the designations and matches are random and anonymous. There is a single round of play in which each participant privately makes one choice.

	L	R
U	4,4	1,5
D	5,1	0,0

Figure 1: Chicken

The Nash equilibria of the game played by any pair of subjects are the two pure-strategy equilibria (D, L) and (U, R), and the mixed equilibrium $\left(U \text{ with probability } \frac{1}{2},\ \ L \text{ with probability } \frac{1}{2}\right)$.

Given the symmetry of the situation, it is doubtful that the asymmetric pure strategy equilibria have much predictive power. Formally, we can reinforce this point by labeling players' actions in the experiment identically (e.g. by replacing L by U and R by D, respectively) and describing the game to each participant in the experiment from the perspective of a row player. In the terminology of Crawford and Haller [1990] we have created a situation where players are not distinguished by a *common language*. Crawford and Haller require that players' strategies treat objects identically for which they lack a common language. In the present case this means that since players themselves are not distinguished by a common language, they use identical strategies. This rules out the two asymmetric pure equilibria in favor of the symmetric mixed equilibrium.

In the experiment just described the set of objects for which participants lack a common language is the player set. In general there are other sets of objects for which players might lack a common language, including the set of players' strategies or some set of elements that enter the description of players' strategies. In our experiment strategies are payoff distinguished and therefore players do have a common language to describe them. But suppose players could observe a public signal from the set $\{*, \#\}$ before making their action choices. It seems fair to expect that the observation of a generic signal $*$ or $\#$ prior to play would have no effect on play in our imagined laboratory.[5]

The irrelevance of observations of generic signals before play is independent of whether or not we think of the chicken game itself being subject to symmetry constraints arising from lack a common language for the player set. In the case where players lack a common language for both the signal set $\{*, \#\}$ and for the player set, the unique Nash equilibrium that respects the lack of a common language requires players to ignore the public signal and to play the mixed strategy Nash equilibrium of the chicken game at the action stage. In the case where players are distinguished, lack of a common language for the signal set $\{*, \#\}$ does not expand the set of Nash equilibria of the chicken game, (D, L), (U, R), and the mixed equilibrium

[5]In introducing this example, we have relied on the intuition that players cannot make useful distinctions between the symbols $*$ and $\#$. This is only for the sake of readability. We could, instead, enforce lack of a common language by using the translation procedure of Blume, DeJong, Kim and Sprinkle [7], where messages are passed through a translator that individualizes them. The same is true for all other examples in this section.

$\left(U \text{ with probability } \frac{1}{2},\ \ L \text{ with probability } \frac{1}{2}\right)$.

Essentially the same observation we made about public signals applies to players exchanging messages before the chicken game. Modify our imaginary experiment so that one participant in each pairing is given the message set $\{*, \#\}$ and the other the message set $\{\bowtie, \circledcirc\}$. Have participants simultaneously send messages to their partners before taking actions in the chicken game. We expect an exchange of generic messages like these prior to play of the chicken game in our imagined laboratory not to have any impact on subsequent behavior. Formally, if players lack a common language for their message spaces the analyst must view them as treating all their messages identically, i.e. to randomize uniformly over messages at the message stage and to ignore messages at the action stage.

Contrast the austere scenario above, where players can observe a public signal prior to play of the Chicken game but signals are designed to be useless generic symbols like $*$ and $\#$, with one where players observe with equal probability one of the two public signals "The suggested pair of actions is (D, L)" or "The suggested pair of actions is (U, R)." Here the participants of our imaginary experiment arguably do have a common language for the signal space, and indeed it is not implausible for them to follow the suggested actions. If they do, they are using the public signal as a correlation device and achieve a correlated equilibrium payoff pair $(3, 3)$ of the Chicken game. By varying the probability distribution of the public signal it is possible to achieve all payoff profiles in the convex hull of the set of Nash equilibrium payoffs. The convex hull of Nash equilibrium payoffs is a subset of the set of correlated equilibrium payoffs. The latter we can informally think of as the set of all equilibrium payoffs of games that have been extended by some communication device or protocol, such as public signals, private signals, face-to-face communication etc.

Again, the observation about public signals has a parallel in a simultaneous exchange of messages. Whereas an exchange of messages that are generic meaningless symbols has no consequences for subsequent play, with the proper context a simultaneous exchange of messages can expand the equilibrium payoff set. Imagine for example a version of our laboratory thought experiment in which each participant has the message set $\{1, 2\}$ and the common instructions include the statements "When the sum is odd, we suggest (D, L)" and "When the sum is even, we suggest (U,R)." It is an equilibrium for players to randomize uniformly over their messages at the message stage and to behave in accordance with the suggestions at the action stage. Note that here the uniform randomization at the communication stage is not necessitated by players lacking a common language to describe messages; they do have a common language as a result of the statements included in the instructions. Instead, here it is incentive compatible to randomize, since given the uniform randomization of the partner, neither player's choice of message has an impact on the probability that the sum of messages is odd. The simultaneous message exchange gives rise to a *jointly controlled lottery*, in which the distribution over the relevant outcomes, odd or even, is independent of the individual choices.

Even with a common language, as is well known, public correlation devices and simultaneous pre-play communication have a limited impact on the equilibrium payoff set. Neither takes us outside the convex hull of the set of Nash equilibrium payoffs. Consider then a variant of our imaginary experiment in which prior to playing the chicken game, each participant belonging to a pair draws a message from an urn that initially contains three slips of paper. One reads "It is your turn to get '5'"; the other two read "It is not your turn to get '5'." Draws are private and without replacement; i.e. after the first participant has drawn, there are two slips left to draw from for the second participant. In this scenario, it is optimal for each participant to take the action that makes it possible for her to get a payoff of 5 if her draw is "It is your turn to get '5'" and to take the other action otherwise. Importantly, the expected payoff pair $\left(\frac{10}{3}, \frac{10}{3}\right)$ is outside the convex hull of the set of Nash equilibria. This is just a retelling of the standard textbook example of a game in which the set of correlated equilibrium payoffs is a strict superset of the convex hull of the set of Nash equilibrium payoffs.

Up until this point, we have recalled a few well-known facts from the literature in the context of a series of imagined experiments. Regarding communication, we have contrasted the extremes of absence of a common language for messages that renders pre-play communication ineffective by design and of correlated equilibrium, which can be thought of as delineating the bound of what communication can achieve. We are now ready to illustrate one of the main results of this paper, that public communication with an imperfectly shared language can replicate what can be achieved via correlated equilibria.

To this end, consider the following variant of our laboratory thought experiment. Suppose that only females have been invited to participate. They are instructed that for the purpose of the experiment each will be given one of the names Ann, Ada or Ali. The assignments of names are private. In every pair of matched participants all pairs of distinct names are equally likely. Thus someone named Ali will know that her partner is equally likely to be named Ann or Ada and that her partner assigns probability one-half to her being named Ali. Once participants have received their names, they play the game of chicken as before. We would argue that although the names are different, it is implausible that the participants would be able to use them to implement the correlated equilibrium with payoffs $\left(\frac{10}{3}, \frac{10}{3}\right)$ from above. There is no natural association of the names with outcomes of the game and the situation seems best formalized as one in which players *a priori*, before they have received their names, fail to have a common language for the set of names; they make no common distinctions among the names that are relevant for the game. Once each participant has received her name, she can privately distinguish the name she has received from those she did not receive, she has a private *language type* and she has beliefs about the language types of her partner. But notice that *a priori* lack of a common language carries over to the language types. All three language types are symmetric and having one language type or another should have no impact on the play of the chicken game. Indeed, if players lack an *a priori* common language for the set of names, then the game in which players are privately assigned names and in which strategies respect the *a priori* lack of a common language has the same set of equilibrium distributions as the chicken base game.

Now amend our laboratory thought experiment further by adding a public announcement. After participants have received their names and before taking actions in the chicken game, there is an announcement of the form "It is X's turn to get '5'," where X is equally likely to be Ann, Ada or Ali. The public announcement breaks the symmetry among names and thus removes the *a priori* absence of a common language. Suppose the realized announcement is "It is Ann's turn to get '5'." Consider a participant who is a row player and has been named Ann. If she believes that her partner will act in accordance with the announcement, she finds it optimal to take action D. If she is named Ali instead, she believes that her partner with probability one-half is named Ann and will play R and with probability one-half is named Ada and will play L. Therefore, if she is named Ali, playing U is optimal for her (she is indifferent between U and D). Thus if we combine the game in which players are given names that by all accounts are meaningless and irrelevant to the game with a public announcement that refers to those names, we can implement the correlated equilibrium payoff pair $\left(\frac{10}{3}, \frac{10}{3}\right)$ as a Bayesian Nash equilibrium payoff of the extended game. One of our results will show that this observation is general: Any (rational) correlated equilibrium payoff of any finite static complete-information game, the base game, can be achieved in a game where there is a public announcement of a signal drawn from a commonly known distribution over signals prior to the base game, players can make no *a priori* common distinctions among signals and players draw language types that allow them to make private distinctions among signals. Thus, while public announcements with a shared language can only implement the convex hull of Nash equilibrium distributions, public announcements with an imperfectly shared language can implement the entire set of (rational) correlated equilibrium distributions.

To prepare for the consideration of an exchange of messages with an imperfectly shared language modify our laboratory thought experiment once more. Again, only females will be invited for the experiment and each will be assigned a *set* of names. Each name will be composed of a first name and a last name. Possible first names are Ann, Ada and Ali; possible last names are Smith, Miller and Baker. The assignments of sets of names are private. The assignment proceeds as follows: First one participant in each pair draws a set of names uniformly from the collection of sets that have the property that each first name appears exactly once and each last name appears exactly once. For example, the first participant might draw the set consisting of Ann Miller, Ada Baker and Ali Smith. Next the second participant draws a set of names uniformly from the collection of sets that have the property that each first name appears exactly once, each last name appears exactly once and none of the names drawn by the first participant appear. So the second participant might draw the set consisting of Ann Smith, Ada Miller and Ali Baker. A key property of this assignment rule is that for each first (last) name each participant attaches equal probability to her partner having either of the two names with the same first (last) name that is not ruled out by her own name. Without communication, if we had participants play the chicken game after learning their sets of names, their language types, players have no *a priori* common language for the sets of first names and last names; that is there is nothing to suggest to participants how to condition their play in the chicken game on their language type. The set of equilibria in this game that respect the *a priori* lack of common language generates the same distributions over action profiles as the set of equilibria of the chicken base game.

We now come to the final modification of our laboratory thought experiment, in which we let participants exchange signals with an imperfectly shared language. Only females are invited to the experiment. Each will be assigned a set of names, consisting of a first and a last name, exactly as in the last version of our experiment; these will be referred to as 'their names.' In addition row players will be given the set of signals {Ann, Ada, Ali} and column players will be given the set of signals {Smith, Miller, Baker}. Before playing the chicken base game, participants simultaneously send signals to each other. They will be told that the pair of resulting signals is the 'agreed upon name.' The instructions for the experiment include a statement of the form "If the agreed upon name is one of your names, it is your turn to get '5'." It is an equilibrium for players to randomize uniformly over their signal sets and then act in accordance with the instructions; this equilibrium respects the *a priori* absence of a common language for signals. Uniform randomization over signals at the communication stage is a consequence of *a priori* absence of common language and the fact that the rule by which sets of names are constructed does not break the symmetry of the signal sets: Every possible set of names of a player can be obtained by some permutation of the set of signals from any other possible set of names. At the action stage, following the suggestion in the instructions is optimal if the partner follows the suggestion because players have the same information they have in the correlated equilibrium that we described above which induces the payoff pair $\left(\frac{10}{3}, \frac{10}{3}\right)$. To see this, consider first a row player for whom the agreed upon name is her name. She knows that the agreed upon name is not her partner's name and therefore expects her partner to take the action L, against which D is uniquely optimal, therefore making it optimal to follow the suggested behavior. Now suppose the agreed upon name is not her name. Without loss of generality, suppose the agreed upon name is Ada Baker and that Ada Miller is one of her names. By construction, the column player must have either Ada Baker or Ada Smith as one of her names and these possibilities are equally likely; i.e., there is an equal chance that the agreed upon name is one of the column player's names and that it is not one of the column player's names. Therefore the row player expects the column player to play each of her actions L and R with equal probability. Against that distribution, following the behavior suggested in the instructions by playing U is optimal.

3 Information structures and language structures

In this section we show that the parallel between correlated equilibria and equilibria of language games that we illustrated above is fully general. To this end, we relate the information structures that are associated with correlated equilibria to language structures that express players' uncertainty about each other's ability to produce and interpret messages. We establish an equivalence between (rational) correlated equilibria with their associated information structures and (rational) language equilibria with their associated language structures.

3.1 Information structures

Consider finite static complete-information games $G = \{I, \{A_i\}_{i\in I}, \{U_i\}_{i\in I}\}$, where I is the player set (we will use I to denote both the set and its cardinality), A_i player i's action set, $A = \times_{i=1}^{I} A_i$ the set of action profiles and $U_i : A \to \mathbb{R}$ player i's payoff function, with the usual extension to mixed strategies. An *information structure* $\mathcal{I} = (\Omega, \{\mathcal{P}^i\}_{i\in I}, p)$ consists of a finite state space Ω, a probability distribution p on Ω and a partition $\mathcal{P}^i$ of the state space for each player i. Denote the game that is obtained by augmenting G with the information structure $\mathcal{I}$ by $G(\mathcal{I}) = \{I, \{A_i\}_{i\in I}, \{U_i\}_{i\in I}\}, \mathcal{I}\}$. In the game $G(\mathcal{I})$ each player's strategy is a function $f_i : \Omega \to A_i$ that is measurable with respect to her information partition $\mathcal{P}^i$. Recall that a *correlated equilibrium* of the game G can be defined as a Bayesian Nash equilibrium of an augmented game $G(\mathcal{I})$ for some information structure $\mathcal{I}$, i.e., each player optimizes at each of the elements of her information partition, given the strategies of others. The interpretation is that a state $\omega \in \Omega$ is drawn according to the common prior p and each player i learns only that the realized state is in $\mathcal{P}^i(\omega)$, the element of her partition that contains ω.

3.2 Language structures for public announcements

In this section we introduce language structures. For now we limit attention to the case of public announcements; i.e., we only have to deal with how players interpret messages and not with which messages they can send. The intent is to model the possibility that different players may interpret the same public announcement differently and that they may be uncertain about other players' interpretations.

Let M be a finite set of messages and for each player i denote by $\mathcal{Q}^i$ an ordered partition of M, her *language type*, and by $\mathfrak{Q}^i$ the set of player i's language types. Player i's language type describes the distinctions she can make among messages; messages belonging to the same partition element she has to treat identically. This has two implications: (1) she draws identical inferences from observing messages that belong to the same partition element, and (2) her strategies must be measurable with respect to the partition that is her language type. For every (ordered) partition $\mathcal{Q}^i$ and every $m \in M$, the set $\mathcal{Q}^i(m)$ is the partition element that satisfies $m \in \mathcal{Q}^i(m)$. A *language state* is a profile of ordered partitions $\mathcal{Q} = (\mathcal{Q}^1, \dots, \mathcal{Q}^n)$. Denote the set of possible language states by $\mathfrak{Q} \subseteq \times_{i=1}^{I} \mathfrak{Q}^i$. A *language structure* $\mathcal{L} = (M, \mathfrak{Q}, q)$ consists of a finite message space M, a set of possible language states $\mathfrak{Q}$ and a probability distribution q over the set of possible language states.

Given a language state $\widehat{\mathcal{Q}}$, denote by $\mathfrak{Q}(\widehat{\mathcal{Q}})$ the set of language states that can be generated from $\widehat{\mathcal{Q}}$ via permutations of the message space M; i.e., $\mathfrak{Q}(\widehat{\mathcal{Q}}) = \{\mathcal{Q} \in \mathfrak{Q} | \mathcal{Q} = \pi(\widehat{\mathcal{Q}})$ for some permutation π of $M\}$. A *symmetric language structure* $\mathcal{L}(\widehat{\mathcal{Q}}) = (M, \mathfrak{Q}(\widehat{\mathcal{Q}}), u)$ generated by a language state $\widehat{\mathcal{Q}}$ consists of a finite message space M, the set $\mathfrak{Q}(\widehat{\mathcal{Q}})$ and a uniform distribution, u, over the set $\mathfrak{Q}(\widehat{\mathcal{Q}})$. Denote the corresponding set of language types of player i by $\mathfrak{Q}^i(\widehat{\mathcal{Q}})$ and note that in general $\mathfrak{Q}(\widehat{\mathcal{Q}}) \subsetneq \times_{i=1}^{I} \mathfrak{Q}^i(\widehat{\mathcal{Q}})$.

Denote the game that is obtained by augmenting G with the language structure $\mathcal{L}$ by $G(\mathcal{L}) = \{I, \{A_i\}_{i \in I}, \{U_i\}_{i \in I}\}, \mathcal{L}\}$. In the game $G(\mathcal{L})$ each player i privately observes her language type $\mathcal{Q}^i$ before choosing her action in A_i; she only learns a partition and not also an element of that partition.

A strategy for player i in the game $G(\mathcal{L})$ is a function $f_i : \mathfrak{Q}^i \to A_i$.

In the game $G(\mathcal{L})$ no messages are received (or sent). Therefore the language structure plays a limited role. Note, however, that it might matter simply because of the possibility of player i using the information about her language type $\mathcal{Q}^i$ as a correlating device. We will rule out this possibility by requiring that players cannot make *a priori* distinctions among messages and must therefore treat all messages symmetrically.

We want a player's language type $\mathcal{Q}^i$ to be the only source of distinctions the player can make among messages. This requires that players are unable to make common *a priori* distinctions among the elements of the message space. In the terminology of Crawford and Haller [12], we require that *a priori* players face *absence of a common language* with which to describe the elements of M. If, for example, there are no *a priori* distinctions among the elements the message space $\{m_1, m_2, m_3\}$, the players cannot distinguish the language types $\{\{m_1\}, \{m_2, m_3\}\}$, $\{\{m_2\}, \{m_1, m_3\}\}$ and $\{\{m_3\}, \{m_1, m_2\}\}$. Therefore we require that a player's strategy is only defined up to permutations of the message space M. For the game $G(\mathcal{L})$ this means that $f_i(\mathcal{Q}^i) = f_i(\pi(\mathcal{Q}^i))$ for every permutation π of the message space M, where $\pi(\mathcal{Q}^i)$ denotes the partition of M that is obtained from the partition $\mathcal{Q}^i$ via the permutation π. An immediate consequence is the following observation:

Observation 1 *If players lack a common language for the message space M and if the language structure $\mathcal{L}$ in the game $G(\mathcal{L})$ is symmetric, the game $G(\mathcal{L})$ has the same set of equilibrium distributions as the game G.*

Let $\Gamma(G)$ be the game obtained by having G preceded by a public draw from a uniform distribution over the message space M, a public announcement. Even without our requirement that players lack a common language for the message space M, it is well known that the set of equilibrium distributions of $\Gamma(G)$ is a subset of the convex hull of the set of Nash equilibrium distributions of G and therefore may be strictly smaller than the set of correlated equilibrium distributions. If in addition players lack a common language for M, then the public announcement can have no effect and we have the following obvious observation.

Observation 2 *If players lack a common language for the message space M, the game $\Gamma(G)$ has the same set of equilibrium distributions as the game G.*

Let $\Gamma(G(\mathcal{L}))$ be the game obtained by having $G(\mathcal{L}))$ preceded by a public draw from a uniform distribution over the message space M.[6] A strategy $\gamma_i : \mathfrak{Q}^i \times M \to \Delta(A_i)$ maps player i's language types and the messages observed by player i into distributions over player i's actions. The constraints imposed by player i's language types $\mathcal{Q}^i$ are captured by the condition that for each $\mathcal{Q}^i \in \mathfrak{Q}^i$ the function $\gamma_i(\mathcal{Q}^i, \cdot) : M \to \Delta(A_i)$ must be measurable with respect to $\mathcal{Q}^i$. *A priori* lack of a common language amounts to the requirement that $\gamma_i(\mathcal{Q}^i, m) = \gamma_i(\pi(\mathcal{Q}^i), \pi(m))$ for all permutations π of the message space M.

For this environment, we get our first key result:

Proposition 1 *For every rational correlated equilibrium g of the game G, there exists a message space M and symmetric language structure $\mathcal{L}$ such that if agents lack a common language for M the game $\Gamma(G(\mathcal{L}))$, in which G is preceded by a public draw from a uniform distribution over M, has a Bayesian Nash equilibrium γ with the same distribution over action profiles as g.*[7]

[6]In *one-shot public mediated talk* (Lehrer and Sorin [31]) players simultaneously and privately send messages to a mediator who then makes a public announcement that is a deterministic function of players' messages. These private messages share with players' private language types in our setting that both make it possible to privately interpret public messages.

Differently from public mediated talk, in our environment the announcer's information at every stage of the game is publicly available information. He receives no signals from the players and never shares information with strict subsets of the player set. As a result he does not know the impact of his announcement, players' privacy is preserved and (provided it can be guaranteed that the announcer sends some message from M), it is immaterial whether the announcer has an interest in the outcome of the game or not.

[7]This result does not require a shared language between the announcer and any of the

Proof: Let g be a rational correlated equilibrium of the game G. Thus g is a Bayesian Nash equilibrium of the game $G(\mathcal{I}) = \{I, \{A_i\}_{i\in I}, \{U_i\}_{i\in I}\}, \mathcal{I}\}$ for some information structure $\mathcal{I} = (\Omega, \{\mathcal{P}^i\}_{i\in I}, p)$, where $p(\omega)$ is rational for every $\omega \in \Omega$. Since g is rational, there will be an integer K such that for every $\omega \in \Omega$ there is an integer $K(\omega)$ with $p(\omega) = \frac{K(\omega)}{K}$. Split each state $\omega_\ell \in \Omega$ into $K(\omega_\ell)$ states $\omega_{\ell,1}, \omega_{\ell,2}, \ldots, \omega_{\ell,K(\omega_\ell)}$. Denote the new state space by $\widetilde{\Omega}$. Player i's information partition $\mathcal{P}^i$ on the original state space Ω naturally induces an information partition $\widetilde{\mathcal{P}}^i$ on the new state space $\widetilde{\Omega}$ via the property that $\omega_{\ell'} \in \mathcal{P}^i(\omega_\ell)$ if and only if $\omega_{\ell',k'} \in \widetilde{\mathcal{P}}^i(\omega_{\ell,k})$ for all $k \in \{1, \ldots, K(\omega_\ell)\}$ and all $k' \in \{1, \ldots, K(\omega_{\ell'})\}$. Consider the information structure $\widetilde{\mathcal{I}} = (\widetilde{\Omega}, \{\widetilde{\mathcal{P}}^i\}_{i\in I}, \tilde{p})$ where $\tilde{p}$ is the uniform distribution on $\widetilde{\Omega}$. Trivially, the game $G(\widetilde{\mathcal{I}}))$ has a Bayesian Nash equilibrium $\tilde{g}$ that induces the same distribution over action profiles A as does g in $G(\mathcal{I})$. Let $M = \widetilde{\Omega}$. For each i, let $\widehat{\mathcal{Q}}^i$ be an ordered version of $\widetilde{\mathcal{P}}^i$. Let $\mathcal{L}$ in the statement of the proposition be the *symmetric language structure* $\mathcal{L}(\widehat{\mathcal{Q}}) = (M, \mathfrak{Q}(\widehat{\mathcal{Q}}), u)$ generated by the language state $\widehat{\mathcal{Q}} = (\widehat{\mathcal{Q}}^1, \widehat{\mathcal{Q}}^2, \ldots, \widehat{\mathcal{Q}}^I)$. For each $\mathcal{Q}^i \in \mathfrak{Q}^i(\widehat{\mathcal{Q}})$ define $\pi_{\mathcal{Q}^i}$ as any permutation of M that satisfies $\mathcal{Q}^i = \pi_{\mathcal{Q}^i}(\widehat{\mathcal{Q}}^i)$, representative for the class of permutations with that property. For all $m \in M$ and $\mathcal{Q}^i \in \mathfrak{Q}^i(\widehat{\mathcal{Q}})$ let $\gamma(\mathcal{Q}^i, m) = \tilde{g}(\pi^{-1}_{\mathcal{Q}^i}(m))$

Since players lack an *a priori* common language for the message space M, player i can make exactly those distinctions among the messages in M that are implied by her language type $\mathcal{Q}^i$. Hence, having observed her language type $\mathcal{Q}^i$, player i deems all language states equally likely that are consistent with the property $\mathcal{Q}^i = \pi(\widehat{\mathcal{Q}}^i)$ for some permutation π of M. Therefore, when observing the public announcement m player i with language type $\mathcal{Q}^i$ has the same information about the distribution of other player's actions as she

players. If we did allow for the possibility that each player shared a common language with the announcer that was not understood by any of the other players, there would be an obvious way for the announcer to use public messages to send private signals: Use a public message composed of multiple statements, each addressed to an individual player in a language only that player and the announcer understand. Privacy of players would not be protected, there would be considerable demands on the announcer's language competence and the announcer would have to be disinterested in the outcome of the game.

does in the correlated equilibrium $\tilde{g}$ at information set $\pi^{-1}_{\mathcal{Q}^i}(\mathcal{Q}^i(m))$ and at that information set $\tilde{g}(\pi^{-1}_{\mathcal{Q}^i}(m))$ is optimal. $\square$

3.3 Language structures for pre-play communication

We now derive a similar result for the case of simultaneous pre-play communication as we did in the preceding section for public announcements. This requires that we model both the constraints that players face when they send and when they interpret messages. Analogous to the previous section, uncertainty about each other's language types leads players to interpret message profiles differently.

For each player i let S^i be a finite set of signals. The set $M := \times_{i=1}^{I} S^i$ of signal profiles is the message set, with typical element $m = (s^1, \ldots, s^I)$. Apart from this modification, the definitions of language types $\mathcal{Q}^i \in \mathfrak{Q}^i$, language states $\mathcal{Q} \in \mathfrak{Q} \subseteq \times_{i=1}^{I} \mathfrak{Q}^i$, language structures $(M, \mathfrak{Q}, q)$ and symmetric language structures $\mathcal{L}(\widehat{\mathcal{Q}}) = (M, \mathfrak{Q}(\widehat{\mathcal{Q}}), u)$ generated by a language state $\widehat{\mathcal{Q}}$ remain the same as above.

We continue to require *a priori* absence of a common language, here for each player i's signal space S^i. Therefore a player's strategy is only defined up to permutations of players' signal spaces. For the game $G(\mathcal{L})$ this means that $f_i(\mathcal{Q}^i) = f_i(\pi(\mathcal{Q}^i))$ for every $\mathcal{Q}^i \in \mathfrak{Q}^i$ and every permutation $\pi = (\pi^1, \ldots, \pi^I)$ of the message space M that can be obtained via permutations π^j of each player j's signal space S^j, where $\pi(\mathcal{Q}^i)$ denotes the partition of M that is obtained from partition $\mathcal{Q}^i$ via the permutation π. Evidently, Observation 1 continues to hold: with *a priori* absence of a common language (for signal spaces) and a symmetric language structure, the augmented game $G(\mathcal{L})$ has the same set of equilibrium distributions as the base game G.

Here we let $\Gamma(G)$ denote the game obtained by having G preceded by one round of simultaneous pre-play communication. In this case a version of Observation 2 holds: if players *a priori* lack a common language for their signal spaces the set of equilibrium distributions of $\Gamma(G)$ is the same as the set of equilibrium distributions of G.

Let $\Gamma(G(\mathcal{L}))$ be the game obtained by having $G(\mathcal{L}))$ be preceded by one round of simultaneous pre-play communication. A strategy $\gamma_i = (\gamma_i^C, \gamma_i^R)$ for player i in this game consists of a communication rule $\gamma_i^C : \mathfrak{Q}^i \to \Delta(S^i)$ that maps player i's language types into distributions over player i's signals and a response rule $\gamma_i^R : \mathfrak{Q}^i \times M \to \Delta(A_i)$ that maps player i's language types and the messages observed by player i into distributions over player i's actions. One implication of the *a priori* absence of a common language for player i's signals is that for each $\mathcal{Q}^i \in \mathfrak{Q}^i$ the distribution $\gamma_i^C(\mathcal{Q}^i)$ is uniform over player i's signal set S^i. The second implication of the *a priori* absence of a common language for players' signals is that $\gamma_i^R(\mathcal{Q}^i, m) = \gamma_i^R(\pi(\mathcal{Q}^i), \pi(m))$ for all i, $\mathcal{Q}^i$, $m \in M$ and permutations π of the message space M that result from permutations of players' signal spaces. The fact that player i's language type dictates the distinctions player i can make among messages is captured by the requirement that for each $\mathcal{Q}^i \in \mathfrak{Q}^i$ the function $\gamma_i^R(\mathcal{Q}^i, \cdot) : M \to \Delta(A_i)$ must be measurable with respect to $\mathcal{Q}^i$.

For this language game we get our second key result:

Proposition 2 *For every rational correlated equilibrium g of the game G, there exist signal spaces S^i, $i = 1, \ldots, I$, and a corresponding symmetric language structure $\mathcal{L}$ such that if agents lack a common language for the signal spaces S^i, the game $\Gamma(G(\mathcal{L}))$, in which G is preceded by one round of simultaneous pre-play communication, has a Bayesian Nash equilibrium γ with the same distribution over action profiles as g.*

Proof: Let g be a rational correlated equilibrium of the game G. Thus g is a Bayesian Nash equilibrium of the game $G(\mathcal{I}) = \{I, \{A_i\}_{i\in I}, \{U_i\}_{i\in I}\}, \mathcal{I}\}$ for some information structure $\mathcal{I} = (\Omega, \{\mathcal{P}^i\}_{i\in I}, p)$, where $p(\omega)$ is rational for every $\omega \in \Omega$. Since g is rational, there will be an integer K such that for every $\omega \in \Omega$ there is an integer $K(\omega)$ with $p(\omega) = \frac{K(\omega)}{K}$. Split each state $\omega_\ell \in \Omega$ into $K(\omega_\ell)$ states $\omega_{\ell,1}, \omega_{\ell,2}, \ldots, \omega_{\ell,K(\omega_\ell)}$. Denote the new state space by $\widetilde{\Omega}$. Player i's information partition $\mathcal{P}^i$ on the original state space Ω naturally induces an information partition $\widetilde{\mathcal{P}}^i$ on the new state space $\widetilde{\Omega}$ via the property that $\omega_{\ell'} \in \mathcal{P}^i(\omega_\ell)$ if and only if $\omega_{\ell',k'} \in \widetilde{\mathcal{P}}^i(\omega_{\ell,k})$ for all $k \in \{1, \ldots, K(\omega_\ell)\}$ and all $k' \in \{1, \ldots, K(\omega_{\ell'})\}$. Consider the information structure $\widetilde{\mathcal{I}} = (\widetilde{\Omega}, \{\widetilde{\mathcal{P}}^i\}_{i \in I}, \tilde{p})$ where $\tilde{p}$ is the uniform distribution on $\widetilde{\Omega}$. Trivially, the game $G(\widetilde{\mathcal{I}}))$ has an equilibrium $\tilde{g}$ that induces the same distribution over action profiles A as does g in $G(\mathcal{I})$. It will suffice to have only two players talking. Without loss of generality, let these be players 1 and 2. It does not matter if the other players communicate as well, given that their signals can always be ignored. For each player $i = 1, 2$, let $S^i = \{s^i_1, \ldots, s^i_{|\widetilde{\Omega}|}\}$ denote a set of signals that has the same cardinality as the expanded state space $\widetilde{\Omega}$. In view of only players 1 and 2 sending messages, we slightly abuse notation and let $M = S^1 \times S^2$. Call a subset ψ of $M = S^1 \times S^2$ a *bijection-set* if it is the graph of a bijection $b_\psi : S^1 \to S^2$. To each state $\omega_j \in \widetilde{\Omega}$ recursively assign a bijection-set $\beta(\omega_j)$ as follows:

1. For ω_1 pick any bijection-set $\beta(\omega_1)$.
2. For ω_j with $j > 1$ pick any bijection-set $\beta(\omega_j)$ that satisfies $\beta(\omega_j) \cap \left(\bigcup_{\ell=1}^{j-1} \beta(\omega_\ell)\right) = \emptyset$

Denote the resulting collection of bijection-sets by Ψ. For each $\psi \in \Psi$ let $\mathcal{W}^i(\psi) := \beta(\widetilde{\mathcal{P}}^i(\beta^{-1}(\psi)))$. For each $m \in M$, define $\psi(m)$ via $m \in \psi(m)$; i.e., $\psi(m)$ is the element of the bijection set Ψ that contains m. For each player i define an ordered partition $\widehat{\mathcal{Q}}^i$ of the message space M via the property that for each $m \in M$ the partition element containing m is given by $\widehat{\mathcal{Q}}^i(m) := \{m' \in S^1 \times S^2 | \exists \xi' \in \mathcal{W}^i(\psi(m)) \text{ with } m' \in \xi'\}$ and by adding an (arbitrary) order of the elements of the partition. Let $\mathcal{L}$ in the statement of the proposition be the *symmetric language structure* $\mathcal{L}(\widehat{\mathcal{Q}}) = (M, \mathfrak{Q}(\widehat{\mathcal{Q}}), u)$ generated from the language state $\widehat{\mathcal{Q}} = (\widehat{\mathcal{Q}}^1, \widehat{\mathcal{Q}}^2, \ldots, \widehat{\mathcal{Q}}^I)$ by those permutations of M that result from permutations of the signal spaces S^i. For each $\mathcal{Q}^i \in \mathfrak{Q}^i(\widehat{\mathcal{Q}})$ define $\pi_{\mathcal{Q}^i}$ as any permutation of the signal sets S^j, $j = 1, \ldots, I$, that satisfies $\mathcal{Q}^i = \pi_{\mathcal{Q}^i}(\widehat{\mathcal{Q}}^i)$. One can also think of $\pi_{\mathcal{Q}^i}$ as the class of permutations with that property. Let $\gamma_i = (\gamma_i^C, \gamma_i^R)$ be such that $\gamma_i^C(\mathcal{Q}^i)$ is the uniform distribution over S^i for all language types $\mathcal{Q}^i \in \mathfrak{Q}^i(\widehat{\mathcal{Q}})$, and $\gamma_i^R(\mathcal{Q}^i, m) = \tilde{g}(\pi_{\mathcal{Q}^i}^{-1}(m))$. Notice that for any bijection set ψ, any permutation π of players' signal sets and for any choice of signal of player 1 there is exactly one signal of player 2 such that the resulting message belongs to $\pi(\psi)$. The same is true with the roles of players 1 and 2 reversed. Therefore, if one of these players randomizes uniformly over her signal set the other cannot affect which $\pi(\psi)$ will be realized, and *vice versa*. Thus players 1 and 2 are indifferent among all of their signals. Since players lack an *a priori* common language for the signal spaces S^j, $j = 1, \ldots, I$, player i cannot make any distinctions among the messages in M that are not implied by her language type $\mathcal{Q}^i$. Hence, having observed her language type $\mathcal{Q}^i$, player i deems all language states equally likely that are consistent with the property $\mathcal{Q}^i = \pi(\widehat{\mathcal{Q}}^i)$ for some permutation π of the signal spaces S^j, $j = 1, \ldots, I$. Therefore, when observing the message $m = (s^1, s^2)$ player i with language type $\mathcal{Q}^i$ has the same information about the distribution of other player's actions as she does in the correlated equilibrium $\tilde{g}$ at information set $\pi_{\mathcal{Q}^i}^{-1}(\mathcal{Q}^i(m))$ and at that information set $\tilde{g}(\pi_{\mathcal{Q}^i}^{-1}(m))$ is optimal. □

4 An Example with Independent Language Types

In the previous section we made deliberate use of correlated language type distributions to highlight the parallels between information and language structures. Players were unable to take advantage of that correlation without communication because communication was needed to break the symmetry associated with lack of a common language for messages or signals. We will now show that correlation of language types, while convenient, is not necessary to achieve at least some of the enlargement of the set of equilibrium outcomes that can be attainted by converting communication games into language games.

Consider a two-player communication game in which players simultane-

	L	R
U	4,4	2,5
D	5,2	0,0

Figure 2: Base Game

ously exchange messages prior to playing the base game shown in Figure 2. Players send messages from a common message space $M = \{*, \#, \&, \$\}$. Player 1, the row player, has language type $\mathcal{Q}_1^1 = \{\{*\}, \{\#\}, \{\&, \$\}\}$ with probability one third and language type $\mathcal{Q}_2^1 = \{\{*\}, \{\#\}, \{\&\}, \{\$\}\}$ with probability two thirds. This can be interpreted as player 1 always knowing the meaning of messages $*$ and $\#$ and with probability two thirds additionally knowing the meaning of the messages & and \$. Similarly, player 2, the column player, has language type $\mathcal{Q}_1^2 = \{\{*, \#\}, \{\&\}, \{\$\}\}$ with probability one third and language type $\mathcal{Q}_2^2 = \{\{*\}, \{\#\}, \{\&\}, \{\$\}\}$ with probability two thirds. This can be interpreted as player 2 always knowing the meaning of messages & and \$ and with probability two thirds additionally knowing the meaning of the messages $*$ and $\#$. Language types are drawn independently from a common prior distribution $p = p^1 \times p^2$ over the set of language states $\mathfrak{Q} = \mathfrak{Q}^1 \times \mathfrak{Q}^2$ with $p^1(\mathcal{Q}_1^1) = p^2(\mathcal{Q}_1^2) = \frac{1}{3}$ and $p^1(\mathcal{Q}_2^1) = p^2(\mathcal{Q}_2^2) = \frac{2}{3}$. It will be convenient to denote the set of messages that player 1 always understands by $M_1 = \{*, \#\}$ and the set of messages that player 2 always understands by $M_2 = \{\&, \$\}$.

We are interested in equilibria of the language game that results from combining the communication game in which players simultaneously send messages from M to each other before playing the base game in Figure 2 with the language structure just described. That is, players must respect their language constraints, i.e. treat those messages identically whose meaning they do not know.

For this language game, consider the following strategy profile $((\gamma_1^C, \gamma_1^R), (\gamma_2^C, \gamma_2^R))$, consisting of a communication rule γ_i^C and a response rule γ_i^R for each player i:

$$\gamma_1^C(\mathcal{Q}_1^1)[*] = \gamma_1^C(\mathcal{Q}_2^1)[*] = \gamma_1^C(\mathcal{Q}_1^1)[\#] = \gamma_1^C(\mathcal{Q}_2^1)[\#] = \frac{1}{2}$$
$$\gamma_2^C(\mathcal{Q}_1^2)[\&] = \gamma_2^C(\mathcal{Q}_2^2)[\$] = \gamma_2^C(\mathcal{Q}_1^2)[\&] = \gamma_2^C(\mathcal{Q}_2^2)[\$] = \frac{1}{2}$$

$$\begin{aligned}
\gamma_1^R(\mathcal{Q}_1^1, (m_1, m_2))[U] &= 1, \forall (m_1, m_2) \in \{(*, \&)), (*, \$)), (\#, \&), (\#, \$)\} \\
\gamma_1^R(\mathcal{Q}_2^1, (m_1, m_2))[U] &= 1, \forall (m_1, m_2) \in \{(*, \$)), (\#, \&)\} \\
\gamma_1^R(\mathcal{Q}_2^1, (m_1, m_2))[D] &= 1, \forall (m_1, m_2) \in \{(*, \&)), (\#, \$)\} \\
\gamma_1^R(\mathcal{Q}_1^1, (m_1, m_2))[U] &= \gamma_1^R(\mathcal{Q}_2^1, (m_1, m_2))[U] = \frac{2}{3}, \\
&\quad \forall (m_1, m_2) \notin \{(*, \&)), (*, \$)), (\#, \&), (\#, \$)\}
\end{aligned}$$

$$\begin{aligned}
\gamma_2^R(\mathcal{Q}_1^2, (m_1, m_2))[L] &= 1, \forall (m_1, m_2) \in \{(*, \&)), (*, \$)), (\#, \&), (\#, \$)\} \\
\gamma_2^R(\mathcal{Q}_2^2, (m_1, m_2))[R] &= 1, \forall (m_1, m_2) \in \{(*, \$)), (\#, \&)\} \\
\gamma_2^R(\mathcal{Q}_2^2, (m_1, m_2))[L] &= 1, \forall (m_1, m_2) \in \{(*, \&)), (\#, \$)\} \\
\gamma_2^R(\mathcal{Q}_1^2, (m_1, m_2))[L] &= \gamma_1^R(\mathcal{Q}_2^2, (m_1, m_2))[L] = \frac{2}{3}, \\
&\quad \forall (m_1, m_2) \notin \{(*, \&)), (*, \$)), (\#, \&), (\#, \$)\}
\end{aligned}$$

Although not required by our formalism, to understand the rationale for this construction, it will help to think of there being a set of meanings in addition to the messages and a player knowing the meaning of messages if she knows the true bijection between messages and meanings. For instance, let the meanings of the four messages be the first four integers and let the bijection between messages and meanings be given by $* \mapsto 1$, $\# \mapsto 2$, $\& \mapsto 3$ and $\$ \mapsto 4$.

Player 1's strategy prescribes that at the communication stage she randomize uniformly over M_1, the set of message that she always understands. At the response stage, if she does not understand the messages in M_2, she sent a message from M_1 and player 2 sent a message from M_2, she takes action U with probability 1; if she does understand the messages in M_2, she sent a message from M_1 and player 2 sent a message from M_2, she takes action U with probability 1 if the sum of the messages is odd and takes action D if the sum of the messages is even; if either player 1 did not send a message from M_1 or player 2 did not send a message from M_2 (i.e. message are unexpected), player 1 randomizes, taking action U with probability $\frac{2}{3}$.

Player 2's strategy is the mirror image of player 1's strategy. In abbreviated form, player 2 uses the strategy: Randomize uniformly over M_2; if 'understand', messages are expected and the sum is odd, play R; if 'understand', messages are expected and the sum is even, play L; if 'not understand' and messages are expected, play L; after unexpected message, randomize according to the mixed equilibrium of the stage game.

Note that if the row player does not understand and messages are as expected, she assigns probability $\frac{2}{3}$ to the event that column understands and therefore $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ to the event that column understands and that the sum is odd. Hence, conditionally on not understanding, she assigns probability $\frac{2}{3}$ to column playing L and probability $\frac{1}{3}$ to column playing R. Conditionally on understanding herself, if the sum is even she assigns probability one to the column player playing L (either because the column player understands and sees that the sum is even or because the column player does not understand); if the sum is odd, she assigns probability $\frac{2}{3}$ to the column understanding and playing R and probability $\frac{1}{3}$ to the column player not understanding and therefore playing L. Given the symmetry of the game and the postulated equilibrium profile, the column player's information and belief following the various combinations of messages, have the same structure and the row player's. From this, it is easily checked that it is incentive compatible to follow the prescribed strategies and that the expected payoff is outside the convex hull of Nash payoffs.

The key intuition driving this observation is that players who do not understand all messages, in equilibrium have an incentive to take the conciliatory action. Since players sometimes simultaneously do not understand all messages, they will on those occasions both take the conciliatory action, which puts positive weight on payoff pairs that are outside of the convex hull of the set of Nash equilibrium payoffs of the base game. The fear of misunderstandings prompts players to reach advantageous compromises.

This contrasts with the fact that merely adding language constraints, with or without uncertainty about those constraints, does not change equilibrium outcomes relative to the base game as along as players do not communicate prior to the base game – after all, language types are drawn independently. Similarly, adding communication to the base game without privately known language constraints achieves nothing more than to expand the equilibrium payoff set to the convex hull of Nash equilibrium payoffs of the base game.

	L	R
U	$\frac{1}{3}$	$\frac{1}{3}$
D	$\frac{1}{3}$	0

Figure 3: A CE outside the convex hull of NE

We have shown that the correlated equilibrium distribution in Figure 3 can be supported as a language equilibrium with independently drawn language types and therefore that there are language equilibria for games with independently drawn language types outside the convex hull of the set of Nash equilibria. This is not, however, the symmetric optimal correlated equilibrium distribution, which is shown in Figure 4, and one may wonder whether there is a language game with independently drawn language types that supports the symmetric optimal correlated equilibrium distribution.

	L	R
U	$\frac{1}{2}$	$\frac{1}{4}$
D	$\frac{1}{4}$	0

Figure 4: Optimal Symmetric CE

We do not know the answer to this puzzle. What we can say, is that the previous construction does not work. Suppose that each player 'understands' with probability $\tilde{p}$ and that Row plays D if and only if she understands and the sum is even. Since, conditional on her understanding, the sum is even with probability $\frac{1}{2}$ and since the CE distribution requires her to play D with probability $\frac{1}{4}$, we must have $\tilde{p} = \frac{1}{2}$. If Row does not understand and plays U, she believes that column plays R with probability $\tilde{p}\frac{1}{2} = \frac{1}{4}$ (the probability that Column understands and the sum is odd) and that Column plays L with probability $1 - \tilde{p} + \tilde{p}\frac{1}{2} = \frac{3}{4}$, the probability that either Column does not understand or that Column understands and the sum is even. With these beliefs, however, she strictly prefers D instead of U, which breaks the putative equilibrium.

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