Market Selection and the Information Content of Prices

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Abstract

We study price formation in an economy where buyers with unit demand decide to purchase one of two possible goods which are traded in two distinct markets. The goods traded within each market are identical, common-value objects and we model the price formation process as a large uniform-price auction. Before the auctions, bidders receive informative but imperfect signals about the state of the world and choose to participate in one of the markets. Our main result shows that if market frictions lead to uncertain gains from trade in any of the two markets, then there is no equilibrium where prices aggregate information. In contrast, if both markets are frictionless, then prices fully aggregate information. These findings are driven by how bidders self-select across markets: Better-informed bidders select frictional markets while uninformed, pessimistic bidders select the safety of frictionless markets. Our results suggest a novel mechanism through which market imperfections in one market can have widespread effects across all linked markets.

Keywords: Auctions, Large markets, Information Aggregation.

JEL Codes: C73, D44, D82, D83.

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1. Introduction.

An important reason to trust markets arises from the belief that market prices accurately summarize the vast array of information held by market participants. Whether this belief is justified, that is, whether prices efficiently aggregate information dispersed among market participants is a central economic question. Prices provide incentives which guide production, investment and consumption decisions. In turn, these decisions determine the allocation of resources. Thus, asset or factor prices that accurately reflect preferences and beliefs are integral for economic efficiency. As such, well functioning securities or commodity markets generate tremendously valuable public goods by impounding new, disperse information into publicly available asset or factor prices. Not surprisingly, there is a large body of research that explores the circumstances under which prices efficiently aggregate disperse information pertaining to beliefs and studies policies that lead to prices that are more responsive to information.

Past work shows that prices in certain auction markets do in fact effectively aggregate dispersed information. Consider a market in which a large number of identical common-value objects are sold through a uniform-price auction. In such an auction, if the bidders each have an independent signal about an unknown state of the world and if this unknown state determines the value of the object, then the equilibrium price converges to the true value of the object as the number of objects and the number of bidders grow arbitrarily large. Therefore, the auction price reveals precisely the unknown state of the world. Wilson (1977), Milgrom (1979), and Pesendorfer and Swinkels (1997) established this remarkable result under quite general assumptions. Kremer (2002) showed that this finding also holds in a number of different auction formats. Reny and Perry (2006) and Cripps and Swinkels (2006) proved that information is also aggregated in certain large double auctions.

In all previous work, the homogeneous (or closely-linked) objects are assumed to trade in a single centralized frictionless auction market. However, such a centralized market is an exception rather than the rule. Fragmentation, the disperse trading of the same security in multiple markets, is common place: many stocks listed on the New York Stock Exchange trade concurrently on the regional exchanges (see Hasbrouck (1995)). Investors, who participate in a primary treasury bond auction, could purchase a bond with similar cashflow characteristics from the secondary market. Labor markets are linked but also segmented according to industry, geography and skill. Buyers in the market for aluminum or steel can choose between the London Metal Exchange or the New York Mercantile Exchange. Such fragmented markets and exchanges also differ in structure, rules and regulations. In particular, markets are heterogeneous in terms of the frictions that participants face. In this paper we study information aggregation when bidders

actually choose between multiple, potentially-frictional auction markets.

More specifically, suppose that n bidders, each with unit demand, receive an informative but imperfect signal about the state of the world. The bidders then choose to participate in one of two auctions. There are $k_m = \kappa_m n$ goods on auction in market m where $\kappa_m < \frac{1}{2}$. In auction market m, bidders choose their bids only as a function of their signal and all bidders who win an object pay a uniform price equal to the highest losing bid. The highest k_m bidders are allocated objects and possible ties are broken randomly. If the number of bidders is less that or equal to the number of goods in market m, then the auction price is equal to the minimum price , $c_m \geq 0$, for market m. The goods on auction are common-value objects for the bidders. To fix ideas, suppose there are two states, low or high, and that a bidder's valuation for an object in either market is equal to one in the high state and zero in the low state.

Frictions refer to market imperfections that lead to uncertain gains from trade for bidders. We focus on frictions that take the form of a minimum price c_m in market m. There are uncertain gains from trade in market m if the object's value in the low state is less that the minimum price, i.e., if $c_m > 0$. The minimum price has various interpretations: (1) It is a *reserve price* set by a single auctioneer selling the k_m goods, i.e., it is a friction caused by imperfect competition in the form of a monopolistic auctioneer exercising market power. For example, in primary sovereign bond auctions the auctioneers (i.e., the sovereign treasuries) set reserve prices (or interest rates) for the bonds on auction. (2) The auction market m is comprised of k_m non-strategic sellers, the reservation value (or the cost) for these sellers is equal to c_m and each seller requires at least c_m in order to participate in the auction, i.e., there are *informational frictions* as in Myerson and Satterthwaite (1983).[1] (3) The *government/regulator imposes a minimum price* in market m. For example, the minimum price is the central bank's interest rate target if one models the secondary bond market as a large auction. Alternatively, the minimum price is the minimum wage if one models the market for unskilled labor as a large auction.

In the model that we study, an outsider who could observe the signals of an arbitrarily large number of bidders would learn the state of the world perfectly. Motivated by such an outsider's perspective, prices are said to *aggregate information* if an outsider can figure out the state of the world almost perfectly just by observing the equilibrium prices in both markets. Consider an environment with two markets, suppose in the frictionless "safe" market there is no minimum price and further suppose that there may be a minimum price $c_r > 0$ in the "risky" market. We prove two main results: if the gains from trade are certain in both markets, i.e., if $c_r = 0$, then information is aggregated in both markets in every equilibrium. In contrast, if the gains from trade are uncertain in even one of the

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[1]The auction is in fact a double auction with non-strategic sellers who simply bid their valuations c_m.

markets, i.e., if $c_r > 0$, then, under certain assumptions, there is no equilibrium where information is aggregated in either market. In other words, market frictions in the risky market inhibit the frictionless market's price from aggregating information.

Intuitively, information aggregation fails if $c_r > 0$ because relatively few, highly optimistic bidders select the risky market in equilibrium. This kind of market selection causes competitive forces, which lead to information aggregation, to remain particularly weak in both markets. More specifically, competition is weak in the risky market because the number of goods for sale exceeds the expected number of bidders who select this market. There are many more bidders than goods in the safe market but competition is nevertheless weak in this market also. This is because more pessimistic bidders opt for the safe market and pessimism depresses prices across all states of the world. These findings sketch a new avenue through which there is informational contagion between the two markets.

For a concrete example, suppose there are n bidders, $n\kappa$ goods on auction in either market (where $\kappa \in (0, \frac{1}{2})$) and suppose $c_r \in (\frac{1}{2}, 1)$. Further, both states are a priori equally likely, bidders receive a signal that is perfectly informative with probability $p < \kappa$ and receive an uninformative signal with the remaining probability. An uninformed bidder's expected value for the good is equal $\frac{1}{2}$ because both states are equally likely. Such an uniformed bidder would never select the risky market. This is because the minimum price, c_r, exceeds the bidder's expected value for the good by assumption. Thus, the expected number of bidders in the risky market is fewer than the number of goods in either state: the uninformed never choose this market and the number of goods, $n\kappa$, exceeds the expected number of informed bidders, np, by assumption. So, the risky market's price is equal to c_r in both states and hence completely uninformative as n grows arbitrarily large. Interestingly, the price in the safe market does not aggregate information either. This is because if it were to aggregate information, then the safe market's price would converge to one, i.e., the object's value, in the high state. Therefore, an informed bidder's profits in the safe market would converge to zero. However, an informed bidder could select the risky market in the high state and in the low state submit a bid equal to zero in the safe market. This strategy guarantees him an expected payoff equal to $\frac{1}{2}(1 - c_r)$. Thus, no informed bidder would submit a positive bid in the safe market if information is aggregated there. However, if only bidders without any information submit positive bids in the safe market, then information cannot be aggregated in the safe market. Our main insight is that an analogous result holds under general signal distributions, with many states, many markets, and for any $c_r > 0$.

1.1. Relation to the literature.

This paper is closely related to earlier work which studies information aggregation in large auctions. Wilson (1977) studied second-price auctions with common value for one object for sale, and Milgrom (1979) extended the analysis to any arbitrary number of objects. Both of these papers show that as the number of bidders gets arbitrarily large, price converges to the true value of the object, but only provided that there are bidders with arbitrarily precise signals about the state of the world. Pesendorfer and Swinkels (1997) further generalize the analysis to the case where there are no arbitrarily precise signals. They show that prices converge to the true value of a common-value object in all symmetric equilibria if and only if both the number of identical objects and the number of bidders who are not allocated an object grow without bound. Pesendorfer and Swinkels (2000) generalize the analysis further to a mixed private, common-value environment. Finally, Kremer (2002) shows that the information aggregation properties of auctions are more general than the particular mechanisms studied before; he does this by providing a unified approach that uses the statistical properties of certain order statistics.[2] Our model is closest to Pesendorfer and Swinkels (1997).

There is also a recent literature, which relates to this paper, on frictional auctions with endogenous participation. Lauermann and Wolinsky (2014), Murto and Valimaki (2014) and Axelson and Makarov (2014) study single-object auctions with common values. The novel feature of Lauermann and Wolinsky's model is that the auctioneer knows the value of the object but must solicit bidders for the auction and soliciting bidders is costly. In Murto and Valimaki's model, potential bidders must pay a cost to participate in the auction. Axelson and Makarov study a model where the winner of the auction must make an ex-post investment in order to put the object into productive use. These papers relate to this paper because participation in the auctions that they study is endogenously determined: by the auctioneer in the first paper, by bidders who pay the participation cost in the second and by bidders who choose to invest in the third. However, these papers differ from ours because they study single-object, single-market auctions and their emphasis is not on information aggregation.

Our work also relates to the literature on costly information acquisition in rational-expectations models, such as Grossman and Stiglitz (1976, 1980) and Grossman (1981). These papers explain the conceptual difficulties in interpreting prices as both allocation devices and information aggregators. Specifically, they argue that if consumers and

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[2]See Hong and Shum (2004) for a calculation of the rate at which price converges to the true value in large common-value auctions. Jackson and Kremer (2007) show that the result of Pesendorfer and Swinkels (1997) does not generalize to an auction with price discrimination. See Kremer and Skrzypacz (2005) for related results concerning the link between information aggregation and the properties of order statistics.

producers need to undertake a costly activity in order to acquire information, then equilibrium prices cannot reveal the state of the world perfectly. Their reasoning is as follows: if prices were to reveal the state perfectly, then no agent would have an incentive to pay for information in the first place; but if no agent acquires information, then the prices cannot reveal the state as there is no information to aggregate. However, as was the case for auction markets, these papers do not explicitly consider how the market choice by informed individuals can affect the information content of prices.

Finally, there is a growing literature which investigates the impact of financial markets on the real economy. Past work in this literature explores situations where speculators' private information about the success prospects of an action to be chosen by a manager (or a CEO or a central banker) is partially revealed through its effect on the prices of financial assets or derivatives. Knowing this, managers also factor in financial asset price information when deciding on their course of action. Papers in this literature then argue that such feedback loops may decrease the information content of prices. A recent paper by Bond et al. (Forthcoming) surveys this literature.

2.   The Model.

There are n bidders who want to purchase one unit of $|M|$ possible different goods that are for sale. Each good $m \in M$ is sold in a separate market. Unless we state otherwise we will take $M = \{s(afe), r(isky)\}$, i.e., we will focus on the case with two markets. The index m designates both the good as well as the market in which the particular good is sold. Buyers have unit demand and therefore will buy a good in only one of the $|M|$ markets. There are $\lfloor \kappa_m n \rfloor$ identical goods available for sale in each market $m \in M$ and there are more bidders $\sum_{m \in M} \kappa_m < 1$.[3]

The state $\omega \in \Omega$ is unknown and the buyer valuation v for each good depends jointly on the state and the market, i.e., $v : \Omega \times M \to \mathbb{R}$. We assume that $v(\omega, m)$ is increasing in ω and $v(\omega, m) \geq 0$ for all $\omega \in \Omega$ and all $m \in M$. For most of the results that we present and unless we state otherwise we assume that $\Omega = \{l, h\}$.

Bidders first choose the auction market in which they will bid and then they choose their bid. A bidder does not observe anything beyond her private signal when choosing her market or when her deciding on her bid.

In each market m, the goods are sold using a closed-bid uniform-price auction where the auction price is equal to the highest losing bid if there are more bidders than there are goods in market m. If the number of bidders is less that or equal to the number of goods in market m, then the auction price is equal to the minimum price for market m. The minimum price for market m is denoted by c_m. We will assume that $c_m \geq 0$.

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[3]The largest integer not greater than x is denoted by $\lfloor x \rfloor$.

The minimum price represents market frictions that can be interpreted in various ways. (1) Frictions resulting from an auctioneer exercising market power by setting a reserve price: the minimum price c_m is a *reserve price* set by a single auctioneer selling the $\kappa_m n$ goods. For example, in primary sovereign bond auction the auctioneers (i.e., the sovereign treasury) sets a reserve price (or a maximum interest rate) for the bonds on auction. (2) A more naturally occurring informational friction resulting from seller costs exceeding buyer values in certain uncertain states of the world: The auction market m is comprised of $\kappa_m n$ non-strategic sellers, the reservation value (or the cost) for these sellers is equal to c_m and each seller require at least c_m in order to participate in the auction. In other words, the market is a double auction where the sellers are non-strategic and simply bid their valuations c_m. (3) Frictions caused by a government intervention in the shape of a price control: The minimum price is a price control imposed on market m by a regulator or governmental body. For example, the minimum price is the target interest rate imposed by a central bank if one models the secondary bond market as a large auction. Alternatively, the minimum price can be regarded as a minimum wage set by the government if one models the market for unskilled labor as a large auction.

Definition 1. We say that the **gains from trade are certain** in market m if $v(\omega, m) - c_m \geq 0$ for all states $\omega \in \Omega$. In other words, if one takes c_m to represent seller costs, then certain gains from trade means that it is efficient for the buyers and sellers to trade irrespective of the state. We say that the **gains from trade are uncertain** in market m if there exists a state ω such that $v(\omega, m) - c_m < 0$.

Each buyer shares a common prior $\pi \in \Delta^{\circ}(\Omega)$ where $\pi(\omega)$ represents the probability that the state is ω. Before choosing a market, each agent receives a signal $\theta \in [0, 1]$ according to a continuous cumulative distribution function $F(\theta|\omega)$ which admits a density function $f(\theta|\omega)$. The signals are identically and independently distributed conditional on the state. For all the results in the paper, we assume that the signals satisfy the monotone likelihood ratio property (MLRP) which we formally define below.

Definition 2. The signals satisfy (strict) MLRP if the likelihood ratio $l(\omega, \omega', \theta) = \frac{\pi(\omega)}{\pi(\omega')} \frac{f(\theta|\omega)}{f(\theta|\omega')}$ is (strictly) increasing in θ whenever $\omega > \omega'$.

After observing their signal, each bidder first chooses a market from the set of markets $m \in M$ and then submits a bid in this market. A symmetric market choice strategy is a function $a_m : [0, 1] \to [0, 1]$ where $a_m(\theta)$ is the probability the agent chooses market m when she receives signal θ. A symmetric bidding strategy in market m is a measure H_m on $[0, 1] \times [0, \infty) \times [0, \infty)$ with marginal distribution $F(\theta) = \sum_{\omega} \pi(\omega) F(\theta|\omega)$ on its first coordinate (see Milgrom and Weber (1985)). A bidding strategy is pure if there is a

function $b_m : [0,1] \to [0,\infty)$ such that $H_m(\{\theta, b_m(\theta)\}_{\theta \in [0,1]}) = 1.$[4] The project focuses on the perfect Bayesian Equilibria (PBE) of this auction with market selection. The term $\Pr_{(a,b)}$ denotes the joint probability distribution over states of the world, signal and bid distributions, allocations, market choices, and prices, where this distribution is induced if all players use the symmetric market selection strategy $a = (a_1, ..., a_{|M|})$ and the pure and symmetric bidding strategy profile $b = (b_1, ..., b_{|M|})$.

The following preliminary observation allows us to work exclusively with pure and non-decreasing bidding strategies. This is a result also found in Pesendorfer and Swinkels (1997) and follows from the MLRP condition. The argument for this lemma closely follows the proof of Pesendorfer and Swinkels (1997, Lemmata 3-7).

Lemma 1. *If signals satisfy MLRP, then any PBE bidding strategy H_m can be represented by an increasing bidding function b_m.*

Proof. The proof follows directly from Lemmata 2-6 in Pesendorfer and Swinkels (1997). □

2.1. Information aggregation, uncertain gains from trade and market selection.

In this section we formal define information aggregation and present our three main results. We consider a sequence of games where the nth game has n bidders who make a choice between the two markets and $n\kappa_m$ objects for sale in each market. In Theorem 1, we show that there is no sequence of equilibrium which aggregates information if gains from trade are uncertain. In Theorem 2, we assume that the gains from trade are uncertain and we construct a sequence of equilibria in which no information is aggregated in market S and information is imperfectly aggregated in market r. Finally, in Theorem 3, we show that if the gains from trade are certain in both markets then there is a sequence of equilibria in which information is aggregated in both markets.

In order to do so, we restrict attention to a simple model with two states: $l(ow)$ and $h(igh)$; and two markets: $s(afe)$ and $r(isky)$. Assume that

$$v_h = v(h,s) = v(h,r) = 1 > v(l,s) = v(l,r) = v_l = 0.$$

In words, the objects for sale in the two markets are symmetric and generate the same state dependent utility for the buyers. Further assume that the minimum price in the safe market is always equal to zero $c_s = 0$ and it is possibly positive in the risky market, i. e., $c_r \in [0,1)$.

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[4]If b represents H, then so does any function that is equal to b at almost every $\theta \in [0,1]$.

The assumption on market frictions is what generates an asymmetry between the two markets. In the safe market, the gains from trade are always nonnegative, i.e., this is a frictionless market. In contrast, the gains from trade are uncertain and therefore there are market frictions in the risky market. The minimum price (or a seller's cost) which is equal c_r exceeds a buyer's valuation of the good, v_l, in the low state l. Ideally, trade should take place only in the high state in market r.

2.1.1. Information aggregation. The object of study is a sequence of games where the nth game has n bidders and $\lfloor n\kappa_m \rfloor$ objects for sale in each market. The behavior in the sequence of games is described by a sequence of bidding functions $\boldsymbol{b} = \{b^n\}_n^\infty$ and a sequence of market selection functions $\mathbf{a} = \{a^n\}_n^\infty$. The sequence $\boldsymbol{b}$ is an equilibrium sequence if $b^n = (b_s^n, b_r^n)$ and $a^n = (a_s^n, a_r^n)$ is a PBE equilibrium (b^n, a^n) of Γ^n for each n.

A bidding function b_m^n and the market selection function a_m^n jointly determine a price p_m for the auction in market m given any realization of signals. Information is said to be aggregated in a particular market if the auction price p_m conveys precise information about the state of the world. More precisely, (i) if the likelihood ratio $\frac{\Pr_{(a^n,b^n)}(p^*|h)}{\Pr_{(a^n,b^n)}(p^*|l)}$ is close to zero (i.e., if it is arbitrarily more probable that we observe such a price when the state is l), then an outsider who observes price p^* learns that the state is l. Alternatively, (ii) if the likelihood ratio $\frac{\Pr_{(a^n,b^n)}(p^*|h)}{\Pr_{(a^n,b^n)}(p^*|l)}$ is arbitrarily large, then an outsider who observes price p^* learns that the state is h. If the probability that we observe a price that satisfies either (i) or (ii) is arbitrarily close to one, then we say that the equilibrium sequence $(\boldsymbol{a}, \boldsymbol{b})$ aggregates information. The precise definition is as follows:

Definition 3. (Atakan and Ekmekci (2014)) An equilibrium sequence $\boldsymbol{b}$ **aggregates information** if, for any $\epsilon > 0$,

$$\lim_{z \to \infty} \Pr_{(a^n,b^n)} \left(p^n \in \left\{ p \in [0, \infty) : \frac{\Pr_{(a^n,b^n)}(p|h)}{\Pr_{(a^n,b^n)}(p|l)} \in [0, \epsilon) \cup (1/\epsilon, \infty)) \right\} \right) = 1.$$

For more intuition, suppose that the number of bidders n is large. In this case, the law of large numbers and strict MLRP jointly imply that the n signals received by the buyers convey highly precise information about the state of the world, ω, i.e., if an outside observer were to see all n signals, she would almost surely learn the state. Again from the perspective of an outside observer, the market price is said to aggregate information if one can learn the state by simply observing the price instead of observing all n signals, i.e., if the price is a "sufficient statistic".

Definition 4. Let $\theta_h^{a_m}$ and $\theta_l^{a_m}$ denote the signals such that

$$\int_{\theta_\omega^{a_m}}^1 a(\theta) f(\theta|\omega)\, d\theta = \kappa_m.$$

In words, $\theta_h^{a_m}$ is the signal such that the expected number of bidders with signals that exceed $\theta_h^{a_m}$ exactly equals the number of objects that are for sale in market m. Thus, type $\theta_h^{a_m}$ is the last bidder who would win an object in market m in a large auction if all bidders used a strictly increasing bidding function.

Definition 5. For any set $A \subset [0,1]$, denote the indicator function for A by $I_A(\theta)$ and let $\theta_\omega^m = \theta_\omega^{a_m}$ where $a_m = I_{[0,1]}$.

Intuitively, type θ_h^r is the last bidder who would win an object in market r in a large auction if all bidders used a strictly increasing bidding function and all bidders selected market r.

Define by $Y^n(k)$ the k$^{\text{th}}$ highest signal out of n signals, i.e., the k$^{\text{th}}$ order statistic, and for the case with multiple markets, let $Y_m^n(k)$ denote the k$^{\text{th}}$ highest signal in market m. Let

$$\beta^n(\theta) = E[v_\omega | Y^{n-1}(k) = \theta, \theta] = \Pr[h | Y^{n-1}(k) = \theta, \theta], \qquad (2..1)$$

that is, $\beta^n(\theta)$ is a bidder's expected value conditional on the event that her signal is equal to θ and that the k$^{\text{th}}$ highest signal received among all bidders except herself is also equal to θ. The second equality follows because $v_h = 1$ and $v_l = 0$. Pesendorfer and Swinkels (1997) showed that in the unique symmetric equilibrium of any auction with n bidders and k objects, a player who receives signal θ submits a bid equal to $\beta^n(\theta)$. In this unique symmetric equilibrium, each player bids her value conditional on being "pivotal", i.e., her valuation for the good conditional on the auction price being equal to her own bid under the assumption that all bidders use a symmetric, strictly monotone bidding strategy.

Definition 6. We refer to the function $\beta_m^n(\theta) = E[v_\omega | Y_m^{n-1}(k) = \theta, \theta]$ as the Pesendorfer and Swinkels bidding function.

Let $u(m, b(\theta')|\theta)$ denote the payoff to type θ from bidding as θ' in market m and let $u(m|\theta) = u(m, b(\theta)|\theta)$ in an equilibrium where player use symmetric bidding strategy $b(\cdot)$.

2.1.2. Information aggregation and gains from trade. In the first main result, we focus on an environment with a single active market (i. e., $\kappa_s = 0$ and $\kappa_r > 0$ in the notation introduced earlier) and show that information is aggregated if and only if the gains from trade are certain in this market, i.e., if this market is free of frictions. We will first present this relatively straight forward result because (1) it will provide the background for why information is aggregated with certain gains from trade and (2) it will form the basis for the main results that we will describe in the next section.

Theorem 1. *Assume that strict MLRP is satisfied, $\kappa_s = 0$, and $\kappa_r > 0$. Information is aggregated in market r if and only if $c_r = 0$.*

The proof is in the appendix. We will first informally argue that information is aggregated if the gains from trade are certain and $c_r = 0$. In this case, all bidders will choose to participate in the auction market r with probability one. This is because they can ensure positive expected payoff by submitting a bid arbitrarily close to zero. Therefore, the auction in market r is a standard auction with n bidders and $k = n\kappa_r$ objects. Pesendorfer and Swinkels (1997) showed that in the unique symmetric equilibrium of any auction with n bidders and k objects, a player who receives signal θ submits a bid equal to $\beta^n(\theta)$.

The function $\beta^n(\theta)$ is strictly increasing in θ because the signal distribution satisfies strict MLRP. The key observation that delivers information aggregation is that $\lim_{n\to\infty} \beta^n(\theta_h^r) = 1$ and $\lim_{n\to\infty} \beta^n(\theta_l^r) = 0$, i.e., prices converge to 1 and 0 in states h and l, respectively. This is a consequence of the "competition effect:" for any given bid, there are more bidders willing to submit an even higher bid in state h than in l because (1) bidders with higher signals submit higher bids (Lemma 1) and (2) there are more bidders who receive higher signals in the high state of the world (MLRP). In a large auction, a proportion κ_r of bids exceed $\beta^n(\theta_h^r)$ in state h and because of the competition effect the proportion of bids that exceed $\beta^n(\theta_h^r)$ is strictly less than κ_r in state l. Thus, type θ_h^r is willing to submit a bid arbitrarily close to one because she realizes that it is arbitrarily more likely that the market clears at her bid in state h. A similar reason implies that it is arbitrarily more likely that the market clears at type θ_l^r's bid in state l and hence type θ_l^r bids zero.

The law of large number tells us that the equilibrium prices in state h and l are almost surely equal to the bids of type θ_h^r and θ_l^r, respectively. As the bids of θ_h^r and θ_l^r are distinct from each other, the auction price reveals which of these types won the last unit and thus was pivotal in determining the price. Therefore, information is aggregated.

We now turn attention to the case of $c_r > 0$. In this case, bidders who bid in market r lose money in state l because the price they pay is at least $c_r > v_l = 0$. Therefore, such bidders must make strictly positive profits in state h to compensate for these losses. However, if bidding in market r provides non-negative profits for some type θ', then profits in market r must be positive for all types who put more weight on state h than θ', i.e., for all $\theta > \theta'$. This line of reasoning implies that there is a cutoff type $\hat{\theta}$ such that all types $\theta > \hat{\theta}$ choose market r with probability one, i.e., participation in auction r has a cut-off structure.

This cutoff structure for the types participating in the auction implies, as in Pesendorfer and Swinkels (1997), that there is a unique symmetric equilibrium bidding strategy given by $\beta^n(\theta)$. Following the same argument as in the case for $c_r = 0$, we can conclude that the bid of type θ_h^r determines the price in state h if $\theta_h^r > \hat{\theta}$. In this case, the price is

equal to one because $\lim_{n\to\infty} \beta^n(\theta_h^r) = 1$. On the other hand, if $\theta_h^r < \hat{\theta}$, then the number of goods exceeds the number of bidder in state h with probability one. Therefore, the price is equal to c_r. Hence, the price in state h is either equal to one or c_r.

The cutoff type $\hat{\theta}$ must be exactly indifferent between bidding in the auction and not bidding. Moreover, the price in state l is at least the minimum price c_r. Therefore,

$$\Pr(h|\hat{\theta})[\lim_n \Pr^n(p = c_r|h)(1 - c_r)] - \Pr(l|\hat{\theta})c_r \geq 0.$$

The equation displayed above shows that the probability that the price is equal to c_r in state h must strictly exceed zero, or more precisely, $\lim_n \Pr^n(p = c_r|h) \geq \frac{\Pr(l|\hat{\theta})}{\Pr(h|\hat{\theta})} \frac{c_r}{1-c_r} > 0$. For this to occur, it must be the case that $\hat{\theta} = \theta_h^r$. But, the MLRP assumption implies that $1 - F(\hat{\theta}|l) < 1 - F(\hat{\theta}|h) = 1 - F(\theta_h^r|h) = \kappa_r$. Therefore, there are more goods than bidders in state l and thus the price is equal to c_r with probability one in in state l. However, the fact that the price is equal to c_r with positive probability in both states implies that information is not aggregated.

2.1.3. Frictions, market selection, and the failure of information aggregation. The previous subsection argued that information is not aggregated in a market if the gains from trade are uncertain because of a market friction resulting from a minimum price. Typically, however, there are many active markets, some of which may not be burdened by frictions. If the state is not revealed by the price in one market, it might be revealed by the price in another, frictionless market. For example, information might not be aggregated in the primary auction market for government bonds because the treasury intervenes in this market by setting a reserve price. But, nevertheless, information pertaining to bond returns could then be learnt from the bond prices in a secondary market which is free from such frictions. In this subsection, I turn attention to such issues of information aggregation by focusing on multiple, linked markets.

In particular, suppose that there are two active auction markets, i.e, $\kappa_s > 0$ and $\kappa_r > 0$, and the gains from trade are certain in market s but uncertain in market r, i.e., $c_s = 0$ and $c_r \in (0, 1)$. Can one, in such an environment, learn the state by observing the prices in both of these markets? Below I will argue that information is not aggregated in any equilibrium in either market if the gains from trade are uncertain in market r and if a certain "selection effect" dominates a "competition effect" both of which I describe below.

In the auctions under consideration, for any given bid, there are many more bidders willing to submit an even higher bid in the high state than in the low state. This is because bidders with higher signals submit higher bids (by Lemma 1) and because there are many more bidders who receive higher signals in the high state of the world (by the

MLRP assumption). We refer to this force as the "competition effect". The competition effect is the primary driver of information aggregation in a frictionless auction market: in the high state it drives price to the good's value which is equal to one.

Uncertain gains from trade in market r implies that optimistic bidders, i.e., those bidders who receive higher signals, are more likely to opt-for the riskier prospects in market r in equilibrium. In contrast, pessimistic traders are more likely to prefer relative safety in market s. We refer to this equilibrium phenomenon as the "selection effect." This selection effect is a countervailing economic force that works against the competition effect and therefore against information aggregation. As a consequence of the selection effect, for any given bid, there can be more bidders willing to submit an even higher bid in the bad state of the world in market s. This is because even though there are more bidders with high signals in state h and these bidders are willing to submit higher bids, bidders with higher signals end up choosing the more risky market r.

Below we formally state the assumption that the selection effect dominates the competition affect at the bids of the pivotal types.

Definition 7. Let $\theta_r = \max\{\theta^*, \theta_h^r\}$ where $\theta^* = \min\{\theta : \Pr(h|\theta) \geq c_r\}$ and $\theta^* = 1$ if $\{\theta : \Pr(h|\theta) \geq c_r\} = \emptyset$.

Recall that in an arbitrarily large market, the prices in states l and h are determined by the bids of $\theta_l^{a_s}$ and $\theta_h^{a_s}$ if the bidder select their market according to a_s.

Assumption 1. *Suppose that only types in $[0, \theta_r]$ select market s or $a_s(\theta) = I_{[0,\theta_r]}(\theta)$. Then the signal $\theta_l^{a_s}$ exceeds signal $\theta_h^{a_s}$, i.e., $\theta_l^{a_s} > \theta_h^{a_s}$.*

If bidders with signals that exceed θ_r opt for market r, then in a large market s, the bids of $\theta_l^{a_s}$ and $\theta_h^{a_s}$ are pivotal in states l and h, respectively. In other work, the bids of types $\theta_l^{a_s}$ and $\theta_h^{a_s}$ determine the prices in the two states. The above assumption requires that the selection effect dominates the competition effect exactly at the bids of $\theta_l^{a_s}$ and $\theta_h^{a_s}$. This is because $\theta_l^{a_s} > \theta_h^{a_s}$ and monotonic bidding together imply that there are more than $n\kappa_s$ bidders in expectation who submit at bid that exceeds the bid of type $\theta_h^{a_s}$ in state l. However, the expected number of bidders who submit a bid that exceeds the bid of type $\theta_h^{a_s}$ in state h is equal to $n\kappa_s$ by definition.

To get a better sense of the restriction imposed by Assumption 1, suppose that c_r is less that $\Pr(h|\theta_h^r)$ so that $\theta_r = \theta_h^r$ in Definition 7. In this case, for any signal distribution, one can choose $\kappa_s + \kappa_r$ sufficiently close to one such that the above assumption is satisfied. Explained differently, if the market tightness is close to one, i.e., if the market is close to being balanced, then the selection effect will dominate the competition effect in market s at the bids of the pivotal bidders.

The following result shows that if the selection effect dominates the competition effect exactly at the pivotal bids (the bids of $\theta_l^{a_s}$ and $\theta_h^{a_s}$), then information is not aggregated in either market. Therefore, the result tells us that if frictions hinder information aggregation in one market, then market selection will hinder information aggregation in any linked market also.

Theorem 2. *Suppose that strict MLRP and Assumption 1 are satisfied. If the gains from trade are uncertain in market r (i.e., if $c_r > 0$), then information aggregation fails in both markets.*

Intuitively, information aggregation fails in market s even though gains from trade are certain in this market because bidders with lower signals, i.e., more pessimistic bidders, self select into market s. This, in turn, implies that there are many more bidders who are willing to pay at least the bid of $\theta_h^{a_s}$ in the bad state, i.e., the demand for goods is higher in the bad state. The fact that the demand is high at the bid of $\theta_h^{a_s}$ exactly when people do not value the goods implies that market s cannot clear properly. Thus, under Assumption 1 information is not aggregated in this market because the selection effect overwhelms the competition effect.

In order to convey more precisely why this result holds, I will restrict attention to the case where c_r is less than $\Pr(h|\theta_h^r)$ so that $\theta_r = \theta_h^r$ in Definition 7. I will argue by contradiction that information cannot be aggregated in market s. On the way to a contradiction, assume that information is aggregated in market s. The first step of the argument establishes that the payoff of bidders in market s converges to zero: The initial assumption that information is aggregated in market s means that the price will converge to one in state h and zero in state l. Therefore, the payoff of anybody who bids in market s must be equal to zero.

The next step in the argument entails showing that the market selection function has a cutoff structure with all types that exceed θ_h^r opting for market r: At the limit, bidders face a choice between market s, where their payoff is equal to zero, and market r, where their payoff is strictly negative in state l. Therefore, the choice is essentially identical to the choice that bidders faced with only one active market as in Proposition 1. Therefore, the argument we provided in the previous subsection implies that all types that exceed θ_h^r opt for market r.

The final step in the argument concludes that all types that exceed θ_h^r opting for market r, Assumption 1 and information aggregation in market s taken together are incompatible with bidders behaving according to a monotone bidding function as we claimed that they must in Lemma 1. More precisely, denote by a the limit of the market selection functions, i.e., $a_r(\theta) = \lim_{n \to \infty} a_r^n(\theta)$. Because all types that exceed θ_h^r opt for market r we have $a_r(\theta) = 1$ for any $\theta > \theta_h^r$. However, $a_r(\theta) = 1$ for any $\theta > \theta_h^r$ and Assumption 1 together

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imply that $\theta_l^a > \theta_h^a$. The law of large number tells us that the equilibrium prices in state h and l are almost surely equal to the bids of type θ_h^a and θ_l^a, respectively. Our assumption of information aggregation in market s implies that $\lim_{n\to\infty} b_s^n(\theta_h^a) = 1$ and $\lim_{n\to\infty} b_s^n(\theta_l^a) = 0$. However, this provides the contradiction that proves the result. The findings that $\lim_{n\to\infty} b_s^n(\theta_h^a) = 1$, $\lim_{n\to\infty} b_s^n(\theta_l^a) = 0$, and $\theta_l^a > \theta_h^a$ together contradict that the bidding function is monotone for all sufficiently large n.

2.1.4. Frictionless markets and Information aggregation. The final result that we will present shows that if there are no frictions in either market, then information aggregated in both markets across all equilibria. The delicacy of this result is to show that the selection effect can never dominate the competition effect for any equilibrium market selection function if there are no frictions.

Theorem 3. *Suppose that strict MLRP is satisfied. If the gains from trade are certain in both markets ($c_s = c_r = 0$), then information is aggregated in both markets.*

The argument for this theorem is based on the repeated application of the following lemma. The lemma shows that if the competition effect dominates the selection effect at the bids of the pivotal bidders in market m, then information is aggregated in m.

Lemma 2. *Suppose that strict MLRP is satisfied. Suppose that $\{b_m^n\}$ is a sequence of equilibrium bidding functions for market m where the participant of the auctions are determined by an arbitrary sequence of market selection functions $\{a^n\}$ and let $a_m = \lim_n a_m^n(\theta)$. If $\theta_h^{a_m} > \theta_l^{a_m}$, then information is aggregated in market m*

The argument for Proposition 3 is by contradiction. Assume that information is not aggregated in market r. First assume that $\theta_h^{a_r} > \theta_l^{a_r}$. However, if $\theta_h^{a_r} > \theta_l^{a_r}$, then the competitive effect dominates the selection effect at the bids of the pivotal types in market r. This would then imply, by Lemma 2, that information is aggregated in market r. Thus, for information aggregation to fail in market r, it must be the case that $\theta_h^{a_r} \le \theta_l^{a_r}$. If $\theta_h^{a_r} \le \theta_l^{a_r}$ in market r, then strict MLRP implies that $\theta_h^{a_s} > \theta_l^{a_s}$ in market s. Therefore, the competitive effect dominates at the pivotal types in market s. This establishes, by Lemma 2, that information is aggregated in market s. If information is aggregated in market s, then the payoff to bidders in market s is equal to zero. The fact that information is not aggregated in market r means that the price is not equal to zero with probability one in state l. Thus, the payoff in market r is strictly negative in state l. But then, an argument similar to the ones provided for Propositions 1 and 2 implies that the market selection function has a cutoff structure and all types that exceed $\theta_h^{a_r}$ opt for market r. Consequently, applying strict MLRP we establish that $1 - F(\theta_h^{a_r}|h) > 1 - F(\theta_h^{a_r}|l)$. This, however, contradicts the fact that $\theta_h^{a_r} \le \theta_l^{a_r}$ because $1 - F(\theta_h^{a_r}|h) = \kappa_r$ implies that $1 - F(\theta_h^{a_r}|l) < \kappa_r$ and thus $\theta_h^{a_r} > \theta_l^{a_r}$.

3. Multiple markets and states.

In this section we explore whether the results extend to the case of multiple markets and states under the following assumption that the set of states is sufficiently rich with respect to the signal space. In this section, we suppose that the state space is $[0,1]$ and $v(\omega, m) = \omega$ for each $\omega \in [0,1]$ and each market m.

Assumption 2 (Rich State Space). *The signal space is finite and for each $\theta > 0$ and $\epsilon > 0$ there exists ω and ω' such that $F(\theta|\omega) > 1 - \epsilon$ and $F(\theta|\omega') < \epsilon$.*

For example this assumption is satisfied if there are two signals $\theta \in \{l, h\}$, a continuum of states $\omega \in [0,1]$, and $\Pr(h|\omega) = \omega$. This is the example presented in Pesendorfer and Swinkels (1997, Section 3.3).

Conjecture 1. *Suppose that strict MLRP and Assumption 2 are satisfied. If $c_m = 0$ in all markets m, then information is aggregated in all n markets. There exists c^* such that if $c_r \in (0, c^*)$ for some market r, then information is not aggregated in any market m.*

4. Equilibrium Existence

In this section we construct a simple equilibrium which exists for certain parameter values. In particular, we will construct an equilibrium where all types $\theta \in [0, \theta_h^r]$ opt for market s and submit the same pooling bid all others choose market r. In such a situation, the probability of winning at object at the pooling bid will converge to

$$\frac{\kappa_s}{F(\theta_h^r|\omega)}$$

in state ω as the number of bidders grows large. Similarly, the probability of failing to win an object will converge to

$$\frac{F(\theta_h^r|\omega) - \kappa_s}{\kappa_s}.$$

Therefore, the likelihoods of state h condition on θ and winning or failing to win an object converge to

$$l(\theta)\frac{F(\theta_h^r|H)}{F(\theta_h^r|L)}$$

and

$$l(\theta)\left(\frac{F(\theta_h^r|h) - \kappa_s}{F(\theta_h^r|l) - \kappa_s}\right)\frac{F(\theta_h^r|l)}{F(\theta_h^r|h)},$$

respectively, i.e.,

$$\lim^n l^n(h|win, \theta) = l(h|win, \theta) = l(\theta)\frac{F(\theta_h^r|h)}{F(\theta_h^r|l)}$$

and

$$l(h|lose, \theta) = l(\theta)\left(\frac{F(\theta_h^r|h) - \kappa_s}{F(\theta_h^r|l) - \kappa_s}\right)\frac{F(\theta_h^r|l)}{F(\theta_h^r|h)},$$

respectively.

We will construct such an equilibrium under the following two assumptions.

Assumption 3. *Suppose that $l(h|win, 0) > l(h|lose, \theta_h^r)$.*

This assumption ensures that type 0 and type θ_h^r is willing to pool at the same pooling at some possible price.

Assumption 4. *Suppose that $c > \frac{l(h|win,0)}{1+l(h|win,0)} \frac{\kappa_s}{F(\theta_h^r|l)}$.*

This assumption ensures that the market selection function indeed has the cut-off structure that we are using in constructing the equilibrium.

The following theorem summarizes the equilibrium construction. The proof of the theorem is in the appendix.

Theorem 4. *Suppose that signals satisfy strict MLRP, $E[v(r, \omega) - c|\theta_h^r] > 0$ and Assumptions 3 and 4 are satisfied. There is a sequence of equilibria where all agents who choose to go to market s submit the same bid b^*, i.e., $b_n^s(\theta) = b^*$ for all θ and the price in market s is equal to b_n^* in both states. In this sequence of equilibria, the price in market r converges to a random variable which is equal to c_r with probability one in state l; and is equal to 1 and c_r with probability $1 - x$ and $x > 0$, respectively in state h.*

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A   The Appendix: Omitted proofs

Lemma 3. *Assume that signals satisfy strict MLRP. For any $\epsilon > 0$, there exists a $\nu > 0$ such that*

$$u(m|\theta) \geq u(m|\theta') + \nu(u(m, b(\theta')|h) - u(m, b(\theta')|l))$$

for any θ' and any $\theta \geq \theta' + \epsilon$. Therefore, if $u(m|\theta') \geq x > 0$, then $u(m|\theta) > x$ for any $\theta > \theta'$.

Proof. Note that $u(m, b(\theta')|\theta', h) = u(m, b(\theta')|h)$. This is because conditioning on one's signal does not add any information beyond the state. For any $\epsilon > 0$, let $\nu = \min_{\theta \in [0, 1-\epsilon]} \Pr[h|\theta + \epsilon] - \Pr(h|\theta) > 0$. This minimization problem is well defined by the Weierstrass Theorem because we are minimizing a continuous function over a compact set. Also, the minimized value is strictly positive because of strict MLRP. The following inequalities delivers the result:

$$u(m|\theta') = u(m, b(\theta')|l) \Pr(l|\theta') + u(m, b(\theta')|h) \Pr(h|\theta')$$
$$u(m|\theta) \geq u(m, b(\theta')|l) \Pr(l|\theta) + u(m, b(\theta')|h) \Pr(h|\theta)$$
$$u(m|\theta) \geq u(m|\theta') + \nu(u(m, b(\theta')|h) - u(m, b(\theta')|l))$$

where the last inequality follows from taking the difference between the first two inequalities and noting that $p(h|\theta) \geq p(h|\theta') + \nu$. The second part of the lemma follows from the fact that $u(m|\theta') \geq x > 0$ implies that $u(m, b(\theta')|h) > 0$ and from the fact that $u(m, b(\theta')|l)) \leq 0$ by definition. $\qquad \square$

Lemma 4. *Assume that strict MLRP is satisfied Let $\Pr_\sigma(r, win|\theta)$ denote the probability that type θ wins an object from the auction in market r in an equilibrium σ.*

i. Let $U = \{\theta : \Pr_\sigma(r, win|\theta) > 0\}$ and $\hat{\theta} = \inf U$. If $u(s|\theta) = 0$ for all θ, then

$$a(\theta) = \begin{cases} 0 & if \ \theta > \hat{\theta}, \\ 1 & if \ \theta < \hat{\theta}. \end{cases}$$

ii. Let $U = \{\theta : \lim_n \Pr^n(r, win|\theta) > 0\}$ and $\hat{\theta} = \inf U$. Suppose that $\lim_n u^n(s|\theta) = 0$. If $\theta > \hat{\theta}$, then there exists an $n(\theta)$ such that $a^n(\theta) = 0$ for all $n > n(\theta)$.

Proof. We begin by proving item i. In order to win a good, any type θ must pick market r with positive probability, i.e., $a(\theta) < 1$ for all $\theta \in U$.[5] Also, note that if $\Pr_\sigma(win|r, \theta) > 0$

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[5]Note that $\Pr_\sigma(r, win|\theta) = \Pr_\sigma(win|r, \theta)a(\theta)$.

for some type θ, then $\Pr_\sigma(win|r,\theta,l) > 0$, i.e., the probability of winning an object in state l is also strictly positive for this type θ. However, the fact that $\Pr_\sigma(win|r,\theta,l) > 0$ implies that $u(r|\theta,l) < 0$ because in state l the object's value is equal to zero but the price is at least $c > 0$. Therefore, because it is individual rational for all $\theta \in U$ to choose $a(\theta) < 1$ it must be that $u(r|\theta,h) > 0$ for all $\theta \in U$. However, then Lemma 3 implies that if $\theta' \in U$, then $u(r|\theta) > 0$ for all $\theta > \theta'$. Therefore, if $\theta' \in U$, then $a(\theta) = 0$ for all $\theta > \theta'$, i.e., if $\theta' \in U$, then$[\theta',1] \subset U$.

The argument above implies that the set U is an interval. Therefore, if $\theta > \hat{\theta}$, then $(\hat{\theta},\theta] \subset U$ because otherwise $\hat{\theta}$ would not be the infimum of the interval U. Hence, if $\theta > \hat{\theta}$, then $a(\theta) = 0$ because there is $\theta' \in U$ such that $\theta' \in (\hat{\theta},\theta)$. Note that if $\theta > 0$ and $a(\theta) < 1$, then $\Pr_\sigma(win|r,\theta) > 0$. Therefore, we conclude that if $\theta' > 0$ and $a(\theta') < 1$, then $a(\theta) = 0$ for all $\theta > \theta'$. However, this implies that if $\theta < \hat{\theta}$, then $a(\theta) = 1$ because otherwise we would have $a(\theta) = 0$ for all $\theta \in (\theta',\hat{\theta})$ which would contradict that $\hat{\theta}$ is the infimum of U.

We now prove item ii. Pick $\theta' \in U$ and note that if $\lim_n \Pr^n(win,r|\theta') > 0$, then $\lim_n \Pr^n(win|r,\theta') > 0$ and $\lim_n \Pr^n(win|r,\theta',l) > 0$, i.e., the probability of winning an object in state l is also strictly positive for this type θ'. However, $\Pr^n(win|r,\theta',l) > 0$ implies that $u^n(r|\theta',l) \leq -c\Pr^n(win|r,\theta',l) < 0$ because in state l the object's value is equal to zero but the price is at least $c > 0$ for any n. Because it is individual rational for $\theta' \in U$ to choose $a^n(\theta') < 1$ it must be that $u^n(r|\theta') \geq 0$. Therefore, Lemma 3 implies that $u^n(r|\theta) \geq u^n(r|\theta') + \nu c\Pr^n(win|r,\theta',l)$ where ν is defined by taking $\epsilon = \theta - \theta'$. Taking limits shows that

$$\lim_n u^n(r|\theta) \geq \lim_n u^n(r|\theta') + \nu c\lim_n \Pr^n(win|r,\theta',l)$$
$$= \nu c\lim_n \Pr^n(win|r,\theta',l) > 0.$$

Therefore, if $\theta' \in U$, then $a(\theta) = 1$ for all $\theta > \theta'$, i.e., if $\theta' \in U$, then$[\theta',1] \subset U$. Moreover, if $\theta \notin U$ and if $\theta' \leq \theta$, then $\theta' \notin U$. Thus, if $\theta > \hat{\theta}$, then $(\hat{\theta},\theta] \subset U$ because otherwise $\hat{\theta}$ would not be the infimum of the interval U. Hence, if $\theta > \hat{\theta}$, then $a(\theta) = 0$ because there is $\theta' \in U$ such that $\theta' \in (\hat{\theta},\theta)$. $\qquad\square$

Proof of Theorem 1. If $c = 0$, then Pesendorfer and Swinkels (1997) implies that information is aggregated in any sequence of symmetric equilibria.

We will show that if $c > 0$, then information is not aggregated. Suppose that $\theta_r < 1$ and note that if $\theta_r = 1$, then nobody would bid in the auction and therefore information is not aggregated by definition.

Let $\hat{\theta}$ be defined as in Lemma 4. Note that $u(s|\theta) = 0$ because there are no goods in market s. Therefore, Lemma 4 item (ii) implies that $a(\theta) = 0$ for all $\theta > \hat{\theta}$, i.e., all such

types come to market r with probability one.

Claim 1. The bidding function b_r is strictly monotone on $[\hat{\theta}, 1]$.

Proof. We need to show that there is no pooling region $P = (\theta', \theta'') \subset (\hat{\theta}, 1]$ such that $b_r(\theta) = b$ for all $\theta \in P$. Recall that $Y(k)$ denotes the kth (where $k = n\kappa_r$) order statistic. MLRP implies that the random variables $\{Y(1), ..., Y(n), \theta, \omega\}$ are affiliated (see Milgrom and Weber (1982, Theorem 2). We can write the probability of losing at the pooling bid as a function $f_P((Y(1), ..., Y(n)))$ of the order statistics. Note that this is an increasing function of the vector of order statistics. Affiliation of the random variables $\{Y(1), ..., Y(n), \theta, \omega\}$ and the fact that f_P is increasing implies that $\Pr\{win|Y(k) \in P, \theta, \omega\} = E[f_P(Y(1), ..., Y(n))|Y(k) \in P, \theta, \omega]$ is increasing in ω (see Milgrom and Weber (1982, Theorem 5)), i.e., losing is a signal in favor of h. In other words $\Pr\{win|Y(k) \in P, \theta, h\} \leq \Pr\{win|Y(k) \in P, \theta, l\}$. The difference between bidding slightly above the pooling bid and bidding the pooling bid is as follows:

$$\lim_{\epsilon \to 0}(u(r, b(\theta) = b + \epsilon|\theta) - u(r, b(\theta) = b|\theta)) = \Pr\{h|Y(k) \in P, \theta\} - b$$
$$- \Pr\{h|Y(k) \in P, \theta\} \Pr\{win|Y(k) \in P, \theta, h\}(1 - b)$$
$$+ \Pr\{l|Y(k) \in P, \theta\} \Pr\{win|Y(k) \in P, \theta, l\}b$$

Note that the fact that $\Pr\{win|Y(k) \in P, \theta, h\} \leq \Pr\{win|Y(k) \in P, \theta, l\}$ implies that if $\Pr\{h|Y(k) \in P, \theta\} - b > 0$ then the above difference is negative, i.e., bidding slightly above the pooling bid is better than pooling the pooling bid. Moreover, if $\Pr\{h|Y(k) \in P, \theta\} - b < 0$ then bidding below the pooling bid is better than bidding the pooling bid. Also, note that if $\Pr\{h|Y(k) \in P, \theta'\} - b = 0$ then $\Pr\{h|Y(k) \in P, \theta''\} - b > 0$ by strict MLRP and because $\theta' < \theta''$. Also, similarly if $\Pr\{h|Y(k) \in P, \theta''\} - b = 0$ then $\Pr\{h|Y(k) \in P, \theta'\} - b < 0$. Therefore, bidding the pooling bid is never optimal and hence pooling is not possible. $\square$

Claim 2. The fact that $b_r(\theta)$ is strictly increasing implies that $b_r(\theta) = \beta(\theta) = E[v|Y(k) = \theta, \theta]$.

Proof. Follows from Pesendorfer and Swinkels (1997, Proposition 1). $\square$

Claim 3. If then $\theta \geq \theta_r$, then $b_r^n(\theta) = \beta^n(\theta) \to 1$.

Proof. This follows from the fact that $E^n[v|Y(k) = \theta, \theta] \to 1$ for any $\theta \geq \theta_r$. To see this first note that

$$E^n[v|Y(k) = \theta, \theta] = \frac{l^n(Y(k) = \theta, \theta)}{1 + l^n(Y(k) = \theta, \theta)}$$

where

$$l^n(Y(k) = \theta, \theta) = \frac{\pi}{1-\pi} \frac{f(\theta|h)^2}{f(\theta|l)^2} \frac{F(\theta|l)(1-F(\theta|l))}{F(\theta|h)(1-F(\theta|h))} \left(\frac{F(\theta|h)^{1-\kappa_r}(1-F(\theta|h))^{\kappa_r}}{F(\theta|l)^{1-\kappa_r}(1-F(\theta|l))^{\kappa_r}} \right)^n.$$

However, if $\theta \geq \theta_r$, then $\frac{F(\theta|h)^{1-\kappa_r}(1-F(\theta|h))^{\kappa_r}}{F(\theta|l)^{1-\kappa_r}(1-F(\theta|l))^{\kappa_r}} > 1$ which[6] implies that $l^n(Y(k) = \theta, \theta) \to \infty$ and thus $E^n[v|Y(k) = \theta, \theta] \to 1$. $\qquad\square$

Claim 4. $\lim_n \hat{\theta}^n = \theta_r$.

Proof. If $\lim_n \hat{\theta}^n \leq \theta_r$, then $b_r^n(\theta) \to 1$ and $\lim_n u^n(r|\theta) < 0$ for any $\theta > \theta_r$. If on the other hand $\lim_n \hat{\theta}^n > \theta_r$, then the number of bidders in the market is less than the number of goods with probability one. Therefore, the price is equal to c with probability one in both states. Therefore, $\lim_n u^n(r|\theta_r) > 0$ by definition. However, this contradicts the assertion that $\lim_n \hat{\theta}^n > \theta_r$. $\qquad\square$

Claim 4 and the law of large numbers implies that the limit price is equal to c with probability one in state l because $1 - F(\theta_r|l) < \kappa_r$. Also, the price in state h is either equal to 1 or c because the bid of all θ converges to one. However, the individual rationality implies that the price must be less than 1 with positive probability in state h also. In particular, individual rationality for θ_r implies that

$$\Pr(h|\theta_r)[\lim_n \Pr^n(p = c|h)(1 - c)] - \Pr(l|\theta_r) = 0.$$

Therefore, the probability that the price is equal to c in state h $\lim_n \Pr^n(p = c|h) = \frac{1}{l(\theta_r)} \frac{1}{1-c}$ and the price is equal to one with the remaining probability, i.e., $\lim_n \Pr^n(p \geq 1 - \epsilon|h) = 1 - \frac{1}{l(\theta_r)} \frac{1}{1-c}$ for any ϵ. Note that $\frac{1}{l(\theta_r)} \frac{1}{1-c} \in (0, 1)$. This shows that information is not aggregated in any equilibrium sequence. $\qquad\square$

Proof of Theorem 2. Assume that $p_s^n \to v(s, \cdot)$ in probability. Note that $p_s^n \to v(s, \cdot)$ implies that $u^n(s|\theta) \to 0$ for all θ. We will show that this assumption leads to a contradiction. Let $u = \lim_{n \to \infty} u^n$. This limit exists (possibly on a subsequence) as u is a monotone function of θ.

Let U and $\hat{\theta}$ denote the set and type as defined in Lemma (4), item ii.

Step 1. We will first argue that $\hat{\theta} \leq \theta_r$ and therefore if $\theta > \theta_r \geq \hat{\theta}$, then $a^n(\theta) = 0$ for all n sufficiently large.

Assume not, i.e., $\hat{\theta} > \theta_r$. Note that $1 - F(\theta_r|h) < \kappa_r$ by definition and therefore, $1 - F(\hat{\theta}|h) < \kappa_r$. This implies that any agent who comes to market r and submits the

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[6]The expression $F(\theta|l)^{1-\kappa_r}(1 - F(\theta|l))^{\kappa_r}$ is maximized at $1 - F(\theta|l) = \kappa_r$ and strictly decreasing after such a θ^*. The expression $F(\theta|h)^{1-\kappa_r}(1 - F(\theta|h))^{\kappa_r}$ is maximized at $1 - F(\theta|h) = \kappa_r$, i.e., at θ_r. MLRP implies that $\theta^* < \theta_r$. Thus $\frac{F(\theta_r|h)^{1-\kappa_r}(1-F(\theta_r|h))^{\kappa_r}}{F(\theta_r|l)^{1-\kappa_r}(1-F(\theta_r|l))^{\kappa_r}} > 1$

minimum bid of c will win an object with probability nearly equal to one in state h if n is large. Therefore, $\lim_n \Pr^n(win, r|\theta) = 0$ implies that $\lim_n a^n(\theta) = 1$ for $\theta < \hat{\theta}$. Also, for any $\theta \in (\theta_r, \hat{\theta})$ the profit from going to market r and bidding $b = c$, i.e., $u^n(r, c|\theta)$ is strictly positive because it ensures that such a type θ wins an object in state h at a price equal to c with probability close to one for sufficiently large n. But, $E[v(r, \omega) - c|\theta] > E[v(r, \omega) - c|\theta_r] \geq 0$ and therefore $u^n(r, c|\theta) > 0$ for sufficiently large n. However, this implies that $a^n(\theta) = 0$ and $\lim_n \Pr^n(win, r|\theta) > 0$ for all n sufficiently large, i.e., $\theta \in U$. This however contradicts that $\hat{\theta} > \theta_r$.

Step 2. $\theta_l = \lim_n \theta_l^{a^n} > \lim_n \theta_h^{a^n} = \theta_h$.

In step 1 we argued that $\lim a^n(\theta) = 0$ for all $\theta > \theta_r$. The definition of $\theta_h^{\chi_{[0,\theta_r]}}$ implies that $\theta_h \leq \theta_h^{\chi_{[0,\theta_r]}}$ because $\lim a^n(\theta) = 0$ for all $\theta > \theta_r$. Consider the following maximization problem

$$V = \max_{b:[0,\theta_r] \to [0,1]} \frac{\int_{[0,\theta_r]} b(\theta) f(\theta|h) d\theta}{\int_{[0,\theta_r]} b(\theta) f(\theta|l) d\theta}$$

$$s.t. \int_{[0,\theta_r]} b(\theta) f(\theta|h) d\theta = \kappa_s$$

Strict MLRP and the fact that $\int_{[\theta_h^{\chi_{[0,\theta_r]}}, \theta_r]} f(\theta|h) d\theta = \kappa_s$ together imply that the maximized value

$$V = \frac{\int_{[\theta_h^{\chi_{[0,\theta_r]}}, \theta_r]} f(\theta|h) d\theta}{\int_{[\theta_h^{\chi_{[0,\theta_r]}}, \theta_r]} f(\theta|l) d\theta}.$$

By Assumption 1 $\theta_h^{\chi_{[0,\theta_r]}} < \theta_l^{\chi_{[0,\theta_r]}}$ and therefore $V < 1$. We have,

$$\frac{\lim_n \int_{[\theta_h^{a^n}, \theta_r]} a^n(\theta) f(\theta|h) d\theta}{\lim_n \int_{[\theta_h^{a^n}, \theta_r]} a^n(\theta) f(\theta|l) d\theta} = \frac{\int_{[\theta_h, \theta_r]} \lim_n a^n(\theta) f(\theta|h) d\theta}{\int_{[\theta_h, \theta_r]} \lim_n a^n(\theta) f(\theta|l) d\theta} = \frac{\kappa_s}{\int_{[\theta_h, 1]} \lim_n a^n(\theta) f(\theta|l) d\theta}.$$

However,

$$\frac{\int_{[\theta_h, \theta_r]} \lim_n a^n(\theta) f(\theta|h) d\theta}{\int_{[\theta_h, \theta_r]} \lim_n a^n(\theta) f(\theta|l) d\theta} \leq V < 1$$

because $b(\theta) = \chi_{[\theta_h, \theta_r]}(\theta)(\lim_n a^n(\theta))$ is a feasible solution to the maximization problem. Thus,

$$\frac{\kappa_s}{\int_{[\theta_h, 1]} \lim_n a^n(\theta) f(\theta|l) d\theta} < 1.$$

This inequality implies that

$$\int_{[\theta_h, 1]} \lim_n a^n(\theta) f(\theta|l) d\theta > \kappa_s$$

and hence that $\theta_l > \theta_h$.

Step 3. $\lim_n \Pr(p_s^n < b_s^n(\theta_h^{a_n}) + \epsilon|h) = 1$ and $\lim_n \Pr(p_s^n > b_s^n(\theta_l^{a_n}) - \epsilon|l) = 1$.

This follows immediately from the weak law of large numbers and our assumption of information aggregation.

Our assumption that that $p_s^n \to 1$ and $p_s^n \to 0$ with probability one in states h and l, respectively, imply that $b_s^n(\theta_h^{a_n}) > 1 - \epsilon$ and $b_s^n(\theta_l^{a_n}) < \epsilon$ for all n sufficiently large. Therefore, $b_s^n(\theta_h^{a_n}) > b_s^n(\theta_l^{a_n})$ for all n sufficiently large. However, Step 2 implies that $\theta_l^{a_n} > \theta_h^{a_n}$ for sufficiently large n which contradicts the fact that b_s^n is increasing for all n. $\qquad\square$

Proof of Theorem 3. Assume that information is not aggregated in market r.

Step 1. Assume that $\theta_h^{a_r} > \theta_l^{a_r}$.

Then information is aggregated in r. Therefore, $\theta_h^{a_r} \le \theta_l^{a_r}$ and thus $\theta_h^{a_s} > \theta_l^{a_s}$.

Step 2. Information is aggregated in market s.

This is because $\theta_h^a > \theta_l^a$.

Step 3. The fact that information is not aggregated implies that if $\theta > \theta_l^{a_r}$ then $a_s(\theta) = 0$.

Step 4. If $a_s(\theta) = 0$ for all $\theta > \theta_l^{a_r}$, then $\theta_h^{a_r} > \theta_l^{a_r}$ which leads to a contradiction. $\quad\square$

Proof of Theorem 5. To be completed. $\qquad\square$

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