Is There Too Much Benchmarking in Asset Management?

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Abstract

We propose a model of asset management in which benchmarking arises endogenously, and analyze its unintended welfare consequences. Fund managers' portfolios are unobservable and they incur private costs in running them. Conditioning managers' compensation on a benchmark portfolio's performance partially protects them from risk, and thus boosts their incentives to invest in risky assets. In general equilibrium, these compensation contracts create an externality through their effect on asset prices. Benchmarking inflates asset prices and gives rise to crowded trades, thereby reducing the effectiveness of incentive contracts for others. Contracts chosen by fund investors diverge from socially optimal ones. A social planner, recognizing the crowding, opts for less benchmarking and less incentive provision. We also show that asset management costs are lower with socially optimal contracts, and the planner's benchmark-portfolio weights differ from the privately optimal ones.

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1 Introduction

Investors worldwide have delegated the investment of almost $100 trillion to asset management firms. Portfolio managers at these firms are invariably paid based on how their fund performs relative to a benchmark.[1] There is little academic research analyzing why the compensation contracts take this form and there is no standard explanation for this phenomenon. We provide a theoretical framework that offers an explanation for the common use of benchmarking in asset management. More importantly, we use this framework to assess the welfare implications of benchmarking and explore its unintended consequences.

To study these questions, we embed an optimal-contracting model into a general-equilibrium setting. We show that when the fund managers incur a private cost in managing portfolios, optimally designed contracts for the managers involve benchmarking. Because of this private cost, managers underinvest. Conditioning the managers' compensation on the performance of a benchmark portfolio partially protects them from risk and thus boosts their incentives to invest. In general equilibrium, the use of such incentive contracts creates a pecuniary externality through their effect on asset prices. Benchmarking inflates asset prices and reduces expected returns. This in turn reduces the marginal benefit of using incentive contracts for others. We show that a constrained social planner, who internalizes this externality, would opt for less incentive provision and less benchmarking.

Here is how our model works. Some agents in the economy—direct investors—manage their own money and others—fund investors—delegate their investment choice to fund (or portfolio) managers. All agents are risk averse. Critically, the managers' portfolios are unobservable to fund investors and the cost of managing a portfolio is private. The managers are paid based on incentive contracts designed by the fund investors.[2] We focus on linear contracts, which include a fixed salary, a fee for absolute performance, and potentially a fee for performance relative to a benchmark.

We assume that the managers can potentially generate superior returns (or "alpha") relative to those of the direct investors through various sophisticated strategies. These include lending securities, conserving on transactions costs (e.g., from crossing trades in-house or by obtaining favorable quotes from brokers) or providing liquidity (i.e., serving as a counterparty to liquidity demanders and earning a premium on such trades). While these

[1] For example, Ma, Tang, and Gómez (2019) report that around 80% of U.S. mutual funds explicitly base compensation on performance relative to a benchmark (usually a prospectus benchmark such as the S&P 500, Russell 2000, etc.).

[2] We abstract from the asset management firm and assume that the firm acts in the interest of the fund investors, so that effectively the fund investors directly control the compensation arrangements for the portfolio managers. This is consistent with the fund trustees having a fiduciary obligation to their investors.

activities augment returns, they are associated with a private cost for a portfolio manager. We assume the costs are increasing in the size of the fund's risky portfolio. The simplest way to justify these assumptions is to appeal to the time costs involved in the activities and to interpret the rising costs as reflecting the additional time required for managing a larger fund/portfolio. An alternative interpretation that we discuss in Section 3 is that the manager has to exert costly effort to augment returns.[3]

Fund investors design the manager's compensation contracts to incentivize the manager to take the risk associated with the sophisticated strategies. The presence of the private cost calls for a contract that rewards the manager based on fund performance and gives her a larger share of the returns than if risk sharing were all the contract was aimed at achieving. However, this element of the contract exposes the manager to additional risk (because stock returns are stochastic). This risk, if unmitigated, means that the manager will underinvest. Adding a benchmark to the contract partially protects the manager from this risk and therefore will be used by fund investors to improve the manager's incentives.

Our paper's main contribution is analyzing the unintended welfare consequences of benchmarking. When all fund investors use incentive contracts, they change the total demand for assets. In particular, benchmarking leads all managers to invest more in assets that are compatible with the return-augmenting strategies and in assets that are in their benchmarks. The managers' demand boosts prices of such assets and lowers their expected returns. In other words, benchmarking contracts give rise to crowded trades.

Importantly, individual fund investors in our model take asset prices as given and do not internalize the effects of contracts they design on equilibrium prices. Crowded trades resulting from the contract-induced incentives are a pecuniary externality. Because of the agency frictions, markets are incomplete, so this pecuniary externality leads to an inefficiency. Specifically, the use of benchmarking contracts by a group of investors reduces the effectiveness of contracts designed by other investors through crowded trades. This happens because asset prices enter the fund managers' incentive constraints. Each manager still has to incur the full private cost of managing assets but the benefits of doing so are reduced because of the crowded trades.

In light of this, a natural question to ask is how does the incentive contract chosen by a social planner, who is subject to the same restrictions as individual investors but recognizes the effect of contracts on prices, differ from the privately optimal one? We show

[3] We show in Appendix C that our main insights carry over to this case, however, we cannot get closed-form solutions for that version of the model. So in the main text we proceed with a simpler model that allows us to focus on the role of the essential friction—the unobservability of the portfolio choice—that is responsible for our main results.

that individual investors underestimate the cost of incentive provision relative to the social planner, who internalizes the negative externality of incentive contracts. As a result, the planner opts for less incentive provision. Specifically, we show that both the performance sensitivity ("skin in the game") as well as the level of benchmarking are lower in the socially optimal contract than in the privately optimal one. This ameliorates the price pressure that portfolio managers exert and reduces the crowdedness of trades.

Our model informs the debate as to whether costs of asset management are excessive and whether returns delivered by the fund managers justify these costs. We use the model to compare the managers' costs and expected returns under privately and socially optimal contracts. We find that, from the socially optimal point of view, fund investors excessively rely on contracts and make their managers invest too much at too high a cost.[4] In the equilibrium with privately optimal contracts, asset prices are higher and consequently expected per-share returns are lower than those under socially optimal contracts. Key to these implications is that, in contrast to fund investors, the planner internalizes the pecuniary externality arising from crowded trades.

While prices under the privately optimal contracts are higher than in the constrained optimum, they are lower than in the first best, where the portfolio choice is observable. Intuitively, it is optimal to invest more in assets with higher abnormal returns when there are no agency costs, so in this case prices would be higher and expected returns lower. If the portfolio choice is observable, crowded trades create no externality, and thus pose no problem.

We also investigate how benchmarks ought to be designed. We show that both privately and socially optimal benchmarks put more weight on assets for which portfolio management adds more value as well as on assets for which incentive misalignment is most severe. The relative tilt in the weights, however, is different in the privately and socially optimal benchmarks. For example, the planner puts relatively less weight on assets with large costs compared to fund investors. This is because the planner understands that contracts are less effective at providing incentives than fund investors perceive, and is therefore less willing to use benchmark weights for incentive provision.

Finally, we discuss the implications of the model for the active regulatory debate regarding the structure of compensation in the asset management industry. Our model shows why there can be a pecuniary externality coming from crowded trades, but this possibility is hardly discussed in regulatory conversations. Instead those debates tend to focus on

[4]While the cost is borne by the manager, it ultimately gets passed on to the fund investor, who needs to compensate the manager enough to ensure her participation.

issues such as the split between variable, at-risk pay, and fixed pay. Absent the externality, there would be no reason why socially and privately optimal compensation contracts would diverge, meaning that in our model there would be no reason for any regulation. So perhaps the possibility of crowded trades ought to get more attention.

The remainder of the paper is organized as follows. In the next section, we review the related literature. Section 3 presents our model, and Section 4 analyzes the model and derives our main results. Section 5 links the model's implications to some ongoing regulatory discussions about compensation in the asset management industry. Section 6 concludes and outlines directions for future research. Omitted proofs and derivations are in the appendices.

2 Related Literature

Our work builds on the vast literature on optimal contracts under moral hazard, and in particular on seminal contributions of Holmstrom (1979) and Holmstrom and Milgrom (1987, 1991). Holmstrom (1979) argues that including in a contract a signal that is correlated with the output of the manager—in our case, such signal is the benchmark's performance—is beneficial to the principal. In our paper, the contract designer optimally chooses the signal to include in the contract. But more importantly, the benefit of including the signal is endogenous through the general-equilibrium effect on prices. To our knowledge, ours is the first paper that endogenizes the effectiveness of including such an additional signal in an incentive contract.

Holmstrom and Milgrom (1991) introduce a tractable contracting setting with moral hazard, with which our model shares many similarities. The standard implication in this literature is that increasing the agent's share in the output of a project helps provide incentives to the agent. In the context of delegated asset management though, giving the agent a larger share of portfolio return encourages her to scale down the risk of the (unobservable) portfolio by reducing risky asset holdings. Stoughton (1993) and Admati and Pfleiderer (1997) show that the manager is able to completely "undo" her steeper incentives by such scaling, and her incentives to collect information on asset payoffs remain unchanged. In our paper, we design a contract that provides desired incentives, despite the endogenous portfolio response of the manager, and show that it involves benchmarking. Another notable difference from the aforementioned literature is that we embed optimal (linear) contracts in a general-equilibrium setting and study interactions between contracts and equilibrium prices, and the implications of these interactions on welfare.

Our work is also related to the literature in asset pricing and corporate finance theory that explores the general-equilibrium implications of benchmarking. The pioneering work of Brennan (1993) shows that benchmarking leads to lower expected returns on stocks included in the benchmark. Cuoco and Kaniel (2011) and Basak and Pavlova (2013) study benchmarking in dynamic models, and show that the positive price pressure on benchmark stocks pushes up their prices and lowers their Sharpe ratios. Basak and Pavlova also show that benchmarking leads to excess volatility and excess co-movement of returns on stocks inside the benchmark. Kashyap, Kovrijnykh, Li, and Pavlova (2020) focus on implications of benchmarking portfolio managers for firms' corporate decisions and demonstrate that firms in the benchmark have a higher valuation for investment projects or merger targets than firms outside the benchmark. These papers take the benchmarking contract of managers to be exogenous. The three papers that do not, Buffa, Vayanos, and Woolley (2014), Cvitanic and Xing (2018), and Sockin and Xiaolan (2020), study asset-pricing implications of benchmarking in an environment with endogenous contracts.[5] In the first two papers, benchmarking helps reduce diversion of cash flows from the fund by managers. Our rationale for benchmarking is to reward activities that generate superior returns. Sockin and Xiaolan study costly information acquisition by managers, and, like us, highlight the pecuniary externality that emerges because of the effect of contracts on equilibrium prices. In contrast to us, they show that a constrained social planner, who internalizes this externality, opts for more incentive provision and more benchmarking.

Our paper also relates to the literature on pecuniary externalities in competitive equilibrium settings with incomplete markets, for example, Lorenzoni (2008), He and Kondor (2016), Gromb and Vayanos (2002), Davila and Korinek (2018), Di Tella (2017), Biais, Heider, and Hoerova (forthcoming), and Acemoglu and Simsek (2012), among others.[6] Lorenzoni (2008) studies a model of credit booms in which a pecuniary externality arises from the combination of limited commitment and asset prices being determined in spot markets. Decentralized equilibria feature over-borrowing relative to what is constrained optimal (although there is always under-borrowing compared to the first best). Borrowing less ex ante is welfare improving because it leads to an increase in asset prices in the low-output

[5] See also Ozdenoren and Yuan (2017) who conduct a related analysis in the context of an industry equilibrium, in a classical moral-hazard setting with many principal-agent pairs. They show that benchmarking is privately optimal but it creates overinvestment and excessive risk-taking at the industry level. Albuquerque, Cabral, and Guedes (2019) present a related model of industry equilibrium, enriched further with strategic interactions among firms in the industry, and show that benchmarking against peer performance induces agents to take correlated actions.

[6] This literature goes back to Hart (1975), Greenwald and Stiglitz (1986), and Geanakoplos and Polemarchakis (1996).

state, which allows entrepreneurs to transfer resources to the low-output state. Both our setting and mechanism are very different, but we share a similar prediction that asset prices in the decentralized equilibrium fall between those in the constrained and unconstrained optima. However, the actual comparison of prices is reversed in our model—decentralized-equilibrium prices are higher than in the constrained optimum (lower in his paper), but lower than in the first best (higher in his paper).

He and Kondor (2016) study a model in which individual firms' liquidity management decisions generate investment waves. These investment waves are constrained inefficient when future investment opportunities are noncontractible, and the social and private value of liquidity differs. In their model, overinvestment occurs during booms and underinvestment occurs during recessions. Gromb and Vayanos (2002) analyze a model in which competitive financially constrained arbitrageurs supply liquidity to the market, and fail to internalize the fact that changing their positions affects prices. A change in prices effectively moves resources across time and states and thus can bring the marginal rates of substitution closer together. A social planner can achieve a Pareto improvement by either reducing or increasing the arbitrageurs' position. Davila and Korinek (2018) highlight a distinction between "distributive externalities" that arise from incomplete insurance markets and "collateral externalities" that arise from price-dependent financial constraints. The externality that we emphasize in our paper falls into the second category, broadly defined, although in our case the inefficiency arises from the incentive problem rather than financial constraints. Di Tella (2017) studies optimal contracts in a model where financial intermediaries trade capital on behalf of households and can divert cash flows. He shows that, due to a pecuniary externality, the competitive equilibrium is constrained efficient and can lead to the concentration of aggregate risk on financial intermediaries' balance sheets.[7]

Biais, Heider, and Hoerova (forthcoming) analyze a model in which protection buyers trade derivatives with protection sellers and there is moral hazard on the side of protection sellers. In their model, although prices enter incentive constraints, a pecuniary externality does not lead to constrained inefficiency as it does in our model. The reason is that in their setup investors optimally supply insurance against the risk of fire sales. In Acemoglu and Simsek (2012), firms trade off providing insurance to workers and incentivizing them to exert effort. The authors show that, under certain conditions, equilibrium prices can tighten incentive constraints. They mainly focus on inefficient sharing of idiosyncratic risk. Instead, our focus is on the inefficient use of an additional signal—return of the benchmark

[7] In a follow-up paper, Di Tella (2019) characterizes the optimal financial regulation policy in such an economy and shows that the socially optimal allocation can be implemented with a tax on asset holdings.

portfolio—in the incentive contract. Fershtman and Judd (1987) look at contract design in an equilibrium setting, but in an oligopoly rather than a competitive equilibrium, as in our model. In their paper, there is strategic manipulation of agents' incentives, because owners realize that the contracts they give to their managers affect contracts chosen by other owners. In our competitive setting (in a very different environment), private agents ignore their effects on others.

Finally, there is some empirical evidence that benchmarking creates crowded trades. Lines (2016) observes that in times of high-market volatility, portfolio tracking error rises. This mechanically leads portfolio managers to rebalance their portfolios towards benchmark stocks. He finds that this trading behavior leads to lower returns for the rebalanced portfolios.

3 Model

We embed a linear optimal-contracting problem into a general-equilibrium asset-pricing framework. In this section we set up the model and provide justification for our main assumptions.

3.1 Economy

Except for portfolio managers and their clients, our environment is standard. There are two periods, $t = 0, 1$. Investment opportunities are represented by N risky assets (stocks) and one risk-free bond. The risky assets are claims to cash flows $\tilde{D}$, realized at $t = 1$, where $\tilde{D} \sim N(\mu, \Sigma)$. The variables $\tilde{D}$ and μ are $N \times 1$ vectors and Σ is an invertible positive semi-definite $N \times N$ matrix. The risk-free bond pays an interest rate that is normalized to zero. The risky assets are in a fixed supply of $\bar{x} > 0$ shares, where $\bar{x}$ is an $N \times 1$ vector. The bond is in infinite net supply. Let S, an $N \times 1$ vector, denote asset prices.

There is a continuum of agents in the economy, of three types. First, there are "direct" investors—constituting a fraction λ_D of the population—who manage their own portfolios. There are also portfolio or fund managers—a fraction λ_M—and fund investors who hire those managers—a fraction λ_F, with $\lambda_D + \lambda_M + \lambda_F = 1$. We assume for simplicity that each fund investor employs one manager, so that $\lambda_M = \lambda_F$.[8] Fund investors can buy the bond directly, but cannot trade risky assets, so they delegate the selection of their portfolios to managers. Managers can invest the delegated funds in risky assets and the bond, but

[8] The extension where one manager is hired by multiple investors acting collectively is straightforward.

are restricted to invest their personal wealth in the bond.

Each agent has a constant absolute risk aversion (CARA) utility function over final wealth (or compensation) W, $U(W) = -e^{-\gamma W}$, where $\gamma > 0$ is the coefficient of absolute risk aversion. Agents of type $j \in \{D, F, M\}$ are endowed with x^j_{-1} shares of risky assets, where $\sum_{j\in\{D,F,M\}} \lambda_j x^j_{-1} = \bar{x}$.

For fund investors, delegating investment to a portfolio manager has costs and benefits. On the one hand, as we will discuss in the next subsection, managers can potentially outperform direct investors. On the other hand, the manager's portfolio choice is unobservable meaning that fund investors cannot perfectly control it. This is a realistic assumption because even when managers are required to disclose their portfolios at particular points in time, their actual portfolios between the disclosure dates typically differ from their reported portfolios (Kacperczyk, Sialm, and Zheng, 2008), and a fund investor cannot obtain detailed information on the manager's trades. Furthermore, the managers incur a private cost in managing a portfolio. The combination of the private cost and the portfolios being unobservable will be the central friction in our model.

We do not model an agent's choice to become a direct investor or a fund investor—the fractions of different investors in the population are exogenous. One could endogenize this choice, for example, by assuming heterogeneous costs of participating in the asset market. In Remark 4 at the end of Section 4 we describe the additional considerations that arise in this kind of an extension, but we do not consider it here to maintain our focus on the central message of the paper.

3.2 Value Added and Costs of Asset Management

We assume that portfolio managers can potentially outperform direct investors. The (per-share) return for a direct investor's portfolio x is given by $x^\top(\tilde{D} - S)$. The manager's returns are

$$r_x = x^\top(\Delta + \tilde{D} - S) + \varepsilon, \tag{1}$$

where $\Delta \geq 0$ is an exogenous vector and $\varepsilon \sim N(0, \sigma_\varepsilon)$ is a (scalar) noise term. The manager incurs a private portfolio-management cost $x^\top\psi$, where $\psi > 0$ is an exogenous vector.[9]

[9] Implicit in our expressions for the returns on the fund in (1) and the portfolio-management cost is that they scale linearly with the size of the portfolio. This is seemingly inconsistent with Berk and Green (2004) who assume that there are decreasing returns to scale in asset management, but it is not. Berk and Green explicitly attribute decreasing returns to scale to the price impact of fund managers. The bigger the portfolio invested in an alpha-opportunity, the smaller the return on a marginal dollar invested. Berk and

So managers in our model incur costs to generate excess returns of $x^\top \Delta + \varepsilon$ that we call "alpha."

In this formulation, the managers' alpha has nothing to do with superior information, which gives rise to stock-selection and market-timing abilities. If it did, then any direct investors who happened to buy the same assets or traded at the same time, without any knowledge of the Δ's, would earn the same returns. So this setup is consistent with the vast literature (e.g., Fama and French, 2010) that casts doubt on the ability to generate abnormal returns by stock picking or market timing.

Instead, the managers' alpha comes from activities such as lending securities, delivering lower transactions costs (e.g., from crossing trades in-house or by obtaining favorable quotes from brokers) or providing liquidity (i.e., serving as a counterparty to liquidity demanders and earning a premium on such trades). We refer to these opportunities as "return-augmenting" activities.

There is a wealth of evidence establishing that securities lending, trading cost minimization, and liquidity provision are profitable activities for asset managers. For example, securities lending contributed 5% of total revenue of both BlackRock and State Street in 2017. While it has recently become possible for some retail investors to participate in securities lending, they earn lower returns for this activity and do not have the same opportunities as a large asset management firm. It is also well established that portfolio managers can profit from providing immediacy in trades, by either buying assets which are out of favor or selling ones that are in high demand.[10] It would be prohibitively expensive for retail investors to try to do this. Finally, Eisele, Nefedova, Parise, and Peijnenburg (2020) present evidence that trades crossed internally within a fund complex are executed more cheaply than comparable external trades.

The noise term ε in (1) captures the fact that the return-augmenting activities do not produce a certain return each period. For example, the demand for liquidity, the opportunities to lend shares and the possibility of crossing trades all fluctuate, so even a very alert and skilled manager will have some randomness in her returns. Also for securities that are lent, there is a risk that they will not be returned in a timely manner or potentially

Green's model is in partial equilibrium and their price impact is simply an exogenous function of fund size. Ours is a general-equilibrium model, in which the price impact *endogenously* arises from a higher aggregate demand of portfolio managers for high-Δ assets.

[10] In a classic paper, Keim (1999) estimates an annual alpha of 2.2% earned by liquidity provision activities of a fund. Rinne and Suominen (2016) document that the top decile of liquidity providing mutual funds outperform the bottom decile by about 60 basis points per year. Anand, Jotikasthira, and Venkataraman (2018) find similar estimates using a different sample of funds over a different time period.

at all.[11]

We close this section by discussing some important assumptions in the model and alternatives that we could have made instead. One is that the manager incurs a private cost in order to deliver the abnormal returns. The existence of the costs seems very clear cut. For instance, to successfully buy and sell at the appropriate times to provide liquidity, the manager has to be actively monitoring market conditions while markets are open. For securities lending, the manager would also have to decide whether to accommodate requests to borrow shares. In some cases, these demands arise because the entity borrowing the shares wants to vote them and the manager must decide whether to pass up that choice.[12]

It is also plausible that the benefits and costs associated with the return-augmenting activities are increasing in the size of the holdings.[13] For example, in terms of the liquidity provision and trade-crossing, the wider the range of securities in the portfolio and/or the more a fund holds on any particular security, the easier it would be to provide liquidity or more likely it would be that a trade can be offset. For securities lending, a larger portfolio opens up additional lending opportunities. As mentioned earlier, it is simplest to think of the costs as being tied to the time it takes to undertake the various activities. Thought of this way, if the opportunities to augment returns increase as the portfolio expands, then the costs of realizing them would naturally grow too.

We could instead assume that the private cost arises because the manager needs to exert costly effort to generate the excess returns, as is often done in the contracting literature (e.g., Holmstrom and Milgrom, 1987, 1991). Incorporating effort makes the algebra much more involved.[14] However, under certain assumptions our main insights extend to this case. Importantly, it is the unobservability of the portfolio holdings and not the unobservability of effort that is central to our mechanism. To make this clear and to focus on the key friction, in our main model we we do not include an effort choice. We analyze an extension that incorporates effort in Appendix C and show that our main insights carry over.

Lastly, for simplicity we assume that Δ (the expected per-share return coming from the

[11] One might wonder what happens of the noise is proportional to x (that is, the noise term is εx instead of ε). This is a special case of the extension that we analyze in Appendix C. The algebra is more involved in this case, but the main mechanism is the same.

[12] Most managers also incur some costs that are observable and can be passed on directly to fund investors. Examples would include custody, audit, shareholder reports, proxies and some external legal fees. Our main results continue to hold in a model in which some costs are observable.

[13] Linearity allows us to solve the model in closed form, but what is important conceptually is that the cost is increasing in x. We show in Appendix C that while the algebra is messier, under some assumptions our main analysis extends to the case of more general specifications of the return and cost.

[14] Our results trivially extend if effort is bounded from above (e.g., if there is a time constraint), and the optimal solution is at the upper bound.

return-augmenting activities) is exogenous. One might argue that because of congestion that arises in general equilibrium when many agents engage in an activity, Δ should decline. Our mechanism would work through an endogenous Δ the same way as it works through the equilibrium price S: fund investors, taking the (per-share) returns $\Delta + \tilde{D} - S$ as given, ignore the fact that their contracts push down the equilibrium returns.[15] Thus endogenizing Δ would, if anything, strengthen our results. In fact, mechanically all of our results go through even if Δ is zero (while the cost ψ is positive, at least for some stocks). However, it seems implausible to assume that the asset management industry would exist if using portfolio managers involved only costs and no benefits.

3.3 Managers' Compensation Contracts

To provide incentives for the managers to invest in the risky assets and to generate alpha, the fund investors design compensation contracts. The managers receive compensation w from fund investors. This compensation has three parts: one is a linear payout based on absolute performance of the manager's portfolio x, the second part depends on the performance relative to the benchmark portfolio, and the third is independent of performance.[16] Then

$$w = \hat{a} r_x + b(r_x - r_{\mathbf{b}}) + c = a r_x - b r_{\mathbf{b}} + c, \tag{2}$$

where r_x is the performance of the manager's portfolio defined in (1) and $r_{\mathbf{b}} = \theta^\top(\tilde{D} - S)$ is the performance of the benchmark portfolio θ. The contract for a manager is $(\hat{a}, b, c, \theta)$—or, equivalently, (a, b, c, θ)—where $\hat{a}$ (or a), b, and c are scalars, and θ is an $N \times 1$ vector of benchmark weights such that $\sum_i \theta_i = 1$. We refer to $\hat{a}$ as the sensitivity to absolute performance and b as the sensitivity to relative performance. Our main analysis and the intuitions that follow will be in terms of a rather than $\hat{a}$. We refer to the variable a as the manager's "skin in the game."[17] The contract for a particular manager is optimally chosen by the fund investor who employs her. As we mentioned earlier, the manager is restricted to investing her personal wealth in the bond and so she cannot "undo" her contract via

[15] A way of explicitly modeling the market for the return-augmented activities would depend on which exact activity is considered. Since we attempt to capture several of such activities, we abstract from fully modeling such a market.

[16] This part captures features such as a fee linked to initial assets under management or a fixed salary or any fixed costs.

[17] We assume here that returns from dividends and return-augmenting activities are not observed separately. In some cases, e.g., in the case of securities lending, it might be possible to observe them separately. We illustrate in Appendix D that our mechanism still applies in that case.

trading in her personal account.[18]

We think of a manager's contract as a compensation contract between a portfolio manager and her investment-advisor firm (e.g., BlackRock, who we assume is acting in the interests of the fund investors). The structure of the contract in (2) is consistent with empirical evidence. For example, Ma, Tang, and Gómez (2019) analyze mandatory disclosures by U.S. mutual funds and find that around 80% of the funds explicitly base managers' compensation on performance relative to a benchmark (usually the prospectus benchmark, e.g., S&P 500, Russell 2000, etc.). Managers also have a fixed salary component, but the fraction of fund managers whose entire compensation consists of only fixed salary is very small.[19]

The restriction to linear contracts warrants discussion. We assume linearity for the purpose of tractability.[20] Characterizing fully optimal contracts is hard in general. It is even harder in our case since we solve for them in a general-equilibrium model in which contracts affect equilibrium prices and thus in turn affect the contract chosen by each fund investor. The restriction to linear contracts allows us to find optimal contracts in closed form. However, the argument behind our main mechanism—individual contracts change demand functions, causing the equilibrium prices to change, which dampens the effect of contracts on demands, and thus makes contracts less effective—is general. We have no reason to believe that this mechanism would not apply with fully optimal contracts.

[18] In practice, portfolio managers have a fiduciary duty to their investors. This precludes them from taking actions that harm the investors, or engaging in any activity that creates a conflict of interest between the manager and the fund investors. Compliance departments at asset management firms attempt to deal with these problems by requiring pre-approval of many types of trades by the manager or banning them altogether, and restricting when trading can occur. A trade such as shorting a manager's benchmark would be blocked by these policies.(See U.S. Securities and Exchange Commission, 2004 for details.)

[19] The performance-based bonus exceeds the fixed salary for 68% of the funds in the sample, constituting more than 200% of fixed salary for 35% of funds. In contrast, Ibert, Kaniel, Van Nieuwerburgh, and Vestman (2017) find surprisingly weak sensitivity of manager pay to performance for Swedish mutual funds.

[20] There is a literature that justifies the use of linear contracts in environments with CARA preferences and normally-distributed returns. Holmstrom and Milgrom (1987) show that, in a specific dynamic setting, the solution of the optimal-contracting problem is as if the problem were a static one and the principal were constrained to use a linear compensation rule that depended on the final outcome. Holmstrom and Milgrom restrict the agent's action to affect only the mean of the random process but not the variance, which is not the case in our model (the portfolio choice affects both the mean and the variance of the return). Sung (1995) establishes the robustness of the Holmstrom and Milgrom's linearity result by allowing the agent to also control the variance.

4 Analysis

4.1 Direct Investors' and Managers' Problems

At $t = 0$, each direct investor chooses a portfolio of risky assets, x, and the risk-free bond holdings to maximize his expected utility $-Ee^{-\gamma W}$. Since his return on the portfolio is $x^\top(\tilde{D} - S)$, the resulting time-1 wealth is $W = \left(x_{-1}^D\right)^\top S + x^\top(\tilde{D} - S)$. It is well known that a direct investor's maximization problem is equivalent to the following mean-variance optimization:

$$\max_x x^\top(\mu - S) - \frac{\gamma}{2} x^\top \Sigma x.$$

Each portfolio manager chooses a portfolio of risky assets x and the risk-free bond holdings to maximize $-E\exp\{-\gamma[ar_x - br_{\mathbf{b}} + c - x^\top\psi]\}$, where the quantity inside the square brackets is her compensation net of private cost. This maximization problem is equivalent to the following mean-variance optimization:

$$\max_x \; ax^\top(\Delta - \psi/a + \mu - S) - b\theta^\top(\mu - S) + c - \frac{\gamma}{2}\left[(ax - b\theta)^\top \Sigma(ax - b\theta) + a^2\sigma_\varepsilon^2\right].$$

Note that the manager receives a fraction a of the per-share abnormal return on the assets, Δ, but pays the entire cost ψ per share. (We later show that $a < 1$.)

Both the direct investors and managers take asset prices as given. Lemma 1 reports the optimal portfolio choices of the direct investors and managers arising from their optimizations.

Lemma 1 (**Portfolio Choice**). *The direct investors' and managers' optimal portfolio choices are as follows:*

$$x^D = \Sigma^{-1}\frac{\mu - S}{\gamma}, \tag{3}$$

$$x^M = \Sigma^{-1}\frac{\Delta - \psi/a + \mu - S}{a\gamma} + \frac{b\theta}{a}. \tag{4}$$

A direct investor's portfolio is the standard mean-variance portfolio, scaled by his risk aversion γ. A manager's portfolio choice differs from that of a direct investor in three dimensions. First, managers split their risky-assets investments between two portfolios: the (modified) mean-variance portfolio and the benchmark portfolio. The latter arises because the manager's compensation is exposed to fluctuations in the benchmark. To mitigate this risk, she holds a hedging portfolio that is (perfectly) correlated with the benchmark, i.e., the benchmark itself.[21] The split between the two portfolios is governed by the strength of the relative-performance incentives, captured by b. The higher b is, the closer the manager's portfolio is to the benchmark. Second, because our managers have access to return-augmenting strategies, they perceive the mean-variance tradeoff differently from the direct investors and tilt their mean-variance portfolios towards high-Δ assets (i.e., they try to produce alpha). Consistent with this result, Johnson and Weitzner (2019) report that fund managers' portfolios in their sample overweight assets with high securities-lending fees. Finally, the manager scales her (modified) mean-variance portfolio by $1/a$ relative to that of a direct investor. The reason for the scaling is that, as we can see from the first term in (2), for each share that a manager holds, she gets a fraction a of the total return.

Substituting portfolio demands from Lemma 1 into the market-clearing condition for assets, $\lambda_M x^M + \lambda_D x^D = \bar{x}$, we find the expression for the equilibrium asset prices:

$$S = \mu - \gamma\Sigma\Lambda\bar{x} + \gamma\Sigma\Lambda\lambda_M \frac{b\theta}{a} + \Lambda\frac{\lambda_M}{a}\left(\Delta - \frac{\psi}{a}\right), \tag{5}$$

where $\Lambda \equiv [\lambda_M/a + \lambda_D]^{-1}$ modifies the market's effective risk aversion.

Because contracts affect the managers' demand functions, the equilibrium asset prices will depend on these contracts. Benchmarking and the tilt towards high-Δ assets push up prices, thus lowering the expected returns. Unlike the social planner, individual fund investors take prices as given and do not account for this pecuniary externality. We turn to the fund investors' problem next.

4.2 Fund Investors' Problem

Each fund investor chooses the contract (a, b, c, θ) and portfolio $x = x^M$ to maximize his expected utility subject to the manager's participation and incentive constraints. The latter is the manager's first-order condition (4), capturing the fact that the portfolio x is the manager's private choice.

To write the fund investor's problem formally, it is convenient to express payoffs in terms of the following variables. Denote $y = ax - b\theta$ and $z = x - y$, which are effective allocations of asset holdings to the manager and fund investor, respectively. Then the fund

[21] This implication is very general, and we share it with other models that analyzed benchmarking, both in two-period and multi-period economies and for other investor preferences specifications. This result first appeared in Brennan (1993) in a two-period model. Cuoco and Kaniel (2011) and Basak and Pavlova (2013), among others, obtain it in dynamic models with different preferences.

investor's and manager's utilities (in the mean-variance form) can be written as follows:

$$U^F\left(a, \frac{b\theta}{a}, y, S\right) = x^\top (1-a)\Delta + z^\top(\mu - S) - \frac{\gamma}{2}\left[z^\top \Sigma z + (1-a)^2\sigma_\varepsilon^2\right] - c + \left[x_{-1}^F\right]^\top S,$$
$$U^M\left(a, \frac{b\theta}{a}, y, S\right) = x^\top (a\,\Delta - \psi) + y^\top(\mu - S) - \frac{\gamma}{2}\left[y^\top \Sigma y + a^2\sigma_\varepsilon^2\right] + c,$$

where x and z are given by

$$x = \frac{y}{a} + \frac{b\theta}{a}, \tag{6}$$
$$z = \frac{1-a}{a} y + \frac{b\theta}{a}. \tag{7}$$

Then the fund investor's problem can then be written as follows:[22]

$$\max_{a,b,c,\theta,y} \quad U^F$$
$$\text{s.t. } U^M \geq u_0, \tag{8}$$
$$y = \Sigma^{-1}\frac{\Delta - \psi/a + \mu - S}{\gamma}. \tag{9}$$

Constraint (8) is the manager's participation constraint, where u_0 is (the mean-variance equivalent of) the value of manager's outside option.[23] Equation (9) is the manager's (modified) incentive constraint.

We next discuss the roles that the contract parameters a, b, and θ play in the fund investor's maximization problem.

4.3 Contracts and Incentives

As a point of reference, consider the first best where the manager's portfolio choice is observable and contractible. The first-best contract involves efficient risk sharing between the (equally risk-averse) fund investor and manager and no benchmarking, $a = 1/2$ and $b = 0$.[24] If the manager were facing the first-best contract but chose the portfolio privately, she would underinvest, especially in assets with high ψ. A higher a reduces the manager's

[22]The formulation of the fund investor's problem in terms of the exponential utilities (rather than in the mean-variance form) can be found in Appendix A.

[23]We do not model explicitly what this outside option is, as it does not matter for our main results. It can be exogenous, or it can be endogenized. Notice also that because of the contract's constant component c, in the mean-variance formulation utility becomes transferable, and the fund investor effectively maximizes the total utility of the fund investor and the manager subject to the manager's incentive constraint. The manager's participation constraint is then trivially satisfied by adjusting the constant c.

[24]See Lemma 5 in Appendix A for the formal analysis.

effective cost ψ/a, which increases her demand for risky assets. However, a higher a also exposes the manager to more risk, which makes her scale down x^M, as reflected in the denominator(s) of (6). Thus the use of performance pay creates a tension between incentive provision and risk sharing.

The use of benchmarking, together with an appropriate benchmark selection, alleviates this tension by mitigating the adverse effect of a. Benchmarking shields the manager from risk by reducing variance in her compensation for a given portfolio choice.[25] As a result, (for the same a) the manager invests more. In addition, if the benchmark portfolio puts a relatively higher weight on certain assets, the manager's exposure to risk is reduced more for those assets, and she will invest proportionally more in them. That is, benchmarking protects the manager from risk, and an appropriate choice of the benchmark portfolio can help to improve incentives for alpha-production.

4.4 Privately Optimal Contracts

Notice that the fund investor fully internalizes the manager's cost of managing the fund.[26] But since the manager bears the cost privately and only receives fraction a of the return, for her the effective cost is higher, which is why ψ/a appears in (9). The actual (from the social perspective) and perceived (by the manager) marginal costs, ψ and ψ/a, being different plays an important role in our analysis.

Notice that b enters into the fund investor's and manager's problems only though $b\theta/a$. Therefore we take the first-order condition with respect to $b\theta/a$, and later derive the expression for b separately. The first-order condition with respect to $b\theta/a$ is given by[27]

$$\frac{\partial(U^F + U^M)}{\partial(b\theta/a)} = \Delta - \psi + \mu - S - \gamma\Sigma z = 0. \tag{10}$$

It captures the marginal effect on the total utility of the fund investor and the manager due to a higher demand by the managers in response to more benchmarking. This equation can

[25] By reducing the manager's risk exposure, benchmarking makes it cheaper for the fund investor to implement any particular portfolio choice.

[26] Formally, this can be seen by taking the first-order condition with respect to c, which implies that the Lagrange multiplier on the participation constraint equals one.

[27] We show in Lemma 6 in Appendix A that the second-order conditions hold in both privately and socially optimal cases.

be rewritten as[28]

$$\gamma\Sigma b\theta = (2a-1)\left(\Delta - \psi + \mu - S\right) + (1-a)\left(\frac{1}{a} - 1\right)\psi. \tag{11}$$

The two terms on the right-hand side of equation (11) capture two considerations that fund investors have in mind when designing the benchmark. Note two extreme cases: $a = 1/2$ when efficient risk-sharing is achieved, and $a = 1$ when the private and social costs are aligned. As we will show later, in the optimal contract $a \in (1/2, 1)$, so both terms on the right-hand side of (11) are positive. The first term, $(2a-1)\left(\Delta - \psi + \mu - S\right)$, arises because the fund investor recognizes that benchmarking increases the total expected surplus net of cost. Since $a > 1/2$, the manager is exposed to more risk than is efficient, so the fund investor uses benchmarking to make her invest more, in particular in assets with a higher value added $\Delta - \psi$. The second term, $(1-a)(1/a-1)\psi$, reflects the incentive-provision role of $b\theta$. By protecting the manager from risk, benchmarking provides her with incentives to invest more. Such incentive provision is especially important for high-ψ assets because the manager is the most reluctant to invest in them.

Next, let us consider the first-order condition with respect to a, which is given by

$$0 = -\,(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}\psi^\top \frac{\partial y}{\partial a} = -(2a-1)\gamma\sigma_\varepsilon^2 + (1-a)\frac{\psi^\top \Sigma^{-1}\psi}{\gamma a^3}. \tag{12}$$

Notice the appearance of $\partial y/\partial a$ in (12). It captures how a marginal increase in a affects the surplus through the direct response of the manager's (modified) demand for the risky assets, y. This is the incentive-provision channel. The other terms in (12) govern how risk is split between the fund investor and the manager (and thus also how much of the risky asset the manager buys). This is the risk-sharing channel.

Unlike in (11), the incentive-provision term and the risk-sharing term have different signs. This means that there is a tradeoff between incentive provision and risk sharing. A higher a is beneficial as it provides incentives for alpha-production, but is also costly because it exposes the manager to too much risk.

Substituting the expression for S, we obtain closed-form expressions for equilibrium contracts given in part (i) of the next lemma.

Lemma 2. *In the equilibrium with the privately optimal contract,*

[28]See the proof of Lemma 2 in Appendix A for the derivations.

(i) $a = a^{private}$, $b = b^{private}$, and $\theta = \theta^{private}$ are given by

$$0 = (1-a)\frac{\psi^\top \Sigma^{-1}\psi}{\gamma a^3} - (2a-1)\gamma\sigma_\varepsilon^2, \tag{13}$$

$$b = (2a-1)\,\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta-\psi)\right] + (1-a)\left[\frac{1}{a} - \frac{\lambda_M}{a} - \lambda_D\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi, \tag{14}$$

$$\theta = \frac{2a-1}{b}\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta-\psi)\right] + \frac{1-a}{b}\left[\frac{1}{a} - \frac{\lambda_M}{a} - \lambda_D\right]\frac{\Sigma^{-1}}{\gamma}\psi; \tag{15}$$

(ii) the asset prices are given by

$$S^{private} = \mu - \gamma\Sigma\bar{x} + \lambda_M\left(2\Delta - \psi - \frac{\psi}{a}\right), \tag{16}$$

and the fund portfolio is

$$x^M_{private} = 2\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D\left(2\Delta - \psi - \frac{\psi}{a}\right). \tag{17}$$

Notice that there is a recursive structure to these conditions. The expression in (13) is a function solely of a. Then given a, (14) delivers the expression for b, and finally, given a and b, (15) gives us expression for the benchmark weights θ. Note that it immediately follows from (13) that $a^{private} \in (1/2, 1)$.

It is worth pointing out that as σ_ε^2 goes to zero, a in the privately optimal contract approaches 1, and the allocation approaches the first-best one (see Lemma 5 in Appendix A.) Indeed, it is crucial for our results that the fund investor does not "sell the project" to the manager, i.e., $a < 1$. As an alternative to the assumption of $\sigma_\varepsilon^2 > 0$, there are other modeling choices that would ensure that $a < 1$, for example, a lower-bound on c, the constant part of the contract.

Let us briefly comment on the expression for the equilibrium prices given by (16). Absent fund managers, the equilibrium prices would be $S = \mu - \gamma\Sigma\bar{x}$. Prices are higher in the presence of managers due to their higher demands as they engage in return-augmenting activities, as captured by the last term in (16). Notice that the term in parentheses is a sum of $\Delta - \psi$ and $\Delta - \psi/a$, which are the (marginal) extra expected returns net of costs as perceived by the fund investors and by the managers, respectively. Similarly, the equilibrium asset holdings of managers in (17) are higher when the opportunities for alpha-production are better. Notice that managers hold exactly $2\bar{x}$ when $\lambda_D = 0$. We will discuss this special case further in subsection 4.5.

For some of our results on benchmarking, we will need to impose the following assumptions.

Assumption 1. *Suppose that*
(i) $\mathbf{1}^\top \left[\bar{x} + \lambda_D \Sigma^{-1}(\Delta - \psi)/\gamma\right] > 0$,
(ii) $\mathbf{1}^\top \Sigma^{-1}\psi > 0$.

These assumptions are mild technical restrictions. They are trivially satisfied when the variance-covariance matrix Σ is diagonal or when Δ's and ψ's are the same for all assets (and given that $\Delta - \psi \geq 0$). When Σ is not diagonal (which implies that cross-price elasticities of the manager's demand function are not zero), it is useful to interpret Assumption 1 as follows. Part (i) is a necessary and sufficient condition for the sum of shares (over all assets) that the manager holds in the first best to be positive (which is trivially satisfied if, for example, there is no short-selling).[29] Part (ii) means that if the private cost ψ increases by the same percentage for all assets, then the sum of shares (over all assets) that the manager holds in equilibrium goes down. In other words, the manager reduces total holdings when the cost is higher.

Using Assumption 1 and the equilibrium expression for b presented in Lemma 2, we have the following result:

Proposition 1 (Benchmarking is Optimal). *Suppose that Assumption 1 holds. Then the privately optimal contract involves benchmarking, that is,* $b^{private} > 0$.

Proposition 1 is essentially a version of Holmstrom's (1979) famous sufficient-statistic result—the use of an additional signal (in this case, the benchmark return) helps the contract designer provide incentives to the manager in a more effective way. Holmstrom's result is general and does not require contract linearity, so while we have restricted our attention to linear contracts, we would expect that the manager's compensation would depend on the benchmark return even with more general contracts. While Holmstrom's result suggests that b is different from zero in general, our Assumption 1 allows us to determine when b is strictly positive, which is the relevant case given this application.

The above proposition helps us understand why benchmarking in the asset management industry is so pervasive. Benchmarking is useful to fund investors because it incentivizes the manager to engage more in risky return-augmenting activities by partially protecting her from risk. In the language of the asset management industry, benchmarked managers are

[29] See the proof of Lemma 5 in Appendix A.

being protected from "beta" (i.e., the fluctuations in the return of the benchmark portfolio) while being rewarded for alpha.

Next, we discuss the properties of the privately optimal benchmark weights. Using equation (15), the lemma below shows how these weights differ across assets with different value added or cost of alpha-production, which are $\Delta - \psi$ and ψ respectively.

Lemma 3. *Consider two assets, i and j, that have the exact same characteristics except $\Delta_i - \psi_i \geq \Delta_j - \psi_j$ and $\psi_i \geq \psi_j$, with at least one inequality being strict. Then in the privately optimal contract, asset i has a larger weight in the benchmark than asset j: $\theta_i^{private} > \theta_j^{private}$.*

The reason for this result is intuitive: fund investors recognize that manipulating benchmark weights allows them to provide more incentives for investment in assets where alpha-production is the most valuable. The effect of a larger ψ on the benchmark weight is ambiguous, as can be seen from (15). On the one hand, the incentive problem is the most severe for assets with a larger ψ, and thus setting higher weight is most valuable for those assets. On the other hand, a larger ψ reduces the total expected return, which reduces the marginal benefit of using $b\theta$ for protecting the manager from extra risk. However, for the same (or a larger) value added, higher-cost assets would have a higher weight in the privately optimal benchmark.

Fund investors design contracts to influence the manager's demand for risky assets. Through the aggregate demand of the managers, contracts influence equilibrium asset prices, as given by (5). Prices then affect the marginal cost/marginal benefit tradeoff of contracts for all fund investors. Since fund investors take prices as given, they do not internalize how their choices of contracts (once aggregated) change the effectiveness of other fund investors' contracts. In other words, fund investors impose an externality on each other through their use of contracts. In the next subsection, we ask what contract a planner, who is subject to the same restrictions as fund investors, would choose to internalize this externality.

4.5 Socially Optimal Contracts

We define the problem of such a constrained social planner as follows. The planner maximizes the weighted average of fund investors' and direct investors' utilities subject to the participation and incentive constraints of the managers, as well as the constraint that direct investors choose their portfolios themselves.[30] As before, this problem can be equivalently rewritten in terms of mean-variance preferences.[31] Define $U^D = \left(x_{-1}^D\right)^\top S + \left(x^D\right)^\top (\mu - S) - (\gamma/2)\left(x^D\right)^\top \Sigma x^D$. Then the social planner's problem is

$$\max_{a,b,c,\theta,y,x^D} \; \omega_F U^F + \omega_D U^D$$

subject to (8), (9), and (3).

The social planner's first-order condition with respect to $b\theta/a$ is

$$\begin{aligned} 0 = &\left[\omega_F \left(x_{-1}^F - x^M\right)^\top + \omega_D \left(x_{-1}^D - x^D\right)^\top\right] \frac{\partial S}{\partial(b\theta/a)} \\ &+ \omega_F \left[\frac{\partial(U^F + U^M)}{\partial(b\theta/a)} + \frac{\partial U^F}{\partial y}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right]. \end{aligned} \tag{18}$$

The terms in the first line of (18) capture what we call the redistribution effect. Depending on the initial endowments and the Pareto weights, the social planner has incentives to use benchmarking to move prices so as to benefit one or the other party based on this redistribution motive. We discuss the redistribution effects in Remark 1 at the end of this section. To isolate the planner's motive to correct the externality, we want to neutralize this redistribution motive. To do this, we set the Pareto weights $\omega_F = \omega_D$.[32] Then by market clearing, $\omega_F\left(x_{-1}^F - x^M\right) + \omega_D\left(x_{-1}^D - x^D\right) = 0$, so the term in the first line of (18) is zero. Rewriting the term in the second line, (18) yields

$$0 = (\Delta - \psi + \mu - S - \gamma\Sigma z)^\top + \frac{1-a}{a}(\Delta + \mu - S - \gamma\Sigma z)^\top \frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}, \tag{19}$$

or, equivalently,

$$\Delta - \psi\frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D} + \mu - S - \gamma\Sigma z = 0. \tag{20}$$

Compare (19) or (20) with (10). The second term in (19) captures the externality that the planner is trying to correct, and it is negative. The planner realizes that benchmarking

[30]Equivalently, instead of imposing the manager's participation constraint, her utility can be included into the planner's objective function with a Pareto weight ω_M. For the transfer c to be finite, we must have $\omega_M = \omega_F$. This is analogous to noticing that the Lagrange multiplier on the participation constraint, which effectively acts as the Pareto weight on the manager, equals ω_F.

[31]We provide the original formulation in terms of exponential utilities in Appendix A.

[32]Choosing Pareto weights to cancel out the redistribution effects is equivalent to allowing the social planner to use transfers. The planner will then use transfers to equate the marginal utilities (weighted by Pareto weights) of different agents.

inflates prices and thus reduces expected returns. Hence for the social planner the benefit of alpha-production is smaller due to this crowded-trades effect, or, equivalently, the cost is higher for the same unit of benefit: $\psi(\lambda_M/a+\lambda_D)/(\lambda_M+\lambda_D) > \psi$ in (20) instead of ψ in (10). So from the planner's point of view, alpha-production is less beneficial/more expensive, which, as we will see, will make her do less of it.

The planner's first-order condition with respect to a can be written as

$$(1-a)\frac{\psi^\top \Sigma^{-1}\psi}{\gamma a^3}\frac{\lambda_D}{\lambda_M+\lambda_D} - (2a-1)\gamma\sigma_\varepsilon^2 = 0. \tag{21}$$

(See the proof of Lemma 4 in Appendix A for the derivations.) Compare this equation to its analog in the private case, equation (12). Notice that the benefit of incentive provision captured by the first term in (21) is smaller than the corresponding term in (12). It is then easy to see that the social planner will use a lower a than individual fund investors. We will formalize this result later in Proposition 2. While this result is immediate given equations (12) and (21), the derivation of (21) is quite involved, so the result about the comparison between the coefficients a in the two cases is rather subtle and crucially relies on the use of benchmarking. We will come back to this in the discussion of Proposition 2.

Substituting the equilibrium prices into the first-order conditions and using the definition of z, we arrive at the following lemma, which describes the equilibrium contract and prices in closed form.

Lemma 4. *In the equilibrium with the socially optimal contract,*
(i) $a = a^{social}$, $b = b^{social}$ and $\theta = \theta^{social}$ are given by[33]

$$0 = (1-a)\frac{\psi^\top \Sigma^{-1}\psi}{\gamma a^3}\frac{\lambda_D}{\lambda_M+\lambda_D} - (2a-1)\gamma\sigma_\varepsilon^2, \tag{22}$$

$$b = (2a-1)\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta-\psi)\right] + (1-a)\left[\frac{1}{a} - \frac{\lambda_M/a+\lambda_D}{\lambda_M+\lambda_D}\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi, \tag{23}$$

$$\theta = \frac{2a-1}{b}\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta-\psi)\right] + \frac{1-a}{b}\left[\frac{1}{a} - \frac{\lambda_M/a+\lambda_D}{\lambda_M+\lambda_D}\right]\frac{\Sigma^{-1}}{\gamma}\psi; \tag{24}$$

(ii) the asset prices are given by

$$S^{social} = \mu - \gamma\Sigma\bar{x} + \lambda_M\left(2\Delta - \frac{\lambda_M/a+\lambda_D}{\lambda_M+\lambda_D}\psi - \frac{\psi}{a}\right), \tag{25}$$

[33] From (22), $1/2 \le a^{social} < 1$, where the first inequality is strict so long as $\lambda_D > 0$.

and the fund portfolio is

$$x^M_{social} = 2\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D \left(2\Delta - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi - \frac{\psi}{a}\right). \tag{26}$$

Equations (22)−(26) are the analogs of (13)−(17). As expected, the two sets of equations coincide when $\lambda_M = 0$, and hence there is no externality. But so long as there are managers, the socially and privately optimal contracts are different. Our analysis below will reveal how exactly they compare to each other.

We are now ready to present the central result of the paper.

Proposition 2 (**Socially vs. Privately Optimal Contracts**). *(i) Compared to the privately optimal contract, the socially optimal contract involves less "skin in the game," that is,* $a^{social} < a^{private}$*;*
(ii) Suppose that Assumption 1 holds. Then compared to the privately optimal contract, the socially optimal contract involves less benchmarking, that is, $b^{social} < b^{private}$*.*[34]

As we have seen in our analysis, the use of contracts inflates prices and thus *reduces the marginal benefit of incentive provision* for everyone else. The social planner internalizes this effect, and opts for less incentive provision than fund investors.

An interesting special case is when there are no direct investors, $\lambda_D = 0$. In this case, each fund will hold exactly $2\bar{x}$ shares and the total alpha in the economy is fixed, equal to $2\bar{x}^\top\Delta$. The planner understands that incentive provision is unnecessary in this case, so there is no tradeoff between incentive provision and risk sharing. Indeed, by substituting $\lambda_D = 0$ into (22)−(23), it immediately follows that the socially optimal contract is $a = 1/2$ and $b = 0$, which coincides with the first-best one (see Lemma 5 in Appendix A). Interestingly, the fund investors ignore the fact that, in equilibrium, their contracts will not help them generate higher returns, and use contracts with $a > 1/2$ and $b > 0$, as can be seen from (13)−(14).

To further emphasize that benchmarking is crucial for the comparison between privately and socially optimal contracts, consider a case where benchmarking is exogenously set to zero, $b = 0$. In this case, all incentive provision and risk sharing has to be done through a. As we discussed earlier, an increase in a has two opposing effects on the managers' demands and hence prices. As a result, it can be shown that with $b = 0$ the comparison between a^{social} and $a^{private}$ is ambiguous. Intuitively, both the marginal benefit of a (incentive provision) as well as its marginal cost (exposing the manager to more risk) are lower for the

[34] We also show in the proof of Proposition 2 that $b^{social}/a^{social} < b^{private}/a^{private}$.

social planner who internalizes the effect of a on prices. Depending on which reduction is bigger, the planner can choose a higher or a lower a than the fund investors do. Thus, only because of benchmarking ($b \neq 0$) can we be sure of the direction of the externality and are able to say that privately optimal contracts deliver excessive incentive provision.

We now show that excessive incentive provision and excessive benchmarking give rise to crowded trades and excessive costs.

Proposition 3 (**Crowded Trades and Excessive Costs of Asset Management**). *Compared to the equilibrium corresponding to the privately optimal contract, in the equilibrium corresponding to the socially optimal contract*
(i) the asset prices are lower, $S^{social} < S^{private}$*;*
(ii) the managers' costs are lower, $\psi^\top x^M_{social} < \psi^\top x^M_{private}$.

As Proposition 3 shows, excessive use of incentive contracts by fund investors inflates prices and reduces returns per share. In addition, the costs of asset management are excessively high. Our model thus contributes to the debate on whether costs of asset management are excessive and whether returns delivered by the managers justify these costs.

Finally, we discuss the benchmark weights. Lemma 3 continues to be valid in the economy with socially optimal contracts. In addition, we can compare the tilts to high value-added and/or high-cost assets in the privately and socially optimal contracts.

Proposition 4 (**Comparison of Benchmark Weights**). *Suppose that Assumption 1 holds. Then the privately optimal benchmark underweights assets with higher value-added and overweights assets with higher costs compared to the socially optimal benchmark. Formally, consider two assets i and j, that have the exact same characteristics except $\Delta_i - \psi_i \geq \Delta_j - \psi_j$ and $\psi_i \leq \psi_j$, with at least one inequality being strict. Then $\theta_i^{social} - \theta_j^{social} > \theta_i^{private} - \theta_j^{private}$.*

The intuition behind this result is a little tricky. Compare (15) and (24), and recall that the role of $b\theta$ is to protect the manager from risk as well as provide incentives. The first term in each equation captures the insurance consideration, while the second relates to incentive provision. The planner understands that the incentives are less powerful than the fund investors believe. The need to provide incentives is driven by ψ, and hence the planner is more reluctant to use benchmark weights to provide incentives for high-ψ assets. And as the role of $b\theta$ in protecting the manager from risk is relatively more important than incentive provision, the planner will tilt the benchmark more towards high-value-added assets than individual fund investors would.

Remark 1 (**Redistribution Effects**)**.** Through our choice of weights in the social welfare function, we have shut down the contracts' redistribution effects and isolated the pecuniary externality that the planner desires to correct. For certain applications, such as those related to wealth inequality, however, it could be interesting to analyze the transfers from one set of agents to another that benchmarking generates. Allowing for redistribution changes outcomes depending on whether an agent is a (net) buyer of assets or a (net) seller. As we have argued, benchmarking boosts asset prices. This benefits (net) sellers of the assets at the expense of (net) buyers. If the social planner favors investors who have high endowments of assets and are planning to sell (e.g., the older generations), she has incentives to use more benchmarking in order to inflate prices to assist them, and vice versa if she favors net buyers (who are typically the younger generations).

Remark 2 (**Prices Relative to the First Best**)**.** According to Proposition 3, $S^{social} < S^{private}$. Surprisingly, the asset prices in the first-best case exceed equilibrium prices under both privately and socially optimal contracts, that is, $S^{social} < S^{private} < S^{FB}$.[35] So, equilibrium prices in the constrained optimum are not closer to the unconstrained-optimum prices than the decentralized-equilibrium ones, but are instead further away.[36] While this might be surprising at first glance, this result is in fact quite intuitive. Under the first best, the portfolio is observable and it is optimal to choose high-alpha portfolios. This, of course, will push up the asset prices and reduce expected returns. But, crowded trades are not a problem per se, because a pecuniary externality does not lead to an inefficiency in this case. In contrast, when the contract needs to provide incentives because the portfolio cannot be observed, a pecuniary externality does lead to an inefficiency, and crowded trades pose a problem as they reduce the effectiveness of incentive provision. While the comparison to the first best may not be relevant for practical purposes (as the first best is unattainable), it is helpful to highlight how exactly the mechanism that we explore works.

Remark 3 (**Achieving Social Optimum with Taxes**)**.** Given that privately optimal contracts result in an externality, it is natural to ask whether some sort of taxes could implement the constrained social optimum. We provide a detailed analysis of this question in Appendix B. We find the following. First, the manager's compensation needs to be

[35] The expression for the first-best asset prices is given in Lemma 5 in Appendix A. Comparing it to $S^{private}$ given in Lemma 2 immediately yields the result.

[36] This result parallels that in Lorenzoni (2008), where the decentralized equilibrium falls between the constrained and unconstrained optima in terms of amount of borrowing and asset prices. However, in Lorenzoni's model the inequality signs in the price comparison are reverse—decentralized-equilibrium asset prices are lower than in the constrained optimum (higher in our model) and higher than in the first best (lower in our model).

(proportionally) taxed to make it more costly for the fund investor to provide incentives to the manager. This type of tax mimics the higher cost of incentive provision for the planner, who internalizes the externality. Second, the fund returns net of the manager's compensation—which is the same as the fund investor's earnings in our model—should be (proportionally) subsidized. While this might be counterintuitive, the subsidy motivates the fund investor to lower a by increasing the benefit of keeping a larger $1 - a$. An alternative to the subsidy is imposing a cap $\bar{a}$ on the fund manager's "skin in the game." Of course, the tax and subsidy rates (or, alternatively, the tax rate and the cap on a) has to be chosen at particular levels that depend on the model parameters (see Appendix B for the formulas).

Remark 4 (**Endogenizing the Choice of Becoming a Fund vs. Direct Investor**). To zero in on the main mechanism we consider in the paper, we exogenously fixed the fractions of different agents in the population. One could endogenize the choice of becoming a fund investor or a direct investor, for example, by assuming a heterogenous cost of participating in the asset market. This type of extension would introduce another channel through which crowded trades matter. The choices of individual investors of whether to be a fund investor or a direct investor, in the aggregate, would determine the size of the asset management sector. This in turn would affect the strength of the externality that we identify in the paper (i.e., how much contracts affect prices and thus effectiveness of contracts designed by others). When making their decisions, the individual agents ignore this effect while the planner would account for this "extensive margin" of the externality when designing contracts.

5 Regulatory Relevance

While we have simplified our model to make our main points as clearly as possible, we believe it still delivers several robust implications that can inform some important ongoing regulatory debates. Probably the most important one relates to the regulation of fund managers' remuneration.

The first robust implication of our analysis is that if fund investors are trying to incentivize managers to take costly actions to generate superior returns (or alpha), then privately and socially optimal contracts differ. The divergence occurs because the privately optimal contracts fail to account for the effect of crowded trades, so that the planner wants to provide less incentives for the manager to undertake the activities that create the superior returns and the crowding.

The second implication is that because managers are risk averse, making their pay completely tied to realized returns would distort their portfolio choices. Benchmarking helps protect the manager from risk, which generates a portfolio that is better for both the manager and the fund investor. Thus, the optimal contracts involve both "skin in the game" and benchmarking. We argue in Remark 3 that in order to replicate the social optimum, one needs a pair of tools to target the choice of the two corresponding coefficients, a and b.

This observation is powerful because most of regulatory discussions of fund managers' remuneration focus on the split of the compensation that is fixed versus variable.[37] For instance, in the United Kingdom, investment funds must have some component of pay that is at risk. Furthermore, there is an ongoing debate between the United Kingdom and the European Union over how much bonus pay should be permitted. In the notation of our model, this kind of regulation could be mapped into a restriction on a. The model tells us that in the presence of benchmarking, the value of a for the social planner will be lower than private agents would choose.

One might be tempted to conclude that if a regulation were to lower a relative to what privately optimal contracts would specify, this restriction would be welfare increasing. In our model, however, merely adjusting the level of a can reduce or increase welfare relative to the equilibrium with privately optimal contracts. It is therefore important to stress that a regulation should also consider the role of the benchmark in determining compensation.

More generally, our analysis reflects a more fundamental problem with the standard regulatory discussion: it ignores how altering the risk-taking incentives via the form of the pay interacts with the crowded-trades problem. This is important, because if it were not for the crowded trades, then socially and privately optimal contracts would not diverge in the first place, so there would be no particular reason to worry about regulating these contracts.

Our model is not designed to provide empirical guidance on the appropriate level to which the regulations should be set. Nonetheless, it tells us that the basis for thinking that privately optimal contracts could be improved upon is that they fail to account for the externality that they can create in trying to generate proper incentives.

[37] There is also quite a bit of attention to the vesting rules for the variable pay, however, given that we have no dynamics in our model, we cannot comment on that.

6 Conclusions

We consider the problem of optimal incentive provision for portfolio managers in a general-equilibrium asset-pricing model. The optimal contacts involve benchmarking. We show that by ignoring the effects of contracts on equilibrium prices, fund investors impose an externality on each other—the effectiveness of their incentive contracts is lower than they perceive it to be. The reason is that contracts incentivize the managers to invest more in stocks with higher alpha as well as stocks in the benchmark. This boosts prices and lowers returns, making the marginal benefit of alpha-production lower for everyone. The social planner, who internalizes the effects of contracts on equilibrium prices, opts for less incentive provision, less benchmarking, and lower asset management costs.

In future work, it would be interesting to incorporate passive asset managers into the model. However, such an extension does present challenges. The existing evidence on the compensation contracts in the asset management industry covers only active funds. Very little is known about contracts of managers in passive funds. Before engaging in modeling of passive managers, it would be important to collect such evidence. A natural starting point would be to analyze the Statements of Additional Information filed by the U.S. mutual funds with the Securities and Exchange Commission, which contain information on managers' compensation structure. If contracts of passive managers turn out to be incentive contracts, it would be interesting to understand the incentive problem they solve. It is not obvious what kind of incentive problem would result in optimal contracts that make the managers closely follow the benchmark. We leave this problem for future work.

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Appendix A: Proofs

Proof of Lemma 1. Equation (3) immediately follows from taking the first-order condition of the direct investor's problem with respect to x. Similarly, (4) follows from taking the first-order condition of the manager's problem with respect to x. $\square$

Lemma 5 (First Best). *If x is observable or if $\psi = 0$, then the optimal contract is $a = 1/2$ and $b = 0$, and the asset prices are given by $S^{FB} = \mu - \gamma\Sigma\bar{x} + 2\lambda_M (\Delta - \psi)$.*

Proof of Lemma 5. When x is observable, the problem of the fund investor is simply to maximize $U^F + U^M$, or

$$\max_{a,b,\theta,x} \quad x^\top(\Delta - \psi + \mu - S)$$
$$-\frac{\gamma}{2}\left\{(ax - b\theta)^\top \Sigma(ax - b\theta) + [(1-a)x + b\theta]^\top \Sigma[(1-a)x + b\theta] + \left[a^2 + (1-a)^2\right]\sigma_\varepsilon^2\right\}.$$

The first-order condition with respect to x is

$$x^M = \Sigma^{-1}\frac{\Delta - \psi + \mu - S}{\gamma[a^2 + (1-a)^2]} + (2a-1)\frac{b\theta}{a^2 + (1-a)^2}.$$

The first-order condition with respect to $b\theta$ is $\gamma\Sigma(y - z) = 0$, where $y = ax - b\theta$ and $z = (1-a)x + b\theta$. The first-order condition with respect to a is $-\gamma\left[\Sigma(y-z)\right]^\top x + \gamma(1-2a)\sigma_\varepsilon^2 = 0$, which, using the first-order condition with respect to $b\theta$, implies $a = 1/2$. Then setting $b = 0$ satisfies the first-order condition with respect to $b\theta$.

The portfolio choice evaluated at the optimal contract is $x^M = \Sigma^{-1}(\Delta - \psi + \mu - S)/(\gamma/2)$. Using this, the first-best equilibrium asset prices are $S^{FB} = \mu - \gamma\Sigma\bar{x} + 2\lambda_M(\Delta - \psi)$. Comparing with (16), $S^{FB} > S^{private}$.

Finally, substituting the equilibrium prices into the demand function, the equilibrium asset holdings of the manager are $x^M_{FB} = 2\left[\bar{x} + \Sigma^{-1}\lambda_D(\Delta - \psi)/\gamma\right]$. Notice that if the manager holds a positive amount of each asset in the first best, then part (i) of Assumption 1 must hold. Therefore part (i) of Assumption 1 is a necessary condition for no short-selling to occur in the first best. $\square$

The Fund Investor's Problem in Terms of Exponential Utilities:

$$\max_{a,b,\theta,c,x=x^M} - E\exp\left\{-\gamma\left[\left(x^F_{-1}\right)^\top S + r_x - (ar_x - br_{\mathbf{b}}) - c\right]\right\}$$

subject to the manager's incentive constraint (4) and her participation constraint

$$-E\exp\left\{-\gamma\left[ar_x - br_{\mathbf{b}} + c\right]\right\} \geq \hat{u}_0, \tag{27}$$

where $\hat{u}_0$ is the exponential-utility version of u_0.[38] It is well known that CARA utility with normally-distributed returns can be rewritten in a mean-variance form, leading to the problem described in Section 4.2.

[38]In particular, if the manager's outside option is risk-free, then $\hat{u}_0 = -\exp(-\gamma u_0)$.

Proof of Lemma 2. (i) The first-order condition with respect to $b\theta/a$ is given by (10). Using (7) and (9), it can be rewritten as

$$0 = \Delta - \psi + \mu - S - \gamma\Sigma\left[\Sigma^{-1}\frac{\Delta - \psi/a + \mu - S}{\gamma}\left(\frac{1}{a} - 1\right) + \frac{b\theta}{a}\right],$$
$$\gamma\Sigma\frac{b\theta}{a} = \left(2 - \frac{1}{a}\right)(\Delta - \psi + \mu - S) + \left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{a}\right)\psi.$$

Using the expression for prices given in (5), this implies

$$\gamma\Sigma\frac{b\theta}{a} = \left(2 - \frac{1}{a}\right)\left[\Delta - \psi + \gamma\Sigma\Lambda\left(\bar{x} - \lambda_M\frac{b\theta}{a}\right) - \frac{\lambda_M}{a}\Lambda\left(\Delta - \frac{\psi}{a}\right)\right] + \left(1 - \frac{1}{a}\right)\left(\psi - \frac{\psi}{a}\right).$$

Rearranging terms and using the expression for Λ gives

$$\begin{aligned}
&\gamma\Sigma\left[1 + \left(2 - \frac{1}{a}\right)\Lambda\lambda_M\right]\frac{b\theta}{a} \\
&\quad = \gamma\Sigma\Lambda\left(2 - \frac{1}{a}\right)\bar{x} + \left(2 - \frac{1}{a}\right)\lambda_D\Lambda(\Delta - \psi) + \left[1 - \frac{1}{a} - \left(2 - \frac{1}{a}\right)\frac{\lambda_M}{a}\Lambda\right]\left(\psi - \frac{\psi}{a}\right), \\
&\gamma\Sigma\Lambda\frac{b\theta}{a} = \Lambda\left(2 - \frac{1}{a}\right)[\gamma\Sigma\bar{x} + \lambda_D(\Delta - \psi)] - \left[\frac{\lambda_M}{a} + \lambda_D\left(\frac{1}{a} - 1\right)\right]\Lambda\left(\psi - \frac{\psi}{a}\right), \qquad (28)
\end{aligned}$$

or

$$b\theta = (2a - 1)\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta - \psi)\right] + (1 - a)\left[\frac{1}{a} - \frac{\lambda_M}{a} - \lambda_D\right]\frac{\Sigma^{-1}}{\gamma}\psi.$$

The expressions for b and θ separately are then given by (14) and (15), respectively.

The first-order condition with respect to a is given by[39]

$$\begin{aligned}
0 &= \frac{\partial(U^F + U^M)}{\partial a} + \frac{\partial U^F}{\partial y}\frac{\partial y}{\partial a} \\
&= -(2a - 1)\gamma\sigma_\epsilon^2 - (\Delta - \psi + \mu - S - \gamma\Sigma z)^\top\frac{y}{a^2} + \frac{1 - a}{a}(\Delta + \mu - S - \gamma\Sigma z)^\top\frac{\partial y}{\partial a} \\
&= -(2a - 1)\gamma\sigma_\epsilon^2 + \frac{1 - a}{a}\psi^\top\frac{\partial y}{\partial a}, \qquad (29)
\end{aligned}$$

where the last equality follows from (10). Using $\partial y/\partial a = \Sigma^{-1}\psi/(\gamma a^2)$ (obtained by differentiating (9) with respect to a), we arrive at (13).

[39]Since the manager's utility is maximized with respect to y, $(\partial U^M/\partial y)(\partial y/\partial a)$ does not appear in (29).

(ii) Plugging (28) into the expression for prices (5), yields

$$\begin{aligned}
S^{private} &= \mu - \gamma\Sigma\Lambda\bar{x}\left[1 - \lambda_M\left(2 - \frac{1}{a}\right)\right] + \lambda_M(\Delta - \psi) + \frac{\lambda_M\Lambda}{a}\left[\lambda_M + a\lambda_D\right]\left(\Delta - \frac{\psi}{a}\right) \\
&= \mu - \gamma\Sigma\bar{x} + \lambda_M\left(2\Delta - \psi - \frac{\psi}{a}\right).
\end{aligned}$$

Substituting (11) into (4) and rearranging terms, implies $\gamma\Sigma x^M = (\Delta - \psi + \mu - S) + (\Delta - \psi/a + \mu - S)$. Substituting the expression for prices $S = S^{private}$ derived above and rearranging terms yields (17). $\square$

Proof of Proposition 1. Immediately follows from (14) and Assumption 1. $\square$

Proof of Lemma 3. Denote the (k, ℓ)-th element of matrix Σ^{-1} by $e_{k,\ell}$, where $e_{k,\ell} = e_{\ell,k}$ by symmetry. Since assets i and j are assumed to be identical (except for Δ's and ψ's), we have $e_{i,i} = e_{j,j}$ and $e_{i,k} = e_{j,k}$ for all $k \neq i, j$ (i.e., assets i and j have the same variance and covariance with other assets). As a result,

$$\theta_i - \theta_j = [e_{i,i} - e_{i,j}]\left\{\frac{2a-1}{b\gamma}\lambda_D[\Delta_i - \psi_i - \Delta_j + \psi_j] + \frac{1-a}{b\gamma}\left[\frac{1}{a} - \frac{\lambda_M}{a} - \lambda_D\right][\psi_i - \psi_j]\right\}.$$

Because Σ^{-1} is positive definite, we have $e_{i,i} > 0,\ e_{i,i}e_{j,j} - e_{i,j}^2 > 0,\ e_{i,i} > |e_{i,j}|$. As a result, $e_{i,i} - e_{i,j} > 0$, and thus $\theta_i > \theta_j$ whenever $\Delta_i - \psi_i \geq \Delta_j - \psi_j$, $\psi_i \geq \psi_j$, and at least one of the inequalities is strict. With a slight modification, this proof also applies to the socially optimal contract. $\square$

The Social Planner's Problem in Terms of Exponential Utilities:

$$\begin{aligned}
\max_{a,b,\theta,c,x=x^M,x^D} \; & -\tilde{\omega}_F E\exp\left\{-\gamma\left[\left(x_{-1}^F\right)^\top S + r_x - (ar_x - br_{\mathbf{b}}) - c\right]\right\} \\
& -\tilde{\omega}_D E\exp\left\{-\gamma\left[\left(x_{-1}^D\right)^\top S + \left(x^D\right)^\top (D - S)\right]\right\}
\end{aligned}$$

subject to (3), (4), and (27), where $\tilde{\omega}_i$, $i = S, C$, are the modified Pareto weights.

From the first-order condition with respect to c it follows that the Lagrange multiplier on the participation constraint equals $\tilde{\omega}_F MU_F/MU_M$, where MU_i denotes the expected marginal utility of agent i. This value is the effective Pareto weight on the manager's utility given that transfers between the fund investor and manager are allowed. Similarly, if transfers between fund investors and direct investors are allowed, then $\tilde{\omega}_F MU_F = \tilde{\omega}_D MU_D$, and the redistribution effects is zero. Without transfers, the Pareto weights that cancel out the redistribution effects (in the formulation with exponential utilities) are equal to inverse marginal utilities, $\tilde{\omega}_i = 1/MU_i$.

Rewriting the objective function and the participation constraint in the mean-variance form gives the problem described in Section 4.5.

Proof of Lemma 4. (i) The planner's first-order condition with respect to $b\theta/a$ is[40]

$$\begin{aligned}
&\left[\omega_F\left(x^F_{-1} - x^M\right)^\top + \omega_D\left(x^D_{-1} - x^D\right)^\top\right]\frac{\partial S}{\partial(b\theta/a)} \\
&+ \omega_F\left[(\Delta - \psi + \mu - S - \gamma\Sigma z)^\top + (\Delta + \mu - S - \gamma\Sigma z)^\top\left(\frac{1}{a} - 1\right)\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right] = 0.
\end{aligned}$$

Canceling out the redistribution effects, using $\partial y/\partial S = -\Sigma^{-1}/\gamma$ and $\partial S/\partial(b\theta/a) = \gamma\Sigma\Lambda\lambda_M$, and the expression for z, the above equation (or (19)) becomes

$$\begin{aligned}
&0 = \Delta - \psi + \mu - S - \gamma\Sigma z - (\Delta + \mu - S - \gamma\Sigma z)\frac{1-a}{a}\Lambda\lambda_M, \\
&0 = \Delta - \psi + \mu - S - \gamma\Sigma\left[\Sigma^{-1}\frac{\Delta - \psi/a + \mu - S}{\gamma}\left(\frac{1}{a} - 1\right) + \frac{b\theta}{a}\right] - \psi\frac{(1/a-1)\Lambda\lambda_M}{1-(1/a-1)\Lambda\lambda_M}.
\end{aligned}$$

Rearranging terms,

$$\begin{aligned}
\gamma\Sigma\frac{b\theta}{a} &= \Delta - \psi + \mu - S + \left(1 - \frac{1}{a}\right)\left(\Delta - \frac{\psi}{a} + \mu - S\right) - \psi\frac{(1-a)/a\Lambda\lambda_M}{1-(1/a-1)\Lambda\lambda_M}, \\
\gamma\Sigma b\theta &= (2a-1)(\Delta - \psi + \mu - S) + (1-a)\left[\frac{1-a}{a} - \frac{\Lambda\lambda_M}{1-(1/a-1)\Lambda\lambda_M}\right]\psi, \\
\gamma\Sigma b\theta &= (2a-1)(\Delta - \psi + \mu - S) + (1-a)\left(\frac{1}{a} - \frac{1}{\lambda_M + \lambda_D}\right)\psi. \qquad (30)
\end{aligned}$$

Alternatively, from (20),

$$\begin{aligned}
\gamma\Sigma\frac{b\theta}{a} &= \Delta - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi + \mu - S + \left(1 - \frac{1}{a}\right)\left(\Delta - \frac{\psi}{a} + \mu - S\right), \\
\gamma\Sigma b\theta &= (2a-1)\left[\Delta - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi + \mu - S\right] + (1-a)\left[\frac{1}{a} - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\right]\psi. \qquad (31)
\end{aligned}$$

Substituting the expression for prices into (30) leads to

$$b\theta = (2a-1)\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta - \psi)\right] + (1-a)\left[\frac{1}{a} - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\right]\frac{\Sigma^{-1}}{\gamma}\psi.$$

The expressions for b and θ separately are then given by (23) and (24).

[40] Because the manager's and direct investor's utilities are maximized with respect to y and x^D, respectively, by the Envelope theorem the only terms from their payoffs that enter the first-order conditions are those entering the redistribution term.

The planner's first-order condition with respect to a is

$$\begin{aligned}0 &= \frac{\partial(U^F + U^M)}{\partial a} + \frac{\partial U^F}{\partial y}\left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial a}\right] \\ &= (1-2a)\gamma\sigma_\epsilon^2 - (\Delta - \psi + \mu - S - \gamma\Sigma z)^\top \frac{y}{a^2} \\ &+ \frac{1-a}{a}(\Delta + \mu - S - \gamma\Sigma z)^\top \left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial a}\right].\end{aligned}$$

Compared to (29), there is an additional term containing $(\partial y/\partial S)(\partial S/\partial a)$. It reflects the planner's understanding that the contract affects prices, which in turn affect the managers' demands and thus the marginal benefit of alpha-production. However, unlike in the first-order condition with respect to $b\theta/a$, we cannot sign this extra term—recall that the effect of a on the manager's incentives is ambiguous. That is, *for a given* $b\theta/a$, the planner's benefit of using a can be higher or lower than that of an individual fund investor. Nonetheless, once the planner takes into account the adjustment in the optimal $b\theta$, the effect of a that reduces x^M (and thus lowers prices) is exactly offset by this adjustment. Thus the additional term that remains in the first-order condition with respect to a is only the part that takes into account how a higher a increases incentives for x^M, which in turn increases prices and reduces returns. Hence, the marginal benefit of a for the planner is lower than for fund investors, and the possibility of benchmarking is crucial for this result.

To see this formally, use (19) to rewrite the above equation as follows:[41]

$$\begin{aligned}0 &= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}\frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi^\top\left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial a} + \frac{y}{a^2}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right] \\ &= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}\frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi^\top\left[\frac{\partial y}{\partial a} - \frac{\partial y}{\partial a}\frac{\lambda_M/a}{\lambda_M/a + \lambda_D}\right].\end{aligned}$$

We can see that the effectiveness of incentive provision for the planner, captured by the term proportional to $\partial y/\partial a$, is smaller than for private fund investors in equation (12). Finally, using $\partial y/\partial a = \Sigma^{-1}\psi/(\gamma a^2)$, we obtain (22).

[41] To get the second line, differentiate the market-clearing condition $\lambda_M(y/a + b\theta/a) + \lambda_D x^D = 0$ with respect to $b\theta/a$ and a and use $\partial x^D/\partial S = \partial y/\partial S$ to get $\left(\frac{\lambda_M}{a} + \lambda_D\right)\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)} + \lambda_M = 0$ and $\left(\frac{\lambda_M}{a} + \lambda_D\right)\frac{\partial y}{\partial S}\frac{\partial S}{\partial a} - \lambda_M\frac{y}{a^2} + \frac{\lambda_M}{a}\frac{\partial y}{\partial a} = 0$ so that $\left(\frac{\lambda_M}{a} + \lambda_D\right)\left[\frac{\partial y}{\partial S}\frac{\partial S}{\partial a} + \frac{y}{a^2}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right] + \frac{\lambda_M}{a}\frac{\partial y}{\partial a} = 0.$

(ii) Following the same steps as in the proof of Lemma 2,

$$\begin{aligned}S^{social} &= \mu - \gamma\Sigma\bar{x} + \lambda_M(\Delta - \psi) + \lambda_M\left(\Delta - \frac{\psi}{a}\right) - \left(\frac{1}{a} - 1\right)\frac{\lambda_M^2}{\lambda_M + \lambda_D}\psi \\ &= \mu - \gamma\Sigma\bar{x} + \lambda_M\left(2\Delta - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi - \frac{\psi}{a}\right). \end{aligned} \tag{32}$$

Substituting (31) into (4) and rearranging terms, gives

$$\gamma\Sigma x^M = \left[\Delta - \frac{\lambda_M/a + \lambda_D}{\lambda_M + \lambda_D}\psi + \mu - S\right] + \left[\Delta - \frac{\psi}{a} + \mu - S\right].$$

Substituting (32) and rearranging terms yields (26). □

Proof of Proposition 2. (i) Comparison $a^{social} < a^{private}$ follows straightforwardly from comparing (13) and (22).

(ii) Denote $a_1 = a^{private}$ and $a_2 = a^{social}$. We first prove that $b^{social}/a^{social} < b^{private}/a^{private}$. From (14) and (23),

$$\begin{aligned}\frac{b_1}{a_1} &= \left(2 - \frac{1}{a_1}\right)\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta - \psi)\right] + \left(\frac{1}{a_1} - 1\right)\left[\frac{1}{a_1} - \frac{\lambda_M}{a_1} - \lambda_D\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi, \\ \frac{b_2}{a_2} &= \left(2 - \frac{1}{a_2}\right)\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta - \psi)\right] \\ &\quad + \left(\frac{1}{a_2} - 1\right)\left[\frac{1}{a_2} - \frac{1}{\lambda_M + \lambda_D}\left(\frac{\lambda_M}{a_2} + \lambda_D\right)\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi.\end{aligned}$$

Under Assumption 1, in order to show that $b_1/a_1 > b_2/a_2$, it is sufficient to show that

$$\left(\frac{1}{a_1} - 1\right)\left[\frac{1}{a_1} - \frac{\lambda_M}{a_1} - \lambda_D\right] > \left(\frac{1}{a_2} - 1\right)\left[\frac{1}{a_2} - \frac{1}{\lambda_M + \lambda_D}\left(\frac{\lambda_M}{a_2} + \lambda_D\right)\right],$$

which (given that both sides of the above inequality are positive) is equivalent to

$$\frac{(1 - a_1)/a_1^2}{(1 - a_2)/a_2^2}\frac{\lambda_M + a_1\lambda_D + (1 - 2a_1)\lambda_D}{(\lambda_M + a_2\lambda_D)\lambda_D/(\lambda_M + \lambda_D) + (1 - 2a_2)\lambda_D} > 1. \tag{33}$$

From (13) and (22) we have

$$\frac{1 - a_1}{a_1^3(2a_1 - 1)} = \frac{1 - a_2}{a_2^3(2a_2 - 1)}\frac{\lambda_D}{\lambda_M + \lambda_D}. \tag{34}$$

Substituting this into inequality (33), obtain

$$\frac{a_1(2a_1-1)}{a_2(2a_2-1)}\frac{\lambda_D}{\lambda_M+\lambda_D}\frac{\lambda_M+a_1\lambda_D+(1-2a_1)\lambda_D}{(\lambda_M+a_2\lambda_D)\lambda_D/(\lambda_M+\lambda_D)+(1-2a_2)\lambda_D}>1.$$

Since $a_1>a_2$, it suffices to show that

$$\frac{\lambda_M+a_1\lambda_D+(1-2a_1)\lambda_D}{(\lambda_M+a_2\lambda_D)\lambda_D/(\lambda_M+\lambda_D)+(1-2a_2)\lambda_D}>\frac{\lambda_D+\lambda_M}{\lambda_D},$$

which is equivalent to

$$\frac{\lambda_M(2a_2-1)}{\lambda_D(a_1-a_2)}>1. \tag{35}$$

To show (35), we will use equation (34). Rearranging (34) yields

$$\frac{1-a_1}{a_1^3(2a_1-1)}\frac{\lambda_M}{\lambda_D}=\frac{1-a_2}{a_2^3(2a_2-1)}-\frac{1-a_1}{a_1^3(2a_1-1)},$$

or, equivalently,

$$\frac{\lambda_M(2a_2-1)}{\lambda_D}=\frac{a_1^3}{1-a_1}\left[\frac{(1-a_2)(2a_1-1)}{a_2^3}-\frac{(1-a_1)(2a_2-1)}{a_1^3}\right].$$

The right-hand side of the above equation equals

$$\begin{aligned}&\frac{-a_1^3+2a_1^4-2a_1^4a_2+a_2a_1^3-(-a_2^3+2a_2^4-2a_2^4a_1+a_1a_2^3)}{(1-a_1)a_2^3}\\&=\frac{(a_1-a_2)}{(1-a_1)a_2^3}[-(1+2a_1a_2)(a_1^2+a_1a_2+a_2^2)+2(a_1+a_2)(a_1^2+a_2^2)+a_1a_2(a_1+a_2)].\end{aligned}$$

Rearranging terms and doing some more algebra, yields

$$\begin{aligned}&\frac{\lambda_M(2a_2-1)}{\lambda_D(a_1-a_2)}\\&=\frac{(2a_1-1)a_1^2(1-a_2)+(2a_2-1)a_2^2(1-a_1)+(2a_1-1)a_1a_2+2a_1a_2^2(1-a_1)}{a_2^3(1-a_1)}.\end{aligned}$$

Since $1/2 < a_2 < a_1 < 1$,

$$\frac{\lambda_M(2a_2-1)}{\lambda_D(a_1-a_2)} > \frac{(2a_1-1)a_1^2(1-a_2)+(2a_2-1)a_2^2(1-a_1)+(2a_1-1)a_1a_2+a_2^3(1-a_1)}{a_2^3(1-a_1)} > 1,$$

and thus (35) holds. Therefore $b_1/a_1 > b_2/a_2$. Using this and $a_1 > a_2 (> 1/2)$, it follows that $b_1 > b_2$. $\square$

Proof of Proposition 3. (i) The result follows immediately from comparing (16) and (25) and using part (i) of Proposition 2:

$$S^{private} - S^{social} = \lambda_M \left(\frac{1}{a^{social}} - \frac{1}{a^{private}}\right)\psi + \left(\frac{1}{a^{social}} - 1\right)\frac{\lambda_M^2}{\lambda_M+\lambda_D}\psi.$$

Since both terms on the right-hand side are strictly positive, $S^{private} > S^{social}$.

(ii) Using (17) and (26),

$$\psi^\top \left(x^M_{social} - x^M_{private}\right) = \lambda_D \psi^\top \frac{\Sigma^{-1}}{\gamma}\psi \left[1 - \frac{\lambda_M/a^{social}+\lambda_D}{\lambda_M+\lambda_D} + \frac{1}{a^{social}} - \frac{1}{a^{private}}\right].$$

Since Σ^{-1} is positive definite and the expression in square brackets is negative (because $a^{social} < a^{private} < 1$), we have $\psi^\top \left(x^M_{social} - x^M_{private}\right) < 0$. $\square$

Proof of Proposition 4. Denote $a_1 = a^{private}$ and $a_2 = a^{social}$, and let $e_{i,j}$ be the (i,j)-th element of matrix Σ^{-1} as defined in the proof of Lemma 3. Then

$$\begin{aligned}
\theta_i^{private} - \theta_j^{private} &= \frac{2a_1-1}{b_1\gamma}\lambda_D(e_{i,i}-e_{i,j})[\Delta_i - \Delta_j - (\psi_i - \psi_j)] \\
&\quad + \frac{1-a_1}{b_1\gamma}\left(\frac{1}{a_1} - \frac{\lambda_M}{a_1} - \lambda_D\right)(e_{i,i}-e_{i,j})(\psi_i-\psi_j), \\
\theta_i^{social} - \theta_j^{social} &= \frac{2a_2-1}{b_2\gamma}\lambda_D(e_{i,i}-e_{i,j})[\Delta_i - \Delta_j - (\psi_i - \psi_j)] \\
&\quad + \frac{1-a_2}{b_2\gamma}\left(\frac{1}{a_2} - \frac{\lambda_M/a_2+\lambda_D}{\lambda_M+\lambda_D}\right)(e_{i,i}-e_{i,j})(\psi_i-\psi_j).
\end{aligned}$$

Using similar steps as in the proof of $b_1 > b_2$ in part (ii) of Proposition 2 we can show that

$b_1/(2a_1-1) > b_2/(2a_2-1)$ and thus $(2a_1-1)/b_1 < (2a_2-1)/b_2$. Furthermore,

$$\begin{aligned}
&\frac{1-a_1}{b_1}\left[\frac{1}{a_1}-\frac{\lambda_M}{a_1}-\lambda_D\right] \\
&=\left[\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi+\frac{2a_1-1}{(1-a_1)(1/a_1-\lambda_M/a_1-\lambda_D)}\mathbf{1}^\top\left(\bar{x}+\frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta-\psi)\right)\right]^{-1}, \\
&\frac{1-a_2}{b_2}\left[\frac{1}{a_2}-\frac{\lambda_M/a_2+\lambda_D}{\lambda_M+\lambda_D}\right] \\
&=\left[\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi+\frac{2a_2-1}{(1-a_2)\left[1/a_2-(\lambda_M/a_2+\lambda_D)/(\lambda_M+\lambda_D)\right]}\mathbf{1}^\top\left(\bar{x}+\frac{\Sigma^{-1}}{\gamma}\lambda_D(\Delta-\psi)\right)\right]^{-1}.
\end{aligned}$$

From the proof of $b_1 > b_2$ in part (ii) of Proposition 2 we know that

$$\frac{2a_1-1}{(1-a_1)(1/a_1-\lambda_M/a_1-\lambda_D)} < \frac{2a_2-1}{(1-a_2)\left[1/a_2-(\lambda_M/a_2+\lambda_D)/(\lambda_M+\lambda_D)\right]},$$

so that

$$\frac{1-a_1}{b_1}\left[\frac{1}{a_1}-\frac{\lambda_M}{a_1}-\lambda_D\right] > \frac{1-a_2}{b_2}\left[\frac{1}{a_2}-\frac{\lambda_M/a_2+\lambda_D}{\lambda_M+\lambda_D}\right].$$

Hence when $\Delta_i-\psi_i \geq \Delta_j-\psi_j$ and $\psi_i \leq \psi_j$, and at least one inequality is strict, we have $\theta_i^{social}-\theta_j^{social} > \theta_i^{private}-\theta_j^{private}$. And conversely, if $\Delta_i-\psi_i \leq \Delta_j-\psi_j$ and $\psi_i \geq \psi_j$, and at least one inequality is strict, then we have $\theta_i^{social}-\theta_j^{social} < \theta_i^{private}-\theta_j^{private}$. That is, the socially optimal contract puts relatively less weight on incentive provision (compared to the privately optimal contract) and thus relatively more weight on protecting the manager from risk. $\square$

Lemma 6. *In both private and social optima, the second-order conditions are satisfied.*

Proof of Lemma 6. Denote by $F_{b\theta/a}$ and F_a (the left-hand sides of) the first-order conditions with respect to $b\theta/a$ and a, respectively. From the proofs of Lemmas 2 and 4, once we plug in the first-order condition with respect to $b\theta/a$ in the first-order condition with respect to a, the remaining terms only depend a. Thus we can write F_a in the following form: $F_a = g(a)+h(a,b\theta/a)^\top F_{b\theta/a}$. The function $g(a)$ is given by (the right-hand sides of) equations (13) and (22) in the privately and socially optimal cases, respectively.

Differentiating F_a with respect to a and $b\theta/a$,

$$F_{aa} = \frac{\partial F_a}{\partial a} = g'(a) + \frac{\partial h^\top(a, b\theta/a)}{\partial a} \underbrace{F_{b\theta/a}}_{=0} + h^\top(a,b) F_{b\theta/a,a},$$

$$F_{a,b\theta/a} = \frac{\partial F_a}{\partial(b\theta/a)} = \frac{\partial h^\top(a, b\theta/a)}{\partial(b\theta/a)} \underbrace{F_{b\theta/a}}_{=0} + h^\top(a, b\theta/a) F_{b\theta/a,b\theta/a}.$$

Notice that $g'(a) < 0$ (this follows from (13) in the privately optimal case and from (22) in the socially optimal case). Suppose we knew that $F_{b\theta/a,b\theta/a}$ is negative semi-definite. Then we can show that the following determinant has opposite sign of $\det(F_{b\theta/a,b\theta/a})$:

$$\begin{aligned}
&\det \begin{pmatrix} F_{b\theta/a,b\theta/a} & F_{b\theta/a,b\theta/a} h \\ h^\top F_{b\theta/a,b\theta/a} & g'(a) + h^\top F_{b\theta/a,b\theta/a} h \end{pmatrix} \\
&= \det(F_{b\theta/a,b\theta/a}) \det \left[h^\top F_{b\theta/a,b\theta/a} h + g'(a) - h^\top F_{b\theta/a,b\theta/a} (F_{b\theta/a,b\theta/a})^{-1} F_{b\theta/a,b\theta/a} h \right] \\
&= g'(a) \det(F_{b\theta/a,b\theta/a}) = -\det(F_{b\theta/a,b\theta/a}),
\end{aligned}$$

where the first equality follows from $\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A)\det(D - CA^{-1}B)$.

It remains to prove that $F_{b\theta/a,b\theta/a}$ is negative semi-definite. In the privately optimal case, $F_{b\theta/a,b\theta/a} = -\gamma\Sigma/a$. Since Σ is positive semi-definite, $F_{b\theta/a,b\theta/a}$ is negative semi-definite. Similarly, in the socially optimal case, $F_{b\theta/a,b\theta/a} = -\gamma\Sigma\Lambda\lambda_M - \gamma\Sigma/a$, also negative semi-definite. □

Appendix B: Achieving the Social Optimum with Taxes (for Online Publication)

This appendix analyzes how imposing taxes can implement the constrained socially optimal allocation and prices in the equilibrium in which contracts are chosen by fund investors. There are multiple ways of doing that, and we consider two alternatives here—one with proportional income taxes (or subsidies) on the managers and fund investors, the other with an income tax on the managers and a cap on a.

First, suppose there are proportional tax rates on the fund investors' and managers' incomes, denoted by t and t', respectively. The tax revenue—which is uncertain, given that the incomes are uncertain—is distributed to the fund investors as a lump-sum transfer T. Denote the constant and stochastic part of the transfer by τ_0 and τ so that $T =$

$\tau_0 + \tau^\top(\tilde{D} - S)$. How τ_0 and τ are determined is discussed later.

Since we want to implement the constrained optimal allocation, the taxes and the lump-sum transfer will be such that $y = (1-t')[ax - b\theta]$ and $z = (1-t)[(1-a)x + b\theta] + \tau$ are the same as in the constrained social optimum.

The utilities of the fund investor and manager with taxes can be written as

$$\begin{aligned}
U^F &= (1-t)(1-a)x^\top\Delta + z^\top(\mu - S) - c(1-t) + \tau_0 - \frac{\gamma}{2}\left[z^\top\Sigma z + (1-t)^2(1-a)^2\sigma_\varepsilon^2\right], \\
U^M &= (1-t')ax^\top\Delta - x^\top\psi + y^\top(\mu - S) + c(1-t') - \frac{\gamma}{2}\left[y^\top\Sigma y + (1-t')^2a^2\sigma_\varepsilon^2\right].
\end{aligned}$$

The manager's demand function is

$$x^M = \Sigma^{-1}\frac{\Delta - \psi/[a(1-t')] + \mu - S}{\gamma a(1-t')} + \frac{b\theta(1-t')}{a(1-t')}. \tag{36}$$

To implement the social optimum, we need $a(1-t') = a^{social}$ and $b\theta(1-t') = (b\theta)^{social}$.

From the first-order condition with respect to c, the Lagrange multiplier on the manager's participation constraint is $\xi = (1-t)/(1-t')$. The fund investor maximizes

$$\begin{aligned}
U^F + \xi U^M = {}& [(1-t)x + \tau]^\top(\Delta + \mu - S) + \tau_0 - \frac{1-t}{1-t'}x^\top\psi \\
& - \frac{\gamma}{2}\left\{z^\top\Sigma z + \frac{1-t}{1-t'}y^\top\Sigma y + (1-t)\left[(1-t)(1-a)^2 + (1-t')a^2\right]\sigma_\varepsilon^2\right\}
\end{aligned}$$

subject to the manager's incentive constraint (36), $y = (1-t')[ax - b\theta]$, and

$$z = (1-t)\left[\frac{1}{1-t'}\frac{1-a}{a}y + \frac{b\theta}{a}\right] + \tau.$$

The first-order condition with respect to $b\theta/a$ is

$$\begin{aligned}
&(1-t)(\Delta + \mu - S - \gamma\Sigma z) - \frac{1-t}{1-t'}\psi = 0, \\
&\Delta + \mu - S - \gamma\Sigma z - \frac{1}{1-t'}\psi = 0. \qquad (37)
\end{aligned}$$

Recall that the planner's first-order condition with respect to $b\theta/a$ is

$$\Delta + \mu - S - \gamma\Sigma z - \psi\frac{\lambda_M/a^{social} + \lambda_D}{\lambda_M + \lambda_D} = 0.$$

To equate the two, we need $1 - t' = (\lambda_M + \lambda_D)(\lambda_M / a^{social} + \lambda_D)$, or

$$t' = \frac{\lambda_M}{\lambda_M / a^{social} + \lambda_D} \frac{1 - a^{social}}{a^{social}}. \tag{38}$$

Intuitively, the positive tax on the manager's income inflates his costs relative to returns, which discourages him from investing in risky assets.

Notice, quite interestingly, that one tax rate, t' equates N first-order conditions (provided that $a(1-t') = a^{social}$), since θ is an $N \times 1$ vector.

The first-order condition with respect to a is

$$(1-t)\left[(1-t)(1-a) - (1-t')a\right]\gamma\sigma_\varepsilon^2 + (\Delta + \mu - S + \gamma\Sigma z)\frac{1-t}{1-t'}\frac{1-a}{a}\frac{\partial y}{\partial a} = 0.$$

Dividing by $1-t$ and using (37), $\partial y / \partial a = \Sigma^{-1}\psi / (\gamma a^2 (1-t'))$, and $a(1-t') = a^{social}$, the above condition can be rewritten as

$$\left[(1-t)(1-a) - (1-t')a\right]\gamma\sigma_\varepsilon^2 + \frac{1-a}{(a^{social})^3}\frac{\psi^\top \Sigma^{-1}\psi}{\gamma} = 0.$$

Recall that the planner's first-order condition with respect to a is

$$(1 - 2a^{social})\gamma\sigma_\varepsilon^2 + \frac{1 - a^{social}}{(a^{social})^3}\frac{\psi^\top \Sigma^{-1}\psi}{\gamma}\frac{\lambda_D}{\lambda_M + \lambda_D} = 0.$$

To equate the two, we need

$$\frac{1-a}{(1-t)(1-a) - (1-t')a} = \frac{\lambda_D}{\lambda_M + \lambda_D}\frac{1 - a^{social}}{1 - 2a^{social}}, \tag{39}$$

From $a = a^{social}/(1-t') = a^{social}(\lambda_M / a^{social} + \lambda_D)/(\lambda_M + \lambda_D)$, $1 - a = (1 - a^{social})\lambda_D / (\lambda_M + \lambda_D)$, and (39) simplifies to $(1-t)(1-a) - (1-t')a = 1 - 2a^{social}$, or

$$t(1-a) + t'a = 0. \tag{40}$$

Using the expression for t' given in (38) and $a = a^{social}/(1-t')$, we have

$$t = -\lambda_M / \lambda_D.$$

That is, in order to implement the constrained social optimum, the fund manager's income tax rate should be negative. Intuitively, in order to discourage the fund investor from setting

a too high, the subsidy should be used so that the fund investor effectively retains a larger share of the return for himself. His after-tax share of the return equals $(1-t)(1-a) = 1-(1-t')a$. That is, it is as if he only has to give $(1-t')a$ instead of a to the manager. Thus the income tax rates t and t' considered here effectively translate into the tax rates of t' imposed directly on a and $b\theta$ such that $(1-t')a = a^{social}$ and $(1-t')b\theta = (b\theta)^{social}$.

Finally, the transfer to the fund investor that balances the budget is

$$\begin{aligned} T &= [t(1-a)+t'a]x^{\top}(\Delta+\tilde{D}-S)+(t-t')[b\theta^{\top}(\tilde{D}-S)-c] \\ &= (t-t')[b\theta^{\top}(\tilde{D}-S)-c], \end{aligned}$$

where the last equality follows from (40), and so $\tau_0 = (t-t')c$ and $\tau = (t-t')b\theta$. Note that while $t-t' < 0$, the expected lump-sum transfer $(t-t)'\left[b\theta^{\top}(\mu-S)-c\right]$ can be negative or positive depending on the value of the manager's outside option, which pins down c.

An alternative scheme that achieves the social optimum is a combination of the income tax rate t' given by (38) imposed on the manager together with a cap (an upper bound) on the sensitivity of the manager's compensation with respect to the fund performance, a, at $\bar{a} = a^{social}/(1-t')$, so that $a \leq \bar{a} = (\lambda_M + a^{social}\lambda_D)/(\lambda_M+\lambda_D)$. As before, the total amount of tax revenue should be paid to the fund investor as a lump-sum transfer.

Appendix C: Incorporating the Manager's Effort Choice (for Online Publication)

In this appendix we extend the model in the main text to incorporate the manager's choice of effort. We will assume here that the effort choice is unobservable to the fund investor (the analysis of the case with observable effort is similar). We still assume, as in the main text, that the manager's portfolio choice is unobservable as well. We will demonstrate that our main insights extend in this case. In particular, the individual fund managers overestimate the effectiveness of incentive provision relative to the planner, which results in crowded trades.

For simplicity, we consider the case with one risky asset (and one risk-free bond). Consider general functional forms, namely, suppose the benefit function is $\tilde{\Delta}(x,e)$, the cost function is $\tilde{\psi}(x,e)$, and the variance of the noise term is $\tilde{\varepsilon}(x,e)$.

The manager's problem is

$$\max_{x,e} \quad a\tilde{\Delta}(x,e) - \tilde{\psi}(x,e) + (ax-b)(\mu - S) - \frac{\gamma}{2}\sigma^2(ax-b)^2 - \frac{\gamma}{2}a^2\tilde{\varepsilon}(x,e) + c.$$

The first-order conditions with respect to e is

$$\frac{\partial\tilde{\Delta}}{\partial e} - \frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} - \frac{\gamma}{2}a\frac{\partial\tilde{\varepsilon}}{\partial e} = 0. \tag{41}$$

Think of the optimal effort solving (41) as $e^*(x,a)$.

We impose the following assumptions.

Assumption 2. *Suppose that for each $a \in [1/2, 1]$, the function*

$$a\tilde{\Delta}(x,e) - \tilde{\psi}(x,e) - \frac{\gamma a^2}{2}\left[x^2 + \varepsilon(x,e)\right]$$

is concave in (x,e). Moreover, denote

$$\frac{df(x, e^*(x,a))}{dx} = \frac{\partial f}{\partial e}\frac{\partial e^*}{\partial x} + \frac{\partial f}{\partial x},$$

where function f is either $\tilde{\Delta}$, $\tilde{\psi}$, or $\tilde{\varepsilon}$, and $e^*(x,a)$ is implicitly defined by (41). Suppose that for each $a \in [1/2, 1]$,

$$\frac{d\psi}{dx} > \frac{\gamma}{2}\left|\frac{d\varepsilon}{dx}\right|, \quad \frac{d^2\psi}{dx^2} \geq \frac{\gamma}{2}\left|-\frac{d^2\varepsilon}{dx^2}\right|.$$

The above inequalities require that the manager's private cost is sufficiently increasing and sufficiently convex in x (once the optimal effort choice is taken into account).

We now proceed with the analysis of the manager's problem. The manager's first-order condition with respect to x (taking into account the fact that x affects the optimal choice of effort according to $e^*(x,a)$) is

$$\mu - S - \gamma\sigma^2(ax-b) + \frac{d\tilde{\Delta}}{dx} - \frac{1}{a}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}a\frac{d\tilde{\varepsilon}}{dx} = 0. \tag{42}$$

Assumption 2 implies that the second-order conditions are satisfied, in particular,

$$SOC_x \equiv -\gamma\sigma^2 a + \frac{d^2\tilde{\Delta}}{dx^2} - \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} - \frac{\gamma a}{2}\frac{d^2\tilde{\varepsilon}}{dx^2} < 0.$$

In what follows, we will use expressions for the effects of b and a on x that we derive below. Differentiating (42) with respect to b,

$$\gamma\sigma^2 + SOC_x \frac{\partial x}{\partial b} = 0,$$

$$\frac{\partial x}{\partial b} = -\frac{\gamma\sigma^2}{SOC_x} = \frac{\gamma\sigma^2}{\gamma\sigma^2 a - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}} > 0.$$

Denote $\frac{dx}{di} \equiv \frac{\partial x}{\partial i} + \frac{\partial x}{\partial S}\frac{\partial S}{\partial i}$, $i \in \{a, b\}$. Taking a full derivative of (42) with respect to b,

$$\gamma\sigma^2 - \frac{\partial S}{\partial b} + SOC_x \frac{dx}{db} = 0. \tag{43}$$

Differentiating the market-clearing condition $\lambda_M x + \lambda_D x^D = \bar{x}$ with respect to b (and using the expression for x^D in the main text),

$$\lambda_M \frac{dx}{db} + \lambda_D \frac{\partial x^D}{\partial S}\frac{\partial S}{\partial b} = \lambda_M \frac{dx}{db} - \lambda_D \frac{1}{\gamma\sigma^2}\frac{\partial S}{\partial b} = 0,$$

$$\frac{\partial S}{\partial b} = \gamma\sigma^2 \frac{\lambda_M}{\lambda_D}\frac{dx}{db}.$$

Substituting this into (43),

$$\frac{dx}{db} = \frac{\gamma\sigma^2}{\gamma\sigma^2\frac{\lambda_M}{\lambda_D} - SOC_x} = \frac{\gamma\sigma^2}{\gamma\sigma^2\left(a + \frac{\lambda_M}{\lambda_D}\right) - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}.$$

Notice that $\frac{dx}{db} \leq \frac{\partial x}{\partial b}$, with strict inequality if $\lambda_M > 0$.

Similarly, differentiating (42) with respect to a,

$$-\gamma\sigma^2 x + \frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx} + SOC_x\frac{\partial x}{\partial a} = 0,$$

$$\frac{\partial x}{\partial a} = \frac{1}{\gamma\sigma^2 a - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}\left[\frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx}\right] - x\frac{\partial x}{\partial b}. \tag{44}$$

The last term captures the negative effect of a on x because the manager is exposed to too much aggregate risk—the effect which b offsets. There is a new effect that we did not have before—a larger a reduces x if $\tilde{\varepsilon}$ is increasing in x because it exposes the manager to more idiosyncratic risk, and this risk cannot be offset by an increase in b. Notice that without it (as in the main text), we would have $\partial x/\partial a + x\partial x/\partial b > 0$, which captures the fact with

b offsetting the negative effect of a on x, we are only left with the positive effect that is coming from reducing the effective cost. We want to make sure that $\partial x/\partial a + x\partial x/\partial b > 0$. Notice that if this was not the case, it would not be optimal for the fund investor to use a for incentive provision purposes. Assumption 2 ensures that, and we have

$$\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b} = \frac{\frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx}}{\gamma\sigma^2 a - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}} > 0.$$

Similarly, we have

$$\frac{dx}{da} + x\frac{dx}{db} = \frac{\frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx}}{\gamma\sigma^2\left(a + \frac{\lambda_D}{\lambda_M}\right) - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}},$$

which is smaller than $\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b}$.

We now turn to the analysis of the fund investor's problem. Denoting $y = ax - b$ and $z = x - y$, this problem is

$$\max_{a,b,c,x} \quad (1-a)\tilde{\Delta}(x, e^*(x,a)) + z(\mu - S) - \frac{\gamma\sigma^2}{2}z^2 - \frac{\gamma(1-a)^2}{2}\tilde{\varepsilon}^2(x, e^*(x,a)) - c$$

subject to the manager's participation constraint and incentive constraint (42) (in which we substituted $e^*(x,a)$ implicitly defined by (41)).

The fund investor's first-order condition with respect to b is

$$\frac{d(U^F + U^M)}{db} = \frac{\partial U^F}{\partial x}\frac{\partial x}{\partial b} + \underbrace{\frac{\partial U^M}{\partial x}}_{=0}\frac{\partial x}{\partial b} + \frac{\partial(U^F + U^M)}{\partial b} = 0. \tag{45}$$

The last term captures how b directly affects the social welfare by linearly transferring from y to z. The first term captures the indirect effect of b on social welfare through its effect on the manager's demand x. Intuitively, notice that $\partial U^F/\partial x$ should be positive, otherwise b would not be positive. We will show that $\partial U^F/\partial x > 0$ formally below. The last term in (45) is

$$\frac{\partial(U^F + U^M)}{\partial b} = -\frac{\gamma\sigma^2}{2}\frac{\partial(y^2 + z^2)}{\partial b} = \gamma\sigma^2(y - z) = \gamma\sigma^2\left[(2a-1)x - 2b\right].$$

We will show below that this term is negative (notice that this term would be zero under perfect risk sharing $a = 1/2$ and $b = 0$.)

Using (42),

$$\frac{\partial U^F}{\partial x} = (1-a)\left[\frac{d\tilde{\Delta}}{dx} + \mu - S - \gamma\sigma^2 z - \frac{\gamma}{2}(1-a)\frac{d\tilde{\varepsilon}}{dx}\right]$$
$$= (1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right].$$

Then the investor's first-order condition with respect to b becomes

$$(1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\frac{\partial x}{\partial b} + \gamma\sigma^2(y-z) = 0, \tag{46}$$

or equivalently

$$\frac{(1-a)\dfrac{\partial x}{\partial b}}{(1-a)\dfrac{\partial x}{\partial b}+1}\left[\frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right] + \gamma\sigma^2(y-z) = 0. \tag{47}$$

Notice that since the first term is strictly positive by Assumption 2, the second term is strictly negative. It then also follows that the term in the square brackets in 46 must be strictly positive, that is, $\partial U^F/\partial x = \partial(U^F + U^M)/\partial x > 0$. Intuitively, it means that it is optimal for the fund investor to use contracts to provide incentives. It also then follows that $b > 0$. Indeed, notice that at $b = 0$ and $a \in [1/2, 1]$, the left-hand side of (47) is strictly positive given Assumption 2, and thus $b \leq 0$ cannot be optimal.

We will now compare the social planner's first-order condition with respect to b to that of an individual fund investor. The planner's first-order condition with respect to b (after canceling out the redistribution effects, as in the main text) is the same as the corresponding first-order condition for an investor, but $\partial x/\partial b$ is being replaced with dx/db, namely

$$\frac{\partial U^F}{\partial x}\frac{dx}{db} + \frac{\partial(U^F + U^M)}{\partial b} = 0,$$

or

$$\frac{(1-a)\dfrac{dx}{db}}{(1-a)\dfrac{dx}{db}+1}\left[\frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right] + \gamma\sigma^2(y-z) = 0.$$

Since $dx/db < \partial x/\partial b$ as long as $\lambda_M > 0$,

$$\frac{(1-a)\dfrac{dx}{db}}{(1-a)\dfrac{dx}{db}+1} < \frac{(1-a)\dfrac{\partial x}{\partial b}}{(1-a)\dfrac{\partial x}{\partial b}+1}.$$

It then follows that under Assumption 2, the additional terms in the planner's first-order condition relative to the investor's first-order condition are strictly negative.

Now consider the first-order condition with respect to a. In the privately optimal case, it is

$$\frac{d(U^F+U^M)}{da} = \frac{\partial U^F}{\partial x}\frac{\partial x}{\partial a} + \frac{\partial U^F}{\partial e}\frac{\partial e}{\partial a} + \frac{\partial(U^F+U^M)}{\partial a} = 0.$$

Rewrite this to get

$$\begin{aligned}\frac{d(U^F+U^M)}{da} &= (1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\frac{\partial x}{\partial a} \\ &\quad + (1-a)\left[\frac{\partial\tilde{\Delta}}{\partial e} - \frac{\gamma}{2}(1-a)\frac{\partial\tilde{\varepsilon}}{\partial e}\right]\frac{\partial e}{\partial a} - \gamma\sigma^2(y-z)x - \gamma\varepsilon^2(2a-1). \\ &= (1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\frac{\partial x}{\partial a} \\ &\quad + (1-a)\left(\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} + \frac{\gamma}{2}(2a-1)\frac{\partial\tilde{\varepsilon}}{\partial e}\right)\frac{\partial e}{\partial a} - \gamma\sigma^2(y-z)x - \gamma\varepsilon^2(2a-1) = 0.\end{aligned}$$

where the second equality uses (41). Then using (46), we can rewrite the above condition as follows:

$$\begin{aligned}&(1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\left(\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b}\right) \\ &\quad + (1-a)\left(\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} + \frac{\gamma}{2}(2a-1)\frac{\partial\tilde{\varepsilon}}{\partial e}\right)\frac{\partial e}{\partial a} - \gamma\varepsilon^2(2a-1) = 0.\end{aligned}$$

Using (47), the fund investor's first-order condition with respect to a becomes

$$\begin{aligned}&\frac{(1-a)\left(\dfrac{\partial x}{\partial a} + x\dfrac{\partial x}{\partial b}\right)}{(1-a)\dfrac{\partial x}{\partial b}+1}\left[\frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right] + (1-a)\left(\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} + \frac{\gamma}{2}(2a-1)\frac{\partial\tilde{\varepsilon}}{\partial e}\right)\frac{\partial e}{\partial a} \\ &- \gamma\varepsilon^2(2a-1) = 0. \qquad (48)\end{aligned}$$

Notice that we need $d\tilde{\psi}/dx > 0$ or $\partial\tilde{\psi}/\partial e > 0$, otherwise $a = 1/2$ is optimal. This is guaranteed by Assumption 2.

The social planner's first-order condition with respect to a is obtained from (48) by replacing

$$\frac{(1-a)\left(\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b}\right)}{(1-a)\frac{\partial x}{\partial b} + 1} = \frac{\left(\frac{1}{a} - 1\right)\left(\frac{1}{a}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}a\frac{d\tilde{\varepsilon}}{dx}\right)}{\gamma\sigma^2 - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}$$

by a strictly smaller term,

$$\frac{(1-a)\left(\frac{dx}{da} + x\frac{dx}{db}\right)}{(1-a)\frac{dx}{db} + 1} = \frac{\left(\frac{1}{a} - 1\right)\left(\frac{1}{a}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}a\frac{d\tilde{\varepsilon}}{dx}\right)}{\gamma\sigma^2\left(1 + \frac{\lambda_D}{\lambda_M}\right) - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}.$$

Recall that the term in square brackets in (48) is strictly positive (by Assumption 2). Therefore in the socially optimal case, there are additional negative terms (or the positive terms are smaller) in the first-order condition with respect to a relative to that in the privately optimal case.

As in the main model in the text, the planner recognizes that incentive provision is weaker than how individual fund investors perceive it. This is captured by additional negative terms in the first-order conditions for a and b. Establishing that this implies that both a and b in the socially optimal case are smaller than those in the privately optimal case is no longer straightforward, and requires imposing additional assumptions on the cross-derivatives and third derivatives of the functions $\tilde{\Delta}$, $\tilde{\psi}$ and $\tilde{\varepsilon}$, which are hard to interpret. Intuitively though, b is used to undo some costs that arise from using a larger a, so we would expect that the planner sets both a and b lower than those in the privately optimal case.

We can still prove the crowded trades result, namely, $S^{social} < S^{private}$. Define $k = (a, b)$, $W(k, S) = U^F(k, S, x(k, S), e^*(k, x(k, S))) + U^M(k, S, x(k, S), e^*(k, x(k, S)))$. The fund investor's problem is to maximize $W(k, S)$ with respect to k taking S as given. Since we cancel out redistribution effects in the social planner's problem, it is equivalent to maximizing $W(k, S(k))$ with respect to k.

Denote the optimal solutions in the privately and socially optimal cases by $k^*_{private}$ and

k^*_{social}, respectively. Notice that

$$W(k^*_{social}, S(k^*_{social})) > W(k^*_{private}, S(k^*_{private})) > W(k^*_{social}, S(k^*_{private}))$$

implying

$$W(k^*_{social}, S(k^*_{social})) > W(k^*_{social}, S(k^*_{private})). \tag{49}$$

Differentiating W with respect to S (and canceling the redistribution effects),

$$\frac{dW}{dS} = \frac{\partial U^F}{\partial x}\frac{dx}{dS} = (1-a)\left\{\gamma\sigma^2\left[(2a-1)x(S) - 2b\right] + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right\}\frac{dx}{dS} < 0.$$

Differentiating with respect to S one more time,

$$\begin{aligned}\frac{d^2W}{dS^2} &= \frac{dW_s}{dx}\left(\frac{dx}{dS}\right)^2 + W_s \underbrace{\frac{d^2x}{dS^2}}_{=0} \\ &= \left[\gamma\sigma^2(2a-1)x + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}(2a-1)\frac{d^2\tilde{\varepsilon}}{dx^2}\right]\left(\frac{dx}{dS}\right)^2 > 0\end{aligned}$$

by Assumption 2. Since $dW(k^*_{social}, S)/dS < 0$ at $S = S^*_{social}$, this implies that $W(k^*_{social}, S(k^*_{social})) < W(k^*_{social}, S)$ for $S < S(k^*_{social})$. Given inequality 49, it must be the case $S(k^*_{social}) < S(k^*_{private})$. It then also follows that $x(k^*_{social}) < x(k^*_{private})$. So the crowded trade results from the main text extends to the case with unobservable effort.

Appendix D: Contractible Revenues of Return-Augmenting Activities (for Online Publication)

In this appendix we consider what happens if the revenue from the return-augmenting activities, $x^\top \Delta + \varepsilon$, is contractible. We will show that our main results extend, namely, benchmarking is still optimal, and socially and privately optimal contracts differ as the planner recognizes that incentive provision is less effective than how fund investors perceive it. This holds unless there is only one risky stock or all stocks are identical. Intuitively, while the investors have one more instrument, this instrument is not enough to fine-tune incentives for multiple stocks and reach the first best.

Suppose the manager receives a fraction $\tilde{a}$ of it, so that her compensation is

$$w = (ax^\top - b\theta^\top)(\tilde{D} - S) + \tilde{a}(x^\top \Delta + \varepsilon) + c.$$

As in the main text, denote $y = ax^\top - b\theta^\top$. Then the manager's problem can be written as follows:

$$\max_{y} \quad \left(\frac{y}{a} + \frac{b}{a}\right)^\top (\tilde{a}\Delta - \psi) + c + y^\top(\mu - S) - \frac{\gamma}{2} y^\top \Sigma y + c - \frac{\gamma \tilde{a}^2 \sigma_\varepsilon^2}{2}.$$

The manager's first-order condition with respect to y is

$$y = \Sigma^{-1} \frac{(\tilde{a}\Delta - \psi)/a + \mu - S}{\gamma},$$

and the equilibrium prices are

$$S = \mu - \gamma \Sigma \Lambda \left(\bar{x} - \lambda_M \frac{b}{a}\right) - \Lambda \frac{\lambda_M}{a} \frac{\tilde{a}\Delta - \psi}{a}.$$

Denoting $z = y(1-a)/a + \theta b/a$, the fund investor's problem is

$$\begin{aligned}
\max_{a,\tilde{a},b,c,y} & \left(\frac{y}{a} + \frac{b}{a}\right)^\top (1 - \tilde{a})\Delta + z^\top(\mu - S) - \frac{\gamma}{2} z^\top \Sigma z - \frac{\gamma(1-\tilde{a})^2 \sigma_\varepsilon^2}{2} - c \\
\text{s.t.} & \left(\frac{y}{a} + \frac{b}{a}\right)^\top (\tilde{a}\Delta - \psi) + y^\top(\mu - S) - \frac{\gamma}{2} y^\top \Sigma y + c - \frac{\gamma \tilde{a}^2 \sigma_\varepsilon^2}{2} + c \geq u_0, \\
& y = \Sigma^{-1} \frac{(\tilde{a}\Delta - \psi)/a + \mu - S}{\gamma}.
\end{aligned}$$

As in the main text, the first-order condition with respect to $b\theta/a$ is

$$\begin{aligned}
& \frac{\partial(U^F + U^M)}{\partial(b\theta/a)} + \frac{\partial U^F}{\partial y} \frac{\partial y}{\partial(b\theta/a)} = 0, \\
& \Delta - \psi + \mu - S - \gamma \Sigma z = 0.
\end{aligned}$$

This is a vector, which is equal to zero element by element.

The planner's first-order condition with respect to $b\theta/a$ (after canceling out redistribution effects, as in the main text) is

$$\frac{\partial(U^F+U^M)}{\partial(b\theta/a)}+\frac{\partial U^F}{\partial y}\left[\frac{\partial y}{\partial(b\theta/a)}+\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right]=0,$$
$$(\Delta-\psi+\mu-S-\gamma\Sigma z)^\top+[(1-\tilde{a})\Delta+(1-a)(\mu-S-\gamma\Sigma z)]^\top\frac{1}{a}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}=0.$$

The additional terms in the planner's first-order condition with respect to $b\theta/a$ evaluated at the privately optimal contract are

$$\frac{1}{a}\left[(1-a)\psi+(a-\tilde{a})\Delta\right]^\top\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}=\frac{1-a}{a^2}\Lambda\lambda_M\left[(1-a)\psi+(a-\tilde{a})\Delta\right]^\top. \tag{50}$$

Notice that this is a vector; for the additional terms to be zero, this vector would have to be zero element by element.

The first-order condition with respect to a in the privately optimal contract (after substituting the first-order condition with respect to $b\theta/a$) can be written as

$$\begin{aligned}0&=\left[(1-a)\psi+(a-\tilde{a})\Delta\right]^\top\frac{\partial y}{\partial a}\\&=\left[(1-a)\psi+(a-\tilde{a})\Delta\right]^\top\frac{1}{a\gamma}\Sigma^{-1}\left(\frac{\tilde{a}\Delta}{a}-\frac{\psi}{a}\right).\end{aligned} \tag{51}$$

The right-hand side is a number. Notice that this equality does *not* imply that (50) is zero element by element unless there is only one stock, or all stocks are identical. The first-order condition with respect to a in the socially optimal contract is

$$\frac{\partial(U^F+U^M)}{\partial a}+\frac{\partial U^F}{\partial y}\left[\frac{\partial y}{\partial a}+\frac{\partial y}{\partial S}\frac{\partial S}{\partial a}\right]=0,$$

which can be rewritten as

$$\begin{aligned}0&=\left[(1-a)\psi+(a-\tilde{a})\Delta\right]^\top\frac{\partial y}{\partial a}\\&=\left[(1-a)\psi+(a-\tilde{a})\Delta\right]^\top\frac{1}{a\gamma}\Sigma^{-1}\left(\frac{\tilde{a}\Delta}{a}-\frac{\psi}{a}\right)\frac{\lambda_D}{\lambda_M+\lambda_D}.\end{aligned}$$

Compared to (51), the right-hand side is only scaled down by a constant, so for the same $\tilde{a}$, the planner's choice of a coincides with the fund investor's. However, we will see that $\tilde{a}^{social}\neq\tilde{a}^{private}$.

The first-order condition with respect to $\tilde{a}$ in the privately optimal contract is

$$\begin{aligned} 0 &= [(1-a)\psi + (a-\tilde{a})\Delta]^\top \frac{\partial y}{\partial \tilde{a}} + (1-2\tilde{a})\gamma\sigma_\varepsilon^2 \\ &= [(1-a)\psi + (a-\tilde{a})\Delta]^\top \frac{1}{a\gamma}\Sigma^{-1}\Delta + (1-2\tilde{a})\gamma\sigma_\varepsilon^2. \end{aligned}$$

Notice that because the vectors $\partial y/\partial \tilde{a}$ and $\partial y/\partial a$ are different from each other, $\tilde{a} = 1/2$ generally does not solve the above equation. The corresponding first-order condition in the socially optimal contract is

$$\frac{\partial(U^F + U^M)}{\partial \tilde{a}} + \frac{\partial U^F}{\partial y}\left[\frac{\partial y}{\partial \tilde{a}} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial \tilde{a}}\right] = 0.$$

This can be rewritten as

$$0 = [(1-a)\psi + (a-\tilde{a})\Delta]^\top \frac{1}{a\gamma}\Sigma^{-1}\Delta\frac{\lambda_D}{\lambda_M + \lambda_D} + (1-2\tilde{a})\gamma\sigma_\varepsilon^2.$$

Notice that this implies that there are additional negative terms in the planner's first-order condition with respect to $\tilde{a}$ as compared to the privately optimal first-order condition. As we saw above, the same is true for the fist-order condition with respect to $b\theta/a$, while the first-order condition with respect to a is undistorted.

Thus we conclude that, as in the main model, benchmarking is optimal and privately and socially optimal contracts differ.