How Credit Constraints Impact Job Finding Rates, Sorting & Aggregate Output*

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Abstract

We examine empirically and theoretically the effect of consumer credit limits on job finding rates and the subsequent replacement earnings of displaced workers. Using new administrative data, we find that in response to an increase in credit limits equal to 10% of prior annual earnings, medium-tenure displaced mortgagors take .5 to 2 weeks longer to find a job and, conditional on finding a job, their replacement rates are 0 to 2.2% greater. They are also more likely to work at larger and more productive firms. We construct a labor sorting model with credit to provide a structural estimate of the duration and replacement rate elasticities, which we find to be .2 weeks and 1.8%, respectively. We use the model to assess what happens if consumer credit limits contract during a recession. We find that when credit limits tighten during a downturn, employment rises but both labor productivity and output exhibit weaker recoveries. The tension between recovery speed and recovery health is due to the fact that when limits tighten, low-asset, low-productivity job losers are unable to self-insure. Subsequently, they search less thoroughly and take relatively more accessible jobs at less productive firms. As a result, standard measures of sorting improve.

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Recent research by Kaplan and Violante [2014] has shown that many households, even those with large amounts of wealth, have very little liquid assets. At the same time, many of these households have significant amounts of credit access (Herkenhoff [2013]). This generates a potentially important consumption smoothing role of consumer credit for ‘hand-to-mouth’ households who lose their jobs. While much is known theoretically and empirically about the impact of unemployment benefits on employment outcomes (inter alia Katz and Meyer [1990], Ljungqvist and Sargent [1998], Acemoglu and Shimer [1998], Chetty [2008], Mitman and Rabinovich [2012], and Hagedorn et al. [2013]), little is known about the role consumer credit plays in the search decisions of unemployed households, and even less is known about how this interaction affects the macroeconomy.\footnote{The nascent but growing literature on the topic has focused on two mechanisms, the self-insurance role of credit (e.g. Athreya and Simpson [2006], Herkenhoff [2013], Athreya et al. [2014]) and labor demand effects of credit (e.g. Bethune et al. [2013], Donaldson et al. [2014], Bethune [2015]). The equally sparse empirical literature on unemployment and borrowing is limited due to data constraints (Hurst and Stafford [2004], Sullivan [2008], Herkenhoff and Ohanian [2012] among others) but recent inroads are being made with new account data (Baker and Yannelis [2015], Gelman et al. [2015] among others).} How does consumer credit affect job finding rates, replacement earnings, or the types of jobs workers take? How does access to consumer credit affect the allocation of workers to firms, and what does this imply for labor productivity, output, and employment dynamics?

By answering these questions, we make several contributions. Our empirical contribution is to merge 5 million individual credit reports with administrative employment records to measure the impact of consumer credit access on job finding rates and re-employment earnings of displaced workers. We find that being able to replace 10\% more of prior annual labor earnings with personal revolving credit allows medium-tenure displaced mortgagors take .5 to 2 weeks longer to find a job, and, among those who find a job, they obtain a 0 to 2.2\% greater annual earnings replacement rate. Moreover, households with greater access to credit find jobs at larger firms and more productive firms. These results are consistent with individuals using personal credit to fund longer unemployment spells so that they can search and find better job matches; however, these results make an important point previously overlooked in existing empirical studies which is that independent of realized borrowing, the potential to borrow affects search decisions regardless if credit lines are actually drawn down.

To our knowledge, we are the first to measure the elasticity of non-employment durations and the elasticity of replacement rates with respect to consumer credit access.

Our theoretical contribution is to develop a general equilibrium labor sorting model with consumer credit. Relative to existing labor sorting models such as Shimer and Smith [2000], Shi [2001], Lise and Robin [2013] and Eeckhout and Sepahsalari [2014], our framework
generates interactions between heterogeneous credit histories and the allocation of workers to firms. While our focus is on households, this framework is tractable enough that it can be used by future researchers to study a variety of questions related to misallocation and credit access, including credit access among firms.

In our model, heterogeneous credit-constrained workers accumulate human capital while working. When unemployed, they direct their search, as in Menzio and Shi [2010, 2011], for jobs among heterogeneous firms. Firms differ with respect to capital and produce output by combining the human capital of workers with their own physical capital (for simplicity we refer to firm capital as physical capital, but this may also be thought of as intellectual capital). We assume supermodularity, meaning that firms with greater amounts of physical capital produce more with workers who have greater amounts of human capital. We therefore measure sorting in the model as the raw correlation coefficient between worker human capital and firm physical capital. Which worker matches with which firm determines both output and labor productivity in this economy, and therefore the ability of unemployed households to self-insure, either through saving or borrowing, and search for higher physical capital jobs has the potential to change the path of a recovery.

Using this new theoretic framework, we make two quantitative contributions. First, we estimate the model using public data in order to provide a set of independent structural estimates of the elasticity of durations and the elasticity of replacement rates with respect to consumer credit access. We find an elasticity of durations with respect to unused credit of .15 (implying about a .2 week longer non-employment duration with a credit line worth 10% of prior income) which falls at the low end of our reduced form estimates. We also estimate an elasticity of earnings with respect to unused credit, among those who find a job, of approximately 1.8%.

Second, given that the model produces earnings and duration elasticities that are consistent with the data, we use the model as a laboratory to answer the question of what

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2Related theoretical work includes sorting models with frictions (inter alia Teulings and Gautier [2004], Bagger and Lentz [2008], Eeckhout and Kircher [2011], Hagedorn et al. [2012], Bonhomme et al. [2014]), frictionless assignment models with borrowing constraints (Fernandez and Gali [1999], Legros and Newman [2002], and Strauss [2013]), and occupational choice under credit constraints (inter alia Neumuller [2014], and Dinlersoz et al. [2015]). Our work is also related to Lentz [2009], Krusel et al. [2010] and Nakajima [2012a] who have studied the impact of savings on search decisions, and Guerrieri and Lorenzoni [2011] among others have looked at the role household borrowing constraints play in models with frictionless labor markets.

3This measure is highly correlated with the Spearman Rank correlation coefficient. We also report the Spearman Rank correlation coefficient for completeness.
happens to labor sorting, productivity, and the ensuing employment recovery if consumer credit limits expand or contract during a recession. The main experiment we conduct using the model is to tighten borrowing limits during the 2007-2009 recession and then study the subsequent recovery. In particular, we simulate the 2007-2009 recession by feeding actual total factor productivity residuals into the model. During the recession, we permanently tighten borrowing limits, delivering roughly a 3 percentage point reduction in the fraction of households borrowing, and a 1 percentage point reduction in the aggregate debt to income ratio. Upon impact and throughout the recovery, the tighter credit limit depresses output per worker (labor productivity) by .28 percentage points and decreases overall output by .11 percentage points.

We find that when debt limits tighten, however, standard measures of sorting improve and remain elevated throughout the recovery. The reason is that constrained households, who are also more likely to have low human capital, take jobs with low physical capital. Households with savings, who are more likely to have high human capital, are able to continue to search thoroughly for jobs with higher physical capital. Since sorting measures the correlation between human capital and physical capital, and since this correlation actually increases when debt limits tighten, standard measures of sorting improve. Even though sorting improves, because the jobs that are being created during periods of tight debt limits are less-physical-capital-intensive jobs, output falls. What disconnects the positive comovement of sorting with labor productivity and output is the presence of firm investment and household credit constraints, which are typically absent in sorting models. These results, which to our knowledge are new, raise important questions about the welfare implications of sorting patterns.

Lastly, when debt limits tighten during the recession, employment recovers more quickly. The mechanism is that when credit limits tighten, unemployed low-human-capital-borrowers lose their ability to self-insure and take relatively abundant jobs at less-capital-intensive firms. In other words, constrained households take lower quality jobs, relatively fast. As a result, there is a tradeoff between the speed of the labor market recovery and the health of the recovery, measured by labor productivity and output.

The paper proceeds as follows. We first describe our conceptual framework in Section 1. Section 2 describes the data. Sections 3 and 4 contain our empirical results. Section 5 presents the model, Section 6 describes the model estimation and structural estimates of the duration elasticity, Section 7 conducts the main counterfactual exercise of tightening debt.
1 Job Finding and Unsecured Credit

Unsecured credit allows unemployed households to augment today’s liquid asset position by borrowing against future income. In McCall models of search, such as those studied by Athreya and Simpson [2006] and Chetty [2008], access to liquid assets allows households to search more thoroughly for higher wage jobs. While this mechanism is at the heart of the unemployment insurance literature, there is limited evidence linking access to liquid assets and job search decisions. In an influential paper, Chetty [2008] shows that workers who receive unemployment benefits take longer to find jobs, with the effect being strongest among low wealth households. He also shows that unemployed households who receive severance payments take significantly longer to find jobs. However, to our knowledge, there are no existing studies documenting the way consumer credit limits impact unemployment durations, subsequent wage outcomes, or the characteristics of the firms where these households ultimately take jobs. To fill this gap in the empirical literature, we test two hypotheses:

**Hypothesis 1:** Ceteris paribus, greater credit access among the unemployed increases non-employment durations.

**Hypothesis 2:** Ceteris paribus, greater credit access among the unemployed increases subsequent re-employment earnings.

It is important to note that because durations increase with greater credit access, the theoretic prediction of credit access on earnings (including zeros) is ambiguous since those who have more credit are taking longer to find a job and have zero earnings. However, conditional on finding a job, we test whether those with greater credit access find higher wage jobs (this is what we mean by re-employment earnings).

2 Data and Definitions

Our main data source is a randomly drawn panel of 5 million TransUnion credit reports which are linked by a scrambled social security number to the Longitudinal Employment and Household Dynamics (LEHD) database. All consumer credit information is taken from
TransUnion at an annual frequency from 2001 to 2008. The TransUnion data includes information on the balance, limit, and status (delinquent, current, etc.) of different classes of accounts held by individuals. The different types of accounts include unsecured credit as well as secured credit on mortgages.

The LEHD database is a quarterly matched employer-employee dataset that covers 95% of U.S. private sector jobs. The LEHD includes data on earnings, worker demographic characteristics, firm size, firm age, and average wages. Our main sample of earnings records includes individuals with credit reports between 2001 and 2008 from the 11 states for which we have LEHD data: California, Illinois, Indiana, Maryland, Nevada, New Jersey, Oregon, Rhode Island, Texas, Virginia, and Washington. Since job dismissal and reason of dismissal are not recorded in the LEHD, we follow Jacobson et al. [1993] and focus on mass layoffs.4

We then define several labor market variables of interest. First, we define non-employment duration to be the number of quarters it takes an individual to find a job following a mass displacement.5 Non-employment duration therefore takes values ranging from 0 (indicating immediate job finding) to 9 (all spells longer than 9 quarters of non-employment are assigned a value of 9).6

Second, we define replacement earnings as the ratio of annual earnings 1 year after layoff over annual pre-displacement earnings. Suppose a worker is displaced in year \( t \), then we define the replacement earnings ratio to be the ratio of annual earnings in the year after layoff, in year \( t + 1 \), to the pre-displacement annual earnings, in year \( t - 1 \). To avoid attributing the duration of non-employment with replacement earnings, when we measure replacement earnings, we condition on households who have a full year of earnings in year \( t + 1 \). We consider alternate definitions of replacement rates that include zeros as well as longer-term measures of replacement earnings (e.g. in year \( t + 2 \)) in Appendix [X].

In terms of TransUnion variables, our main measure of credit access is an individual’s unused credit limit across all types of revolving debt (excluding any mortgage related revolving debt) over annual earnings (hereafter, ‘unused revolving credit limit ratio’), measured prior to displacement.7 We focus on revolving credit because it can be drawn down on short no-

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4The data appendix, Appendix A, includes a detailed explanation of how we identify mass layoffs.
5We follow Abowd et al. [2009] (Appendix A, Definitions of Fundamental LEHD Concepts) to construct our measures of job accessions and employment at end-of-quarter.
6Very few workers in our sample of displaced workers remain non-employed for longer than 4 quarters. Changing the censored value to 8 or 10 has no impact on the results.
7In the main text, to control for housing wealth, we will control directly for HELOCs and mortgage credit.
tice following job loss and paid-off slowly over time without any additional loan-applications or income-checks. The main components of revolving credit include bank revolving (bank credit cards), retail revolving (retail credit cards), and finance revolving credit (other personal finance loans with a revolving feature).

3 Empirical Approach

The goal of this section is to estimate the impact of credit access on employment outcomes of displaced workers. While many authors, including Jacobson et al. [1993], have argued that mass layoffs are exogenous to worker characteristics, credit access upon layoff certainly is not. To solve this issue, we need to find a characteristic of workers that impacts credit limits and only impacts employment prospects through its impact on credit limits. To isolate such exogenous variation in credit limits, we use three sets of orthogonal instruments which we discuss in detail in the following sections: (i) geographic instruments, (ii) bankruptcy flag removals, and (iii) individual-specific account-age instruments.

3.0.1 Saiz Instrument

The first approach is to follow Saiz [2010] and Mian and Sufi [2012] who exploit variation in geography to answer a wide variety of questions. The idea is to instrument the unused credit limit ratio of workers with the geographic constraints of the MSA in which the worker lives and then use this exogenous component of unused credit limits to measure the impact of credit on various employment outcomes such as job finding rates and earnings replacement rates. Mian and Sufi [2012] have laid much of the groundwork for this instrument by showing that geographic constraints significantly impact house price growth as well as leverage and are orthogonal to labor markets except through their impact on leverage (their samples always exclude construction workers and real-estate related sectors). Our analysis relies on

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The results are very similar if we instrument revolving credit including HELOCs, or total credit including HELOCs, which is shown in Appendix B.2. The standard measure of credit limits in this paper corresponds to the TransUnion variable ‘Revolving High Credit/Credit Limit.’ This variable is constructed as the sum of the ‘High Credit/Credit Limit’ across all types of revolving debt. The ‘High Credit/Credit Limit’ is defined as the actual credit limit if such a limit is recorded or the highest historical balance if no credit limit is recorded. We then subtract the total current revolving credit balance to arrive at the unused credit limit. We exclude HELOCs in our benchmark specifications.

8We provide a detailed discussion of identification assumptions in Section 4.4.
the arguments made in Mian and Sufi [2012], but, rather than focusing on realized leverage (realized borrowing), the channel we emphasize is that geographic constraints impact house price growth, and house price growth is a determinant of credit access, and in particular, credit limits.

There are two reasons why house prices determine access to revolving credit: (i) workers have more access to capital and are less likely to default, increasing the propensity of lenders to extend any type of credit, and (ii) lenders expect workers to consume more, and therefore offer more credit cards since they profit from transaction volume (not just balances). In the first-stage regression, from a purely statistical point of view, we show that the Saiz [2010] geographic constraint instrument is a strong predictor of the unused credit limit ratio of individual workers for the 38 MSAs present in our sample. In the second stage regression, the predicted unused credit limit ratios from the first stage are used to measure the impact of credit on non-employment durations and annual earnings replacement rates.

Consider the sample of workers laid-off due to plant closure in year $t$. Let $D_{i,t}$ denote the non-employment duration (in quarters and capped at 9 quarters) of individual $i$ who is laid off in year $t$. Let $l_{i,t-1}$ denote the unused limit ratio of individual $i$ in year $t-1$, the year prior to layoff. Let $X_{i,t}$ include static demographic controls as well as year dummies and MSA-level aggregate economic controls. The first-stage regression is to predict the unused credit limit ratio as a function of the housing supply elasticity, $s_{i,t}$.

$$l_{i,t-1} = \pi s_{i,t} + BX_{i,t} + u_{i,t}$$ (1)

These first-stage estimates of $\pi$ and $B$ are used to isolate the exogenous component of the unused credit limit ratio, $\hat{l}_{i,t-1}$. The second stage regression is then used to estimate how this exogenous variation in credit impacts employment outcomes such as duration, $D_{i,t}$.

$$D_{i,t} = \gamma \hat{l}_{i,t-1} + \beta X_{i,t} + \epsilon_{i,t}$$ (2)

9Due to the frequency of the credit reports, we use annual credit limit information. Likewise, the earnings information is annual earnings. Durations of non-employment are measured in quarters.
3.0.2 Bankruptcy Flag Removals

Our second strategy is to follow Musto [2004] who exploits the fact that bankruptcy flags are removed after ten years, and this gives rise to a large exogenous increase in credit access which does not reflect changes in underlying credit worthiness or unobserved quality of the household.\(^\text{10}\) Consider the set of displaced workers who have a bankruptcy flag on their record prior to displacement, and let \(Removal_{i,t}\) equal 1 if the worker has their bankruptcy flag removed in the year of displacement. Our empirical approach is to compare those who have their bankruptcy flag removed in the year of displacement (the treatment group) to those whose flags remain on their record in the year of displacement (the control group). We therefore implement the following specification in order to isolate the impact of increased credit access on outcome variables such as duration, \(D_{i,t}\),

\[
D_{i,t} = \gamma Removal_{i,t} + \beta X_{i,t} + \epsilon_{i,t}
\] (3)

In this specification \(\gamma\) represents how much longer a displaced worker takes to find a job if their flag is removed relative to the control group of households whose flag is not removed.

3.0.3 Gross and Souleles Instrument

The last instrument is based on the identification strategy of Gross and Souleles [2002] who exploit the fact that credit card limits increase automatically as a function of the length of time an account has been open.\(^\text{11}\) We exploit these time-contingent changes in credit access by using the age of the oldest account as the instrument \(s_{i,t}\) for credit limits in the first stage regression of equation (1). The main challenge to exogeneity for this instrument is that account ages are related to physical ages. Unlike credit scoring companies, however, we observe physical age. We argue and provide supporting evidence with overidentification tests that, after controlling for physical age as well as a host of other individual characteristics, that account age satisfies the exclusion restriction. See Appendix [X] for more discussion.

\(^\text{10}\) Chapter 7 Bankruptcy flags (the most pervasive type of bankruptcy) remain on a worker’s credit report for a statutory 10 years, whereas Chapter 13 flags are removed after 7 years. We cannot differentiate between these types of flag removal, but what matters for us is that the removal is statutory.

\(^\text{11}\) As they explain, credit issuers revise account limits regularly, and the length of time an account was open is a determinant of these credit limit revisions. They write, “many issuers will not consider (or are less likely to consider) an account for a line change if it has been less than six months or less than one year since the last line change” (p.7).
3.1 Samples

We use two samples in this paper.

i. **Displaced Mortgagor Sample:** Our first sample includes displaced workers with mortgages who had at least 3 years of tenure at the time of displacement, worked in a non-construction or non-real-estate industry, and worked at a firm with at least 25 employees. These are standard restrictions used in the literature, e.g. Davis and Von Wachter [2011], to mitigate any issues associated with seasonal employment or weak labor-force attachment. Given these criteria we end up with a sample (to the nearest thousand) of 32,000 individuals.\(^\text{12}\) Given the way we identify displacements, and the use of lagged credit prior to displacement, this sample covers the years 2002-2006.

ii. **Displaced Bankrupt Sample:** Our second sample includes all displaced households who had a bankruptcy flag on their record in the year prior to displacement. We then split this sample into a treatment group of 1,000 individuals whose flags are removed in the year of displacement, and a control group of 17,000 individuals whose flags remain on their records throughout the displacement. In order to garner enough observations to disclose this sample, we had to drop the tenure requirement to 1 year. This sample covers the years 2002-2006.

3.2 Descriptive Statistics and Raw Correlations

Table 1 includes summary statistics for the displaced mortgagor sample. All variables are deflated by the CPI, and the top 1% (and bottom 1% if the variable is not bound below) of continuous variables are winsorized. Column (1) of Table 1 shows that among all displaced workers, the average age is X years old and they worked at their prior job for about X years before the mass layoff. On average, their annual labor earnings were about $X prior to layoff. Workers can replace on average X% of their prior annual labor earnings with unused revolving credit.\(^\text{13}\) Workers involved in mass displacements take roughly X quarters to find a new job. Annual earnings replacement rates are X% one year after mass displacement,\(^\text{12}\) Census requires sample numbers to be rounded off to the nearest hundred to ensure no individual data is disclosed or can be inferred. We round to the nearest thousand to allow for quicker disclosure of results.\(^\text{13}\) The distribution of replacement rates is skewed. In the SCF, unused credit card limits to annual family income among the unemployed peaks at 38% in 1998, and among the employed it peaks at 33% in 2007.
including zeros, similar to what Davis and Von Wachter [2011] find. Finally, the age of the oldest account is approximately 14 years on average in our sample.

Column (2) of Table 1 shows that if we condition on households who have a job in year \( t + 1 \), the average duration is quite short at X quarters, and the average replacement rate is close to 1. This measure of replacement rates, conditional on employment, drops ‘zeros’ and parses out any effects of duration on earnings.

Figure 1 plots the duration of non-employment by unused revolving credit to income decile prior to layoff for displaced mortgage sample, and Figure 2 plots the replacement rate by unused revolving credit to income decile prior to layoff for the displaced mortgage sample, conditional on finding a job. The deciles of unused revolving credit to income range from approximately zero to roughly 200%. In other words, those in the top decile can approximately replace 2x their annual income with revolving credit. Both figures reveal a generally monotone increasing relationship between unused credit prior to layoff and both durations and replacement rates, with a pronounced rise in the last decile of unused credit.

In our displaced bankrupt sample, on average the demographic characteristics are quite close, with imputed years of education being X for those who have their flag removed (the treatment group) and Y for those who do not have their flag removed (the control group) and average tenure being X in the treatment group and Y in the control group. Those whose flags are removed are naturally older, with an average age of X in the treatment group and an average age of Y in the control group. The treatment group earned X before displacement whereas the control group earned Y; while there are level differences in earnings, we show in a companion paper that in general, those who have flag removals and those who do not have flag removals have parallel earnings trends [CITE BK PAPER]. The treatment group takes X quarters to find a job on average after displacement, but their replacement rate is Y, whereas the control group takes X quarters on average to find a job after displacement, but their replacement rate is Y. We show in the regression analysis that follows that the difference in duration is significant after adjusting for composition, whereas the difference in replacement rates is not.
4 Empirical Results

For each dependent variable, we show four sets of results corresponding to the baseline OLS estimate, firstly, and then the three instruments, (i) geography, (ii) flag removals, and (iii) account ages. In every specification, our vector of controls \( (X_{i,t}) \) includes quadratics in age and tenure as well as sex, race and education dummies, lagged annual income, cumulative lagged earnings (to proxy for assets), 1-digit SIC industry dummies, a dummy for the presence of auto loans, an equity proxy (the highest mortgage balance observed less the current balance), HELOC limits, as well as year dummies, the MSA unemployment rate, MSA income per capita, and lagged characteristics of the previous employer including the size, age, and wage per worker.

4.1 Duration Results

Table 2 illustrates the impact of unused credit on durations. Column (1) of Table 2 illustrates the results from a simple OLS regression of duration on unused credit limits. The estimates reveal a duration elasticity of .3, which implies that being able to replace 10% more of prior annual income with unused credit is associated with an increase in duration of 2 days (.36 weeks). In Column (2), we instrument the unused credit limit with the Saiz [2010] instrument. We find a duration elasticity of 1.4 which implies that being able to replace 10% more of prior annual income with unused credit allows workers to take roughly 1.5 weeks longer to find a job. In Column (3) we instrument the unused credit limit with the Gross and Souleles [2002] instrument. We find a duration elasticity of .6 which implies that being able to replace 10% more of prior annual income with unused credit allows workers to take roughly .5 weeks longer to find a job. Column (4) uses the displaced bankrupt sample, and shows the impact of a derogatory flag drop on duration. Column (4) shows that if an individual’s bankruptcy flag is dropped in the year of displacement, households take on average .15 quarters longer to find a job (about 1.5 weeks longer).

To convert the flag removal result into an elasticity, Table 3 shows that a bankruptcy flag is associated with a contemporaneous increase in revolving credit limits equal to 7% of prior annual income and an increase in credit scores of 140 pts. This implies an elasticity of durations with respect to credit access of nearly 2 weeks.

Our estimates imply that $1 of additional unused credit limit is about half as potent for
unemployment durations as $1 of unemployment benefit. Being able to replace 5% of annual earnings on a credit card is equivalent to a 10% increase in UI replacement rates for the typical 6-month unemployment benefit. In the empirical UI literature, the impact of a 10% increase in the UI replacement rate for 6 months is to increase unemployment durations by .3 to 2 weeks with the modal estimate lying between .5 and 1 for the US (see Nakajima [2012b] and Card et al. [2015] for a summary of recent empirical and quantitative elasticities). Our IV estimates imply an equivalent elasticity with respect to credit of .25 to 1 week.

For robustness, Appendix B.5 merges our sample with Schedule C tax records to adjust the non-employment spells for self-employment. Appendix B.5 also uses the earnings gap method to infer partial quarters of non-employment. Under either of these definitions of non-employment duration, we find that the main results are robust.

4.2 Earnings Replacement Rates

**Earnings Replacement Rates Including Zeros:** Table 4 illustrates the impact of unused credit on replacement rates of annual earnings, including zeros. In general, the impact of additional credit access is ambiguous, and statistically indistinguishable from zero. There are two competing forces generating this result: (i) durations increase with more credit access, depressing replacement earnings, (ii) of those who find a job, those who have more credit access find higher wages, increasing replacement rates. In the remainder of the section, we isolate the second effect.

**Earnings Replacement Rates Excluding Zeros:** To avoid confounding annual replacement earnings with durations, in Table 5 we isolate the set of households who have positive earnings in each quarter during the year after layoff. Table 5 reveals that conditional on finding a job, those with greater credit access find higher wage jobs.

Table 5 shows that being able to replace 10% more of prior annual income increases the replacement rate of job finders by .4% in the OLS regressions, by 2.2% in using the Saiz Instrument, and by .5% using the Gross and Souleles instrument. Column (4) shows that if an individual’s bankruptcy flag is dropped in the year of displacement, there is a small negative, but statistically insignificant impact of credit access on replacement rates.

These results are, in general, in line with US estimates in the UI literature. Studies that have considered the impact of unemployment benefits on re-employment earnings have found
positive and significant but mixed-magnitude effects in US data (see Addison and Blackburn [2000] for a summary), whereas European studies have found both positive and insignificant effects, as well as negative effects in one case (see Nekoei and Weber [2015] for a summary).

4.3 Size and Productivity of Firms

In this section, we show that of those who find a job in the year after displacement, those with greater credit access are more likely to work at larger and more productive firms. Our main dependent variables include an indicator function if the worker finds a job at a firm in the 99th percentile of the firm size distribution or better (‘Large Firm Dummy’), measured 1 year after displacement, as well as an indicator function if the worker finds a job at a firm in the 75th decile of the wage-per-worker distribution (aggregate wage bill divided by total employees), which is our proxy for productivity. These deciles were chosen for comparability: i.e. firms in the 99th percentile of the size distribution comprise approximately 1/3 of employment, and firms in the 75th percentile of the wage-per-worker distribution comprise approximately 1/3 of employment.

Table 6 illustrates the impact of unused credit on the odds that a worker finds a job at a firm in the 99th decile of the size distribution or greater. Being able to replace 10% more of prior annual income increases the odds that a worker finds a job at a large firm by X% in the OLS regressions, by Y% in using the Saiz Instrument, by Z% using the Gross and Souleles instrument, and X% following a bankruptcy flag removal.

Table 7 illustrates the impact of unused credit on the odds that a worker finds a job at a firm in the 75th decile of the wage-per-worker (our proxy for labor productivity) distribution or greater. Being able to replace 10% more of prior annual income increases the odds that a worker find a job at a productive firm by X% in the OLS regressions, by Y% using the Saiz Instrument, by Z% using the Gross and Souleles instrument, and X% following a bankruptcy flag removal.

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14 What we call firms in the text are State Employment Identification Numbers (SEINs) in the LEHD. SEINs aggregate all plants within a state.
4.4 Discussion of Identifying Assumptions for Saiz Instrument

There are two main challenges to exogeneity of the Saiz [2010] instrument: (i) aggregate conditions, and (ii) housing wealth. We conduct a thought experiment to address the first challenge. While Mian and Sufi [2012] argue that there is no correlation between the supply elasticity and aggregate conditions except through leverage, if there is a correlation, it should bias our results toward zero. Suppose MSAs with low supply elasticities have quickly rising house prices and have better labor markets, then credit should expand and non-employment durations should be shorter in those MSAs. This is the exact opposite of what our IV estimates reveal.

To mitigate concerns about housing wealth, we include an equity proxy (the highest mortgage balance ever observed less the current balance) and HELOC limits (home equity lines of credit) in each table. When trying to capture wealth effects coming from house price appreciation, HELOC limits isolate the relevant portion of housing wealth for job loss episodes. HELOC limits indicate what portion of the home can be used as an ATM immediately following a job loss. We argue that the HELOC credit limit just prior to a job loss is a good proxy for access to liquid housing assets during the non-employment spell. The idea is that whether the house is worth 200k or 220k should not affect short term job search decisions directly. Only if the job loser can use the equity of the home to smooth consumption, should the value of the home impact short term job search decisions.

Moreover, since the average job loss spell in our data is quite short, it is unlikely that a worker who is laid off will be able to secure additional home equity lines, or vacate the home immediately and sell the house. As Piazzesi et al. [2015] show empirically, it takes over 1 quarter for the median homeowner to sell their home, once it is listed. In our sample, we do not see workers disproportionately selling their homes during or after layoff.

Moreover, in Appendix B.4, we conduct OLS regressions of unemployment duration on unused credit, directly controlling for the OFHEO house price index. We show that the inclusion of house prices does not affect our point estimates.

To further address the role of housing wealth and wealth more generally, we turn to the Survey of Consumer Finances (SCF) in Appendix B.3. Using the 1998 to 2007 SCF surveys, we show that the relationship in the SCF between non-employment duration and unused credit card limits (the only limit available in the SCF) is similar to our IV estimates and unaffected by the direct inclusion of self-report home values, liquid assets, or illiquid assets.
Lastly, in terms of direct tests of exogeneity, there are none, but in Appendix B.1 we conduct overidentification tests. Table 12 uses both the housing supply elasticity and the age of the oldest account, to achieve over-identification. In all cases, the instruments pass overidentification tests.

### 4.5 Borrowing by Displaced Workers

One important point of our empirical section is that regardless of realized borrowing, the potential to borrow affects job search decisions, regardless if the credit line is actually drawn down. Workers know that if their buffer stock of liquid assets is depleted, they can borrow, and this affects their job search decisions even if they never borrow. Existing work by Sullivan [2008] using the PSID and SIPP has shown that about 20% of workers borrow during unemployment, and it is precisely low wealth workers who borrow during unemployment. Unfortunately we do not have wealth information, but we plot the distribution of bankcard borrowing among displaced workers. Figure 3, which is a smoothed density, plots the change in bankcard balance among displaced workers 1 year after layoff relative to one year before layoff. The graph reveals significant heterogeneity in borrowing responses among displaced workers. Some workers borrow, consistent with Sullivan [2008], whereas some workers save, also consistent with Sullivan [2008]'s regression results. As a result, the net amount borrowed among displaced workers is close to zero, consistent with recent findings (e.g. Gelman et al. [2015]). However, this masks quite large and economically significant heterogeneity in borrowing by displaced workers.

We further explore the role of borrowing by displaced workers in Figure 4 which illustrates the change in real revolving debt \( [X] \) in the year of layoff relative to 1 year before layoff as a function of duration.\(^{15}\) Figure 4 shows that borrowing is a weakly increasing function of unemployment duration, so it appears that those who were able to take the longest to find a job were those who were able to borrow the most. The graph is a raw mean, and the standard error is a standard error for the mean, so there is significant composition bias still present in the graph. To address these concerns, Table 8 illustrates regression results for the relationship between non-employment duration and borrowing, controlling for as many characteristics of workers as possible. Column (1) omits controls, and Column (2)

\(^{15}\)To obtain more power in the graph, durations are recoded to increase sample sizes, (i.e. durations of length 0 or 1 are coded as 1, durations length 2 to 3 are coded as 3, 3-4=4,5-6=6,7-8=8, 9=9).
includes controls. The coefficient in Column (2) implies that for every additional quarter of non-employment, workers borrow on average $200. As discussed above, however, this regression masks significant heterogeneity in how much displaced workers borrow, which we are exploring in future research.

5 Model

To get around the inherently local nature of our IV estimates, we now turn to a structural model which we use to obtain independent ‘global’ estimates of the unemployment duration and replacement rate elasticities. We then use the model to conduct our main experiment which is to consider how changes in aggregate borrowing limits impact the allocation of workers to firms, output, and productivity.

Let \( t = 0, 1, 2, \ldots \) denote time. Time is discrete and runs forever. There are three types of agents in this economy. A unit measure of risk averse finitely-lived households, a continuum of risk neutral entrepreneurs that run the endogenously chosen measure of operating firms, and a unit measure of risk neutral lenders.

As in Menzio et al. [2012], there are \( T \geq 2 \) overlapping generations of risk averse households that face both idiosyncratic and aggregate risk. Each household lives \( T \) periods deterministically and discounts the future at a constant rate \( \beta \in (0, 1) \). Every period households first participate in an asset market where they make asset accumulation, borrowing, and bankruptcy decisions. After the asset market closes, households enter the labor market where they direct their search for jobs.\(^{16}\) Let \( c_{t,t+t_0} \) and \( L_{t,t+t_0} \) respectively denote the consumption and hours worked of an agent born at date \( t \) in period \( t + t_0 \). The objective of a household is to maximize the expected lifetime flow utility from non-durable consumption and leisure.

\[
E_t \left[ \sum_{t_0=1}^{T} \beta^{t_0} u(c_{t,t+t_0}, 1 - L_{t,t+t_0}) \right]
\]

From this point on we will drop time subscripts and focus on a recursive representation

\(^{16}\)The way directed search is modeled in this paper rules out the possibility that wage gains may simply reflect differences in bargaining power and outside options.
of the problem. We assume that labor is indivisible, such that the household consumes its entire time endowment while employed \( L = 1 \), and vice versa for the unemployed.

Households are heterogeneous along several dimensions. Let \( b \in B \equiv [b, \bar{b}] \subset \mathbb{R} \) denote the net asset position of the household, where \( b > 0 \) denotes that the household is saving, and \( b < 0 \) indicates that the household is borrowing. Let \( h \in H \subset \mathbb{R}_+ \) denote the human capital of the worker. Workers also differ with respect to the capital \( k \in K \subset \mathbb{R}_+ \) of the firm with which they are matched, and with respect to their credit access status \( a \in \{G, B\} \) where \( a = G \) denotes good standing, and \( a = B \) denotes bad standing. Let \( \mathbb{N}_T = \{1, 2, \ldots, T\} \) denote the set of ages.

The aggregate state of the economy includes three components: (i) total factor productivity (TFP) \( y \in \mathcal{Y} \subset \mathbb{R}_+ \) and (ii) the borrowing limit \( b \subset \mathbb{R}_- \), and (iii) the distribution of agents across states \( \mu : \{W, U\} \times \{G, B\} \times B \times H \times K \times \mathbb{N}_T \to [0, 1] \). Let \( \Omega = (y, b, \mu) \in \mathcal{Y} \times \mathbb{R}_- \times M \) summarize the aggregate state of the economy where \( M \) is the set of distributions over the state of the economy. Let \( \mu' = \Phi(\Omega, b', y') \) be the law of motion for the distribution, and assume productivity and the borrowing limit follow a Markov process. It is important to note that even though there is an exogenously imposed borrowing limit \( b \), debt will be individually priced as in Chatterjee et al. [2007], and many workers will have ‘effective borrowing limits’ where the bond price reaches zero well before \( b \).

Let \( M(u, v) \) denote the matching function, and define the labor market tightness to be the ratio of vacancies to unemployment. Since there is directed search, there will be a separate labor market tightness for each submarket. In each submarket, there is a job finding rate for households, \( p(\cdot) \), that is a function of the labor market tightness \( \theta_t(h, k; \Omega) \), such that \( p(\theta_t(h, k; \Omega)) = \frac{M(u_t(h, k; \Omega), v_t(h, k; \Omega))}{u_t(h, k; \Omega)} \). On the other side of the market, the hiring rate for firms \( p_f(\cdot) \) is also a function of the labor market tightness and is given by \( p_f(\theta_t(h, k; \Omega)) = \frac{M(u_t(h, k; \Omega), v_t(h, k; \Omega))}{v_t(h, k; \Omega)} \). Once matched with a firm, a worker produces \( f(y, h, k) : \mathcal{Y} \times H \times K \to \mathbb{R}_+ \) and keeps a share \( \alpha \) of this production.\(^{17}\)

At the beginning of every period, households with debt positions \( b < 0 \) make a default decision. In the present formulation, the default punishment is similar to Ch. 7 bankruptcy

\(^{17}\)This a similar assumption to Kaplan and Menzio [2013], and is only made for tractability purposes. Directed search models with commitment to one submarket, including Shi [2001], find that firms optimally post unique wages that are monotone in workers’ types, but other models in which firms do not commit to any given submarket, such as Shimer [2001], find non-monotone wages in workers’ types within any given job, in some cases. Empirically, wage profiles are concave in education and decreasing for higher levels of education. We can allow for this by introducing flexible functional forms for production.

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in the United States. A household in bankruptcy has a value function scripted by $B$ and cannot save or borrow. With probability $\lambda$, a previously bankrupt agent regains credit access. If a household is in good standing (i.e. they have regained credit access), its value function is scripted with a $G$, and the household can freely save and borrow.

The problem of an unemployed household in good standing is given below. To suppress an additional state variable, we allow unemployment benefits $z(k)$ to be a function of the worker’s prior wage, but only through its dependence on $k$.

$$U^G_t(b, h, k; \Omega) = \max_{b' \geq b} u(c, 1) + \beta \mathbb{E} \left[ \max_k p(\theta_{t+1}(h', \tilde{k}; \Omega')) W_{t+1}(b', h', \tilde{k}; \Omega') \right]$$

$$+ \left( 1 - p(\theta_{t+1}(h', \tilde{k}; \Omega')) \right) U_{t+1}(b', h', k; \Omega'), \quad t \leq T$$

$$U^G_{T+1}(b, h, k; \Omega) = 0$$

Such that

$$c + q_{U,t}(b', h, k; \Omega) b' \leq z(k) + b$$

We assume that human capital abides by the following law of motion (note that the process is indexed by employment status $U$):

$$h' = H(h, U)$$

And the shock processes and aggregate law of motion are taken as given:

$$y' \sim F(y' \mid y), \quad b' \sim F(b' \mid b), \quad \mu' = \Phi(\Omega, y', b'), \quad \Omega' = (y', b', \mu')$$

For households who default, they are excluded from both saving and borrowing. There is an exogenous probability $\lambda$ that they regain access to asset markets:

$$U^B_t(b, h, k; \Omega) = u(c, 1) + \lambda \beta \mathbb{E} \left[ \max_k p(\theta_{t+1}(h', \tilde{k}; \Omega')) W_{t+1}(0, h', \tilde{k}; \Omega') \right]$$

$$+ \left( 1 - p(\theta_{t+1}(h', \tilde{k}; \Omega')) \right) U_{t+1}(0, h', \tilde{k}; \Omega')$$

$$+ (1 - \lambda) \beta \mathbb{E} \left[ \max_k p(\theta_{t+1}(h', \tilde{k}; \Omega')) W^B_{t+1}(0, h', \tilde{k}; \Omega') \right]$$

$$+ \left( 1 - p(\theta_{t+1}(h', \tilde{k}; \Omega')) \right) U^B_{t+1}(0, h', k; \Omega'), \quad t \leq T$$

\footnote{Shocks to $k$ during unemployment could proxy expiration of unemployment benefits.}
Such that
\[ c \leq z(k) \]
and the law of motion for human capital and aggregates are taken as given. For households in good standing, at the start of every period, they must make a default decision:
\[ U_t(b, h, k; \Omega) = \max \left\{ U^G_t(b, h, k; \Omega), U^B_t(b, h, k; \Omega) - \chi \right\} \]

Let \( D_{U,t}(b, h, k; \Omega) \) denote the unemployed household’s default decision. Due to the finite life cycle, a utility penalty of default, \( \chi \), is necessary to support credit in equilibrium.

A similar problem holds for the employed. The value functions are denoted with a \( W \) for employed households, and at the end of every period, employed households face layoff risk \( \delta \). If they are laid off, since the period we will ultimately use is 1 quarter, we must allow the workers to search immediately for a new job.\(^{19} \) We relegate the employed value functions to Appendix C.

5.1 Lenders

There is a continuum of potential lenders who are risk neutral and can obtain funds, without constraint, at the risk free rate \( r_f \). Lenders may lend to households or firms. Recall \( e \in \{E, U\} \) denotes employment status. The price of debt for households must therefore satisfy the inequality below:
\[ q^e_{t} (b', h, k; \Omega) \leq \frac{E \left[ 1 - D^e_{t} (b', h', k'; \Omega') \right]}{1 + r_f} \] \hspace{1cm} (5)

Under free entry, the price of debt must yield exactly the risk free rate, \( r_f \), and this equation holds with equality.

The price of debt for firms follows a similar form. For the sake of brevity, and the necessity for additional notation, this bond price will be shown below in the firm section. Since lenders earn zero profit for each contract in equilibrium, lenders are indifferent between lending to a firm or a household.

\(^{19} \) This allows the model to match labor flows in the data.
5.2 Firms

There is a continuum of risk neutral entrepreneurs that operate constant returns to scale production functions. The entrepreneurs invest in capital \( k \in \mathcal{K} \subset \mathbb{R}_+ \) and post vacancies to attract workers in the frictional labor market. We assume capital is denominated in units of the final consumption good.

The entrepreneur, when attempting to create a firm, is subject to a financing constraint. When a firm is not yet operational, the firm does not have access to perfect capital markets. The firm must borrow the money, \( b_f < 0 \) to finance the initial capital investment. We assume the firm is not subject to the aggregate debt limit. If the firm fails to find an employee, the firm defaults and the capital is lost.

When deciding whether or not to post a vacancy, the firm solves the following problem. It chooses capital \( k \in \mathcal{K} \) and what types of workers, indexed by human capital and age \((h, t) \in \mathcal{H} \times \mathbb{N}_T \), to hire. In the event that the worker is hired, the firm has access to perfect capital markets and repays \( b_f \) immediately. In the event that no worker can be found, the firm defaults. Let \( J_t(h, k; \Omega) \) be the profit stream of a firm that has \( k \) units of physical capital and is matched with an age \( t \) worker with human capital \( h \). Let \( q_{f,t}(b, k, h; \Omega) \) denote the bond price faced by the firm. Then the problem a firm solves when attempting to recruit a worker is given below (recall \( b \) is negative if borrowing),

\[
\kappa \leq \max_{k,h,t} \{ p_f(\theta_t(h, k; \Omega)) [J_t(h, k; \Omega) + b_f] + (1 - p_f(\theta_t(h, k; \Omega))) \cdot 0 \} \tag{6}
\]

such that

\[
-k \geq q_{f,t}(b_f, k, h; \Omega)b_f \tag{7}
\]

With free entry in the lending market, the price of debt must be given by (note that \( k \) is

\footnote{This is because we want to isolate the impact of household credit limits on the macroeconomy. In future work, we are exploring the role of firm constraints on sorting.}

\footnote{We are envisioning specific assets with low liquidation value, however, in Appendix F.4 we allow for an explicit partial liquidation by the lender (capital is denoted in units of the final consumption good, and so this amounts to capital reversibility).}
implicitly related to $b_f$ in the equation above),

$$q_{f,t}(b_f, k, h; \Omega) = \frac{p_f(\theta_t(h,k;\Omega))}{1 + r_f}$$

Using the fact that Equation (6) holds with equality under free entry and that Equation (7) must also hold with equality, the market tightness in each submarket which is entered with positive probability is given by,

$$\theta_t(h, k; \Omega) = p_f^{-1}\left(\frac{\kappa + (1 + r_f)k}{J_t(h, k; \Omega)}\right)$$

For tractability, we assume that workers and firms split output according to a constant piece-rate $\alpha$. We assume the firm keeps a share $1 - \alpha$ of its production, and workers receive the remaining share $\alpha$ of production. Of that remaining output, firms must then pay a fixed cost $f_c$. The value function for the firm is given by,

$$J_t(h, k; \Omega) = (1 - \alpha)f(y, h, k) - f_c + \beta E[(1 - \delta)J_{t+1}(h', k; \Omega')], \quad \forall t \leq T$$

$$J_{T+1}(h, k; \Omega) = 0$$

There are three stark assumptions implicit in this value function, (i) zero liquidation value of capital, (ii) static capital, and (iii) no on-the-job search. In Appendix F we allow capital to have a nonzero liquidation value and we allow firms to dynamically invest in capital. We do not explicitly model on-the-job search due to tractability (it would require firms knowing workers’ asset policy functions, see Herkenhoff [2013] for a model with one sided heterogeneity, credit, and OJS), but by allowing firms to invest in capital, we mitigate workers’ incentives to switch jobs; in fact, with frictionless capital adjustment, firms set capital to the surplus maximizing value and workers have no incentive to leave the firm.

### 5.3 Equilibrium: Definition, Existence and Uniqueness

Let $\mathbf{x}$ summarize the state vector of a household. An equilibrium in this economy is a set of household policy functions for saving and borrowing ($\{b_{e,t}(\mathbf{x})\}_{t=1}^T$), bankruptcy

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22The representative entrepreneur will make exactly zero profits across plants and over time, even if some firms are temporarily making negative profits. When calibrating the model this fixed cost will serve to generate a small surplus for firms, and help the model match quantitative features of the data.
Given the law of motion for aggregates, the bond price, and market tightness function, households’ decision rules are optimal.

ii. Given the law of motion for aggregates and the bond price, the free entry condition in the labor market (9) holds.

iii. Given household policy functions, the labor market tightness function, and the law of motion for aggregates, the free entry conditions for lenders making loans to households (5) and firms (8) both hold.

iv. The aggregate law of motion is consistent with household policy functions.

In what follows below, we use the same tools as Menzio and Shi [2011] to solve for a Block Recursive Equilibrium in which policy functions and prices do not depend on the aggregate distribution µ (even though it fluctuates over time and can be recovered by simulation). However, policy functions still depend on aggregate productivity, y, and the borrowing limit, b. As we show below, a Block Recursive Equilibrium exists in this economy, and thus to solve the model economy, we only need to solve the first ‘block’ of the equilibrium i.-iii. ignoring iv., and then we can simulate to recover the dynamics of µ. Furthermore, we establish that certain classes of production functions yield uniqueness.

We begin with Proposition 5.1 which is the existence result for a Block Recursive Equilibrium. Without loss of generality, we set the firm fixed cost f_c to zero.

**Proposition 5.1.** Assume that the utility function meets standard conditions \( u' > 0, u'' < 0, \lim_{c \to 0} u'(c) = \infty, \lim_{c \to \infty} u'(c) = 0, \) and \( u \) is invertible), the matching function is invertible and constant returns to scale, and there is a bounded support (which can be non-binding) for the choice set of debt \( b \in \mathbb{B} \subseteq [\underline{b}, \overline{b}] \) and the capital of firms \( k \in \mathcal{K} \subseteq [\underline{k}, \overline{k}] \), then a Block Recursive Equilibrium exists.

**Proof.** Appendix D
A simple corollary follows in which one can establish the existence of an equilibrium with debt.

**Corollary 5.2.** Under the hypotheses of Proposition 5.1, so long as $\chi > 0$ and $B$ contains a neighborhood of debt around 0, a Block Recursive Equilibrium with credit exists.

*Proof. Appendix D*

Now, we turn to uniqueness. In Lemma (5.3) we provide sufficient conditions for the economy to admit a unique, Block Recursive Equilibrium.

**Lemma 5.3.** In addition to the assumptions in Proposition 5.1, let the production function be Cobb-Douglas, i.e. $f(y, h, k) = yh^{1-a}k^a$ ($0 < a < 1$), let the matching function be given by $M(u, v) = u^{\frac{1}{2}}v^{\frac{1}{2}}$, let $\chi \to \infty$ (no default for households), the value of leisure is zero, and assume there is no uncertainty over human capital $h$, aggregate productivity $y$, or the borrowing limit $b$. Then if the utility function is negative, increasing, and concave (e.g. $c^{1-\sigma-1}$ for $\sigma > 1$ or $u(c) = -e^{-c}$), the household labor search problem (equation (10)) admits a unique solution.

*Proof. Appendix D*

Lemma 5.3 demonstrates that for a broad range of production functions and utility functions, the model admits a unique solution, and so there is no equilibrium selection implicitly taking place in the computation below.\(^{23}\) Removing uncertainty in the proof is only for the sake of closed form solutions to the firm problem, and as long as the utility function of the household is additively separable in leisure, the proof holds.

### 6 Calibration

The parameters are calibrated so that the model’s stochastic steady state is consistent with 1970-2007 averages. Stochastic steady state means that aggregate total factor productivity

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\(^{23}\)We use value function iteration over discrete grids to compute the model. Appendix G describes the solution algorithm in detail.
(y) still fluctuates but that the borrowing limit (b) is constant forever.\footnote{A long sequence of productivity shocks is drawn according to the AR(1) process for y and held fixed. A large number of agents (N=30,000) is then simulated for a large number of periods (T=270 quarters, burning the first 100 quarters). Averages are reported over the remaining 170 quarters across R = 10 repetitions.} The period is set to one quarter. We calibrate the productivity process to match the Fernald et al. [2012], non-utilization adjusted total factor productivity series. The series is logged and band pass filtered to obtain deviations from trend with periods between 6 and 32 quarters. Aggregate productivity deviations are assumed to fluctuate over time according to an AR(1) process:

\[ \ln(y') = \rho \ln(y) + \epsilon_1 \quad \text{s.t.} \quad \epsilon_1 \sim N(0, \sigma_e^2) \]

Estimation yields \( \rho = 0.894 \) and \( \sigma_e = 0.00543 \), and the process is discretized using Rouwenhurst’s method.

We set the annualized risk free rate to 4%. In stochastic steady state, we set \( b = -0.5 \), which is non-binding for all agents in our simulations. We set the job destruction rate to a constant 10\% per quarter, \( \delta = 0.1 \) (Shimer [2005]). For the labor market matching function, we use a constant returns to scale matching function that yields well-defined job finding probabilities:

\[ M(u, v) = \frac{u \cdot v}{(u^{\zeta} + v^{\zeta})^{1/\zeta}} \in [0, 1] \]

The matching elasticity parameter is chosen to be \( \zeta = 1.6 \) as in Schaal [2012].

Preferences are given below, where \( \eta \) is the flow from leisure, and \( L=1 \) for employed persons and \( L=0 \) otherwise:

\[ u(c, 1 - L) = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \eta(1 - L) \]

We set the risk aversion parameter to a standard value, \( \sigma = 2 \). The life span is set to \( T = 80 \) quarters (20 years), and newly born agents are born unemployed, with zero assets, in good credit standing, and with a uniform draw over the grid of human capital. The household share of income, \( \alpha \), is set to \( \frac{2}{3} \), and the production function is Cobb-Douglas, \( f(y, h, k) = yh^a k^{(1-a)} \) with parameter \( a = \frac{2}{3} \). The bankruptcy re-access parameter \( \lambda = 0.036 \) generates the statutory 7 year exclusion period.

The remaining 8 parameters including the discount factor \( \beta \), the unemployment benefit \( z \), the utility penalty of bankruptcy \( \chi \), the entry cost of firms \( \kappa \), the fixed cost of opera-
tions $f_c$, the flow from leisure $\eta$, the human capital appreciation $p_{+\Delta}$ rate, and the human capital depreciation $p_{-\Delta}$ rate are calibrated jointly to match 8 moments: the fraction of households with liquid asset to income ratios less than 1%, the immediate consumption loss from unemployment, the bankruptcy rate, the unemployment rate, the relative volatility of unemployment to productivity, the autocorrelation of unemployment, the wage growth of 25 year olds, and the long term consumption losses from layoff. We do not directly target the duration elasticity or replacement rate elasticity.

The household discount factor $\beta = .988$, which implies a discount rate of about 5% per annum, is calibrated to match the fact that 25.4% of households have a ratio of liquid assets to annual gross income less than one percent.\(^{25}\)

The unemployment benefit is set to a constant, $z(k) = .101 \forall k$, in order to match the observed consumption losses following job loss.\(^{26}\) This value of $z$ yields an average replacement rate of approximately 40% for the lowest human capital workers (Shimer [2005]), but implies significantly lower replacement rates of 10% for higher human capital workers, in line with Chodorow-Reich and Karabarbounis [2013].

The labor vacancy posting cost $\kappa = .034$ is chosen to target a mean U6 unemployment rate of 8.9% which is the 1994-2007 average.\(^{27}\)

We set the bankruptcy utility penalty $\chi = .077$ to generate the average bankruptcy rate in the US from 1970-2007 of approximately .1% per quarter.\(^{28}\)

The processes for human capital are calibrated to generate 1.05% wage growth per quarter while employed, as well as the long term consumption losses of displaced households.\(^{29}\) These

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\(^{25}\)See Herkenhoff [2013]. The data is from the SCF (and its predecessor survey the Survey of Consumer Credit). For each household, we sum cash, checking, money market funds, CDS, corporate bonds, government saving bonds, stocks, and mutual funds less credit card debt over annual gross income. We take the mean of this liquid asset to income ratio across households in each survey year, and then we average over 1970 to 2007 to arrive at the moment.

\(^{26}\)Browning and Crossley [2001] find 16% consumption losses after 6 months of unemployment for Canadians, and as they explain, scaling food consumption losses in Gruber [1994] results in 15% consumption losses in the year of layoff for US households in the PSID. We therefore target a 15% consumption loss from the quarter prior to initial displacement until the end of the 1st year of layoff, 4 quarters after initial displacement.

\(^{27}\)Since there is no concept of “marginally-attached” workers or part-time employment in the model, U6 is a better measure of unemployment for the model. The data is available from 1994:Q1 to present.

\(^{28}\)The bankruptcy rate is .41% per annum from 1970-2007 according to the American Bankruptcy Institute (accessed via the Decennial Statistics).

\(^{29}\)Our measure of wage growth in the data is the median 2-year real-income growth rate for households aged between 25 and 30 in the PSID between 2005 and 2007. In the data, the median growth rate among
processes are governed by two parameters $p_\Delta$ and $p_+\Delta$.

$$H(h, U) = h' = \begin{cases} h - \Delta & \text{w/ pr. } p_-\Delta \text{ if unemployed} \\ h & \text{w/ pr. } 1 - p_-\Delta \text{ if unemployed} \end{cases}$$

$$H(h, W) = h' = \begin{cases} h + \Delta & \text{w/ pr. } p_+\Delta \text{ if employed} \\ h & \text{w/ pr. } 1 - p_+\Delta \text{ if employed} \end{cases}$$

In the calibration below, the grid for human capital, $h \in [.5, .6, .7, .8, .9, 1]$, as well as the step size, $\Delta = .1$, between grid points are taken as given. Our estimates are $p_-\Delta = .143$ and $p_+\Delta = .077$, which produce similar human capital processes as Ljungqvist and Sargent [1998]. Once every year-and-a-half, unemployed agents in the model expect to fall one rung on the human capital ladder. This implies between 10% to 20% earnings losses (depending on the initial human capital), which is smaller than the 30% per year Ljungqvist and Sargent [1998] target.

In terms of the flow utility of leisure, we follow most of the quantitative search and matching literature by setting $\eta$ to target a labor market moment. We choose $\eta = .237$ to match the autocorrelation of unemployment since the flow utility of leisure determines unemployed households’ willingness to remain out of work.

We calibrate the fixed cost of operations for firms $f_c = .100$, which determines how sensitive firms are to productivity shocks, to match the observed volatility of unemployment to productivity.

Table 9 summarizes the parameters, and Table 10 summarizes the model’s fit relative to the targeted moments. As we show in the next section, the model will succeed at replicating the new empirical facts on debt and duration of unemployment; however, the model is unable to match several moments, largely due to the inclusion of business cycle moments as targets in the calibration. The fundamental tension in the model is between generating this subset of households was 8.8% (we condition on at least 1k of earnings in each year). Converting this estimate to quarters yields a quarterly income growth rate of 1.05%. Assuming agents are born at 25, our model measure of wage growth is at the midpoint of that interval, measured among 27.5 year olds in the model. Using the 2005-2011 PSID, we calculate a full consumption recovery (1% higher relative to pre-layoff consumption) 2 years after layoff for unemployed households who have zero duration spells. For distressed layoffs, Saporta-Eksten [2013]’s estimates long-run consumption losses of approximately 8% two years after initial displacement. We take the average of these two estimates and target a 3% consumption loss.
borrowing/bankruptcies and matching the business cycle facts: intuition would suggest that
lowering the discount factor would be the best way to generate borrowing/bankruptcies
but doing so only exacerbates the models ability to match business cycle facts. The more
impatient are agents, the more they want to work immediately, regardless of productivity,
which dampens business cycle dynamics. Because of the two sided heterogeneity, we cannot
deploy the simple fixes in Hagedorn and Manovskii [2008]; raising the value of leisure will, at
best, make only the lowest-human-capital agents indifferent between working and not, and
reducing firm surplus will, at best, make only the lowest-capital firms sensitive to productivity
movements. The remainder of the distribution of workers and firms will not, in general,
respond to productivity shocks. We discuss this more in Appendix E.

6.1 Non-Targeted Moments: Model Estimates of Duration and
Replacement Rate Elasticities

In this section, we use the model generated policy functions to estimate the duration and
replacement earnings elasticities with respect to credit access.\textsuperscript{30} Since the debt pricing
schedule does not have an explicit credit limit, we define the credit limit to be the maximum
of either the level of debt where the bond interest rate first exceeds 30\% (denote this level of
debt $b_{30}(\cdot)$) or the exogenous debt limit $\bar{b}$.\textsuperscript{31} Therefore, we define the credit limit for an agent
with state vector $x$ as $L(x) = \min\{-b_{30}(x), -\bar{b}\}$. We isolate newly laid off agents (let $I_\delta$
denote this set of agents, and let $N_\delta$ denote its cardinality), and then we compute each agent’s
optimal search decision under loose ($\bar{b} = b_\ell$) and tight exogenous debt limits ($\bar{b} = b_H > b_\ell$),
ceteris paribus. What makes this calculation feasible is that the policy function of each
agent is contingent on the realization of $\Omega$ which includes the exogenous debt limit $\bar{b}$. So
at each decision node, encoded in this policy function is the search decision of the agent
if debt limits tighten as well as if debt limits remain slack. What makes this experimental
design valid is the block recursive nature of the model; the menu of job choices faced by
the household is not a function of $\bar{b}$. This allows us to determine the impact of changing
debt limits, holding all else constant, including the set of jobs from which households can

\textsuperscript{30}We calculate the duration and replacement elasticities using 30,000 agents simulated for 200 periods
(burning the first 100 periods), while holding the aggregate state fixed at $y = 1$, and defining $b_H = -0.1$ and
$b_\ell = -0.5$. Agents hold the same rational beliefs over the transition rate $P_b$ between $b_H$ and $b_\ell$ as Section 7.

\textsuperscript{31}Since virtually no agents borrow at rates above 30\%, this is a robust definition of the credit limit.
Changing the threshold to the point at which agents face a 50\% interest rate does not alter the results either.
choose.\textsuperscript{32} We compute the change in unemployment duration, weighted by the distribution of job losers after moving from an exogenous limit \( b = b_L \) to \( b = b_H \) as follows,\textsuperscript{33}

\[
\Delta Dur_t = \sum_{i \in I_b} \frac{Dur(b_{i,t}, h_{i,t}, k_{i,t}; y_{t}, b_H) - Dur(b_{i,t}, h_{i,t}, k_{i,t}; y_{t}, b_L)}{N_b}
\]

Define \( \frac{\Delta(L_t + b_t)}{Y_{t-1}} \) as the change in the unused credit to income ratio that the agent faces if the exogenous debt limit is tightened.\textsuperscript{34} The model implied duration elasticity is therefore given by,

\[
\epsilon_{dur} = \frac{\Delta Dur_t}{\left( \frac{\Delta(L_t + b_t)}{Y_{t-1}} \right)} = 0.15
\]

In other words, if unused credit to income increased by 10\%, then agents would take .2 weeks longer to find a job. This falls in the lower range of our IV estimates. However, the elasticity calculated in the model is a ‘global’ elasticity and is conceptually different from the local average treatment effect identified by the IV.

Next, we calculate the elasticity of replacement earnings with respect to credit, including zeros. Let \( e_i \in \{ W, U \} \) denote employment status and \( I(I) \) be the indicator function. Then define \( R_t(b) \) as the replacement rate, \( R_t(b) = \frac{1}{N_b} \sum_{i \in I_b} \sum_{j \in I_t} [I(e_i = W)] \frac{\alpha f(y_{t-1}, h_{i,t-1}, k_{i,t-1}, y_{t-1}; y_{t-1}, h_{i,t-1}, k_{i,t-1}; y_{t-1})}{\alpha f(y_{t-1}, h_{i,t-1}, k_{i,t-1}, y_{t-1})} \)

The model implied replacement earnings elasticity is therefore given by,

\[
\epsilon_{Rep} = \frac{R_t(b_H) - R_t(b_L)}{\left( \frac{\Delta(L_t + b_t)}{Y_{t-1}} \right)} = -0.024
\]

The model produces a near-zero, slightly negative, earnings replacement rate elasticity of -.026, whereas in the data, the earnings replacement rate elasticity is positive and lies anywhere between zero and +.22. To understand why this is the case, we can decompose earnings losses into two offsetting components: (i) access to additional credit depresses job finding rates which tend to lower replacement earnings, and (ii) access to additional credit

\textsuperscript{32}The intuition is simple and is formally shown in the existence proof. \( J_T(h, k; y) = f(y, h, k) \) does not depend on \( b \), and working back, neither does \( J_t(h, k; y) \) for arbitrary \( t \). Therefore, using the free entry condition, \( \theta_t(h, k; y) \), which pins down the menu of operating submarkets, will not either.

\textsuperscript{33}The expected duration is based on the 1-quarter ahead implied job finding rate, based on the search policy function. In quarters, for large M, the expected duration is given by, \( Dur(b_t, h_t, k_t; y_t, b_H) = \sum_{m=1}^{M} mp(h_t, k_t; y_t, b_H) \sum_{i \in I_b} \frac{(1 - p(h_t, k_t; y_t, b_H))}{(1 - p(h_t, k_t; y_t, b_H))} \)

\textsuperscript{34}Let \( Y_{t-1} \) denote earnings prior to layoff. Define \( \frac{\Delta(L_t + b_t)}{Y_{t-1}} \) as follows,
increases the capital intensity of submarkets searched by agents which tends to raise replacement earnings. We can compute each of these components separately. Define the job finding rate for agents as
\[ JF_t(b) = \frac{1}{N} \sum_{i \in I_b} \theta_i (h_{i,t}, k^*(b_{i,t}, h_{i,t}, k_{i,t}; y_t, b); y_t, b) \]. Then the model implied job finding elasticity is given by,
\[ \epsilon_{JF} = \frac{JF_t(b_H) - JF_t(b_L)}{\frac{\Delta(L + b)}{Y_{t-1}}} = -0.11 \]
This implies that when debt limits expand by 10% of prior annual income, job finding rates fall by 1.1% as workers can better self-insure while searching more thoroughly for jobs. This tends to decrease the replacement earnings of agents, since unemployed workers have a replacement rate of zero.

Turning to the second component of replacement earnings, define the capital intensity rate of submarkets in which agents search as
\[ K_t(b) = \frac{1}{N} \sum_{i \in I_b} k^*(b_{i,t}, h_{i,t}, k_{i,t}; y_t, b) d\mu. \] Then the model implied capital intensity elasticity is given by,
\[ \epsilon_K = \frac{K_t(b_H) - K_t(b_L)}{\frac{\Delta(L + b)}{Y_{t-1}}} = .27 \]
In other words, being able to replace 10% more of prior income with credit allows agents in the model to search in submarkets with 2.7% greater intellectual or physical capital intensity. This tends to increase the replacement earnings of agents. The combination of the two effects, namely the negative influence of job finding rates and positive influence of capital intensity on replacement earnings, yields the near-zero replacement earnings elasticity observed in the model.

Next, we calculate the elasticity of replacement earnings with respect to credit, among job finders. Let \( I_{e}(b) \) denote the set of job finders at the end of period \( t \). Let \( N_{b,e} \) denote the cardinality of \( I_{b} \cap I_{e}(b) \), which is the set of laid off households who find a job at the end of period \( t \). Define replacement earnings among this set of households as \( R_{t,e}(b) = \frac{1}{N_{b,e}} \sum_{i \in I_{b} \cap I_{e}(b)} \frac{4\alpha f(y_t, h_{i,t}, k^*(b_{i,t}, h_{i,t}, k_{i,t}; y_t, b))}{4\alpha f(y_{t-1}, h_{i,t-1}, k_{i,t-1})}. \) Lastly, define \( \frac{\Delta(L_t + b_t)}{Y_{t-1,e}} \) to be the change in credit limits to income of those who find a job at the end of period \( t \) under borrowing limit \( b_{t,e} \). The model implied replacement earnings elasticity, among the employed, is therefore given
\[ \Delta(L_t + b_t) \] is insensitive to our choice of denominator, and are very similar using \( \frac{\Delta(L_t + b_t)}{Y_{t-1}} \).
by,

$$\epsilon_{\text{Rep},e} = \frac{R_{t,e}(b_H) - R_{t,e}(b_L)}{\left(\frac{\Delta(L_{t,e}+b_{t,e})}{Y_{t-1,e}}\right)} = .18$$

This implies that in the model, among job finders, being able to replace 10% more of prior income with credit results in a 1.8% greater replacement rate. This elasticity falls in the middle of our IV estimates.

7 Main Quantitative Experiment

Our main experiment is designed to understand how fluctuations in consumer credit limits impact the macroeconomy. In particular, we study the way changes in borrowing limits impact the path of output, productivity, and employment during the 2007-2009 recession. We do so by comparing aggregate outcomes across two economies, both of which have the same beliefs about debt limit transitions $P_b$:

1. **Tight Debt Limit Economy**: The debt limit tightening from $b = -.5$ (a non-binding value) to $b = -.1$ in 2008-Q4 (the first quarter in which the aggregate consumer credit limit declined), and stays there permanently.

2. **Constant Debt Limit Economy**: The debt limit $b = -.5$ remains constant throughout the simulation.

In the experiments below, both economies are simulated in their ergodic stochastic steady states with the non-binding debt limit, $b = -.5$, for a large number of periods. We then feed in a realized set of shocks that replicates, as approximated on a grid, the path of the Fernald et al. [2012] productivity residuals from 1974-Q1 to 2012-Q4. The borrowing limit is held constant at $b = -.5$ through 2008-Q4 in both economies, for simplicity. In 2008-Q4, one economy has the limit tighten to $b = -.1$, and it remains there permanently.

We impose that both economies have the same beliefs over debt transitions. Let $p_{l,l}$ be the probability of remaining in the ‘low’ debt limit state, $b = -.1$, and let $p_{h,h}$ be the probability of remaining in the ‘high’ debt limit state, $b = -.5$. Then the transition matrix for the debt limit $b$ is given by

$$P_b = \begin{pmatrix} p_{l,l} & 1 - p_{l,l} \\ 1 - p_{h,h} & p_{h,h} \end{pmatrix}.$$
Agents understand that if the debt limit tightens, it is permanent, so we set \( p_{l,t} = 1 \). And, agents also understand that once every 34 years (from 1974 to 2008), debt limits will tighten, so we set \( p_{h,h} = .9926 \). On average, therefore, agents are rational.

### 7.1 Model Results

Figure 5 illustrates the path for the exogenous component of productivity \( y \) and the path for the borrowing limit \( b \). These are the two inputs in the experiment. Each plot contains two dashed lines that correspond to differing degrees of debt limit tightening. The dashed blue line corresponds to the economy where limits tighten to \( b = -.1 \), and the dash-dot red line corresponds to the economy where limits tighten to \( b = -.2 \).

Firstly, Table 11 illustrates what the tightening of debt limits does to borrowing in the model economies. The model economies see reductions in the fraction of households borrowing of 3.09 percentage points, and 1.21 percentage points, respectively. Economy-wide debt to income ratios fall by 1.09 percentage points and .53 percentage points respectively. In the data, the fraction of households that stopped borrowing fell by 6.77 percentage points from 2007 to 2010 (measured in the SCF) while the debt to income ratio fell by .86 percentage points from 2007 to 2010 (measured in the SCF).\(^{36}\)

Figure 6 plots the percentage change in employment during the 2007-2009 recession across the economy with a tighter debt limit versus the economy with a fixed debt limit. When debt limits tighten, employment tends to increase, persistently. The mechanism is that with looser credit limits, unemployed households borrow to smooth consumption while thoroughly searching for capital-intensive jobs. If debt limits tighten, they lose their ability to self-insure, and, as a result, take low-capital-intensity jobs that are relatively quick to find. In other words, when limits tighten, low-asset job losers take relatively less productive employment opportunities. This introduces a strong tension between recovery speed and recovery health, as workers find jobs more quickly but these jobs are of lower quality.

As Figure 9 shows, the aggregate capital stock held by entrepreneurs, which is a function of existing matches and new matches, drops severely relative to the economy in which debt limits are held constant. This is entirely driven by new entrepreneurial entrants posting more vacancies in submarkets with less capital, and constrained households searching for

\(^{36}\)While not reported here for the sake of space, the bankruptcy rate reaches .97% in the model in the quarter in which limits are tightened, which is in line with ABI bankruptcies per capita.
jobs in those submarkets. The time it takes for the aggregate capital stock to recover to its pre-recession levels is as much as 6 quarters longer in the economy in which debt limits tighten.

Because households become more constrained and take jobs in which there is less capital per unit of labor, Figure 7 shows that measured labor productivity, defined as output over employment, declines when debt limits are tightened. The economy in which debt limits tighten the most has .28 percentage points lower labor productivity as compared to the economy with constant debt limits, and this productivity gap persists throughout the recovery. In Appendix F, when we allow for capital investment to change over the course of a match, the labor productivity decline is slightly less pronounced. The relative labor productivity decline reaches .24 percentage points 16 quarters after the start of the recession as firms subsequently invest more in capital in existing matches during the recovery.

While the impact of tighter debt limits on capital per worker is unambiguous, the impact of tighter debt limits on aggregate output is theoretically ambiguous: households find jobs faster, but the jobs workers find are less productive. However, Figure 8 shows that quantitatively the reduction in capital per worker is so severe that output falls by .11 percentage points.

The mechanism at the heart of the output decline involves a reallocation of workers from high capital firms to low capital firms. To understand this reallocation in greater detail, we now turn to standard measures of sorting. Figure 10 plots the percentage change in the correlation between human capital, \( h \), and firm capital, \( k \), during the 2007-2009 recession. Figure 11 plots the corresponding percentage change in the Spearman rank correlation coefficient between human capital, \( h \), and firm capital, \( k \), as well (workers are ranked by \( h \), and firms are ranked by \( k \), and the Spearman Rank correlation coefficient is the resulting correlation between the numeric ranks of workers and firms). The raw correlation coefficient between worker human capital and firm physical capital is approximately +.33.

Figures 10 and 11 show that in the economy in which debt limits are tighter, these standard measures of sorting improve. The mechanism behind this sorting improvement is that in the economy in which debt limits tighten, unemployed agents with low-human-capital cannot borrow to smooth consumption while thoroughly searching for jobs. Therefore, they take jobs that are less-capital-intensive, but more abundant. On average, since low human capital workers are less productive (recall the assumption of supermodularity), tighter debt
limits force these ‘low quality’ workers to take ‘low quality’ jobs. As such, standard measures of sorting improve, even as output falls, since they do not take into account the investment decisions of firms. In this economy, these standard measures of sorting are not good proxies for either productivity or output, even with a supermodular production function.

The fact that firms can invest or that workers can save are standard assumptions in most neoclassical business cycle models but are often ignored in search theoretic models. The fact that sorting moves in the opposite direction of output and productivity under these mild assumptions raises important questions about the welfare implications of sorting patterns derived from search theoretic models with linear utility or under the assumption of fixed firm types.

7.2 Evidence on Countercyclical Sorting

The empirical literature on the cyclicality of sorting is very thin. In a model with linear utility, Lise and Robin [2013] find similar countercyclical sorting patterns among certain subgroups of households. Likewise, Bagger et al. [2013] compute long-run sorting trends, and their time series reveal that sorting accelerated during the Danish recession in the early 1990s, and then declined in the tranquil 2000s. On the other hand, Moscarini and Vella [2008] use the CPS to study occupational sorting, and they find that there is less occupational sorting in recessions. In general, the available data supports the broad patterns observed in the model and over the business cycle, but much research remains to be done on the topic.

7.3 Robustness: Capital Investment and Liquidation Value

We conduct two robustness exercises in Appendix F. First, we allow for the entrepreneurs to invest in capital over time, mitigating concerns about both quits and on-the-job-search. With costless adjustments to entrepreneur capital, there would never be a reason to quit or change jobs. We find that our main results are largely unchanged, but the ability to invest in capital during recoveries marginally dampens the response of capital, productivity, output, and sorting to business cycle shocks. Second, we allow for a liquidation value of firm capital, and again, the main predictions of the model still hold.
8 Conclusions

In this paper, we provide the first estimates of the impact of credit constraints on job finding rates and subsequent replacement wages of displaced workers. Using new administrative data, we find that medium-tenure displaced mortgagors, in response to being able to replace 10% of their annual income with revolving credit, take .3 to 1 week longer to find a job but obtain an earnings replacement rate that is .5% to 1.5% greater.

We then develop, to our knowledge, the first labor sorting model with credit. We estimate the model on a set of independent public moments in order to derive structural estimates of the duration and replacement rate elasticities, yielding estimates of approximately .2 and zero, respectively. We then use the model as a laboratory to understand how fluctuations in debt limits impact productivity, output, and employment. We find that tighter debt limits during recessions may increase employment during the recovery, but depress both productivity and output. This tension between the speed of recovery and health of recovery is at the heart of the mechanism: tighter debt limits force constrained households to cut their job search short, taking relatively unproductive jobs that are more abundant.

The estimates and quantitative exercises in this paper have implications for the way both policy-makers and economists think about the optimal provision of unemployment insurance and the response of labor markets to monetary policy. The fact that increases in credit access can actually reduce job finding rates by easing household credit constraints brings into question the ability of the Federal Reserve Bank to effectively meet the dual mandate of “maximum employment, stable prices and moderate long-term interest rates.” Furthermore, the theory we advance in this paper is tractable enough that it can be used by future researchers to study a variety of questions related to misallocation and credit access, including credit access among firms.

We view this paper as the beginning of a research agenda which uses new micro data and new theory to understand how consumer credit impacts the allocation of households to firms. The next step in our research agenda is to measure the impact of consumer credit constraints on the other side of the market, on the Schedule C entrepreneurs.
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Table 1: Summary Statistics for Displaced Mortgagor Sample and Displaced Bankrupt Sample (Source: LEHD/TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>Mortgagor Sample</th>
<th>Bankrupt Sample</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Employed at t+1</td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td></td>
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<tr>
<td>Tenure</td>
<td></td>
<td></td>
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<td>Imputed Years of Education</td>
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<tr>
<td>Lagged Annual Earnings</td>
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<tr>
<td>Lagged Unused Revolving Credit to Income</td>
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<td></td>
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<tr>
<td>Lagged Unused Total Credit to Income</td>
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<td></td>
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<tr>
<td>Duration of Non-Employment (In Quarters)</td>
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<td></td>
</tr>
<tr>
<td>Replacement Rate (Annual Earnings Year t+1/Annual Earnings Year t-1)</td>
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<td></td>
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<tr>
<td>Lagged Months Since Oldest Account Opened</td>
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</tbody>
</table>

Observations (Rounded to 000s)
Table 2: Dependent Variable is Duration. Column (1) OLS, Column (2) IV using the Saiz instrument, Column (3) IV using the Gross and Souleles Instrument, Column (4) OLS with Bankruptcy Flag Drop. (Source: 2002-2006 LEHD/TransUnion)

<table>
<thead>
<tr>
<th>Flag Drop (d)</th>
<th>(1) OLS</th>
<th>(2) IV-Saiz</th>
<th>(3) IV-GS</th>
<th>(4) OLS-Flag Drop</th>
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</thead>
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<tr>
<td></td>
<td>Unused Revolving Credit to Income Ratio</td>
<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
<td>HELOC Limits and Equity Proxy</td>
<td>R2 (1st Stage for IVs)</td>
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<tr>
<td>Flag Drop (d)</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
<td>HELOC Limits and Equity Proxy</td>
<td>R2 (1st Stage for IVs)</td>
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<tr>
<td></td>
<td>Y</td>
<td>Y</td>
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<td>Y</td>
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</tbody>
</table>

Notes. Std. errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Col. (1), (3), and (4) use robust std. errors, and Col. (2) uses clustered std. errors at MSA Level. Unused Revolving Credit to Income measured 1 year prior to layoff. Demographic controls include quadratic in age & tenure, race, sex and education dummies as well as year & auto loan dummies. Industry controls include 1-digit SIC dummies and lagged size, age, and wage per worker of prior firm. MSA controls include real per capita GDP and the MSA unemployment rate. Lagged earnings controls include both lagged real annual earnings, and cumulative lagged real annual earnings to proxy for assets. Equity proxy is highest observed mortgage balance less current mortgage balance. HELOC limits include combined home equity limits.

Table 3: Impact of Bankruptcy Flag Removal on Credit Access, OLS. Column (1) Dependent Variable is Revolving Credit Limit to Income, and Column (2) Dependent Variable is Credit Score. (Source: 2002-2006 LEHD/TransUnion)

<table>
<thead>
<tr>
<th>Flag Drop (d)</th>
<th>(1) Credit Limit to Inc</th>
<th>(2) Credit Score</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>Flag Drop (d)</td>
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</table>

Notes. Robust Std. errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Same control definitions as Table 2.
Table 4: Dependent Variable is Replacement Rate, Measured 1 Year After Layoff Relative to 1 Year Before Layoff. (Source: 2002-2006 LEHD/TransUnion)

<table>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tr>
<td>Unused Revolving Credit to Income Ratio</td>
<td>OLS</td>
<td>IV-Saiz</td>
<td>IV-GS</td>
<td>OLS-Flag Drop</td>
</tr>
<tr>
<td>Flag Drop (d)</td>
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<tr>
<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
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<tr>
<td>HELOC Limits and Equity Proxy</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
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<tr>
<td>R2 (1st Stage for IVs)</td>
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<tr>
<td>Pval Weak Id Null Weak</td>
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<td>Round N</td>
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</table>

Notes. Same as Table 2.

Table 5: Dependent Variable is Replacement Rate, Measured 1 Year After Layoff Relative to 1 Year Before Layoff. (Source: 2002-2006 LEHD/TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tbody>
<tr>
<td>Unused Revolving Credit to Income Ratio</td>
<td>OLS</td>
<td>IV-Saiz</td>
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<td>OLS-Flag Drop</td>
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<td>Flag Drop (d)</td>
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<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
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<tr>
<td>R2 (1st Stage for IVs)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. Same as Table 2.

Table 6: Dependent Variable is Dummy if Firm is in 99th Decile of Size Distribution or Greater (‘Large Firm Dummy’). Sample Includes Those With Jobs 1 Year After Layoff. (Source: 2002-2006 LEHD/TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unused Revolving Credit to Income Ratio</td>
<td>OLS</td>
<td>IV-Saiz</td>
<td>IV-GS</td>
<td>OLS-Flag Drop</td>
</tr>
<tr>
<td>Flag Drop (d)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>HELOC Limits and Equity Proxy</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>R2 (1st Stage for IVs)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pval Weak Id Null Weak</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. Same as Table 2.
Table 7: Dependent Variable is Dummy if Firm is in 75th Decile of Wage Per Worker Distribution or Greater (‘Productive Firm Dummy’). Sample Includes Those With Jobs 1 Year After Layoff. (Source: 2002-2006 LEHD/TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>IV-Saiz</td>
<td>IV-GS</td>
<td>OLS-Flag Drop</td>
</tr>
<tr>
<td>Unused Revolving Credit to Income Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flag Drop (d)</td>
<td></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>HELOC Limits and Equity Proxy</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>R2 (1st Stage for IVs)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pval Weak Id Null Weak</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. Same as Table 2.

Table 8: Dependent Variable is Change in Real Revolving Debt 1 Year After Layoff Minus 1 Year Before Layoff. (Source: LEHD/TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>(1) Change in Revolving Debt</th>
<th>(2) Change in Revolving Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration of Unemployment</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demographic, Industry, MSA, &amp; Lagged Earnings Controls</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>HELOC Limits and Equity Proxy</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>R2</td>
<td>0.000249</td>
<td>0.0102</td>
</tr>
<tr>
<td>Round N</td>
<td>19000</td>
<td>19000</td>
</tr>
</tbody>
</table>

Notes. Robust Std. errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Same control definitions as Table 2.
Table 9: Summary of Model Parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Calibrated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.894</td>
<td>Autocorrelation of Productivity Process</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>0.00543</td>
<td>Std. Dev. Of Productivity Process</td>
</tr>
<tr>
<td>( r_f )</td>
<td>4%</td>
<td>Annualize Risk Free Rate</td>
</tr>
<tr>
<td>( \delta )</td>
<td>10%</td>
<td>Quarterly Layoff Rate</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>1.6</td>
<td>Matching Function Elasticity</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>Risk Aversion</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.66</td>
<td>Household share of income</td>
</tr>
<tr>
<td>( a )</td>
<td>0.66</td>
<td>Cobb-Douglas Labor Share</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.036</td>
<td>Bankruptcy Re-Access</td>
</tr>
<tr>
<td>( b )</td>
<td>-0.5</td>
<td>Non-binding debt limit</td>
</tr>
<tr>
<td>Jointly-Estimated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.034</td>
<td>Firm Entry Cost</td>
</tr>
<tr>
<td>( z )</td>
<td>0.101</td>
<td>UI</td>
</tr>
<tr>
<td>( p-\Delta )</td>
<td>0.143</td>
<td>Depreciation Rate of Human Cap.</td>
</tr>
<tr>
<td>( p+\Delta )</td>
<td>0.077</td>
<td>Appreciation Rate of Human Cap.</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.988</td>
<td>Discount Factor</td>
</tr>
<tr>
<td>( f_c )</td>
<td>0.100</td>
<td>Fixed Cost</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.237</td>
<td>Flow Utility of Leisure</td>
</tr>
<tr>
<td>( \chi )</td>
<td>0.077</td>
<td>Bankruptcy Utility Penalty</td>
</tr>
</tbody>
</table>

Table 10: Model Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment Rate</td>
<td>8.93%</td>
<td>8.90%</td>
<td>BLS, U6 1994-2007</td>
</tr>
<tr>
<td>Consumption Drop 1 Yr After Layoff</td>
<td>0.84</td>
<td>0.84</td>
<td>Browning &amp; Crossley (2001)</td>
</tr>
<tr>
<td>Consumption Drop 2 Yrs After Layoff</td>
<td>0.971</td>
<td>.97</td>
<td>Saporta-Eksten (2013) / PSID 2005-2011</td>
</tr>
<tr>
<td>Quarterly Income Growth Rate 25yo</td>
<td>1.078%</td>
<td>1.05%</td>
<td>PSID, 2005-2007</td>
</tr>
<tr>
<td>Fraction of Households with Liquid assets to Income Ratio&lt;1%</td>
<td>0.093</td>
<td>0.254</td>
<td>SCF, 1974-2007</td>
</tr>
<tr>
<td>Autocorr Unempl</td>
<td>0.730</td>
<td>0.94</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Bankruptcy rate</td>
<td>0.01%</td>
<td>0.10%</td>
<td>ABI, 1970-2007</td>
</tr>
</tbody>
</table>
Table 11: Reduction in Borrowing When Borrowing Limit Tightens, Model v. Data.

<table>
<thead>
<tr>
<th></th>
<th>Δ Fraction of HHs Borrowing</th>
<th>Δ DTI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt limit tightened from $b = -.5$ to $b = -.1$</td>
<td>-3.09%</td>
<td>-1.09%</td>
</tr>
<tr>
<td>Debt limit tightened from $b = -.5$ to $b = -.2$</td>
<td>-1.21%</td>
<td>-0.53%</td>
</tr>
<tr>
<td>Data</td>
<td>-6.77%</td>
<td>-0.86%</td>
</tr>
</tbody>
</table>

Notes: All Differences Computed using 2007 and 2010 SCF. DTI is Change Unsecured Revolving Consumer Credit to Annual Family Income. Fraction Borrowing Change is Difference in Fraction of Households Carry Positive Balances. Means Weighted Using Survey Weights. Model statistics calculated as difference in average of quarterly values over same corresponding years.
Figure 1: Non-Employment Duration by Unused Revolving Credit to Income Decile, *prior to layoff* (Source: LEHD/TransUnion)

Figure 2: Replacement Earnings 1 Year After Layoff (Including 0s) by Unused Revolving Credit to Income Decile, *prior to layoff* (Source: LEHD/TransUnion)

Figure 3: Change in Real Bankcard Debt 1 Year After Layoff Minus 1 Year Before Layoff (Source: LEHD/TransUnion)

Figure 4: Change in Real Revolving Debt 1 Year After Layoff Minus 1 Year Before Layoff as Function of Non-Employment Duration (Source: LEHD/TransUnion)
Figure 5: Experiment Input: Exogenous Aggregate Productivity (y) and Borrowing Limit $b$, 2007-2009 Recession

Figure 6: Percentage Change in Employment Per Capita

Figure 7: Labor Productivity, 2007-2009 Recession

Figure 8: Aggregate Output
A Data Appendix

Employer reports are based on the ES-202 which is collected as part of the Covered Employment and Wages (CEW) program (run by BLS). One report per establishment per quarter is filed. On this form, wages subject to statutory payroll taxes are reported.

The employment records are associated with a firm’s State Employment Identification Number (SEIN). This is an identifier based on an employer within a given state, and it is, in general, not an identifier of the establishment of the worker. Minnesota is the only state to collect establishment identifiers, and in all other states, an imputation based on place-of-work is used to generate establishment level identifiers. In general, workers are included in the dataset if they earn at least one dollar from any employer.

The Quarterly Census of Employment and Wages (QCEW) contains firm level data which is collected in each state. This dataset includes information on industry, ownership, and worksite.

The demographic data in the LEHD comes from the 2000 census as well as social security records, and tax returns. These are linked by social security number with the unemployment insurance data. In the LEHD, social security numbers are not present, rather there is a scrambled version called a Protected Identification Key (PIK).

The main demographic information database is the Person Characteristic File (PCF). Information on sex, date of birth, place of birth, citizenship, and race are included here.

A.1 Identifying Mass Layoffs

To identify mass layoffs, we combine data from the Longitudinal Business Dynamics (LBD) database on establishment exits with the LEHD. In each state, employers are assigned a State Employment Identification Number (SEIN) in the LEHD database. This is our unit of analysis for mass layoffs. We define a mass layoff to occur when an SEIN with at least 25 employees reduces its employment by 30% or more within a quarter and continues operations, or exits in the LEHD with a contemporaneous plant exit in the LBD. In California, we do not have LBD establishment exit information, however. To ensure that the there was actually a mass layoff, we then verify that fewer than 80% of laid-off workers move to any other single SEIN using the Successor Predecessor File (SPF). This allows us to remove mergers, firm
name-changes, and spin-offs from our sample.
B  Additional Robustness Checks

B.1 Over-Identification Tests

In this section we discuss the assumptions underlying the Gross and Soueleles instrument, and we use the fact that we have two different instruments in order to conduct over-identification tests. In summary, the Saiz and Gross and Souleles instruments pass identification tests, suggesting that they are satisfying exclusion restrictions. However, there is not true test of exogeneity.

For the Gross and Souleles instrument to be valid, it must be both relevant and exogenous. What makes the Gross and Souleles instrument relevant is that a large component, approximately 15%, of a credit score is solely based on length of credit history.\(^{37}\) By simply having an account open, you credit score increases and affects your credit limits. Empirically, the first stage is very strong, as evidenced by the small p-values in the Stock-Yogo weak identification tests.

For the age of the oldest account to be a valid instrument for credit limits, it must not only be a strong determinant of credit limits, but it can only have an impact on employment prospects through credit limits (exogeneity). The main challenge to exogeneity is that the age of an account is related to the physical age of the individual. Since the age of an account is how scoring companies proxy for physical age, by conditioning on physical age (which we observe but scoring companies do not), we are able to isolate changes in credit scores simply due to variation in account age, that have nothing to do with physical age.\(^{38}\)

Lastly, since we have multiple instruments, we show that both the Gross and Souleles instrument and Saiz instrument pass over-identification tests. Table 12 illustrates the main results using both instruments in first stage. We see that our main results hold, with credit limits positively impacting durations, replacement rates, firm size and firm productivity [X] check.

\(^{37}\)See ‘Your Credit Score,’ prepared by Fair Isaac Corporation and available from http://www.consumerfed.org/pdfs/yourcreditscore.pdf

\(^{38}\)The reason they do this is that the Equal Credit Opportunity Act bans the use of age, race, or sex in determining credit score, and so the credit scoring companies use account age as a proxy for physical age.
Table 12: Over Identification Tests. (Source: 2002-2006 LEHD/TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OverID</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Replacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large Firm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OverID</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OverID</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Unused Revolving Credit to Income

Flag Drop (d)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographic Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Industry Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>MSA Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Lagged Earnings Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>HELOC Limits and Equity Proxy</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

R² (1st Stage for IVs)

Angirst Pischke FStat Pval

Pval Weak Id Null Weak

Jtest Pval Null Valid

Round N

Notes. Clustered std. errors at MSA level in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Same control definitions as Table 2.

B.2 Alternate Measures of Personal Financial Constraints: Total Credit, Revolving Credit Including HELOCs, and Credit Scores

In Table 13, we use alternate endogenous regressors: (i) total unused credit, including all types of secured (including HELOCs and mortgage debt) and unsecured debt, and (ii) credit scores (this corresponds to TransUnion’s bankruptcy model, and ranges from 0 to 1000). We define ‘total unused credit to income’ as the total credit limit less the amount currently borrowed over annual earnings, where the ratio is measured 1 year prior to layoff. Columns (3)-(6) of Table 13 illustrate the results. In general, total unused credit to income is less potent than revolving credit. Installment loans and other auto loans are difficult to adjust upon job loss, even if the household has had large balances in the past (indicating a large borrowing capacity). Credit scores produce a similar sign and significance level as unused credit, but the magnitudes of the coefficients are difficult to interpret. Overall, we interpret these robustness checks as corroborating our estimates reported in the main body of the

39The Total Credit Limit is formally the TransUnion variable “Total High Credit/Credit Limit” which is sum of actual credit limits across all types of debt, or if the credit limit is not stated, it is the highest observed prior balance. This measure of credit includes secured credit lines like home equity lines of credit and installment credit, as well as auto loans, and other personal finance loans.
Table 13: Alternate Endogenous Regressors. (Source: 2002-2007 LEHD/TransUnion)

<table>
<thead>
<tr>
<th>Dep Var</th>
<th>Saiz House Supply Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Duration Rate</td>
</tr>
<tr>
<td>Revolving Unused Credit to Income</td>
<td>0.699*** (0.193)</td>
</tr>
<tr>
<td>Total Unused Credit to Income</td>
<td></td>
</tr>
<tr>
<td>Credit Score</td>
<td></td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>Y Y Y Y Y Y</td>
</tr>
<tr>
<td>Industry Controls</td>
<td>Y Y Y Y Y Y</td>
</tr>
<tr>
<td>MSA Controls</td>
<td>Y Y Y Y Y Y</td>
</tr>
<tr>
<td>Lagged Earnings Controls</td>
<td>Y Y Y Y Y Y</td>
</tr>
<tr>
<td>R2 First Stage</td>
<td>0.0699 0.0699</td>
</tr>
<tr>
<td>Angrist Pischke FStat Pval</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>Pval Weak Id Null Weak</td>
<td>19000 19000 19000 19000 19000</td>
</tr>
</tbody>
</table>

Notes. Clustered standard errors at MSA level in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Revolving Unused Credit to Income measured 1 year prior to layoff. Demographic controls include quadratic in age & tenure, race, sex and education dummies as well as year & auto loan dummies. Industry controls include 1-digit SIC dummies, MSA controls include real per capita GDP and the MSA unemployment rate. Lagged earnings controls include both lagged real annual earnings, and cumulative lagged real annual earnings to proxy for assets.
B.3 Correlation of Unemployment Durations and Credit Limits in the SCF, Controlling for Assets

In the SCF between 1998 and 2007 (which includes the 1998, 2001, 2004, and 2007 surveys), we can compute the raw correlation between unused credit limits and unemployment durations, controlling for a host of assets, including home values. Table 14 demonstrates a strong correlation between unused credit card limits and unemployment durations, subject to attenuation bias. The ‘Unused Unsecured Limit to Income’ refers to unused credit card limits (as of the survey date) over annual gross family income (over the prior year). Unemployment duration measures weeks spent unemployed over the past 12 months prior to the survey. It is measured in weeks, and does not distinguish individual unemployment spells.

Column 1 of Table 14 shows that simple regressions of unemployment duration on unused credit card limits reveal a strong positive correlation, even after controlling for income and liquid assets. Columns 2 and 3 impose age restrictions and add basic demographic controls, but the positive and significant relationship persists. Column 4 adds in all available categories of illiquid assets, and finally Column 5 restricts the dataset to mortgagors (as is the case in the LEHD/TransUnion sample considered in the text). The strong positive and significant relationship between unused credit limits and unemployment durations persists. An unused credit limit worth 10% of prior annual family income is associated with 1 week longer unemployment spells, very similar to the IV estimate in the LEHD/TransUnion sample considered in the text.
Table 14: Correlation between Durations (in Weeks) and Unused Credit, Controlling for Assets (Source: 1998-2007 SCF)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(5.85)</td>
<td>(4.87)</td>
<td>(4.02)</td>
<td>(4.31)</td>
<td>(3.75)</td>
<td>(2.66)</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Demographics and Income</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Liquid Assets to Inc (Checking/Savings plus Stocks and Bonds)</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Illiquid Assets to Inc (Homes, Vehicles, etc.)</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Mortgagors Only</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Observations</td>
<td>764</td>
<td>764</td>
<td>764</td>
<td>759</td>
<td>759</td>
<td>421</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.052</td>
<td>0.130</td>
<td>0.144</td>
<td>0.137</td>
<td>0.148</td>
<td>0.157</td>
</tr>
</tbody>
</table>

Notes: SCF 24 to 65yo Heads of Household with Positive Unemployment Spell over Prior 12 months and Positive Limit. Restrict to Mortgagors in Col 6. Demographics include quadratic in age, dummies for education, and dummies for race and Income refers to gross annual family income. Liquid Assets include cash, checking, money market funds, CDS, corporate bonds, government saving bonds, stocks, and mutual funds less credit card debt. Unused Credit Limit to Income refers to total credit card limits less credit card balances. Illiquid Assets includes Homes, Vehicles, Retirement, Annuities, Life Insurance at self-reported market values.
B.4 House Prices and the Relationship Between Credit, Non-Employment Durations, and Replacement Rates

Table 15 illustrates the main regressions estimated with OLS, with and without housing price controls. The house price control we include in the regression is the OFHEO All-Transaction House Price Index for MSAs. We control for house prices at the time of the layoff, and we find that the correlation between the unused credit limit ratio and non-employment durations changes very little. Likewise, the replacement rate 1 year after layoff is hardly impacted by the inclusion of house prices as controls. As discussed in the text, the magnitude of the coefficients are significantly smaller than the IV estimates. This likely due to the fact that high-type/high-earner households have higher amounts of credit on average, but, orthogonal to credit access, take longer to find jobs (the PSID from 2005-2013 confirms a strong significant negative relationship between duration and income). By isolating an exogenous component of income, we mitigate this offsetting force due to largely unobserved heterogeneity in worker types.

The fact that house prices increase does not necessarily imply households are wealthier, nor that they should consume more, and this topic is actively debated in the literature (see both sides of the literature in Calomiris et al. [2009], among others). As most theoretic studies show, a household may attempt to sell the house, but they must buy a new one or rent thereafter, mitigating housing wealth effects. There is also mixed evidence regarding housing lock (see both sides of the literature discussed in Karahan and Rhee [2011]), suggesting that housing wealth may not matter for selling decisions as well. We find very little evidence of interstate movers or intrastate movers in our sample around job loss. If we drop movers, our IV regression results remain unchanged in terms of sign, significance, and magnitude.

---

40This is publicly available from the OFHEO website. The index is normalized to 100 in 1995.
Table 15: Baseline OLS regressions with Direct Controls for OFHEO House Prices (Source: LEHD/TransUnion 2002-2007)

<table>
<thead>
<tr>
<th>Dep Var</th>
<th>(1) Duration</th>
<th>(2) Duration</th>
<th>(3) Duration</th>
<th>(4) Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolving Unused Credit to Income</td>
<td>0.0796***</td>
<td>0.0546**</td>
<td>0.0526**</td>
<td>0.0529**</td>
</tr>
<tr>
<td></td>
<td>(0.0269)</td>
<td>(0.0229)</td>
<td>(0.0223)</td>
<td>(0.0223)</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Industry Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>MSA Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Direct Control for House Prices</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Lagged Earnings Controls</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>R2</td>
<td>0.00199</td>
<td>0.0486</td>
<td>0.0490</td>
<td>0.0491</td>
</tr>
<tr>
<td>Round N</td>
<td>19000</td>
<td>19000</td>
<td>19000</td>
<td>19000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dep Var</th>
<th>(5) Replacement Rate</th>
<th>(6) Replacement Rate</th>
<th>(7) Replacement Rate</th>
<th>(8) Replacement Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolving Unused Credit to Income</td>
<td>0.0310***</td>
<td>0.0267***</td>
<td>0.0264***</td>
<td>0.0259***</td>
</tr>
<tr>
<td></td>
<td>(0.00870)</td>
<td>(0.00825)</td>
<td>(0.00826)</td>
<td>(0.00810)</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Industry Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>MSA Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Direct Control for House Prices</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Lagged Earnings Controls</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>R2</td>
<td>0.00799</td>
<td>0.0758</td>
<td>0.0760</td>
<td>0.0816</td>
</tr>
<tr>
<td>Round N</td>
<td>19000</td>
<td>19000</td>
<td>19000</td>
<td>19000</td>
</tr>
</tbody>
</table>

Notes. Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Revolving Unused Credit to Income measured 1 year prior to layoff. Demographic controls include quadratic in age & tenure, race, sex and education dummies as well as year & auto loan dummies. Industry controls include 1-digit SIC dummies. MSA controls include real per capita GDP and the MSA unemployment rate. Lagged earnings controls include both lagged real annual earnings, and cumulative lagged real annual earnings to proxy for assets. House Price control is OFHEO All Transaction MSA level house price index.
### B.5 Self-Employment and the Earnings Gap Method

Table 16 redoes the main analysis in two different ways. Column (1) is a regression of duration on unused credit where the self-employed with more than 5k in annual Schedule C earnings are counted as employed. Column (2) infers the length of unemployment duration using the earnings gap method. Using quarterly earnings prior to layoff as the base ($E_{q-1}$), then those who find a job within the first quarter of layoff will have spent $1 - E_q/E_{q-1}$ fraction of the quarter unemployed. Table 16 illustrates that the main results are robust to these alternate definitions.

Table 16: Column (1) is duration of non-employment, counting the self-employed who earn more than 5k in a year as employed, and Column (2) is duration of non-employment with partial duration values inferred using the earnings gap method. (Source: LEHD / TransUnion)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration (Self-Employment)</td>
<td>0.616**</td>
<td>0.810***</td>
</tr>
<tr>
<td></td>
<td>(0.282)</td>
<td>(0.280)</td>
</tr>
<tr>
<td>Revolving Unused Credit to Income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Industry Controls</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>MSA Controls</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Lagged Earnings Controls</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Round N</td>
<td>19000</td>
<td>19000</td>
</tr>
<tr>
<td>R2 First Stage</td>
<td>0.0525</td>
<td>0.0525</td>
</tr>
<tr>
<td>Angrist Pischke FStat Pval</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pval Weak Id Null Weak</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes. Robust standard errors in parentheses, *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Revolving Unused Credit to Income measured 1 year prior to layoff. Demographic controls include quadratic in age & tenure, race, sex and education dummies as well as year & auto loan dummies. Industry controls include 1-digit SIC dummies. MSA controls include real per capita GDP and the MSA unemployment rate. Lagged earnings controls include lagged real annual earnings.
C Employed Value Functions

For employed households, value functions are denoted with a $W$, and at the end of every period, employed households face layoff risk $\delta$. If they are laid off, since the period is 1 quarter, we must allow the workers to search immediately for a new job.\footnote{This allows the model to match labor flows in the data.}

\[
W_t^G(b, h, k; \Omega) = \max_{b' \geq b} \left[ u(c) + \beta \mathbb{E}\left[ (1 - \delta)W_{t+1}(b', h', k; \Omega') + \delta \left\{ \max_{\tilde{k}} p\left(\theta_{t+1}(h', \tilde{k}; \Omega')\right)W_{t+1}(b', h', \tilde{k}; \Omega') + (1 - p\left(\theta_{t+1}(h', \tilde{k}; \Omega')\right))U_{t+1}(b', h', \tilde{k}; \Omega') \right\} \right], \quad t \leq T
\]

\[
W_{T+1}^G(b, h, k; \Omega) = 0
\]

Such that the aggregate laws of motion are given by equation (4), human capital evolves according to the law of motion below,

\[
h' = H(h, W)
\]

and the budget constraint holds,

\[
c + q_{W,t}(b', h, k; \Omega)b' \leq \alpha f(y, h, k) + b
\]

The value functions for employed borrowers who default as well as the discrete default
decision are formulated in an identical fashion to that of the unemployed.

\[ W_t^B(b, h, k; \Omega) = u(c, 0) + \lambda \beta \mathbb{E} \left[ (1 - \delta)W_{t+1}(0, h', k; \Omega') + \delta \left\{ \max_{\tilde{k}} p(\theta_{t+1}(h', \tilde{k}; \Omega')) W_{t+1}(0, h', \tilde{k}; \Omega') \right\} \right] + (1 - \lambda) \beta \mathbb{E} \left[ (1 - \delta)W_{t+1}^B(0, h', k; \Omega') + \delta \left\{ \max_{\tilde{k}} p(\theta_{t+1}(h', \tilde{k}; \Omega')) W_{t+1}(0, h', \tilde{k}; \Omega') \right\} \right], \quad t \leq T \]

\[ W_{T+1}^B(b, h, k; \Omega) = 0 \]

Such that the aggregate laws of motion are given by equation (4), human capital evolves such that \( h' = H(h, W) \) and the budget constraint is given by,

\[ c \leq \alpha f(y, h, k) \]

For employed households in good standing, at the start of every period, they must make the following default decision,

\[ W_t(b, h, k; \Omega) = \max \left\{ W_t^G(b, h, k; \Omega), W_t^B(b, h, k; \Omega) - \chi \right\} \]

Let \( D_{W,t}(b, h, k; \Omega) \) denote the employed household’s default decision.

\section*{D Proofs}

\textbf{Restatement of Proposition 5.1:} Assume that the utility function meets standard conditions (\( u' > 0, u'' < 0, \lim_{c \to 0} u'(c) = \infty, \lim_{c \to \infty} u'(c) = 0 \), and \( u \) is invertible), the matching function is invertible and constant returns to scale, and there is a bounded support (which can be non-binding) for the choice set of debt \( b \in B \subseteq [b, \overline{b}] \) and the capital of firms \( k \in K \subseteq [k, \overline{k}] \), then a Block Recursive Equilibrium exists.
Proof. The proof will follow backward induction. Let $t = T$, and consider an unemployed household for the sake of brevity (an identical argument follows for employed households). Since the household’s continuation value is zero from $T+1$ onward, the household dynamic programming problem trivially does not depend on the aggregate distribution $\mu$ across states in the last period of life,

$$U_T^G(b, h, k; \Omega) = u(z(k) + b, 1) + \beta \cdot 0$$

$$= U_T^G(b, h, k; y, \bar{b})$$

$$W_T^G(b, h, k; \Omega) = u(\alpha f(y, h, k) + b, 1) + \beta \cdot 0$$

$$= W_T^G(b, h, k; y, \bar{b})$$

In this last period of life, the saving and borrowing policy function $b'_{e,T}(b, h, k; y, \bar{b})$ is trivially zero (for both employed $e = W$ and unemployed agents $e = U$). Likewise, for households in bad standing in the last period of life, the value of unemployment (and nearly identical conditions hold for the employed, and so are omitted) is given by,

$$U_T^B(b, h, k; y, \bar{b}) = u(z(k), 1) + \beta \cdot 0$$

Stepping back to the default decision, $U_T$ will also not depend on the aggregate distribution $\mu$,

$$U_T(b, h, k; y, \bar{b}) = \max \left\{ U_T^G(b, h, k; y, \bar{b}), U_T^B(0, h, k; y, \bar{b}) - \chi \right\}$$

Let $D_{U,T}(b, h, k; y, \bar{b})$ denote the policy function of the household. Since there is a utility penalty $\chi$ of defaulting, debt can be supported in equilibrium, and $D_{U,T}$ will not be trivially zero.

Now stepping back to the labor search problem, the firm’s value function will be independent of $\mu$ as well,

$$J_T(h, k; \Omega) = (1 - \alpha) f(y, h, k) + \beta \cdot 0$$

$$= J_T(h, k; y, \bar{b})$$
And the labor market tightness will also be independent of $\mu$,

$$\theta_T(h, k; \Omega) = p_f^{-1} \left( \frac{\kappa + (1 + r_f)k}{J_T(h, k; y, b)} \right) = \theta_T(h, k; y, b)$$

The household at age $T - 1$ (note that the primes below simply note that age $T - 1$ risk over $y$ and $b$ has already been realized and human capital has already evolved to $h'$) must therefore make the following labor market search choice over $k$, the capital of firms,

$$\max_{k \in K} \ p(\theta_T(h', k; y', b'))W_T(b', h', k; y', b') + (1 - p(\theta_T(h', k; y', b')))U_T(b', h', k; y', b') \quad (10)$$

So long as $k$ lies in a bounded interval, the extreme value theorem guarantees at least one solution to this problem. As we will see below, for certain classes of production functions, only one solution exists. For the current exposition, assume the production function lies within this class, and a unique solution exists.

Given the household policy functions for labor search $k'_{T-1}(h', k; y', b')$ and default $D'_{e,T}(h', k; y', b')$, the bond price $q_{U,T}(b', h, k; \Omega)$ is given by,

$$q_{U,T-1}(b', h, k; \Omega) = \frac{\mathbb{E}\left[1 - D'_{e,T}(b', h', k'; y', b')\right]}{1 + r_f} = q_{U,T-1}(b', h, k; y, b)$$

Clearly the bond price does not depend on the aggregate distribution $\mu$.

Stepping back from $t = T - 1, \ldots, 1$, and repeating the above procedure completes the proof.

\[\square\]

**Restatement of Corollary 5.2:** Under the hypotheses of Proposition 5.1, so long as $\chi > 0$, and $\mathcal{B}$ contains a neighborhood of debt around 0, a Block Recursive Equilibrium with credit exists.

**Proof.** Because of the inada conditions, for every positive $\chi \in \mathbb{R}_+$, there exists a sufficiently
small debt in an $\epsilon$-neighborhood around zero, $b \in N_\epsilon(0)$, such that the household strictly prefers repayment in the last period of life. The households repayment choice is given by,

$$\max \left\{ U^G_T(b, h, k; y, b), U^B_T(0, h, k; y, b) - \chi \right\}$$

This holds with equality at the cutoff debt $b^*$,

$$U^G_T(b^*, h, k; y, b) = U^B_T(0, h, k; y, b) - \chi$$

Substituting,

$$u(z(k) + b^*, 1) = u(z(k), 1) - \chi$$

The minimum supportable debt is given by,

$$b^* = u^{-1}(u(z(k), 1) - \chi, 1) - z(k) < 0$$

\[ \square \]

**Restatement of Lemma (5.3):** In addition to the assumptions in Proposition 5.1, let the production function be Cobb-Douglas, i.e. $f(y, h, k) = yh^{1-a}k^a$ ($0 < a < 1$), let the matching function be given by $M(u, v) = u^{\frac{1}{2}}v^{\frac{1}{2}}$, let $\chi \to \infty$ (no default for households), the value of leisure is zero, and assume there is no uncertainty over human capital $h$, aggregate productivity $y$, or the borrowing limit $b$. Then if the utility function is negative, increasing, and concave (e.g. $\frac{c^{1-\sigma-1}}{1-\sigma}$ for $\sigma > 1$ or $u(c) = -e^{-c}$), the household labor search problem (equation (10)) admits a unique solution.

**Proof.** The non-stochastic firm problem can be solved by hand, and under the hypotheses of the present lemma, it is directly proportional to capital,

$$J_t(h, k) = \frac{(1 - \alpha)f(y, h, k)}{1 - \beta(1 - \delta)} - \frac{(\beta(1 - \delta))^{T-t+1}(1 - \alpha)f(y, h, k)}{1 - \beta(1 - \delta)} \propto k^a$$

Under the assumption $M(u, v) = u^{\frac{1}{2}}v^{\frac{1}{2}}$, the equilibrium market tightness $\theta_t(h, k)$ can be solved by hand.

$$\kappa = \theta_t(h, k; \Omega)^{-\frac{1}{2}} \left[ J_t(h, k; \Omega) - (1 + r_f) \cdot \frac{k}{\theta_t(h, k; \Omega)^{-\frac{1}{2}}} \right]$$

63
Solving for $\theta_t$ yields,

$$\left(\frac{\kappa + (1 + r_f)k}{J_t(h, k; \Omega)}\right)^{-2} = \theta_t(h, k; \Omega)$$

The household job finding rate is therefore given by,

$$\left(\frac{J_t(h, k; \Omega)}{\kappa + (1 + r_f)k}\right) = p(\theta_t(h, k; \Omega))$$

For $\kappa$ and $r_f$ sufficiently small,

$$p(\theta_t(h, k; \Omega)) \propto k^{a-1}$$

The constant worker share $\alpha$ in combination with the non-negative and increasing production function implies that the wage a worker receives is concave and increasing in $k$. Note that the composition of two non-decreasing concave functions in $k$ preserves concavity in $k$, i.e. $\tilde{u}(k) = u(w(h, k) + \mu)$ is concave in $k$ for arbitrary $\mu$. Let $u$ be the outside option of the household if they remain unemployed. Since the probability of finding a job is directly proportional to $k^{a-1}$, the household chooses $k$ to maximize

$$k^{a-1}\tilde{u}(k) + (1 - k^{a-1})u$$

Since $-k^{a-1}$ is concave, we ignore the second term (the idea will be to show the first term is concave, and then use the fact that the sum of two concave functions is concave). The condition for the first term to be concave is given by,

$$\frac{(a - 1)(a - 2)k^{a-3}\tilde{u}(k)}{(-)} + 2(a - 1)k^{a-2}\tilde{u}'(k) + k^{a-1}\tilde{u}''(k) < 0$$

Under the hypotheses that $u < 0$, $u' > 0$, $u'' < 0$ (note, these properties transfer to $\tilde{u}$), and $0 < a < 1$, the labor search problem of the household is strictly concave and one solution exists for $k$. 

$\square$
Table 17 displays the business cycle moments for the main model in the text versus the data. The table makes the shortcomings of the model quite clear: the model is unresponsive to productivity shocks (Shimer [2005] and more recently Chodorow-Reich and Karabarbounis [2013]). Why does the Hagedorn and Manovskii [2008] calibration not work in this context? They noticed that the flow utility from non-employment must be large enough to make workers nearly indifferent between working and not working: workers then become sensitive to small movement in productivity and wages. It is impossible to make every type of worker indifferent between working and not working with significant heterogeneity and a constant unemployment benefit or flow utility of leisure. The only paper to our knowledge to address this issue is Lise and Robin [2013] who make the flow utility of non-employment a function of the workers type, the workers type squared, the aggregate state, and interactions between the workers type and the aggregate state. One alternate approach used by Christiano et al. [2013] to generate cyclical responses in the economy could be to squeeze firm surplus so that vacancy posting becomes very sensitive to small movements in productivity. However, since firms are heterogeneous as well, only the lowest type firm will be sensitive to productivity movements.
Table 17: Business Cycle Moments for Model During Main Simulation (1974 to 2012) vs. Data

<table>
<thead>
<tr>
<th>Model</th>
<th>x</th>
<th>u₁</th>
<th>v</th>
<th>θ</th>
<th>y</th>
<th>k</th>
<th>UE Rate</th>
<th>Default Rate*</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD(x)/SD(y)</td>
<td>2.69</td>
<td>1.66</td>
<td>1.91</td>
<td>1.00</td>
<td>0.97</td>
<td>1.66</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Autocorr(x)</td>
<td>0.73</td>
<td>0.39</td>
<td>0.78</td>
<td>0.82</td>
<td>0.79</td>
<td>0.40</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Corr(y,x)</td>
<td>1.00</td>
<td>-0.19</td>
<td>-0.74</td>
<td>-0.72</td>
<td>-0.73</td>
<td>-0.87</td>
<td>-0.14</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data</th>
<th>x</th>
<th>u₁</th>
<th>v</th>
<th>θ</th>
<th>y</th>
<th>k</th>
<th>UE Rate</th>
<th>Default Rate*</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD(x)/SD(y)</td>
<td>9.50</td>
<td>10.10</td>
<td>19.10</td>
<td>1.00</td>
<td>-</td>
<td>5.90</td>
<td>6.07</td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.88</td>
<td>-</td>
<td>0.91</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>Corr(y,x)</td>
<td>1.00</td>
<td>-0.89</td>
<td>-0.97</td>
<td>-0.41</td>
<td>-</td>
<td>-0.95</td>
<td>0.55</td>
<td></td>
</tr>
</tbody>
</table>

Notes: HP filtered with smoothing parameter 10⁵ to be consistent with Shimer [2005]. Data are from Shimer [2005], except (*) the default rate which is taken from Equifax (1999-2012). As in the data, u₁ is calculated as the fraction of unemployed households at the end of a quarter. θ = v/(u₁+u₂) includes the measure of households that immediately found jobs (u₂), hence the low volatility as that mass is quite large and very stable.
F Model Robustness: Capital Investment and Liquidation

F.1 Model with Firm Investment

Now assume that Firms can invest in capital, depending on the worker's type. The problem of an unemployed household is unchanged. The value functions for employed borrowers who default as well as the discrete default decision are formulated in an identical fashion to that of the unemployed.

Timing assumption: New capital is not operable immediately.

As a result, the Bellman equation for a household in bad standing is given below (good standing is extremely similar):

\[ W_t^B(b, h, k; \Omega) = u(c(0)) + \lambda \beta \mathbb{E}\left[ (1 - \delta)W_{t+1}(0, h', k'; \Omega') + \delta \left\{ \max_k p(\theta_{t+1}(h', \tilde{k}; \Omega'))W_{t+1}(0, h', \tilde{k}; \Omega') + (1 - p(\theta_{t+1}(h', \tilde{k}; \Omega')))U_{t+1}(0, h', k; \Omega') \right\} \right] + (1 - \lambda) \beta \mathbb{E}\left[ (1 - \delta)W_{t+1}^B(0, h', k'; \Omega') + \delta \left\{ \max_k p(\theta_{t+1}(h', \tilde{k}; \Omega'))W_{t+1}^B(0, h', \tilde{k}; \Omega') + (1 - p(\theta_{t+1}(h', \tilde{k}; \Omega')))U_{t+1}^B(0, h', k; \Omega') \right\} \right], \quad t \leq T \]

\[ W_{T+1}^B(b, h, k; \Omega) = 0 \]

Such that the aggregate laws of motion are given by equation (4), human capital evolves such that \( h' = H(h, W) \) and the budget constraint is given by,

\[ c \leq \alpha f(y, h, k) \]
And, additionally
\[ k' = k_t^*(h, k; \Omega) \]

This final condition \( k' = k_t^*(h, k; \Omega) \) means that households have rational expectations over what the entrepreneurs's optimal investment decision is.

### F.2 Lenders

Lenders’ bond prices are update to reflect changes in capital, since it may affect the wage of the worker and hence their repayment probability.

### F.3 Entrepreneurs

We now allow entrepreneurs to invest in capital subject to an adjustment cost \( \Gamma(k' - k) \).

Therefore the value function for the firm is given by,
\[
J_t(h, k; \Omega) = \max_{k'} (1 - \alpha)f(y, h, k) - i - \Gamma(k' - k) - f_c + \beta \mathbb{E}[(1 - \delta)J_{t+1}(h', k'; \Omega')] \]

Subject to a unit investment cost (i.e. the MRT of output and capital is 1, excluding the adjustment cost),
\[
i = k' - k \]
\[
J_{T+1}(h, k; \Omega) = 0 \]

In the results below, we choose a quadratic adjustment cost \( \Gamma(x) = x^2 \). We see that the presence of firm investment does not significantly alter the main set of results. Figure 12 illustrates employment with the quadratic adjustment cost, Figure 13 illustrates sorting (the correlation between human capital and capital), Figure 14 illustrates firm capital, and Figure 15 illustrates labor productivity. Figure 14 shows that firm capital recovers much faster as firms invest in more capital per worker as productivity recovers and human capital grows.
Figure 12: Allowing for Capital Investment: Employment

Figure 13: Allowing for Capital Investment: Corr. B/w Human Capital (h) and Firm Capital (k)

Figure 14: Allowing for Capital Investment: Agg. Firm Capital, 2007-2009 Recession

Figure 15: Allowing for Capital Investment: Labor Productivity, 2007-2009 Recession
F.4 Liquidation

We also allow for the baseline model to have a liquidation value of capital, $\chi_f$. The continuation value of the firm becomes,

$$J_t(h, k; \Omega) = (1 - \alpha)f(y, h, k) - f_c + \beta\mathbb{E}[(1 - \delta)J_{t+1}(h', k; \Omega') + \delta\chi_f k]$$

In the results below, we choose $\chi_f = .25$ which is relatively low, but it allows us to preserve the calibration, approximately. For larger values of $\chi_f$, the same aggregate patterns emerge, except we must significantly expand the capital grid to a point that it become computationally infeasible. Figures 16 and 17 illustrate the model’s main results with liquidation values. Employment rises while productivity falls in both cases, which is the same pattern that emerged when tighter debt limits were imposed in an economy with no liquidation value.

Figure 16: Liquidation Value Experiment: Employment, 2007-2009 Recession

Figure 17: Liquidation Value Experiment: Labor Productivity, 2007-2009 Recession
G Solution Algorithm

We solve the model using value function iteration on a discrete grid. Capital lies between \([0.25,1]\) with 40 evenly spaced grid points including the ends of the grid. Bonds lies on the grid \([-0.5,1.5]\) with 81 evenly spaced grid points. With the discount factor given in the paper, .04% of all simulated agent-time observations ever hit the top of the asset grid. The human capital grid is 6 evenly spaced grid points including the end of the grid over \([.5,1]\). The aggregate shock is discretized with 4 states using Rouwenhorst’s method. The bond limit is discretized with 2 possible values.

Starting at \(t = T\) and working backwards, the solution method is given below:

i. Recover \(J_t(h, k; \Omega)\) using value function iteration.

ii. Recover \(\theta_t(h, k; \Omega)\), the market tightness, by free entry, \(\theta_t(h, k; \Omega) = p^{-1} \left( \frac{r + (1 + r_f)k}{J_t(h, k; \Omega)} \right)\)

iii. Solve the household default decision to recover \(D_{e,t}(b, h, k; \Omega)\).

iv. Solve the household maximization problem over the grid of \(k\)'s to recover \(k_t(b, h, k; \Omega)\) using the market tightness and the implied job finding rates in step ii.

v. Use realized search behavior and default outcomes to recover the bond price \(q_{e,t}(b, h, k; \Omega)\) (in the last period of life, this is simply zero).

vi. Solve the household maximization problem over the grid of \(b\)'s to recover \(b'_{\epsilon,t}(b, h, k; \Omega)\), taking the bond price from step v as given.

vii. Repeat i to vi until \(t=1\).

viii. Fix the aggregate shock path. Use policy functions from household problem to simulate 30,000 households for 270 periods, 10 times, burning the first 100 periods. Report average over the 10 simulations.