Expected inflation and other determinants of Treasury yields

Gregory R. Duffee
Johns Hopkins University

First version April 2013
Current version February 2014

Abstract

A standard factor model is used to estimate the magnitude of inflation risk in nominal bonds. At a quarterly frequency, news about expected average inflation over a bond's life accounts for between 10 to 20 percent of shocks to nominal Treasury yields. This result is robust statistically, stable across time, and insensitive to the number of factors used in the model. Shocks to real rates and term premia account for the remainder of shocks to nominal yields.

Voice 410-516-8828, email duffee@jhu.edu. Address correspondence to 440 Mergenthaler Hall, 3400 N. Charles St., Baltimore, MD 21218. Thanks to seminar participants at the Booth School of Business, the Shanghai Advanced Institute of Finance, the University of Lausanne, participants at the Bank of Canada "Advanced in Fixed Income Modeling" conference, George Constantinides, and Mike Chernov for helpful comments. This version is still preliminary. Comments are encouraged.

1 Introduction

A large and expanding literature explores the relation between nominal bond yields and inflation. Ang and Piazzesi (2003) make a particularly important contribution by introducing Gaussian macro-finance dynamic term structure models. This framework ensures internally consistent prices of nominal and real bonds. No-arbitrage pricing determines the compensation investors require to face shocks to inflation. The related literature has quickly branched out to include unspanned macro risks, non-Gaussian dynamics, and fundamental explanations for inflation risk premia that are grounded in investor preferences and New Keynesian macro models.

This paper takes a step back from these advances to focus on a simple question. How much inflation risk is there in nominal Treasury bonds? To put more structure on this question, recall that nominal yields can be decomposed into the sum of three components: expected inflation over the life of the bond, expected short-term real rates over the life of the bond, and a nominal term premium. The question here is how much of the short-term variance of a long-term bond yield is attributable to news about the expected inflation component. Every dynamic term structure model that includes inflation as well as yields necessarily answers this question. However, the macro-finance literature devotes relatively little attention to measures of the quantity of inflation risk. Much greater emphasis is placed on risk premia—the product of the quantity of risk and the price of risk.

Estimates of the quantity of inflation risk embedded in nominal bonds vary wildly across prominent term structure models in the literature. (These estimates are typically not reported but can be computed from the model parameters.) Some estimated models imply that almost all of the variance of short-term shocks to nominal yields is attributable to news about average inflation over the life of the bond. Others imply that almost none of the variance stems from this source.

Perhaps these differences are attributable to the variety of restrictions on risk premia dynamics that appear in the literature. These restrictions tie physical dynamics to cross-sectional covariances. Not all of the restrictions can be correct, and false restrictions distort estimates of the physical dynamics.

To avoid imposing false restrictions, I estimate dynamic models of yields and expected inflation without imposing any restrictions on risk premia dynamics. In fact, I do not impose any no-arbitrage restrictions. The models explored here pin down physical dynamics but not equivalent-martingale dynamics. The focus is on the quantity of inflation risk, not the pricing of risk. Since yields depend on expected inflation rather than realized inflation, I follow the path pioneered by Pennacchi (1991) by linking yields to survey forecasts of inflation.

Using 45 years of quarterly data, I conclude that empirically, shocks to expected inflation are a small part of shocks to the nominal bond yields. Roughly 10 to 20 percent of variances of quarterly shocks to long-term Treasury bond yields are attributable to news about expected inflation over the life of the bond. This conclusion is robust statistically, holds across subsamples, and is satisfied for both parsimonious and highly flexible factor specifications.

The properties of the data that underlie this conclusion are easy to summarize. Although expectations of future inflation are highly persistent, they fluctuate little over time. The standard deviation of shocks to survey forecasts of three-quarter-ahead inflation is much less than standard deviations of shocks long-term bond yields. Even if inflation expectations are extremely persistent, these relative volatilities do not allow shocks to expected average inflation to drive much of the innovation in long-term yields.

Thus mechanically, innovations to expected short-term real rates and term premia are the primary drivers of yield shocks. There is insufficient information in the data to disentangle the relative contributions of these two components. Again, the relevant properties of the data are easy to summarize. Shocks to short-term real rates are large, and long-term nominal yields covary strongly with them. If short-term real rates are highly persistent, then the variation in long-term yields is explained by shocks to average expected future short-term real rates. If short-term real rates die out quickly, the variation is explained by term premia that positively covary with short-term real rates. Point estimates of the persistence are consistent with the latter account, but statistical uncertainty in these estimates cannot rule out the former account.

The uncertainty about persistence is consistent with asset-pricing research that predates the active use of dynamic term structure models. It is also consistent with recent work in the applied cointegration literature. This work typically produces a double negative: we cannot reject the hypothesis that inflation and nominal short-term yields are both nonstationary and not cointegrated.

The next section describes how I measure the quantity of inflation risk and makes some preliminary calculations based on the existing literature. Section 3 describes the dynamic model I use to estimate inflation risk. The main results are in Section 4. Section 5 takes a deeper look at these and related results. Section 6 concludes.

2 The question and some preliminary evidence

Identifying the amount of inflation risk in a nominal bond is inherently ambiguous because shocks cannot be cleanly decomposed into those that affect the path of expected inflation and those that do not. This is obvious in New Keynesian models such as the one developed in Rudebusch and Swanson (2012). There are no exogeneous shocks to inflation. Instead, there are shocks to productivity, monetary policy, and government spending, each of which affects the paths of expected inflation, real rates, and risk premia. All risk is inflation risk, in the sense that every shock contains news about expected future inflation.

The same conclusion holds for typical reduced-form macro-finance models. In the canonical macro-finance setting of Joslin, Le, and Singleton (2013), all shocks affect the paths of all state variables. In fact, if the model includes inflation, it is straightforward to rotate the length N state vector to

$$X_t = \begin{pmatrix} \pi_t & E_t\pi_{t+1} & \dots & E_t\pi_{t+n-1} \end{pmatrix}'. \tag{1}$$

It is clear from this rotation that all state variables and all shocks affect expected future inflation and thus nominal yields.

Models in which a monetary authority follows a Taylor rule typically satisfy the property. Inflation is endogenous, and absent strong assumptions, no shocks that affect nominal yields without also affecting expected inflation. This holds even for exogeneous shocks to risk preferences, as discussed in Gallmeyer, Hollifield, Palomino, and Zin (2008).

Sufficiently strong assumptions help to reduce the identification problem. The real business cycle term structure model of van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) exhibits money neutrality. Exogeneous shocks to inflation affect nominal yields but not real yields or risk premia. In a reduced-form setting, similar restrictions can be imposed on model dynamics. For example, Ang and Ulrich (2012) assume the existence of monetary policy shocks that affect nominal yields but are independent of inflation at all leads and lags. Such shocks are clearly not inflation shocks. However, it is not useful to define a measure of inflation risk that can be applied to a handful of models.

The approach here defines the quantity of inflation risk through the statistical properties of yields and inflation. It is model-free, in the sense that quantity is well-defined regardless of the underlying model. Thus we can think of the quantity of inflation risk as a moment to help evaluate macro-finance models.

The focus on statistical properties does not circumvent the problem of identifying the quantity of inflation risk. For example, we might consider a simple regression of yields on the contemporaneous expectation of j-period-ahead future inflation,

$$y_t^{(n)} = \beta_{0,n,j} + \beta_{1,n,j} E_t(\pi_{t+j}) + e_{n,j}. \tag{2}$$

Then shocks to inflation expectations affect yields through the regression coefficient. But why stop with a single forecast horizon on the right side? We can add additional inflation expectations at different horizons to capture more inflation risk in bonds. However, this approach runs up against the rotation invariance of Joslin et al. (2013). In a canonical macro-finance model with inflation, the state vector (1) explains everything. In the model, adding sufficient forecast horizons to the right side produces an R^2 of one. This does not lead to a useful measure of inflation risk.

The approach here uses an accounting identity to decompose shocks to yield into shocks to expected real rates, expected inflation, and term premia. The next subsection describes the methodology.

2.1 Measuring inflation risk

Formal analysis of the relation between nominal yields and expected inflation begins with Fisher (1930). He describes the decomposition of a short-term nominal interest rate into expected inflation and a real rate. Macaulay (1938) extended this logic to long-term yields, interpreting them in terms of expected future short-term yields. Decades of ensuing research has refined their intuition.

The following notation helps clarify the relevant concepts.

$y_t^{(n)}$: Continuously compounded yield, nominal zero-coupon bond maturing at $t+n$.

π_t : log change in the price level from $t-1$ to t.

π_t^e : Period-t expectation of next period's inflation, $\pi_t^e \equiv E_t\left(\pi_{t+1}\right)$.

r_t : one-period real rate for nominal bonds, $r_t \equiv y_t^{(1)} - \pi_t^e$.

The one-period real rate r_t, also known as the ex-ante real rate, differs from the yield on a one-period real bond owing to both Jensen's inequality and the compensation investors require to face uncertainty in next period's price level. Inflation expectations should be thought of as investors' forecasts.

Yields on multiperiod bonds can be written as the sum of average expected short-term yields and term premia. Formally, the term premium on an n-period nominal bond, denoted

$ntp_t^{(n)}$, satisfies

$$y_t^{(n)} \equiv \frac{1}{n} \sum_{j=0}^{n-1} E_t \left(y_{t+j}^{(1)} \right) + ntp_t^{(n)}. \tag{3}$$

Given the yield, the term premium depends on the expectations. We treat the expectation operator in (3) as investors' expectations and thus the term premium is investors' perception of the term premium. Standard manipulations express this nominal yield in a variety of useful forms. One replaces the expected short-term nominal yield in (3) with its components:

$$y_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} E_t \left(r_{t+j} \right) + \frac{1}{n} \sum_{j=0}^{n-1} E_t \left(\pi_{t+j}^e \right) + ntp_t^{(n)}. \tag{4}$$

Nominal yields are the sum of expected average inflation and average real rates over the life of the bond, plus a term premium. I use some shorthand to refer to the first two components of this decomposition:

$$y_t^{(n)} = E_t(\overline{r}_{t,t+n-1}) + E_t(\overline{\pi}_{t+1,t+n}) + ntp_t^{(n)}. \tag{5}$$

Yield variance is the sum of the variance of the individual components of yields and twice their covariances:

$$\begin{aligned}
\text{Var}\left(y_t^{(n)}\right) =& \text{Var}\left(E_t(\overline{r}_{t,t+n-1})\right) + \text{Var}\left(E_t(\overline{\pi}_{t+1,t+n})\right) + \text{Var}\left(ntp_t^{(n)}\right) \\
&+ 2\text{Cov}\left(E_t(\overline{r}_{t,t+n-1}), E_t(\overline{\pi}_{t+1,t+n})\right) \\
&+ 2\text{Cov}\left(E_t(\overline{r}_{t,t+n-1}), ntp_t^{(n)}\right) + 2\text{Cov}\left(E_t(\overline{\pi}_{t+1,t+n}), ntp_t^{(n)}\right).
\end{aligned} \tag{6}$$

The direct contribution of expected average inflation to yield variance is the ratio of the second term on the right to the total variance. The indirect contribution is the sum of the first and third covariance terms divided by total variance.

The variance decomposition (6) is unconditional. It does not tell us about the uncertainty bond investors face from t to $t+1$. This uncertainty is captured by a conditional version, which replaces the unconditional variances and covariances in (6) with conditional versions. Similarly, the direct and indirect contributions of expected inflation are

$$\text{direct measure} \equiv \frac{\operatorname{Var}_t\left(E_{t+1}(\overline{\pi}_{t+2,t+n+1})\right)}{\operatorname{Var}_t\left(y_{t+1}^{(n)}\right)}, \tag{7}$$

$$\text{indirect measure} \equiv \frac{2\left[\operatorname{Cov}_t\left(E_{t+1}(\overline{r}_{t+1,t+n}), E_{t+1}(\overline{\pi}_{t+2,t+n+1})\right) + \operatorname{Cov}_t\left(E_{t+1}(\overline{\pi}_{t+2,t+n+1}), ntp_{t+1}^{(n)}\right)\right]}{\operatorname{Var}_t\left(y_{t+1}^{(n)}\right)}. \tag{8}$$

These are the measures of the quantity of inflation risk that are emphasized in the empirical analysis that follows.

2.2 Some measurements from the literature

Dynamic term structure models that include inflation characterize fully the dynamics of nominal yields and inflation. Therefore given a parameterized stationary model, the population regression properties of (2) can be calculated. Similarly, the direct and indirect measures of (7) and (8) can be calculated, both conditionally and unconditionally.

This subsection applies these calculations to four well-known contributions to the literature. Campbell and Viceira (2001) estimate two-factor Gaussian no-arbitrage models of nominal yields and inflation for two sample periods. Ang, Bekaert, and Wei (2008) estimate a four-factor model with an additional factor that captures changes in regimes. Chernov and Mueller (2012) estimate a variety of four-factor and five-factor Gaussian models, and Haubrich, Pennacchi, and Ritchken (2012) estimate a seven-factor model with stochastic volatility.

Table 1 reports various population properties of five-year nominal bond yields computed from the reported parameters.[1] The direct contribution ratio (7) is reported in the column

[1]For Ang et al., I use the parameters of their benchmark IV^C model. For Chernov and Mueller, I use

labeled “Avg conditional variance ratio.” Here, “Avg” refers to the numerator, which is average expected inflation over the life of the bond. The corresponding column labeled as unconditional reports the unconditional version of the same ratio. The regression coefficient and R^2 for the regression (2), using three-quarter-ahead inflation expectations as the explanatory variable, are reported in the final two columns.

The models disagree substantially about the relation between yields and expected inflation. Estimates of the conditional direct contribution of inflation expectations range from almost none of the yield variance (0.03) to most of the yield variance (0.60). The range of estimates for the unconditional direct contribution is even greater. Similarly, estimates of the regression (2) differ widely, with coefficients ranging from less than -1.0 to more than 2.0, and R^2s ranging from 0.05 to 0.99.

Which estimates are more plausible? We cannot answer this question cleanly because because we do not have direct observations of either forecasts of five-year average inflation or shocks to these forecasts. We do, however, have comprehensive survey forecasts of inflation at shorter horizons. I therefore use these same models to calculate modified versions of the direct contribution of expected inflation. The new measures replace the numerator in (7) and its unconditional counterpart with the variance of three-quarter-ahead expected inflation. When inflation is mean-reverting, the variance of three-quarter-ahead expected inflation should be larger than the variance of average expected inflation over a five-year horizon. Thus the resulting variance ratios should be larger, a conclusion that is verified in Table 1.

The columns labeled “3-Q” are the modified variance ratios. Models that imply extremely persistent inflation, such as the first model of Campbell and Viceira, exhibit only a tiny difference between the modified variance ratios and the original variance ratios. Models with less persistent inflation, such as Ang et al., have much larger modified ratios than original ratios.

---

the parameters of their $AO5$ model. Thanks to Mike and Philippe for helping me with their model.

I compare these model-implied moments to sample moments. I use quarterly observations of a five-year Treasury bond yield and survey forecasts of three-quarters-ahead inflation. Section 3.3 describes the data in detail. Conditional variances and covariances are proxied by sample variances and covariances of residuals from forecasting regressions. I fit both the five-year yield and the inflation forecast to univariate AR(1) models. The variances and covariances of the fitted residuals are used as crude proxies for true conditional variances and covariances.

Results for three sample periods are displayed in Table 2. The first row uses the full sample 1968Q4 through 2012Q4. The second row uses only the sample through the Federal Reserve's monetarist experiment period, ending in 1982Q4. The third row uses the remaining sample of 1983Q1 through 2012Q4.

The sample variance ratios are all on the low side of the model-implied counterparts in Table 1. They are more consistent with the models of Chernov and Mueller (2012) and Haubrich et al. (2012) than those of Campbell and Viceira (2001) and Ang et al. (2008). However, the sample explanatory power of the regression (2) are unlike those of either Chernov and Mueller or Haubrich et al. The former model implies a small R^2 between yields and expected inflation, while the latter model implies an almost perfect fit. In the samples, the R^2s are large but not close to perfect.

To be fair, none of these models is designed expressly to fit the moments considered here. The proper interpretation of this discussion is that we cannot look to the existing literature to pin down the amount of inflation risk embedded in nominal bonds. We need a methodological approach that focuses on this measurement.

3 A dynamic model of yields and expected inflation

This section describes a state-space Gaussian model that describes the joint dynamics of nominal yields and expected inflation. It then discusses the data used to estimate the model.

3.1 The framework

State-space models are standard in the dynamic term structure literature.[2] A vector of observables are linked through their loadings on an N-vector of common factors, where there are more observables than factors. The common factors have the Gaussian VAR(1) dynamics

$$x_{t+1} = \mu + Kx_t + \Sigma\epsilon_{t+1}, \qquad \epsilon_{t+1} \sim MVN(0, I). \tag{9}$$

Relevant observables are assumed to be affine functions of the state vector. The notation for nominal yields is

$$y_t^{(n)} = A_n + B_n' x_t + \eta_{n,t}, \tag{10}$$

where $\eta_{n,t}$ represents measurement error or some other deviation from an exact affine representation.

The absence of arbitrage is not imposed. Therefore the coefficients of (10) are unrestricted. By itself, the assumption of no-arbitrage is probably unimportant here. Joslin, Le, and Singleton (2013) show that when risk premia dynamics are not constrained, Gaussian no-arbitrage macro-finance models are close to factor-VAR models such as (9) and (10). No-arbitrage is typically imposed to allow researchers to impose economically-motivated restrictions on risk premia dynamics. All of the research discussed in the context of Table 1 imposes such additional restrictions.

Since these restrictions link equivalent-martingale dynamics to physical dynamics, they affect estimates of the magnitude of inflation risk in nominal yields. The evidence in Table 1 suggests that at least some of these restrictions produce estimates of inflation risk that are at odds with what we observe in the data. I choose to impose no pricing restrictions rather than attempt to determine which of these restrictions work better.

[2] The first use of these models in the real-rate literature is Hamilton (1985), although his motivation differs from that in the dynamic term structure literature.

3.2 Survey forecasts of inflation

Like many other macro-finance researchers, I follow Pennacchi (1991) by using survey forecasts of inflation, including Chernov and Mueller (2012) and Haubrich et al. (2012). The main reason to include survey forecasts is their accuracy. Ang, Bekaert, and Wei (2007) conclude that inflation forecasts from surveys are more accurate than econometric forecasts. Faust and Wright (2009) and Croushore (2010) draw the same conclusion. The notation for period-t survey forecasts of inflation at period $t + j$ is

$$E_t^s(\pi_{t+j}) = A_{\pi,j} + B'_{\pi,j} x_t + \eta_{\pi,j,t}. \tag{11}$$

I interpret the survey forecast in (11) as investors' expectation of j-ahead inflation, possibly contaminated by some measurement error. In other words, the model imposes

$$E_t(\pi_{t+j}) = A_{\pi,j} + B'_{\pi,j} x_t. \tag{12}$$

I break from the usual approach (including Pennacchi, Chernov and Mueller, and Haubrich et al.) by not including realized inflation among the observables. There are two reasons. First, they are unlikely to improve the fit of the model. Ang et al. find no evidence that using realized inflation in addition to survey forecasts helps reduce forecast errors. In a comprehensive handbook chapter, Faust and Wright (2012) concur: "…purely judgmental forecasts of inflation are right at the frontier of our forecasting ability." Including realized inflation among the observables increases the number of free parameters and raises the likelihood of overfitting.

Second, I am not interested in estimating the compensation investors require to face shocks to inflation. If investors are not risk-neutral with respect to the shock $\pi_t - E_{t-1}\pi_t$, then the one-period nominal rate will include a risk premium. This risk premium cannot be pinned down without observing realizations of inflation. However, risk premia are not the focus of this analysis.

3.3 The data

Inflation forecasts are from the Survey of Professional Forecasters. Near the beginning of the second month in quarter t, respondents provide predictions of the GDP price level for quarters t, $t+1$, $t+2$, $t+3$, and $t+4$. These imply expected inflation for quarters $t+1$ through $t+4$. I construct the mean cross-sectional prediction following the procedure of Bansal and Shaliastovich (2012), which discards outlier responses from individual forecasters.

I use the forecasts for $t+1$ and $t+3$ in the empirical analysis that follows, capturing both short-horizon and medium-horizon inflation forecasts. I do not use the forecasts for quarter $t+4$ because there are a few missing observations for this series.

Seven zero-coupon Treasury bond yields are observed at each quarter. They are for maturities of three months, one through five years, and ten years. The ten-year yield is constructed by bond constructed by staff at the Federal Reserve Board following the procedure of Gurkaynak, Sack, and Wright (2007). All other artificial yields are constructed by the Center for Research in Security Prices (CRSP). To align the yields with inflation survey data, I use yields as of the end of the second month in the quarter.

The first observation of the survey data is 1968Q4. The final observation used in this empirical analysis is 2012Q4, for a total of 177 quarters.

3.4 Estimation details

The length-N state vector is latent and thus underidentified. For convenience, I rotate the vector such that the first two elements are the survey forecasts of one-quarter-ahead inflation and three-quarter-ahead inflation. The third element is the one-quarter nominal yield. For model versions with more than three factors, the remaining $N-3$ elements of the state vector are bond yields.[3] The parameters are constrained to satisfy (12) for both the one-quarter-ahead and three-quarter-ahead inflation forecasts. In practice, this means that the model-implied prediction of the two-quarter-ahead survey forecast of one-quarter-ahead inflation equal the current three-quarter-ahead survey forecast. For the chosen state vector rotation, this corresponds the N nonlinear restrictions

$$e_1' K^2 = e_2$$

and the single additional linear restriction on mu,

$$e_1'(I + K)\mu = 0,$$

where e_i is a length-N vector with element i equal to one and all others equal to zero.

The free parameters are estimated with the Kalman filter, which corresponds to maximum likelihood under the model's assumptions. The covariance matrix of parameter estimates is constructed with the outer product of first derivatives. Confidence bounds on nonlinear functions of the parameters are calculated using Monte Carlo simulations, randomly drawing parameter vectors from a multivariate Gaussian distribution with a mean equal to the parameter estimates.

Three, four, and five-factor versions of the model are estimated. Estimation is performed over three sample periods: the full sample 1968Q4 through 2012Q4, the monetarist experiment subsample 1968Q4 through 1982Q4, and the post-experiment sample 1983Q1 through 2012Q4.

Given the model's parameters, expected average ex ante real rates and inflation are calculated using standard VAR mathematics. The mean of expected values of the state

[3]More precisely, the factors are yields survey forecasts uncontaminated by the measurement error in (11) and (10) respectively. Thus in principle the state vector is unobserved.

vector from t to $t+n-1$ is

$$\frac{1}{n}\sum_{j=0}^{n-1} E_t(x_{t+j}) = \left(I - \frac{1}{n}(I-K^n)(I-K)^{-1}\right)(I-K)^{-1}\mu + \frac{1}{n}(I-K^n)(I-K)^{-1}x_t.$$

Write this as

$$\frac{1}{n}\sum_{j=0}^{n-1} E_t(x_{t+j}) = W_0 + W_1 x_t.$$

Since one-step-ahead inflation is the first element of the state vector, the expected inflation rate from 1 to n is

$$\frac{1}{n}\sum_{j=0}^{n-1} E_t\pi_{t+1+j} = e_1' W_0 + e_1' W_1 x_t.$$

Similarly, since the one-period nominal yield is the third element of the state vector, the average expected ex ante real rate from 0 to $n-1$ is

$$\frac{1}{n}\sum_{j=0}^{n-1} E_t r_{t+j} = (e_3 - e_1)' W_0 + (e_3 - e_1)' W_1 x_t.$$

The term premium on the n-period nominal bond is the bond's yield less the average expected real rate and the average inflation rate,

$$ntp_t^{(n)} = A_n - e_3' W_0 + (B_n' - e_3' W_1)\, x_t.$$

Unconditional and conditional population covariances matrices of these three components of the bond's yield are easily calculated from the loadings of the components on the state vector x_t and the corresponding covariance matrices of the state vector.

4 Results

The first subsection documents the main conclusion that inflation shocks account for a small fraction of the variance of shocks to nominal yields. The second subsection discusses this result in more detail.

4.1 Variance decompositions

The most important message contained in these results is that inflation shocks account for a small fraction of the total variance of shocks to nominal yields. Table 4 presents detailed results for a four-factor model estimated over the entire sample 1968Q4 through 2012Q4. Variance decompositions are reported in Panel A for bonds with maturities of one, five, and ten years. Panel B sums the direct and indirect contributions of expected inflation.

There are four observations from the table worth emphasizing. First, for all the bonds, less than 15 percent of the total variance of nominal yield shocks is statistically explained by the direct contribution of news about inflation expectations. The 95 percent confidence bounds are tight. For each bond, we can confidently reject the hypothesis that a quarter of the variance is attributable to inflation expectations.

Second, there is insufficient information in the data to decompose accurately the remainining variance of long-maturity yields into news about expected future real rates and news about term premia. The point estimates suggest that the latter source is relatively more important, but the confidence bounds do not rule out the possibility that either source dominates the other.

Third, point estimates imply a positive covariance between news about expected real rates and news about term premia. The estimates indicate that between 25 and 40 percent of the variance of yield shocks is attributable to this covariance. The confidence bounds are very large. The next subsection shows that this observation is closely related to the second observation above.

Fourth, point estimates imply a negative covariance between news about expected inflation and news about term premia. Thus Panel B shows that the total direct and indirect contribution of inflation to the variance of nominal yields is close to zero. The confidence bounds are wider than those for the direct contribution in Panel A because of the uncertainty in the covariance between news about inflation expectations and term premia.

The result that inflation expectations contribute little to the variance of nominal yields is not dependent on the choice of a four-factor model. Table 4 reports estimates of the direct and total contributions of inflation expectations for three-factor and five-factor models estimated over the same sample. (To conserve space, other variance ratios are not reported in the table.) All of the reported point estimates of the direct contribution of inflation expectations are less than 0.2. Even the largest confidence bound in the table allows us to reject the hypothesis that more than 35 percent of the variance of a yield's shocks is attributable to news about inflation expectations. Sums of the direct and indirect contributions of inflation expectations are, on average, a little larger in Table 4 than in Table 3, and the confidence bounds are not quite as tight, but on balance the differences between the tables are small.

The sample period includes the monetarist experiment, the Great Moderation, and the financial crisis. It is well-known that inflation (and inflation expectations) were more volatile in the early part of the sample than in the latter. See, e.g., the discussion in Campbell, Shiller, and Viceira (2009). This pattern suggests that the variance ratios reported in Tables 3 and 4 are not stable over time. To examine stability, I split the sample at 1982Q4 and estimate the three-factor and four-factor models over the two subperiods. (There are too few observations in the samples to reliably estimate the five-factor version.)

The point estimates are displayed in Table 5. Confidence bounds are not reported because they are not relevant to the main conclusion to draw from the table.[4] The direct contribution of inflation is not noticeably different across the two subsamples. Only one of the point estimates exceeds 0.15.

However, there is evidence of instability in other components of the variance decomposition. In particular, covariances between news about expected inflation and news about term premia are strongly negative in the first part of the sample, and close to zero in the second part. This pattern and other features of the data are explored in more detail in the next

[4]Moreover, there are too few observations in the first subsample to calculate confidence bounds.

subsection.

4.2 Interpreting features of the variance decompositions

How can the monetarist experiment period, characterized by volatile inflation expectations, have such a small fraction of yield innovations attributable to news about expected inflation? The reason is that all of the components of yields were more volatile during the monetarist experiment period. The evidence is in Figure 1.

Figure 1 displays filtered estimates of shocks to the three components of a five-year yield. The time series in the top row are based on a single estimation over the full sample, while the time series in the bottom row are spliced together from separate estimation over the two subsamples. Both rows provide visual evidence of the main empirical observation. Shocks to the five-year yield owing to average expected inflation (the middle panels) are much less volatile than shocks owing to either average expected real rates or term premia.

Consistent with our intuition, the sample standard deviation of shocks to average expected inflation in the first subsample is much larger than it is in the second subsample. Using full-sample filtered estimates, the ratio of standard deviations is about 1.7. Using spliced-sample results, the ratio is about 1.9.

According to the two sets of filtered estimates, the volatilities of the other two components were also higher in the first subsample. For the full-sample estimates, most of the action is in changes in the volatility of average expected real rates. Its standard deviation in the first subsample is more than twice as large as it is in the second subsample. For the spliced sample estimates, the action is concentrated in changes in the volatility of term premia shocks. This discrepancy supports an observation made in the previous subsection: the data do not allow us to distinguish clearly between the effects of average expected real rates and term premia.

Impulse responses to shocks provide some additional information about the small contribution of inflation expectations to yield shocks. Figure 2 displays responses to a one standard deviation quarterly shock to the three-quarter-ahead inflation forecast, based on full-sample results for the four-factor model. Panel A reveals that the shock is small—about 30 basis points—and highly persistent. The point estimate of the immediate response by the five-year bond yield is about 20 basis points, and is also highly persistent. Responses by the real short rate and the term premium on a five-year bond are economically small and statistically insignificant.

This figure contrasts sharply with Figure 3, which displays responses to a one standard deviation shock to the real short rate. (The shocks in Figures 2 and 3 are not orthogonalized.) The initial shock is large—about a full percentage point and the point estimates imply that it dies out quickly. The immediate response of the five-year yield is about 50 basis points. Because the shock dies out so quickly, this response of the five-year yield substantially exceeds the shock to the average expected real rate over the next five years. Thus the term premium for the bond also immediately jumps by almost 30 basis points.

The pattern of these responses underlies an observation made in the previous subsection: the covariance between average expected real rates and term premia is large. The confidence bounds on these responses underly another earlier observation: the data do not allow us to distinguish statistically between the roles played by average expected real rates and term premia. The point estimates imply that shocks to real rates die out quickly, but the confidence bounds in Panel A allow for the possibility that real rates are actually highly persistent. If real rates are highly persistent, then the immediate response of the five-year yield to the shock to real rates is in line with the shock to the average expected real rate over the next five years. Hence the confidence bounds on the response of the term premium includes the possibility that term premia do not react at all.

Another way to say this is that in the sample, shocks to real rates are volatile and not persistent. Long-term bond yields covary strongly with these shocks and these responses die out quickly as well. There are two ways to explain this pattern. One is that term premia are also volatile, covary strongly with real rates, and die out quickly. The other is that the sample pattern is at odds with the population properties of the data. In the population, shocks to real rates are highly persistent. Investors know this and price long-term bonds accordingly. Investors were subsequently surprised by the speed at which the shocks died out in the sample. There is not enough information in the sample to reject either explanation.

Subsample impulse responses to expected inflation shocks are displayed in Figures 4 and 5. Confidence bounds are not included for the pre-1983 sample because the sample is too short to compute them. The responses for the post-1982 period in Figure 5 are similar to those for the full sample in Figure 2. The responses in Figure 4 differ from these other responses in one striking way. A positive shock to the three-quarters-ahead inflation forecast corresponds (at least according to the point estimates) to a *drop* in the bond yield. This drop is reversed within a few quarters. The model therefore requires a large negative shock to term premia that quickly dies out.

This pattern suggests that the joint behavior of yields and inflation expectations during the turbulent monetarist experiment is probably too complex to capture with the VAR(1) model used here. For example, between 1981Q2 and 1981Q3, the five-year yield rose by 165 basis points. At the same time, the one-quarter-ahead and three-quarters-ahead inflation forecasts fell by 85 and 100 basis points respectively. Perhaps risk premia increased substantially owing to the recession that began in July 1981. But another plausible explanation is that the five-year inflation expectation rose even as the recession triggered a drop in short-horizon inflation forecasts.

4.3 Unconditional properties of yields and expected inflation

Inflation risk is created by shocks. Therefore the empirical analysis in this section emphasizes the behavior of shocks to yields and expected inflation. Nonetheless, it is worth devoting some attention to unconditional properties. Consider regressing yields on contemporaneous expectations of future inflation, as in (2). Table 6 reports population point estimates, R^2s, and their confidence bounds of regressions, based on full-sample estimation of models with three, four, and five factors.

The main conclusion to draw from the reported results is that the confidence bounds are much too large to say anything concrete about the unconditional regression. The point estimates might exceed two, or might be close to zero. The population R^2s might be close to one, or they might be close to zero.

The large confidence intervals are another manifestation of the inability to pin down the persistence of shocks to real rates. Shocks to expected inflation are small and highly persistent. Shocks to real rates are large. If they are also highly persistent in population, then unconditional yield variability will be driven primarily by real rate variability. By contrast, if the population persistence of real rates matches their persistence in the sample, unconditional yield variability will be driven primarily by expected inflation variability.

5 Extensions

Litterman and Scheinkman (1991) show that almost all of the cross-sectional variation in bond returns can be characterized by level, slope, and curvature factors.[5] Returns are closely related to yield shocks, thus it is not surprising that the same decomposition holds for shocks.

Panel A of Figure 6 displays the first two principal components of the model-implied population covariance matrix of yield shocks.[6] The model uses four factors and is estimated over the full sample. For clarity the third factor is not shown. The first two components, which the figure shows can be called level and slope, explain 99.9 percent of the population covariance matrix.

Panel B reports the first two principal components for shocks to the real part of nominal yields, defined as the nominal yield less expected inflation during the life of the bond. The first two principal components explain 99.8 percent of the model-implied population covariance matrix. Again, they can be called level and slope. Note that the real part of nominal

[5]They are often cited for documenting the same three factors in the levels of yields. Although level, slope, and curvature also explain levels, that evidence is not in Litterman and Scheinkman.

[6]The bond maturities are the same as those used to estimate the model; three months, one through five years, and ten years.

yields is not the same as the yield on a real bond. Nominal term premia need not equal real term premia. Model-implied real bond yields cannot be calculated because the model does not impose no-arbitrage.

With this four-factor, four-shock model, each of these components can be expressed as an exact linear combination of four arbitrarily rotated shocks. For example, they can be written as functions of shocks to expected inflation at horizons of one to four quarters. This rotational indeterminacy suggests that we should not attempt to expend much effort at mapping the principal components to specific types of shocks. Nonetheless, some simple correlations are intriguing.

Table 7 reports correlations among the four principal components, the shock to three-quarter-ahead expected inflation, the shock to the real rate, and the shock to the term premium on a ten-year nominal bond. The correlations suggest the following description of shocks to yields. There are two types of level shocks to yields. One is a shock to expected inflation. The other is a shock to real rates. The table reports that these two shocks are close to orthogonal, and combined they explain, in an R^2 sense, almost the entire first principal component of yield shocks. When expected inflation is stripped out of nominal yields, almost all that remains is the real yield shock. The correlation between the first principal components of the real part of nominal yields and the shock to the real rate is close to one.

As emphasized in the discussion of Section 4.2, the level response to real rate shocks is consistent with investors that view shocks to real rates as highly persistent. It is also consistent with a combination of low-persistence real-rate shocks and term premia that are positively correlated with real-rate shocks. The latter explanation is more consistent with the point estimates of the model, which is why term premia shocks are also highly correlated with the first principal components of nominal yields and their real parts.

The slope principal components are most closely associated with shocks to term premia. This is not surprising, given that a long line of research beginning with Fama and Bliss

(1987) and Campbell and Shiller (1991) finds strong evidence that the slope of the term structure has forecast power for excess returns.

5.1 Shifting endpoints

It is worth mentioning another interpretation of the relation between the term premium and the slope. Following Kozicki and Tinsley (2001), many researchers have interpreted part of the variation in long-term nominal yields as the response to shifting forecasts of very long-run inflation. Examples include Rudebusch and Wu (2008), Gurkaynak, Sack, and Swanson (2005), and (briefly) Rudebusch and Swanson (2012).

The factor model of Section 3 is compatible with the spirit of shifting endpoints. The model allows inflation expectations to contain both low-persistence and high-persistence components. The model also allows for long-horizon inflation expectations to be inferred from both long-term yields and short-horizon inflation expectations. In other words, long-horizon expectations do not need to move in lockstep with short-horizon expectations. In fact, these are features of the estimated models. We've seen in Figure 2 that shocks to inflation expectations are highly persistent. In addition, much of the shock to ten-year-ahead inflation expectations is not attributable to shocks to either one-quarter-ahead or three-quarter-ahead inflation expectations. The population R^2 of a regression of the ten-year ahead expectation shocks on the two short-horizon shocks is less than 0.65.[7] Yield shocks explain the remainder.

Nonetheless, it is possible that the model does not capture fully the shifting endpoints. If so, yield responses to shocks to long-horizon inflation expectations will be interpreted by the model as term premia shocks. For example, a positive shock to long-run expectations unaccompanied by changes in short-run expectations will produce a positive slope shock accompanied by a positive term premia shock.

The principal component analysis here suggests that any such misspecification is eco-

[7] This information is not reported in any table. The R^2 is for the four-factor model estimated over the full sample.

nomically small. Term premia are included in the real part of nominal bond yields. The first principal component displayed in Panel B of Figure 6 explains more than 95 percent of the variance of shocks. Moreover, Table 7 reports that the correlation between slope shocks and term premia shocks is about 0.5. Put differently, three-fourths of the variance of slope shocks is unexplained by variations in term premia. This puts a tight upper bound on the importance of long-horizon inflation expectations not captured elsewhere by the model.

5.2 Inflation compensation and TIPS yields

This paper does not attempt to estimate the compensation investors require to face inflation risk. We might guess that inflation compensation must be small because the quantity of inflation risk is small. However, if Sharpe ratios for facing inflation risk are sufficiently high, risk compensation for holding nominal bonds will be high.

An indirect way to get at this issue is to compare the real part of nominal yields, defined in the previous subsection, to observed yields on Treasury Inflation Protected Securities (TIPS). This is a highly informal exercise. The real parts of nominal yields are model-dependent and filtered from inflation expectations, while TIPS yields are contaminated by illiquidity. D'Amico, Kim, and Wei (2008) discuss the problems this illiquidity poses for inferring inflation risk premia.

Yields on artificially-constructed zero-coupon TIPS bonds are available for maturities between five and ten years for the sample 1999Q1 through 2012Q12.[8] For this exercise I focus on the five-year yield; results for others are similar. Figure 7 displays the TIPS yield as a black line. The blue dotted line is the filtered real part of a nominal five-year yield implied by a three-factor model estimated over the full sample.

Aside from the post-Lehman failure period when TIPS yields temporarily jumped, the two series move reasonably closely together. The effect of the bankruptcy on the TIPS market is discussed by Campbell et al. (2009). Excluding the 2008Q4 observation, their

[8] I use yields as of the end of the middle month in the quarter, which matches the timing of the nominal yields.

correlation is 0.92. However, there are large persistent differences between the two yields, especially during the last two years of the sample.

Again excluding 2008Q4, the two series have sample means that differ by a single basis point. But this should not be interpreted as evidence that mean inflation compensation is roughly zero. Instead, it is evidence that I can pick a model that matches the sample mean of the TIPS yield. The sample mean of the filtered real part of the five-year yield is sensitive to the choice of the number of factors and the sample period.

The relevant evidence is in Table 8. I estimated six different models that vary in the number of factors (3, 4, 5) and sample period (full sample, post-1982 sample). Each model can be used to produce the time series displayed in Figure 7. The sample means and other statistics for these time series are calculated for each estimated model. Table 8 reports the minimum and maximum values across the six different estimated models.

The minimum sample mean is more than 80 basis points below the TIPS yield sample mean, while the maximum is more than 50 basis points above the TIPS yield sample mean. There is a smaller relative range in the standard deviation of quarterly changes in yields. The maximum standard deviation is 1.2 times the minimum standard deviation. The models also roughly agree on the correlation of quarterly changes between the TIPS yield and the real part of the nominal yield. The correlations are all around 0.5.

A reasonable interpretation of this evidence is that although long-run variations in the real part of nominal yields lines up broadly with TIPS yields, at higher frequencies the relation is much weaker. We cannot say anything reliable about their relative levels.

5.3 Unit roots and cointegration

As noted in Section 4.3, the evidence from the estimated factor models is consistent with two highly persistent processes affecting nominal yields: inflation expectations and real rates. This result is consistent with asset-pricing research that predates the no-arbitrage revolution.

Fama (1975) initiates the asset-pricing approach to studying the dynamics of the real rate. He infers properties of the real rate from its ex-post counterpart constructed with Treasury bill yields and CPI inflation. The early literature, most prominently Nelson and Schwert (1977), Garbade and Wachtel (1978), Mishkin (1981), and Fama and Gibbons (1982), concludes that the real rate varies through time and is highly persistent.

Similarly, early research notes that both short-term nominal Treasury yields and U.S. inflation appear to have unit roots.[9] This work struggled to find econometric tools appropriate to analyze a real rate process that is derived from two highly persistent processes (bill yields and inflation) and obscured by substantial noise (inflation shocks embedded in ex-post real rates).

The early literature takes a step forward with the development of cointegration. If short-term nominal yields have a unit root and term premia are stationary, then yields at all maturities are cointegrated. The earliest comprehensive empirical analysis of cointegration among nominal yields is in Campbell and Shiller (1987). If both yields and inflation have unit roots, but they are not cointegrated, then either real rates or term premia must also have a unit root. Rose (1988) is the first to use cointegration logic to study the properties of real rates.

In this subsection I use standard cointegration tools to check whether it is plausible that both inflation and real rates are highly persistent. Yield and expected inflation data are described in Section 3.3. I also use measures of realized inflation. They are quarter-to-quarter log changes in the NIPA GDP price index and the CPI.

Table 9 displays test statistics and p-values for Augmented Dickey-Fuller (ADF) tests of unit roots. The tests do not come close to rejecting the hypothesis of a unit root for any yield. The t-statistics decline with maturity, indicating that the evidence against unit roots shrinks as maturity increases.

[9] Fama (1975) reports the high autocorrelations of Treasury bill yields are consistent with nonstationarity. Nelson and Schwert (1977) make the same point about inflation. Schwert (1986) combines these observations, but does not use cointegration tools.

By contrast, yield spreads are stationary. The second set of results evaluate the stationarity of differences between the five-year yield and the other yields. Most of the reported p-values are less than one percent. The standard interpretation of this result is that yields are cointegrated with a single cointegrating vector.[10] handbook treatment by Martin, Hall, and Pagan (1996) provide a handbook treatment of this evidence. The third set of results shows that each inflation measure, the null hypothesis of a unit root cannot be rejected at the five percent level.

Panel A of Table 10 reports ADF tests of hypotheses that yields less inflation have a unit root. The alternative hypothesis is that the cointegration vector is $[1, -1]$. The main result is that the null hypothesis of no cointegration cannot be rejected. In the cointegration literature, Lardic and Mignon (2004) and Hjalmarsson and Österholm (2010) reach similar conclusions.

Panel B of Table 10 reports results for the Engle-Granger cointegration test. Rather than assume a cointegrating vector of $[1, -1]$, this procedure first regresses yields on inflation, then applies the ADF test to the residuals. The critical values differ from standard ADF critical values. Using this test, the hypothesis of no cointegration cannot be rejected for any combination of yield and inflation measure.

These results are consistent with the evidence from the factor model. The properties of observed yields and inflation do not allow us to rule out the possibility that both inflation and real rates are extremely persistent processes.

6 Concluding comments

This paper studies the joint dynamics of nominal yields and inflation expectations from 1968 through 2012. For this sample as well as subsamples, quarterly shocks to nominal yields are primarily shocks to real rates and term premia.

[10] Kozicki and Tinsley (2001) argue a more accurate characterization is that there is a single source of nonstationarity in all yields, but that this source is not $\mathcal{I}(1)$.

This is a robust result that can be used to help evaluate dynamic term structure models. In particular, some estimated models in the literature imply that inflation shocks drive much of the variation in nominal yields. The evidence here serves as both a specification test and a moment to help guide the development of macro-finance term structure models.

References

Ang, Andrew, Geert Bekaert, and Min Wei, 2007, Do macro variables, asset markets or surveys forecast inflation better?, *Journal of Monetary Economics* 54, 1163-1212.

Ang, Andrew, Geert Bekaert, and Min Wei, 2008, The term structure of real rates and expected inflation, *Journal of Finance* 63, 797-849.

Ang, Andrew, and Monika Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* 50, 745-787.

Ang, Andrew, and Maxim Ulrich, 2012, Nominal bonds, real bonds, and equity, Working paper, Columbia GSB.

Bansal, Ravi, and Ivan Shaliastovich, 2012, A long-run risks explanation of predictability in bond and currency markets, Working paper, Duke University.

Campbell, John Y., and Robert J. Shiller, 1987, Cointegration and tests of present value models, *Journal of Political Ecoomy* 95, 1062-1088.

Campbell, John Y., and Robert J. Shiller, 1991, Yield spreads and interest rate movements: A bird's eye view, *Review of Economic Studies* 58, 495-514.

Campbell, John Y., Robert J. Shiller, and Luis M. Viceira, 2009, Understanding inflation-indexed bond markets, *Brookings Papers on Economic Activity*, Spring, 79-120.

Campbell, John Y., and Luis M. Viceira, 2001, Who should buy long-term bonds?, *American Economic Review* 91, 99-127.

Chernov, Mikhail, and Philippe Mueller, 2012, The term structure of inflation expectations, *Journal of Financial Economics* 106, 367-394.

Croushore, Dean, 2010, An evaluation of inflation forecasdts from surveys using real-time data, *B.E. Journal of Macroeconomics* 10, 1-32.

D'Amico, Stefania, Don H. Kim, and Min Wei, 2008, Tips from TIPS: The informational content of Treasury Inflation-Protected Security prices, Federal Reserve Board Discussion Paper 2008-30.

Fama, Eugene F., 1975, Short-term interest rates as predictors of inflation, *American Economic Review* 65, 269-282.

Fama, Eugene F., and Robert R. Bliss, 1987, The information in long-maturity forward rates, *American Economic Review* 77, 680-692.

Fama, Eugene F., and Michael R. Gibbons, 1982, Inflation, real returns and capital investment, *Journal of Monetary Economics* 9, 297-323.

Faust, Jon, and Jonathan H. Wright, 2009, Comparing Greenbook and reduced form forecasts using a large real-time dataset, *Journal of Business and Economic Statistics* 27, 468-479.

Faust, Jon, and Jonathan H. Wright, 2012, Forecasting inflation, *Handbook of Forecasting*, forthcoming.

Fisher, Irving, 1930, *Theory of Interest.* New York: Macmillan.

Gallmeyer, Michael, Burton Hollifield, Francisco Palomino, and Stanley Zin, 2008, Term premium dynamics and the Taylor rule, Working paper, University of Michigan.

Garbade, Kenneth, and Paul Wachtel, 1978, Time variation in the relationship between inflation and interest rates, *Journal of Monetary Economics* 4, 755-765.

Gurkaynak, Refet S., Brian Sack, and Eric Swanson, 2005, The sensitivity of long-term interest rates to economic news: evidence and implications for macroeconomic models," *American Economic Review* 95, 425-436.

Gurkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, The U.S. Treasury yield curve: 1961 to the present, *Journal of Monetary Economics* 54, 2291-2304.

Hamilton, James D., 1985, Uncovering financial market expectations of inflation, *Journal of Political Economy* 93, 1224-1241.

Haubrich, Joseph, George Pennacchi, and Peter Ritchken, 2012, Inflation expectations, real rates, and risk premia: evidence from inflation swaps, *Review of Financial Studies* 25, 1588-1629.

Hjalmarsson, Erik, and Pär Österholm, 2010, Testing for cointegration using the Johansen methodology when variables are near integrated: size distortions and partial remedies, *Empirical Economics* 39, 51-76.

Joslin, Scott, Anh Le, and Kenneth J. Singleton, 2013, Why Gaussian macro-finance term structure models are (nearly) unconstrained factor-VARs, *Journal of Financial Economics*.

Kozicki, Sharon, and Peter A. Tinsley, 2001, Shifting endpoints in the term structure of interest rates, *Journal of Monetary Economics* 47, 613-652.

Lardic, Sandrine, and Valérie Mignon, 2004, Fractional cointegration and the term structure, *Empirical Economics* 29, 723-736.

Litterman, Robert, and Jose Scheinkman, 1991, Common factors affecting bond returns, *Journal of Fixed Income* 1, 54-61.

Macaulay, Frederick, 1938, *The Movements of Interest Rates, Bond Yields, and Stock Prices in the United States since 1856.* New York: National Bureau of Economic Research.

Martin, Vance, Anthony D. Hall, and Adrian R. Pagan, 1996, Modelling the term structure, *Handbook of Statistics* 14, G.S. Maddala and C.R. Rao, Eds., 91-118.

Mishkin, Frederic S., 1981, The real rate of interest: an empirical investigation, *Carnegie-Rochester Conference Seies on Public Policy* 15, 151-200.

Nelson, Charles R., and G. William Schwert, 1977, Short-term interest rates as predictors of inflation: On testing the hypothesis that the real rate of interest is constant, *American Economic Review* 67, 478-486.

Pennacchi, George G., 1991, Identifying the dynamics of real interest rates and inflation: evidence using survey data, *Review of Financial Studies* 4, 53-86.

Rose, Andrew K., 1988, Is the real interest rate stable?, *Journal of Finance* 43, 1095-1112.

Rudebusch, Glenn D., and Eric Swanson, 2012, The bond premium in a DSGE model with long-run real and nominal risks, *American Economic Journal: Macroeconomics* 4, 105-143.

Rudebusch, Glenn D., and Tao Wu, 2008, A macro-finance model of the term structure, monetary policy and the economy, *Economic Journal* 118, 906-926.

Schwert, G. William, 1986, The time series behavior of real interest rates: A comment, *Carnegie-Rochester Conference Series on Public Policy* 24, 275-288.

van Binsbergen, Jules H., Jesús Fernández-Villaverde, Ralph S.J. Koijen, and Juan F. Rubio-Ramírez, 2012, The term structure of interest rates in a DSGE model with recursive preferences, *Journal of Monetary Economics* 59, 634-648.

Table 1. Properties of nominal yields and expected inflation implied by term structure models

The table reports model-implied population properties of a five-year bond yield and inflation expectations. The source of the models and the sample period used to estimate the parameters are reported in the first two columns. The first reported variance ratio is the unconditional variance of expected average inflation over five years divided by the unconditional variance of the five-year yield. The second ratio has the same denominator, while the numerator is the unconditional variance of the three-quarter-ahead inflation forecast. The other variance ratios are for one-step-ahead shocks to inflation expectations and yields. The final two columns report the population coefficient and R^2 of a regression of the bond yield on the contemporaneous three-quarter-ahead expectation of inflation.

		Variance ratios				Projections on expected inflation	
	Sample	Unconditional		Conditional			
Source	Period	Avg	3-Q	Avg	3-Q	Coef	R^2
Campbell and Viceira (2001)	1952–1996	0.99	1.01	0.60	0.61	1.00	0.99
Campbell and Viceira (2001)	1983–1996	0.01	0.04	0.10	0.48	−1.15	0.05
Ang et al. (2008)	1952–2004	0.50	0.85	0.50	1.46	1.02	0.88
Chernov and Mueller (2012)	1971–2008	0.17	0.24	0.03	0.06	0.96	0.22
Haubrich et al. (2012)	1982–2010	0.15	0.18	0.17	0.21	2.35	0.99

Table 2. Sample properties of nominal yields and expected inflation

The table reports sample properties of a five-year Treasury bond yield and survey expectations of inflation. The quarter-t expectation is the mean, across those responding to the Survey of Professional Forecasters, of predicted GDP inflation in quarter $t+3$. The first reported variance ratio is the sample variance of this inflation expectation to the sample variance of the five-year yield. The other ratio is defined similarly, using fitted residuals to univariate AR(1) regressions as proxies of quarterly shocks to inflation expectations and bond yields. The final two columns report the estimated coefficient and R^2 of a regression of the bond yield on the contemporaneous survey forecast of inflation.

Sample	Variance ratio		Regression	
Period	Unconditional	Conditional	Coefficient	R^2
1968Q4 – 2012Q4	0.38	0.22	1.29	0.64
1968Q4 – 1982Q4	0.53	0.29	0.99	0.52
1983Q1 – 2012Q4	0.14	0.14	2.44	0.82

Table 3. Decompositions of population variances of yield shocks

The table reports model-implied standard deviations of quarterly shocks to nominal Treasury bond yields. Yields are expressed in percent per year. The table also reports decompositions of the corresponding variances. The model uses four factors to describe the joint dynamics of nominal yields and expected inflation. Yields are the sum of average expected inflation and short-term real rates over the life of the bond, plus a term premium. The contributions to total variance sum to one (aside from rounding). The sample period is 1968Q4 through 2012Q4. Brackets display [2.5% 97.5%] percentile bounds. Panel B reports the sum of the variance column and two covariance columns in Panel A that involve average expected inflation.

A. Full decomposition

Maturity (years)	Std dev	1. Average expected real rate	2. Average expected inflation	3. Term premium	2Cov([1], [2])	2Cov([1], [3])	2Cov([2], [3])
One	0.96	0.61	0.13	0.03	−0.02	0.26	−0.01
	[0.80 1.20]	[0.35 1.10]	[0.07 0.18]	[0.01 0.17]	[−0.20 0.13]	[−0.21 0.46]	[−0.12 0.10]
Five	0.69	0.23	0.10	0.30	0.04	0.42	−0.09
	[0.58 0.88]	[0.05 1.25]	[0.05 0.17]	[0.07 0.97]	[−0.16 0.27]	[−0.77 0.51]	[−0.39 0.10]
Ten	0.57	0.12	0.11	0.53	0.08	0.31	−0.14
	[0.49 0.73]	[0.02 1.37]	[0.05 0.24]	[0.15 1.45]	[−0.26 0.44]	[−1.21 0.50]	[−0.73 0.19]

B. Components related to inflation

	Point estimate	95th Percentile Bounds	
One-yr	0.09	−0.14	0.26
Five-yr	0.05	−0.25	0.26
Ten-yr	0.05	−0.49	0.37

Table 4. Decompositions of population variances of yield shocks, for different factor models

Two different factor models describe the joint dynamics of nominal yields and expected inflation. One uses three factors and the other uses five factors. The sample period is 1968Q4 through 2012Q4. The table reports a subset of the variance decomposition information reported in Table 3. It reports model-implied standard deviations of quarterly shocks to nominal Treasury bond yields, the fraction of variance attributable to expected inflation over the life of the bond, and the fraction of variance attributable to expected inflation and its covariance with expected future real short rates and term premia. Brackets display [2.5% 97.5%] percentile bounds.

A. Three factors

Maturity (years)	Std dev	Average expected inflation	All components related to expected inflation
One	0.96	0.15	0.07
	[0.81 1.15]	[0.09 0.21]	[−0.14 0.24]
Five	0.68	0.16	0.16
	[0.57 0.82]	[0.09 0.26]	[−0.15 0.36]
Ten	0.57	0.18	0.29
	[0.49 0.67]	[0.08 0.33]	[−0.15 0.53]

B. Five factors

Maturity (years)	Std dev	Average expected inflation	All components related to expected inflation
One	0.94	0.13	0.01
	[0.76 1.25]	[0.07 0.22]	[−0.28 0.27]
Five	0.68	0.13	0.08
	[0.55 0.92]	[0.06 0.24]	[−0.24 0.33]
Ten	0.52	0.16	0.21
	[0.42 0.72]	[0.06 0.35]	[−0.22 0.51]

Table 5. Decompositions of population variances of yield shocks: subsamples

The table reports model-implied decompositions of sample variances of quarterly shocks to nominal Treasury bond yields. The model describes the joint dynamics of nominal yields and expected inflation. Yields are the sum of average expected inflation and short-term real rates over the life of the bond, plus a term premium. Rows sum to one (aside from rounding).

Sample Period	Number of Factors	Maturity (years)	1. Average expected real rate	2. Average expected inflation	3. Term premium	2Cov([1], [2])	2Cov([1], [3])	2Cov([2], [3])
1968Q4–1982Q4	3	One	0.36	0.10	0.18	−0.02	0.48	−0.11
		Five	0.06	0.14	1.16	0.05	0.22	−0.63
		Ten	0.03	0.16	1.58	0.09	−0.02	−0.83
1968Q4–1982Q4	4	One	0.77	0.08	0.07	−0.08	0.27	−0.11
		Five	0.18	0.12	0.73	0.04	0.36	−0.44
		Ten	0.06	0.06	0.86	0.00	0.32	−0.31
1983Q1–2012Q4	3	One	0.61	0.13	0.03	0.13	0.04	0.07
		Five	0.33	0.12	0.04	0.24	0.19	0.09
		Ten	0.25	0.13	0.05	0.29	0.17	0.10
1983Q1–2012Q4	4	One	0.62	0.12	0.02	0.11	0.15	−0.02
		Five	0.31	0.09	0.29	0.18	0.13	0.00
		Ten	0.21	0.09	0.41	0.20	0.06	0.02

Table 6. Model-implied regressions of nominal yields on expected inflation

The table reports model-implied population properties of regressions of nominal yields on contemporaneous expectations of three-quarter-ahead expected inflation. The factor model describes the joint dynamics of nominal yields and expected inflation. Three different models are estimated using the sample period 1968Q4 through 2012Q4; they differ in the number of factors. Brackets display 95th percentile bounds.

Factors	Maturity (years)	Regression Coefficient	R^2
3	1	1.67	0.80
		[0.81 2.73]	[0.19 0.98]
3	5	1.56	0.84
		[0.80 2.39]	[0.26 0.98]
3	10	1.39	0.84
		[0.74 2.03]	[0.30 0.99]
4	1	1.69	0.67
		[0.30 3.19]	[0.04 0.96]
4	5	1.60	0.70
		[0.27 2.94]	[0.04 0.97]
4	10	1.44	0.71
		[0.27 2.55]	[0.05 0.97]
5	1	1.54	0.71
		[0.04 2.87]	[0.02 0.97]
5	5	1.53	0.78
		[0.22 2.60]	[0.04 0.98]
5	10	1.40	0.80
		[0.21 2.32]	[0.05 0.98]

Table 7. Model-implied correlations of shocks to yields and expected inflation

A four-factor model describes the joint dynamics of nominal yields and expected inflation. It is estimated over the sample 1968Q4 through 2012Q4. The model's parameters imply a decomposition of nominal yields into expected inflation over the life of the bond and the remainder, called the real part of nominal yields. The table reports population correlations among various quarterly shocks. They are (a,b) the first two principal components of the population covariance matrix of shocks to nominal yields; (c,d) the first two principal componetns of the population covariance matrix of the real part of nominal yields; (e) the shock to three-quarter-ahead expected inflation; (f) the shock to the ex ante real rate, and (g) the shock to the term premium on a ten-year nominal bond.

	1st PC of nominal yields	2nd PC of nominal yields	1st PC of real part	2nd PC of real part	expected inflation	ex ante real rate
1st PC of nom yields	1.00					
2nd PC of nom yields	0.00	1.00				
1st PC of real part	0.93	0.03	1.00			
2nd PC of real part	0.03	0.95	0.00	1.00		
expected inflation	0.35	−0.18	0.00	−0.01	1.00	
ex ante real rate	0.89	−0.18	0.97	−0.24	−0.04	1.00
term premium	0.65	0.57	0.80	0.52	−0.31	0.66

Table 8. Inflation-indexed yields and model-implied real parts of nominal yields

A factor model describes the joint quarterly dynamics of nominal Treasury yields and expected inflation. The model's parameters imply a decomposition of nominal yields into expected inflation over the life of the bond and the remainder, called the real part of nominal yields. Six versions of the model are estimated with the Kalman filter, using two choices of sample period (1968Q4–2012Q4, 1983Q1–2012Q4) and three choices of the number of factors (3, 4, and 5). For each estimated model, filtered values of the real part of the five-year nominal yield are compared with the five-year inflation-indexed Treasury yield. The sample period for comparision is 1999Q1 through 2012Q4. Inflation-indexed statistics exclude 2008Q4. The table reports minimum and maximum values, across the six models, of various statistics.

	Minimum	Maximum
Mean of the real part of the nominal yield	0.75	2.13
Std dev of quarterly changes, real part of the nominal yield	0.40	0.48
Correlation of quarterly changes between inflation-indexed yield, real part of the nominal yield	0.45	0.54
Mean of the inflation-indexed yield	1.59	
Std dev of quarterly changes of the inflation-indexed yield	0.47	

Table 9. Tests of nonstationarity

The table reports results of augmented Dickey-Fuller tests that yields, yield spreads, and inflation have unit roots. The table reports t-statistics and their p-values under the hypothesis of a unit root. Yield spreads are all relative to the five-year yield. GDP inflation is the log change in the GDP index from quarter $t-1$ to quarter t. CPI inflation is the log change in the CPI index from the last month of quarter $t-1$ to the last month of quarter t. Expected inflation is from the Survey of Professional Forecasters. It is the mean forecast of GDP inflation from quarter t to quarter $t+1$. The sample period is 1968Q4 through 2012Q4. All ADF tests use six lags.

Variable	Coef	p-val
Three-mon yield	−2.17	0.223
One-yr yield	−1.60	0.475
Two-yr yield	−1.22	0.641
Three-yr yield	−0.95	0.758
Four-yr yield	−0.78	0.821
Five-yr yield	−0.67	0.849
Ten-yr yield	−0.47	0.892
Three-mon less five-yr	−4.30	0.001
One-yr less five-yr	−4.23	0.001
Two-yr less five-yr	−4.14	0.001
Three-yr less five-yr	−3.75	0.005
Four-yr less five-yr	−3.77	0.004
Ten-yr less five-yr	−3.12	0.027
GDP infl	−2.06	0.270
CPI infl	−2.46	0.128
Expected GDP infl	−1.57	0.488

Table 10. Tests of cointegration

Panel A reports results of augmented Dickey-Fuller (ADF) tests of the hypotheses that yields less inflation have unit roots. The panel reports t-statistics and their p-values under the hypothesis of a unit root. Panel B reports results of Engle-Granger tests (using ADF) that yields and inflation are cointegrated. For these tests yields are regressed on inflation. Data are described in Table 9. The sample period is 1968Q4 through 2012Q4. All ADF tests use six lags.

Yield	Inflation measure	Coef	p-val
A.			
Three-mon	GDP	−2.11	0.249
One-yr	GDP	−1.80	0.385
Five-yr	GDP	−1.62	0.463
Ten-yr	GDP	−1.77	0.397
Three-mon	Expected	−2.42	0.138
One-yr	Expected	−1.94	0.324
Five-yr	Expected	−1.18	0.657
Ten-yr	Expected	−1.28	0.615
Three-mon	CPI	−2.29	0.177
One-yr	CPI	−2.31	0.169
Five-yr	CPI	−2.56	0.105
Ten-yr	CPI	−2.74	0.070
B.			
Three-mon	GDP	−2.11	0.475
One-yr	GDP	−1.71	0.660
Five-yr	GDP	−1.13	0.875
Ten-yr	GDP	−1.01	0.901
Three-mon	Expected	−1.92	0.562
One-yr	Expected	−1.73	0.647
Five-yr	Expected	−1.39	0.802
Ten-yr	Expected	−1.37	0.809
Three-mon	CPI	−2.50	0.299
One-yr	CPI	−2.12	0.472
Five-yr	CPI	−1.34	0.820
Ten-yr	CPI	−1.22	0.853

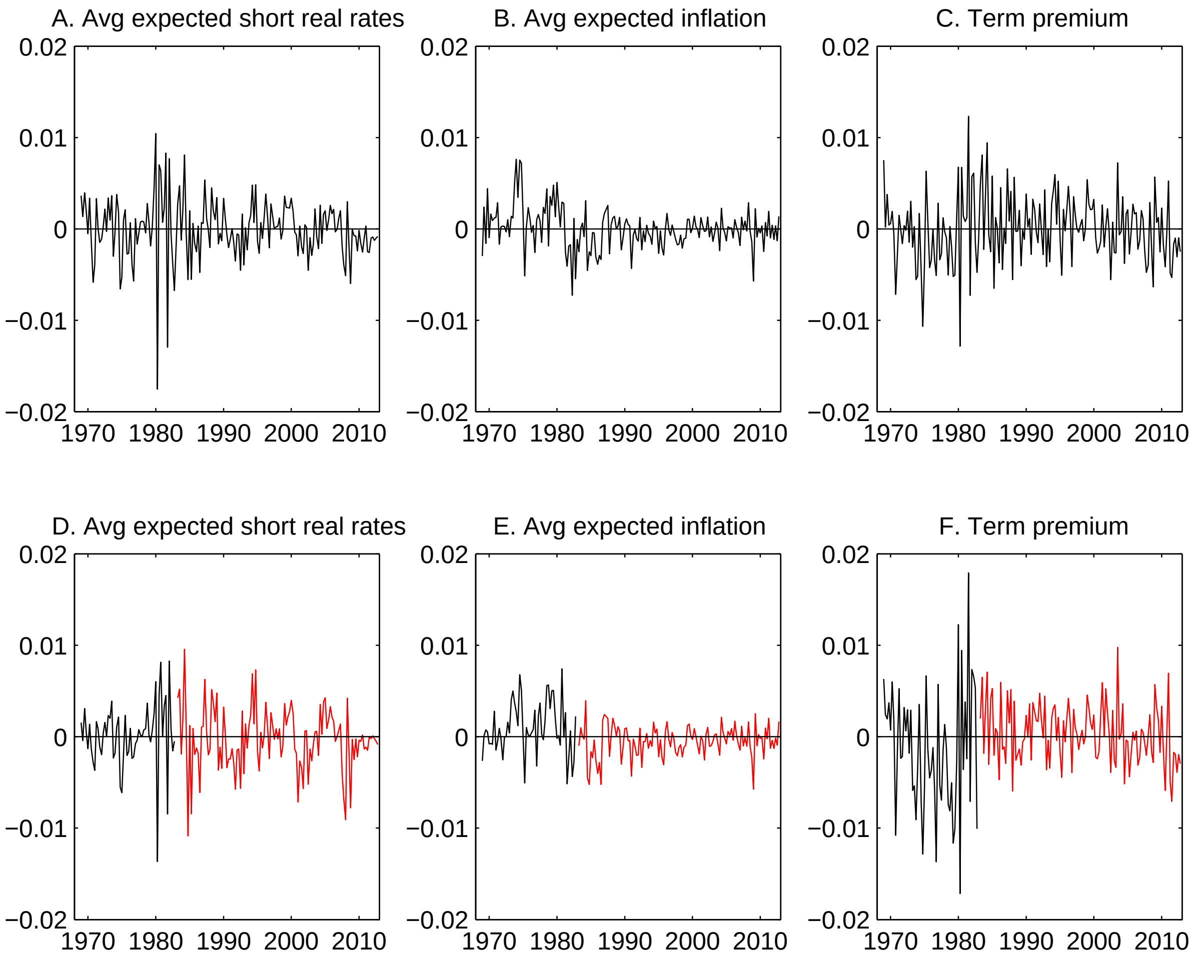

Figure 1. Shocks to components of the five-year nominal Treasury yield

Quarterly shocks to nominal yields are the sum of shocks to average expected real rates during the life of the bond, average expected inflation during the life of the bond, and term premia. The figure plots fitted shocks for a five-year zero-coupon Treasury bond, where the shocks are inferred from an estimated four-factor dynamic model of yields and expected inflation. Panels A through C use a model estimated over the sample 1968Q4 through 2012Q4. Panels D, E, and F splice two series. The earlier series uses data from 1968Q4 to 1982Q4 and the latter series use data from 1983Q1 through 2012Q4.

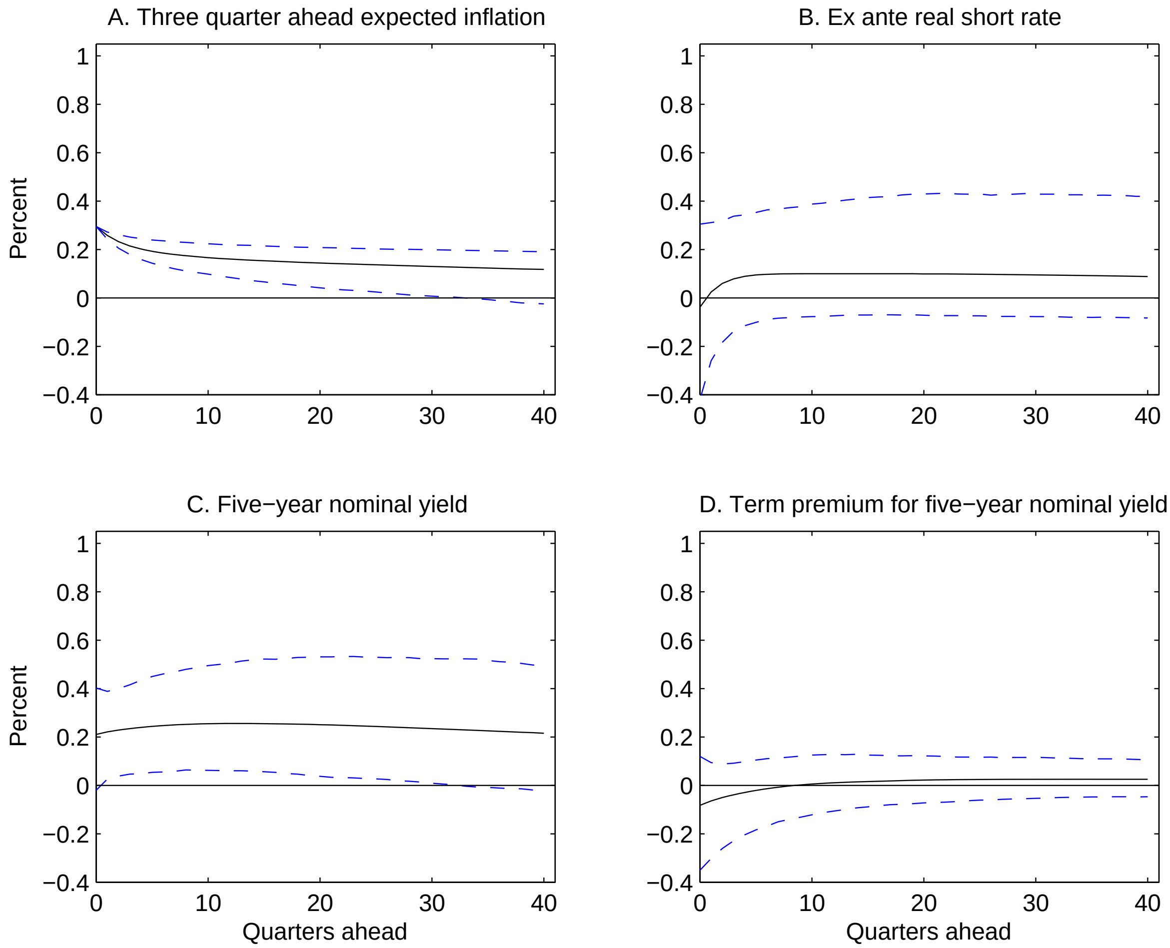

Figure 2. Impulse responses for a shock to expected inflation

A four-factor dynamic model of nominal yields and expected inflation is estimated over the sample 1968Q4 through 2012Q4. The figure displays model-implied impulse responses to a shock to three-quarter-ahead expected inflation. The initial shock is 30 basis points, which is the model-implied population standard deviation of the shock. The yield and term premium responses are for a five-year nominal bond. Also displayed are 95 percentile confidence bounds on the impulse responses.

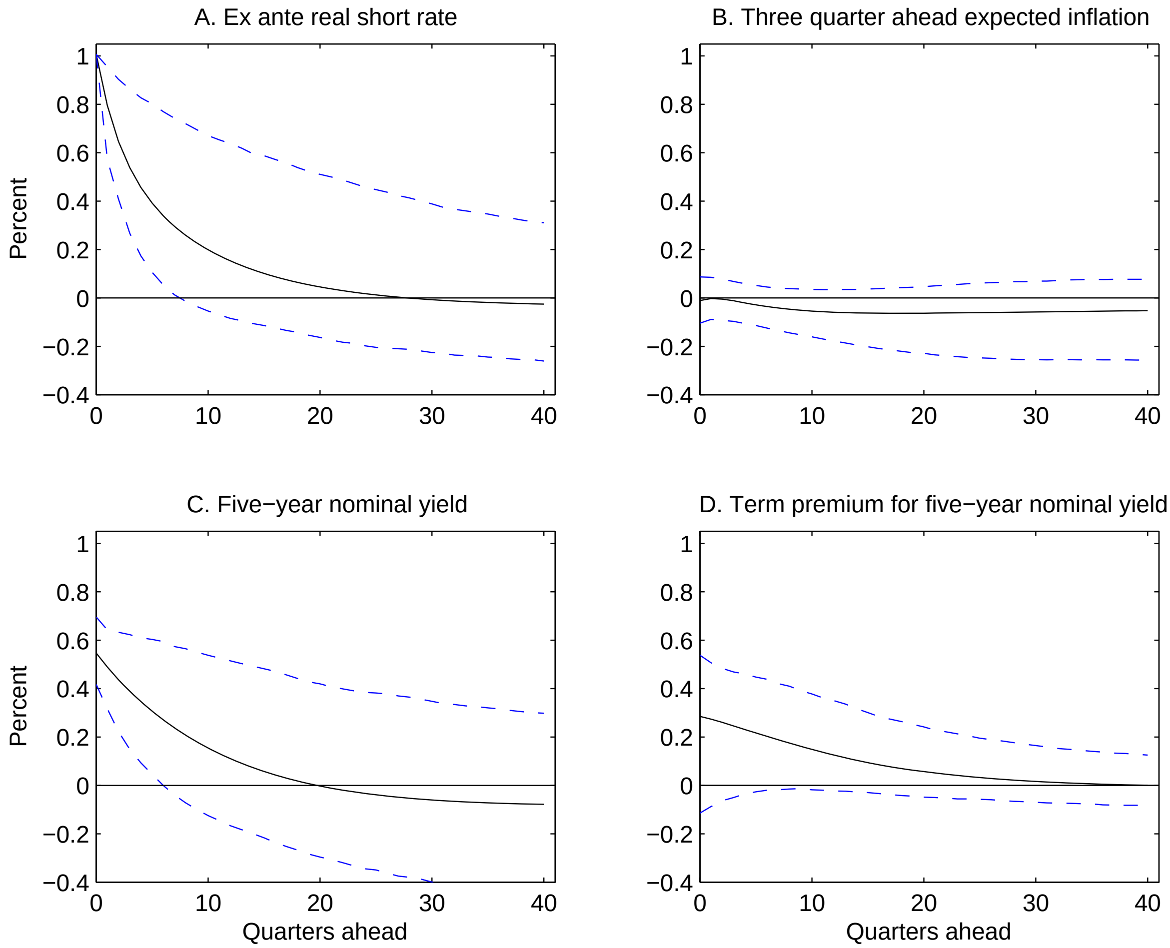

Figure 3. Impulse responses for a shock to the ex ante real rate

A four-factor dynamic model of nominal yields and expected inflation is estimated over the sample 1968Q4 through 2012Q4. The figure displays model-implied impulse responses to a shock to the ex ante real rate, defined as the three-month nominal yield less expected inflation during the next quarter. The initial shock is 100 basis points, which is the model-implied population standard deviation of the shock. The yield and term premium responses are for a five-year nominal bond. Also displayed are 95 percentile confidence bounds on the impulse responses.

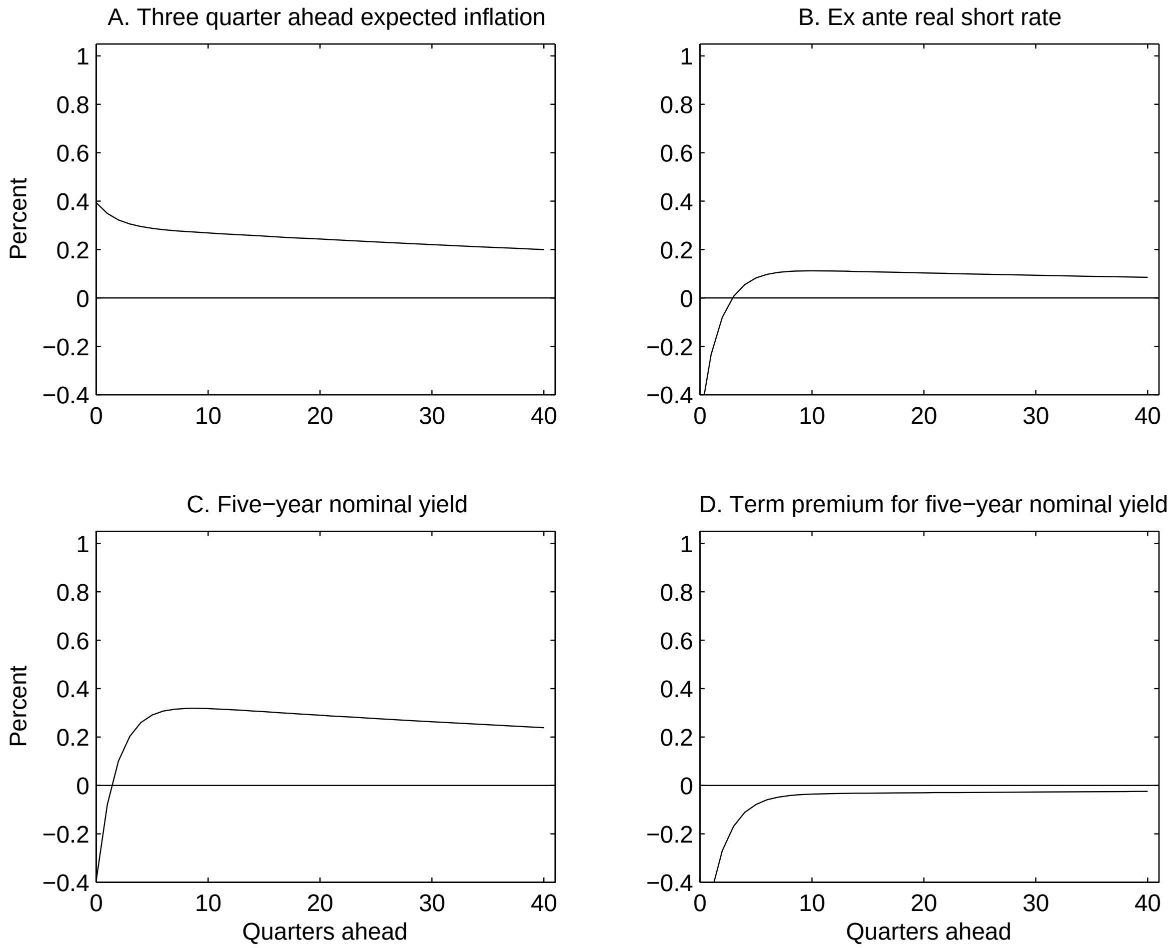

Figure 4. Impulse responses for a shock to expected inflation, pre-1983 data

A four-factor dynamic model of nominal yields and expected inflation is estimated over the sample 1968Q4 through 1982Q4. The figure displays model-implied impulse responses to a shock to three-quarter-ahead expected inflation. The initial shock is 30 basis points, which is the model-implied population standard deviation of the shock. The yield and term premium responses are for a five-year nominal bond.

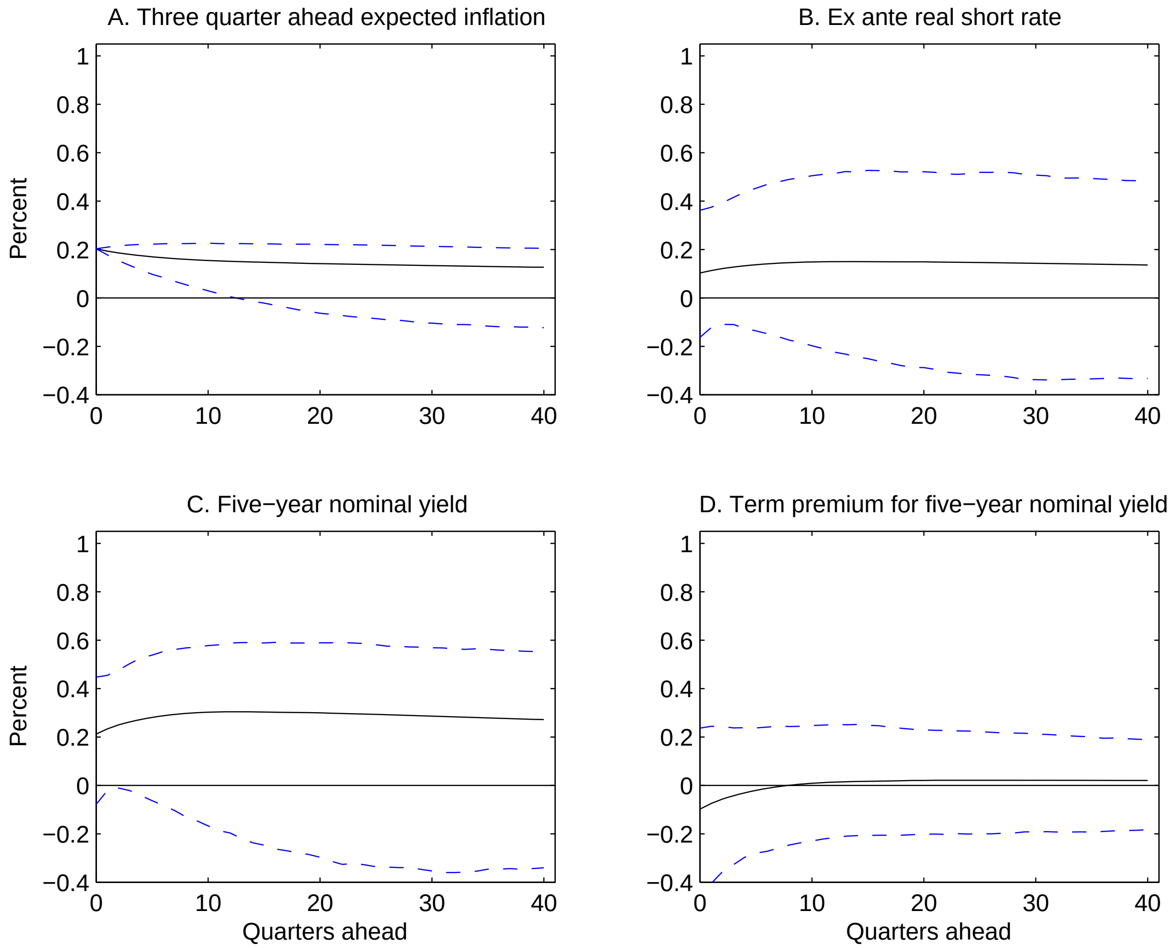

Figure 5. Impulse responses for a shock to expected inflation, post-1982 data

A four-factor dynamic model of nominal yields and expected inflation is estimated over the sample 1983Q1 through 2012Q4. The figure displays model-implied impulse responses to a shock to three-quarter-ahead expected inflation. The initial shock is 30 basis points, which is the model-implied population standard deviation of the shock. The yield and term premium responses are for a five-year nominal bond. Also displayed are 95 percentile confidence bounds on the impulse responses.

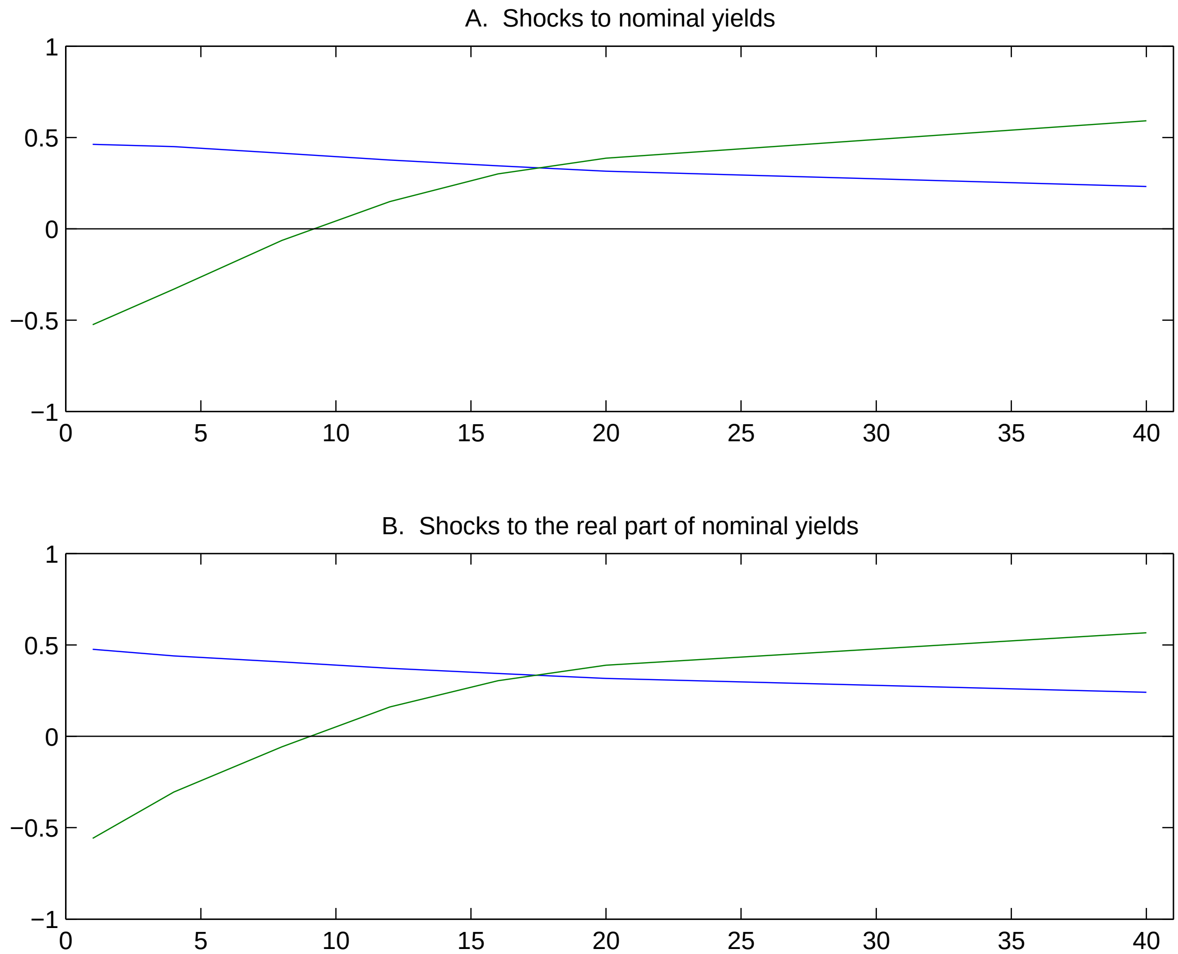

Figure 6. Principal components of yield shocks

A four-factor model describes the joint quarterly dynamics of nominal Treasury yields and expected inflation. The model's parameters imply a decomposition of nominal yields into expected inflation over the life of the bond and the remainder, called the real part of nominal yields. The model is estimated over the period 1968Q4 through 2012Q4. Panel A displays the first two principal components of the population covariance matrix of quarterly nominal yield shocks. Panel B displays the first two principal components of the population covariance matrix of quarterly shocks to the real part of nominal yields.

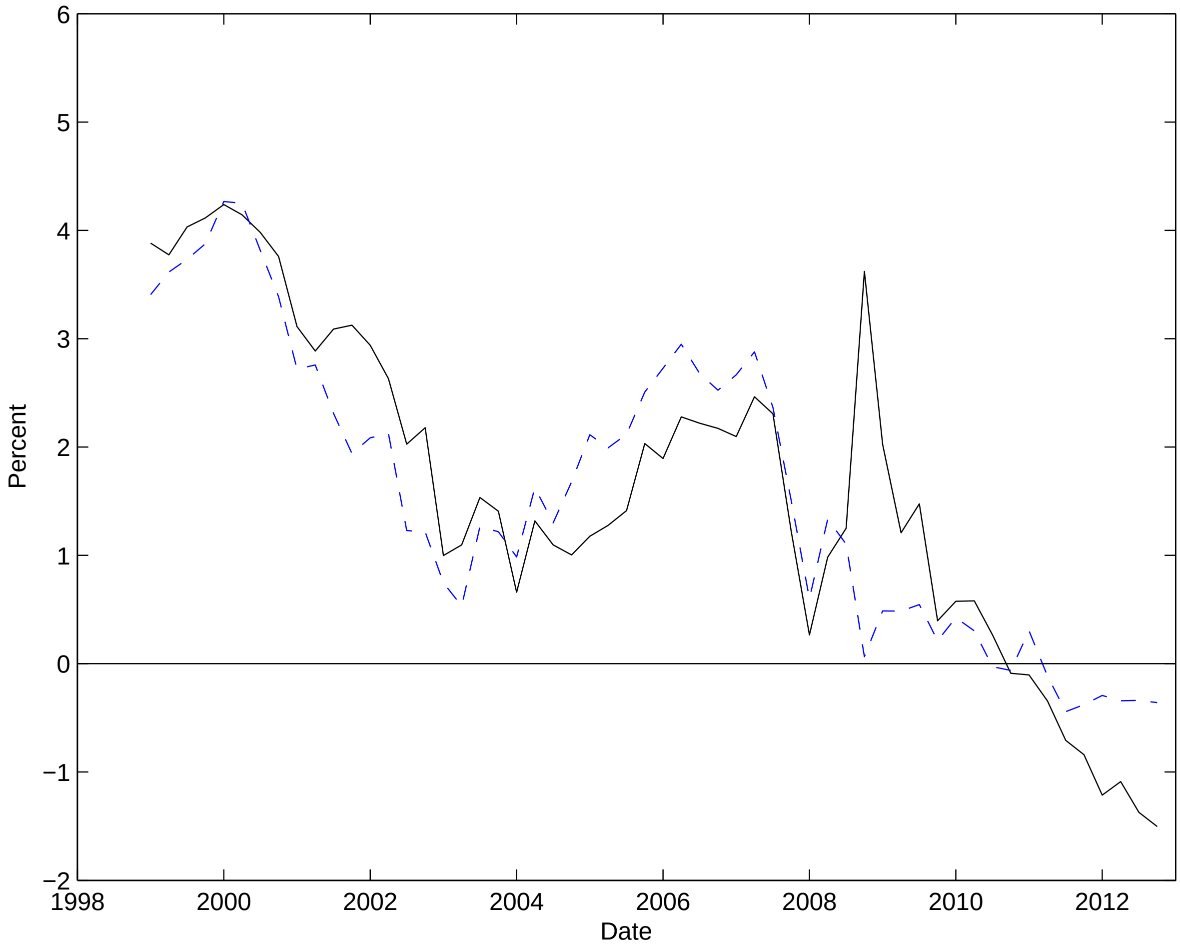

Figure 7. The five-year TIPs yield and the five-year nominal yield less expected inflation

A three-factor model describes the joint quarterly dynamics of nominal Treasury yields and expected inflation. The model's parameters imply a decomposition of nominal yields into expected inflation over the life of the bond and the remainder, called the real part of nominal yields. The model is estimated with the Kalman filter over the period 1983Q1 through 2012Q4. The blue dashed line is the filtered real part of the five-year nominal yield. The black line is the five-year inflation-indexed Treasury yield.