Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade

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Abstract

This paper develops and applies a framework to quantify both the aggregate gains from trade and their distribution across different groups of workers. The framework combines a multi-sector gravity model of trade with a Roy-type model of the allocation of workers across sectors. Workers belong to groups that are differentiated by characteristics such as education, age, gender, or region of employment, and by assumption groups differ in their relative abilities across sectors. By opening to trade, a country gains in the aggregate by specializing according to its comparative advantage, but the distribution of these gains is unequal as labor demand increases (decreases) for groups that are specialized in export-oriented (import-oriented) sectors. In fact, some groups of workers can lose from trade even when the whole economy gains. As such, the model generalizes the specific-factors intuition to a setting with labor reallocation, while maintaining analytical tractability for any number of groups and countries. Our new notion of “risk-adjusted gains from trade” evaluates the full distribution of welfare changes in one measure, as the counterfactual scenario is evaluated by a risk-averse agent behind the veil of ignorance regarding the group to which she belongs. In the baseline analysis each group corresponds to a region, and the quantitative analysis uses trade and labor allocation data from the US and Germany to compute the aggregate gains from trade and their distribution across regions. Depending on the value of the (Roy) dispersion parameter for worker comparative advantage across sectors, the standard deviation of the gains across groups varies widely, going from 1/2 of the mean gains to a very small number. Even in the case of high standard deviation of gains, however, the risk-adjusted gains from trade are positive under commonly used measures of risk aversion. In fact, we find that risk-adjusted gains from trade are larger than the aggregate gains, as income risk coming from income inequality among groups actually falls with trade relative to autarky.

1 Introduction

This paper jointly analyzes the aggregate gains from trade and the distribution of these gains across different groups of workers. Our analysis rests on a quantitative general-equilibrium model that combines a multi-sector gravity model of trade with a Roy (1951) model for the sectoral allocation of workers. We derive a simple expression for $\hat{W}_{ig}$, with $\hat{W}_{ig} = W_{ig}'/W_{ig}$ the change in real income due to a trade-shock (i.e. a change in iceberg trade costs) for group $g$ in country $i$. Using exact-hat algebra (Dekle et al., 2008), where $\hat{x} = x'/x$, we show that this expression is the product of two terms.

$$\hat{W}_{ig} = \prod_{s} \hat{\lambda}_{iis}/\theta \cdot \prod_{s} \hat{\pi}_{igs}/\kappa$$

Multi-sector ACR  Roy reallocation

We are grateful to seminar participants at Rochester, UC Berkeley, UC Merced and USC for helpful comments.
The first, “Multi-sector ACR” term is a weighted geometric average of \( \lambda_{iis}^{-1/\theta} \), with \( \lambda_{iis} \) the expenditure shares on domestic goods for sector \( s \), and \( \theta \) the Fréchet shape parameter of the gravity-side of the model.

While the first component is standard in the trade-gravity literature (Arkolakis et al. (2012) - ACR), the second, “Roy reallocation” component is new. As a geometric average of changes in sectoral employment shares, this term summarizes the gains from labor market reallocation. Here, \( \pi_{igs} \) is the employment share of group \( g \) in sector \( s \), and \( \kappa \) is the Fréchet shape parameter of the Roy-side of the model. For both the first and second term, the averaging weights are the Cobb-Douglas expenditure shares \( \beta_{is} \).

This framework extends the existing analysis of Ricardian sector-level comparative advantage in Costinot et al. (2012a) - henceforth CDK - to incorporate the labor market.\(^1\) In fact, as \( \kappa \to \infty \), our model collapses to the CDK structure. For non-degenerate cases, however, a new source for the aggregate gains from trade appears, since when \( \kappa \) is finite, our gains from trade are strictly higher than in CDK. Intuitively, a lower \( \kappa \) - i.e. more worker heterogeneity - leads to higher aggregate gains from trade (given trade shares) because it makes it more costly for countries to reallocate resources from the export to the import-competing sectors. In other words, the curvature of the PPF decreases with \( \kappa \). This increased cost from reallocation arising from more worker heterogeneity disproportionately affects export-oriented groups. Therefore, given data on the degree of import-competition (or export-orientation) across groups, a lower \( \kappa \) also leads to higher variance in the group-level gains from trade.

Our baseline model of the labor market has a Roy-structure.\(^2\) This Roy approach captures the basic specific-factors intuition that workers are a better fit for one sector compared to others. The model therefore serves as a natural generalization of the benchmark specific-factors model for analyzing the distributional labor-market consequences of trade. Importantly, our labor-market model delivers tractability both on the aggregate and the distributional side, for any number of groups and countries. At the same time, the model is flexible in terms of micro-foundations. For instance, when groups are defined along a geographical basis, the model can be shown to nest Kovak (2013), which serves as a theoretical foundation for recent work on the differential effect of trade across regions within countries.

This paper’s methodology can be applied to several different categorizations of workers into “groups”, which adds further analytical flexibility to the model. For instance, one can define workers based on education, age or gender. Our leading empirical exercise utilizes a geographical categorization. This choice is motivated by a growing body of recent empirical work documenting substantial variation in local labor-market outcomes in response to national-level trade shocks (Autor et al., 2013a; Dauth et al., 2014; Dix-Carneiro and Kovak, 2014; Kovak, 2013; McLaren and Hakobyan, 2010; Topalova, 2010). In this literature, variation in local labor market outcomes generally depends on the degree of local import competition. We provide a tractable general-equilibrium framework to analyze this heterogeneous impact of counterfactual trade-shocks, which makes our paper a structural complement to the existing set of empirical papers.\(^3\)

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\(^1\)CDK extend the seminal Eaton and Kortum (2002) framework to a multi-sector environment. The analysis in ACR implies that any quantitative trade model with a gravity equation (e.g. the Armington model or Krugman (1980)) would be a workable substitute for the CDK-side of the model. The Pareto-versions of Melitz (2003), as in Eaton et al. (2011), would also work, though these models would introduce extra terms because of entry effects. The standard results from the gravity literature, extensively discussed by Costinot and Rodríguez-Clare (2014), are reflected in the multi-sector ACR term, which affects all groups equally. Analogous to ACR, the gains from trade are inversely related to the sectoral expenditure shares on domestic goods. The gains from trade also decrease with \( \theta \), since the dispersion of comparative advantage across countries falls with \( \theta \).

\(^2\)This paper belongs to the Ricardian revival in international trade, nicely surveyed by Costinot and Vogel (2014). Their terminology of Ricardo-Roy models succinctly summarizes the framework of our model: Ricardo on the trade-side and Roy on the labor-side, capturing the source of comparative advantage at the country and worker-level respectively.

\(^3\)Kovak (2013) also provides a theoretical framework for his findings on the Brazilian labor market, in the setting of a small open economy. Compared to his model, we have a complete general equilibrium structure for the world economy, and we have tractable expressions for the full distribution of the group-level welfare changes.
Importantly, trade reforms generally lead to both negative and positive labor-demand shocks, from the import and the export-side respectively. Except for Dauth et al. (2014), who analyze the impact of both intensified import-competition in certain sectors as well as the increased scope for export in other sectors, most of the reduced-form empirical work restricts its attention to the downsides of increased import-competition. Our general-equilibrium framework naturally includes both changes on the import and the export-side, thereby integrating the full spectrum of distributional consequences of trade-shocks. Moreover, our project allows for the counterfactual analysis of different trade-shocks, and is thereby able to compare the distribution of winners and losers across a variety of trade reforms. For instance, we can explore how the distribution of gains from a North-South liberalization compares to the one from a North-North liberalization.

Our simulations demonstrate that the extent to which groups gain or lose from returning to autarky, perfectly correlates with their relative specialization in import-competing versus export-oriented sectors. Groups specialized in sectors with positive net exports tend to lose when going to autarky, whereas groups mainly employed in net-importing sectors will lose less. In fact, in a number of extreme cases the import-competing groups will gain as the country returns to autarky.

We are able to incorporate both the level and the distribution of the gains from trade into one summary measure of the aggregate welfare-consequences of trade. These risk-adjusted gains from trade (RAGT) evaluate free trade from behind the veil of ignorance for a risk-averse agent, when there is no compensation for group-level losses from trade. In principle, the RAGT can be both higher or lower than the regular gains from trade, depending on the comparison of risk-levels under autarky and trade. The RAGT are therefore a convenient, though optional, measure for the evaluation of free trade in general, or certain specific trade-reforms in particular. In practice, our simulations will show that the RAGT are higher than the regular gains from trade, for both the US and Germany.

1.1 Relation to the Literature

In addition to the above-mentioned research on trade and local labor markets (Autor et al., 2013a; Dauth et al., 2014; Dix-Carneiro and Kovak, 2014; Kovak, 2013; McLaren and Hakobyan, 2010; Topalova, 2010), there is also a broader literature on the unequal effects of trade on labor market outcomes (e.g. Burstein and Vogel, 2011; Costinot et al., 2012b; Autor et al., 2013b). Within this literature, there is a strand that studies the link between trade and wage inequality, both at the aggregate level and within groups (e.g. Burstein and Vogel, 2012; Goldberg and Pavcnik, 2007; Helpman et al., 2010, 2012; Krishna et al., 2012, 2014). A literature that focuses more on sectoral reallocation is surveyed in Goldberg and Pavcnik (2007) and in Wacziarg and Seddon-Wallack (2004) who find limited evidence for the role of reallocation. Such a lack of reallocation has also been found by Menezes-Filho and Muendler (2011), in the case of Brazil. While these papers tend to find limited evidence of sectoral reallocation, this is potentially a result of the high level of aggregation in the data. Other studies, such as Gourinchas (1999) and Kline (2008) do find substantial reallocation in response to sectoral price shocks.

Dix-Carneiro (2014) offers a structural analysis of the dynamic adjustment to trade liberalization. His project is particularly related to ours since his model features Roy-type selection into different sectors. However, he focuses on the transitional dynamics of sectoral reallocation, with a single national labor market for a small number of sectors, whereas we employ a gravity model and study the distribution of gains across regions or groups. Other structural analyses of trade liberalization are Artuç et al. (2010),

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4In this set of papers, the final two subsets focus on inequality arising from firm-worker matches, and in doing so relate to the Melitz (2003) analysis of unequal outcomes of international trade on firms.
Coşar (2013), Coşar et al. (2013), Kambourov (2009) and Ritter (2012). While the above papers all focus on the impact of trade on the earnings channel, another approach to studying the distributional consequences of trade focuses on the expenditure channel, as in Atkin and Donaldson (2014), Faber (2014), Fajgelbaum and Khandelwal (2014) and Porto (2006). Although this approach is only indirectly related to ours, in terms of methodology we have more in common with Fajgelbaum and Khandelwal (2014), since both our projects feature a structural analysis that primarily employs aggregate data.

Finally, our paper also relates to the renewed attention to Roy-models in the broader labor economics literature. One of the first recent papers to use a Roy framework is Hsieh et al. (2013), with a focus on allocative efficiency for worker-talent. Analogous to our setting, Burstein et al. (2014) utilize such a model to analyze the distribution of earnings. However, they focus on the impact of technological change on the skill premium and apply the Roy model to worker selection into occupations instead of sectors. Given our focus on the distributional consequences of international trade, we study worker selection into sectors instead of occupations. The reasons for this different approach are first, the benchmark-role of the specific factors model in international trade, mentioned above. Second, in our framework trade-induced sectoral shocks map directly into changes of sectoral labor demand. Hence, there is a direct link between trade-shocks and the worker-level variation in welfare from these shocks, which makes the sectoral Roy-model a natural option in the context of international trade. A third paper, Young (2014) applies the Roy model to allocation of workers to different sectors, which is analogous to us, although we study reallocation within the manufacturing sector whereas he analyzes the growth of the service sector’s workforce share. An earlier and related analysis is performed by Lagakos and Waugh (2013), who study the effects of worker-selection into agriculture on differences in productivity between the agricultural and manufacturing sector for developing countries.

2 Trade and Sectoral Reallocation in the Data

To understand the relation between trade and sectoral reallocation, we start by a short exploration of the related empirical patterns in Germany. First, we provide descriptives on the changing composition of output across sectors and how these compositional changes are related to trade. Specifically, we decompose the changes in sectoral shares of total output into changes in domestic demand and changes in net exports. This descriptive exercise will demonstrate the substantial magnitude of sectoral reallocation, and at the same time quantify the relative importance of changes in net exports in this reallocation. We then examine how the observed changes in output shares translate into shifts in sectoral employment shares.

In a second step, we move beyond the descriptive exercise, and present evidence on how trade-shocks affect sectoral output and employment shares. This is a first illustration of the relevance of the model, where sectoral reallocation in response to trade shocks will have smaller or larger welfare effects depending on the dispersion of comparative advantage.

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5 There is also a broad literature on the impact of trade on poverty and the income distribution using a Computable General Equilibrium (CGE) methodology. Savard (2003) offers an overview of the different approaches for counterfactual analysis of the income distribution within this CGE literature, while Cockburn et al. (2008) integrate multiple chapters on methodology and empirical findings of the CGE approach into a book-length discussion.

6 Costinot and Vogel (2014) provide an extensive survey and guide for Ricardo-Roy models in international trade. One of the most salient spill-overs of the recent “Ricardian revival” is into labor economics, where the Roy model originated.

7 A paper that combines an analysis of trade-data with detailed skill-data on workers is Bombardini et al. (2012), but they focus on skill substitutability within sectors whereas we look at worker selection into sectors.
2.1 Decomposition of Sectoral Reallocation

We start from the following accounting identity.

\[ C_{is}^t = Y_{is}^t - X_{is}^t + M_{is}^t \]

where \( C_{is}^t \) is consumption of goods from sector \( s \) in country \( i \) at time \( t \), \( Y_{is}^t \) is production, \( X_{is}^t \) is exports and \( M_{is}^t \) is imports. Rearranging and dividing both sides by total consumption in country \( i \):

\[ \frac{Y_{is}^t}{C_{is}^t} = \frac{C_{is}^t - M_{is}^t}{C_{i}^t} + \frac{X_{is}^t}{C_{is}^t} = \beta_{is}^t \lambda_{is}^t + \frac{X_{is}^t}{C_{is}^t} \]

where \( \beta_{is}^t \equiv \frac{C_{is}^t}{C_{i}^t} \) is the consumption share and \( \lambda_{is}^t \equiv \frac{C_{is}^t - M_{is}^t}{C_{is}^t} \) is the expenditure share on domestic goods for sector \( s \). If we assume balanced trade, then \( Y_{is}^t = C_{is}^t \), which implies that \( \frac{Y_{is}^t}{C_{i}^t} = \frac{Y_{is}^t}{Y_{i}^t} \) is the share of sector \( s \) in total national output, while \( \frac{X_{is}^t}{C_{i}^t} = \frac{X_{is}^t}{Y_{i}^t} \) is that sector’s exports as a share of national output. For consistency with the data, we do not impose trade balance and continue with the above identity. First define \( y_{is}^t \equiv \frac{Y_{is}^t}{C_{i}^t} \) and then find that changes over time in \( y_{is}^t \) can be decomposed as:

\[ \frac{y_{is}^t - y_{is}^{t-1}}{y_{is}^{t-1}} = (\beta_{is}^t - \beta_{is}^{t-1}) \lambda_{is}^t + (\lambda_{is}^t - \lambda_{is}^{t-1}) \beta_{is}^{t-1} + \frac{X_{is}^t}{C_{i}^t} - \frac{X_{is}^{t-1}}{C_{i}^{t-1}} \]

To bring this equation to the data, we focus on Germany and set \( t = 2007, t - 1 = 2000 \). We first visualize the decomposition of changes in output shares in Figure 1, for 15 manufacturing sectors at the 2-digit level of aggregation. There, we see that both trade-induced and home-induced reallocation are strongly correlated with output-share reallocation. Note that the sector with the highest output-share reallocation, with an increase of 3.9 percentage points, is the sector producing “Motor Vehicles, Trailers, and Semi-Trailers.”

Figure 1: Decomposition of Changes in Output Shares

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8We select these years to be consistent with the estimation in Section 6.

9Section 4 provides a detailed discussion of the data.
We now quantify the share of trade-induced and home-induced reallocation in the output-share reallocation. Define $G_{t}^{i} = y_{t}^{i} - y_{t-1}^{i}$, $H_{t}^{i} = (\beta_{t}^{i} - \beta_{t-1}^{i})\lambda_{t}^{i}$, $T_{t}^{i} = (\lambda_{t}^{i} - \lambda_{t-1}^{i})\beta_{t}^{i-1} + \frac{\lambda_{t}^{i}}{C^{i}} - \frac{\lambda_{t-1}^{i}}{C^{i}}$, such that $G_{t}^{i} = H_{t}^{i} + T_{t}^{i}$. We want to know what share of the variance of changes in output shares ($G_{t}^{i}$) is home-induced (related to $H_{t}^{i}$), and what share is trade-induced (related to $T_{t}^{i}$). We can answer this question by running two separate regressions where we either regress $H_{t}^{i}$ on $G_{t}^{i}$, or $T_{t}^{i}$ on $G_{t}^{i}$.  

Table 1: Decomposition of Changes in Output Shares

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output-share Reallocation</td>
<td>0.643***</td>
<td>0.357***</td>
</tr>
<tr>
<td></td>
<td>(0.0583)</td>
<td>(0.0583)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.00174</td>
<td>-0.00174</td>
</tr>
<tr>
<td></td>
<td>(0.000927)</td>
<td>(0.000927)</td>
</tr>
<tr>
<td>Observations</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1 shows that around 64% of the variance of changes in output shares is due to changes in trade-induced reallocation. The remainder is related to home-induced reallocation.

As a final step, we ask to what extent changes in output-shares are related to changes in employment shares. Empirically, we find that there is a correlation of 56.8% between growth rates of sectoral output shares and growth rates of employment shares. We visualize this relation in Figure 2.

Figure 2: Growth Rates of Sectoral Output and Employment Shares

2.2 Sectoral Reallocation In Response to Trade Shocks

The next step is to examine if we can document a causal effect of trade on sectoral reallocation. After all, the observed changes in sectoral output and employment shares could in principle be unrelated to

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10Formally, we run the following regressions: $H_{t}^{i} = \alpha + \beta_{1}G_{t}^{i} + \epsilon$; $T_{t}^{i} = \alpha + \beta_{2}G_{t}^{i} + \epsilon$. Since $\beta_{1} = \text{cov}(G_{t}^{i}, H_{t}^{i})/\text{var}(G_{t}^{i})$ and $\beta_{2} = \text{cov}(G_{t}^{i}, T_{t}^{i})/\text{var}(G_{t}^{i})$. It then follows that $\beta_{1}$ is the share of the variance in $G_{t}^{i}$ explained by $H_{t}^{i}$, while $\beta_{2}$ is the share explained by $T_{t}^{i}$, because of this relation: $\beta_{1} + \beta_{2} = \text{cov}(H_{t}^{i}, G_{t}^{i})/\text{var}(G_{t}^{i}) + \text{cov}(T_{t}^{i}, G_{t}^{i})/\text{var}(G_{t}^{i}) = \text{cov}(H_{t}^{i} + T_{t}^{i}, G_{t}^{i})/\text{var}(G_{t}^{i}) = \text{var}(G_{t}^{i})/\text{var}(G_{t}^{i}) = 1$. 

trade and be caused by domestic technological trends. To examine the causal effect of trade, we utilize the trade-shock variable constructed by Dauth et al. (2014). Specifically, for each sector \( s \), we construct an import penetration measure \( \Delta M_{st}^{East \rightarrow Other} \) as the change in net import flows, normalized by sectoral employment, from China and Eastern Europe to a group of “similar countries” during time period \( t \). Formally,

\[
\Delta IP_{st}^{East \rightarrow Other} = \frac{\Delta M_{st}^{East \rightarrow Other}}{E_{st}^{Germany}}
\]

where \( E_{st}^{Germany} \) is the number of workers in Germany employed in industry \( s \) at the beginning of time period \( t \). We run the following regressions:

\[
\Delta z_{st} = \gamma \Delta IP_{st}^{East \rightarrow Other} + \nu_{st}
\]

\[
g(z_{st}) = \gamma \Delta IP_{st}^{East \rightarrow Other} + \nu_{st}
\]

where \( \Delta z_{st} = z_{st} - z_{st-1} \) and \( g(z_{st}) = \frac{z_{st} - z_{st-1}}{z_{st-1}} \) for \( z_{st} = y_{st}, \pi_{st} \), where \( \pi_{st} \) is the employment share of sector \( s \) at time \( t \).

We find that a negative trade shock has a negative effect on sectoral output shares, at borderline levels of statistical significance. In addition, these increases in net imports in other countries also negatively affect growth rates of sectoral employment shares, at strong levels of statistical significance.

### Table 2: Output and Labor Reallocation in Response to Trade Shock

<table>
<thead>
<tr>
<th></th>
<th>Output Shares (%)</th>
<th>Employment Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>( \Delta ) Net Imports</td>
<td>-0.0128 (0.0156)</td>
<td>-0.00304 (0.00184)</td>
</tr>
<tr>
<td>Observations</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.046</td>
<td>0.164</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
The independent variable is measured in 1000 EURO per worker.
The output and employment share are expressed in percentage terms.
* \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \)

This section has made the case that trade leads to a reallocation of sectoral output and employment shares. In the theoretical section, we present a model that predicts the observed reallocation patterns. In addition, the model allows to understand and quantify the aggregate and distributional welfare consequences of a given trade reform through its impact on reallocation. Moreover, the model will enable counterfactual welfare analysis of different trade reforms. After laying out the model, we will return to the data on trade and reallocation in order to causally identify the central parameters of the model.

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\(^{11}\)The instrument group employed by Dauth et al. (2014) consists of Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom. Countries were selected based on having a similar income level as Germany, but all direct neighbors and members of the European Monetary Union were excluded. The logic of the instrument is that all the “rise of the East” is an exogenous event, affecting trade for all countries at comparable levels of development as Germany in a similar way.
3 Theory: Baseline Model

We present a multi-sector, multi-country, Ricardian model of trade with heterogeneous workers. There are \(N\) countries indexed by \(i, j\) and \(S\) sectors indexed by \(s, k\), each with a continuum of goods indexed by \(\omega \in [0,1]\). Each sector is modeled as in Eaton and Kortum (2002 – henceforth EK); preferences across the continuum of goods in each sector are CES with elasticity of substitution \(\sigma\) and technologies have constant returns to scale and productivities that are distributed Fréchet with shape parameter \(\theta > \sigma - 1\) and level parameters \(T_{is}\) in country \(i\) and sector \(s\). Preferences across sectors are Cobb-Douglas with shares \(\beta_{is}\). There are iceberg trade costs \(\tau_{ij} \geq 1\) to export goods in sector \(s\) from country \(i\) to country \(j\).

We model heterogeneous workers by assuming that there are \(G\) types of workers indexed by \(g\) and \(h\), with workers within each group differing in their ability across sectors, as in Roy (1951). A worker from group \(g\) in country \(i\) has a number of efficiency units \(z\) in sector \(s\) drawn from a Fréchet distribution with shape parameter \(\kappa > 1\) and level parameters \(A_{igs}\). Note that workers within a group are identical ex-ante (i.e., before they draw their efficiency units) whereas workers differ across groups because of differences in the \(A_{igs}\) across \(g\). We let \(L_{ig}\) denote the measure of workers of type \(g\) in country \(i\). Labor supply is inelastic – workers simply choose the sector to which they supply their entire labor endowment.

If \(\kappa \to \infty\) and \(A_{igs} = 1\) for all \(g\) and \(s\), the model collapses to the multi-sector EK model developed in CDK. On the other hand, if \(\tau_{ij} \to \infty\) for all \(j\) and \(s\) then economy \(i\) is in autarky and collapses to the Roy model developed by Hsieh et al. (2013), except that here workers have different productivities across sectors rather than occupations.\(^{12}\)

3.1 Equilibrium

To determine the equilibrium of the model, it is useful to separate the analysis into two parts: the determination of labor demand in each sector in each country as a function of wages, which comes from the EK part of the model; and the determination of labor supply to each sector in each country as a function of wages, which comes from the Roy part of the model.

Since workers are heterogeneous in their sector productivities, the supply of labor to each sector is upward sloping, and hence wages can differ across sectors. However, since technologies are national, wages cannot differ across groups. Let wages per efficiency unit in sector \(s\) of country \(i\) be denoted by \(w_{is}\). From EK we know that the demand for efficiency units in sector \(s\) in country \(i\) is

\[
\frac{1}{w_{is}} \sum_{j} \lambda_{ijs} \beta_{js} Y_j,
\]

with \(Y_j\) total income for country \(j\) and sectoral trade shares are given by

\[
\lambda_{ijs} = \frac{T_{is} (\tau_{ij} w_{is})^{-\theta}}{\sum_k T_{ks} (\tau_{kjs} w_{ks})^{-\theta}}.
\]

\(^{12}\)There are two sources of comparative advantage in this model: first, as in CDK, differences in \(T_{is}\) drive sector-level (Ricardian) comparative advantage; second, differences in \(l_{ig} \equiv L_{ig}/L_i\) and \(A_{igs}\) lead to factor-endowment driven comparative advantage. Given the nature of our comparative statics exercise, however, the source of comparative advantage will not matter for the results, only the actual sector-level specialization as revealed by the trade data will be relevant.
For future purposes, also note that the price index in sector $s$ in country $i$ is

$$P_{js} = \eta^{-1} \left( \sum_j T_j (\tau_{ij} w_{is})^{-\theta} \right)^{-1/\theta} \quad (3)$$

where $\eta \equiv \Gamma(1 - \frac{1}{\theta})^{1/(1 - \sigma)}$.

Labor supply is determined by workers’ choices regarding which sector to work in. Let $w_i \equiv (w_{i1}, ..., w_{iS})$, let $z = (z_1, z_2, ..., z_S)$ and let $\Omega_s(w_i) \equiv \{ z \text{ s.t. } w_{is} z_s \geq w_{ik} z_k \text{ for all } k \}$. A worker with productivity vector $z$ in country $i$ will choose sector $s$ if $z \in \Omega_s(w_i)$. Let $F_{ig}(z)$ be the joint probability distribution of $z$ for workers of group $g$ in country $i$. The following lemma characterizes the labor supply side of the economy:

**Lemma 1.** The share of workers in group $g$ in country $i$ that choose to work in sector $s$ is

$$\pi_{igs} \equiv \int_{\Omega_s(w_i)} dF_{ig}(z) = \frac{A_{igs} w_{is}^{\kappa}}{\Phi_{ig}},$$

where $\Phi_{ig} \equiv \sum_k A_{igk} w_{ik}^{\kappa}$. The efficiency units supplied by this group in sector $s$ are given by

$$E_{igs} \equiv L_{ig} \int_{\Omega_s(w_i)} z_s dF_{ig}(z) = \gamma L_{ig} \frac{A_{igs} w_{is}^{\kappa} \Phi_{ig}}{\Phi_{ig} w_{is}},$$

where $\gamma \equiv \Gamma(1 - 1/\kappa)$.\(^{13}\)

One implication of this lemma is that income levels per worker are equalized across sectors. That is, for group $g$, we have

$$\frac{w_{is} E_{igs}}{\pi_{igs} L_{ig}} = \gamma \Phi_{ig}.$$

This is a special implication of the Frechet distribution and it implies that the share of income obtained by workers of group $g$ in country $i$ in sector $s$ (i.e., $w_{is} E_{igs} / \sum w_{ik} E_{igk}$) is also given by $\pi_{igs}$. Note also that total income of group $g$ in country $i$ is $Y_{ig} \equiv \sum_s w_{is} E_{igs} = \gamma L_{ig} \Phi_{ig}$. In turn, total income in country $i$ is $Y_i \equiv \sum_g Y_{ig}$.

Putting the supply and demand sides of the economy together, we see that excess demand for efficiency units in sector $s$ of country $i$ is

$$ELD_{is} \equiv \frac{1}{w_{is}} \sum_j \lambda_{ij} \beta_{j} Y_j - \sum_g E_{igs}.$$  \(^{(4)}\)

Noting that $\lambda_{ij}, Y_j$ and $E_{igs}$ are functions of the whole matrix of wages $w \equiv \{ w_{is} \}$, the system $ELD_{is} = 0$ for all $i, s$ is a system of equations in $w$ whose solution gives the equilibrium wages given some choice of numeraire.

### 3.2 Comparative Statics

Consider some change in trade costs. We proceed as in Dekle et al. (2008) and solve for the proportional change in the endogenous variables. Formally, using notation $\dot{x} \equiv x'/x$, we consider a shock with $\dot{\tau}_{ij}$ for

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\(^{13}\)Lemma 1 generalizes easily to a setting with correlation in workers’ ability draws across sectors. In this case, the dispersion parameter $\kappa$ is replaced by $\kappa/(1 - \rho)$, where $\rho$ measures the correlation parameter of ability draws. This change in modeling dispersion is then maintained throughout the analysis, analogous to Hsieh et al. (2013).
i \neq j \) while keeping all other parameters constant \((i.e., \hat{A}_{igs} = 1 \) for all \( g, s \) and \( \hat{T}_{is} = 1 \) for all \( i, s \)). The counterfactual equilibrium entails \( ELD'_{is} = 0 \) for all \( i, s \). The equation \( ELD'_{is} = 0 \) can be rewritten as

\[ \sum_g \hat{w}_{is} \hat{w}_{is} \hat{E}_{igs} E_{igs} = \sum_j \lambda_{ij} \lambda_{ij} \beta_{js} Y_j Y_j. \]

This can be shown to be equivalent to

\[ \sum_g \hat{w}_{is} \hat{w}_{is} \Phi_{ig} \pi_{igs} Y_{ig} = \sum_j \lambda_{ij} \lambda_{ij} \beta_{js} \sum_k \Phi_{ig} Y_{ig} \]

with

\[ \Phi_{ig} = \sum_k \pi_{igk} \hat{w}_{ik}. \]

This equation can be solved for \( \hat{w}_{is} \) given data on income levels, \( Y_{ig} \), trade shares, \( \lambda_{ij} \), expenditure shares, \( \beta_{is} \), labor allocation shares \( \pi_{igr} \), and labor endowments, \( L_{ig} \), and the trade-cost shocks, \( \hat{T}_{ij} \). From the \( \hat{w}_{is} \) we can then solve for all other relevant changes, including changes in trade shares,

\[ \hat{\lambda}_{ij} = \frac{\hat{T}_{ij} \hat{w}_{is}}{\sum_k \hat{\lambda}_{ij} \hat{T}_{ij} \hat{w}_{ks}} \]

and employment shares,

\[ \hat{\pi}_{igs} = \frac{\hat{w}_{is} \hat{\pi}_{igs}}{\sum_k \hat{\pi}_{igk} \hat{w}_{ik}}. \]

### 3.3 Welfare Effects

Our measure of welfare is ex-ante real income, \( W_{ig} \equiv \frac{Y_{ig}/L_{ig}}{\bar{P}} \). Given \( \hat{w}_{is} \), the following proposition characterizes the impact of such a shock on welfare for different groups of workers.

**Proposition 1.** Given some trade shock, the ex-ante percentage change in the real wage of group \( g \) in country \( i \) is given by

\[ \hat{W}_{ig} = \prod_s \hat{\lambda}_{sis}^{-\beta_{is}/\theta} \cdot \prod_s \hat{\pi}_{igs}^{-\beta_{is}/\kappa}. \]

The RHS of the expression in (8) has two components: the term \( \prod_s \hat{\lambda}_{sis}^{-\beta_{is}/\theta} \) is common across groups, while all the variation across groups comes from the second term, \( \prod_s \hat{\pi}_{igs}^{-\beta_{is}/\kappa} \). If \( \kappa \rightarrow \infty \), this second term converges to one, and the gains for all groups are equal to \( \prod_s \hat{\lambda}_{sis}^{-\beta_{is}/\theta} \), which is the multi-sector formula for the welfare effect of a trade shock in ACR once we note that \( \theta \) is the trade elasticity in all sectors in this model.

The term \( \prod_s \hat{\pi}_{igs}^{-\beta_{is}/\kappa} \) is related to the change in the degree of specialization of group \( g \). We use the Kullback-Leibler (KL) divergence as a way to define the degree of specialization of a group. Formally, the KL divergence of \( \pi_{ig} = \{\pi_{ig1}, \pi_{ig2}, ..., \pi_{igs}\} \) from \( \beta_i = \{\beta_{i1}, \beta_{i2}, ..., \beta_{iS}\} \) is given by \( D_{KL}(\pi_{ig} || \beta_i) = \sum_s \beta_{is} \ln(\beta_{is}/\pi_{igs}) \). Note that if group \( g \) in country \( i \) was in autarky (i.e., not trading with any other group or country) then \( \pi_{igs} = \beta_{is} \). Thus, \( D_{KL}(\pi_{ig} || \beta_i) \) is a measure of the degree of specialization as reflected in the actual distribution \( \pi_{ig} \) relative to \( \beta_i \). We have

\[ \prod_s \hat{\pi}_{igs}^{-\beta_{is}/\kappa} = \exp \frac{1}{\kappa} \left[ D_{KL}(\pi'_{ig} || \beta_i) - D_{KL}(\pi_{ig} || \beta_i) \right]. \]

This implies that the welfare effect of a trade shock on a particular group is determined by the change
in the degree of specialization of that group as measured by the KL divergence (modulo \( \prod_s \hat{\lambda}_{is}^{-\beta_{is}/\theta} \)). Consider a group \( g \) in country \( i \) that happens to have efficiency parameters \( (A_{ig1}, \ldots, A_{igS}) \) that give it a strong comparative advantage in a sector \( s \) for which the country as a whole has a comparative disadvantage, as reflected in positive net imports in that sector. Group \( g \) would be highly specialized in \( s \) when the country is in autarky (but groups trade among themselves) but that specialization would diminish as the country starts trading with the rest of the world. As a consequence, the KL degree of specialization falls with trade for group \( g \), implying lower gains relative to other groups in the economy.

The aggregate welfare effect can be obtained from Proposition 1 as

\[
\hat{W}_i = \prod_s \hat{\lambda}_{is}^{-\beta_{is}/\theta} \cdot \sum_g \left( \frac{Y_{ig}}{Y_i} \right) \prod_s \hat{\pi}_{igs}^{-\beta_{is}/\kappa}
\]

where \( Y_{ig}/Y_i \) is group \( g \)'s share of income. The relative welfare effect is

\[
\frac{\hat{Y}_{ig}}{\hat{Y}_i} = \frac{\prod_s \hat{\pi}_{igs}^{-\beta_{is}/\kappa}}{\sum_h \left( \frac{Y_{ih}}{Y_i} \right) \prod_s \hat{\pi}_{ish}^{-\beta_{is}/\kappa}}.
\]

The aggregate welfare effect of a trade shock is no longer given by the multi-sector ACR term (i.e., \( \hat{W}_i \neq \prod_s \hat{\lambda}_{is}^{-\beta_{is}/\theta} \)). This is because a trade shock will in general affect wages \( w_{is} \), and this in turn will affect welfare through its impact on income and sector-level prices.

### 3.4 Gains from Trade

A particularly relevant trade-shock is the move to autarky. This entails \( \hat{\tau}_{ij} = \infty \) for all \( s \) and all \( i \neq j \). Conveniently, solving for changes in wages in country \( i \) (i.e., solving for \( \hat{w}_{is} \) for \( s = 1, \ldots, S \)) from Equation (5) only requires knowing the values of trade and employment shares for country \( i \), namely \( \lambda_{iis} \) for all \( s \) and \( \pi_{igs} \) for all \( g, s \). This can be seen by letting \( \hat{\tau}_{ij} \to \infty \) in Equation (5), which yields

\[
\sum_g \hat{w}_{is}^{\kappa} \left( \sum_k \pi_{i gs} \hat{w}_{ik}^{\kappa} \right)^{1/\kappa - 1} \pi_{igs} Y_{ig} = \beta_{is} \sum_g \left( \sum_k \pi_{i gs} \hat{w}_{ik}^{\kappa} \right)^{1/\kappa} Y_{ig}.
\]

Following ACR, we define the aggregate gains from trade as the negative of the change in aggregate real income for a shock that takes the economy back to autarky as a percentage of the \( G_T_i = 1 - \hat{W}_i \).

**Proposition 2.** For a finite \( \kappa \), the gains from trade are higher than those in the multi-sector gravity model (e.g., CDK), and become equal as \( \kappa \to \infty \).

To understand this further, it is useful to consider the simpler case with a single group of workers, \( G = 1 \). Then,

\[
\frac{\hat{Y}_i}{\hat{P}_i} = \prod_s \hat{\lambda}_{is}^{-\beta_{is}/\theta} \cdot \prod_s \hat{\pi}_{is}^{-\beta_{is}/\kappa}.
\]

If the trade shock is to move back to autarky, then it is easy to confirm that \( \pi'_{is} = \beta_{is} \), hence

\[
\prod_s \hat{\pi}_{is}^{-\beta_{is}/\kappa} = \left( \prod_s \pi'_{is} / \prod_s \beta_{is} \right)^{1/\kappa}.
\]
But we know that
\[
\max_{\pi_1, \ldots, \pi_i, \ldots, \pi_S} \prod_{s} \pi_s^{\beta_i} = \prod_{s} \beta_i^{\pi_s},
\]
hence \(\prod_s \hat{\pi}_is^{\beta_is} < 1\) whenever \(\pi_is \neq \beta_is\) for all \(s\). This implies that the welfare losses associated with a move to autarky are higher with a finite \(\kappa\) than with \(\kappa \to \infty\). Intuitively, a finite \(\kappa\) introduces more "curvature" to the PPF, making it harder for the economy to adjust as it moves to autarky. This leads to higher losses if the economy were to move to autarky, implying higher gains from trade – see Costinot and Rodriguez-Clare (2014). Proposition 2 establishes that this result generalizes to the case \(G > 1\).

The group-specific gains from trade are \(GT_{gi} = 1 - \hat{Y}_{gi}/\hat{P}_i\). In the data we will find that such gains are negative for some groups. Intuitively, this happens to groups that happen to be specialized in industries that face strong import competition. To capture this formally, note that, since \(\beta_is\) is the expenditure share of country \(i\) in industry \(s\) and \(\hat{\pi}_is \equiv \sum_s \pi_isY_is/Y_is\) is the corresponding output share, then \(\beta_is/\hat{\pi}_is\) is a measure of the degree of import competition in industry \(s\). We can then construct a measure of import competition of group \(g\) in country \(i\) as \(I_{ig} \equiv \sum_s \pi_is\beta_is/\hat{\pi}_is\).

### 3.5 Risk-Adjusted Welfare Effects

Consider an agent "behind the veil of ignorance" who doesn’t know what group she will belong to. Since there are \(L_{ig}\) workers in group \(g\), the probability that our agent behind the veil will end up in group \(g\) is \(l_{ig} \equiv L_{ig}/L_i\). Let \(\rho\) denote the degree of relative risk aversion. The certainty-equivalent real income of an agent behind the veil is
\[
U_i \equiv \left(\sum_g l_{ig}W_{ig}^{1-\rho}\right)^{1/(1-\rho)}.
\]
Let \(R_i \equiv W_i/U_i\) be the adjustment factor associated with risk aversion for an agent behind the veil. Consistent with this idea, \(R_i = 1\) at \(\rho = 0\) and is increasing in \(\rho\), reaching \(W_i/\min_g W_{ig}\) when \(\rho \to \infty\).

In the quantitative section below we will present results for "risk-adjusted gains from trade" defined as \(RAGTi \equiv 1 - \hat{U}_i\) for a shock that takes the economy back to autarky, and compare such gains with the standard aggregate gains, \(GT_i = 1 - \hat{W}_i\). Given our definition of \(R_i\), we have \(\hat{U}_i = \hat{W}_i/R_i\), and hence
\[
RAGTi = 1 - \frac{1 - GT_i}{R_i}.
\]
If there is more risk in autarky than in the trade equilibrium then \(\hat{R}_i > 1\) and \(RAGTi > GT_i\), while \(\hat{R}_i < 1\) implies \(RAGTi < GT_i\).

### 3.6 Alternative Models and Extensions

In this section we consider a series of alternative models and extensions. In Section 3.1 we show that, once extended to a setting with trade gravity, the model proposed in Kovak (2013) is isomorphic to ours, with his regions corresponding to our groups, and with the inverse the labor share playing the role of our parameter \(\kappa\). In Section 3.2 we then explore how our results above change when we consider the extreme case in which labor is specific to each sector (i.e., no labor mobility across sectors). Section 3.3 extends our model to allow for tradable intermediate goods, while Section 4.4 extends the model to a setting where each group corresponds to a region, and there is labor mobility across regions.

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\(^{14}\)Related welfare measures are examined by Cordoba and Verdier (2008); Heathcote et al. (2008) and Jones and Klenow (2015), who incorporate income risk into the analysis of aggregate welfare in macro models without trade.
3.6.1 Kovak (2013)

Kovak (2013) postulates a multi-sector model of a small economy that is composed of multiple regions. He shows that the impact of a trade shock on a region is determined by the interaction of the region’s ex-ante industry shares and the changes in the sector-level prices. We now show that when extended to a gravity model of trade, this model is isomorphic to ours, with Kovak’s regions corresponding to our groups. Kovak assumes that labor is homogeneous and freely mobile across sectors, but that each region has a fixed quantity of a factor that is specific to each sector. We will present this model using groups rather than notation and keeping the notation as close as possible to the notation used in the previous section.

Let $K_{igs}$ be the endowment of a factor specific to sector $s$ owned by group $g$ in country $i$, and assume that labor and capital are combined according to a Cobb-Douglas production function to produce efficiency units specific to sector $s$, $E_{igs} = K_{igs}^{\alpha} L_{igs}^{1-\alpha}$. Let $w_{is}$ be the unit cost of efficiency units in sector $s$ in country $i$. As in Section 3, these wages must be common across groups. Given wages $w$, we can think of the determination of $L_{igs}$ as coming from the following program:

$$\max_{L_{igs}} \sum_s w_{is} K^{\alpha}_{igs} L^{1-\alpha}_{igs} \quad \text{subject to} \quad \sum_s L_{igs} = L_{ig}. \quad (10)$$

This program determines $L_{igs}$, $E_{igs}$ and $Y_i = \sum_s \sum_g w_{is} E_{igs}$ as a function of wages, $w$. Just as in Section 3.1, the equilibrium is a $w$ such that the excess labor demand in (4) is zero for all $i, s$.

Letting $K_{ig} = \sum_s K_{igs}$ and $k_{igs} = K_{igs}/K_{ig}$, it is straightforward to verify that the program in (10) implies

$$\pi_{igs} = \frac{L_{igs}}{L_{ig}} = \frac{k_{igs} w_{is}^{1/\alpha}}{\Psi^{1/\alpha}_{ig}},$$

where $\Psi^{1/\alpha}_{ig} = \sum_s k_{igs} w_{is}^{1/\alpha}$. In addition, letting $E_{ig} = K^{\alpha}_{ig} L^{1-\alpha}_{ig}$, one can easily check that

$$E_{igs} = (1 - \alpha)^{1/\alpha} E_{ig} \frac{k_{igs} w_{is}^{1/\alpha}}{\Psi^{1/\alpha}_{ig}} \frac{\Psi^{1/\alpha}_{ig}}{w_{is}}.$$ 

These two results are isomorphic to those in Lemma 1, with $1/\alpha, k_{igs}$ and $E_{ig}$ now playing the role of $\kappa, A_{gs}$ and $L_{ig}$ in the model of Section 3. Since the trade side of the model remains the same, the model here is isomorphic to the one in Section 3, except that the critical parameter is now $1/\alpha$ rather $\kappa$.

3.6.2 Specific Factors

We now consider an extreme case in which labor is immobile across sectors. Let $L_{igs}$ be the measure of workers of group $g$ work in sector $s$ in country $i$. Total employment in sector $s$ in country $i$ is $\sum_g L_{igs}$. The excess labor demand is now simply

$$ELD_{is} = \frac{1}{w_{is}} \sum_j \lambda_{ijs} \beta_{js} Y_j - \sum_g L_{igs}.$$ 

As above, this is a system in $w$. Comparative statics can be explored as in Section 3.2. The equation analogous to (5) is

$$\sum_g w_{is} w_{ks} L_{ig} = \sum_j \sum_k \lambda_{ijs} (\hat{\tau}_{iks} \hat{\tau}_{iks})^{-\theta} \beta_{js} \sum_k \sum_g \hat{w}_{jk} w_{jk} L_{jjk}.$$
Turning to welfare, the change in price of industry $s$ in country $i$ is $\tilde{w}_{is}\lambda_{iis}^{1/\theta}$, so

$$\tilde{P}_i = \prod_s \left(\tilde{w}_{is}\lambda_{iis}^{1/\theta}\right)^{\beta_{is}}.$$ \hspace{1cm}

Letting $\pi_{igs} = w_{is}L_{igs}/Y_{ig}$, the real income change for group $g$ is then

$$\frac{\dot{Y}_{ig}}{\tilde{P}_i} = \prod_s \lambda_{iis}^{\beta_{is}/\theta} \cdot \sum_s \pi_{igs}\tilde{w}_{is} \prod_s \tilde{w}_{is}^{\beta_{is}}.$$ \hspace{1cm}

Again, this is composed of two terms, the multi-sector ACR term, and a term that varies across groups.

In the case in which the shock entails a move to autarky we can fully solve for $\tilde{w}_{is}$ for $s = 1, \ldots, S$ for a particular country $i$. Setting $\tilde{Y}_i = 1$ by choice of numeraire, we would now have $\tilde{w}_{is} = \beta_{is}/r_{is}$, where $r_{is} = \sum_g \pi_{igs}Y_{ig}/Y_i$ is the share of income accruing nationally to workers in sector $s$. Plugging in the above expression for $\dot{Y}_{ig}/\tilde{P}_i$ we get

$$\frac{\dot{Y}_{ig}}{\tilde{P}_i} = \prod_s \lambda_{iis}^{\beta_{is}/\theta} \cdot \sum_s \pi_{igs} \left(\beta_{is}/r_{is}\right) \sum_k \beta_{ik}/r_{ik}^{\beta_{ik}}.$$ \hspace{1cm}

Note that since here workers in a region are stuck in a sector, then this should lead to more variance in $\dot{Y}_{ig}/\tilde{P}_i$ than the Roy model. Loosely speaking, the Roy model is something in between the classic specific factors model and the case with $\kappa \to \infty$. Note also that letting

$$\frac{\dot{Y}_{ig}}{Y_i} = \sum_s \sum_g \pi_{igs} \frac{\tilde{Y}_{ig}}{Y_i} = I_{ig}.$$ \hspace{1cm}

The point here is that the change in relative income is given by the index of import competition used earlier.

### 3.6.3 Intermediate Goods

Consider again the basic model but now with an input-output structure as in Caliendo and Parro (2014). This extension is important because a significant share of the value of production in a sector originates from other sectors, and taking this into account may affect the effects of trade on wages $\tilde{w}_{is}$ and hence the welfare effects across groups.

The labor supply of the model is exactly as in the main model (as characterized by Lemma 1), and trade shares and the price indices are given as in (2) and (3), except that instead of $w_{is}$ we now have $c_{is}$, where $c_{is}$ is given by

$$c_{is} = w_{is}^{1-\alpha_{is}} \prod_k P_{ik}^{\alpha_{iks}}. \tag{11}$$ \hspace{1cm}

Here the $\alpha_{iks}$ are the Cobb-Douglas input shares: a share $\alpha_{iks}$ of the output of industry $s$ in country $i$ is used buying inputs from industry $k$, and $1 - \alpha_{is}$ is the share spent on labor, with $\alpha_{is} = \sum_k \alpha_{iks}$. Combining this expression for $c_{is}$ with (3) (but with $w_{is}$ replaced by $c_{is}$) yields

$$P_{js} = \eta^{-1} \left(\sum_t T_{jis} \tilde{r}_{jst}^{-\theta} w_{is}^{-(1-\alpha_{is})\theta} \prod_k (P_{ik}^{-\theta})^\alpha_{iks}\right)^{-1/\theta}.$$ \hspace{1cm}

Given wages, this equation represents a system of $N \times S$ equations in $P_{js}$ for all $j$ and $s$, which can be
used to solve for $P_{js}$ and hence $c_{is}$ and $\lambda_{ij}$. This implies that trade shares are an implicit function of wages.

Let $X_{js}$ and $R_{js}$ be total expenditure and total revenues for country $j$ on sector $s$. We know that $R_{js} = \sum_{j=1}^{n} \lambda_{ij} X_{js}$ while Cobb-Douglas preferences and technologies imply $X_{js} = \beta_{js} Y_{j} + \sum_{k=1}^{S} \alpha_{jsk} R_{jk}$. Combining these equations we get a system of linear equations that we can use to solve for revenues given income levels and trade shares,

$$R_{js} = \sum_{j} \lambda_{ij} \left( \beta_{js} Y_{j} + \sum_{k=1}^{S} \alpha_{jsk} R_{jk} \right).$$

Since trade shares and income levels themselves are a function of wages, this implies that revenues are a function of wages. The excess demand for efficiency units in sector $s$ of country $i$ is now

$$ELD_{ls} = R_{ls} - \sum_{g} E_{igs}.$$

As in the baseline model, the system $ELD_{ls} = 0$ for all $i, s$ is a system of equations that we can use to solve for wages. In turn, given wages we can solve for all the other variables of the model.

The next step is to write the hat algebra system. From $ELD'_{ls} = 0$ we get

$$\sum_{g} \hat{w}_{ls}^{\kappa} \hat{\Phi}_{lg}^{1-\kappa} \pi_{igs} Y_{ig} = (1 - \alpha_{is}) \sum_{j=1}^{n} \lambda_{ij} \hat{\lambda}_{ij} \left( \beta_{js} \sum_{g} \hat{\Phi}_{tg} Y_{tg} + \sum_{k=1}^{S} \alpha_{jsk} \hat{R}_{jk} R_{jk} \right),$$

where

$$\hat{\Phi}_{lg}^{\kappa} = \sum_{k} \pi_{igk} \hat{w}_{ik}^{\kappa},$$

$$\hat{\lambda}_{ij} = \frac{\left( \hat{t}_{ij} w_{js}^{1-\alpha_{is}} \prod_{k} \hat{P}_{ik}^{\alpha_{jsk}} \right)^{-\theta}}{\sum_{j} \lambda_{ij} \left( \hat{t}_{ij} w_{js}^{1-\alpha_{is}} \prod_{k} \hat{P}_{ik}^{\alpha_{jsk}} \right)^{-\theta}},$$

$$\hat{P}_{js}^{-\theta} = \sum_{i} \lambda_{ij} \hat{\lambda}_{ij} \hat{\lambda}_{ij} \left( 1 - \alpha_{is} \right) \prod_{k} \left( \hat{P}_{ik}^{-\theta} \right)^{\alpha_{jsk}},$$

and

$$\hat{R}_{is} R_{is} = \sum_{j} \lambda_{ij} \hat{\lambda}_{ij} \left( \beta_{js} \sum_{g} \hat{\Phi}_{tg} Y_{tg} + \sum_{k=1}^{S} \alpha_{jsk} \hat{R}_{jk} R_{jk} \right).$$

Analogous to Proposition 1, from the hat algebra we find the following result:

**Proposition 3.** Given some trade shock, the ex-post percentage change in the real wage of group $g$ in country $i$ is given by

$$\hat{W}_{ig} = \prod_{s,k} \lambda_{ik}^{\beta_{is} \hat{a}_{is}} / \theta \cdot \prod_{s,k} \hat{w}_{igk}^{\beta_{is} \hat{a}_{is} (1 - \alpha_{ik}) / \kappa}$$

where $\hat{a}_{is}^{sk}$ is the typical element of matrix $(I - Y_{i}^{T})^{-1}$ with $Y_{i} \equiv \{ \alpha_{iks} \}_{k,s=1,...,S}$.
3.6.4 Mobility Across Regions

In our model, the ability of workers can be interpreted as being determined by the fundamentals of the region where they work, in addition to innate characteristics particular to the worker’s region of origin.\footnote{Specifically, there are two ways to interpret our baseline model. First, one could think that the \( z \) is inherent to the worker, something that the worker is born with, and that if she were to migrate to another region this \( z \) would not change. Since wages vary across sectors but not across regions, this interpretation would imply that there are no incentives for workers to migrate. Second, one could think that all workers draw an \( x \) in each sector from a Frechet distribution with parameters \( 1 \) and \( \kappa \), and that their efficiency units if they work in \((g,s)\) are \( A_{igs}^{1/\kappa} x_s \) (note that this is isomorphic to our current specification because \( \Pr(z \leq a) = \Pr(A_{igs}^{1/\kappa} x_s \leq a) \)). In this interpretation, \( A_{igs}^{1/\kappa} \) is a region-sector specific shifter that is common to all workers, and \( z \) is an worker-specific idiosyncratic term that is distributed the same everywhere. If we adopt the second interpretation, then labor income would differ across regions for the same worker, and there would be an incentive to migrate. For example, workers would want to move to regions that have a comparatively high common shifter in sectors whose relative wage increases after the trade shock.}

Under this interpretation, workers have an incentive to move across regions in response to trade shocks, which is something we have not modeled thus far.\footnote{There is limited empirical evidence of geographic mobility in response to trade shocks. Autor et al. (2013a), Dauth et al. (2014), and Topalova (2010) find that trade shocks induced only small population shifts across regions in the US, Germany, and India, respectively. These studies focus on the short and medium run, while ours focuses on the long run.}

Here we consider an extension of the benchmark model where workers can move across regions but not across countries. Assume that each worker gets a draw in each sector and each region. Workers also have an “origin region.” We say that a worker with origin region \( g \) is “from region \( g \).” Each worker gets a draw \( z \) in each region-sector combination \((h, s)\) from a Frechet distribution with parameters \( \kappa \) and \( A_{ikh_s} \). Workers are fully described by a matrix \( z = \{z_{hs} \} \) and an origin region \( g \). A worker from region \( g \) in country \( i \) that wants to work in region \( h \) of country \( i \) has to pay an iceberg cost \( \zeta_{igh} \), with \( \zeta_{igg} = 1 \) and \( \zeta_{igh} \leq 1 \) for all \( i, g, h \). Thus, a worker from \( g \) that works in region \( h \) in sector \( s \) has income of \( w_{is} \zeta_{igh} z_{hs} \).

We assume that \( \zeta_{gg} = 1 \) and \( \zeta_{gf} \leq 1 \).

Let wages per efficiency unit in sector \( s \) of country \( i \) be denoted by \( w_{is} \). Let \( w_i \equiv (w_{i1}, ..., w_{iS}) \) and let

\[
\Omega_{g,fs}(w_i) = \{z \text{ s.t. } w_{is} \gamma_{gf} z_{fs} \geq w_{ik} \gamma_{igh} z_{hk} \text{ for all } h, k \}.
\]

A worker with productivity matrix \( z \) from region \( g \) in country \( i \) will choose region-sector \((f, s)\) iff \( z \in \Omega_{g,fs}(w_i) \). The following lemma characterizes the labor supply side of the economy:

**Lemma 2.** The share of workers in group \( g \) in country \( i \) that choose to work in \((f, s)\) is

\[
\pi_{ig,fs} = \int_{\Omega_{g,fs}(w_i)} dF(z) = \frac{A_{fs}(z^g w_{is})^\kappa}{\Phi_{ig}(w_i)},
\]

where

\[
\Phi_{ig}(w_i) = \sum_{h,k} A_{hk}(z^g w_{ik})^\kappa,
\]

the efficiency units supplied by this group in sector \((f, s)\) are given by

\[
E_{ig,fs}(w_i) = L_{ig} \int_{\Omega_{g,fs}(w_i)} z_{fs} dF_{f}(z) = \zeta_{ig} \frac{A_{fs}(z^g w_{is})^\kappa (\Phi_{ig}(w_i))^{1/\kappa}}{w_{is} \zeta_{igf}},
\]

where \( \gamma \equiv \Gamma(1 - 1/\kappa) \), the total income of group \( g \) in country \( i \) is

\[
Y_{ig}(w_i) = \sum_{f,s} w_{is} \zeta_{gf} E_{ig,fs}(w_i) = \gamma L_{ig} (\Phi_{ig}(w_i))^{1/\kappa}.
\]
and total income in country \(i\) is

\[
Y_i(w_i) = \sum_g Y_{ig}(w_i) = \gamma L_i \sum_g l_{ig}(\Phi_{ig}(w_i))^{1/\kappa}.
\]

Moreover, the share of income obtained by workers in group \(g\) in country \(i\) in region-sector \((f,s)\) is also given by \(\pi_{ig,fs}\), while (ex-ante) per capita income for workers of group \(g\) in country \(i\) is

\[
Y_{ig}/L_{ig} = \gamma A_{ig}^{1/\kappa} w_{ig} \zeta_{ig} \pi_{ig,fs}^{-1/\kappa} \text{ for any } (f,s).
\]

Let \(\mu_{igh} \equiv \sum_s \pi_{ighs}\) be the share of workers from \(g\) that work in \(h\). It is easy to verify that \(\pi_{ighs}/\mu_{igh} = \pi_{ihhs}/\mu_{ihh}\) for all \(i, g, h, s\). Thus, conditional on locating in region \(h\), all workers irrespective of their origin have sector employment shares given by \(\pi_{ihhs} \equiv \pi_{ighs}/\mu_{igh}\). The shares \(\pi_{ihhs}\) and \(\mu_{igh}\) will be enough to characterize the equilibrium below.

The labor demand side of the model is exactly as in the case with no labor mobility across regions. Putting the supply and demand sides of the economy together, we see that excess demand for efficiency units in sector \(s\) of country \(i\) is

\[
ELD_{is} = \frac{1}{w_{is}} \sum_j \lambda_{ijs} \beta_j Y_j - \sum_{g,h} E_{ghs}.
\]

Noting that \(\lambda_{ijs}, Y_j\) and \(E_{ghs}\) are functions of the whole matrix of wages \(w \equiv \{w_{is}\}\), the system \(ELD_{is} = 0\) for all \(i, s\) is a system of equations in \(w\) whose solution gives the the equilibrium wages given some choice of numeraire.

Turning to comparative statics, the implications of a trade shock can be characterized in similar fashion to what we did in Section 3.2. Changes in wages can be obtained as the solution to the system of equations given by

\[
\sum_{g,h} \hat{\Phi}_{ig}^{1/\kappa} \mu_{igh} \pi_{ihhs} Y_{ig} = \sum_j \frac{\lambda_{ijs} (\hat{\tau}_{ijs} \hat{w}_{is})^{-\theta}}{\sum_k \lambda_{kjs} (\hat{\tau}_{kjs} \hat{w}_{ks})^{-\theta}} \beta_is \sum_g \hat{\Phi}_{jg} Y_{jg}
\]

with \(\hat{\Phi}_{ig}^{1/\kappa} = \sum_{h,k} \mu_{ih} \pi_{ihk} \hat{w}_{ik}^{1/\kappa}\).

Equation (5) can be solved for \(\hat{w}_{is}\) given data on income levels, \(Y_{ig}\), trade shares, \(\lambda_{ijs}\), migration shares \(\mu_{igh}\), employment shares \(\pi_{ihhs}\), and labor endowment, and the shocks, \(\hat{\tau}_{ijs}\). In turn, given \(\hat{w}_{is}\), changes in trade shares can be obtained from (6), while changes in migration and employment shares can be obtained from

\[
\hat{\pi}_{ighs} = \frac{\hat{w}_{is}^{\kappa}}{\sum_{h,k} \mu_{ihf} \pi_{ifjs} \hat{w}_{ik}^{\kappa}},
\]

combined with \(\hat{\mu}_{igh} = \sum_s \pi_{ihhs} \hat{\pi}_{ighs}\) and

\[
\hat{\pi}_{ihhs} = \hat{\pi}_{ighs}/\hat{\mu}_{igh}.
\]

Given \(\hat{w}_{ik}\), the following proposition analogous to Proposition 1 characterizes the impact of a trade shock on ex-ante real wages for different groups of workers.

**Proposition 4.** Given some trade shock, the ex-ante percentage change in the real wage of group \(g\) in
country $i$ is given by

$$
W_{ig} = \prod_s \left( \frac{\lambda_{is}^{\beta_{is}}}{\theta} \right) \cdot \prod_s \left( \mu_{ig} \pi_{igs} \right)^{\beta_{is}/\kappa}
$$

Letting $\nu_{igs} = \sum_h \mu_{igh} \pi_{ih}$ be the share of workers from region $g$ that work in sector $s$, we can show that

$$
\hat{Y}_{ig} = \left( \sum_s \nu_{igs} \hat{w}_{is} \right)^{1/\kappa}.
$$

This says that $\hat{Y}_{ig}$ is a power mean of the wage changes, $\hat{w}_{is}$, with power $\kappa$ and weights $\nu_{igs}$ given by the share of people from $g$ that work in industry $s$, which are obtained from migration and industry shares as $\nu_{igs} = \sum_h \mu_{igh} \pi_{ih}$. When we construct the index $I_g$, now we should use $\nu_{igs}$ rather than $\pi_{igs}$, so that $I_g \equiv \sum_s \nu_{igs}^\beta / \mu_{igs}$.

4 Data

We employ German administrative data to obtain sectoral employment shares at the regional-level. Data on bilateral trade flows and sectoral output are from the OECD Database for Structural Analysis (STAN). Due to the limited availability of compatible data for trade flows and sectoral employment shares (at the regional), we restrict our simulation analysis to the year 2003. Our choice of industry classification is also driven by the availability of the data. In the current version of this paper, we aggregate manufacturing industries into 15 groups which roughly correspond to two-digit ISIC Rev. 3 codes ($S = 15$)

<table>
<thead>
<tr>
<th>ISIC Rev. 3 Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-16</td>
<td>C15T16 Food products, beverages and tobacco</td>
</tr>
<tr>
<td>17-19</td>
<td>C17T19 Textiles, textile products, leather and footwear</td>
</tr>
<tr>
<td>20</td>
<td>C20 Wood and products of wood and cork</td>
</tr>
<tr>
<td>21-22</td>
<td>C21T22 Pulp, paper, paper products, printing and publishing</td>
</tr>
<tr>
<td>23</td>
<td>C23 Coke, refined petroleum products and nuclear fuel</td>
</tr>
<tr>
<td>24</td>
<td>C24 Chemicals and chemical products</td>
</tr>
<tr>
<td>25</td>
<td>C25 Rubber and plastics products</td>
</tr>
<tr>
<td>26</td>
<td>C26 Other non-metallic mineral products</td>
</tr>
<tr>
<td>27</td>
<td>C27 Basic metals</td>
</tr>
<tr>
<td>28</td>
<td>C28 Fabricated metal products, except machinery and equipment</td>
</tr>
<tr>
<td>29</td>
<td>C29 Machinery and equipment, n.e.c.</td>
</tr>
<tr>
<td>30-33</td>
<td>C30T33 Electrical and optical equipment</td>
</tr>
<tr>
<td>34</td>
<td>C34 Motor vehicles, trailers and semi-trailers</td>
</tr>
<tr>
<td>35</td>
<td>C35 Other transport equipment</td>
</tr>
<tr>
<td>36-37</td>
<td>C36T37 Manufacturing n.e.c. and recycling</td>
</tr>
</tbody>
</table>

For Germany, we obtain the employment shares $\pi_{igs}$ employing detailed data from the German Social Security System. The geographical units of observation $g$ are German Kreise, which are roughly the German equivalent of US counties. Each of these regions contains a minimum of 100,000 inhabitants as of December of 2008. In the current version of the data, we observe 265 of these regions (all located in West Germany).\footnote{The employment counts are based on the job in which workers spent the longest spell during 2003. Additional restrictions were also imposed.\footnote{In cases where $\pi_{igs} = 0$, we imputed a small value to make the data consistent with our model.}}\footnote{In cases where $\pi_{igs} = 0$, we imputed a small value to make the data consistent with our model.}

Our measures of trade flows are taken from the OECD STAN Database (ISIC Rev.3). To arrive at
our measures, we combine values of national sectoral output \(^{19}\), and total import and export figures by sector (to the entire world). This allows us to obtain consistent values of import penetration by sector \((\lambda_{iis})\).

\[
\lambda_{iis} = \frac{Y_{is} - X_{iis}^{\text{WORLD}}}{Y_{is} - X_{iis}^{\text{WORLD}} + N_{iis}^{\text{WORLD}}}
\]

Employing the sectoral output and trade flow data from the OECD STAN Database, we obtain the consumption shares \(\beta_{iis}\) as follows:

\[
\beta_{iis} = \frac{Y_{is} - X_{iis}^{\text{WORLD}} + N_{iis}^{\text{WORLD}}}{\sum_s Y_{is} - X_{iis}^{\text{WORLD}} + N_{iis}^{\text{WORLD}}}
\]

Lastly, to obtain the measures of regional output shares for Germany \((\hat{Y}_{i}^{\text{gs}})\), we make use of EuroStat regional statistics.

5 Benchmark counterfactual: Germany’s return to autarky

Based on our baseline model, we perform a counterfactual exercise by solving for Germany’s autarky equilibrium as described in Section 3.2. We then compute how much the country as a whole, as well as each individual region, loses from going to autarky. Both at the aggregate and the regional level, the results are as expected. The country as a whole loses from a move to autarky. At the regional level, more export-oriented regions lose the most, while import-competing regions lose the least, and sometimes even gain from increased trade costs. In all the ensuing exercises, we follow Costinot & Rodriguez-Clare (2014) in assuming a value of \(\theta = 5\).

5.1 Autarky

Table 4 summarizes the results on both the aggregate and the distributional side. For a value of \(\kappa = 2\), our results indicate an average loss of 12.6%, with a significant dispersion in these losses across regions (a standard deviation of 7.1 percentage points).\(^{20}\) In fact, for intermediate values of \(\kappa\) the most affected regions experience losses of 10%, while the least affected regions experience gains of 17 percentage points or more. The loss from a return to autarky decreases with \(\kappa\), with an aggregate loss of 12.6% when \(\kappa = 2\) and 8.7% when \(\kappa = 100\). The intuition is that a decreasing \(\kappa\) introduces more curvature to the PPF, making it harder to adjust to autarky.

The region-level losses deviate considerably from the aggregate loss, with some regions gaining substantially from the move to autarky. This can be seen clearly in Figure 3, which plots the distribution of regional losses for different values of \(\kappa\). A lower \(\kappa\) leads to higher dispersion in the distribution of the group-level gains from trade. Intuitively, lowering \(\kappa\) introduces more variation in worker-level comparative advantage, and therefore a given trade-shock will have a more heterogeneous impact across the distribution of workers. As \(\kappa\) approaches infinity, workers are perfectly substitutable across sectors, and the variance in regional gains from trade gradually disappears.

\[^{19}\text{Output measures } Y_{is} \text{ are based on STAN variable PROD “Production (gross output)” (see Appendix for detailed description).}

\[^{20}\text{The table displays both } \hat{W}_i \text{ and the mean value for } \hat{W}_{i}^{\text{gs}}. \text{ The difference between these two values is that the former is a weighted mean across groups, while the latter is an unweighted mean. In general, the two values are closely related, with a maximum difference of 0.9 percentage points, corresponding to the specific-factors simulation.}\]
In our simulations, regions specialized in import-competing sectors tend to lose less than average whereas initially export-oriented regions will lose more than average. Here, import-competing or export-oriented sectors are defined relative to the share of the workforce employed in that sector under autarky. It can be shown that in autarky, $\beta_s^a = \pi_s^a$, i.e. the expenditure-share on a certain sector has to equal the share of the workforce employed in this sector. Hence, if $\beta_s > \pi_s$, this sector will expand at the national level as Germany moves to autarky, and vice versa. As such, $s/\pi_s$ is a measure of the necessary expansion/contraction that a sector has to undergo at the national level as Germany moves to autarky. In other words, $s/\pi_s > 1$ ($< 1$) indicates an import-competing (export-oriented) sector, as the sector’s workforce share is smaller (larger) under trade than it is under autarky. It is then intuitive that regions specialized in import-competing sectors, i.e. sectors whose relative price will increase under autarky, will lose less from the return to autarky than in the opposite case. Table 5 shows the national deviations from autarky sectoral shares, which vary considerably across sectors. The measure reaches a maximum for sector 2300 “Coke, refined petroleum products and nuclear fuel”, with $s/\pi_s = 9.16$. Therefore, this sector will have to grow by more than 900% as Germany moves to autarky. On the other side of the spectrum, Sector 2900 “Machinery and equipment” is the most export-oriented sector, with a value of $s/\pi_s = 0.65$. Taken together, this sizable variation in $s/\pi_s$ implies considerable sectoral reallocation under the return to autarky.
Table 5: Index of sectoral import competition

<table>
<thead>
<tr>
<th>$\beta_{Is}/\pi_{Is}$</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.224</td>
<td>Food products, beverages and tobacco</td>
</tr>
<tr>
<td>1.26</td>
<td>Textiles, textile products, leather and footwear</td>
</tr>
<tr>
<td>0.865</td>
<td>Wood and products of wood and cork</td>
</tr>
<tr>
<td>0.838</td>
<td>Pulp, paper, paper products, printing and publishing</td>
</tr>
<tr>
<td>9.159</td>
<td>Coke, refined petroleum products and nuclear fuel</td>
</tr>
<tr>
<td>1.342</td>
<td>Chemicals and chemical products</td>
</tr>
<tr>
<td>0.715</td>
<td>Rubber and plastics products</td>
</tr>
<tr>
<td>0.989</td>
<td>Other non-metallic mineral products</td>
</tr>
<tr>
<td>1.11</td>
<td>Basic metals</td>
</tr>
<tr>
<td>0.706</td>
<td>Fabricated metal products, except mach. and equip.</td>
</tr>
<tr>
<td>0.647</td>
<td>Machinery and equipment, n.e.c.</td>
</tr>
<tr>
<td>0.93</td>
<td>Electrical and optical equipment</td>
</tr>
<tr>
<td>1.408</td>
<td>Motor vehicles, trailers and semi-trailers</td>
</tr>
<tr>
<td>1.162</td>
<td>Other transport equipment</td>
</tr>
<tr>
<td>0.826</td>
<td>Manufacturing n.e.c. and recycling</td>
</tr>
</tbody>
</table>

Figure 4 confirms our intuition. It plots the change in real income on the Y-axis and the import-competition index $I_g$ on the X-axis. The figure indicates that a region’s proportional change in real income almost perfectly correlates with our measure of regional import-competition. In fact, the most import-competing regions have positive changes in real income due to the move to autarky. Gelsenkirchen is the region that gains the most, with an increase in real income of 40% when $\kappa = 2$, mainly because it has 18% of its manufacturing workforce employed in sector 2300.

So far, we have only discussed the results for our standard Roy-model exercise. In addition, we have also performed specific-factors simulations, where each worker is constrained to remain in his initial sector when Germany moves to autarky. The results are qualitatively similar to the Roy-model results, except that the average loss is stronger and the variance of regional losses is higher as well (see Table 4 and Figure A.1).
5.2 Risk-Adjusted Gains from Trade

Given the distribution of group-level gains from trade, we can compute the risk-adjusted gains of trade ($RAGT$), as described in Section 2.5. Figure 3 shows that for a strictly positive coefficient of relative risk aversion, the $RAGT$ for Germany are higher than the standard aggregate gains from trade, which is based on a lower risk-level under trade than under autarky. For instance, for $\kappa = 2.5$ ($\kappa = 7$), the gains from trade are 11.9% (9.9%), while $RAGT = 13.6\%$ (10.4%) for $\rho = 2$. Furthermore, the $RAGT$ tends to increase, though not monotonically, with the coefficient of relative risk aversion. For reasonable values of risk aversion (CRRA smaller than 10), the more risk averse the agent behind the veil of ignorance, the more she values trade.

Figure 5: Trade as Lottery - Autarky - Germany
5.3 Multiple Country Simulations

In addition to the autarky exercise, we also plan to perform simulations for the counterfactual world equilibrium where cross-country trade costs universally increase. This type of exercise has much stronger data requirements than the autarky exercise described above. Specifically, our model requires the sectoral employment shares and bilateral trade flows for all relevant trading countries in the world, all with consistent definitions and industry classifications. This is work in progress.

6 Estimation of Parameter $\kappa$

We have shown theoretically and with simulations how two parameters affect the distributional effects of trade: $\theta$ and $\kappa$.

Since Caliendo and Parro (2014); Head and Mayer (2014) present and discuss estimates for $\theta$, our focus will be on estimating $\kappa$. To do so, we propose a novel approach based on our structural model and the relationship between $\kappa$ and the elasticity of employment shares with respect to sectoral wages.

As in section 5, our analysis defines groups in terms of geographical units. Due to data limitations, we plan to focus on Germany.

6.1 Main Estimation Procedure

From our model, the parameter $\kappa$ governs the response of sectoral employment shares to changes in efficiency-unit wages. Note that fixing a numeraire sector $s_0$, Lemma 1 can be rewritten as:

$$\tilde{\pi}_{gst} = \frac{\pi_{gst}}{\pi_{gst}} = \frac{A_{gst}}{A_{gst}} \left( \frac{w_{st}}{w_{s_0 t}} \right)^{\kappa}$$

Taking logs and first differences, we obtain the following estimating equation:

$$\Delta \ln \tilde{\pi}_{gst} = \kappa \Delta \ln \tilde{w}_{st} + \varepsilon_{gst}$$

(14)

where $\tilde{w}_{st} \equiv \frac{w_{st}}{w_{s_0 t}}$ is the relative sectoral wage for sector $s$, and $\varepsilon_{gst} \equiv \Delta \frac{A_{gst}}{A_{gst}}$. Equation 14 will form the basis of our estimation procedure, and requires three types of data: regional employment shares for each sector (which can be readily observed from the data), measures of sectoral wages per efficiency unit ($w_{s_0 t}$'s, which are unobserved), and a credible instrument for $\Delta \ln \tilde{w}_{st}$ to address endogeneity issues. Specifically, correlation between $\tilde{w}$ and $\varepsilon_{gst}$ as a result of supply shocks would bias our estimate of $\kappa$.

Our estimation procedure will therefore consist of several steps. First, we will employ worker-level data to obtain estimates of changes in sectoral efficiency-unit wages. Second, we will generate measures of sectoral trade shocks to use as instruments. Lastly, we will integrate all these components to estimate $\kappa$ from the response of sectoral employment shares to changes in efficiency-unit wages. Below, we detail each step of our analysis.

---

21 We have imposed that $\theta$ is equal across sectors. Relaxing this assumption would affect the aggregate gains of trade, but not the distribution of gains.

22 Hsieh et al. (2013) and Burstein et al. (2014) obtain values of $\kappa$ for a related Roy-framework. However, their values of $\kappa$ are calibrated and apply to occupations instead of industries. We view our methodology as complementary to theirs. As a robustness test, we will later employ the methodology from Hsieh et al. (2013) to calibrate alternative measures of $\kappa$. 

23
6.1.1 Efficiency unit wages ($w_{st}$)

To obtain values for $\Delta \ln \bar{w}_{st}$, we follow our model closely and employ a simplified version of the method developed by Heckman and Scheinkman (1987). In this procedure, we employ individual-level panel data and assume that the sectoral productivity draw $z_{ns}$ for individual $n$ is fixed over time. From our model, observed wages for worker $n$ in sector $s$ at periods $t_0$ and $t_1$ can be written as:

$$y_{nst} = w_{st} z_{ns} + \xi_{nst} \text{ for } t = t_0, t_1$$

where $\xi_{nst}$ is random noise. Solving for $z_{ns}$ we get the estimating equation:

$$y_{nst_1} = \frac{w_{st_1}}{w_{st_0}} y_{nst_0} + \left[ \frac{w_{st_1}}{w_{st_0}} \xi_{nst_0} \right]$$

(15)

The coefficient from Equation (15) can be estimated separately for workers in each sector $s$. To ensure consistency, we restrict the sample to workers present in sector $s$ on both period $t_0$ and $t_1$. Lastly, we follow Heckman and Scheinkman (1987) and instrument for $y_{nst}$ using lagged earnings. The intuition behind this procedure is simple. If realized wages for workers is a combination of their unobserved sectoral ability and a sectoral wage for their ability, then changes in the observed wages for a fixed sample of workers will reflect changes in $w_{st}$.

Equation 15 above assumes that wage changes depend solely on changes in the returns to a one dimensional sectoral ability ($z_{ns}$). In reality, changes in the returns to other worker characteristics are likely to influence wage changes over time. To address this concern, we plan to first estimate a standard Mincer regression, regressing observed wages for all workers on gender, education, experience and experience squared. We then use the residuals from this regression as measures of realized wages in equation 15.

To implement this estimation we plan to employ rich administrative data from the German Social Security System. Specifically, we will use a weakly anonymous version of the Sample of Integrated Labour Market Biographies (SIAB). The SIAB contains a 2% random sample of the working histories of the vast majority of the German labor force. In addition to individual information on wages and demographic characteristics, the SIAB allows us to observe industry affiliation over time with a high level of precision. We expect to gain access to the SIAB in the near future.

6.1.2 Measures of import competition

We instrument for changes in sectoral wages with trade shocks as in Autor et al. (2013b) and Dauth et al. (2014). Specifically, we will construct national measures of import competition faced by each sector in Germany that are caused by the rise of China and Eastern Europe in the world economy. To address endogeneity concerns, we follow the intuition of the aforementioned papers and use trade flows from China and Eastern Europe to a group of countries “similar” to Germany. As in Autor et al. (2013b),

23In Heckman and Scheinkman (1987), there are many productive attributes, observed and unobserved, priced differently in each sector. Our application of their methodology is a simplified version in which we assume there is a single one-dimensional unobserved attribute ($z_{ns}$) with price $w_{st}$ that determines the observed wages of workers.

24This is necessary to account for the correlation of $y_{nst}$ and the error term in 15.

25We plan to run this wage regression for workers in all sectors together, restricting the coefficients on each control variable to be the same across sectors, but allowing them to vary by year. The details of this estimation procedure are presented in Appendix C.

26Eastern Europe is comprised of the following countries: Bulgaria, Czech Republic, Hungary, Poland, Romania, Slovakia, Slovenia, and the former USSR or its succession states Russian Federation, Belarus, Estonia, Latvia, Lithuania, Moldova, Ukraine, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, and Uzbekistan.

27We follow Dauth et al. (2014) in defining this set of countries to include Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom.
the idea behind this approach is to capture the effect of import demand shocks arising from growth in China and Eastern Europe on German wages, which is uncorrelated to unobserved local shocks. The specifics of this instrument have been discussed in Section 2.2.

6.1.3 Estimating $\kappa$

Having obtained estimates for the changes in efficiency unit wages and trade shocks, we can use equation 14 to obtain estimates of $\kappa$. Formally, our system of equations is the following:

\[
\begin{align*}
\Delta \ln \tilde{\pi}_{gst} &= \kappa \Delta \ln \tilde{w}_{st} + \varepsilon_{gst} \\
\Delta \ln \tilde{w}_{st} &= \gamma \Delta IP_{st}^{East \to Other} + \nu_{st}
\end{align*}
\]

Employment shares will be obtained directly from our German administrative data. For consistency with the simulations in section 5, industries will be aggregated to match those presented in table 3. In all cases, we cluster standard errors at the industry-State level.\footnote{Given our use of instruments for $\Delta \ln \tilde{w}_{st}$, it is not necessary to adjust our standard errors to account for the fact that the $\Delta \ln \tilde{w}_{st}$’s are themselves estimates.}
References


Faber, B. (2014). Trade Liberalization, the Price of Quality, and Inequality: Evidence from Mexican Store Prices.


A Specific Factors

Our measure of import-competition ($\beta_s/\pi_s$) also predicts gains and losses under the simulations for the specific factors model. However, the correlation is lower than in the Roy-model. Intuitively, the reason is that in the Roy-model, there is a reallocation of workers across sectors such that $\beta_s = \pi_s$, whereas there is no reallocation of workers in the specific-factors model.

Figure A.1: Distributional Gains by Region - Autarky

B US version

B.1 Data

For the US, we combine employment data from the County Business Patterns (CBP) dataset and sectoral output data from the NBER CES database. We also employ data on trade flows and regional earnings that were kindly provided by Gordon Hanson.

We follow Autor, Hanson and Dorn (2013) in defining regional economies using the concept of Commuting Zones (CZs). Our industry classification follows the 1987 SIC classification codes aggregated to the 2-digit level by an algorithm also provided employed by AHD, and restricted to manufacturing industries only. This leaves us with a total of 722 CZs and 20 industries. All current figures are for the year 2000.

For employment shares $\pi_{ig}$, we apply the same algorithm as AHD to obtain commuting zone employment shares from the CBP county level data. As in the German case, we currently imput very
low values \( (\pi_{10s} = e^{-10}) \) to CZ-industry cells with zero values. Our figures for national sectoral output \( Y_{is} \) come directly from the NBER-CES database variable \( vship \), which represents the total value of industry shipments. To obtain aggregate earnings in manufacturing at the CZ leve \( (Y_{10s}) \), we employ publicly available data from AHD’s China Syndrome paper. Specifically, we multiply each commuting zone’s weekly average wages in manufacturing by their employment count in manufacturing.

B.2 Autarky Exercise - US

Figure B.1: Distributional Gains by Region - Autarky - US

Variation in Regional Losses – US

Autarky Case

Last updated 3 Sep 2014, 20 sectors. tables_graphs_aut_US.do.
Figure B.2: Distributional Gains by Region - Autarky - US

Going to autarky: winners and losers
Specific Factors Model

US Going to autarky: winners and losers

Last updated 3 Sep 2014, 20 sectors. tables_graphs_aut_US.do.
Figure B.3: RAGT - US

Coefficient of Relative Risk Aversion

RAGT

Coefficient of Relative Risk Aversion

- kappa = 2
- kappa = 3
- kappa = 5
- kappa = 100
C Estimation of sectoral efficiency unit wages

The following are our estimation equations:

\[ m_{nst} = \alpha + X'_{nt}\beta_t + y_{nst} \]

where \( m_{nst} \) is the observed daily wage (in levels) of individual \( n \) in sector \( s \) at time \( t \), and \( y_{nst} \) is the residual. \( X_{nt} \) is a vector of control variables that includes dummies for gender, 7 education categories, and experience and experience squared, all interacted by year dummies. Note that we do not allow for a year-specific intercept. We also restrict the coefficients on each control variable to be the same across sectors, but we allow them to vary by year.