Quantifying Confidence*

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Abstract

We enrich workhorse macroeconomic models with a mechanism that proxies strategic uncertainty and that manifests itself as waves of optimism and pessimism about the short-term economic outlook. We interpret this mechanism as variation in “confidence” and show that it provides a parsimonious account of multiple salient features of the data; it drives a significant fraction of the business cycle in estimated models that allow for multiple structural shocks; and it captures a type of fluctuations in “aggregate demand” that does not rest on nominal rigidities.

Keywords: Business cycles, strategic uncertainty, higher-order beliefs, confidence, aggregate demand, coordination failure, DSGE models.

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1 Introduction

Recessions are often described as periods of "low confidence" and "weak aggregate demand," sometimes as symptoms of "coordination failure." Yet, it is not clear what these concepts mean, how to quantify them, or how to identify their empirical counterparts.

In this paper, we develop a formalization of these notions which is grounded on the role of strategic uncertainty in games but can be readily incorporated into workhorse macroeconomic models. This permits us to accommodate a certain kind of waves of optimism and pessimism that regard the short-run economic outlook and that can be attributed to a friction in coordination. We interpret these waves as variation in "confidence", we quantify their importance within RBC and NK models of either the textbook or the medium-scale DSGE variety, and we argue that they offer a parsimonious yet powerful explanation of multiple salient features of the macroeconomic data.

Background and methodological contribution. Our starting point is a literature that has already highlighted the potential role of strategic uncertainty and higher-order beliefs in macroeconomics. This literature goes back at least to Phelps (1971) and Townsend (1983) and has been revived recently by the influential contributions of Morris and Shin (2001, 2002) and Woodford (2003). Within this literature, the closest precursors to our paper are Angeletos and La’O (2013) and Benhabib, Wang and Wen (2015). These works have shown that incomplete information can help unique-equilibrium, rational-expectations models accommodate forces akin to “animal spirits”. More broadly, the literature has indicated the potential significance of incomplete information as a source of both volatility and persistence.

Despite these advances, there has been limited progress on the quantitative front. This is because the introduction of incomplete information in dynamic general-equilibrium models raises a number of technical difficulties that hinder their solution and estimation. These difficulties include large state spaces in order to keep track of the dynamics of higher-order beliefs in the cross-section of the population; and a high-dimensional fixed point between the equilibrium law of motion of the state and the Kalman filter that each agent uses to update his beliefs of it. In practical terms, this means that it can take hours to simulate a small-scale macroeconomic model even if one knows the parameters of the model—never mind to estimate a large-scale DSGE model!

The methodological contribution of our paper lies in the bypassing of these difficulties. This is achieved by relaxing the common-prior assumption: we develop a heterogeneous-prior specification that allows us to engineer aggregate waves in higher-order beliefs while abstracting from both noisy learning and action heterogeneity. Literally taken, this entails a systematic bias in beliefs. But it can also be seen as a convenient proxy for the higher-order uncertainty that can be sustained in common-prior, incomplete-information settings at the expense of significantly lower tractability.

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1 A key difference between the two papers is that the former imposes a unique equilibrium, whereas the latter uses the incompleteness of information to open the door to multiple equilibria. Nevertheless, we view these papers, along with the more abstract work by Bergemann and Morris (2013) and Bergemann, Heumann and Morris (2014), as complementary common-prior foundations of the type of fluctuations studied and quantified in the present paper.
To illustrate, consider the textbook RBC model. The equilibrium dynamics of this model take the form $K_{t+1} = G(K_t, A_t)$, where $K_t$ is the endogenous capital stock and $A_t$ is the exogenous technology shock. In general, adding incomplete information to this model causes the dimension of its state space to explode to infinity. By contrast, our formulation guarantees a minimal change: the equilibrium dynamics are given by $K_{t+1} = G(K_t, A_t, \xi_t)$, where $\xi_t$ is an exogenous variable that encapsulates the desired aggregate variation in higher-order beliefs. The belief-augmented model can thus be solved, simulated, and estimated with the same ease as the standard one.

**Applied contribution.** In abstract games, higher-order uncertainty helps rationalize the uncertainty that each player may face about the actions of other players for given payoffs. In the macroeconomic models we are interested in, it helps rationalize the uncertainty that firms may face about consumer spending and demand, or the one that consumers face about firm hiring and income, for given fundamentals such as preferences and technologies. But whereas the observable implications of higher-order uncertainty can be arbitrary in abstract games, they take a very specific form in the models we study. The applied contribution of our paper rests on spelling out these observable implications; on contrasting them to those of other structural shocks and mechanisms employed in the macroeconomics literature; and ultimately on offering a novel, parsimonious, yet powerful, structural interpretation of the business cycle data.

To this goal, we start with a variant of the textbook RBC model that features only two sources of volatility: a persistent technology shock, $A_t$, which moves the production possibilities of the economy; and a transitory belief shock, $\xi_t$, which moves higher-order beliefs of $A_t$ for given $A_t$. In the equilibrium of this model, variation in $\xi_t$ manifests as waves of optimism and pessimism about the short-term economic outlook, or as variation in the “confidence”. With this in mind, we henceforth refer to $\xi_t$ as the “confidence shock”.

In spite of its parsimony, the model has indeed no difficulty in matching a variety of key business-cycle moments in the US data. Importantly, this success is not trivial: the match between the model and the data deteriorates significantly if we replace the confidence shock with any of a variety of other structural shocks that have been deployed in the literature as proxies for shifts in either “supply” or “demand”, including technology shocks, news shocks, investment-specific shocks, and discount-rate shocks. What is more, the superior performance of the confidence shock extends to a New-Keynesian variant that adds sticky prices and a realistic Taylor rule for monetary policy.

In order to elaborate on what lies behind this success, consider a negative innovation in $\xi_t$. This causes pessimism about the short-term economic outlook, without affecting the medium- to long-term prospects. As firms expect the demand for their products to be low over the next few quarters, they find it optimal to lower their own demand for labor and capital. As a consequence, households experience a transitory fall in wages, capital returns, and overall income. Because this entails relatively weak wealth effects, they react by working less and by cutting down on both consumption

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2The precise definition and the economic meaning of the variable $\xi_t$ will become clear in the sequel. For now, we only wish to emphasize the tractability afforded by our approach.
and saving. Therefore, variation in $\xi_t$ leads to strong co-movement between employment, output, consumption, and investment at the business-cycle frequency, without commensurate movements in labor productivity, TFP, and inflation at any frequency. It is precisely these patterns that best characterize the US data and that competing structural mechanisms have difficulty matching.

By the same token, our mechanism offers a formalization of the notion that recessions are period of “weak aggregate demand” without the need to rest on either nominal rigidities or frictions in the conduct of monetary policy. Indeed, a drop in confidence has similar incentive effects on a firm’s hiring and investment decisions as a joint tax on labor and capital. Seen through the lenses of the NK framework, a drop in confidence may therefore register as an increase in measured markups and in the “output gap”. Yet, there are no nominal rigidities at work, no constraints on monetary policy, and no commensurate drop in prices. Unlike what required by the New-Keynesian framework, demand-driven recessions therefore do not have to be deflationary episodes.

Notwithstanding these points, one may wonder whether our mechanism remains potent once it is embedded in richer DSGE models that allow for multiple structural shocks. We study this question in the context of two “medium-scale” DSGE models, which we estimate on US data. In both models, our mechanism has to compete against many alternative shocks such TFP, news, investment and consumption specific, fiscal and monetary policy. In the one model, prices are sticky; in the other, they are flexible.

Despite the presence of these competing forces, our mechanism is estimated to account for about one half of GDP volatility at business-cycle frequencies (6-32 quarters). Furthermore, the observable properties of the confidence shock are similar across the two models, underscoring the robustness of our mechanism across RBC and NK settings, a quality not shared by other structural mechanisms. Last but not least, the model-based confidence shock tracks well empirical counterparts such as the University of Michigan Index of Consumer Sentiment—a fact that lends support to our interpretation of this structural shock.

All these findings support the view that a significant part of the business cycle can be understood as the outcome of shifts in market sentiment or of frictions in coordination. They also indicate the broader quantitative potential of enriching the belief structure in macroeconomic models.

Layout. The rest of the paper is organized as follows. Section 2 relates our paper to the literature. Section 3 sets up the baseline model. Section 4 explains our solution method. Section 5 explores the quantitative performance of the baseline model. Section 6 extends the analysis to two richer, estimated models. Section 7 discusses how our approach undermines existing interpretations of the observed recessions. Section 8 concludes. The Appendix contains the details of the solution method; a number of auxiliary results; and a more detailed discussion of the related literature.

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3It seems interesting to analyze the recent recession using our approach. In particular, we favor a structural interpretation of the recession that attributes the start of the recession to a transient financial shock and the slow recovery to a drop in confidence, or a coordination failure. We leave this investigation to future research because the models used in this paper do not contain any financial frictions.
2 Related literature

The idea that incomplete information and higher-order uncertainty help formalize frictions in the coordination of beliefs and actions has a long history in game theory. Similarly, the idea of using heterogeneous priors as a short-cut for engineering variation in higher-order beliefs is not completely new. Prior applications include, inter alia, Allen, Morris and Postlewaite (1993), Scheinkman and Xiong (2003), Yildiz and Izmalkov (2009), and Section 7 of Angeletos and La’O (2013). Our methodological contribution is found in the development of a tractable yet flexible methodology for enriching the belief structure in a large class of DSGE models.

In so doing, our paper adds to a blossoming literature that studies the macroeconomic effects of informational frictions. Some works focus on decision-theoretic mechanisms (Sims, 2001; Reis, 2006; Alvarez, Lippi and Pacciello, 2011); other works emphasize the role of strategic interactions (Morris and Shin, 2002, Woodford, 2003). Our paper is closer to the latter: we enrich the beliefs that agents face about one another’s choices, and thereby about endogenous economic outcomes, for given beliefs of preferences and technologies.

Attempts to push the quantitative frontier on this literature include the following: Nimark (2013), who studies the asymptotic accuracy of finite-state-space approximations to a class of dynamic models with private information; Rodina and Walker (2013) and Huo and Takayama (2014), who obtain analytic solutions in certain models using frequency-domain techniques; Melosi (2014), who estimates a version of Woodford (2002); Mackowiak and Wiederholt (2014), who study the quantitative performance of a particular DSGE model augmented with rational inattention; and David, Hopenhayn, and Venkateswaran (2014), who use firm-level data to gauge the cross-sectional misallocation caused by informational frictions on the side of firms.

We view our methodological contribution as complementary to all these attempts. The benefits concern the accommodation of a flexible form of strategic uncertainty and the straightforward applicability to DSGE models. The costs are that we abstract from the endogeneity of how agents collect, exchange, and digest information and that we bypass the restrictions that the common-prior assumption, potentially in combination with micro-level data, can impose on the size and dynamics of higher-order uncertainty and thereby on the observable implications of the theory.

An ingenious method for characterizing the set of such restrictions that are robust across all information structures is provided by Bergemann and Morris (2013) and Bergemann, Heumann and Morris (2014). Unfortunately, their characterization only applies to static models, providing limited guidance for the kind of dynamic settings we are interested in.

By emphasizing the role of expectations, our paper connects to the literature on news and noise shocks that followed the influential contribution of Beaudry and Portier (2006); see, Christiano et al (2008), Jaimovich and Rebelo (2009), Lorenzoni (2009), and Barsky and Sims (2012). Yet, there exist some important differences. That literature formalizes a form of optimism and pessimism that is tied to signals on future technology and that concerns the medium- to long-run macroeconomic outlook. By contrast, we introduce a form of optimism and pessimism that is disconnected from
technology at any frequency and that concerns the short-run outlook. As we explain in due course, this is key to the distinct predictions of our theory and its superior quantitative performance.

By emphasizing the role of coordination, our paper connects to the voluminous literature on multiple equilibria and sunspot fluctuations. The latter includes seminal contributions such as Benhabib and Farmer (1994), Cass and Shell (1983), and Diamond (1982), as well as more recent work by Benhabib, Wang and Wen (2014) and Farmer (2012). One can view our contribution, along with that of Angeletos and La’O (2013), as a bridge that extends some of the spirit of that literature to the class of unique-equilibrium DSGE models that dominate modern research.

At a broader level, our paper relates to work that relaxes the concept of rational-expectations, such as that on robustness and ambiguity (Hansen and Sargent, 2007, Ilut and Schneider, 2014), learning (Evans and Honkapohja, 2001, Eusepi and Preston, 2011), and educative stability (Guesnerie, 1992); see also Woodford (2013) and the references therein. Although the mechanism we study and the methods we develop are distinct, we share with this literature the desire to enrich the belief dynamics of macroeconomic models.

Putting aside all these methodological connections, what most distinguishes our paper from the extant literature is its applied contribution: the formalization and quantification of a novel type of structural volatility, which can help explain multiple salient features of the business cycle.

3 An RBC Prototype with a tractable form of higher-order beliefs

In this section we set up our baseline model: an RBC prototype, augmented with a tractable form of higher-order belief dynamics. We first describe the physical environment, which is quite standard. We then specify the structure of beliefs, which constitutes the main novelty of our approach.

**Geography, markets, and timing.** There is a continuum of islands, indexed by $i$, and a mainland. Each island is inhabited by a firm and a household, which interact in local labor and capital markets. The firm uses the labor and capital provided by the household to produce a differentiated intermediate good. A centralized market for these goods operates in the mainland, alongside a market for a final good. The latter is produced with the use of the intermediate goods and is itself used for consumption and investment. All markets are competitive.

Time is discrete, indexed by $t \in \{0, 1, \ldots\}$, and each period contains two stages. The labor and capital markets of each island operate in stage 1. At this point, the firm decides how much labor and capital to demand—and, symmetrically, the household decides how much of these inputs to supply—on the basis of incomplete information regarding the concurrent level of economic activity on other islands. In stage 2, the centralized markets for the intermediate and the final goods operate, the actual level of economic activity is publicly revealed, and the households make their consumption and saving decisions on the basis of this information.

**Households.** Consider the household on island $i$. Its preferences are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_{it}, n_{it})$$
where $\beta \in (0, 1)$ is the discount factor, $c_{it}$ denotes consumption, $n_{it}$ denotes employment (hours worked), and $U$ is the per-period utility function. The latter takes the form

$$U(c, n) = \log c - \frac{n^{1+\nu}}{1 + \nu}$$

where $\nu > 0$ is the inverse of the Frisch elasticity of labor supply. The household’s budget constraint is $P_t c_{it} + P_t i_{it} = w_{it} n_{it} + r_{it} k_{it} + \pi_{it}$, where $P_t$ is the price of the final good, $i_{it}$ is investment, $w_{it}$ is the local wage, $r_{it}$ is the local rent on capital, and $\pi_{it}$ is the profit of the local firm. Finally, the law of motion for capital is $k_{i,t+1} = (1 - \delta) k_{it} + i_{it}$, where $\delta \in (0, 1)$ is the depreciation rate.

**Intermediate-good producers.** The output of the firm on island $i$ is given by

$$y_{it} = A_t (n_{it})^{1-\alpha} (k_{it} u_{it})^\alpha$$

where $A_t$ is aggregate TFP, $k_{it}$ is the local capital stock, and $u_{it}$ is utilization. The latter is chosen in stage 2 and entails a cost equal to $\Psi(u_{it}) k_{it}$ units of the final good. The firm’s profit is $\pi_{it} = p_{it} y_{it} - w_{it} n_{it} - r_{it} k_{it} - P_t \Psi(u_{it}) k_{it}$. We let $\Psi(u) = \psi_0 u^{1-\psi}$, with $\psi_0, \psi > 0$ and $\psi \in (0, 1)$.

**Final-good sector.** The final good is produced with a Cobb-Douglas technology. It follows that its quantity is given by

$$\log Y_t = \int_0^1 \log y_{it} \, di$$

and the demand for the island goods satisfies

$$\frac{p_{it}}{P_t} = \frac{Y_t}{y_{it}}. \quad (1)$$

**Technology shocks.** TFP follows a random walk: $\log A_t = \log A_{t-1} + v_t$, where $v_t$ is the period $t$ innovation. The latter is drawn from a Normal distribution with mean 0 and variance $\sigma_a^2$.

A tractable form of higher-order uncertainty. We open the door to strategic uncertainty by removing common knowledge of $A_t$ in stage 1 of period $t$: each island $i$ observes in that stage only a private signal of the form $x_{it} = \log A_t + \varepsilon_{it}$, where $\varepsilon_{it}$ is an island-specific error. We then engineer the desired variation in higher-order beliefs by departing from the common-prior assumption and letting each island believe that the signals of others are biased: for every $i$, the prior of island $i$ is that $\varepsilon_{it} \sim N(0, \sigma^2)$ and that $\varepsilon_{jt} \sim N(\xi_t, \sigma^2)$ for all $j \neq i$, where $\xi_t$ is a random variable that becomes commonly known in stage 1 of period $t$ and that represents the perceived bias in one another’s signals. These priors are commonly known: the agents “agree to disagree”. Finally, we assume that the actual signals are unbiased and focus on the limit case in which $\sigma = 0$.

These modeling choices strike a balance between the need for tractability and the desire to enrich the dynamics of beliefs. Under a common prior, large and persistent divergences between first- and higher-order beliefs are possible only if the former are ridden with idiosyncratic noise and learning is imperfect. As a result, accommodating higher-order uncertainty in dynamic models typically comes at the cost of technical complications, including Kalman filtering and large state

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4We have in mind a sequence of games in which first- and higher-order beliefs converge to Dirac measures as $\sigma \to 0$. But instead of studying the equilibria of these games, we only study the equilibrium of the game with $\sigma = 0$. 
spaces. By contrast, our heterogeneous-prior specification allows for higher-order beliefs to diverge from first-order beliefs even when the noise in the latter vanishes. The variable $\xi_t$ then permits us to engineer the desirable aggregate belief dynamics, while letting $\sigma \to 0$ guarantees that agents act in equilibrium as if they were perfectly informed about the state of the economy, thus abstracting from noisy learning and bypassing the aforementioned technical difficulties.

We close the model by assuming that $\xi_t$ follows an $AR(1)$ process:

$$\xi_t = \rho \xi_{t-1} + \zeta_t,$$

where $\rho \in (0,1)$ and $\zeta_t$ is drawn from a Normal distribution with mean 0 and variance $\sigma^2_{\xi}$. This specification is in line with standard DSGE practice. More importantly for our purposes, it helps capture within our framework the basic idea that learning is likely to reduce strategic uncertainty over time—and therefore that the belief waves we are after are likely to be short-lived.

**Remark 1.** Although we model $\xi_t$ as an exogenous shock, we think of it as a proxy of a deeper mechanism from which we must abstract in order to make progress in understanding its consequences. As already explained, this mechanism regards the uncertainty that firms and consumers face about one another’s choices, and thereby about the short-term outlook of the economy due to frictions in coordination. In richer settings, the nature of these frictions and the resulting uncertainty can be endogenous to the network of markets or to other social interactions; to the details of how agents collect, digest, and exchange information; and to a variety of forces that influence the formation of expectations about the actions of others. In this paper, we take for granted variation in this type of expectations, which we henceforth interpret as variation in the level of “confidence” about the state of the economy, and focus on gauging its quantitative implications. In so doing, we help formalize and quantify a certain type of waves of optimism and pessimism that can be thought of as a symptom of frictions in coordination, but remain agnostic about the deeper determinants and the triggers of these waves.

**Remark 2.** In general, adding heterogeneous priors to a game entails the introduction of systematic biases in the beliefs that the players form about the actions of the others and therefore about the outcomes of the game. In our model, this boils down to the following property: although the firms and consumers predict correctly the sign of the impact of the shock $\xi_t$ on aggregate output and all other macroeconomic outcomes, they tend to overestimate its magnitude.

We do not wish to take this property literally. Common-prior settings such as those studied in Angeletos and La’O (2013), Benhabib, Wang and Wen (2014), Nimark (2013), Huo and Takayama (2014), and Rodina and Walker (2013), can accommodate similar waves in higher-order beliefs and in equilibrium outcomes as our approach, without invoking a systematic bias in beliefs. In effect, what is “bias” in our approach becomes “rational confusion” in those settings.

To illustrate this, consider the baseline setting of Angeletos and La’O (2013). This is essentially a variant of our model that abstracts from capital, adds island-specific productivity shocks, and assumes that trade takes place in a decentralized fashion: in the end of each period, islands are
randomly matched into pairs, and their households consume a good that is a composite of only the goods produced by the two islands within the match. Because of the absence of capital, this model reduces in effect to a static game between the different islands.

Consider next two possible specifications of the information structure. In the one, which is similar to the specification used in the present paper, each island receives a single private signal of the productivity of her trading partner, but the islands have heterogeneous priors about the noises of these signals. In the other, which extends the specification considered in Angeletos and La’O (2013), the islands share a common prior, but are allowed to observe arbitrary Gaussian signals about one another’s productivity and signals.

In the Appendix we show that the joint distribution of productivity, output, employment, and consumption—as well as of the forecasts of these variables—that is predicted by the model under the second specification can always be replicated under the first specification. The converse is also true, but only insofar as the perceived bias is small enough.

This reveals the two key characteristics of our formulation. On the one hand, our formulation can be viewed merely as a convenient short-cut for the fluctuations sustained in common-prior incomplete-information model. On the other hand, our formulation does away with some of the restrictions that the common-prior assumption can impose on the magnitude of the belief waves we are engineering.

Accordingly, we wish to invite the following broader, but complementary, interpretation of our approach: we are departing from the standard rational-expectations solution concept in order to accommodate a certain kind of “mistakes” in the beliefs that agents form about one another’s actions, but do not necessarily wish to take a stand on whether these mistakes are “fully” rational.

These mistakes introduce stochastic deviations between the observable outcomes of our model and those of the standard RBC model. Our analysis takes the existence of such deviations for granted and, in this sense, it only assumes the presence of an additional source of volatility. Nonetheless, unlike the case, say, of arbitrary trembles or measurement error, these deviations are not entirely free. Instead, they are disciplined by two requirements. First, all the additional variation in the observables of the model is spanned by the variation in beliefs, because there is no change in the payoff structure (preferences, technology, resource constraints, etc) of the model. And second, the beliefs agents form about all kinds of economic outcomes (output, employment, consumption, investment, wages, interest rates, etc) and at all horizons (current quarter, next quarter, 5 years later on, etc) are ultimately anchored to a particular form of higher-order uncertainty.

The fluctuations we engineer in this paper are therefore disciplined by certain “cross-equation restrictions”, which themselves manifest as certain co-movement patterns in the observables of the model. This property is akin to any other form of structural volatility: in the absence of direct measures of the underlying structural shocks, the “testability” of macroeconomic models rests on such cross-equation restrictions. Accordingly, the applied contribution of our paper rests on spelling out the observable patterns of the particular kind of belief-related volatility we have introduced, and on contrasting them with those of other structural shocks that are popular in the literature.
4 Equilibrium characterization and solution method

In this section we develop a recursive representation of the equilibrium. This serves two goals. First, it clarifies how the belief enrichment we propose in this paper enters the equilibrium determination and the restrictions this entails on the observables of the theory. Second, it illustrates the logic behind the solution method developed in Appendix E, which contains our broader methodological contribution and facilitates the quantitative evaluations in the rest of the paper. To simplify the exposition, we momentarily abstract from utilization and, without any loss, normalize $P_t = 1$.

**Recursive equilibrium.** Our formulation implies that an island’s hierarchy of beliefs is pinned down by two objects: the local signal, which itself coincides with $A_t$, the true TFP shock; and the perceived bias in the signals of others, which is given by $\xi_t$. This indicates that, relative to the standard RBC model, the state space of our model has been extended only by the $\xi_t$ variable. We thus study a recursive equilibrium in which the aggregate state vector is given by $(A, \xi, K)$.

To this goal, we first note that the equilibrium allocations of any given island can be obtained by solving the problem of a fictitious local planner. The latter chooses local employment, output, consumption and savings so as maximize local welfare subject to the following resource constraint:

$$c_{it} + k_{i,t+1} = (1 - \delta)k_{it} + p_{it}y_{it}$$

(2)

Note that this constraint depends on $p_{it}$, and thereby on the economy’s aggregate output, both of which are taken as given by the fictitious local planner. This dependence represents the type of aggregate-demand externalities that is at the core of many modern DSGE models.

To make his optimal decisions, the aforementioned planner must form beliefs about the evolution of $p_{it}$ (or of $Y_t$) over time. These beliefs encapsulate the beliefs that the local firm forms about the evolution of the demand for its product and of the costs of its inputs, as well as the beliefs that the local consumer forms about the dynamics of local income, wages, and capital returns. The fact that all these kinds of beliefs are tied together underscores the cross-equation restrictions that discipline the belief enrichment we are considering in this paper: if expectations were “completely” irrational, the beliefs of different endogenous objects would not have to be tied together. The observable implications of this kind of restrictions, both in our baseline model and in richer DSGE models, are discussed below. For now, we emphasize that $\xi_t$ matters for equilibrium outcomes because, and only because, it triggers co-movement in this kind of expectations.

Finally, note that beliefs of $p_{it}$ (or of $Y_t$) are themselves tied to beliefs of how the aggregate economy evolves over time. In a recursive equilibrium, the latter kind of beliefs are encapsulated in the law of motion of the aggregate capital stock, the endogenous state variable. We thus define a recursive equilibrium as a collection of functions $P$, $G$, $V_1$, and $V_2$, such that the following is true:

- $P(x, \xi, K)$ gives the price expected by an island in stage 1 of any given period when the local signal is $x$, the confidence shock is $\xi$, and the capital stock is $K$; and $G(A, \xi, K)$ gives the aggregate capital stock next period when the current realized value of the aggregate state is $(A, \xi, K)$;
- $V_1$ and $V_2$ solve the following Bellman equations:

\[
\begin{align*}
V_1(k; x, \xi, K) &= \max_n V_2(\hat{m}; x, \xi, K) - \frac{1}{1+\nu}n^{1+\nu} \\
\text{s.t.} \quad \hat{m} &= \hat{y} + (1-\delta)k \\
\hat{y} &= xk^{\alpha}n^{1-\alpha} \\
\hat{p} &= P(x, \xi, K) \\
V_2(m; A, \xi, K) &= \max_{\{c, k\}^\prime} \log \frac{c}{c^\prime} + \beta \int V_1(k^\prime; A^\prime, \xi^\prime, K^\prime)\, \text{d}f(A^\prime, \xi^\prime | A, \xi) \\
\text{s.t.} \quad c + k^\prime &= m \\
K^\prime &= G(A, \xi, K)
\end{align*}
\] (3) (4)

- $P$ and $G$ are consistent with the policy rules that solve the local planning problem in (3)-(4).

To interpret (3) and (4), note that $V_1$ and $V_2$ denote the local planner’s value functions in, respectively, stages 1 and 2 of each period; $m$ denotes the quantity of the final good that the island holds in stage 2; and the hat symbol over a variable indicates the stage-1 belief of that variable. Next, note that the last constraint in (3) embeds the island’s belief that the price of the local good is governed by the function $P$, while the other two constraints embed the production function and the fact that the quantity of the final good that the island holds in stage 2 is pinned down by the sales of the local good plus the non-depreciated part of the local capital. The problem in (3) therefore describes the optimal employment and output choices in stage 1, when the local capital stock is $k$, the local signal of the aggregate state is $(x, \xi, K)$, and the local beliefs of “aggregate demand” are captured by the function $P$. The problem in (4), in turn, describes the optimal consumption and saving decisions in stage 2, when the available quantity of the final good is $m$, the realized aggregate state is $(A, \xi, K)$, and the island expects aggregate capital to follow the policy rule $G$.

The decision problem of the local planner treats the functions $P$ and $G$ as exogenous. In equilibrium, however, these functions must be consistent with the policy rules that solve this problem. To spell out what this means, let $u(k; x, \xi, K)$ be the optimal choice for employment that obtains from (3) and $g(m; A, \xi, K)$ be the optimal policy rule for capital that obtains from (4). Next, let $y(x; A, \xi, K) \equiv An(x, \xi, K)^{1-\alpha}K^\alpha$ be the output level that results from the aforementioned employment strategy where the realized TFP is $A$ and the local capital stock coincides with the aggregate one. The relevant equilibrium-consistency conditions can then be expressed as follows:

\[
P(x, \xi, K) = \frac{y(x + \xi, x, \xi, K)}{y(x, x, \xi, K)}
\] (5)

\[
G(A, \xi, K) = g \left( y(A, A, \xi, K) + (1-\delta)K^{1-\alpha} \right) : A, \xi, K
\] (6)

To interpret condition (5), recall that, in stage 1, each island believes that, with probability one, TFP satisfies $A = x$ and the signals of all other islands satisfy $x' = A + \xi = x + \xi$. Together with the fact that all islands make the same choices in equilibrium and that the function $y$ captures their equilibrium production choices, this implies that the local beliefs of local and aggregate output are
given by, respectively, \( \hat{y} = y(x, x, \xi, K) \) and \( \hat{Y} = y(x + \xi, x, \xi, K) \). By the demand function in (1), it then follows that the local belief of the price must satisfy \( \hat{p} = \hat{Y}/\hat{y} \), which gives condition (5). To interpret condition (6), on the other hand, recall that all islands end up making identical choices in equilibrium, implying that the available resources of each island in stage 2 coincide with \( Y + (1 - \delta)K \), where \( Y \) is the aggregate quantity of the final good (aggregate GDP). Note next that the realized production level of all islands is given by \( y(A, A, \xi, K) \) and, therefore, \( Y \) is also given by \( y(A, A, \xi, K) \). Together with the fact that \( g \) is the optimal savings rule, this gives condition (6).

Summing up, an equilibrium is given by a fixed point between the Bellman equations (3)-(4) and the consistency conditions (5)-(6). In principle, one can obtain the global, non-linear solution of this fixed-point problem with numerical methods. As in most of the DSGE literature, however, we find it useful to concentrate on the log-linear approximation of the solution around the steady state. Once we do this, we obtain the equilibrium dynamics of our model as a tractable transformation of the equilibrium dynamics of the standard RBC model.

**Log-linear solution.** The linearized equilibrium dynamics of our model satisfy the following:

\[
(\tilde{Y}_t, \tilde{N}_t, \tilde{I}_t; \tilde{K}_{t+1}) = \Gamma_K \tilde{K}_t + \Gamma_A \tilde{A}_t + \Gamma_\xi \xi_t \tag{7}
\]

where the tilde symbol indicates the log-deviation of a variable from its steady-state value and where \( \Gamma_K, \Gamma_A \) and \( \Gamma_\xi \) are vectors that are pinned down by exogenous parameters and that regulate the model’s impulse response functions. Importantly, \( \Gamma_K \) and \( \Gamma_A \) are the same as those in the standard RBC model, whereas \( \Gamma_\xi \) is obtained by solving an equation that contains \( \Gamma_K \) and \( \Gamma_A \).

In Appendix E, we explain how the type of solution obtained in (7) generalizes to a large class of DSGE models. This facilitates the simulation, calibration, and estimation of the type of belief-augmented macroeconomic models we are interested in as in the case of workhorse DSGE models. The results developed in that appendix are therefore instrumental for the broader methodological contribution of our paper, as well as for the quantitative exercises we conduct next.

**Remark.** The fact that \( \Gamma_\xi \) solves an equation that itself depends on \( \Gamma_K \) and \( \Gamma_A \) underscores a more general principle: the dynamic effects of higher-order beliefs in any given model are tightly connected to the payoff-relevant shocks and the propagation mechanisms that are embedded in the complete-information version of that model. This in turn explains our choice to focus on higher-order uncertainty of TFP as opposed to higher-order uncertainty of, say, discount rates: the former has a better chance to generate realistic waves of optimism and pessimism within the RBC framework, because TFP shocks in the first place do a better job in generating realistic business cycles than discount-rate shocks.

### 5 Quantitative evaluation

Notwithstanding the theoretical motivation behind our modification of the RBC prototype, all that one ultimately sees in (7) is a model with two structural shocks. From this perspective, our applied
contribution ultimately rests on assessing the observable properties of the confidence shock and on contrasting them with other structural shocks proposed in the literature. In this section, we take a first pass at this task by studying the empirical fit of a calibrated version of our baseline model.

Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Role</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Rate</td>
<td>0.99</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Inverse Elasticity of Labor Supply</td>
<td>0.50</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital Share in Production</td>
<td>0.30</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>0.015</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse Elasticity of Utilization</td>
<td>0.30</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>St.Dev. of Technology Shock</td>
<td>0.67</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>St.Dev. of Confidence Shock</td>
<td>2.65</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence of Confidence Shock</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Calibration.** Table 1 reports the values we select for the parameters of the model. The preference and technology parameters of the model are set at conventional values: the discount factor is 0.99; the Frisch elasticity of labor supply is 2; the capital share in production is 0.3; the elasticity of utilization is 3; and the depreciation rate is 0.015. These values guarantee that our quantitative exercise is directly comparable to the literature as well as that the steady state values of our model are consistent with the long-run patterns in the data.

Since the technology shock follows a random walk, three parameters remain to be set in order to complete the parameterization of the model: the standard deviations of the two shocks and the persistence of the confidence shock. For the latter, we set $\rho = 0.75$. This choice is somewhat arbitrary, but it is motivated by the following considerations. In our setting, $\rho$ pins down the persistence of the deviations between first- and higher-order beliefs. In common-prior settings, such deviations cannot last for ever, but can be quite persistent insofar as learning is slow. By setting $\rho = 0.75$, we assume that the half-life of these deviations is less than 2.5 quarters, which does not sound implausibly large. Furthermore, to the extent that the fluctuations induced by $\xi_t$ in our model resembles either the “demand shock” identified in Blanchard and Quah (1989) or the “main business cycle shock” identified in our companion paper (a property that remains to be seen), our parameterization of $\rho$ is broadly consistent with the evidence in those paper.

Turning to $\sigma_a$ and $\sigma_\xi$, the standard deviations of the two shocks, we set them so as to minimize the distance between the volatilities of output, consumption, investment and hours found in the data and those generated by the model. This yields $\sigma_a = 0.67$ and $\sigma_\xi = 2.64$. Clearly, there is no compelling empirical justification for this parameterization. Furthermore, the relatively high value for $\sigma_\xi$ begs the question of whether the belief fluctuations we accommodate in this paper are

---

5By “volatilities” we always refer to the Bandpass-filtered variances at frequencies corresponding to 6-32 quarters. Also, in the minimization objective, each of the model-based volatilities is weighted by the precision of its estimator.
perhaps too large to be reconcilable with the common-prior assumption. We revisit this issue in Section 6 and expand on it in Appendix D. Notwithstanding these points, we believe that the chosen calibration strategy is useful because it facilitates the comparison of our mechanism to competing structural mechanisms in the literature. Furthermore, although the predicted magnitudes are of course sensitive to the chosen values for $\sigma_a$ and $\sigma_\xi$, the shapes of the IRFs and hence the predicted co-movement patterns are entirely invariant to these values.

The effects of the confidence shock. Figure 1 reports the IRFs of the model’s key variables to a positive innovation in $\xi_t$. As is evident from this figure, an increase in “confidence” is associated with a transitory boom in output, consumption, hours, and investment. At the same time, labor productivity stays nearly constant, falling a bit in the beginning and increasing a bit later on.

Figure 1: Impulse responses to a positive confidence shock

These co-movement patterns encapsulate key testable predictions of our theory. Importantly, these patterns are not easily shared by other structural mechanisms in the literature. We next elaborate on the economic forces that shape these patterns and that explain the difference from alternative theories.

In our setting, a positive innovation in $\xi_t$ signals a transitory boom in aggregate output and, in this sense, captures optimism about aggregate demand in the short run. In response to this particular kind of optimism, firms find it optimal to raise their demand for both labor and capital, which in turn pushes up wages and capital returns. Other things being equal, this motivates the households to supply more labor as well as to invest more, because of the familiar static and intertemporal substitution effects. A countervailing wealth effect is also at work, because the household experiences a boom in its income. However, because the boom is transitory, the wealth effect is weak relative to the substitution effects, guaranteeing that households raise the labor supply and split their additional income between consumption and saving. All in all, our mechanism therefore induces employment, output, consumption and investment to move in tandem.

The IRFs to the technology shock are the same as in the standard RBC model and are thus omitted. Also note that the shape of the IRFs, and therefore the co-movement regularities we document for the confidence shock do not depend on the values of $\sigma_a$ and $\sigma_\xi$. These values matter only when we compute the model’s second moments.

---

6The IRFs to the technology shock are the same as in the standard RBC model and are thus omitted. Also note that the shape of the IRFs, and therefore the co-movement regularities we document for the confidence shock do not depend on the values of $\sigma_a$ and $\sigma_\xi$. These values matter only when we compute the model’s second moments.
Alternative mechanisms. Let us contrast the aforementioned patterns with those generated by “news shocks”, that is, by signals of future shifts in technology and productivity. The literature has used such shocks to capture the idea that optimism about the future growth prospects of the economy could lead to a boom in the present. This idea, however, does not square well with the canonical RBC framework. In this framework, positive news or noise shocks raise the consumers' expectations of “permanent income”. In response to this, consumers raise their demand, not only for goods, but also for leisure. In general equilibrium, this leads to a drop in hours, output, and investment, and therefore to negative co-movement between consumption and any of these variables.

Jaimovich and Rebelo (2006) seek to overturn this negative co-movement by considering two modifications of the baseline RBC model: a particular form of internal habit, which implies that the anticipation of higher consumption in the future increases the supply of labor today for any given wage; and an adjustment cost in investment, which makes the investment today increase in anticipation of higher investment in the future. The first feature, which amounts in effect to a perverse short-run income effect on labor supply, helps undo the negative response in employment, while the second features helps undo the negative response of investment. Lorenzoni (2009), on the other hand, proposes a resolution within the NK framework. The resolution relies on a monetary policy that “accommodates” consumer optimism, in the sense of letting the shift in expectations of permanent income induce pro-cyclical deviations from the underlying flexible-price allocations; and on shutting down, or dampening enough, the countervailing behavior of the latter. 7

Consider, next, the case of discount-rate shocks. Such shocks are often deployed in the literature in order to account for the impact of financial constraints on consumer spending. The story goes as follows: tighter credit leads consumers to cut down on their spending, which reduces “aggregate demand” and triggers employment losses. We believe that neither the RBC nor the NK model can capture this story in a satisfactory way. In the RBC model, discount-rate shocks make investment and hours move in the opposite direction from consumption: as resources are freed up by the drop in consumption, investment increases; and as a higher discount factor makes the consumers more eager to work, employment also goes up. This negative co-movement may be mitigated in NK settings via the combination of nominal rigidity, pro-cyclical output gaps, and certain forms of adjustment costs in investment and consumption. Nevertheless, as illustrated in Section 6, discount-rate shock are still unable either to generate realistic co-movement or to capture a significant fraction of the business-cycle volatility in an estimated NK that embeds these features. In contrast, our mechanism has no difficulty in generating realistic business-cycle patterns. It can therefore also help deliver

7Lorenzoni’s model assumes away capital, guaranteeing that employment and output move mechanically in the same direction as consumption. As illustrated in Appendix C, this is not an innocuous simplification: signals of future TFP cause consumption to move in the opposite direction from employment, output and investment in a calibrated version of the textbook NK model with capital. This is because the accommodating role of monetary policy is not sufficiently strong to offset the countervailing dynamics of the underlying flexible-price allocations once investment is free to adjust. Blanchard et al (2013) seek to fix this problem by adding significant adjustment costs to investment and assuming a sufficiently accommodative monetary-policy response. See Barsky and Sims (2011) for a complementary discussion.
the results attributed to discount rate factor shocks within both the RBC and the NK models as long as the drop in consumer spending arises from, or comes with, a drop in “confidence”.

Finally, consider the recent work on uncertainty shocks. The class of linear models we study in this paper cannot accommodate this kind of shocks. Nevertheless, it is worth noting that existing formalizations of the macroeconomic effects of these shocks, such as those in Bloom (2009) and Bloom et al (2013), appear to hinge on strong pro-cyclical movements in aggregate TFP, a property that is not shared by our mechanism.

In short, in addition to generating an interesting narrative about the business cycle, this paper contributes to identifying a structural mechanism whose ability to capture salient features of the macroeconomic data does not appear to be shared by competing mechanisms. We provide additional support to this claim in the sequel, in Appendix C, and in Section 6.

Business-Cycle Moments. Complementing the preceding discussion, we now evaluate our model’s ability to match standard business-cycle moments. Table 2 reports some key moments in the data (column 1), in our model (column 2), and in four competing models (columns 3-6). Each of the competing models replaces the confidence shock with one of the following alternative structural shocks, which have been considered in the literature: a news shock; a discount-rate shock; an investment-specific shock; and a transitory TFP shock.

Our model does a very good job in matching the relevant moments in the data. The only shortcoming is that it underestimates the correlations of consumption with output and hours. As explained in detail in Appendix B, the overall fit owes to a delicate balance between the contribution of the two structural shocks: if we shut down either shock, the model will fail to match moments well. But once the two shocks are combined, all the moments fall in place, as if by magic.

Importantly, this “magic” is not a trivial consequence of adding a second shock to the RBC prototype: none of the competing two-shock models is able to replicate the empirical fit of our model. By trying to attribute the variation in the data to structural shocks that do not exhibit the appropriate co-movement patterns, these models end up missing, not only certain key correlations, but also the relative volatilities of certain variables.

Appendix C shows that a similar property characterizes baseline versions of the NK framework. We interpret this as a further indication of the superior ability of the structural mechanism we propose in this paper to capture salient features of the data.

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8 A refined version of the narrative above is that financial frictions depress the demand, not only for consumption, but also for investment. This version, which seems plausible in the context of the Great Recession, is easier to reconcile with the NK model: a combination of a discount-rate and an investment-specific shock can generate a realistic recession, even if each one of the shocks by itself would not. Note, however, that the recession would have to come with a commensurate deflation episode, a prediction that seems prima-facie inconsistent with the US experience. For a possible resolution that adds a countervailing inflationary shock, see Christiano et al (2014).

9 To ensure a fair ponyrace, the parameterization of the competing models is in line with that of our model; see Appendix C for details. Also, the moments are computed on Bandpass-filtered series, at frequencies 6-32 quarters. This filter is preferable to the simpler HP filter because it removes not only low-frequency trends but also high-frequency “noise” such as seasonal fluctuations and measurement error; see Stock and Watson (1999).
Table 2: Bandpass-filtered Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Our Model</th>
<th>TFP</th>
<th>Invt</th>
<th>Disc</th>
<th>News</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stddev($y$)</td>
<td>1.42</td>
<td>1.42</td>
<td>1.54</td>
<td>1.16</td>
<td>1.16</td>
<td>1.32</td>
</tr>
<tr>
<td>stddev($h$)</td>
<td>1.56</td>
<td>1.52</td>
<td>1.04</td>
<td>1.05</td>
<td>1.05</td>
<td>1.09</td>
</tr>
<tr>
<td>stddev($c$)</td>
<td>0.76</td>
<td>0.76</td>
<td>0.73</td>
<td>0.95</td>
<td>0.95</td>
<td>0.82</td>
</tr>
<tr>
<td>stddev($i$)</td>
<td>5.43</td>
<td>5.66</td>
<td>6.76</td>
<td>6.94</td>
<td>6.94</td>
<td>7.15</td>
</tr>
<tr>
<td><strong>Correlations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr($c, y$)</td>
<td>0.85</td>
<td>0.77</td>
<td>0.68</td>
<td>0.22</td>
<td>0.22</td>
<td>0.30</td>
</tr>
<tr>
<td>corr($i, y$)</td>
<td>0.94</td>
<td>0.92</td>
<td>0.93</td>
<td>0.79</td>
<td>0.79</td>
<td>0.86</td>
</tr>
<tr>
<td>corr($h, y$)</td>
<td>0.88</td>
<td>0.85</td>
<td>0.94</td>
<td>0.79</td>
<td>0.79</td>
<td>0.88</td>
</tr>
<tr>
<td>corr($c, h$)</td>
<td>0.84</td>
<td>0.34</td>
<td>0.39</td>
<td>-0.41</td>
<td>-0.41</td>
<td>-0.17</td>
</tr>
<tr>
<td>corr($i, h$)</td>
<td>0.82</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>corr($c, i$)</td>
<td>0.74</td>
<td>0.47</td>
<td>0.38</td>
<td>-0.42</td>
<td>-0.42</td>
<td>-0.21</td>
</tr>
<tr>
<td><strong>Correlations with productivity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr($y, y/h$)</td>
<td>0.08</td>
<td>0.15</td>
<td>0.84</td>
<td>0.44</td>
<td>0.45</td>
<td>0.56</td>
</tr>
<tr>
<td>corr($h, y/h$)</td>
<td>-0.41</td>
<td>-0.37</td>
<td>0.60</td>
<td>-0.18</td>
<td>-0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>corr($y, sr$)</td>
<td>0.82</td>
<td>0.85</td>
<td>0.98</td>
<td>0.90</td>
<td>0.90</td>
<td>0.93</td>
</tr>
<tr>
<td>corr($h, sr$)</td>
<td>0.47</td>
<td>0.47</td>
<td>0.86</td>
<td>0.47</td>
<td>0.47</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: The first column reports moments in the US data, bandpass-filtered, over the 1960-2007 period. The second column reports the moments in our model. The rest of the table reports the moments in the four competing models discussed in the text. Red color indicates a significant difference between a model’s predicted moment and the corresponding moment in the data.
Output gaps, markups, and aggregate demand. Within NK models, the notion of fluctuations in “aggregate demand” has been tied to deviations from the model’s underlying flexible-price allocations. These deviations manifest as variation in markups and in measured “output gaps”, and come together with commensurate movements in inflation.

In our setting, there are no nominal rigidities. Nevertheless, because firms make their input choices prior to observing the demand for their products, a drop in confidence manifests as an increase in the realized markup. Furthermore, the resulting recession will register as a negative “output gap” insofar as the latter is measured relative to the underlying RBC benchmark.

In this regard, the notion of aggregate demand formalized here with the help of higher-order uncertainty has a flavor and empirical content that are similar as those in the NK framework. There is, however, an important difference: in our setting, the fluctuations in the “output gap” may arise without any movements in inflation. Our formalization therefore bypasses the “inflation puzzles” of the NK framework and may help explain, inter alia, why the severe contraction in output and employment during the recent recession were not accompanied by severe deflation.\textsuperscript{10} We discuss additional distinguishing aspects of our formalization in Section 7.

6 Extension and estimation

The preceding analysis showed that the inclusion of particular form of waves of optimism and pessimism in an RBC prototype helps produce a parsimonious account of multiple salient features of the data, most notably the co-movement patterns documented in Section 2. In this section, we extend the analysis to a pair of richer, “medium scale”, DSGE models, which we estimate on US data.

The two models differ with regard to the existence of of nominal rigidity, but are similar in that they accommodate a multitude of structural shocks. They therefore permit us to address the following questions. Does our mechanism account for a significant fraction of the observed business-cycle volatility once multiple other structural forces are allowed to drive the business cycle? And how does our formalization of “aggregate demand” compare to the one already embedded in the NK framework?

In what follows, we first describe briefly the features and estimation of these two models. We then review the key results and provide the answers to the questions raised above. Many of the details as well as several additional results are delegated to Appendix D.

The two models. The two models we study in this section share the same backbone as our baseline model, but add a number of competing structural shocks, along with habit persistence in consumption, adjustment costs in investment, and, in one of two models, nominal rigidity.

\textsuperscript{10} For a discussion of the “inflation puzzles” faced by NK models and an alternative resolution, see Beaudry and Portier (2013). That paper develops a formalization of non-monetary “demand shocks” that rests on the interaction of incomplete markets and news shocks. The quantitative potential of this formalization remains unexplored.
To accommodate price-setting behavior, we now let each island contain a large number of monopolistic firms, each of which produces a differentiated commodity. These commodities are combined through a CES aggregator into an island-specific composite good, which in turn enters the production of the final good in the mainland. In one of the two models we study in this section, firms are free to adjust their price in each and every period, after observing the realized demand for their product; we refer to this model as the flexible-price model. In the other model, firms can instead adjust prices only infrequently, in the familiar Calvo fashion; we refer to this model as the sticky-price model and we close it by adding a conventional Taylor rule for monetary policy.

To accommodate the possibility that the business cycle is explained by multiple structural forces, and to let these forces compete with our mechanism, we consider the following shocks in addition to our confidence shock: a permanent and a transitory TFP shock; a permanent and a transitory investment-specific shock; a news shock regarding future productivity; a transitory discount-rate shock; a government-spending shock; and, in the sticky-price model, a monetary shock.

This menu of shocks is motivated by various considerations. First, previous research has argued that investment-specific technology shocks are at least as important as neutral, TFP shocks (Fischer, 2006). Second, as already noted, news shocks are obvious competitors to our mechanism and have been on center stage in recent business-cycle research. Third, these kinds of shocks, as well as monetary, fiscal, and transitory discount-rate or investment-specific shocks, have been proposed as formalizations of the notion of “aggregate demand shocks” within the NK framework. Fourth, transitory TFP, investment-specific, or discount-rate shocks are often used as proxies for financial frictions that lead to, respectively, misallocation, a wedge in the firm’s investment decisions, or a wedge in the consumer’s saving decisions. Fifth, the introduction of multiple transitory shocks, whatever their interpretation, maximizes the chance that these shocks, rather than our confidence shock, will pick up the transitory fluctuations in the data. All in all, although the menu we have considered does not exhaust all the shocks that have appeared in the literature, we believe it permits us to embed a variety of mechanisms that seem a priori plausible and have also been found to be quantitatively significant in prior structural estimations.

Finally, we allow for standard modeling features such as adjustment costs in investment (IAC) and habit persistence in consumption (HP) popularized by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). These modeling devices lack compelling micro-foundations, but have played an important dual role in the DSGE literature: as sources of persistence; and as mechanisms that help improve the empirical performance of certain shocks, including monetary, investment-specific, discount-rate, and news shocks. Our preceding analysis has already established that these features are not needed for our mechanism to deliver realistic fluctuations. Here, we incorporate them in order to keep our exercise as close as possible to standard DSGE practice, as well as to give a better chance to the aforementioned competing shocks to outperform the confidence shock.

\[11\] See Christiano et al (2014) for a recent example of using these shocks as proxies for financial shocks, and Buera and Moll (2012) for a careful analysis of how different types of financial frictions map to different wedges.
Estimation. We estimate our models with Bayesian maximum likelihood in the frequency domain, as in Christiano and Vigfusson (2002) and Sala (2013); see Appendix D for details. The advantage of this method, relative to estimation in the time domain, is that it guides the estimation of a model on the basis of its performance at frequencies that correspond to business-cycle phenomena (between 6 and 32 quarters) as opposed to medium- or long-run trends. The underlying justification is that the model is indeed designed to address business-cycle phenomena; to put it differently, the researcher is relatively more concerned about mispecification in the medium- to low-frequencies. A side-benefit then is that this approach dispenses with some ad-hoc features that other works (e.g., Smets and Wouters, 2007) need to assume in order to accommodate the observed low-frequency movements in inflation and hours.

The data used in the estimation include GDP, consumption, investment, hours worked, the inflation rate, and the federal fund rate for the period 1960Q1 to 2007Q4. Our sticky-price model is estimated on the basis of all these six variables. By contrast, our flexible-price model is estimated on the basis of real quantities only (GDP, consumption, investment, and hours). The rationale is that this kind of model is not designed to capture the properties of nominal data. Nonetheless, obtaining predictions about nominal variables in this model may serve some purposes. To allow this we augment the estimated flexible-price model with a simple monetary policy rule that stabilizes inflation but exclude the exogenous random monetary disturbance.

The priors used in the estimation are reported in Tables 9 and 10 in Appendix D. The priors for the preference, technology, and monetary parameters are in line with the pertinent literature. The priors for the shocks are guided by two principles: first, we do not a priori favor the confidence shock; and second, we impose a tight prior only on the persistence of the transitory shocks, in order to help the estimation disentangle them from the permanent shocks.

Posterior distributions were obtained with the MCMC algorithm. The resulting estimates of the parameters are reported in the aforementioned tables. The estimated values of the preference, technology, and monetary parameters are close to previous estimates in the literature. As for the shock parameters, the following point is worth making. To the extent that the $\xi_t$ shock represents higher-order uncertainty, its estimated size should not be unreasonably high relatively to the estimated size of the payoff-relevant uncertainty. As we discuss in Appendix D, this does not appear to be a serious problem: the level of strategic uncertainty we have accommodated in our estimations with the help of a heterogeneous-prior formulation does not appear to be implausibly large, especially so in the sticky-price model. Having said this, it would certainly be desirable to further discipline the type of quantitative exercises we introduce in this paper by taking specific stands

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12 The first four variables are in logs and linearly de-trended. Following Justiniano, Primiceri and Tambalotti (2010), we do not include the price of investment in our data so as to accommodate both supply-side and demand-side interpretations of either the permanent or the transitory investment shock.

13 This policy is equivalent to allowing the intercept in the Taylor rule to track the underlying natural rate. Furthermore, when this rule is appended to the sticky-price model, the allocations in the latter coincide with those in the flexible-price model. Hence, one can readily re-interpret our flexible-price model as the sticky-price model augmented with the aforementioned policy rule instead of the more standard Taylor rule, whose intercept is a constant.
on the micro-structure of how agents update their beliefs and interact with one another and/or by utilizing survey evidence on expectations of economic activity. We leave this to future research.

**The confidence shock.** Figure 2 reports the estimated IRFs to a positive confidence shock. As far as real quantities are concerned, the IRFs are similar across the two models, as well as similar to those in our baseline model. The introduction of investment-adjustment costs and consumption habit adds a hump-shaped property, but does not alter the co-movement patterns found in the baseline model. This underscores the robustness of the key positive implications our mechanism as we move between RBC and NK settings, or as we add the aforementioned modeling ingredients.

Figure 2: Theoretical IRFs to Confidence Shock

![Figure 2: Theoretical IRFs to Confidence Shock](image)

What differs, however, is the behavior of inflation and interest rates. In response to the confidence shock, as well as to other shocks, the flexible-price model predicts implausibly large movements in the real interest rate, due to the inclusion of the particular types of investment-adjustment costs and habit persistence. This compromises the model’s performance vis-a-vis inflation and interest rates. By contrast, because nominal rigidity permits the actual real interest to deviate from its natural level, the sticky-price model is able to accommodate simultaneously modest movements in both the real and the nominal interest rate, as well as in inflation.

Table 3: Variance Contribution of Confidence Shock (6–32 Quarters)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>C</th>
<th>I</th>
<th>h</th>
<th>π</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible P</td>
<td>50.98</td>
<td>43.72</td>
<td>54.63</td>
<td>76.04</td>
<td>0.00</td>
<td>99.15</td>
</tr>
<tr>
<td>Sticky P</td>
<td>47.73</td>
<td>40.89</td>
<td>44.24</td>
<td>65.66</td>
<td>11.95</td>
<td>32.64</td>
</tr>
</tbody>
</table>

Table 3 turns to the estimated contribution of the confidence shock to the volatility of the key macroeconomic variables at business-cycle frequencies (6–32 quarters). Despite all the competing shocks, the confidence shock emerges as the single most important source of volatility in real

\[14\] Since we have augmented our flexible-price model with a monetary policy that stabilizes inflation, the large volatility of the real rate manifests fully in the nominal rate. But even if we had assumed a different monetary policy, the model would not manage to produce modest movements in both inflation and the nominal interest rate.
quantities. For example, the confidence shock accounts for 51% of the business-cycle volatility in output in the flexible-price model, and for 48% in the sticky-price model.

Table 4 completes the picture by looking at the estimated contribution of the confidence shock to the covariances of output, hours, investment, and consumption. The confidence shock is, by a significant margin, the main driving force behind the co-movement of all these variables.

<table>
<thead>
<tr>
<th></th>
<th>Cov(Y, h)</th>
<th>Cov(Y, I)</th>
<th>Cov(Y, C)</th>
<th>Cov(h, I)</th>
<th>Cov(h, C)</th>
<th>Cov(I, C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible Price</td>
<td>75.80</td>
<td>60.06</td>
<td>56.34</td>
<td>75.67</td>
<td>96.53</td>
<td>84.75</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>68.53</td>
<td>53.23</td>
<td>58.40</td>
<td>62.64</td>
<td>106.30</td>
<td>107.41</td>
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</table>

These findings are not driven by the priors in the estimation: the variance contribution of the confidence shock at the priors is less than 3% for output in either model. Rather, what seems to explain these findings is, first, that the data favor a mechanism that triggers strong procyclical movements in hours, investment, and consumption without commensurate movements in labor productivity, TFP, inflation, and interest rates and, second, that our mechanism is well positioned to generate this kind of co-movement within workhorse macroeconomic models.

The other shocks. To economize on space, the estimated role of the other shocks is reported in Appendix D. One findings is nevertheless worth reporting. The co-movement implications of investment-specific, discount-rate, and news shocks change substantially depending on whether we allow or shut down the particular forms of adjustment costs to investment and habit persistence in consumption that are popular in the NK literature. This explains why the estimated contribution of these shocks, whether in our own models or in the existing literature (e.g., Justiniano et al, 2010, Blanchard et al, 2014), depends heavily on the inclusion of these modeling features. By contrast, neither the co-movement implications of the confidence shock nor its estimated contribution are unduly sensitive to the inclusion or exclusion of these modeling ingredients, as well as to the presumed degree of nominal rigidity—a kind of robustness that we view as an advantage of our formalization of the notion of fluctuations in aggregate demand.

Business-cycle moments. We now evaluate the empirical fit of our estimated models with regard to business-cycle moments. Table 5 reports some key moments of the data (first column); those predicted by our estimated models (second and third column); and, for comparison purposes, those predicted by the model in Smets and Wouters (2007) (fourth column) and our own baseline model (last column). Inspection of this table leads to the following conclusions.

First, both of the estimated models do a good job on the real side of the data. Perhaps this is not surprising given that our baseline model had attained a good fit under a more rigid theoretical structure. But this only underscores the good empirical performance of our mechanism.

Second, in comparison to Smets and Wouters (2007), our sticky-price model does a good job in
Table 5: Band-pass Filtered Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>FP</th>
<th>SP</th>
<th>SW</th>
<th>Baseline</th>
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<td><strong>Standard Deviations</strong></td>
<td></td>
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<tr>
<td>$y$</td>
<td>1.42</td>
<td>1.46</td>
<td>1.43</td>
<td>1.42</td>
<td>1.42</td>
</tr>
<tr>
<td>$i$</td>
<td>5.43</td>
<td>5.12</td>
<td>5.64</td>
<td>4.86</td>
<td>5.66</td>
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<tr>
<td>$h$</td>
<td>1.56</td>
<td>1.73</td>
<td>1.87</td>
<td>0.97</td>
<td>1.52</td>
</tr>
<tr>
<td>$c$</td>
<td>0.76</td>
<td>0.92</td>
<td>0.91</td>
<td>1.11</td>
<td>0.76</td>
</tr>
<tr>
<td>$y/h$</td>
<td>0.75</td>
<td>0.97</td>
<td>1.08</td>
<td>0.84</td>
<td>0.79</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.23</td>
<td>0.22</td>
<td>0.27</td>
<td>0.34</td>
<td>–</td>
</tr>
<tr>
<td>$R$</td>
<td>0.35</td>
<td>6.42</td>
<td>0.36</td>
<td>0.35</td>
<td>–</td>
</tr>
<tr>
<td><strong>Correlations with Output</strong></td>
<td></td>
<td></td>
<td></td>
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<td>$\pi$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: FP and SP: our estimated flexible- and sticky-price models, SW: the model in Smets and Wouters (2007), Baseline: the calibrated RBC prototype studied in Section 5.

matching, not only the real, but also the nominal side of the data. In this regard, the inclusion of our mechanism in NK models does not seem to interfere with their ability to match the nominal side of the data. Nonetheless, as we elaborate below, it does call into question the structural interpretation that existing versions of these models offer for the observed recessions.

**Shocks vs empirical proxies.** To further corroborate our theory, we now consider the following question: do the technology and confidence shocks in our theory have any resemblance to empirical measures of, respectively, TFP and “market psychology” in the real world?

We address this question in Figure 3 by comparing the estimated series of the technology and the confidence shocks in our models with, respectively, the series of Fernald’s (2014) utilization-adjusted TFP measure and the University of Michigan Consumer Sentiment Index. Even though we did not use any information on these two empirical measures in the estimation of the models, the theoretical shocks turn to be highly correlated with their empirical counterparts. The same property holds if we replace the Michigan Sentiment Index with the Conference Board’s Indices of Consumer or Producer Confidence, because these indices are highly correlated at business-cycle frequencies. Notwithstanding the inherent difficulty of interpreting such indices and of mapping them to the theory, we view this finding as providing additional validation to our mechanism and its interpretation as variation in “confidence”.

22
7 Nominal rigidities, aggregate demand, and the US recessions

We conclude our analysis by asking the following question: what explains the apparent deficiency in aggregate demand during recessions?

To address this question, one must first define the “potential” level of economic activity, relative to which the aforementioned “deficiency” is to be measured. Our approach shares similarities with the NK model regarding what constitutes natural output: it is the level of economic activity predicted by the underlying RBC benchmark and in the absence of from both nominal rigidities and imperfections in beliefs. The two approaches differ, however, with regard to the friction that creates “output gaps”: in the NK model, the key friction is a nominal rigidity; in our approach, it is a flaw in the agents’ coordination of their beliefs and actions.

The two approaches are not mutually inconsistent. Both types of frictions may be important for understanding business cycles. Furthermore, the NK mechanism appears to complement our mechanism by enhancing the impact of higher-order beliefs in the following manner: although the estimated contribution of our mechanism to the volatility of macroeconomic outcomes is similar across the two models studied in the previous section, the estimated variance of the confidence shock is much smaller in the sticky-price model than that in the flexible-price model.

Notwithstanding this point, we now evaluate the relative significance of the two mechanisms, when each one operates in isolation. We do this by computing the posterior odds that the data are generated by either of three possible models: the variant of our flexible-price model that shuts down the confidence shock; the variant of our sticky-price model that also shuts down the confidence shock; and finally the flexible-price model that contains the confidence shock. The first model serves as a benchmark, capturing the RBC core of the other two models. The second model adds a nominal rigidity to the first model, isolating the NK mechanism. The third model replaces the nominal rigidity with a confidence shock, isolating our proposed mechanism. To assess their fit vis-a-vis real economic activity, all three models are estimated on the basis of real quantities only. We then compute the posterior odds that the data are generated by the sticky-price model rather
than either of the flexible-price models, starting with a uniform prior over the three models. These odds provide a metric of how well a given model captures the data relative to another model.\[^15\]

Consider first the pair-wise comparison between the sticky-price model and the standard flexible-price model. In this case, the sticky-price model wins: the posterior odds that the data are generated by that model are 90%. Consider next how this comparison is affected once the flexible-price model is augmented with the confidence shock. The odds are now completely reversed: the probability that the data are generated by the sticky-price model are only 2%. By this metric, nominal rigidity is essential for the ability of the theory to match the real data when our mechanism is absent, but not once it is present: our mechanism appears to be more potent than the NK mechanism when their relative performance is evaluated in terms of likelihood, as described above.

We interpret these results as providing further confirmation, not only of our mechanism’s ability to capture salient aspects of the data, but also to operationalize the notion of fluctuations in aggregate demand. Our approach can serve as an alternative to the NK approach—but it can also complement it by providing what, in our view, is a more appealing structural interpretation of the observed business-cycle phenomena.

To illustrate this last point, we now take a closer look at the structural interpretation of the US recessions offered by two alternative models: our preferred version of the NK model, which contains the confidence shock; and the “canonical” NK model of Smets and Wouters (2007). Note that both models give a prominent role to nominal rigidity, but differ in the way they formalize and quantify the fluctuations in aggregate demand.

In each of these models, we ask the following counterfactual: how would the historical recessions have looked if nominal rigidities were shut down?\[^16\] Figure 4 answers this question in terms of the dynamics of output: the red dotted lines in the figure give the counterfactual path of output in our model; the blue dashed lines give the counterfactual path in the model of Smets and Wouters (2007); the black solid lines give the actual data. Note that, by construction, both models match perfectly the actual data when the nominal rigidity is at work. The gap between the aforementioned counterfactuals and the actual data therefore reveals the precise role that nominal rigidity plays within each model.

In our version of the NK model, which contains the confidence shock, recessions look qualitatively similar whether the nominal rigidity is shut down or not. In particular, our model predicts that the nominal rigidity exacerbated most of the recessions, which seems consistent with some economists’ priors, but attributes the bulk of the recessions to forces that would have remained potent even if the nominal rigidity were absent. By contrast, the model of Smets and Wouters

\[^15\]See Table 13 in Appendix C for a tabulation of the posterior odds of the aforementioned three models along with those of the model that combines our mechanism with the NK mechanism.

\[^16\]More precisely, the counterfactual is constructed as follows. For each of the two models, we fix the estimated paths of all the structural shocks, as well as of all the estimated preference and technology parameters. We also maintain the estimated nominal parameters until the onset of the recession under consideration. We then compute the counterfactual path of output that obtains when the nominal rigidity is shut down from that point on.
(2007) attributes the recessions to forces that rely on nominal rigidity so heavily that most of the recessions are predicted to turn into booms when the nominal rigidity is shut down.

There is no obvious way to test these counterfactuals, or to know which model is “right”. We nevertheless find it hard to accept a structural interpretation of the business cycle that hinges on the prediction that the observed recessions would turn into booms in the absence of nominal rigidities. Instead, we favor an interpretation that attributes a role to “coordination failures” and “lack of confidence”, while also accommodating a meaningful role for monetary policy.

8 Conclusion

By relying on a particular solution concept together with complete information, standard macroeconomic models impose a rigid structure on how agents form beliefs about endogenous economic outcomes and how they coordinate their actions. In this paper, by contrast, we introduced a certain relaxation of this structure and evaluated its quantitative implications. In particular, we augmented DSGE models with a tractable from of higher-order belief dynamics that proxies the aggregate effects of strategic uncertainty and captures a certain kind of waves of optimism and pessimism about the short-term outlook of the economy. We believe that this adds to our understanding of business-cycle phenomena along the following dimensions:

- It offers a parsimonious explanation of salient features of the macroeconomic data.
- It appears to outperform alternative structural mechanisms that are popular in the literature.
• It offers a potent formalization of the notion of fluctuations in “aggregate demand” that can serve as either an alternative or a complement to the NK formalization of this notion.

• It leads to a structural interpretation of the observed recessions that is not unduly sensitive to the degree of nominal rigidity (whose bite at the aggregate level remains debatable) and that attributes a potentially significant role to forces that can be interpreted as “coordination failures” or “lack of confidence”.

These findings naturally raise the question of where the drop in confidence during a recession, or more generally the waves of optimism and pessimism in the agents’ beliefs about one another’s actions come from. Having treated the “confidence shock” as exogenous, we can not offer a meaningful answer to this question. This limitation, however, is not specific to what we do in this paper: any formal model must ultimately attribute the business cycle to some exogenous trigger, whether this is a technology shock, a discount-rate shock, a financial shock, or even a sunspot. Therefore, although we remain agnostic about the “micro-foundations” of the aforementioned belief waves, we hope to have provided a useful gauge of their potential quantitative importance along with a re-evaluation of prevailing theories of the business cycle.

We think that our results set a useful reference point, which could invite future research either in the direction of further quantitative evaluations of the macroeconomic effects of strategic uncertainty, or in the direction of developing novel structural interpretations of the business-cycle data.
APPENDICES

A. Data

In this appendix we describe the data we use in this paper to (i) obtain the various moments we use to assess the empirical relevance of our models, and (ii) to estimate the extended versions in Section 6.

The data is from the Saint–Louis Federal Reserve Economic Database. The sample ranges from the first quarter of 1960 to the last quarter of 2007. We dropped the post-2007 data because the models we study are not designed to deal with the financial phenomena that appear to have played a more crucial role in the recent recession as opposed to earlier times. All quantities are expressed in real, per capita terms—that is, deflated by the implicit GDP deflator (GDPDEF) and by the civilian non-institutional population (CNP16OV). Because the latter is reported monthly, we used the last month of each quarter as the quarterly observation.

Table 6 summarizes information about the data. GDP, Y, is measured by the seasonally adjusted GDP. Consumption, C, is measured by the sum of personal consumption expenditures in nondurables goods (CND) and services (CS). Investment, I, is measured by the sum of personal consumption expenditures on durables goods (CD), fixed private investment (FPI) and changes in inventories (DI). Government Spending, G, is measured by government consumption expenditures (GCE). Hours worked, H, are measured by hours of all persons in the nonfarm business sector. Labor productivity, Y/H, is measured by Real Output Per Hour of All Persons in the nonfarm business sector. The inflation rate, \( \pi \), is the log-change in the implicit GDP deflator. The nominal interest rate, R, is the effective federal funds rate measured on a quarterly basis. Given that the effective federal funds rate is available at the monthly frequency, we use the average over the quarter (denoted FEDFUNDS).

The relative price of investment was built in the same way as in Benati (2014), and we thank him for his help in doing so. This involves constructing a chained price index for the sum of gross private investment and consumption of durables, along with a chained price index for the sum of consumption of non-durables and services. The relative price of investment is then given by the ratio of these two quantities. For a detailed description, see \url{http://economics.mit.edu/files/10307}.

B. An Anatomy of our Baseline Model

Here we elaborate on the distinct quantitative roles that the technology and the confidence shock play in our RBC prototype. To this goal, Table 7 reports the business-cycle moments predicted by the versions of our model that shut down either of the two shocks (see the last two columns) and compares them to those in the data (first column) and in our full model (second column).

By isolating the role of the technology shock, the third column in this table revisits, in effect, the baseline RBC model. The most noticeable, and well known, failures of this model are its inability to
### Table 6: Description of the Data

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<th>Data</th>
<th>Formula</th>
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<td>GDP</td>
<td>( Y = \frac{\text{GDP}}{\text{GDPDEF} \times \text{CNP16OV}} )</td>
</tr>
<tr>
<td>Consumption</td>
<td>( C = \frac{(\text{CND}+\text{CS})}{\text{GDPDEF} \times \text{CNP16OV}} )</td>
</tr>
<tr>
<td>Investment</td>
<td>( I = \frac{(\text{CD}+\text{FPI}+\text{DI})}{\text{GDPDEF} \times \text{CNP16OV}} )</td>
</tr>
<tr>
<td>Government Spending</td>
<td>( G = \frac{\text{GCE}}{\text{GDPDEF} \times \text{CNP16OV}} )</td>
</tr>
<tr>
<td>Hours Worked</td>
<td>( H = \frac{\text{HOANBS}}{\text{CNP16OV}} )</td>
</tr>
<tr>
<td>Labor Productivity</td>
<td>( \frac{\text{GDP}}{H} )</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>( \pi = \log(\text{GDPDEF}) - \log(\text{GDPDEF})_{-1} )</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>( R = \frac{\text{FEDFUNDS}}{4} )</td>
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<table>
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Table 7: Bandpass-filtered Moments

<table>
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<th>Our Model</th>
<th>A only</th>
<th>ξ only</th>
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<tr>
<td>stddev(y)</td>
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<td>1.42</td>
<td>1.46</td>
<td>1.18</td>
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<tr>
<td>stddev(h)</td>
<td>1.56</td>
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<td>0.49</td>
<td>1.70</td>
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<td>stddev(c)</td>
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<td>0.76</td>
<td>1.13</td>
<td>0.14</td>
</tr>
<tr>
<td>stddev(i)</td>
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<td>5.66</td>
<td>3.01</td>
<td>6.08</td>
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<td>0.77</td>
<td>0.99</td>
<td>0.84</td>
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<td>corr(i, y)</td>
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<td>0.92</td>
<td>0.99</td>
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<tr>
<td>corr(c, h)</td>
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<td>0.80</td>
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<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
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<tr>
<td>corr(c, i)</td>
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<td>0.47</td>
<td>0.99</td>
<td>0.81</td>
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<td><strong>Correlations with productivity</strong></td>
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<tr>
<td>corr(y, y/h)</td>
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<td>0.15</td>
<td>0.99</td>
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<td>0.99</td>
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<tr>
<td>corr(y, sr)</td>
<td>0.82</td>
<td>0.85</td>
<td>0.99</td>
<td>0.98</td>
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<tr>
<td>corr(h, sr)</td>
<td>0.47</td>
<td>0.47</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

generate a sufficiently high volatility in hours; its prediction of a counterfactually strong correlation between hours and either labor productivity or the Solow residual. An additional failure is that the model generates a counterfactually low volatility in investment.

Consider now the fourth column, which isolates the confidence shock. The key failures are now the counterfactually high volatility in hours, the perfectly negative correlation between labor productivity and either hours or output, and the counterfactually low volatility in consumption. The first two properties follow directly from the fact that technology is fixed and exhibits diminishing returns in labor, while the last property is driven by the transitory nature of the confidence shock.

To sum up, neither the standard RBC mechanism nor our mechanism deliver a good fit when working in isolation. But once they work together, the fit is great.

C. Competing Structural Shocks: A Ponyrace

This appendix studies the “ponyrace” introduced in Section 5 when we compared the empirical fit of our baseline model to that obtained in other two-shock models.

Recall that the competing models were variants of our baseline model that replace the confidence shocks with one of the following: a transitory technology shock; a transitory investment-specific shock; a transitory discount-factor shocks; or a news shocks about future TFP. Also recall that the

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17 This version is similar to the model studied in Section 7 of Angeletos and La’O (2013).
analysis was limited to an RBC model. Here, we complement that analysis in two ways: we revisit the ponyrace in the context of a NK model; and we add a monetary shock to the set of shocks contained in the ponyrace.

The NK extension of our analysis is standard and is described in the beginning of Section 6. For the calibrated versions considered in this appendix, the markup rate is set to 15%; the Calvo probability of resetting prices is set so that the average length of an unchanged price is 4 quarters; and monetary policy is assumed to follow a Taylor rule with coefficient on inflation of 1.5 and on output of 0.05, and a degree of interest rate smoothing of 0.8. For illustration purposes, we also consider the alternative case in which the monetary authority completely stabilizes the nominal interest rate. The preference and technology parameters remain the same as before. Likewise the standard deviations of the shocks are obtained by minimizing the weighted distance between the volatility of output, consumption, investment and hours between the data and the model, assuming the same persistence for each alternative shock as that of the confidence shock — with the exception of the monetary shock which has persistence 0.15 which lies within the range of values usually found in the literature (see for example Smets and Wouters, 2007).

Table 8 reports the same kind of information as Table 2, that is, the business-cycle moments of the different models in the ponyrace, but now for the NK versions of these models. Figure 5 reports the IRFs for both the RBC and the NK versions of the models.

Four key lessons emerge. First, the superior empirical performance of our baseline model survives when we introduce sticky prices. Second, with the exception of the monetary shock, none of the aforementioned competing structural shocks is able to generate positive co-movement patterns in the real data whether one considers the RBC or the NK version of the models. Third, the monetary shock can match these patterns quite well, but only at the expense of requiring an implausibly large contribution of monetary shocks to the overall business cycle and of having counterfactual movements on the nominal side (strongly counter-cyclical interest rates and strongly pro-cyclical inflation). Finally, the similarity between the real effects of the confidence shock and those of the monetary shock provide further justification for reinterpreting the confidence shock as some short of “aggregate demand shock”.

These lessons are not unduly sensitive the parameterizations chosen. They survive when we move to estimated versions of richer RBC and NK models that allow multiple structural shocks to coexist and that also introduce two propagation mechanism that have played a crucial role in the DSGE literature, namely investment-adjustment costs and habit. See the discussion in Section 6 and especially the estimated IRFs in Figures 6 and 7 in Appendix D.
<table>
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<tr>
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<th>Data</th>
<th>Our Model</th>
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<th>Disc</th>
<th>News</th>
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<td>0.92</td>
<td>0.92</td>
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<td>0.85</td>
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<td>0.02</td>
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<tr>
<td>$\rho_{\pi, y}$</td>
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<td>$\rho_{R, y}$</td>
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<td>0.69</td>
<td>0.89</td>
<td>0.75</td>
<td>0.61</td>
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Figure 5: Ponyrace IRFs (I)

(a) Confidence Shock

(b) Technology Shock

(c) Investment-Specific Shock

(d) Discount-Factor Shock

(e) News Shock

Real Model; NK Model with Taylor Rule; NK Model with Constant Interest Rate
D. Estimated Models

In this appendix we discuss the estimation of the multi-shock RBC and NK models considered in Section 6. In particular, we first fill in the formal details of the two models; we next explain the estimation method, the priors assumed for the parameters, and the posteriors obtained from the estimation; we finally review a number of findings that were omitted, or only briefly discussed, in the main text.

The details of the two models. As mentioned in the main text, the two models we study in Section 6 share the same backbone as our baseline model, but add a number of structural shocks along with certain forms of habit persistent in consumption and adjustment costs in investment, as in Christiano et al. (2005) and Smets and Wouters (2007). To accommodate monopoly power and sticky prices, we also introduce product differentiation within each island.

Fix an island $i$. Index the firms in this island by $j \in [0,1]$ and let $y_{ijt}$ denote the output produced by firm $j$ in period $t$. The composite output of the island given by the following CES aggregate:

$$y_{it} = \left( \int_0^1 y^{1+\eta}_{ijt} \, dj \right)^{1+\eta},$$

where $\eta > 0$ is a parameter that pins down the monopoly power. The technology is the same as before, so that the output of firm $j$ in island $i$ is

$$y_{ijt} = A_t(u_{ijt}k_{ijt})^{\alpha}h_{ijt}^{1-\alpha};$$

but now TFP is given by the sum of two permanent components, one corresponding to a standard unanticipated innovation and another corresponding to a news shock, plus a temporary shock. More specifically,

$$\log A_t = a_t^\tau + a_t^p,$$

where $a_t^\tau$ is the transitory component, modeled as an AR(1), and $a_t^p$ is the sum of the aforementioned two permanent components, namely,

$$a_t^p = a_{t-1}^p + \varepsilon_t^p + \zeta_t^n$$

where $\varepsilon_t^p$ is the unanticipated innovation and $\zeta_t^n$ captures all the TFP changes that we anticipated in earlier periods. The latter is given by a diffusion-like process of the form

$$\zeta_t^n = \frac{1}{8} \sum_{j=1}^{8} \varepsilon_{t-j}^n,$$

where $\varepsilon_{t-j}^n$ is the component of the period-$t$ innovation in TFP that becomes known in period $t-j$.\footnote{We have experimented with alternative forms of diffusion, as well as with specifications such as $\zeta_t^n = \varepsilon_t^{n-4}$, and we have found very similar results.}

In line with our baseline model, the confidence shock is now modeled as a shock to higher-order beliefs of $a_t^p$. 

To accommodate for a form of habit in consumption as well as discount-rate shocks, we let the per-period utility be as follows:

$$u(c_{it}, n_{it}; \zeta^c_t, C_{t-1}) = \exp(\zeta^c_t) \left( \log(c_{it} - bC_{t-1}) + \theta \frac{n_{it}^{1+\nu}}{1+\nu} \right)$$

where $\zeta^c_t$ is a transitory preference shock, modeled as an AR(1), $b \in (0, 1)$ is a parameter that controls for the degree of habit persistence, and $C_{t-1}$ denotes the aggregate consumption in the last period.\(^{19}\)

To accommodate permanent shocks to the relative price of investment, we let the resource constraint of the island be given by the following:

$$c_{it} + \exp(Z_t) i_{it} + G_t + \exp(Z_t) \Psi(u_{it}) k_{it} = p_{ityt}$$

where $Z_t$ denotes the relative price of investment, $G_t$ denotes government spending (the cost of which is assumed to equally spread across the islands), and $\exp(Z_t) \Psi(u_{it})$ denotes the resource cost of utilization per unit of capital. The latter is scaled by $\exp(Z_t)$ in order to transformed the units of capital to units of the final good, and thereby also guaranteed a balanced-growth path. $Z_t$ is modeled as a random walk: $Z_t = Z_{t-1} + \varepsilon_t$. Literally taken, this represents an investment-specific technology shock. But since our estimations do not include data on the relative price of invest, this shock can readily be re-interpreted as a demand-side shock. Government spending is given by $G_t = \bar{G} \exp(\tilde{G}_t)$, where $\bar{G}$ is a constant and

$$\tilde{G}_t = \zeta^g_t + \frac{1}{1-\alpha} a_t - \frac{\alpha}{1-\alpha} Z_t.$$

In the above, $\zeta^g_t$ denotes a transitory shock, modeled as an AR(1), and the other terms are present in order to guarantee a balanced-growth path. The utilization-cost function satisfies $u \Psi''(u)/\Psi'(u) = \frac{\psi}{1-\psi}$, with $\psi \in (0, 1)$.

Finally, to accommodate adjustment costs to investment as well as transitory investment-specific shocks, we let the law of motion of capital on island $i$ take the following following form:

$$k_{it+1} = \exp(\zeta^i_t) i_{it} \left( 1 - \Phi \left( \frac{i_{it}}{i_{it-1}} \right) \right) + (1-\delta)k_{it}$$

We impose $\Phi'(\cdot) > 0$, $\Phi''(\cdot) > 0$, $\Phi(1) = \Phi'(1) = 0$, and $\Phi''(1) = \varphi$, so that $\varphi$ parameterizes the curvature of the adjustment cost to investment. $\zeta^i_t$ is a temporary shock, modeled as an AR(1) and shifting the demand for investment, as in Justiniano et al (2010).

The above description completes the specification of the flexible-price model of Section 6. The sticky-price model is then obtained by embedding the Calvo friction and a Taylor rule form monetary policy. In particular, the probability that any given firm resets its price in any given period is given by $1 - \chi$, with $\chi \in (0, 1)$. As for the Taylor rule, the reaction to inflation is given by $\kappa_\pi > 1$.

\(^{19}\)Note that we are assuming that habit is external. We experimented with internal habit, as in Christiano et al (2007), and the results were virtually unaffected.
the reaction to the output gap is given by $\kappa_y > 0$, and the parameter that controls the degree of interest-rate smoothing is given by $\kappa_R \in (0, 1)$; see condition (18) below.

In the sticky-price model, the log-linear version of the set of the equations characterizing the general equilibrium of the economy is thus given by the following:

$$\lambda_t = \zeta^e - \frac{1}{1 - b} \tilde{c}_{it} + \frac{b}{1 - b} \tilde{C}_{t-1}$$

$$\zeta^e + \nu \tilde{n}_{it} = \mathbb{E}_{it}[\lambda_{it} + \tilde{s}_{it} + \tilde{Y}_t - \tilde{n}_{it}]$$

$$Z_t + \frac{1}{1 - \psi} \tilde{u}_{it} = \tilde{s}_{it} + \tilde{Y}_t - \tilde{k}_t$$

$$\tilde{y}_{it} = a_t + \alpha(\tilde{u}_{it} + \tilde{k}_it) + (1 - \alpha)\tilde{n}_{it}$$

$$\tilde{Y}_t = s_c \tilde{c}_{it} + (1 - s_c - s_g)(Z_t + \tilde{u}_{it}) + \tilde{G}_t + \alpha \tilde{u}_{it}$$

$$\tilde{k}_{it+1} = \delta \tilde{s}_{it} + (1 - \delta)\tilde{k}_it$$

$$\tilde{q}_{it} = (1 + \beta) \tilde{\varphi} \tilde{n}_{it} - \varphi \tilde{n}_{it-1} - \beta \varphi \mathbb{E}_{it} \tilde{n}_{it+1} + Z_t - \zeta^i_t$$

$$\tilde{R}_t = \tilde{\lambda}_{it} - \mathbb{E}_{it}[\tilde{\lambda}_{it+1} - \tilde{\pi}_{it+1}]$$

$$\tilde{\lambda}_{it} + \tilde{q}_{it} = \mathbb{E}_{it}' \left[ \tilde{\lambda}_{it+1} + (1 - \beta(1 - \delta)) \left( \tilde{s}_{it+1} + \tilde{Y}_{it+1} - \tilde{u}_{it+1} - \tilde{k}_{it+1} \right) + \beta(1 - \delta) \tilde{q}_{it+1} \right]$$

$$\tilde{X}_t = s_c \tilde{c}_t + (1 - s_c - s_g)(Z_t + \tilde{I}_t) + s_g \tilde{G}_t$$

$$\tilde{R}_t = \kappa_R \tilde{R}_{t-1} + (1 - \kappa_R) \left( \kappa_x \tilde{\pi}_t + \kappa_y \left( \tilde{X}_t - \tilde{X}^F_t \right) \right) + \zeta^m_t$$

$$\tilde{\pi}_{it} = (1 - \chi)(1 - \beta \chi) \mathbb{E}_{it} \tilde{s}_{it} + (1 - \chi) \mathbb{E}_{it} \tilde{P}_t + \beta \chi \mathbb{E}_{it} \tilde{\pi}_{it+1}$$

where uppercases stand for aggregate variables, $\lambda_{it}$ and $s_{it}$ denote, respectively, the marginal utility of consumption and the realized markup in island $i$, $\tilde{\pi}_{it} \equiv \tilde{p}_{it} - \tilde{p}_{it-1}$ and $\tilde{N}_t \equiv \tilde{P}_t - \tilde{P}_{t-1}$ denote, respectively, the local and the aggregate inflation rate, $X_t$ denotes the measured aggregate GDP, $X^F_t$ denotes the GDP that would be attained in a flexible price allocation, and $s_c$ and $s_i$ denote the steady-state ratios of consumption and government spending to output.

The interpretation of the above system is familiar. Condition (8) gives the marginal utility of consumption. Conditions (9) and (10) characterizes the equilibrium employment and utilization levels. Condition (11) and (12) give the local output and the local resource constraint. Conditions (13) and (14) give the local law of motion of capital and the equilibrium investment decision. Conditions (15) and (16) are the two Euler equations: the first corresponds to optimal bond holdings and gives the relation between consumption growth and the interest rates, while the second corresponds to optimal capital accumulation and gives the evolution of Tobin’s Q. Condition (17) gives the measured aggregate GDP. Conditions (18) gives the Taylor rule for monetary policy. Finally, condition (19) gives the inflation rate in each island; aggregating this condition across islands gives our model’s New Keynesian Phillips Curve. The only essential novelty in all the above is the presence of the subjective expectation operators in the conditions characterizing the local equilibrium outcomes of each island.

Finally, the flexible-price allocations are obtained by the same set of equations, modulo the following changes: we set $s_{it} = 0$, meaning that the realized markup is always equal to the optimal
markup; we restate the Euler condition (15) in terms of the real interest rate; and we drop the nominal side of this system, namely conditions (18) and (19).

**Estimation.** As mentioned in the main text, we estimate the model on the frequency domain. This method amounts to maximizing the following posterior likelihood function:

\[ L(\theta|Y_T) \propto f(\theta) \times L(\theta|Y_T) \]

where \( Y_T \) denotes the set of data (for \( t = 1 \ldots T \)) used for estimation, \( \theta \) is the vector of structural parameters to be estimated, \( f(\theta) \) is the joint prior distribution of the structural parameters, and \( L(\theta|Y_t) \) is the likelihood of the model expressed in the frequency domain. Note that the log-linear solution of the model admits a state-space representation of the following form:

\[
Y_t = M_y(\theta)X_t \\
X_{t+1} = M_x(\theta)X_t + M_e\varepsilon_{t+1}
\]

Here, \( Y_t \) and \( X_t \) denote, respectively, the vector of observed variables and the underlying state vector of the model; \( \varepsilon \) is the vector of the exogenous structural shocks, drawn from a Normal distribution with mean zero and variance-covariance matrix \( \Sigma(\theta) \); \( M_y(\theta) \) and \( M_x(\theta) \) are matrices whose elements are (non-linear) functions of the underlying structural parameters \( \theta \); and finally \( M_e \) is a selection matrix that describes how each of the structural shocks impacts on the state vector. The model spectral density of the the vector \( Y_t \) is

\[ S_Y(\omega, \theta) = \frac{1}{2\pi}M_y(\theta)(I - M_x(\theta)e^{-i\omega})^{-1}M_e \Sigma(\theta) M'_e(I - M_x(\theta)'e^{i\omega})^{-1}M_y(\theta)\]

where \( \omega \in [0, 2\pi] \) denotes the frequency at which the spectral density is evaluated. The likelihood function is asymptotically given by

\[ \log(L(\theta|Y_T)) \propto -\frac{1}{2} \sum_{j=1}^{T} \gamma_j \left( \log(\det S_Y(\omega_j, \theta)) + \text{tr} \left( S_Y(\omega_j, \theta)^{-1} I_Y(\omega_j) \right) \right) \]

where \( \omega_j = 2\pi j/T, j = 1 \ldots T \) and where \( I_Y(\omega_j) \) denotes the periodogram of \( Y_T \) evaluated at frequency \( \omega_j \). Following Christiano and Vigfusson (2002) and Sala (2013), we include a weight \( \gamma_j \) in the computation of the likelihood in order to select the desirable frequencies: this weight is 1 when the frequency falls between 6 and 32 quarters, and 0 otherwise.

**Parameters: priors.** The first three columns in Tables 9 and 10 report the priors used in the estimation of the parameters of the two models. The logic behind our choice of priors for the shock processes was discussed in the main text; we now briefly discuss the rest of the priors.

The inverse labor supply elasticity, \( \nu \), is Gamma distributed around 0.5 with standard deviation 0.25. The capital share, \( \alpha \), is Beta distributed around 0.3 with standard deviation 0.05. The utilization elasticity parameter, \( \psi \), is Beta distributed around 0.3 with standard deviation 0.25. The parameter governing the size of investment adjustment costs, \( \varphi \), is Gamma distributed around
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<td>Normal</td>
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<td>0.050</td>
<td>–</td>
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Note: 95% HPDI into brackets.
Table 10: Estimated Parameters, Part II

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<td><strong>Shocks: volatilities</strong></td>
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<td>4.000</td>
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<td>0.586</td>
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<td>[0.274, 1.021]</td>
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</table>

Note: 95% HPDI into brackets.
2 with a standard deviation 1; the relatively high standard deviation reflects our own uncertainty about this modeling feature, but also allows the estimation to accommodate the higher point estimates required by the pertinent DSGE literature, in case that would improve the empirical performance of the model. The Calvo probability of not resetting prices, $\chi$, is Beta distributed around 0.5 with standard deviation 0.25. The persistence parameter in the Taylor rule, $\kappa_R$, is Beta distributed around 0.75 with standard deviation 0.1; the reaction coefficient on inflation, $\kappa_\pi$, is Normally distributed around 1.5 with standard deviation 0.25; and the reaction coefficient on the output gap, $\kappa_y$, is also Normally distributed with mean 0.125 and standard deviation 0.05. Finally, the following three parameters are fixed: the discount factor $\beta$ is 0.99; the depreciation rate $\delta$ is 0.015; and the CES parameter $\eta$ is such that the monopoly markup is 15%.

**Parameters: posteriors.** Posterior distributions were obtained with the MCMC algorithm. We generated 2 chains of 100,000 observations each. The posteriors for all the parameters of our two models are reported in the last two columns of Tables 9 and 10. The posteriors for the preference, technology, and monetary parameters are broadly consistent with other estimates in the literature. Below, we find it useful only to comment on the estimated size of $\sigma_\xi$, the standard deviation of the innovation in the confidence shock.

*The estimated $\sigma_\xi$. To the extent that we want to think of the confidence shock as a proxy for the type of higher-order uncertainty that can obtain in common-prior settings, one would like to have a theorem that provides a tight bound on the size of this higher-order uncertainty as a function of the underlying payoff uncertainty. By comparing the estimated value of $\sigma_\xi$ to that bound, we could then judge the empirical plausibility of the presumed level of strategic uncertainty even if we don’t know anything about the details of the underlying information structure.*

In static settings, such a bound can be obtained with the methods developed in Bergemann and Morris (2013) and Bergemann, Haumman and Morris (2014). Unfortunately, analogous methods are not available for dynamic settings: one would like to have bounds on the volatility generated by higher-order uncertainty at different frequencies, but it is unclear how to obtain such bounds.

Having said this, the following seems a reasonable guess. Due to the persistence of the shocks and the forward-looking aspects of our model, it is not appropriate to measure the relative magnitude of different shocks by simply comparing the standard deviations of the corresponding innovations. Instead, some kind of present-value metric seems desirable.

In want of a better alternative, we propose the following rough metric. Let $F_t^T$ denote the typical agent’s first-order belief of the present value of TFP from period $t$ to period $t + T$, discounted by $\beta$, and evaluated conditional on the information available at the end of period $t - 1$; let $S_t^T$ denote the corresponding second-order belief; and let $B_t^T \equiv S_t^T - F_t^T$ denote the difference between the two, which obtains only because of the $\xi_t$ shock. We can think of the variance of $F_t^T$ as a measure of the TFP uncertainty faced by the agent, as of period $t - 1$ and over the next $T$ periods; of the variance of $B_t^T$ as a measure of the corresponding higher-order uncertainty; and of the ratio of the latter to the former as a metric of their relative importance. For the flexible-price model, this ratio
turns out to be 0.26 when $T = 8$ years and 0.07 when $T = \infty$. For the sticky-price model, the corresponding ratios are are 0.08 and 0.02. Finally, these ratios would be even lower if we were to take into account the other shocks in the model (investment-specific, fiscal, monetary, etc). This explains the metric by which we view the estimated higher-order uncertainty as modest relative to the estimated payoff uncertainty.

**IRFs.** Figures 6–7 report the IRFs of our estimated models with respect to all the structural shocks. In the main text, we mentioned that the inclusion of investment adjustment costs (IAC) and habit (HP) plays a crucial role in the existing DSGE literature, but has only a modest effect on the performance of our own mechanism. To illustrate this point, Figures 6–7 report the IRFs of the various shocks both in the versions of our models that include these propagation mechanisms and in those that shut them down.

**Variance/Covariance Decompositions.** Tables 11 and 12 report the estimated contribution of the shocks to, respectively, the variances and the co-variances of the key variables at business-cycle frequencies. (The confidence shock is omitted here, because its contributions were reported in the main text.) For comparison purposes, we also include the estimated contributions that obtain in the variants of the models that remove the confidence shock. Three findings are worth mentioning:

First, unlike the case of the confidence shock, the variance/covariance contributions of some of the other shocks changes significantly as we move from the flexible-price to the sticky-price model.

Second, in the models that assume away the confidence shocks, the combination of permanent and transitory investment shocks emerge as the main driver of the business cycle. This is consistent with existing findings in the DSGE literature (e.g., Justiniano et al, 2010) and confirms that, apart from the inclusion of the confidence shock, our DSGE exercises are quite typical.

Finally, in all models, neither the investment-specific shocks, nor the news or discount-rate shocks are able to contribute to a positive covariation between all of the key real quantities (output, consumption, investment, hours) at the same time. This illustrates, once again, the superior ability of our mechanism to generate the right kind of co-movement patterns.
Figure 6: Theoretical IRFs, Part I

Confidence Shock

TFP: Permanent Shock

TFP: Transitory Shock

Investment: Permanent Shock

Investment: Transitory Shock

Flex. Price Model

Flex. Price Model without (HP,IAC)

Sticky Prices Model

Sticky Prices Model without (HP,IAC)
Figure 7: Theoretical IRFs, Part II
Table 11: Contribution of Shocks to Volatilities (6–32 Quarters)

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<th></th>
<th>Y</th>
<th>C</th>
<th>I</th>
<th>h</th>
<th>π</th>
<th>R</th>
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<td>0.05</td>
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<td>5.67</td>
<td>2.54</td>
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<td>Flexible Price</td>
<td>11.13</td>
<td>13.48</td>
<td>1.55</td>
<td>1.98</td>
<td>0.90</td>
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<tr>
<td>Sticky Price</td>
<td>8.81</td>
<td>10.95</td>
<td>1.79</td>
<td>2.51</td>
<td>0.90</td>
<td>0.90</td>
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<td><strong>Permanent Investment Shock</strong></td>
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<td>0.31</td>
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<td><strong>Transitory TFP Shock</strong></td>
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<td>2.99</td>
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<td>0.05</td>
<td>4.70</td>
<td>0.31</td>
<td>2.99</td>
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<td>0.39</td>
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<td>1.98</td>
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<td>0.00</td>
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<tr>
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<td>0.40</td>
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Table 12: Contribution of Shocks to Comovements (6–32 Quarters)

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<td>(Y, I)</td>
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<td>(I, C)</td>
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<td>14.12</td>
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<tr>
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<td>10.12</td>
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<tr>
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<tr>
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<td>Sticky Price</td>
<td>4.76</td>
<td>20.14</td>
</tr>
<tr>
<td><strong>Transitory Investment Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>3.33</td>
<td>5.26</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>16.33</td>
<td>25.21</td>
</tr>
<tr>
<td>In the absence of Belief shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>4.48</td>
<td>5.47</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>58.76</td>
<td>56.53</td>
</tr>
<tr>
<td><strong>News Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>5.93</td>
<td>11.36</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>1.50</td>
<td>5.76</td>
</tr>
<tr>
<td>In the absence of Belief shock</td>
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<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>12.54</td>
<td>16.80</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>7.39</td>
<td>8.57</td>
</tr>
<tr>
<td><strong>Discount Shock</strong></td>
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<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>0.36</td>
<td>1.38</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>0.41</td>
<td>-0.24</td>
</tr>
<tr>
<td>In the absence of Belief shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>12.54</td>
<td>10.58</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>7.39</td>
<td>8.57</td>
</tr>
<tr>
<td><strong>Fiscal Shock</strong></td>
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<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>4.10</td>
<td>-0.83</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>4.57</td>
<td>-0.39</td>
</tr>
<tr>
<td>In the absence of Belief shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>9.65</td>
<td>11.69</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>1.10</td>
<td>0.41</td>
</tr>
<tr>
<td><strong>Monetary Policy Shock</strong></td>
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<td></td>
</tr>
<tr>
<td>Flexible Price</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>2.20</td>
<td>1.63</td>
</tr>
</tbody>
</table>

In the absence of Belief shock:

<table>
<thead>
<tr>
<th></th>
<th>(Y, h)</th>
<th>(Y, I)</th>
<th>(Y, C)</th>
<th>(h, I)</th>
<th>(h, C)</th>
<th>(I, C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible Price</td>
<td>4.48</td>
<td>5.47</td>
<td>4.00</td>
<td>5.53</td>
<td>3.85</td>
<td>5.11</td>
</tr>
<tr>
<td>Sticky Price</td>
<td>58.76</td>
<td>56.53</td>
<td>-33.34</td>
<td>68.13</td>
<td>-559.56</td>
<td>-203.03</td>
</tr>
</tbody>
</table>
**Posterior odds.** Table 13 reports the posterior odds of four alternative models, starting from a uniform prior and estimating them on the real data only. The models differ on whether they assume flexible or sticky prices, and on whether they contain a confidence shock or not. Since we have dropped the nominal data for this exercise, the nominal parameters of the sticky-price models are now well identified. We have thus chosen to fix these parameters at the values that obtained when the models were estimated on both real and nominal data. We nevertheless re-estimate the preference and technology parameters and the shock processes in order to give a fair chance to each model to match the real data.

<table>
<thead>
<tr>
<th>Model B ↓</th>
<th>Model A →</th>
<th>flex prices</th>
<th>sticky prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>flex prices, with confidence</td>
<td>0.00</td>
<td>0.98</td>
<td>0.02</td>
</tr>
<tr>
<td>flex prices, without confidence</td>
<td>–</td>
<td>1.00</td>
<td>0.90</td>
</tr>
<tr>
<td>sticky prices, with confidence</td>
<td>0.00</td>
<td>–</td>
<td>0.00</td>
</tr>
</tbody>
</table>
E. Log-Linear Solution

In this appendix we explain how to augment a large class of DSGE models with our proposed type of higher-order belief dynamics and how to obtain the solution of the augmented model as a simple transformation of the solution of the original model.

The log-linear version of our baseline model. Before we consider the general case, it is useful to review the linearized version of our baseline model. This helps fix some key ideas.

Thus consider the FOCs of the local planning problem we studied in Section 4 and log-linearize them around the deterministic steady state. Let a tilde ($\sim$) over a variable denote the log-deviation of this variable from its steady-state value. The log-linearized equilibrium conditions take the following form:

\[
\tilde{y}_{it} = \tilde{a}_t + \alpha \tilde{k}_{it} + (1 - \alpha) \tilde{n}_{it} \tag{20}
\]
\[
(1 + \nu) \tilde{n}_{it} = E_{it} \tilde{Y}_t - E_{it} \tilde{c}_{it} \tag{21}
\]
\[
E'_{it} \tilde{c}_{i,t+1} - \tilde{c}_{it} = (1 - \beta(1 - \delta)) E'_{it} \left[ \tilde{Y}_{t+1} - \tilde{k}_{i,t+1} \right] \tag{22}
\]
\[
\tilde{Y}_t = (1 - s) \tilde{c}_{it} + s \tilde{i}_{it} \tag{23}
\]
\[
\tilde{k}_{i,t+1} = \delta \tilde{i}_{it} + (1 - \delta) \tilde{k}_{it} \tag{24}
\]

where $s$ is the investment share.

The interpretation of these equations should be familiar: (20) is the local production function; (21) is the optimality condition for labor; (22) is the Euler condition; (23) is the local resource constraint, with $s$ denoting the saving rate in steady state; and finally (24) is the law of motion for local capital. The only peculiarity in the above system is the presence of two distinct expectation operators $E_{it}$ and $E'_{it}$, which denote the local expectations, respectively, stage 1 and stage 2 of period $t$. The difference between these two expectation operators derives from the fact that islands form beliefs about one another’s signals and thereby about $Y_t$ in stage 1 on the basis of their mis-specified priors, but observe the true state of nature and the true realized $Y_t$ in stage 2. Along with the timing convention we have adopted, this explains why the first expectation shows up in the optimality condition for labor, while the second shows up in the optimality condition for consumption/saving.

At this state, it is important to keep in mind the following. The aggregate-level variables are, of course, obtained from aggregating individual-level variables. Since all islands are identical (recall that we are focusing on the limit with $\sigma \to 0$), the equilibrium values of the aggregate variables coincide with the equilibrium values of the corresponding individual variables. E.g., in equilibrium, it is ultimately the case that $y_{it} = Y_t$ for all $i, t$ and all states of nature. This is because all islands receive the same signals and the same fundamentals. However, this does not mean that one can just replace the island-specific variables in the above conditions with the aggregate ones, or vice versa. This is for two reasons. First, when each island picks its local outcomes, it takes the aggregate
and, accordingly, reasons that stage 1 of each period each island believes that the signals of other can differ from each own signal and, accordingly, reasons that \( y_{it} \) can differ form \( Y_t \) even when all other islands follow the same strategy as itself. Keeping track of this delicate matter is key to obtaining the (correct) solution to the model.

The same principle applies to the general class of DSGE models we consider below. Accordingly, the solution method we develop in this appendix deals with this delicate matter by (i) using appropriate notation to distinguish the signal received by each agent/island from either the average signal in the population or the true underlying shock to fundamentals; and (ii) choosing appropriate state spaces for both the individual policy rules and the aggregate ones.

In the sequel, we first set up the general class of log-linear DSGE models that our solution method handles. We next introduce a class of linear policy rules, which describe the behavior of each agent as a function of his information set. Assuming that all other islands follow such a policy rules, we can use the equilibrium conditions of the model to obtain the policy rules that are optimal for the individual island; that is, we can characterize the best responses of the model. Since the policy rules are linear, they are parameterized by a collection of coefficients (matrices), and the aforementioned best responses reduce to a system of equations in these coefficients. The solution to this system gives the equilibrium of the model.

A “generic” DSGE model. We henceforth consider an economy whose equilibrium is represented by the following linear dynamic system:

\[
M_{yy}y_{it} = M_{yx}x^h_{it} + M_{yX}X_t + M_{yY}E_{it}Y_t + M_{yF}E_{it}x^f_{it} + M_{yE}E_{it}X^f_t + M_{yy}z_{it}
\]

\[
M_{xx0}x^h_{it+1} = M_{xx1}x^h_{it} + M_{eX}X_t + M_{eY}y_{it} + M_{eF}x^f_{it} + M_{eE}X^f_t + M_{xs1}s_t
\]

\[
M_{ff0}E_{it}x^f_{it+1} = M_{ff1}x^f_{it} + M_{fY}y_{it} + M_{fF}x^f_{it} + M_{fE}x^h_{it+1} + M_{fX}X_t + M_{fs1}s_t + \Delta
\]

\[s_t = R_{t-1} + \varepsilon_t\]

\[\xi_t = Q_{t-1} + \nu_t\]

Beliefs. We assume that, as of stage 2, the realizations of \( s_t \), of all the signals, and of all the stage-1 choices become commonly known, which implies that \( y_{it}, x^f_{it}, x^h_{it+1} \) and \( Y_t, X^f_t, X_{t+1} \) are also commonly known in equilibrium). Furthermore, the actual realizations of the signals satisfy \( z_{it} = s_t \) for all \( t \) and all \( i \). However, the agents have misspecified belief in stage 1. In particular, for all \( i \), all \( j \neq i \), all \( t \), and all states of nature, agent \( i \)’s belief during stage 1 satisfy

\[E_{it}[s_t] = z_{it},\]

\[E_{it}[E_{it}s_t] = E_{it}[z_{jt}] = z_{it} + \Delta \xi_t,\]

where \( z_{it} \) is the signal received by agent \( i \), \( \xi_t \) is the higher-order belief shocks, and \( \Delta \) is a loading matrix. We next let \( \bar{z}_t \) denote the average signal in the economy and note that the “truth” is that
$z_{it} = ar{z}_t = s_t$. Yet, this truth is publicly revealed only in stage 2 of period $t$. In stage 1, instead, each island believes, incorrectly, that

$$E_{it} \bar{z}_t = z_{it} + \Delta \xi_t.$$ 

Note next that the stage-1 variables, $y_{it}$, can depend on the local signal $z_{it}$, along with the commonly-observed belief shock $\xi_t$ and the backward-looking (predetermined) state variables $x^b_t$ and $X_t$, but cannot depend on either the aggregate signal $\bar{z}_t$ or the underlying fundamental $s_t$, because these variables are not known in stage 1. By contrast, the stage-2 decisions depend on the entire triplet $(z_{it}, \bar{z}_t, s_t)$. As already mentioned, the truth is that these three variable coincide. Nevertheless, the islands believe in stage 1 that the average signal can differ from either their own signal or the actual fundamental. Accordingly, it is important to write stage-2 strategies as functions of the three conceptually distinct objects in $(z_{it}, \bar{z}_t, s_t)$ in order to do specify the appropriate equilibrium beliefs in stage-1. (Note that this is equivalent to expressing the stage-2 strategies as functions of the realized values of the stage-1 variables $y$ and $Y$, which is the approach we took in the characterization of the recursive equilibrium in Section 4.) In what follows, we show how this belief structure facilitates a tractable solution of the aforementioned general DSGE model.

**Preview of key result.** To preview the key result, let us first consider the underlying “belief-free” model, that is, of the complete-information, representative-agent, counterpart of the model we are studying. The equilibrium system is given by the following:

\[
\begin{align*}
Y_t &= M_X X_t + M_{EY} Y_t + M_F X^f_t + M_s s_t \\
X_{t+1} &= N_X X_t + N_Y Y_t + N_F X^f_t + N_s s_t \\
(P_{f0} - P_{F0})E_{it} X^f_{t+1} &= P_{F1} X^f_t + P_{Y0} E_{it} Y_{t+1} + P_{X} X_t + P_{Y1} Y_t + P_s s_t \\
s_t &= R s_{t-1} + \xi_t \\
\xi_t &= Q \xi_{t-1} + \nu_t
\end{align*}
\]

(This system can be obtained from the one we introduced before once we impose the restriction that all period-$t$ variables are commonly known in period $t$, which means that $E_{it}[x_t] = E_{it}[x_t] = x_t$ for any variable $x$.) It is well known how to obtain the policy rules of such a representative-agent model. Our goal in this appendix is to show how the policy rules of the belief-augmented model that we described above can be obtained as a simple, tractable transformation of the policy rules of the representative-agent benchmark.

In particular, we will show that the policy rules for our general DSGE economy are as follows:

$$X_t = \Theta_X X^b_t + \Theta_s s_t + \Theta_\xi \xi_t,$$

where $X_t = (Y_t, X^f_t, X^b_{t+1})$ collects all the variables, $\Theta_X$ and $\Theta_s$ are the same matrices as those that appear in the solution of the underlying belief-free model, and $\Theta_\xi$ is a new matrix, which encapsulates the effects of higher-order beliefs.
The model, restated. To ease subsequent algebraic manipulations, we henceforth restate the model as follows:

\[
y_{it} = M_x(x_{it} - X_t) + M_X X_t + M_{EY} E_{it} Y_t + M_f E_{it} (x_{it} - X_t) + M_F E_{it} X_t^f + M_s z_{it} \tag{25}
\]

\[
x_{it+1}^b = N_x(x_{it} - X_t) + N_X X_t + N_y(y_{it} - Y_t) + N_Y Y_t + N_f(x_{it} - X_t) + N_F X_t^f + N_s s_{it} \tag{26}
\]

\[
P_{i0} E_{it}^f x_{it+1}^f = P_{f1}(x_{it}^f - X_t^f) + P_{f0} E_{it}^f X_t^f + P_{F1} X_t^f + P_x(x_{it}^b - X_t) + P_X X_t^b + P_y(\bar{E}_{it}^f Y_{it+1}^f) + P_{Y0} E_{it}^f Y_{it+1} + P_{y1}(y_{it} - Y_t) + P_{Y1} Y_t + P_s s_{it} \tag{27}
\]

where

\[
M_x = M_{yy}^{-1} M_{yx}, \quad M_X = M_{yy}^{-1} (M_{yx} + M_{yX}), \quad M_{EY} = M_{yy}^{-1} M_{yY},
\]

\[
M_f = M_{yy}^{-1} M_{yf}, \quad M_F = M_{yy}^{-1} (M_{yf} + M_{yF}), \quad M_s = M_{yy}^{-1} M_{ys}
\]

\[
N_x = M_{xx0}^{-1} M_{xx1}, \quad N_X = M_{xx0}^{-1} (M_{xx1} + M_{xX1}), \quad N_y = M_{xy0}^{-1} M_{xy1}, \quad N_Y = M_{xy0}^{-1} (M_{xy1} + M_{XY1}),
\]

\[
N_f = M_{xx0}^{-1} M_{xf1}, \quad N_F = M_{xx0}^{-1} (M_{xf1} + M_{xF1}), \quad N_s = M_{xx0}^{-1} M_{xs1}
\]

\[
P_{f0} = M_{f0f}, \quad P_{f1} = M_{f1f} + M_{f0N_f}, \quad P_{F0} = M_{F0f}, \quad P_{F1} = M_{F1f} + M_{F0N_F}
\]

\[
P_x = M_{fx0} N_x, \quad P_X = M_{fx1} + M_{fx0} N_x,
\]

\[
P_{y0} = M_{f0y}, \quad P_{Y0} = M_{FY0} + M_{f0y}, \quad P_{y1} = M_{fy1} + M_{fx0} N_y, \quad P_Y = M_{fy1} + M_{FY1} + M_{fx0} N_Y,
\]

\[
P_s = M_{f0R} + M_{fs1} + M_{fx0} N_s
\]

Proposed Policy Rules. We propose that the equilibrium policy rules take the following form:

\[
y_{it} = \Lambda^y_x (x_{it}^b - X_t) + \Lambda^y_X X_t + \Lambda^y_s z_{it} + \Lambda^y_{\xi_t} \tag{28}
\]

\[
x_{it}^f = \Gamma^f_x (x_{it}^b - X_t) + \Gamma^f_X X_t + \Gamma^f_s z_{it} + \Gamma^f_{\xi_t} s_{it} \tag{29}
\]

where the \(\Lambda\)'s and \(\Gamma\)'s are coefficients (matrices), whose equilibrium values are to be obtained in the sequel. Following our earlier discussion, note that the stage-2 policy rules are allowed to depend on the triplet \((z_{it}, \bar{z}_i, s_t)\), while the stage-1 policy rules are restricted to depend only on the local signal \(z_{it}\). It is also useful to note that we would obtain the same solution if we were to represent the stage-2 policy rules as functions of \(y_{it}\) and \(Y_t\) in place of, respectively, \(z_{it}\) and \(\bar{z}_i\): the latter two variables enter the equilibrium conditions that determine the stage-2 decisions, namely conditions (26) and (27), only through the realized values of the stage-1 outcomes \(y_{it}\) and \(Y_t\).

Obtaining the solution. We obtain the solution in three steps. In step 1, we start by characterizing the equilibrium determination of the stage-1 policy rules, taking as given the stage-2 rules. Formally, we fix an arbitrary rule in (29); we assume that all islands believe that the stage-2 variables are determined according to this rule; and we then look for the particular rule in (28) that solves the fixed-point relation between \(y_{it}\) and \(Y_t\) described in (25) under this assumption. This step, which we can think of as the “static” component of the equilibrium, gives as a mapping from
Along with the fact that $i$ island rules, taking as given the stage-2 policy rules. As noted above, we start by studying the equilibrium determination of the stage-1 policy fixed-point between these two mappings to obtain the overall solution to the model. Of this step as solving for the “dynamic” component of the equilibrium. In step 3, we use the fixed-point between these two mappings to obtain the overall solution to the model.

**Step 1.** As noted above, we start by studying the equilibrium determination of the stage-1 policy rules, taking as given the stage-2 policy rules.

Thus suppose that all islands follow a policy rule as in (29) and consider the beliefs that a given island $i$ forms, under this assumption, about the stage-2 variables $x_{it}^f$ and $X_t^f$. From (29), we have

$$x_{it}^f = \Gamma_{it}^f (x_{it}^b - X_t) + \Gamma_{it}^f z_{it} + \Gamma_{it}^f \bar{z}_t + \Gamma_{it}^f s_t + \Gamma_{it}^f \xi_t$$

$$X_t^f = \Gamma_{X_t}^f X_t + (\Gamma_{X_t}^f + \Gamma_{\bar{z}_t}^f) \bar{z}_t + \Gamma_{X_t}^f s_t + \Gamma_{X_t}^f \xi_t$$

Along with the fact that $E_{it}[s_t] = z_{it}$ and $E_{it}[\bar{z}_t] = z_{it} + \Delta \xi_t$, the above gives

$$E_{it} x_{it}^f = \Gamma_{it}^f (x_{it}^b - X_t) + \Gamma_{it}^f z_{it} + (\Gamma_{it}^f + \Gamma_{\bar{z}_t}^f) \bar{z}_t + (\Gamma_{it}^f + \Gamma_{s_t}^f + \Gamma_{\xi_t}^f) \Delta \xi_t$$

$$E_{it} X_t^f = \Gamma_{X_t}^f X_t + (\Gamma_{X_t}^f + \Gamma_{\bar{z}_t}^f + \Gamma_{s_t}^f) \bar{z}_t + (\Gamma_{X_t}^f + (\Gamma_{\bar{z}_t}^f + \Gamma_{s_t}^f) \Delta \xi_t$$

which also implies that

$$x_{it}^f - X_t^f = \Gamma_{it}^f (x_{it}^b - X_t) + \Gamma_{it}^f (z_{it} - \bar{z}_t)$$

$$E_{it} (x_{it}^f - X_t^f) = \Gamma_{it}^f (x_{it}^b - X_t) - \Gamma_{it}^f \Delta \xi_t$$

Plugging the above in (25), the equilibrium equation for $y_{it}$, we get

$$y_{it} = M_X (x_{it}^b - X_t) + M_X X_t + M_{EX} E_{it} Y_t + M_f E_{it} (x_{it}^f - X_t^f) + M_{F} E_{it} X_t^f + M_s z_{it}$$

$$= M_X (x_{it}^b - X_t) + M_X X_t + M_{EX} E_{it} Y_t + M_f \left[ \Gamma_{it}^f (x_{it}^b - X_t) - \Gamma_{it}^f \Delta \xi_t \right]$$

$$+ M_{F} \left[ \Gamma_{X_t}^f X_t + (\Gamma_{\bar{z}_t}^f + \Gamma_{s_t}^f + \Gamma_{\xi_t}^f) \bar{z}_t + (\Gamma_{\bar{z}_t}^f + (\Gamma_{s_t}^f + \Gamma_{\xi_t}^f) \Delta \xi_t \right] + M_s z_{it}$$

Equivalently,

$$y_{it} = (M_X + M_f \Gamma_{it}^f) (x_{it}^b - X_t) + (M_X + M_f \Gamma_{X_t}^f) X_t + M_{EX} E_{it} Y_t$$

$$+ (M_s + M_{F} (\Gamma_{\bar{z}_t}^f + \Gamma_{s_t}^f + \Gamma_{\xi_t}^f)) \bar{z}_t + \left( M_{F} \Gamma_{\bar{z}_t}^f + M_{F} \Gamma_{s_t}^f \Delta + (M_{F} - M_f) \Gamma_{\xi_t}^f \Delta \right) \xi_t$$

Note that the above represents us a static fixed-point relation between $y_{it}$ and $Y_t$. This relation is itself determined by the $\Gamma$ matrices (i.e., by the presumed policy rule for the stage-2 variables). Notwithstanding this fact, we now focus on the solution of this static fixed point.

Thus suppose that this solution takes the form of a policy rule as in (28). If all other island follow this rule, then at the aggregate we have

$$Y_t = \Lambda_{X_t}^Y X_t + \Lambda_{\bar{z}_t}^Y \bar{z}_t + \Lambda_{\xi_t}^Y \xi_t$$

50
and therefore the stage-1 forecast of island \( i \) about \( Y_t \) is given by

\[ \mathbb{E}_i Y_t = \Lambda^y_X X_t + \Lambda^y_z z_{it} + \left( \Lambda^y_{\xi} + \Lambda^y_\Delta \right) \xi_t \]

Plugging this into (30), we obtain the following best response for island \( i \):

\[
y_{it} = (M_s + M_f \Gamma^I_{x}) (x_{it}^b - X_t) + (M_X + M_F \Gamma^I_X) X_t + M_{EY} \left( \Lambda^y_X X_t + \Lambda^y_z z_{it} + \left( \Lambda^y_{\xi} + \Lambda^y_\Delta \right) \xi_t \right)
\]

\[ + \left( M_s + M_F (\Gamma^I_z + \Gamma^I_{\xi} + \Gamma^I_{\Delta}) \right) z_{it} + \left( M_F (\Gamma^I_{\xi} + \Gamma^I_{\Delta}) + (M_F - M_f) \Gamma^I_{z} \Delta \right) \xi_t \]

For this to be consistent with our guess in (28), we must have

\[
\begin{align*}
\Lambda^y_x &= M_s + M_f \Gamma^I_x \\
\Lambda^y_{\xi} &= (I - M_{EY})^{-1} (M_X + M_F \Gamma^I_X) \\
\Lambda^y_z &= (I - M_{EY})^{-1} \left[ M_s + M_F (\Gamma^I_z + \Gamma^I_{\xi} + \Gamma^I_{\Delta}) \right] \\
\Lambda^y_\xi &= (I - M_{EY})^{-1} \left\{ M_F (\Gamma^I_{\xi} + \Gamma^I_{\Delta}) + (M_F - M_f) \Gamma^I_{z} \Delta + M_{EY} \Lambda^y_\Delta \right\}
\end{align*}
\]

This completes the first step of our solution strategy: we have characterized the “static” component of the equilibrium and have thus obtained the \( \Lambda \) coefficients as functions of primitives and of the \( \Gamma \) coefficients.

**Step 2.** We now proceed with the second step, which is to characterize the equilibrium behavior in stage 2, taking as given the behavior in stage 1.

Recall that, once agents enter stage 2, they observe the true current values of the triplet \((z_{it}, \bar{z}_t, s_t)\) along with the realized values of the past stage-1 outcomes, \(y_{it}\) and \(Y_t\). Furthermore, in equilibrium this implies common certainty of current choices, namely of the variables \(x_{it}^f\) and \(X_t^f\), and thereby also of the variables \(x_{it+1}^b\) and \(X_{t+1}^b\). Nevertheless, agents face uncertainty about the next-period realizations of the aforementioned triplet and of the corresponding endogenous variables. In what follows, we thus take special care in characterizing the beliefs that agents form about the relevant future outcomes.

Consider first an agent’s beliefs about the aggregate next-period stage-1 variables:

\[
Y_{t+1} = \Lambda^y_X X_{t+1} + \Lambda^y_z \bar{z}_{t+1} + \Lambda^y_{\xi} \xi_{t+1}
\]

\[
\mathbb{E}_{it+1} Y_{t+1} = \Lambda^y_X X_{t+1} + \Lambda^y_z z_{t+1} + \left( \Lambda^y_{\xi} + \Lambda^y_\Delta \right) \xi_{t+1}
\]

\[
\mathbb{E}_{it} Y_{t+1} = \Lambda^y_X X_{t+1} + \Lambda^y_z R_{st} + \left( \Lambda^y_{\xi} + \Lambda^y_\Delta \right) Q \xi_t
\]

Consider next his beliefs about his own next-period stage-1 variables:

\[
y_{it+1} = \Lambda^y_x (x_{it+1}^b - X_{t+1}) + \Lambda^y_X X_{t+1} + \Lambda^y_z z_{it+1} + \Lambda^y_{\xi} \xi_{t+1}
\]

\[
\mathbb{E}_{it} y_{it+1} = \Lambda^y_X (x_{it+1}^b - X_{t+1}) + \Lambda^y_X X_{t+1} + \Lambda^y_z R_{st} + \Lambda^y_{\xi} Q \xi_t
\]
It follows that

$$\mathbb{E}_{it}^f (y_{it+1} - Y_{t+1}) = \Lambda_y^b (x_{it+1}^b - X_{t+1}) - \Lambda_x^b \Delta Q \xi_t$$

Consider now his beliefs about his own next-period forward variables:

$$x_{it+1}^f = \Gamma_x^f (x_{it}^b - X_t) + \Gamma_X^f X_t + \Gamma_z^f z_{it+1} + \Gamma_{z}^f \bar{z}_{t+1} + \Gamma_s^f s_{t+1} + \Gamma_{\xi}^f \xi_{t+1}$$

$$\mathbb{E}_{it+1} x_{it+1}^f = \Gamma_x^f (x_{it}^b - X_t) + \Gamma_X^f X_t + (\Gamma_z^f + \Gamma_{z}^f + \Gamma_s^f) z_{it+1} + (\Gamma_{\xi}^f + \Gamma_{\xi}^f \Delta) \xi_{t+1}$$

$$\mathbb{E}_{it}^f x_{it+1}^f = \Gamma_x^f (x_{it}^b - X_t) + \Gamma_X^f X_t + (\Gamma_z^f + \Gamma_{z}^f + \Gamma_s^f) R_{st} + (\Gamma_{\xi}^f + \Gamma_{\xi}^f \Delta) Q \xi_t$$

For the aggregate next-period forward variables we have

$$\mathbb{E}_{it+1} X_{t+1}^f = \Gamma_X^f X_t + (\Gamma_z^f + \Gamma_{z}^f + \Gamma_s^f) z_{it+1} + (\Gamma_{\xi}^f + \Gamma_{\xi}^f \Delta) \xi_{t+1}$$

$$\mathbb{E}_{it}^f X_{t+1}^f = \Gamma_X^f X_t + (\Gamma_z^f + \Gamma_{z}^f + \Gamma_s^f) R_{st} + (\Gamma_{\xi}^f + \Gamma_{\xi}^f \Delta) Q \xi_t$$

and therefore

$$\mathbb{E}_{it}^f (x_{it+1}^f - X_{t+1}^f) = \Gamma_X^f (x_{it}^b - X_t) - \Gamma_{\xi}^f \Delta Q \xi_t$$

Next, note that our guesses for the policy rules imply the following properties for the current-period variables:

$$y_t - Y_t = \Lambda_y^b (x_t^b - X_t) + \Lambda_z^b (z_t - \bar{z}_t)$$

$$x_t^f - X_t^f = \Gamma_x^f (x_t^b - X_t) + \Gamma_z^f (z_t - \bar{z}_t)$$

$$Y_t = \Lambda_X^b X_t + \Lambda_z^b \bar{z}_t + \Lambda_{\xi}^b \xi_t$$

$$X_t^f = \Gamma_X^f X_t + (\Gamma_z^f + \Gamma_{\xi}^f) \bar{z}_t + \Gamma_s^f s_t + \Gamma_{\xi}^f \xi_t$$

Plugging these results in the law of motion of backward variables, we get

$$x_{it+1}^b = \Omega_x (x_{it}^b - X_t) + \Omega_X X_t + \Omega_z z_{it} + \Omega_{\xi} \xi_t$$

and hence

$$x_{it+1}^b - X_{t+1} = \Omega_x (x_{it}^b - X_t) + \Omega_z (z_{it} - \bar{z}_t)$$

where

$$\Omega_x = N_x + N_Y \Lambda_y^b + N_f \Gamma_z^f \quad \Omega_z = N_y \Lambda_z^b + N_f \Gamma_{\xi}^f$$

$$\Omega_X = N_X + N_Y \Lambda_X^b + N_F \Gamma_X^f \quad \Omega_{\xi} = (N_Y - N_y) \Lambda_{\xi}^b + (N_F - N_f) \Gamma_{\xi}^f$$

$$\Omega_s = N_s + N_F \Gamma_s^f \quad \Omega_{\xi} = N_y \Lambda_{\xi}^b + N_F \Gamma_{\xi}^f$$
Similarly, the expectation of the corresponding aggregate variable is given by:

\[
\mathbb{E}_t^t x^f_{t+1} = \Gamma_f^f (x^b_{it+1} - X_t) + \Gamma_f^f X_{t+1} + (\Gamma_f^f + \Gamma_f^f + \Gamma_f^f) R_s + (\Gamma_f^f + \Gamma_f^f) Q \xi_t
\]

or equivalently

\[
\mathbb{E}_t^t x^f_{it+1} = \Phi_x (x^b_{it} - X_t) + \Phi_X X_t + \Phi_z z_{it} + \Phi_s s_t + \Phi_\xi \xi_t
\]

where

\[
\begin{align*}
\Phi_x &= \Gamma_f^f \Omega_x \\
\Phi_z &= \Gamma_f^f \Omega_z \\
\Phi_s &= \Gamma_f^f \Omega_s + (\Gamma_f^f + \Gamma_f^f + \Gamma_f^f) R \\
\Phi_\xi &= \Gamma_f^f \Omega_\xi + (\Gamma_f^f + \Gamma_f^f) Q
\end{align*}
\]

Similarly, the expectation of the corresponding aggregate variable is given by

\[
\mathbb{E}_t^t x^f_{t+1} = \Phi_x X_t + \Phi_z z_{it} + \Phi_s s_t + (\Phi_\xi + \Gamma_f^f \Delta Q) \xi_t
\]

With the above steps, we have calculated all the objects that enter the Euler condition \([27]\). We can thus proceed to characterize the fixed-point relation that pins down the solution for the stage-2 policy rule.

To ease the exposition, let us repeat the Euler condition \([27]\) below:

\[
P_{f0} \mathbb{E}_t^t x^f_{it+1} = P_{f1} (x^f_{it} - X^f_t) + P_{F0} \mathbb{E}_t^t X^f_{t+1} + P_{F1} X^f_t + P_x (x^b_{it} - X_t) + P_X X_t + \\
+ P_{y0} (\mathbb{E}_t^t y_{it+1} - \mathbb{E}_t^t Y_{t+1}) + P_{y0} \mathbb{E}_t^t Y_{t+1} + P_{y1} (y_{it} - Y_t) + P_{Y1} Y_t + P_Y s_t
\]

Use now \([35]\) to write the left-hand-side of the Euler condition as

\[
P_{f0} \mathbb{E}_t^t x^f_{it+1} = P_{f0} \left\{ \Phi_x (x^b_{it} - X_t) + \Phi_X X_t + \Phi_z z_{it} + \Phi_s s_t + \Phi_\xi \xi_t \right\}
\]

Next, use our preceding results to replace all the expectations that show up in the right-hand-side of the Euler condition, as well as the stage-1 outcomes. This gives

\[
P_{f0} \mathbb{E}_t^t x^f_{it+1} = P_{f1} \left\{ \Gamma_f^f (x^b_{it} - X_t) + \Gamma_f^f (z_{it} - \bar{z}_t) \right\} + \\
+ P_{F0} \left\{ \Phi_x X_t + (\Phi_z + \Phi_\xi) \bar{z}_t + \Phi_s s_t + (\Phi_\xi + \Gamma_f^f \Delta Q) \xi_t \right\} \\
+ P_{F1} \left\{ \Gamma_f^f X_t + (\Gamma_f^f + \Gamma_f^f) \bar{z}_t + \Gamma_f^f s_t + \Gamma_f^f \xi_t \right\} + P_x (x^b_{it} - X_t) + \\
+ P_X X_t + P_{y0} \left\{ \Lambda^y (\Omega_x (x^b_{it} - X_t) + \Omega_z (z_{it} - \bar{z}_t)) - \Lambda^y \Delta Q \xi_t \right\} + \\
+ P_{y0} \left\{ \Lambda^y (\Omega_X X_t + (\Omega_z + \Omega_\xi) \bar{z}_t + \Omega_s s_t + \Omega_\xi \xi_t) + \Lambda^y R_s + (\Lambda^y + \Lambda^y \Delta) Q \xi_t \right\} + \\
+ P_{y1} \left\{ \Lambda^y (x^b_{it} - X_t) + \Lambda^y (z_{it} - \bar{z}_t) \right\} + P_{Y1} \left\{ \Lambda^y X_t + \Lambda^y \bar{z}_t + \Lambda^y \xi_t \right\} + P_Y s_t
\]

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For our guess to be correct, the above two expressions must coincide in all states of nature, and the following must therefore be true:

\[ P_{f0} \Phi_x = P_x + P_{f1} \Gamma^f_x + P_{y0} \Lambda^y_x \Omega_x + P_{y1} \Lambda^y_x \]  
\[ (P_{f0} - P_{F0}) \Phi_X = P_{F1} \Gamma^f_X + P_X + P_{Y0} \Lambda^y_X \Omega_X + P_{Y1} \Lambda^y_X \]  
\[ P_{f0} \Phi_z = P_{f1} \Gamma^f_z + P_{y0} \Lambda^y_z \Omega_z + P_{y1} \Lambda^y_z \]  
\[ (P_{f0} - P_{F0}) \Phi_{\bar{z}} = P_{F0} \Phi_z + (P_{F1} - P_{f1}) \Gamma^f_{\bar{z}} + P_{Y0} \Lambda^y_{\bar{z}} (\Omega_{\bar{z}} + \Omega_{\bar{z}}) \]  
\[ - P_{y0} \Lambda^y_{\bar{z}} \Omega_\bar{z} + (P_{Y1} - P_{y1}) \Lambda^y_{\bar{z}} \]  
\[ (P_{f0} - P_{F0}) \Phi_s = P_{F1} \Gamma^f_s + P_{Y0} (\Lambda^y_X \Omega_s + \Lambda^y_z R) + P_s \]  
\[ (P_{f0} - P_{F0}) \Phi_\xi = P_{F0} \Gamma^f_\xi \Delta_Q + P_{F1} \Gamma^f_\xi + P_{Y0} \left\{ \Lambda^y_X \Omega_\xi + \Lambda^y_\xi Q \right\} + (P_{Y0} - P_{y0}) \Lambda^y_\xi \Delta_Q + P_{Y1} \Lambda^y_\xi \]

Recall that the \( \Phi \) and \( \Omega \) matrices are themselves transformations of the \( \Gamma \) and \( \Lambda \) matrices. Therefore, the above system is effectively a system of equations in \( \Gamma \) and \( \Lambda \) matrices. This completes Step 2.

**Step 3.** Steps 1 and 2 resulted in two systems of equations in the \( \Lambda \) and \( \Gamma \) matrices, namely system (31)-(34) and system (37)-(42). We now look at the joint solution of these two systems, which completes our guess-and-verify strategy and gives the sought-after equilibrium policy rules.

First, let us write the solution of the underlying representative-agent model as

\[ Y_t = \Lambda^y_X X_t + \Lambda^y_s s_t \]
\[ X^f_t = \Gamma^f_X X_t + \Gamma^f_s s_t \]

It is straightforward to check that the solution to our model satisfies the following:

\[ \Lambda^y_X = \Lambda^y_s \]
\[ \Gamma^f_X = \Gamma^f_s \]
\[ \Gamma^f_z = \Gamma^f_{\bar{z}} + \Gamma^f_s \]

That is, the solution for the matrices \( \Lambda^y_X \), \( \Lambda^y_z \), and \( \Gamma^f_X \), and for the sum \( \tilde{\Gamma}^f_s \equiv \Gamma^f_z + \Gamma^f_{\bar{z}} + \Gamma^f_s \), can readily be obtained from the solution of the representative-agent model.

With the sum \( \tilde{\Gamma}^f_s \equiv \Gamma^f_z + \Gamma^f_{\bar{z}} + \Gamma^f_s \) determined as above, we can next obtain each of its three components as follows. First, \( \Gamma^f_s \) can be obtained from (41):

\[ (P_{f0} - P_{F0}) \Phi_s = P_{F1} \tilde{\Gamma}^f_s + P_{Y0} (\Lambda^y_X \Omega_s + \Lambda^y_z R) + P_s \]

Plugging the definition of \( \Phi_s \) and \( \Omega_s \) in the above, we have

\[ - \left\{ \left( (P_{f0} - P_{f0}) \Gamma^f_X + P_{Y0} \Lambda^y_X \right) N_F + P_{F1} \right\} \tilde{\Gamma}^f_s = \left[ \begin{array}{c} \Lambda^y_x \end{array} \right] \right\} \Gamma^f_s = \left[ \begin{array}{c} s_t + P_{Y0} \left( \Lambda^y_z R + \Lambda^y_X N_s \right) + (P_{f0} - P_{f0}) (\tilde{\Gamma}^f_z R + \Gamma^f_{\bar{z}} N_s) \right] \]

and therefore \( \tilde{\Gamma}^f_s = A_s^{-1} B_s \). Next, \( \Gamma^f_z \) can be obtained from (39):

\[ P_{f0} \Phi_z = P_{f1} \Gamma^f_z + P_{y0} \Lambda^y_z \Omega_z + P_{y1} \Lambda^y_z \]
Plugging the definition of $\Phi_z$ and $\Omega_z$ in the above, we have

$$\left( (P_{f0}\Gamma_z^f - P_{g0}\Lambda_z^y)N_f - P_{f1}\right) \Gamma_z^f = P_{g1}\Lambda_z^y - (P_{f0}\Gamma_z^f - P_{g0}\Lambda_z^y)N_y\Lambda_z^y$$

and therefore $\Gamma_z^f = A_{Z}^{-1}B_{Z}$. Finally, we obtain $\Gamma_z^f$ simply from the fact that $\Gamma_z^f = \bar{\Gamma}_s^f - \Gamma_z^f - \Gamma_z^s$.

Consider now the matrices $\Lambda_z^y$ and $\Gamma_z^f$. These are readily obtained from (31) and (37) once we replace the already-obtained results. It is also straightforward to check that these matrices correspond to the solution of the version of the model that shuts down all kinds of uncertainty but allows for heterogeneity in the backward-looking state variables (“wealth”).

To complete our solution, what remains is to determine the matrices $\Gamma_z^f$ and $\Lambda_z^y$. These matrices solve conditions (34) and (42), which we repeat below:

$$\Lambda_z^y = (I - M_{EY})^{-1}\{M_F(\Gamma_z^f + \Gamma_z^s \Delta) + (M_F - M_f)\Gamma_z^f \Delta + M_{EY}\Lambda_z^y \Delta\}$$

$$(P_{f0} - P_{F0})\Phi_z = P_{F0}\Gamma_z^f \Delta Q + P_{F1}\Gamma_z^f + P_{Y0}\left\{\Lambda_z^y \Omega_z + \Lambda_z^y Q\right\} + (P_{Y0} - P_{g0})\Lambda_z^y \Delta Q + P_{Y1}\Lambda_z^y$$

Let us use the first condition to substitute away $\Lambda_z^y$ from the second, and then the facts that

$$\Omega_z = N_Y\Lambda_z^y + N_F\Gamma_z^f$$

$$\Phi_z = \Gamma_z^f (N_Y\Lambda_z^y + N_F\Gamma_z^f) + (\Gamma_z^f + \Gamma_z^s \Delta)Q$$

to substitute away also $\Omega_z$ and $\Phi_z$. We then obtain a single equation in $\Gamma_z^f$, which takes the following form:

$$B\Gamma_z^f + A\Gamma_z^f Q + C = 0$$

where

$$A \equiv (P_{F0} - P_{f0}) + P_{Y0}(I - M_{EY})^{-1}M_F$$

$$B \equiv ((P_{F0} - P_{f0})\Gamma_z^f N_Y + P_{Y0}\Lambda_z^y N_Y + P_{Y1})(I - M_{EY})^{-1}M_F + (P_{F0} - P_{f0})\Gamma_z^f N_F + P_{F1} + P_{Y0}\Lambda_z^y N_F$$

$$C \equiv \left( P_{F0}\Gamma_z^f \Delta Q + (P_{Y0} - P_{g0})\Lambda_z^y + (P_{F0} - P_{f0})\Gamma_z^f + P_{Y0}(I - M_{EY})^{-1}\left[ M_F\Gamma_z^f + (M_F - M_f)\Gamma_z^f + M_{EY}\Lambda_z^y \right]\right) \Delta Q$$

$$+ \left( (P_{F0} - P_{f0})\Gamma_z^f N_Y + P_{Y0}\Lambda_z^y N_Y + P_{Y1}\right)(I - M_{EY})^{-1}\left[ M_F\Gamma_z^f + (M_F - M_f)\Gamma_z^f + M_{EY}\Lambda_z^y \right] \Delta$$

Note that $A$, $B$, and $C$ are determined by primitives, plus some of the coefficients that we have also characterized. The above equation therefore gives us the unique solution for the matrix $\Gamma_z^f$ as a function of the primitives of the model. $\Lambda_z^y$ is then readily obtained from (34).

This completes the solution.
References


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